NAZARIY MEXANIKA MISOL VA MASALALARDA

2-QISM

KINEMATIKA

O‘ZBEKISTON RESPUBLIKASI VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

TOSHKENT ARXITEKTURA QURILISH INSTITUTI

K. KENJAYEV

NAZARIY MEXANIKA

Misol va masalalarda

II qism

KINEMATIKA

Cho‘lpon nomidagi nashriyot-matbaa ijodiy uyi
Toshkent — 2018

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O'zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligining 2017-yil 24-avgustdagi 603-sonli buyrug'iga asosan 5340200 «Bino va inshootlar qurilishi (sanoat va fuqaro binolari) ta'lim yo'nalishining talabalari uchun o'quv qo'llanma sifatida tavsiya etilgan.

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O'quv qo'llanmada «Nazariy mexanika» fanining «Kinematika» bo'limining nuqta kinematikasi, qattiq jismning ilgarilanma va qo'zg'almas o'q atrofidagi aylanma harakati, nuqtaning murakkab harakati, qattiq jismning tekislikka parallel harakati boblari bo'yicha qisqacha nazariy ma'lumotlar, masalalar yechish tartibi, masalalar yechish namunalari hamda mustaqil o'rganish uchun ko'p variantli masalalar taqdim etilgan.

O'quv qo'llanma TAQI o'quv ishlari bo'yicha prorektori tomonidan 2016-yil 29-avgustda tasdiqlangan «Nazariy mexanika» fani bo'yicha ishchi o'quv dasturi asosida tuzilgan.

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Volidayi muhtaramam Oxunjonova
Gulbining yorqin xotirasiga bag'ishlayman

SO'Z BOSHI

Nazariy mexanikaning kinematika bo'limida moddiy nuqtaning harakati harakatni yuzaga keltiruvchi sabablarga bog'lanmagan holda o'rganiladi. Shuning uchun, odatda, kinematika «harakat geometriyasi» ham deyiladi.

Kinematika bo'limi Statika va Dinamika bo'limlaridan keyinroq, XIX asrda nazariy mexanikaning alohida bo'limi sifatida shakllangan. Kinematikaning rivojlanishida mashinasozlikning rivojlanishi, turli xil mashina va mexanizmlarning yaratilishi va ishlatilishi asosiy sabablardan hisoblanadi.

Qurilish yo'nalishlarida ta'lim oluvchi talabalar uchun turli xil mashinalarni tashkil etuvchi mexanizmlarning kinematik va dinamik xususiyatlarni o'rganish, harakat qonunlarini keltirib chiqarish muhim vazifa hisoblanadi. Buning uchun talabalar kursning nazariy asoslarini chuqur o'rganishi va amaliy masalalarni yechish malakalariga ega bo'lishlari lozim.

O'quv qo'llanmani tuzishda J.L. Meriam, L.G. Kraige «Engineering mechanics statics» (2007), R.C. Hibbeler «Statics and Dynamics» (2013), Vasile Szolge «Theoretical mechanics» (2010), R.S. Khurmi «Engineering mechanics» (2011) kabi xorijiy adabiyotlardan foydalanildi.

Taqdim etilayotgan o'quv qo'llanmada nuqta kinematikasi, qattiq jismning ilgarilanma va qo'zg'almas o'q atrofidagi aylanma harakati, moddiy nuqtaning murakkab harakati, qattiq jismning tekislikka parallel harakati mavzulari bo'yicha qisqacha nazariy ma'lumotlar, masalalar yechish tartibi, namunalari va talabalar mustaqil ishlashlari uchun ko'p variantli masalalar keltirilgan.

Qo'llanma qo'lyozmasini o'qib chiqib, undagi kamchiliklarni tuzatishdagi qimmatli maslahatlarini ayamaganliklari uchun texnika fanlari doktori prof. T.M. Mavlonovga, texnika fanlari doktori prof. Q.S. Abdurashidovga, Toshkent Arxitektura qurilish instituti «Qurilish mexanikasi va inshhootlar zilzilabardoshligi» kafedrasi mudiri, texnika fanlari nomzodi Z.S. Shadmanovaga, fizika-matematika fanlari nomzodi, dotsent S.A. Abduqodirovga hamda qo'llanma qo'lyozmasini tayyorlashdagi beg'araz yordami uchun muhandis-quruvchi N. Xatamovga muallif o'zining chuqur minnatdorchiligini bildiradi.

Qo'llanmada yo'l qo'yilgan kamchliklarni bartaraf etish va uning o'quv-uslubiy qiymatini yanada oshirish borasida bildirilgan fikr va mulohazalari uchun muallif kitobxonlarga o'zining chuqur minnatdorchligini bildiradi.

I BOB

NUQTA KINEMATIKASI

1-§. Kinematikaning asosiy tushunchalari

Nazariy mexanikaning kinematika bo'limida moddiy nuqta va absolut qattiq jismning harakati shu harakatni vujudga keltirgan sabablarga bog'lanmagan holda faqat geometrik nuqtayi nazardan o'rganiladi.

Harakat tushunchasi harakatlanuvchi moddiy nuqta (yoki absolut qattiq jism), vaqt va fazo tushunchalari bilan chambarchas bog'liqdir.

Ko'chish va harakat tushunchalari nazariy mexanikaning asosiy tushunchalari hisoblanadi. *Moddiy nuqtaning ma'lum vaqt ichida fazoda biror sanoq sistemasiga nisbatan bir holatdan boshqa holatga ixtiyoriy ravishda o'tishi ko'chish deyiladi.*

Nuqtaning boshlang'ich holatdan oxirgi holatga aniq bir usulda vaqtga bog'liq holda o'tishi esa harakat deyiladi.

Fazo bir vaqtda mavjud bo'lgan obyektlarning joylashish tartibini ifodalaydi.

Klassik mexanikada fazo uch o'lchovli, absolut qo'zg'almas Evklid fazosi deb qaraladi va undagi barcha o'lchamlar Evklid geometriyasi asosida olib boriladi.

Vaqt obyektiv borliqda ro'y beruvchi hodisalarning qancha davom etishini ifodalaydi va u absolut deb qaraladi. Vaqt barcha sanoq sistemalarida bir xil o'tadi va bir sistemaning ikkinchi sistemaga nisbatan harakatiga bog'liq bo'lmaydi. SI sistemasida sekund vaqt birligi hisoblanadi.

Harakatlanayotgan moddiy nuqtaning fazoda biror sanoq sistemasiga nisbatan holati bilan vaqt orasidagi bog'lanishni ifodalovchi tenglama nuqtaning harakat qonunini ifodalaydi. Agar moddiy nuqtaning biror sanoq sistemasiga nisbatan harakat qonuni berilgan bo'lsa, uning trayektoriyasi, tezligi va tezlanishini aniqlash mumkin bo'ladi. *Trayektoriya deb, moddiy nuqta yoki absolut qattiq jismning harakatlanishi tufayli tekislik yoki fazoda qoldirgan iziga aytiladi.*

Kinematikaning asosiy vazifasi moddiy nuqta va absolut qattiq jismning harakat qonunlarini o'rganishdan iborat.

2-§. Moddiy nuqta harakatining berilish usullari

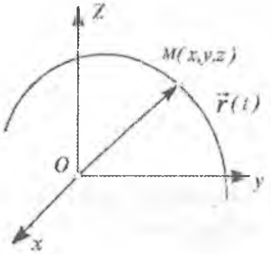
Kinematikada nuqtaning harakati vektor, koordinatalar va tabiiy usulda beriladi.

1. Vektor usuli.

Harakatdagi *M* nuqtaning *Oxyz* sanoq sistemasiga nisbatan holati *O* markazdan o'tkazilgan $\vec{r}$ radius-vektor bilan aniqlanadi *(1.1-rasm)*.

M nuqta harakatlanganda vaqt o'tishi bilan uning radius-vektori $\vec{r}$ miqdor va yo'nalish jihatdan o'zgaradi, ya'ni skalyar argument t ning vektorli funksiyasidan iborat bo'ladi:

$$\vec{r} = \vec{r}(t). \qquad (1.1)$$



1.1-rasm

Agar $\vec{r}\,(t)$ funksiyasi ma'lum bo'lsa, nuqtaning fazodagi holati vaqtning har bir payti uchun aniq bo'ladi. Shu sababli (1.1) tenglama nuqta harakatining vektor ko'rinishdagi kinematik tenglamasi deyiladi. Ko'riladigan masalalarda $\vec{r}(t)$ funksiya bir qiymatli, uzluksiz va kamida ikkinchi tartibli hosilaga ega bo'lishi lozim.

2. Koordinatalar usuli.

M nuqta *Oxyz* sanoq sistemasiga nisbatan harakatlanayotgan bo'lsin. Nuqtaning holatini uning uchta *x*, *y*, *z* Dekart koordinatalari orqali aniqlash mumkin *(1.2-rasm)*.

Nuqta harakatlanganda uning koordinatalari vaqt o'tishi bilan o'zgaradi, ya'ni ular t vaqtning funksiyasidan iborat bo'ladi:

$$\begin{cases} x = x(t), \\ y = y(t), \\ z = z(t). \end{cases} \quad (1.2)$$

Agar nuqta koordinatalari bilan vaqt orasidagi munosabatlar berilgan bo'lsa, nuqtaning istalgan paytdagi holatini aniqlash mumkin bo'ladi. Shu sababli (1.2) tenglamalar nuqta harakatining Dekart koordinatalaridagi kinematik tenglamalarini ifodalaydi.

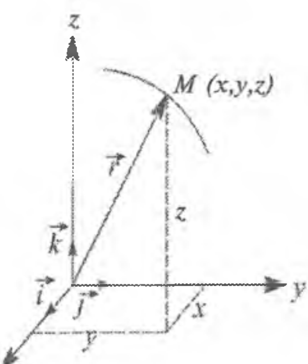
(1.2) tenglamalar nuqta trayektoriyasining parametrik tenglamalarini ham ifodalaydi. Bunda parametr sifatida t vaqt olingan.

(1.2) tenglamalardan t vaqtni yo'qotib, nuqtaning koordinatalar formasidagi trayektoriya tenglamasi aniqlanadi. M nuqtaning O koordinatalar boshiga nisbatan radius-vektorini $\vec{r}$, koordinata o'qlarining birlik yo'naltiruvchi vektorlarini $\vec{i}, \vec{j}, \vec{k}$ bilan belgilasak *(1.2-rasm),* harakatning vektor va Dekart koordinatalari orqali aniqlash usullari orasidagi bog'lanishni ifodalovchi quyidagi tenglama o'rinli bo'ladi:

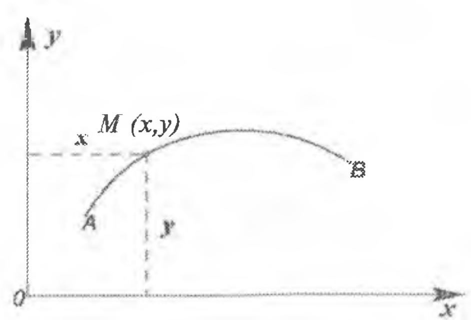
$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}. \quad (1.3)$$

Agar nuqta xy tekisligida harakatlansa *(1.3-rasm),* nuqtaning tekislikdagi harakat tenglamalari quyidagi ko'rinishda bo'ladi:

$$\begin{cases} x = x(t), \\ y = y(t). \end{cases} \quad (1.4)$$



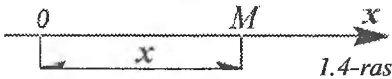
1.2-rasm



1.3-rasm

Nuqta to‘g‘ri chiziqli harakatda bo‘lsa *(1.4-rasm)*, harakat trayektoriyasi bo‘ylab x o‘qini yo‘naltiramiz. Bu holda nuqtaning to‘g‘ri chiziqli harakat tenglamasi quyidagi ko‘rinishda yoziladi:

$$x = x(t). \qquad (1.5)$$



1.4-rasm

3. Tabiiy usul.

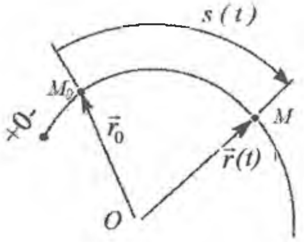
Harakatlanayotgan nuqtaning trayektoriyasi oldindan ma’lum bo‘lsa, nuqta harakatini tabiiy usulda aniqlash qulay. Nuqtaning trayektoriyasi to‘g‘ri chiziqdan yoki egri chiziqdan iborat bo‘ladi. Trayektoriyada qo‘zg‘almas O nuqtani olib, bu nuqtaga nisbatan yoy koordinatasini o‘tkazamiz *(1.5-rasm)*. Harakatlanayotgan M nuqtaning trayektoriyadagi holatini O nuqtadan trayektoriya bo‘yicha hisoblangan $OM=S$ yoy koordinatasi bilan aniqlaymiz. O nuqtadan bir tomonga qo‘yilgan masofani musbat, ikkinchi tomonga qo‘yilgan masofani manfiy deb hisoblaymiz.

Vaqtning o‘tishi bilan harakatlanayotgan nuqtadan qo‘zg‘almas O nuqtagacha bo‘lgan OM masofa o‘zgaradi, ya’ni nuqtaning yoy koordinatasi vaqtning funksiyasidan iborat:

$$S = f(t). \qquad (1.6)$$

Bu munosabatga **nuqtaning tabiiy usuldagi harakat tenglamasi** yoki **harakat qonuni** deyiladi. Agar $f(t)$ funksiya ma’lum bo‘lsa, u holda t vaqtning har bir payti uchun OM ni aniqlab, O nuqtadan trayektoriya bo‘yicha qo‘yamiz. Natijada, M nuqtaning berilgan t paytdagi holati aniqlanadi. Shunday qilib, nuqtaning harakatini tabiiy usulda aniqlash uchun uning trayektoriyasida O qo‘zg‘almas nuqta (hisoblash boshi) va yoy koordinatasining hisoblash yo‘nalishi hamda $S = f(t)$ harakat tenglamasi ma’lum bo‘lishi kerak ekan. Nuqtaning S yoy koordinatasi bilan trayektoriya bo‘yicha o‘tgan OM yo‘li doimo bir xil bo‘lavermaydi. Agar M nuqtaning harakati O qo‘zg‘almas nuqtadan boshlanib, $\Delta t = t - t_0$ vaqt oralig‘ida doimo musbat yo‘nalishi bo‘yicha yuz bersa, t vaqtda nuqtaning yoy koordinatasi bilan Δt vaqt oralig‘ida o‘tilgan yo‘l o‘zaro teng.

Agar t_0 boshlang‘ich vaqtda nuqta M_0 holatda bo‘lib, Δt vaqtdan keyin M holatni egallasa, u holda Δt oralig‘ida nuqtaning bir to-



1.5-rasm

monga harakatlanishi natijasida o‘tilgan yo‘l $S = \int_{t_0}^{t} f(t)dt$ formula bilan aniqlanadi.

Takrorlash uchun savollar:

1. *Kinematika fani nimani o‘rgatadi?*
2. *Kinematika asosiy tushunchalarini ta’riflab bering.*
3. *Nuqtaning harakati qanday usullarda beriladi?*
4. *Nuqtaning vektor ko‘rinishdagi harakat tenglamasini yozing.*
5. *Nuqtaning harakati koordinata usullarida berilganda harakat tenglamalari qanday ko‘rinishda yoziladi?*
6. *Nuqtaning harakati tabiiy usulda berilganda harakat tenglamasi qanday ko‘rinishda yoziladi?*
7. *Trayektoriya nima?*
8. *Harakat deb nimaga aytiladi?*
9. *Ko‘chish deb nimaga aytiladi?*
10. *Nuqtaning harakat qonunini ta’riflang.*

3-§. Nuqtaning tezligi

Tezlik deb berilgan sanoq sistemasida har qanday vaqt onida moddiy nuqta harakatining qanchalik ildamligi va uning yo‘nalishini ifodalaydigan vektor kattalikka aytiladi.

3.1. Harakat qonuni vektor usulida berilgan nuqtaning tezligi

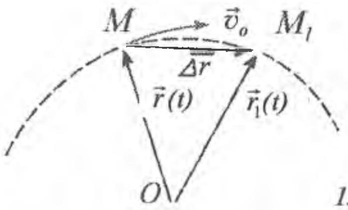
Agar nuqtaning harakati vektor usulda $\vec{r} = \vec{r}(t)$ tenglama bilan berilgan bo‘lsa, nuqtaning berilgan ondagi tezlik vektori uning radius vektoridan vaqt bo‘yicha olingan birinchi tartibli hosilaga teng bo‘ladi:

$$\vec{v} = \frac{d\vec{r}}{dt}. \quad (1.7)$$

Tezlik vektori nuqta trayektoriyasiga harakat yo'nalishi bo'yicha o'tkazilgan urinma bo'ylab yo'naladi *(1.6-rasm).*

Nuqtaning Δt vaqt oralig'idagi o'rtacha tezligi quyidagicha aniqlanadi:

$$\vec{v}_o' = \frac{\Delta r(t)}{\Delta t}.$$



1.6-rasm

3.2. Harakati koordinatalar usulida berilgan nuqtaning tezligi

Agar nuqtaning harakati koordinatalar usulida

$$\begin{cases} x = x(t), \\ y = y(t), \\ z = z(t) \end{cases} \quad (1.8)$$

tenglamalar bilan berilgan bo'lsa, nuqta tezligining biror qo'zg'almas Dekart koordinata o'qidagi proyeksiyasi mos koordinatasidan vaqt bo'yicha olingan birinchi tartibli hosilaga teng bo'ladi.

Shuning uchun:

$$v_x = \frac{dx}{at}, \quad v_y = \frac{dy}{at}, \quad v_z = \frac{dz}{at}. \quad (1.9)$$

Agar tezlikning koordinata o'qlaridagi proyeksiyalari ma'lum bo'lsa, uning moduli

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad (1.10)$$

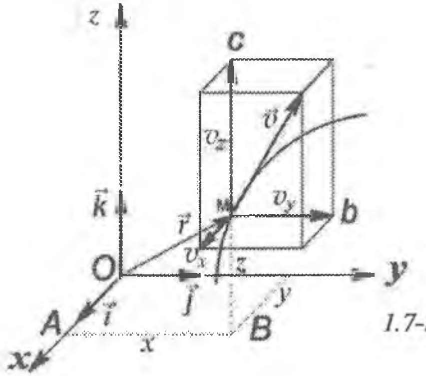
formula bilan, yo'nalishi esa

$$\cos(\vec{v}\wedge\vec{i}) = \frac{v_x}{v},\ \cos(\vec{v}\wedge\vec{j}) = \frac{v_y}{v},\ \cos(\vec{v}\wedge\vec{k}) = \frac{v_z}{v} \qquad (1.11)$$

formulalar yordamida aniqlanadi. Bunda $\vec{i}, \vec{j}, \vec{k}$ lar Dekart koordinata o'qlarining birlik vektorlari *(1.7-rasm)*.

Agar nuqta tekislikda harakatlansa, uning harakati

$$\begin{cases} x = x(t), \\ y = y(t) \end{cases} \qquad (1.12)$$

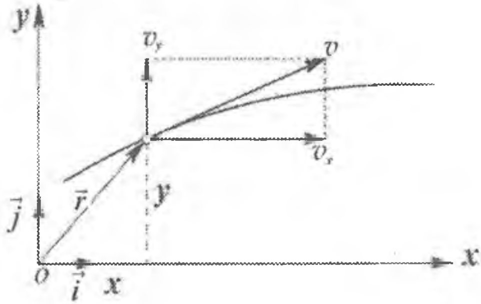


1.7-rasm

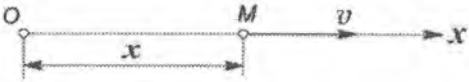
tenglamalar bilan beriladi. Bunday holda tezlik moduli va yo'nalishi quyidagicha aniqlanadi *(1.8-rasm)*:

$$v = \sqrt{v_x^2 + v_y^2}, \qquad (1.13)$$

$$\cos\vec{v}\wedge\vec{i} = \frac{v_x}{v},\ \cos\vec{v}\wedge\vec{j} = \frac{v_y}{v}.$$



1.8-rasm



1.9-rasm

Nuqtaning *Ox* o‘qi bo‘ylab to‘g‘ri chiziqli harakati

$$x = x(t) \tag{1.14}$$

tenglama bilan beriladi.

Bunday holda nuqta tezligining moduli tezlik vektorining koordinata o‘qidagi proyeksiyasining absolut qiymatiga teng bo‘ladi *(1.9-rasm):*

$$v = |v_x| = \left|\frac{dx}{at}\right|. \tag{1.15}$$

3.3. Harakati tabiiy usulda ifodalangan nuqtaning tezligi

Agar nuqta berilgan trayektoriya bo‘ylab $s{=}s(t)$ qonun asosida harakatlansa, tezlik vektori quyidagi formula orqali ifodalanadi:

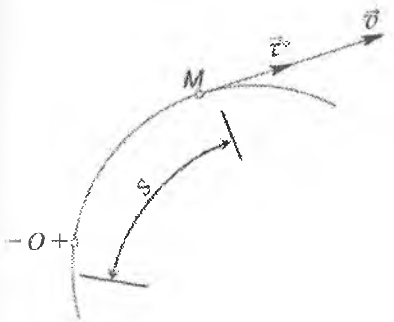
$$\vec{v} = \frac{ds}{dt}\vec{\tau}^{\,0}. \tag{1.16}$$

(1.16) da $\frac{ds}{dt}$ hosila $\vec{v}$ tezlikning urinmadagi proyeksiyasi V_τ ni ifodalaydi va tezlikning algebraik qiymati deyiladi.

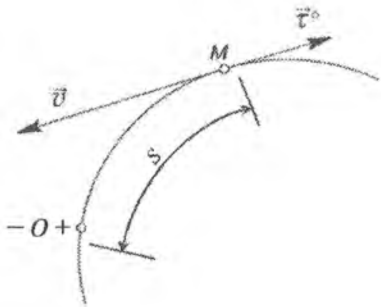
v_τ ning absolut qiymati tezlikning moduliga teng bo‘ladi:

$$v = |v_\tau| = \left|\frac{ds}{dt}\right|. \tag{1.17}$$

Bunda $\frac{ds}{dt} > 0$ bo‘lsa, yoy koordinatasi s orta boradi va nuqta tezligi $\vec{v}$ ning yo‘nalishi $\vec{\tau}^{\,0}$ bilan ustma-ust tushadi. Agar



1.10-a rasm



1.10-b rasm

$\frac{ds}{dt} < 0$ bo‘lsa, yoy koordinatasi s kamaya boradi va $\vec{v}$ tezlik vektori

$\vec{\tau}^0$ ga qarama-qarshi yo‘naladi *(1.10-a, b rasmlar)*.

Takrorlash uchun savollar:

1. *Nuqta tezligi tushunchasini ta’riflang.*
2. *Vaqt onidagi tezlik va o‘rtacha tezlik qanday yo‘naladi?*
3. *Tezlik vektorining Dekart o‘qlaridagi proyeksiyalari qanday aniqlanadi?*
4. *Tezlik vektorining tabiiy o‘qlardagi proyeksiyalari qanday aniqlanadi?*
5. *Tezlik moduli qanday aniqlanadi?*
6. *Xorijiy adabiyotlarda harakatini xarakterlovchi «speed» va «velosity» tushunchalarini sharhlab bering.*

4-§. Nuqtaning tezlanishi

Harakatdagi nuqta tezligining vaqt o‘tishi bilan miqdor va yo‘nalish jihatidan o‘zgarishini ifodalovchi vektor kattalik tezlanish deyiladi.

4.1. Harakati vektor usulida berilgan nuqtaning tezlanishi

Nuqtaning harakati vektor usulida

$$\vec{r} = \vec{r}(t) \tag{1.18}$$

tenglama bilan berilganda, uning tezligi

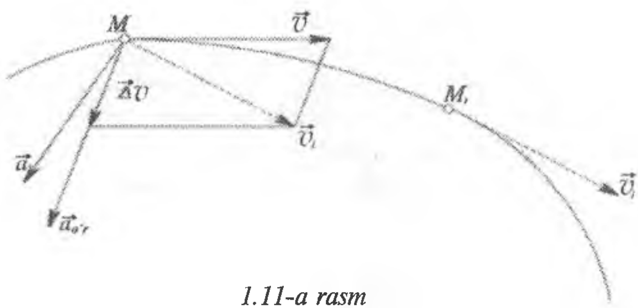
$$\vec{v} = \frac{d\vec{r}}{dt} \qquad (1.19)$$

bo'lishini e'tiborga olsak, nuqtaning tezlanish vektori uning tezlik vektoridan vaqt bo'yicha olingan birinchi tartibli hosilaga yoki radius vektoridan vaqt bo'yicha olingan ikkinchi tartibli hosilaga teng bo'ladi:

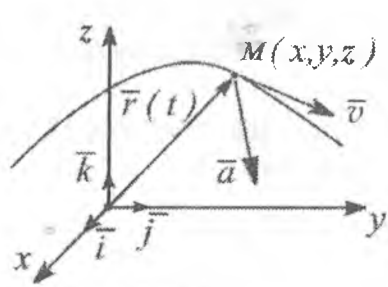
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}. \qquad (1.20)$$

Nuqta bir tekislikda yotuvchi trayektoriya bo'ylab harakatlansa, tezlanish vektori, o'rtacha tezlanish $\vec{a}_{ur}$ kabi trayektoriya tekisligida yotadi hamda trayektoriyaning botiq tomoniga yo'naladi.

Agar nuqtaning trayektoriyasi bir tekislikda yotmaydigan egri chiziqdan iborat bo'lsa, tezlanish vektori egrilik tekisligida yotadi va trayektoriyaning botiq tomoniga yo'naladi *(1.11-a, b rasm).*



1.11-a rasm



1.11-b rasm

1.2. Harakati koordinatalar usulida berilgan nuqtaning tezlanishi

Nuqtaning harakati koordinatalar usulida berilganda nuqta tezligining koordinata o'qlaridagi proyeksiyalari

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt} \tag{1.21}$$

formulalar yordamida aniqlangan edi.

Nuqta tezlanishining biror o'qdagi proyeksiyasi nuqta tezligining mazkur o'qdagi proyeksiyasidan vaqt bo'yicha olingan birinchi tartibli hosilaga yoki radius vektoridan vaqt bo'yicha olingan ikkinchi tartibli hosilaga teng bo'ladi.

Shuning uchun:

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}, \quad a_y = \frac{v_y}{dt} = \frac{d^2y}{dt^2}, \quad a_z = \frac{dv_z}{dt} = \frac{d^2z}{dt^2}. \tag{1.22}$$

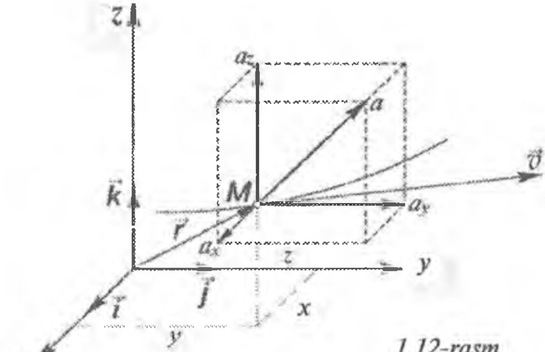
Tezlanishning koordinata o'qlaridagi proyeksiyalari ma'lum bo'lsa, uning moduli

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2} \tag{1.23}$$

formula bilan, yo'nalishi esa

$$\cos(\vec{a} \wedge \vec{i}) = \frac{a_x}{a}, \quad \cos(\vec{a} \wedge \vec{j}) = \frac{a_y}{a}, \quad \cos(\vec{a} \wedge \vec{k}) = \frac{a_z}{a} \tag{1.24}$$

formulalar yordamida aniqlanadi. Bunda $\vec{i}, \vec{j}, \vec{k}$ lar koordinata o'qlarining birlik vektorlari *(1.12-rasm)*.



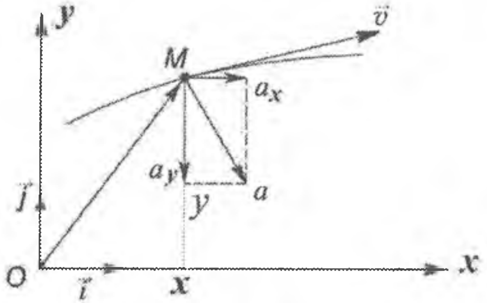
1.12-rasm

Agar nuqta *Oxy* tekisligida harakatlansa *(1.13-rasm)*, $a_z = v_z = \ddot{z} = 0$ bo'lib, tezlanish miqdori va yo'nalishi quyidagi formulalar bilan aniqlanadi:

$$a = \sqrt{ax^2 + ay^2} = \sqrt{\ddot{x}^2 + \ddot{y}^2},$$

$$\cos\left(\vec{a} \wedge \vec{i}\right) = \frac{a_x}{a}, \cos\left(\vec{a} \wedge \vec{j}\right) = \frac{a_y}{a}. \qquad (1.25)$$

Agar nuqta *Ox* o'qi bo'ylab to'g'ri chiziqli harakat qilsa *(1.14-rasm)*, tezlanish moduli



1.13-rasm

$$a = |a_x| = |\ddot{x}| \qquad (1.26)$$

formula bilan aniqlanadi. Agar $\ddot{x} > 0$ bo'lsa, tezlanish vektori $\vec{a}$ *Ox* o'qining musbat yo'nalishi bo'yicha, $\ddot{x} < 0$ bo'lsa, manfiy yo'nalishi bo'yicha yo'naladi.

1.14-rasm

4.3. Harakati tabiiy usulda berilgan nuqtaning tezlanishi

Nuqtaning harakati tabiiy usulda berilganda uning tezligi quyidagicha ifodalanar edi:

$$\vec{v} = \frac{ds}{dt}\vec{\tau}^0 = v\vec{\tau}^0. \qquad (1.27)$$

Tezlanish vektori, tezlik vektoridan vaqt bo‘yicha olingan birinchi tartibli hosilaga teng bo‘ladi:

$$\vec{a}=\frac{d\vec{v}}{dt}=\frac{dv}{dt}\vec{\tau}^0+v\frac{d\vec{\tau}^0}{dt}=\frac{dv}{dt}\vec{\tau}^0+v\frac{d\vec{\tau}^0}{ds}\frac{ds}{dt}. \qquad (1.28)$$

Analitik geometriyadan ma’lumki,

$$\frac{d\vec{\tau}^0}{ds}=\frac{1}{\rho}\vec{n}^0, \qquad (1.29)$$

bunda ρ — trayektoriyaning egrilik radiusi, $\vec{n}^0$ — trayektoriyaga o‘tkazilgan bosh normal birlik vektori.

Bularni e’tiborga olsak,

$$\vec{a}=\frac{dv}{dt}\vec{\tau}^0+\frac{v^2}{\rho}\vec{n}^0. \qquad (1.30)$$

Bu ifodada $\frac{dv}{at}\vec{\tau}^0$ vektor kattalik trayektoriyaga *M* nuqtada o‘tkazilgan urinma bo‘ylab yo‘naladi va *urinma tezlanish* deyiladi:

$$\vec{a}_\tau=\frac{dv}{dt}\vec{\tau}^0. \qquad (1.31)$$

$\frac{v^2}{\rho}\vec{n}^0$ vektor kattalik trayektoriyaga *M* nuqtada o‘tkazilgan bosh normal bo‘ylab yo‘naladi va *normal tezlanish* deyiladi:

$$\vec{a}_n=\frac{v^2}{\rho}\vec{n}^0. \qquad (1.32)$$

Urinmaning birlik vektori $\vec{\tau}^0$ va bosh normalning birlik vektori $\vec{n}^0$ trayektoriyaning *M* nuqtasiga o‘tkazilgan egrilik tekisligida yotganligi tufayli, tezlanish vektori ham mazkur egrilik tekisligida yotadi. Shu sababli tezlanishning binormaldagi tashkil etuvchisi nolga teng bo‘ladi.

Tezlanishning tabiiy koordinata o‘qlaridagi proyeksiyalari quyidagicha aniqlanadi:

$$a_\tau = \frac{dv}{dt} = \frac{d^2S}{dt^2}, \quad a_n = \frac{v^2}{\rho}. \tag{1.33}$$

Tezlanish vektori urinma tezlanish $\vec{a}_\tau$ va normal tezlanish $\vec{a}_n$ larning geometrik yig'indisiga teng bo'ladi:

$$\vec{a} = \vec{a}_\tau + \vec{a}_n. \tag{1.34}$$

Bu tezlanishlar o'zaro perpendikular yo'nalganidan, to'la tezlanish moduli

$$a = \sqrt{a_\tau^2 + a_n^2} \tag{1.35}$$

yoki

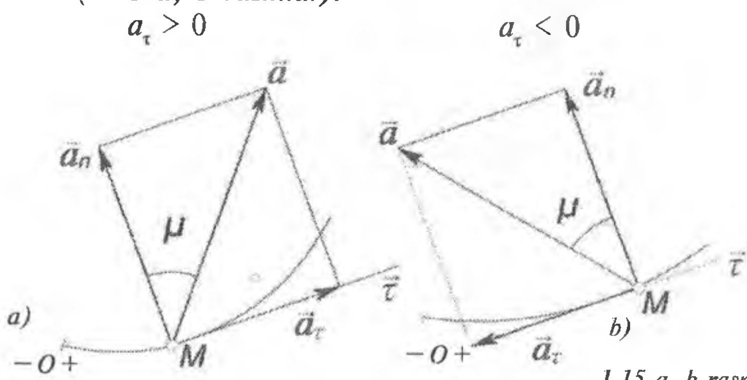
$$a = \sqrt{\left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2} \tag{1.36}$$

formula bilan, yo'nalishi esa

$$tg\mu = \frac{|a_\tau|}{a_n} \tag{1.37}$$

formula bilan aniqlanadi *(1.15-a, b rasmlar).*

Bunda $\vec{a}_n$ har doim trayektoriyaning botiq tomoniga yo'naladi ($a_n > 0$), $\vec{a}_\tau$ proyeksiyasining ishorasiga bog'liq holda nuqtaning urinma tezlanishi M_τ o'qning musbat yoki manfiy tomoniga qarab yo'naladi *(1.15-a, b rasmlar).*



1.15-a, b rasmlar

5-§. Nuqta harakatining xususiy hollari

Nuqtaning tezlanishi tabiiy koordinata o'qlaridagi tashkil etuvchilari orqali quyidagicha yoziladi:

$$\vec{a} = \frac{dv}{dt}\vec{\tau}^0 + \frac{v^2}{\rho}\vec{n}^0. \qquad (1.38)$$

Nuqtaning tezlanishiga qarab harakat turlarini aniqlash mumkin.

5.1. To'g'ri chiziqli tekis harakat

Nuqtaning trayektoriyasi to'g'ri chiziqdan iborat bo'lsa, $\rho = \infty$ bo'ladi.

Bunday holda

$$a_n = \frac{v^2}{\rho} = 0 \qquad (1.39)$$

bo'lib, nuqtaning tezlanishi faqat urinma tezlanishdan iborat bo'ladi:

$$a = a_\tau = \frac{dv_\tau}{dt}. \qquad (1.40)$$

Bunday holda nuqtaning tezligi faqat miqdor jihatdan o'zgaradi ($\rho = \infty$). *Shuning uchun ham urinma tezlanish tezlikning son qiymati jihatdan o'zgarishini ifodalaydi.*

Nuqtaning harakati davomida doimo $\vec{a}_\tau = 0$, $\vec{a}_n = 0$, ya'ni $\vec{a} = 0$ bo'lsa, $\frac{dv_\tau}{dt} = 0$ bo'lib, $v = |v_\tau| = \text{const}$ bo'ladi.

$\frac{v^2}{\rho} = 0$ bo'lganidan $\rho = \infty$ ekanligi kelib chiqadi.

Bunday holda nuqta to'g'ri chiziqli tekis harakatda bo'ladi.

5.2. Egri chiziqli tekis harakat

Agarda tezlikning son qiymati harakat davomida doimo o'zgarmas holda saqlansa, nuqta egri chiziqli tekis harakatda bo'ladi:

$$v = \text{const}.$$

Bunday holda

$$a_\tau = \frac{dv}{dt} = 0 \tag{1.41}$$

bo'lib, nuqtaning tezlanishi faqat normal tezlanishdan iborat bo'ladi:

$$a = a_n = \frac{v^2}{\rho}. \tag{1.42}$$

Nuqtaning normal tezlanishi $\vec{a}_n$ doimo egri chiziqning botiq tomoniga yo'nalgan bosh normal bo'ylab yo'naladi. $v = \text{const}$ bo'lgani uchun bu tezlanish nuqtaning tezligi vaqt o'tishi bilan faqat yo'nalishini o'zgartirishidan hosil bo'ladi.

Shu sababli normal tezlanish nuqta tezligining yo'nalish jihatdan o'zgarishini ifodalaydi.

Agar $v = \frac{ds}{dt}$ ekanligini e'tiborga olsak, $(v = v_0)$

$$ds = vdt. \tag{1.43}$$

Bu tenglikni mos chegaralar bo'yicha integrallasak,

$$\int_{s_0}^{s} ds = \int_{0}^{t} v_0 dt$$

yoki

$$s = s_0 + v_0 t \tag{1.44}$$

tenglama hosil bo'ladi.

Agar $s_0 = 0$ bo'lsa, $s = v_0 t$.

(1.44) tenglama nuqtaning egri chiziqli tekis harakati tenglamasini ifodalaydi.

5.3. Egri chiziqli tekis o‘zgaruvchan harakat

Agar nuqtaning harakati davomida doimo $a_\tau = \text{const}$ bo‘lsa, bunday harakat tekis o‘zgaruvchan harakat deyiladi.

Agar $t = 0$ da $s = s_0$ va $v = v_0$ bo‘lsa,

$$a_\tau = \frac{dv}{dt} = \frac{d^2s}{dt^2} \qquad (1.45)$$

tenglamadan

$$dv = a_\tau ds \qquad (1.46)$$

tenglik hosil bo‘ladi. $a_\tau = \text{const}$ ekanligini e’tiborga olib, (1.46) tenglikni mos chegaralar bo‘yicha integrallasak,

$$\int_{v_0}^{v} dv = \int_0^t a_\tau ds$$

yoki

$$v = v_0 + a_\tau t. \qquad (1.47)$$

(1.47) ifoda egri chiziqli tekis o‘zgaruvchan harakatdagi nuqtaning tezligini ifodalaydi.

Agar

$$v = \frac{ds}{dt}$$

ekanligini e’tiborga olsak, (1.47) tenglama quyidagicha yoziladi:

$$\frac{ds}{dt} = v_0 + a_\tau t. \qquad (1.48)$$

Bu tenglamaning har ikkala tomoni mos chegaralar bo‘yicha integrallansa, tekis o‘zgaruvchan harakat tenglamasi hosil bo‘ladi:

$$s = s_0 + v_0 t + \frac{a_\tau t^2}{2}. \qquad (1.49)$$

To‘g‘ri chiziqli tekis o‘zgaruvchan harakat tezligi va harakat tenglamasi quyidagi ko‘rinishda ifodalanadi:

$$\dot{x} = v_0 + a_x t, \quad (1.50)$$

$$x = x_0 + v_0 t + \frac{a_x t^2}{2}. \quad (1.51)$$

Bunda $a = |a_x| = |\dot{x}|$.

Takrorlash uchun savollar:

1. *Nuqtaning tezlanishi vektori qanday ifodalanadi va nuqta trayektoriyasiga nisbatan qanday yo'naladi?*
2. *Egri chiziqning har bir nuqtasida tabiiy koordinata o'qlari qanday yo'naladi?*
3. *Tezlanish vektori qaysi tekislikda yotadi va uning tabiiy o'qlardagi proyeksiyalari qanday aniqlanadi?*
4. *Nuqtaning qanday harakatida urinma tezlanish va qanday harakatida normal tezlanish nolga teng bo'ladi?*
5. *Nuqtaning tezlanishiga qarab nuqta harakatining xususiy hollarini ta'riflang.*

6-§. Nuqta harakatining tenglamalari va trayektoriyasini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Nuqta kinematikasida nuqtaning harakat tenglamalari berilgan bo'lib, uning trayektoriyasi, tezligi, tezlanishi kabi kinematik kattaliklarni aniqlash talab etiladi.

Nuqta harakatining tenglamalari va trayektoriyasini aniqlashga doir masalalar quyidagi tartibda yechiladi:

1) qo'zg'almas o'qlar sistemalari (to'g'ri burchakli, qutb va h.k.), ularning boshi (qo'yilish nuqtalari) tanlab olinadi;

2) masala shartiga ko'ra, tanlab olingan koordinatalar sistemasi uchun nuqtaning harakat tenglamalari tuziladi;

3) tuzilgan harakat tenglamalariga ko'ra, istalgan vaqt oni uchun nuqtaning o'rni, harakatining yo'nalishi, trayektoriyasi aniqlanadi.

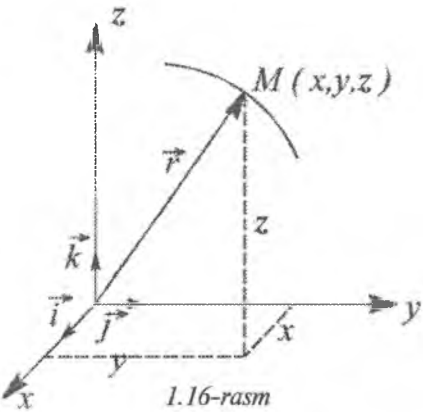
Nuqta kinematikasiga doir masalalarni yechishda quyidagilarga e'tibor berish maqsadga muvofiq bo'ladi:

— harakatdagi nuqtaning fazoda qoldirgan izi uning trayektoriyasi deyiladi. Nuqtaning trayektoriyasi tekislikda yoki fazoda yotuvchi chiziq bo‘lishi mumkin;

— nuqtaning harakati uning harakat qonuni orqali ifodalanadi. Nuqtaning harakat qonuni (tenglamasi) uning tekislikda yoki fazodagi o‘rni va vaqt oralig‘i orasidagi bog‘lanishni ifodalaydi:

$$\vec{r} = \vec{r}(t); \tag{1.52}$$

— nuqtaning harakati vektor usulida berilganida ixtiyoriy vaqt onidagi o‘rni koordinatalar boshidan harakatdagi nuqtaga o‘tkazilgan radius vektor orqali aniqlanadi *(1.16-rasm)*.



1.16-rasm

Nuqtaning harakati koordinatalar usulida berilganda uning ixtiyoriy vaqt oralig‘idagi o‘rni:

a) fazoda $x = f_1(t)$, $y = f_2(t)$, $z = f_3(t)$;

b) tekislikda $x = f_1(t)$, $y = f_2(t)$; (1.5?)

c) nuqta to‘g‘ri chiziqli harakatda bo‘lganda $-x = f(t)$ koordinatalari orqali aniqlanadi.

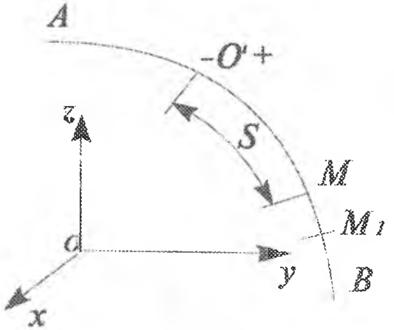
Nuqtaning harakati qutb, slindrik va sferik koordinatalarda ham beriladi. Agar nuqta harakatining trayektoriyasi oldindan ma’lum bo‘lsa, uning harakatini tabiiy usulda berish qulay bo‘ladi.

Bunday holda nuqtaning trayektoriyadagi o'rni

$$S = f(t) \tag{1.54}$$

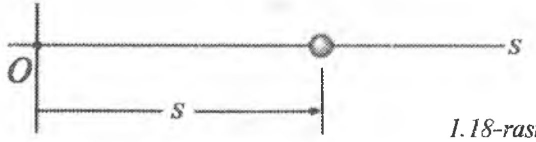
tenglama orqali aniqlanadi.

Bu ifodada S egri chiziqli koordinata bo'lib, trayektoriya bo'ylab tanlab olingan biror O nuqtadan hisoblanadi *(1.17-rasm)*.



1.17-rasm

Bunda nuqtaning trayektoriyasi to'g'ri chiziq bo'lishi ham mumkin *(1.18-rasm)*.

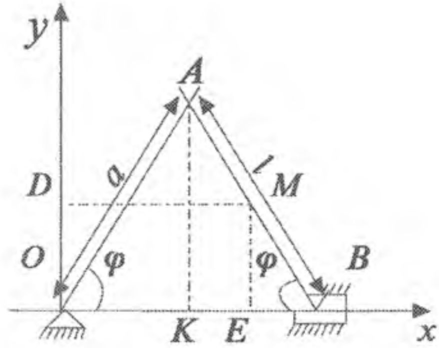


1.18-rasm

7-§. Nuqta harakatining tenglamalari, trayektoriyasini aniqlashga doir masalalar

1-masala.

Krivoship shatun mexanizmida OA krivoship doimiy φ burchak tezlik bilan aylanadi; $OA=l$, $l=a$. Shatun o'rtasidagi M nuqtaning harakat tenglamasi va trayektoriya tenglamasini aniqlang. Shuningdek, B polzunning harakat tenglamasi va trayektoriya tenglamasini aniqlang. Harakat boshlanishida B polzun o'ngdagi eng chetki holatda bo'lgan koordinata o'qlari va krivoship hamda shatunning Ox o'qi bilan hosil qilgan burchaklari rasmda ko'rsatilgan *(1.19-rasm)*.



1.19-rasm

Yechish: *M* nuqtaning harakat tenglamasi quyidagi shaklda yoziladi:

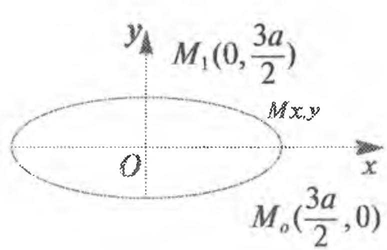
$$\left.\begin{aligned} x_M &= f_1(t), \\ y_M &= f_2(t). \end{aligned}\right\} \qquad (1.55)$$

1.19-rasmdan M nuqtaning Dekart o'qlari sistemasidagi koordinatalarini aniqlaymiz. Buning uchun *M* nuqtadan koordinata o'qlariga *MD* va *ME* perpendikular chiziqlarni o'tkazamiz.

Mazkur perpendikular uzunliklari *M* nuqtaning koordinatalarini ifodalaydi:

$$\left.\begin{aligned} x_M &= OE = OK + KE = OA\cos\varphi + AM\cos\varphi, \\ y_M &= ME = \frac{a}{2}\sin\varphi. \end{aligned}\right\} \qquad (1.56)$$

Mexanizmda $l = a$ bo'lgani uchun $\varphi = \varphi$. Shuning uchun



1.20-rasm

$$\left.\begin{aligned} x_M &= a\cos\varphi + \frac{a}{2}\cos\varphi = \frac{3a}{2}\cos\varphi, \\ y_M &= \frac{a}{2}\sin\varphi. \end{aligned}\right\} \tag{1.57}$$

Masala shartiga ko'ra,

$$\varphi = \omega t.$$

Bunday holda

$$\left.\begin{aligned} x_M &= \frac{3a}{2}\cos\omega t, \\ y_M &= \frac{a}{2}\sin\omega t. \end{aligned}\right\} \tag{1.58}$$

bo'ladi.

(1.58) tenglamalar sistemasi M nuqtaning harakat tenglamalarini ifodalaydi.

M nuqta trayektoriyasining tenglamasini tuzish uchun (1.58)dan vaqt-parametr t ni yo'qotish lozim. Sinus va kosinus funksiyalarining argumentlari bir xil bo'lsa, vaqt t ni yo'qotish uchun quyidagi trigonometrik ayniyatdan foydalanamiz:

$$\sin^2\alpha + \cos^2\alpha = 1. \tag{1.59}$$

(1.58) dan

$$\left.\begin{aligned} \cos\omega t &= \frac{2x}{3a}, \\ \sin\omega t &= \frac{2y}{a}. \end{aligned}\right\} \tag{1.60}$$

(1.60)ning har ikkala tomonlarini kvadratga ko'taramiz.

$$\left.\begin{aligned} \cos^2\omega t &= \frac{4x^2}{9a^2}, \\ \sin^2\omega t &= \frac{4y^2}{a^2}. \end{aligned}\right\} \tag{1.61}$$

(1.61)ning chap va o‘ng tomonlarini qo‘shsak, quyidagi ifoda kelib chiqadi:

$$\frac{4x^2}{9a^2}+\frac{4y^2}{a^2}=1$$

yoki

$$\frac{x^2}{\left(\frac{3a}{2}\right)^2}+\frac{y^2}{\left(\frac{a}{2}\right)^2}=1.$$

(1.62) tenglama M nuqta trayektoriyasining tenglamasini ifodalaydi. M nuqta trayektoriyasi yarim o‘qlari $\frac{3a}{2}$ va $\frac{a}{2}$ bo‘lgan ellipsdan iborat ekan *(1.20-rasm).*

B polzunning harakat tenglamasini aniqlaymiz *(1.19-rasmdan)*:

$$x_B = OB = a\cos\varphi + l\cos\varphi = 2a\cos\varphi = 2a\ \cos\omega t. \quad (1.62)$$

B nuqta krivoship shatunli mexanizmda $\cos\omega t$ qonuni asosida ilgarilanma-qaytalanma harakatda bo‘ladi.

2-masala. Nuqtaning harakati

$$x = v_0 t\ \cos\alpha,$$
$$y = v_0 t\sin\alpha - \frac{gt^2}{2}$$

tenglamalar bilan berilgan. Bundagi v_0 va g lar doimiy miqdorlar.

Nuqtaning trayektoriyasi, maksimal ko‘tarilish balandligi va bunday holatda gorizontal yo‘nalishda s siljishi hamda qancha uzoqqa borishi aniqlansin *(1.21-rasm).*

Yechimi:

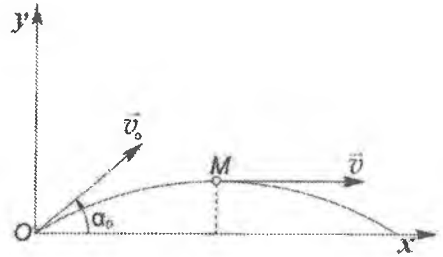
trayektoriyaning tenglamasini aniqlash uchun nuqtaning harakat tenglamalarining biridan t vaqtni topib, ikkinchi tenglamaga qo‘yamiz:

$$t = \frac{x}{v_0\cos\alpha}, \qquad (1.63)$$

$$y = \operatorname{tg}\alpha \cdot x - \frac{g}{2v_0^2 \cos^2\alpha} x^2 \quad (1.64)$$

(1.64) ifoda parabola tenglamasidir.

Nuqtaning trayektoriyasi mazkur parabolaning $x \geq 0$ shartni qanoatlantiruvchi qismidan iborat *(1.21-rasm).*



1.21-rasm

Nuqta eng yuqori holatga koʻtarilguncha oʻtgan vaqt va maksimal koʻtarilish balandligini aniqlash uchun tezlikning koordinata oʻqlaridagi proyeksiyalarini aniqlaymiz:

$$v_x = \dot{x} = v_0 \cos\alpha,$$

$$v_y = \dot{y} = v_0 \sin\alpha - gt. \quad (1.65)$$

Nuqta maksimal balandlikni egallaganda, uning tezligi x oʻqiga parallel boʻladi. Shu sababli

$$v_y = 0$$

yoki

$$v_0 \sin\alpha - gt_1 = 0 \quad (1.66)$$

boʻladi. Bunda t_1 nuqta eng yuqori holatga koʻtarilguncha oʻtgan vaqt. (1.66) dan

$$t_1 = \frac{v_0 \sin\alpha}{g}. \quad (1.67)$$

Vaqt t_1 ning qiymatini (1.64)ga qoʻyib, nuqtaning maksimal koʻtarilish balandligini aniqlaymiz:

$$h = y_{max} = \frac{v_0^2 \sin^2 \alpha}{g} - \frac{g v_0^2 \sin^2 \alpha}{2g^2} = \frac{v_0^2 \sin^2 \alpha}{2g}. \qquad (1.68)$$

Nuqta maksimal balandlikka ko'tarilganda boshlang'ich holatidan gorizontal yo'nalishda s siljishini aniqlash uchun vaqt t_1 ning qiymatini (1.63) ga qo'yamiz:

$$s_1 = x_1 = v_0 \cos\alpha \cdot \frac{v_0 \sin\alpha}{g} = \frac{v_0^2 \sin 2\alpha}{2g}. \qquad (1.69)$$

Nuqtaning maksimal uchish uzoqligi (qancha uzoqqa borishi) trayektoriya tenglamasidan $y = 0$ bo'lgan holatda (harakatlanayotgan jism yerga tushganda) aniqlanadi:

$$\operatorname{tg}\alpha \cdot x - \frac{gx^2}{2v_0^2 \cos^2 \alpha} = 0. \qquad (1.70)$$

Bu tenglamadan x ning ikki qiymati

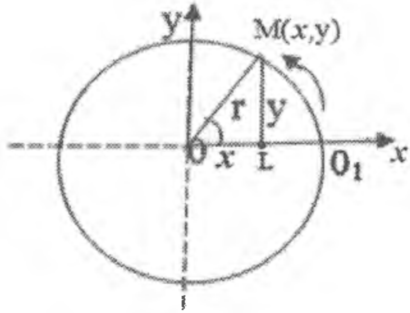
$$x_1 = 0, \; x_2 = \frac{v_0^2 \sin 2\alpha}{g} \qquad (1.71)$$

aniq bo'ladi. Bunda x_1 nuqtaning boshlang'ich holatini, x_2 esa nuqtaning gorizontal yo'nalishda uchish uzoqligini ifodalaydi. Binobarin, nuqtaning maksimal uchish uzoqligi quyidagiga teng bo'lar ekan:

$$x_2 = s_{max} = \frac{v_0^2 \sin 2\alpha}{g}. \qquad (1.72)$$

3-masala. Nuqta radiusi r bo'lgan aylana bo'ylab soat strelkasi yo'nalishiga teskari yo'nalishida $s=kt$ qonunga ko'ra harakatlanadi (k = const). Ox gorizontal o'q nuqtaning boshlang'ich holatidan o'tadi deb qarab, koordinata boshi aylana markazidan o'tuvchi xOy sistemaga nisbatan nuqtaning harakat qonuni topilsin.

Yechish: koordinata boshini r radiusli aylana markazida olib, xOy koordinata sistemasini o'tkazamiz *(1.22-rasm)*. Masala shartiga ko'ra nuqta trayektoriyasida sanoq boshi sifatida O_1 nuqtani olib, bu nuqtadan trayektoriya bo'ylab soat strelkasi harakatiga teskari yo'nalishi musbat yo'nalishi deb qabul qilamiz.



1.22-rasm

$O_1M=S=k\,t$ qonun bo‘yicha harakatlanuvchi M nuqtaning xOy koordinata sistemasidagi koordinatalarini x, y bilan belgilaymiz. *1.22-rasmdan*:

$$\left.\begin{aligned} x &= OL, \\ y &= LM. \end{aligned}\right\} \tag{1.73}$$

Agar M nuqta harakatlanganda uning koordinatalari $\varphi = O_1M$ burchak funksiyasi sifatida o‘zgarishini e’tiborga olsak, (1.73) quyidagi ko‘rinishda yoziladi:

$$\left.\begin{aligned} x &= OL = OM\cos\varphi = r\cos\varphi, \\ y &= LM = OM\sin\varphi = r\sin\varphi. \end{aligned}\right\} \tag{1.74}$$

Yoy uzunligini hisoblash formulasiga ko‘ra $O_1M = r\varphi$; bundan

$$\varphi = \frac{O_1M}{r} = \frac{kt}{r}. \tag{1.75}$$

Aniqlangan φ burchak qiymatini (1.74)ga qo‘ysak, M nuqtaning xOy koordinata sistemasiga nisbatan harakat qonuni quyidagi tenglamalar bilan aniqlanadi:

$$\left.\begin{aligned} x &= r\cos\frac{kt}{r}, \\ y &= r\sin\frac{kt}{r}. \end{aligned}\right\} \tag{1.76}$$

5-masala. M nuqta harakati $\vec{r} = (2t+1)\vec{i} + (2-3t)\vec{j}$ tenglama bilan ifodalanadi (r — metrda, t — sekundda o'lchanadi). M nuqta trayektoriyasi aniqlansin hamda harakat boshlangandan so'ng qancha vaqt o'tgach, u abssissa o'qida bo'lishi topilsin.

Yechish. M nuqtaning x, y, z koordinatalari shu nuqta radius vektorining koordinata o'qlaridagi proyeksiyalari hisoblanadi:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}. \tag{1.77}$$

(1.77)ni e'tiborga olsak, M nuqtaning harakati

$$\vec{r} = (2t+1)\vec{i} + (2-3t)\vec{j} \tag{1.78}$$

tenglama bilan ifodalanadi. (1.77) va (1.78)larni solishtirsak, nuqta harakatining koordinata usulida

$$\left.\begin{aligned} x &= 2t+1, \\ y &= 2-3t \end{aligned}\right\} \tag{1.79}$$

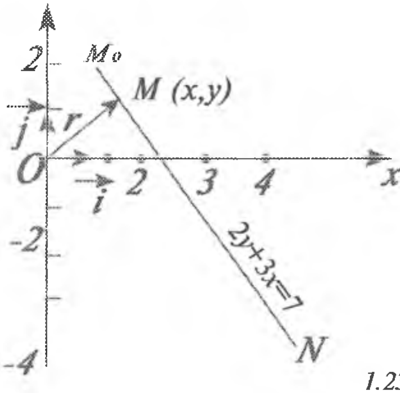
tenglamalar bilan berilishi kelib chiqadi.

M nuqta trayektoriyasini topish uchun (1.79) sistemadan vaqt t ni yo'qotish kerak. Buning uchun (1.79)ning birinchisini t ga nisbatan yechamiz:

$$t = \frac{x-1}{2}. \tag{1.80}$$

(1.80) ifodani (1.79)ning ikkinchi tenglamasiga qo'ysak,

$$2y+3x=7 \tag{1.81}$$



1.23-rasm

to‘g‘ri chiziq tenglamasi hosil bo‘ladi. $t \geq 0$ bo‘lishi sharti (1.180)dan $x \geq 1$ kelib chiqadi.

Shuning uchun, M nuqta trayektoriyasi $2y+3x=7$, $x \geq 1$ tenglamalar bilan ifodalanuvchi M_0N to‘g‘ri chiziqdan iborat bo‘lar ekan *(1.23-rasm)*.

Nuqta $t=0$ vaqtda koordinatalari $x=1$, $y=2$ dan iborat M_0 holatda bo‘ladi.

M nuqta abssissa o‘qida bo‘lganda $y=0$ bo‘ladi.

Binobarin,

$$2-3t=0$$

tenglikdan $t = \frac{2}{3}c$ vaqtda M nuqta abssissa o‘qida bo‘lishi va

$x = 2\frac{1}{3}m$ ekanligi ma’lum bo‘ladi.

6-masala. Uzunligi a bo‘lgan ON krivoship O nuqtadan o‘tuvchi rasm tekisligiga perpendikular bo‘lgan o‘q atrofida aylanadi. Qo‘zg‘almas Ox o‘q va krivoship orasidagi φ burchak vaqtga proporsional holda o‘zgaradi: $\varphi = kt$;

$$t = \frac{2}{3}c.$$

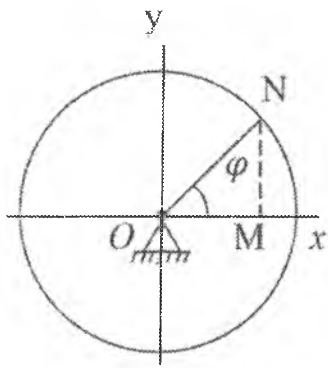
N nuqtaning Dekart koordinata o‘qlaridagi harakat tenglamalari tuzilsin. N nuqtaning to‘liq aylanish vaqti hamda N nuqtaning har ikki koordinatasining qiymatlarini o‘zaro teng bo‘lish vaqti aniqlansin *(1.24-rasm)*.

Yechish. N nuqtaning Dekart koordinata o‘qlaridagi harakat tenglamalarini tuzish uchun *1.24-rasmdan* foydalanib, uning koordinatalari x va y larni aniqlaymiz:

$$\left.\begin{aligned} x = ON\cos\varphi, \\ y = ON\sin\varphi. \end{aligned}\right\} \qquad (1.82)$$

yoki

$$\left.\begin{aligned} x = a\cos kt, \\ y = a\sin kt. \end{aligned}\right\} \qquad (1.83)$$



1.24-rasm

Hosil bo'lgan (1.83) tenglamalar sistemasi *N* nuqtaning harakat tenglamalarini ifodalaydi.

N nuqta trayektoriyasining tenglamasini tuzish uchun (1.83)dan vaqt *t* ni yo'qotamiz. Buning uchun (1.83)ning har birini kvadratga ko'taramiz:

$$\left.\begin{aligned} x^2 &= a^2 \cos^2 kt, \\ y^2 &= a^2 \sin^2 kt. \end{aligned}\right\} \qquad (1.84)$$

Hosil bo'lgan tenglamalarni chap va o'ng tomonlarini qo'shsak,

$$x^2 + y_2 = a^2 \qquad (1.85)$$

tenglama hosil bo'ladi. Ko'rinib turibtiki, *N* nuqta trayektoriyasi radiusi *a*, markazi koordinata boshida bo'lgan aylanadan iborat ekan.

N nuqtaning to'liq aylanishi uchun ketadigan *T* vaqtni aniqlaymiz. *N* nuqta bir marta to'liq aylanganda φ burchak *O* dan 2π radianga o'zgaradi:

$$\varphi = 2\pi = kT. \qquad (1.86)$$

Bundan:

$$T = \frac{2\pi}{k}. \qquad (1.87)$$

N nuqtaning boshlang'ich holati koordinatalarini aniqlaymiz. Buning uchun (1.83) tenglamalarga $t = 0$ ni qo'yamiz. Bunday holda

$$\left.\begin{aligned} x_0 &= a, \\ y_0 &= 0. \end{aligned}\right\} \qquad (1.88)$$

Nuqtaning har ikki koordinatalari o'zaro teng bo'lgan t_1 vaqtni aniqlaymiz:

$$x, \quad a \cos kt_1 = a \sin kt_1$$

yoki

$$\operatorname{tg} kt_1 = 1. \qquad (1.89)$$

Bu munosabat o'rinli bo'ladi, agar

$$kt_1 = \pi n + \frac{\pi}{4} \qquad (1.90)$$

shart bajarilsa, bunda n=0, 1, 2, 3,...
(1.90) ifodadan t_1 vaqtni aniqlaymiz:

$$t_1 = \frac{\pi}{k} n + \frac{\pi}{4k}. \qquad (1.91)$$

8-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan masalalar

1-masala. Nuqtaning koordinata usulida berilgan harakat tenglamasiga ko'ra uning trayektoriya tenglamasi topilsin va rasmda harakat yo'nalishi ko'rsatilsin:

$$x = 3t - 5, \; y = 4 - 2t.$$

2-masala. Nuqtaning koordinata usulida berilgan harakat tenglamasiga ko'ra uning trayektoriya tenglamasi topilsin va rasmda harakat yo'nalishi ko'rsatilsin:

$$x = 5 \sin 10t, \; y = 3 \cos 10t.$$

3-masala. Nuqta harakatining berilgan tenglamalariga qarab, uning trayektoriya tenglamasi topilsin, shuningdek, masofani nuqtaning boshlang'ich holatidan hisoblab, nuqtaning trayektoriya bo'ylab harakatlanish qonuni ko'rsatilsin:

$$x = 3t^2, \; y = 4t^2.$$

4-masala. Nuqta harakatining berilgan tenglamalariga qarab, uning trayektoriya tenglamasi topilsin, shuningdek, masofani nuqtaning boshlang'ich holatidan hisoblab, nuqtaning trayektoriya bo'ylab harakatlanish qonuni ko'rsatilsin:

$$x = a\cos^2 t,\ y = a\sin^2 t.$$

5-masala. Nuqtaning harakati $x = 2a\cos^2\frac{kt}{2}$, $y = a\sin kt$ tenglamalar bilan berilgan, bundagi a va k musbat o'zgarmaslar. Masofani nuqtaning boshlang'ich holatidan hisoblab, harakat trayektoriyasi va trayektoriya bo'ylab harakat qonuni aniqlansin.

6-masala. Moddiy nuqtaning harakati $S = (2t^2 - 8t + 6)m$ tenglama orqali berilgan (t sekundlarda o'lchanadi).

1.25-rasm

Qanday vaqt momentida nuqtaning tezligi nolga teng bo'ladi? Harakat boshlangan paytdan $t = 3$ s vaqt davomida bosib o'tgan yo'l aniqlansin *(1.25-rasm)*.

9-§. Nuqtaning tezligini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Nuqtaning tezligi deb berilgan sanoq sistemasida har qanday vaqt onida nuqta harakatining qanchalik ildamligi va yo'nalishini ifodalovchi vektor kattalikka aytiladi:

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x \cdot \vec{i} + v_y \cdot \vec{j} + v_z \cdot \vec{k}. \qquad (1.92)$$

Bunda $\vec{i}$, $\vec{j}$, $\vec{k}$ lar koordinata o'qlari birlik vektorlari.

Tezlik vektorining Dekart o'qlaridagi proyeksiyalari quyidagicha aniqlanadi:

$$v_x = \frac{dx}{dt} = x',\ v_y = \frac{dy}{dt} = y',\ v_z = \frac{dz}{dt} = z'.$$

Tezlik moduli

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \qquad (1.93)$$

formula asosida, uning yo'nalishi esa

$$\cos(\vec{v}\,\hat{}\,\vec{i}) = \frac{v_x}{v}, \quad \cos(\vec{v}\,\hat{}\,\vec{j}) = \frac{v_y}{v}, \quad \cos(\vec{v}\,\hat{}\,\vec{k}) = \frac{v_z}{v} \qquad (1.94)$$

formulalar asosida aniqlanadi.

Ko'pincha masalalarda harakatdagi nuqtaning ma'lum vaqt oralig'idagi «averagy velo sity» o'rtacha sur'atini aniqlash talab etiladi:

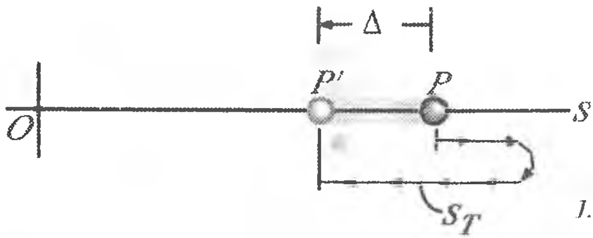
$$v_{o'rt} = \frac{\Delta S}{\Delta t}.$$

Ba'zi hollarda harakatdagi nuqtaning «average speed»i – o'rtacha tezligini topish ham ma'lum qiziqish uyg'otadi:

$$v_{o'rt} = \frac{S_T}{\Delta t}.$$

O'rtacha tezlik har doim musbat kattalik hisoblanadi.

O'rtacha sur'at va o'rtacha tezlik quyidagi rasmdan yaqqol ko'rinadi:



1.26-rasm

Agar nuqtaning harakati tabiiy usulda berilgan bo'lsa, uning tezligi quyidagicha aniqlanadi:

$$\vec{v} = \frac{ds}{dt}\vec{\tau} = v_\tau\,\vec{\tau}. \qquad (1.95)$$

Bunda $\vec{\tau}$ urinmaning birlik vektori, u yoy koordinatasi S ning o'sishi tomon yo'naladi.

Tezlik moduli quyidagi formula yordamida aniqlanadi:

$$\vec{v} = \frac{ds}{st} = \acute{s}.$$

Bunda: $v_\tau > 0$ bo'lsa, nuqta yoy koordinatasining o'sish tomoniga harakatlanadi.

$v_\tau < 0$ bo'lsa, nuqta yoy koordinatasining kamayishi tomoniga harakatlanadi;

Nuqta kinematikasida nuqtaning tezligini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1. Koordinata o'qlari sistemasi tanlab olinadi.
2. Tanlab olingan koordinata o'qlari sistemasida nuqta harakatining tenglamalari tuziladi.
3. Nuqta harakatining tenglamalariga ko'ra tezlik vektorining o'qlaridagi proyeksiyalari aniqlanadi.
4. Nuqtaning tezligining o'qlaridagi proyeksiyalariga ko'ra miqdori va yo'nalishi aniqlanadi.

10-§. Nuqtaning tezligini aniqlashga doir masalalar

1-masala. Sinov paytida raketaning dvigateli u yerdan 40 *m* balandlikka ko'tarilganda ishdan chiqqan. U paytda raketa tezligi 75 *m/s* bo'lgan. Raketaning maksimal ko'tarilish balandligi va u qaytib yerga tushganda qanday tezlikka ega bo'lishi aniqlansin. Erkin tushish tezlanishi $g = 9{,}81\ m/s^2$, u vertikal pastga yo'nalgan, havo qarshiligi e'tiborga olinmasin *(1.27-rasm)*.

Yechish:

koordinata boshi sifatida yer sirtidagi *O* nuqtani tanlab, koordinata o'qini raketa harakati tomon vertikal yuqoriga yo'naltiramiz.

Raketaning maksimal ko'tarilish balandligini aniqlaymiz. Raketa maksimal balandligi *B* nuqtaga yetganda uning tezligi quyidagicha ifodalanadi:

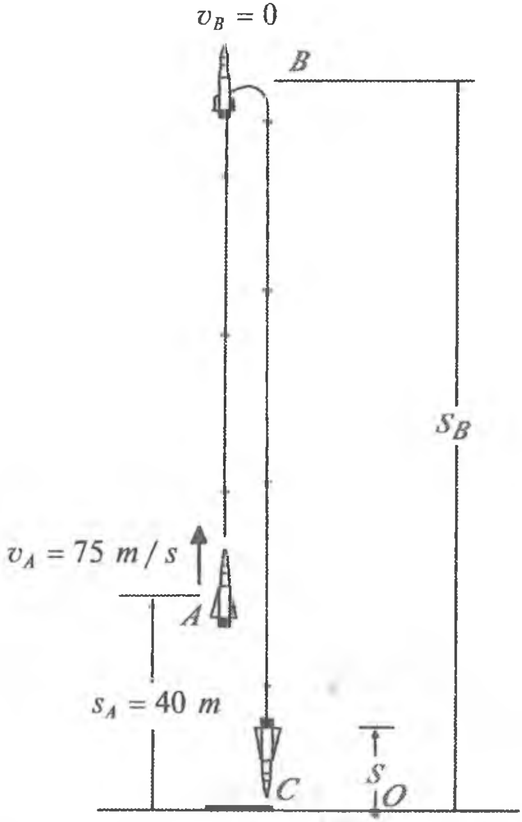
$$v_B^2 = v_A^2 + 2g(S_B - S_A).$$

Raketaning maksimal balandlikdagi tezligi $v_B = 0$ bo'ladi. Shuning uchun

$$0 = (75\ m/s)^2 + 2(-9{,}81\ m/s^2)(S_B - 40\ m).$$

Bu ifodadan $S_B = 327\ m$.

Raketa *C* nuqtaga tushganda uning tezligi quyidagiga teng bo'ladi:



1.27-rasm

$$v_C^2 = v_B^2 + 2g(S_C - S_B) = 0 + 2\left(-9{,}81\frac{m}{s^2}\right)(0 - 327).$$

Bu ifodadan

$$v_c = -80{,}1\ m/s.$$

$\vec{v}_c$ ning (–) ishorasi vertikal pastga yo'nalganligidan darak beradi.

Raketaning yerga tushgandagi tezligi uning AC hududdagi harakatini o'rganishdan ham aniqlanadi.

$$v_C^2 = v_A^2 + 2g(S_C - S_B) = \left(75\frac{m}{s}\right)^2 + 2\left(-9{,}81\frac{m}{s^2}\right)(0 - 40).$$

Bundan

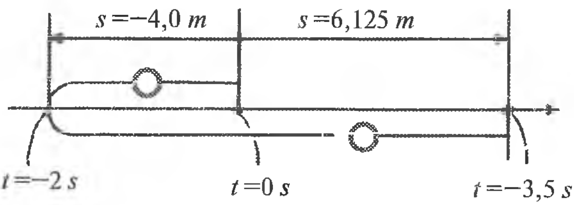
$$v_C = -80{,}1\frac{m}{s}, \quad |v_C| = 80{,}1\frac{m}{s}.$$

2-masala. Moddiy nuqta yo'lning rasmda ko'rsatilgan qismida $v = (3t^2 - 6t)\ m/s$ tezlik bilan harakatlanmoqda, bunda t sekundlarda o'lchanadi.

Agar, dastlab nuqta O holatda bo'lsa, 3,5 s davomida nuqta bosib o'tgan masofa va shu vaqt orasidagi o'rtacha sur'at va o'rtacha tezlik aniqlansin.

Yechish: koordinata o'qini nuqtaning to'g'ri chiziqli harakati trayektoriyasi bo'ylab o'ng tomon yo'naltiramiz.

Koordinata boshi sifatida nuqtaning boshlang'ich (t=0) holatini tanlaymiz *(1.28-rasm)*.



1.28-rasm

Nuqtaning berilgan trayektoriyadagi o'rnini aniqlash usuli:

$$v = \frac{ds}{dt};$$

$$ds = vdt = (3t^2 - 6t)dt;$$

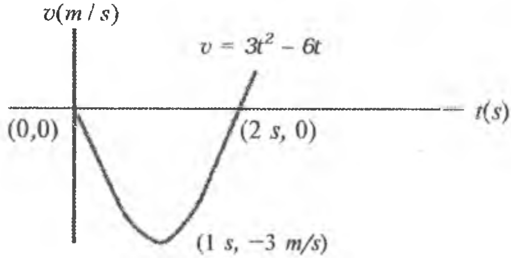
$$\int_0^S dS = \int_0^t (3t^2 - 6t)dt.$$

Tenglamani integrallasak va harakatning boshlang'ich shartlaridan foydalansak, nuqtaning istalgan vaqt momentida trayektoriyadagi o'rnini aniqlash uchun quyidagi tenglama (munobsabat)ga ega bo'lamiz:

$$S = (t^3 - 3t^2)\ m.$$

Moddiy nuqtaning $t = 3{,}5\ s$ vaqt onida trayektoriyada egallagan o'rnini aniqlash uchun harakat grafigini tuzamiz *(1.29-rasm)*.

Harakat grafigidan ko'rinib turibdiki, $0<t<2\ s$ vaqt oralig'ida,



1.29-rasm

nuqta tezligi manfiy ishoraga ega bo'lar ekan va u O nuqtadan chap tomonga harakatlanar ekan.

$t>2\ s$ dan boshlab, nuqta tezligi musbat ishoraga ega bo'lib, u o'ng tomonga harakatlanar ekan. Nuqta tezligi grafigida $t = 0$, $t=2\ s$ va $t=3{,}5\ s$ vaqt onlari uchun tezliklari ko'rsatilgan *(1.29-rasm)*.

Nuqtaning mazkur vaqt oralig'ida trayektoriyadagi o'rnini aniqlash uchun:

$$S = (t^3 - 3t^2)$$

munosabatdan foydalanilamiz:

$$S_{t=0}=0;\ S_{t=2\,s}= -4\ m;\ S_{t=3,5\,s}=6{,}125\ m.$$

Nuqtaning $t=3{,}5\ s$ vaqt davomida bosib o'tgan **masofasi** quyidagicha aniqlanadi:

$$S_T = 4{,}0 + 4{,}0 + 6{,}125 = 14{,}125 = 14{,}1\ m\,.$$

Nuqta $t=0$ dan $t=3{,}5\ s$ vaqt oralig'ida **ko'chishi** quyidagiga teng:

$$\Delta S = S\,|_{t=3,5s} - S\,|_{t=0} = 6{,}125\ m - 0 = 6{,}125\ m.$$

Buni e'tiborga olsak, shu vaqt orasidagi o'rtacha sur'at quyidagiga teng bo'ladi:

$$v = \frac{\Delta S}{\Delta t} = \frac{6{,}125\ m}{3{,}5\ s - 0} = 1{,}75\ m/s.$$

O'rtacha tezlik esa

$$v = \frac{S_T}{\Delta t} = \frac{14{,}125\ m}{3{,}5\ s - 0} = 4{,}04\ m/s.$$

3-masala. Nuqtaning harakati

$$x = v_0 t \cos\alpha_0, \tag{1.96}$$

$$y = v_0 t \sin\alpha_0 - \frac{1}{2} g t^2 \tag{1.97}$$

tenglamalar bilan berilgan; *Ox* o'q gorizontal, *Oy* o'q vertikal bo'yicha yuqoriga yo'nalgan v_0, g va $\alpha_0 < \frac{\pi}{2}$ o'zgarmas miqdorlar.

Nuqta trayektoriyasi, uning eng yuqori holatidagi koordinatalari, nuqta *Ox* o'qda bo'lgan vaqtda tezlikning koordinata o'qlaridagi proyeksiyalari topilsin.

Yechish:

Nuqtaning trayektoriyasini aniqlaymiz.

Masala shartida nuqta trayektoriyasining parametrik tenglamalari berilgan. Koordinatalar formasidagi trayektoriya tenglamasini tuzish uchun berilgan tenglamalardan parameter *t* ni qisqartiramiz. Buning uchun (1.96) tenglamadan *t* ni aniqlab, (1.97) ga qo'yamiz.

Natijada, quyidagi ko'rinishdagi trayektoriya tenglamasiga ega bo'lamiz.

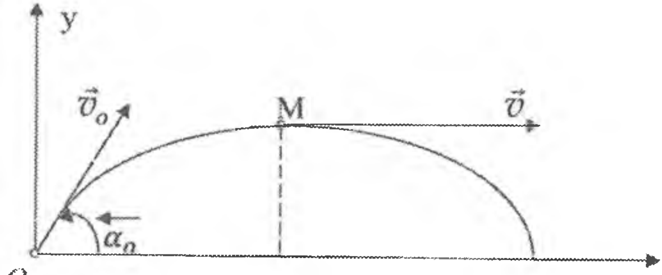
$$y = x g t \alpha_0 - \frac{g}{2 v_0^2 \cos^2\alpha_0} x^2. \tag{1.98}$$

Mazkur tenglama parabolaning tenglamasidir. Nuqtaning trayektoriyasi parabolaning $x \geq 0$ shartni qanoatlantiruvchi qismidan iborat *(1.30-rasm)*.

Tezlikning koordinata o'qlaridagi proyeksiyalari uning mos koordinatasidan vaqt bo'yicha hisoblangan birinchi hosilaga teng. Masala shartiga ko'ra tezlikning koordinata o'qlaridagi proyeksiyalari uchun quyidagi ifodalarni olamiz:

$$v_x = x' = v_0 \cos\alpha_0, \tag{1.99}$$

$$v_y = y' = v_0 \sin\alpha_0 - g t. \tag{1.100}$$



1.30-rasm

Nuqta trayektoriyaning eng yuqori holatini egallaganda tezligi *Ox* o'qiga parallel bo'ladi. Shuning uchun $v_y = 0$ yoki $v_0 \sin \alpha_0 - gt_1 = 0$ bo'ladi, bunda t_1 bilan nuqta eng yuqori holatga ko'tarilguncha ketgan vaqt belgilangan. Oxirgi tenglikdan

$$t_1 = \frac{v_0 sin\alpha_0}{g}.$$

Aniqlangan vaqt t_1 ning qiymatini (1.96) va (1.97)ga qo'yib, nuqta eng yuqori holatining koordinatalarini aniqlaymiz:

$$x = \frac{v_0^2}{2g}\sin 2\alpha_0, \tag{1.101}$$

$$y = \frac{v_0^2}{2g}\sin^2 \alpha_0. \tag{1.102}$$

Nuqta *Ox* o'qda bo'lgan vaqitda tezlikning koordinata o'qlaridagi proyeksiyalarini aniqlaymiz. Nuqta *Ox* o'qda yotgan vaqtda

$$y = 0$$

yoki

$$v_0 T \sin \alpha_0 - \frac{1}{2} g T^2 = 0 \tag{1.103}$$

bo'ladi. Mazkur ifodada t bilan nuqta *Ox* o'qda bo'lgan vaqt belgilangan.

(1.103) tenglikdan

$$T_1 = 0, \; T_2 = \frac{2v \sin \alpha_0}{g}$$

vaqtlar aniqlanadi. T_1=0 vaqt nuqtaning boshlang'ich holatiga mos keladi. T_1 va T_2 ning qiymatini (1.99) va (1.100)ga qo'yib, nuqta Ox o'qda bo'lgan vaqtdagi tezlikning koordinata o'qlaridagi proyeksiyalarini aniqlaymiz.

$$v_x = v_0 \cos\alpha_0 \tag{1.104}$$

$$v_y = \pm v_0 \sin\alpha_0 \tag{1.105}$$

(1.105)da musbat ishora nuqtaning boshlang'ich holatiga mos keladi.

4-masala. Nuqtaning harakati

$$x = 2t, \tag{1.106}$$
$$y = t^2 \tag{1.107}$$

tenglamalar bilan berilgan (t — sekundlarda, x va y — santimerlarda o'lchanadi).

$t = 1$ s vaqt uchun tezlik qiymati topilsin va trayektoriyada ko'rsatilsin.

Yechish:

trayektoriya tenglamasini tuzish uchun harakat tenglamalarining biridan vaqt t ni aniqlab, ikkinchisiga qo'yamiz:

$$t = \frac{x}{2}, \tag{1.108}$$

$$y = \frac{x^2}{4}. \tag{1.109}$$

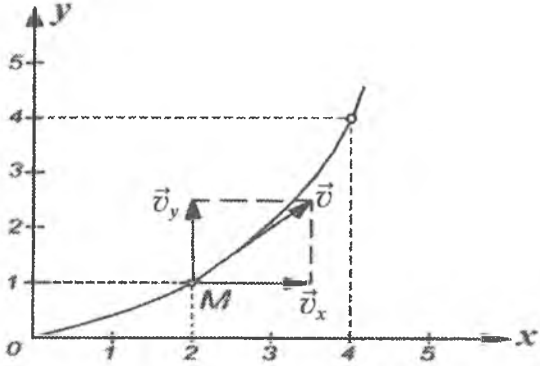
Bu tenglama parabola tenglamasi. Binobarin, nuqtaning trayektoriyasi paraboladan iborat ekan.

Trayektoriyani chizish uchun (1.109) tenglamada x ga qiymatlar berib, unga mos y ning qiymatlarini topamiz *(1.31-rasm):*

x	0	2	4
y	0	1	4

$t = 1$ sekundda nuqtaning trayektoriyadagi o'rnini topamiz.

Buning uchun berilgan harakat tenglamalaridagi t ning o'rniga uning qiymatini qo'yib, nuqtaning koordinatalarini topamiz.



1.31-rasm

$$t = 1\ s \text{ da, } x = 2\ sm,\ y = 1\ sm.$$

Demak, $t = 1$ sekundda M nuqtaning koordinatalari (2, 1) bo'lar ekan.

Nuqtaning tezligini koordinata o'qlaridagi proyeksiyalari orqali aniqlaymiz:

$$v_x = x' = 2\ sm/s,\ (v_x = \text{const}),\ v_y = y' = 2t\ sm/s. \quad (1.110)$$

$$t = 1\ s \text{ da, } v_x = 2\ sm/s,\ v_y = 2 \cdot 1 = 2\ sm/s.$$

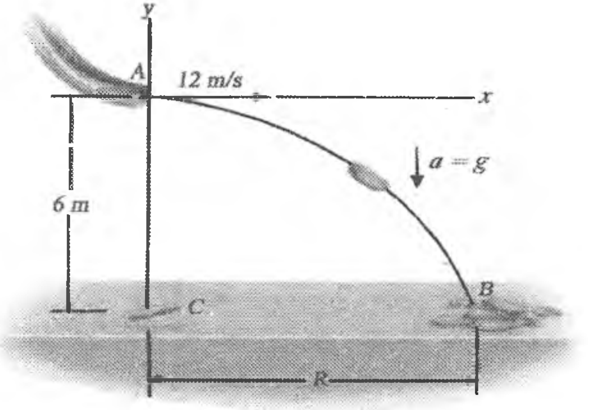
Tezlik miqdori koordinata o'qlaridagi proyeksiyalari orqali quidagicha aniqlanadi:

$$v = \sqrt{v_x^2 + v_y^2} = 2\sqrt{2}\ sm/s. \quad (1.111)$$

Tezlik uchun masshtabni 1 *sm* da 2 *sm/s* deb tanlab, tezlik vektorining yo'nalishini aniqlaymiz *(1.31-rasm)*. Tezlik vektori nuqta trayektoriyasiga urinma holda yo'nalar ekan.

11-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

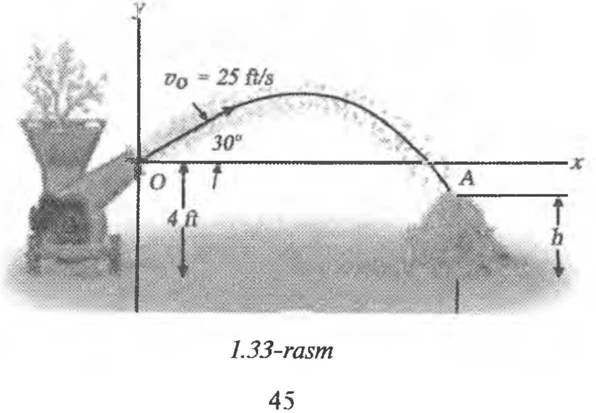
1-muammo. Qop nishablikda harakatlanib, A nuqtada $v_A = 12\ m/s$ gorizontal tezlikka ega bo'ladi. Agar A nuqtaning poldan balandlikgi 6 m bo'lsa, qopning B nuqtaga tushishi uchun ketadigan vaqt va B nuqtaning C nuqtadan qanday uzoqlikda yotishi aniqlansin *(1.32-rasm)*.



1.32-rasm

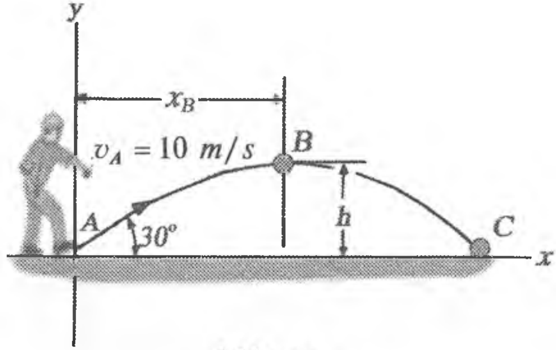
2-muammo. Yog'och qirqadigan mashinaning a nuqtasidan yog'och qirindisi $v_0 = 25\ m/s$ tezlik bilan otilib chiqadi.

Agar qirindining otilib chiqish tezligi gorizontal bilan 90° burchak tashkil etsa, uning A tushish nuqtasining yerdan qanday h balandlikda bo'lishi aniqlansin. A nuqta qirindining otilib chiqishi nuqtasidan yer bo'ylab hisoblanganda 20 m uzoqlikda joylashgan *(1.33-rasm)*.



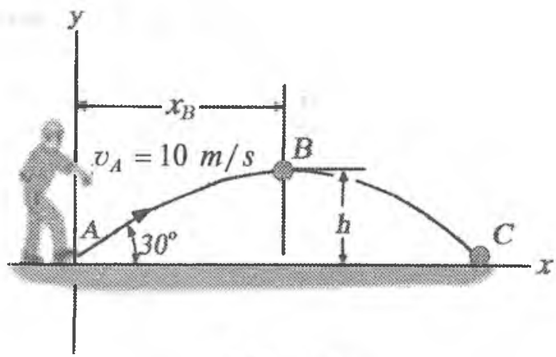
1.33-rasm

3-muammo. Koptok *A* nuqtadan $v_A = 10\ m/s$ tezlik bilan tepiladi. Koptokning maksimal ko'talish balandligini aniqlang *(1.34-rasm).*



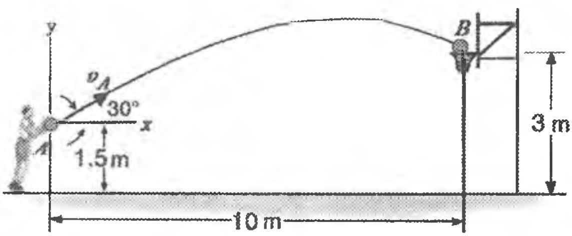
1.34-rasm

4-muammo. Koptok *A* nuqtada $v_A = 10\ m/s$ tezlik bilan tepiladi. Koptokning uchish uzoqligini va yerga tushgandagi tezligi aniqlansin *(1.35-rasm).*



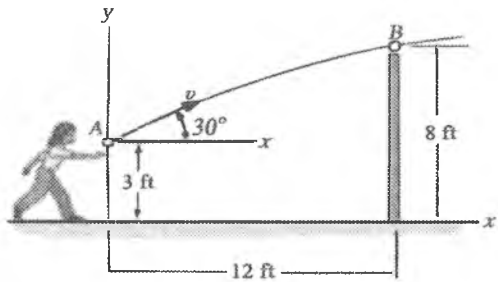
1.35-rasm

5-muammo. Basketbol to'pi *A* nuqtadan gorizont bilan $\alpha = 30°$ burchak hosil qiluvchi $\vec{v}_A$ tezlik bilan otilib, yerdan 3 *m* balandlikda turuvchi basketbol setkasiga tushadi. Basketbol to'pining otilish tezligi v_A aniqlansin *(1.36-rasm).*



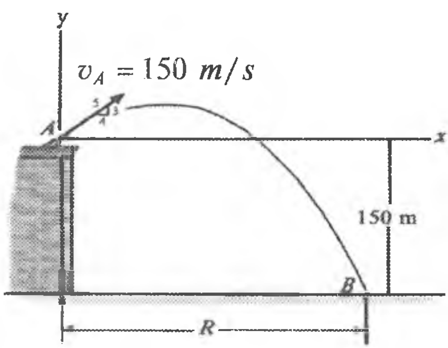
1.36-rasm

6-muammo. To'p *A* nuqtadan otiladi. U yerdan 8 *m*, otilish nuqtasidan 12 m masofada joylashgan *B* nuqtaga tushish uchun qanday v_A tezlik bilan otiladi? *(1.37-rasm).*



1.37-rasm

7-muammo. Reaktiv snaryad *A* nuqtadan $v_A = 150\ m/s$ tezlik bilan otiladi. Agar *A* nuqta yerdan 150 *m* balandlikda joylashgan bo'lsa, snaryadning uchish uzoqligi aniqlansin *(1.38-rasm).*



1.38-rasm

8-muammo. Shar vertikal holda yuqoriga yerdan 15 *m/s* tezlik bilan harakatlana boshlagan. Shar yerga qancha vaqt o'tgach qaytib tushadi? *(1.39-rasm).*

9-muammo. Harakatdagi nuqtaning trayektoriyadagi o'rni $S=(2t^2-8t+6)$ *m* masofa orqali aniqlanadi. Harakat boshlangandan qanday vaqt o'tgach nuqta tezligi 0 ga teng bo'ladi? Nuqta $t = 3$ *s* vaqt davomida qanday masofani bosib o'tadi? *(1.40-rasm)*

1.40-rasm

1.39-rasm

12-§. Nuqtaning tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Nuqtaning tezlanishi deb, nuqta tezligining vaqt o'tishi bilan miqdor va yo'nalish jihatdan o'zgarishini ifodalovchi vektor kattalikka aytiladi:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}. \qquad (1.112)$$

Bu ifodada

$$a_x = \frac{dv_x}{dt} = x'', \; a_y = \frac{dv_y}{dt} = y'', \; a_z = \frac{dv_z}{dt} = z''.$$

Tezlanishning koordinata o'qlaridagi proyeksiyalari aniqlangan bo'lsa, tezlanish moduli quyidagicha aniqlanadi:

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}.$$

Tezlanish vektorining yo'nalishi esa uning yo'naltiruvchi kosinuslari orqali aniqlanadi:

$$\cos(\vec{a} \wedge \vec{i}) = \frac{a_x}{a}, \ \cos(\vec{a} \wedge \vec{j}) = \frac{a_y}{a}, \ \cos(\vec{a} \wedge \vec{k}) = \frac{a_z}{a}.$$

Ba'zan, masalalar yechishda nuqtaning ma'lum vaqt oralig'idagi o'rtacha tezlanishini aniqlash talab etiladi.

$$a_{o'r.} = \frac{\Delta v}{\Delta t},$$

bunda

$\Delta v = v' - v$ nuqataning tezligining Δt vaqt oralig'ida o'zgarishi.

Nuqtaning harakati tabiiy usulda berilganda uning tezlanishi

$$\vec{a} = \vec{a}_\tau + \vec{a}_n = \frac{dv}{dt}\vec{\tau}_0 + \frac{v^2}{\rho}\vec{n}_0 \qquad (1.113)$$

formula asosida aniqlanadi.

Bu ifodada $\vec{a}_\tau$ va $\vec{a}_n$ lar nuqtaning urinma va normal tezlanishlarini ifodalaydi.

Bunday holda tezlanish moduli

$$a = \sqrt{a_\tau^2 + a_n^2}$$

formula asosida hisoblanadi.

Tezlanishning yo'nalishi esa quyidagi formuladan aniqlanadi:

$$tg\mu = \frac{|a_\tau|}{a_n}.$$

Nuqta kinematikasida nuqtaning tezlanishini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1) koordinata o'qlari sistemasi tanlab olinadi;

2) tanlab olingan koordinata o'qlari sistemasida nuqta harakatining tenglamalari tuziladi;

3) nuqta harakatining tenglamalariga ko'ra tezlanish vektorining o'qlardagi proyeksiyalari aniqlanadi;

4)nuqtaning tezlanishining o'qlardagi proyeksiyalariga ko'ra uning miqdori va yo'nalishi aniqlanadi.

Agar moddiy nuqtaning tezlanishi mavhum bo'lsa, u orqali nuqta harakatining tenglamalari va trayektoriyasini aniqlash mumkin.

Nuqta harakatining tezlanishi orqali uning harakati tenglamalarini va trayektoriyasini aniqlashda quyidagi amallarni bajarish tavsiya etiladi:

1) koordinata o'qlari sistemasi tanlab olinadi;

2) tezlanishning tanlab olingan o'qlardagi proyeksiyalari aniqlanadi;

3) hosil bo'lgan tenglamani integrallab, nuqta tezligining o'qlardagi proyeksiyalari aniqlanadi;

4) nuqta tezligining ma'lum vaqt oni uchun mumkin bo'lgan qiymatlaridan foydalanib, hosil bo'lgan ifodalarda ishtirok etuvchi integrallash o'zgarmaslari aniqlanadi;

5) hosil bo'lgan tezlikning o'qlardagi proyeksiyalari bo'lmish ifodalarni integrallab, nuqtaning harakat tenglamalari aniqlanadi;

6) nuqtaning biror vaqt uchun ma'lum bo'lgan koordinatalaridan foydalanib, integrallash o'zgarmaslari aniqlanadi;

7) hosil bo'lgan nuqtaning harakat tenglamalaridan vaqtni yo'qotib (qisqartirib), koordinatalar formasidagi trayektoriya tenglamasi tuziladi.

13-§. Nuqtaning tezlanishini aniqlashga doir masalalar

1-masala.

Samolyotdan $h = 320\ m$ balandlikdan tashlangan yuk

$$x = 60t,\ y = 5t^2 \qquad (1.114)$$

tenglamalarga muvofiq harakatlanadi, bunda x, y lar metrlarda, t sekundlarda o'lchanadi.

Yukning trayektoriyasi, samolyotdan tashlash va yerga tushish nuqtalari orasidagi gorizontal masofa, yerga tushish paytidagi tezligi va tezlanishi, tushish nuqtasida trayektoriyaning egrilik radiusi aniqlansin *(1.41-a rasm).*

Yechish:

yukning trayektoriyasini aniqlaymiz. Buning uchun harakat tenglamalarining biridan t vaqtni topib, ikkinchi tenglamaga qo'yamiz:

$$t = \frac{x}{60}; \; y = 5\left(\frac{x}{60}\right)^2 = \frac{1}{720}x^2.$$

Natijada,

$$y = \frac{1}{720}x^2 \qquad (1.115)$$

ko'rinishdagi parabola tenglamasi hosil bo'ladi. Demak, yukning trayektoriyasi y o'qiga simmetrik, uchi koordinata boshida bo'lgan parabola ekan *(1.41-a rasm).*

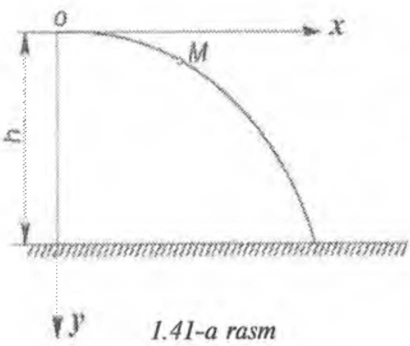
Yukning samolyotdan tashlash va yerga tushish nuqtalari orasidagi gorizontal masofani aniqlaymiz. Yukning M_1 tushish nuqtasidagi $y_1 = h$, $x_1 = l$ koordinatalarni aniqlash uchun yukning harakat tenglamalaridan foydalanamiz:

$$y = 5t^2, \; t_1 = \sqrt{\frac{y_1}{5}} = \sqrt{\frac{h}{5}} = 8 \; s. \qquad (1.116)$$

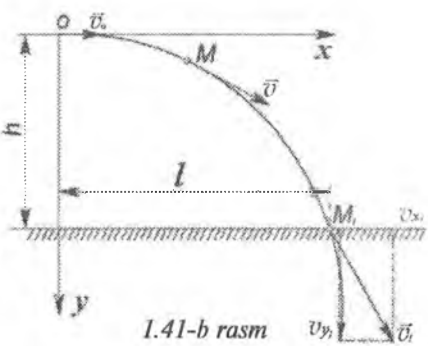
Shuning uchun

$$l = x_1 = 60t = 60 \cdot 8 = 480 \; m.$$

Demak, yukning samolyotdan tashlash va yerga tushish nuqtalari orasidagi gorizontal masofa 480 m ekan.



1.41-a rasm



1.41-b rasm

Yukning tushish nuqtasidagi tezligi va tezlanishini aniqlaymiz.

Yukning tezligi uning koordinata o'qlaridagi proyeksiyalari orqali aniqlanadi *(1.41-b rasm)*:

$$v_1 = \sqrt{v_{x_1}^2 + v_{y_1}^2} = \sqrt{60^2 + (10t)^2}. \tag{1.117}$$

Yuk yerga tushganda $t_1 = 8$ *s*, shuning uchun

$$v_1 = \sqrt{3600 + 6400} = 100 \ m/s.$$

Yukning tezlanishi ham uning tezligi kabi aniqlanadi.

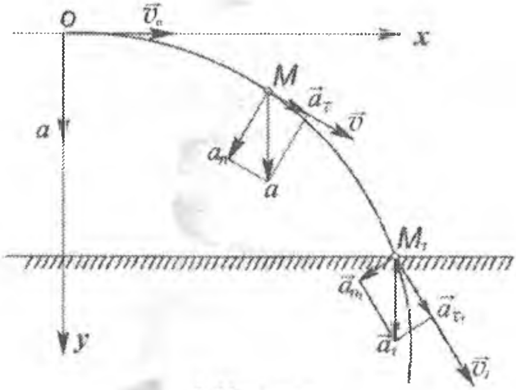
$$a_1 = \sqrt{a_{x_1}^2 + a_{y_1}^2},$$

$$a_{x_1} = \frac{dv_{x_1}}{at} = 0, \ a_{y_1} = \frac{dv_{y_1}}{at} = 10.$$

Shuning uchun

$$a_1 = \sqrt{a_{x_1}^2 + a_{y_1}^2} = 10 \ m/s^2. \tag{1.118}$$

Yukning yerga tushish nuqtasida trayektoriyaning egrilik radiusini aniqlash uchun uning *urinma* va *normal* tezlanishini aniqlaymiz *(1.41-c rasm)*.



1.41-c rasm

Yukning urinma tezlanishini quyidagi formula yordamida aniqlaymiz:

$$a_\tau = \left|\frac{dv}{at}\right| = \frac{v_x a_x + v_y a_y}{v} = 8 \ m/s^2. \tag{1.119}$$

Yukning normal tezlanishi quyidagicha aniqlanadi:

$$a^2 = a_\tau^2 + a_n^2,$$

bunda

$$a_n = \sqrt{a^2 - a_\tau^2} = \sqrt{100 - 64} = 6\ m/s^2. \qquad (1.120)$$

Trayektoriyaning yuk tushgan M_1 nuqtasining egrilik radiusi

$$\rho = \frac{v^2}{a_n} = \frac{100^2}{6} = 1667\ m. \qquad (1.121)$$

2-masala.

Poyezd radiusi $R = 1$ *km* bo'lgan aylana yoyi bo'ylab tekis sekinlanuvchan harakat qiladi va $s = 560$ *m* yo'l bosadi.

Uning boshlang'ich tezligi $v_0 = 36$ *km/soat* $=10$ *m/s*, boshlang'ich tezlanishi esa $a_0 = 0{,}125$ m/s^2. Poyezdning yoy oxiridagi tezligi va tezlanishi aniqlansin.

Yechish:

poyezd nuqtalaridan birining, masalan, og'irlik markazining harakatini o'rganamiz.

Poyezdning harakat tenglamasini yozish uchun yoy koordinatasining sanoq boshini tanlashimiz kerak. Bunday nuqta sifatida poyezdning boshlang'ich holatini olamiz va poyezdning harakat yo'nalishini ***musbat yo'nalish*** deb qabul qilamiz. Bu holda $s_0 = 0$.

Nuqtaning tekis sekinlanuvchan harakatida uning harakat tenglamasi va tezligi quyidagi formulalar asosida ifodalanadi:

$$s = v_0 t - \frac{a_\tau t^2}{2}, \qquad (1.122)$$

$$v = v_0 - a_\tau t, \qquad (1.123)$$

bunda a_τ — urinma tezlanish moduli.

Masala shartidan harakatdagi *M* nuqtaning yoy oxiridagi yoy koordinatasi $s = 560$ *m*, boshlang'ich tezligi $v_0 = 36$ *km/soat* = $=10$ *m/s*, boshlang'ich tezlanish $a_0 = 0{,}125$ m/s^2 hamda trayektoriyaning egrilik radiusi $R = 1000$ *m* berilgan.

M nuqtaning yoy boshidagi normal tezlanishini quyidagi formula asosida aniqlaymiz:

$$a_n o = \frac{(v_0^2)}{\rho} = \frac{100}{1000} = 0{,}1\ m/s^2.$$

M nuqtaning yoy boshidagi to'la tezlanishini bilgan holda, uning yoy boshidagi urinma tezlanishini aniqlaymiz:

$$a_0^2 = a_{n0}^2 + a_\tau^2,$$

bunda

$$a_\tau = \sqrt{a_0^2 - a_{n0}^2} = \sqrt{0{,}125^2 - 0{,}1^2} = 0{,}075\ m/s^2.$$

Nuqtaning harakati tekis sekinlanuvchan bo'lganligi uchun

$$a_\tau = \text{const}.$$

(1.22) va (1.23) tenglamalarga aniqlangan kattaliklarning qiymatlarini qo'yamiz:

$$560 = 10t - 0{,}075t^2, \qquad (1.24)$$

$$v = 10 - 0{,}075t. \qquad (1.25)$$

Bu tenglamalardan harakatlanish vaqti t aniqlanadi:

$$0{,}075t^2 - 20t + 1120 = 0,$$

$$t = \frac{10 \pm \sqrt{100 - 1120 \cdot 0{,}075}}{0{,}075} = \frac{10 \pm 4}{0{,}075}\ s.$$

Harakatlanish vaqti uchun kichik ildiz qiymatini tanlaymiz:

$$t = \frac{6}{0{,}075} = 80\ s, \qquad (1.26)$$

chunki katta ildiz qiymati (187 s) nuqtaning to'xtashi uchun ($v = 0$) ketgan vaqtdan katta

$$\left(t_{to'xtash} = \frac{v_0}{a} = \frac{10}{0{,}075} = 133\ sek.\right) \qquad (1.27)$$

(1.23) tenglamadan nuqtaning yoy oxiridagi tezligini aniqlaymiz:

$$v = v_0 - a_\tau t = 10 - 0{,}075 \cdot 80 = 4\ m/s. \qquad (1.28)$$

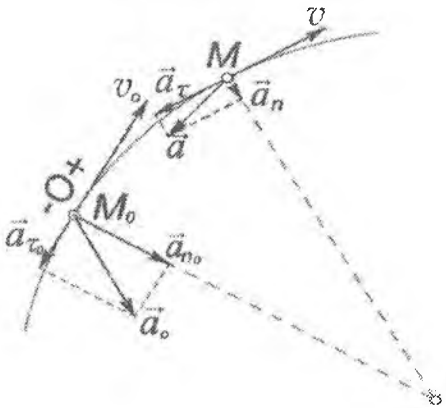
Nuqtaning yoy oxiridagi normal tezlanishi quyidagiga teng bo'ladi:

$$a_n = \frac{v^2}{R} = \frac{v^2}{1000} = 0{,}016\ m/s^2. \qquad (1.29)$$

Nuqtaning yoy oxiridagi to'la tezlanishi quyidagi formuladan aniqlanadi:

$$a = \sqrt{a_\tau^2 + a_n^2} = \sqrt{(0{,}075)^2 + (0{,}016)^2} = 0{,}0767\ m/s^2. \qquad (1.30)$$

Aylana yoyi bo'ylab tekis sekinlanuvchan harakatda nuqtaning urinma tezlanishining moduli o'zgarmaydi, to'la tezlanish moduli esa normal tezlanish modulining kamayishi tufayli kamayadi. Aniqlangan tezlik va tezlanishlar *1.42-rasmda* tasvirlangan.



1.42-rasm

3-masala.

M nuqtaning berilgan harakat tenglamalariga ko'ra trayektoriyasining ko'rinishi aniqlansin va $t = t_1$ vaqt oni uchun nuqtaning trayektoriyadagi o'rni, uning tezligi, to'la, urinma va normal tezlanishlari hamda trayektoriyaning egrilik radiusi topilsin:

$$x = 4\cos^2\left(\frac{\pi t}{3}\right) + 2\ (sm), \qquad (1.131)$$

$$y = 4\sin^2\left(\frac{\pi t}{3}\right)\ (sm), \qquad (1.132)$$

$$t = 1/2\ \text{s}.$$

Yechish:

nuqtaning trayektoriyasini aniqlaymiz. Trayektoriya tenglamasini tuzish uchun harakat tenglamalaridan t vaqtni yo'qotamiz. Buning uchun berilgan masalada quyidagi ayniyatdan foydalanamiz:

$$\sin^2\left(\frac{\pi t}{3}\right)+\cos^2\left(\frac{\pi t}{3}\right)=1. \qquad (1.133)$$

Masalada:

$$\sin^2\left(\frac{\pi t}{3}\right)=\frac{y}{4},\ \cos^2\left(\frac{\pi t}{3}\right)=\frac{x-2}{4}, \qquad (1.134)$$

(1.33)ni e'tiborga olsak,

$$\frac{y}{4}+\frac{x-2}{4}=1,$$

yoki

$$y=6-x.$$

Nuqtaning trayektoriyasi to'g'ri chiziqdan iborat ekan.

Harakat tenglamalaridan foydalanib, nuqtaning $t = 1/2$ sekunddagi koordinatalarini topamiz va shaklda ko'rsatamiz *(1.43-rasm):*

$$x=4\cos^2\frac{\pi}{6}+2=4\left(\frac{\sqrt{3}}{2}^2\right)+2=5\ sm,$$

$$y=4\sin^2\frac{\pi}{6}=4\cdot 0{,}25=1\ sm. \qquad (1.135)$$

Demak, $t = 1/2$ sekundda nuqtaning koordinatalari $x = 5$, $y = 1$ bo'lar ekan.

Nuqtaning tezligini uning koordinata o'qlaridagi proyeksiyalari orqali aniqlaymiz:

$$v=\sqrt{v_x^2+v_y^2}.$$

Buning uchun nuqta harakat tenglamalaridan vaqt bo'yicha birinchi tartibli hosila olamiz:

$$v_x = \dot{x} = -\frac{8\pi}{3}\cos\left(\frac{\pi t}{3}\right)\sin\left(\frac{\pi t}{3}\right) = -\frac{4\pi}{3}\sin\left(\frac{2\pi t}{3}\right), \qquad (1.136)$$

$$v_y = \dot{y} = \frac{8\pi}{3}\sin\left(\frac{\pi t}{3}\right)\cos\left(\frac{\pi t}{3}\right) = \frac{4\pi}{3}\sin\left(\frac{2\pi t}{3}\right). \qquad (1.137)$$

$t = 1/2$ s da,

$$v_x = -\frac{4\pi^2}{3}\sin\left(\frac{2\pi}{6}\right) = -\frac{4\pi}{3}\cdot\frac{\sqrt{3}}{2} = -3{,}6\ sm/s,$$

$$v_y = \frac{4\pi^2}{3}\sin\left(\frac{2\pi}{6}\right) = \frac{4\pi}{3}\cdot\frac{\sqrt{3}}{2} = 3{,}6\ sm/s.$$

Binobarin,

$$v = \sqrt{v_x^2 + v_y^2} = 5{,}1\,sm/s.$$

Tezliklar uchun masshtab tanlab, ularni shaklda ko‘rsatamiz *(1.43-rasm).*

Nuqtaning tezlanishini uning koordinata o‘qlaridagi proyeksiyalari orqali aniqlaymiz:

$$a = \sqrt{a_x^2 + a_y^2}. \qquad (1.138)$$

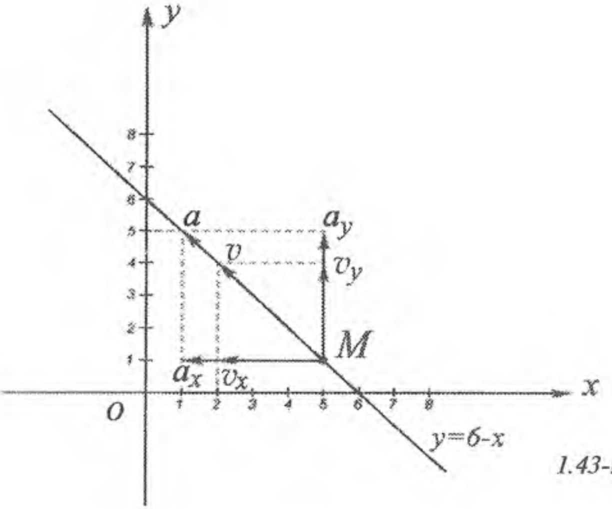
Buning uchun v_x, v_y lardan vaqt bo‘yicha birinchi tartibli hosila olamiz:

$$a_x = v_x' = \ddot{x} = -\frac{8\pi^2}{9}\cos\left(\frac{2\pi t}{3}\right),$$

$$a_y = v_y' = \frac{8\pi^2}{9}\cos\left(\frac{2\pi t}{3}\right). \qquad (1.39)$$

$t = 1/2\ s$ da,

$$a_x = -\frac{8\pi^2}{9}\cos\left(\frac{2\pi}{6}\right) = -\frac{8\pi^2}{9}\cdot 0{,}5 = -4{,}4\ sm/s^2,$$



1.43-rasm

$$a_y = \frac{8\pi^2}{9}\cos\left(\frac{2\pi}{6}\right) = \frac{8\pi^2}{9}\cdot 0{,}5 = 4{,}4\ sm/s^2.$$

Binobarin,

$$a = \sqrt{a_x^2 + a_y^2} = 6{,}2\ m/s^2. \qquad (1.140)$$

Nuqtaning urinma tezlanishi quyidagiga teng bo'ladi:

$$a_\tau = \left|\frac{dv}{at}\right| = \frac{v_x a_x + v_y a_y}{v} = 6{,}2\ sm/s^2.$$

Nuqtaning normal tezlanishi quyidagicha aniqlanadi:

$$a^2 = a_\tau^2 + a_n^2;\ a_n = \sqrt{a^2 - a_\tau^2} = 0. \qquad (1.141)$$

Trayektoriyaning egrilik radiusi quyidagi formula asosida aniqlanadi:

$$a_n = \frac{v^2}{\rho};\ \rho = \frac{v^2}{a_n} = \infty. \qquad (1.42)$$

Masalada, nuqtaning trayektoriyasi to'g'ri chiziq bo'lganligi uchun, egrilik radiusi ∞ ga teng.

Hisoblash natijalarini quyidagi jadvalda joylashtiramiz:

Aniqlangan kattaliklar *1.43-rasmda* ko'rsatilgan.

Nuqta koordinatalari (sm)		Nuqta tezligi (sm/s)			Nuqta tezlanishi (sm/s²)					Egrilik radiusi
x	y	v_x	v_y	v	a_x	a_y	a	a_τ	a_n	ρ
5	1	–3,6	3,6	5,1	–4,4	4,4	6,2	62	0	∞

4-masala.

Avtomobil yo'lning to'g'ri chiziqli uchastkasida ma'lum qisqa vaqt harakatlanib, $v = (3t^2 + 2t)\ m/s$ tezlikka ega bo'ladi (ifodada t sekundda o'lchanadi), harakat boshlangan vaqtdan $t_1 = 3\ s$ vaqtgacha avtomobilning bosib o'tgan yo'li va tezlanishi aniqlansin. $t = 0$ da $S = 0$ bo'lgan.

Yechimi:

1. Koordinata o'qini avtomobil harakati tomon yo'naltiramiz.

2. Avtomobilning $t_1 = 3\ s$ da bosib o'tgan yo'lini aniqlaymiz. Koordinata boshi sifatida avtomobilning boshlang'ich holatini tanlaymiz. Nuqtaning trayektoriyadagi o'rni $v = \frac{dS}{dt}$ formuladan aniqlanadi. $t = 0$ da $S = 0$ bo'lgan.

$$v = \frac{dS}{dt} = (3t^2 + 2t),$$

$$\int_0^s dS = \int_0^t (3t^2 + 2t)dt,$$

$$S\Big|_0^s = (t^3 + t^2)\Big|_0^t$$

Demak,

$$S = t^3 + t^2.$$

Harakat boshlangandan $t_1 = 3\ s$ vaqt o'tgach, avtomobil bosib o'tgan yo'l quyidagiga teng bo'ladi:

$$S = 3^3 + 3^2 = 36\ m.$$

3. Avtomobilning $t_1 = 3\ s$ dagi tezlanishini aniqlaymiz:

$$a = \frac{dv}{dt} = \frac{d(3t^2 + 2t)}{dt} = 6t + 2.$$

Harakat boshlangandan $t_1 = 3$ *s* vaqt o‘tgach,

$$a = 6(3) + 2 = 20 \ m/s^2$$

bo‘ladi.

5-masala. Nuqta radiusi 800 *m* bo‘lgan aylana yoyi bo‘ylab tekis o‘zgaruvchan harakat qiladi. Uning boshlang‘ich tezligi $v_0 = 5$ *m/s* bo‘lib, $s = 800$ *m* masofani o‘tgandan keyingi tezligi $v_T = 15$ *m/s*.

Nuqtaning boshlang‘ich tezlanishi a_0, 800 *m* masofani o‘tish vaqti *T* va harakat boshlangandan keyin *T* vaqt o‘tganda qanday a_T tezlanishga ega bo‘lishi topilsin.

Yechish: nuqta egri chiziqli harakatda bo‘lgani uchun uning tezlanishi quyidagi formulaga ko‘ra topiladi:

$$\vec{a} = \vec{a}_\tau + \vec{a}_n.$$

Masala shartiga ko‘ra nuqta tekis o‘zgaruvchan harakatda bo‘lgani uchun egri chiziqli tekis o‘zgaruvchan harakatdagi tezlikni va harakat qonunini ifodalovchi tenglamalardan foydalaning:

$$v = v_0 + a_\tau t, \qquad (1.143)$$

$$S = S_0 + v_0 t + a_\tau \frac{t^2}{2}. \qquad (1.144)$$

Sanoq boshini nuqtaning boshlang‘ich holatida olsak, $S_0 = 0$. Masala shartiga berilganlarni (1.143) va (1.144)ga qo‘yamiz:

$$15 = 5 + a_\tau \cdot T,$$

$$800 = 5T + a_\tau \cdot \frac{T^2}{2}.$$

Bu tenglamalar sistemasini yechsak, $T = 80$ *s*; $a_\tau = 0{,}125 \ m/s^2$ kelib chiqadi.

Nuqtaning boshlang‘ich va *T* paytdagi normal tezlanishlarini topamiz. Nuqta trayektoriyasi aylana bo‘lgani uchun $r = R = 800$ m.

Shuning uchun

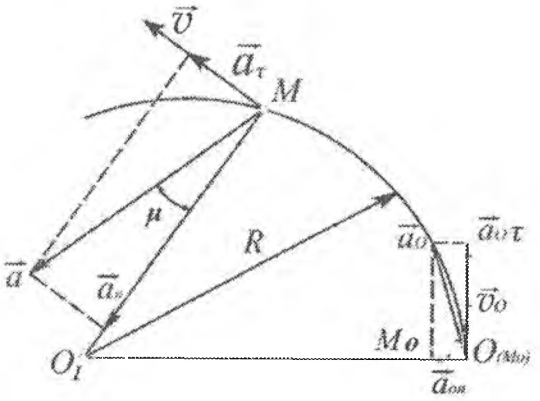
$$a_{no} = \frac{v_0^2}{r} = 0,029\frac{m}{s^2}, \quad a_{nT} = \frac{v_T^2}{r} = 0,281\frac{m}{s^2}.$$

Nuqtaning tezlanishi $a = \sqrt{a_\tau^2 + a_n^2}$ formuladan topiladi. Shunga ko'ra, $t = 0$, $t = T$ vaqtlar uchun mos ravishda $a_0 = 0,129$ m/s^2, $a_T = 0,308$ m/s^2 kelib chiqadi. Har ikki payt uchun tezlanish yo'nalishini quyidagi formula yordamida topamiz:

$$\mu_0 = \text{arctg}\frac{|a_\tau|}{a_{nO}}\text{arctg}4,310, \; \mu_0 \approx 77^0;$$

$$\mu_0 = \text{arctg}\frac{|a_\tau|}{a_{nT}}\text{arctg}0,0444, \; \mu_T \approx 24^0.$$

Tezlanish vektori yo'nalishi *1.44-rasmda* tasvirlangan.



1.44-rasm

6-masala. Harakati $\vec{r} = 2\sin\frac{\pi t}{3}\vec{i} + \left(3\cos\frac{\pi t}{3} + 4\right)\vec{j}$ tenglama bilan ifodalangan nuqtaning trayektoriyasi va $t = 1$ paytdagi tezligi, tezlanishi hamda trayektoriyaning shu vaqtga mos keluvchi egrilik radiusi topilsin (r – metrda, t – sekundda o'lchanadi).

Yechish. Nuqta harakati tenglamasidan kelib chiqib, koordinata usulida harakatni quyidagicha ifodalaymiz:

$$x = 2\sin\frac{\pi t}{3},\ y = 3\cos\frac{\pi t}{3} + 4. \qquad (1.145)$$

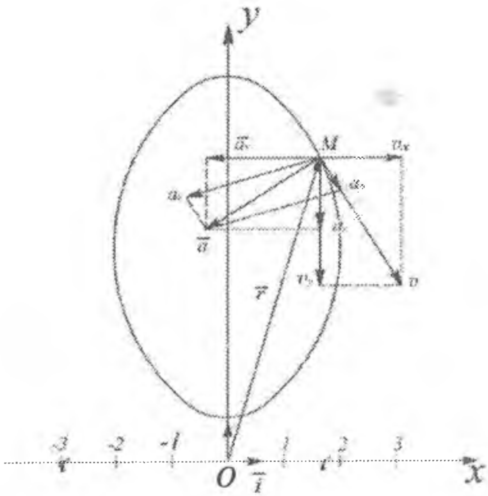
(1.145) tenglamalar sistemasi nuqta trayektoriyasining parametrik tenglamalari bo‘lib, ulardan vaqt t ni yo‘qotsak, trayektoriyaning qanday chiziq bo‘lishi aniqlanadi. Buning uchun (1.145)ni

$$\frac{x}{2} = \sin\frac{\pi t}{3},\ \frac{y-4}{3} = \cos\frac{\pi t}{3}$$

ko‘rinishda yozib, ularning har birini kvadratga oshirib, hadlab qo‘shamiz:

$$\frac{x^2}{4} + \frac{(y-4)^2}{9} = 1 \qquad (1.146)$$

(1.146)dan ko‘rinib turibdiki, nuqta trayektoriyasi ellips shaklida ekan *(1.45-rasm).*



1.45-rasm

$t = 1\ s$ paytda nuqta trayektoriyaning M nuqtasida bo‘ladi. Nuqta tezligini quyidagi formulalar yordamida topamiz:

$$v_x = x' = \frac{2\pi}{3}\cos\frac{\pi t}{3},\; v_y = y' = -\pi\sin\frac{\pi t}{3},$$

$$v = \pi\sqrt{\frac{4}{9}\cos^2\frac{\pi t}{3} + \sin^2\frac{\pi t}{3}}$$

yoki

$$v = \frac{\pi}{3}\sqrt{4 + +5\sin^2\frac{\pi}{3}t}. \qquad (1.147)$$

$t = 1$ s payt uchun $v_x = \frac{\pi}{3} \approx 1{,}05\frac{m}{s}$, $v_y = -\frac{\sqrt{3}}{2}\pi \approx -2{,}72\frac{m}{s}$,

$v = \frac{\pi}{6}\sqrt{31} \approx 2{,}92\frac{m}{s}$, $\cos(\vec{v},\hat{}\vec{i}) = \frac{v_x}{v} \approx 0{,}3584$, $\cos(\vec{v},\hat{}\vec{j}) = \frac{v_y}{v} \approx$

$\approx -0{,}9312$.

Bu kattaliklarni rasmda tasvirlab, $\vec{v}$ vektori trayektoriyaga M nuqtada o'tkazilgan urinma bo'yicha yo'nalganiga iqror bo'lamiz.

Nuqtaning tezlanishini topamiz:

$$a_x = v_x' = -\frac{2\pi^2}{9}\sin\frac{\pi t}{3},\; a_y = v_y' = -\frac{\pi^2}{3}\cos\frac{\pi t}{3};$$

$$a = \frac{\pi^2}{3}\sqrt{\frac{4}{9}\sin^2\frac{\pi}{3}t + \cos^2\frac{\pi}{3}t} = \frac{\pi^2}{9}\sqrt{9 - 5\sin^2\frac{\pi}{3}t}.$$

$t = 1$ s payt uchun:

$$a_x \approx -1{,}9\frac{m}{s^2},\; a_y \approx -1{,}65\frac{m}{s^2},\; a \approx 2{,}51\frac{m}{s^2},$$

$$\cos(\vec{a},\hat{}\vec{i}) = \frac{a_x}{a} \approx 0{,}7570,\; \cos(\vec{a},\hat{}\vec{j}) = \frac{a_y}{a} \approx -0{,}6573.$$

Ma'lum masshtab tanlab olib, bu kattaliklarni ham rasmda tasvirlaymiz *(1.45-rasm)*.

Trayektoriyaning egrilik radiusini aniqlashda $a_n = \frac{v^2}{p}$ formuladan foydalanish mumkin. Buning uchun avval urinma va normal tezlanishlarni topish kerak.

$a_r = \frac{dv}{dt}$ bo‘lgani uchun (3)dan vaqt bo‘yicha hosila olamiz:

$$a_r = v = \frac{\pi}{3}\cdot\frac{\frac{10}{3}\pi\cos\frac{\pi}{3}t\cdot\sin\frac{\pi}{3}t}{2\sqrt{4+5\sin^2\frac{\pi}{3}t}} = \frac{5}{18}\pi^2\cdot\frac{\sin\frac{2\pi}{3}t}{\sqrt{4+5\sin^2\frac{\pi}{3}t}}.$$

Bundan $t = 1$ s payt uchun $a_\tau \approx 0{,}85\frac{m}{s^2}$ kelib chiqadi. Urinma tezlanish tezlik vektori bo‘yicha yo‘nalgan, $a^2 = a_\tau^2 + a_n^2$ formuladan foydalanib, $t = 1$ s vaqt uchun normal tezlanishni aniqlaymiz:

$$a_n = \sqrt{a^2 - a_\tau^2} \approx 2{,}36\frac{m}{s^2}.$$

Normal tezlanish $\vec{a}_\tau$ ga perpendikular ravishda trayektoriyaning botiq tomoniga yo‘nalgan *(1.45-rasm)*.

$t = 1$ s paytda nuqtaning trayektoriyada egallagan holati uchun egrilik radiusini topamiz:

$$\rho = \frac{v^2}{a_n} \approx 2{,}69\ m.$$

14-§. Mustaqil o‘rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo. Yo‘lovchi to‘g‘ri yo‘lda $v = (1{,}2 - 3t)$ m/s tezlik bilan velosipedda ketmoqda. Harakat boshlangandan $t = 1$ s vaqt o‘tgach yo‘lovchi boshlang‘ich nuqtadan 10 m chapda bo‘ladi. Yo‘lovchining $t_1 = 4$ s vaqtdagi tezlanishini aniqlang. Yo‘lovchining $t = 0$ dan $t - 10$ sek. vaqt oralig‘ida bosib o‘tgan yo‘li aniqlansin.

2-muammo. Yo‘lovchi to‘g‘ri chiziq bo‘ylab $v = (-4\ s^2)\ m/s$ tezlik bilan harakatlanmoqda (vaqt sekundlarda o‘lchanadi). Agar $t = 0$ vaqt momentida $s = 2\ m$ bo‘lsa, yo‘lovchining tezligi va tezlanishini vaqt funksiyasi sifatida aniqlang.

3-muammo. Nuqtaning to‘g‘ri chiziqli harakatida tezlanishi $a = (0{,}02\ t^2)$ (vaqt sekundlarda o‘lchanadi). Agar $t = 0$ da $v = 0$, $s = 0$ bo‘lsa, nuqta $s = 4\ m$ masofani bosib o‘tgach, qanday tezlik va tezlanishga ega bo‘ladi?

4-muammo. Nuqta to‘g‘ri chiziq bo‘ylab $a = 5/(3S^{1/3} + S^{5/2})\ m/s^2$ tezlanish bilan harakatlanmoqda (masofa s metrlarda o‘lchanadi). Agar nuqta harakatlana boshlagan vaqtda $s = 1\ m$ bo‘lsa, $s = 2\ m$ masofani bosib o‘tgach, qanday tezlikka erishadi?

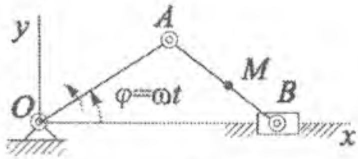
5-muammo. Nuqta to‘g‘ri chiziq bo‘ylab $a = (2t-1)\ m/s^2$ tezlanish bilan harakatlanadi (vaqt t sekundlarda o‘lchanadi). Agar $t = 0$ vaqt momentida $s = 1$ m, $v = 2\ m/s$ bo‘lsa, $t = 8\ s$ vaqt momentidagi nuqtaning tezligi va holati aniqlansin. Nuqta harakatlanishi davomida bosib o‘tgan masofa ham aniqlansin.

6-muammo. Nuqta to‘g‘ri chiziq bo‘ylab $v = (200\ s)\ mm/s$ tezlik bilan harakatlanmoqda (masofa s millimetrlarda o‘lchanadi). Nuqta $s = 2000\ mm$ masofa o‘tgach, qanday tezlikka ega bo‘ladi? Nuqta $s = 500\ mm$ masofani qancha vaqt davomida bosib o‘tishi ham aniqlansin.

7-muammo. Nuqta to‘g‘ri chiziq bo‘ylab $a = \left(12t - 3t^{\frac{1}{2}}\right) m/s^2$ tezlanish bilan harakatlanmoqda, bunda t sekundlarda o‘lchanadi. Nuqtaning tezligi va to‘g‘ri chiziqdagi holati (o‘rni) vaqt funksiyasi sifatida aniqlansin. $t = 0$ da $v = 0$, $S = 15\ m$ bo‘lgan.

8-muammo. OA krivoship ω o‘zgarmas burchak tezlik bilan aylanadi. Krivoship polzunli mexanizm shatunining o‘rtasidagi M nuqtaning tezligi va polzunning tezligi vaqt funksiyasi sifatida topilsin:

$$OA = AB = a \quad (1.46\text{-}rasm).$$



1.46-rasm

9-muammo. Nuqtaning harakati $\vec{r} = 3ti + 3tj$ radius-vektor orqali berilgan bo'lsa, $r = 5$ *m* bo'lganda, *y* koordinatasini hisoblang.

10-muammo. Nuqta harakatining qonuni koordinata usulida: $x = t^2$, $y = \sin \pi t$, $z = \cos \pi t$ berilgan bo'lsa, $t = 1$ *s* paytda uning tezligini hisoblang.

11-muammo. To'g'ri chiziqli harakatdagi nuqtaning tezlanishi $a = 0{,}5$ m/s^2 bo'lib, boshlang'ich paytida $t_0=0$ da, $s_0=0$ bo'lsa, 9 *m* masofa bosib o'tishi uchun qancha vaqt o'tadi?

12-muammo. Qo'nayotgan samolyot yerga 180 *km/soat* tezlik bilan tushadi va 1000 *m* masofa bosib o'tib to'xtaydi. Samolyotning o'rtacha sekinlanish modulini hisoblang.

13-muammo. Berilgan trayektoriya bo'ylab $v = 5$ *m/s* tezlik bilan harakat qilayotgan nuqta boshlang'ich paytda, $t_0=0$ da $s_0=26$ *m* yo'l bosib o'tgan bo'lsa, $t = 18$ *s* da egri chiziqli koordinata *S* ni toping.

14-muammo. Egri chiziqli harakatdagi nuqtaning tezligi $v = 0{,}2\,t$ bo'lib, boshlang'ich holatida, $t_0 = 0$ da, bosib o'tgan yo'li $s_0 = 0$ bo'lsa, $t = 10$ *s* da qancha yo'l bosadi?

15-muammo. Harakatlanayotgan nuqta tezligining proyeksiyalari $v_x = 0{,}2t^2$, $v_y = 3$ *m/s* bo'lsa, $t = 2{,}5$ *s* vaqtda uning urinma tezlanishi nimaga teng?

15-§. Talabalar mustaqil o'rganishi uchun keyslar

Nuqta harakatining berilgan tenglamalariga ko'ra uning tezligi va tezlanishini aniqlash

M nuqtaning berilgan harakat tenglamalariga ko'ra trayektoriyasining ko'rinishi aniqlansin va $t = t_1$ (s) vaqt oni uchun nuqtaning trayektoriyadagi o'rni, uning tezligi, to'la, urinma va normal tezlanishlari hamda trayektoriyaning egrilik radiusi topilsin.

Topshiriqni yechish uchun zarur bo'lgan ma'lumotlar quyidagi jadvalda keltirilgan:

Variantlar raqamlari	Harakat tenglamalari		Vaqt
	$x=x\ (t)$, *sm*	$y=y\ (t)$, *sm*	$t1$, *s*
1.	$x=3t$	$y=4t^2+1$	$^1/_2$
2.	$x=7\sin^2\left(\frac{\pi t}{6}\right)-5$	$y=-7\cos^2\left(\frac{\pi t}{6}\right)$	1
3.	$x=1+3\cos\left(\frac{\pi t^2}{3}\right)$	$y=3\sin\left(\frac{\pi t^2}{3}\right)+3$	1
4.	$x=-5t^2-4$	$y=3t$	1
5.	$x=2-3t-6t^2$	$y=3-\frac{3}{2}t-3t^2$	0
6.	$x=6\sin\left(\frac{\pi t^2}{6}\right)-2$	$y=6\cos\left(\frac{\pi t^2}{6}\right)+3$	1
7.	$x=7t^2-3$	$y=5t$	$^1/_4$
8.	$x=3-3t^2+1$	$y=4-5t^2+\frac{5}{3}t$	1
9.	$x=-4\cos\left(\frac{\pi t}{3}\right)-1$	$y=-4\sin\left(\frac{\pi t}{3}\right)$	1
10.	$x=-6t$	$y=-2t^2-4$	1
11.	$x=8\cos^2\left(\frac{\pi t}{6}\right)+2$	$y=-8\sin^2\left(\frac{\pi t}{6}\right)-7$	1
12.	$x=4t^2+1$	$y=-3t$	1
13.	$x=5t^2+\frac{5}{3}t-3$	$y=3t^2+t+3$	1
14.	$x=2\cos\left(\frac{\pi t^2}{3}\right)-2$	$y=-2\sin\left(\frac{\pi t^2}{3}\right)+3$	1
15.	$x=4\cos\left(\frac{\pi t}{3}\right)-2$	$y=-3\sin\left(\frac{\pi t}{3}\right)$	1
16.	$x-5\cos\left(\frac{\pi t^2}{3}\right)$	$y=-\frac{2}{(t+1)}$	2
17.	$x=-2t-2$	$y=-5\sin\left(\frac{\pi t^2}{3}\right)$	1

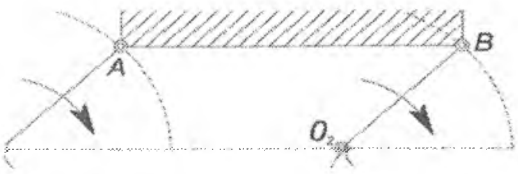
18.	$x=5\sin^2\left(\frac{\pi t}{6}\right)$	$y=-5\cos^2\left(\frac{\pi t}{6}\right)-3$	1
19.	$x=-4t^2+1$	$y=-3t$	1/2
20.	$x=-4\cos\left(\frac{\pi t}{3}\right)$	$y=-2\sin\left(\frac{\pi t^2}{3}\right)-3$	1
21.	$x=-\frac{3}{(t+2)}$	$y=3t+6$	2
22.	$x=7\sin\left(\frac{\pi t^2}{6}\right)+3$	$y=2-7\cos\left(\frac{\pi t^2}{6}\right)$	1
23.	$x=3t^2-t+1$	$y-5t^2-\frac{5}{3}t-2$	1
24.	$x=3t^2+1$	$y=-4t$	$^1/_2$
25.	$x=2\sin\left(\frac{\pi t}{3}\right)$	$y=-3\cos\left(\frac{\pi t}{3}\right)+4$	1
26.	$x=4t+4$	$y=-\frac{4}{(t+1)}$	2
27.	$x=-2t^2+3$	$y=-5t$	$^1/_2$
28.	$x=4\cos^2\left(\frac{\pi t}{3}\right)+2$	$y=4\sin^2\left(\frac{\pi t^2}{3}\right)-1$	1
29.	$x=-\cos\left(\frac{\pi t^2}{3}\right)+3$	$y=\sin\left(\frac{\pi t^2}{3}\right)-1$	1
30.	$x=-3-9\sin\left(\frac{\pi t^2}{6}\right)$	$y=-9\cos\left(\frac{\pi t^2}{6}\right)+5$	1

II BOB
QATTIQ JISMNING ILGARILANMA VA QO‘ZG‘ALMAS O‘Q ATROFIDA AYLANMA HARAKATI

16-§. Qattiq jismning ilgarilanma harakati

Jismda olingan har qanday kesma jism harakati davomida doimo o‘zining boshlang‘ich holatiga parallel qolsa, jismning bunday harakati ilgarilanma harakat deyiladi.

Jismning ilgarilanma harakatini uning to‘g‘ri chiziqli harakati bilan aralashtirib bo‘lmaydi. Ilgarilanma harakatdagi jism nuqtasining trayektoriyasi egri chiziqdan iborat bo‘lishi ham mumkin. Masalan, *2.1-rasmda* ko‘rsatilgan AB sparnikning harakati davomida O_1A va O_2B kripovshiplar O_1, O_2 nuqtalardan o‘tuvchi o‘qlar atrofida aylanadi, sparnik esa hamma vaqt o‘z-o‘ziga parallel qoladi, ya’ni ilgarilanma harakatda bo‘ladi.



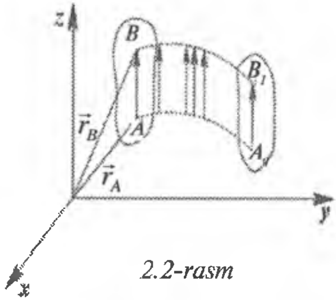
2.1-rasm

Ilgarilanma harakatda bo‘lgan qattiq jismning kinematik xarakteristikalari quyidagi teoremada o‘z ifodalarini topgan:

Teorema: *ilgarilanma harakatdagi jismning hamma nuqtalari bir xil trayektoriya chizadi va har onda bir xil tezlik hamda bir xil tezlanishga ega bo‘ladi.*

Teoremani isbotlash uchun jismning berilgan *Oxyz* qo‘zg‘almas koordinatalar sistemasiga nisbatan ilgarilanma harakatini o‘rganamiz *(2.2-rasm)*. Jismda ixtiyoriy A va B nuqtalarni olib, ularning radius vektorlarini $\vec{r}_A$ va $\vec{r}_B$ bilan belgilaymiz. Rasmdan:

$$\vec{r}_B = \vec{r}_A + \overrightarrow{AB}. \tag{2.1}$$



2.2-rasm

Jism harakatlanganda $\vec{r}_A$, $\vec{r}_B$ o'zgaradi, ammo AB kesmaning uzunligi va yo'nalishi o'zgarmaydi.

B nuqtaning tezligini aniqlash uchun (2.1)dan vaqt t bo'yicha hosila olamiz:

$$\frac{d\vec{r}_B}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\overrightarrow{AB}}{dt}, \qquad (2.2)$$

bunda, $\frac{d\overrightarrow{AB}}{dt} = 0$ bo'lgani uchun

$$\frac{d\vec{r}_B}{dt} = \frac{d\vec{r}_A}{dt}$$

yoki

$$\vec{\upsilon}_B = \vec{\upsilon}_A \qquad (2.3)$$

bo'ladi.

Bu tenglik ilgarilanma harakatdagi jism barcha nuqtalarining har ondagi tezliklari bir xil bo'lishini ifodalaydi.

Agar (2.3)dan vaqt t bo'yicha hosila olsak:

$$\frac{d\upsilon_B}{dt} = \frac{d\upsilon_A}{dt}$$

yoki

$$\vec{a}_B = \vec{a}_A \qquad (2.4)$$

bo'ladi.

(2.4) tenglik ilgarilanma harakatdagi jism barcha nuqtalarining har ondagi tezlanishlari bir xil bo'lishini ifodalaydi.

Shunday qilib, teorema isbotlandi.

Isbotlangan teoremadan jismning ilgarilanma harakati uning biror nuqtasininig harakati bilan aniqlanishi mumkinligi ma'lum bo'ladi. Odatda, bunday nuqta sifatida jismning og'irlik markazi C nuqta olinadi.

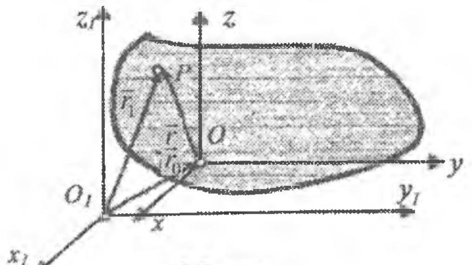
Olingan nuqtaning harakat tenglamalarini koordinata usulida quyidagicha yozish mumkin:

$$x_C = f_1(t),\ y_C = f_2(t),\ z_C = f_3(t). \tag{2.5}$$

(2.5) tenglama C nuqtaning harakat tenglamasi bo'lib, jismning ilgarilanma harakat tenglamasini ham ifodalaydi.

Jismning ilgarilanma harakatida hamma nuqtalari uchun bir xil bo'lgan tezlik jismning ilgarilanma harakat tezligi deyiladi, tezlanish esa — jismning ilgarilanma harakat tezlanishi deyiladi. Ilgarilanma harakat tezlik vektori $\vec{v}$ va tezlanish vektori $\vec{a}$ larni jismning ixtiyoriy nuqtasiga qo'yilgan holda ko'rsatish mumkin. Bu hol qattiq jismning faqat ilgarilanma harakatida o'rinli bo'ladi. Boshqa harakatlarda jismning turli nuqtalari turlicha tezlik va turlicha tezlanishga ega bo'ladi.

Qattiq jismning ilgarilanma harakatida jism O nuqtasining tezligi $\vec{v}_0 \neq 0$ bo'lib, burchak tezlik $\vec{\omega}_0 = 0$ bo'ladi *(2.3-rasm).*



2.3-rasm

Rasmdan

$$\vec{r}_1 = \vec{r}_0 + \vec{r}\,.$$

Ilgarilanma harakat ta'rifidan

$$\vec{r} = \text{const}.$$

Shuning uchun $\frac{d\vec{r}}{dt}=0$,

$$\frac{d\vec{r}_1}{dt}=\frac{d\vec{r}_0}{dt}$$

bo'ladi.
Bundan

$$\vec{v}_p=\vec{v}_0$$

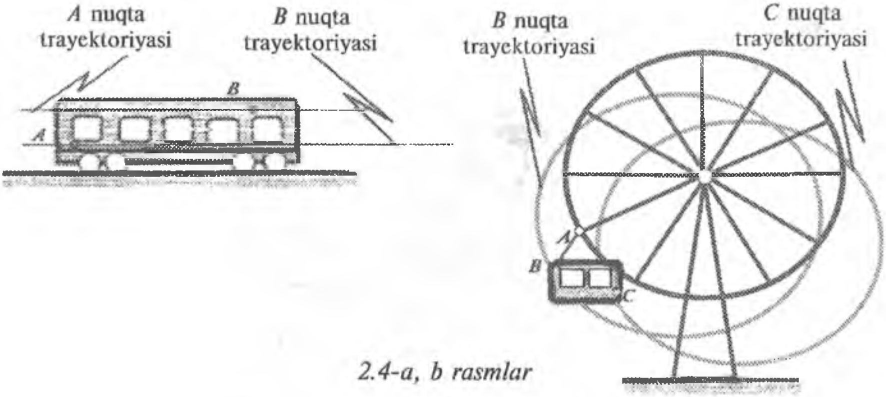
natija kelib chiqadi.

Yuqoridagi ifodadan qattiq jismning ilgarilanma harakatida uning barcha nuqtalarining tezlanishini ham bir xil miqdor va yo'nalishga ega bo'lishi ma'lum bo'ladi:

$$\vec{a}_p=\vec{a}_0.$$

2.4-a rasmda tramvayning ilgarilanma harakatida uning *A* va *B* nuqtalarining trayektoriyasi ko'rsatilgan.

2.4-b rasmda charxpalak *A, B, C* nuqtalarining trayektoriyasi ko'rsatilgan.



2.4-a, b rasmlar

17-§. Qattiq jismning ilgarilanma harakatiga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Qattiq jismning ilgarilanma harakatiga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1. Koordinatalar sistemasi tanlab olinadi, bunda koordinata o'qlaridan birini jismning ilgarilanma harakati yo'nalishida o'tkazish maqsadga muvofiq bo'ladi.

2. Masala shartidan ilgarilanma harakatda bo'ladigan jism tanlab olinadi.

3. Tanlab olingan koordinata o'qlari sistemasida jismning ilgarilanma harakati tenglamasi tuziladi.

4. Jismning ilgarilanma harakat tenglamalariga ko'ra tezlik vektorining o'qlardagi proyeksiyalari aniqlanadi.

5. Jism ilgarilanma harakat tezligining o'qlardagi proyeksiyalariga ko'ra uning miqdori va yo'nalishi aniqlanadi.

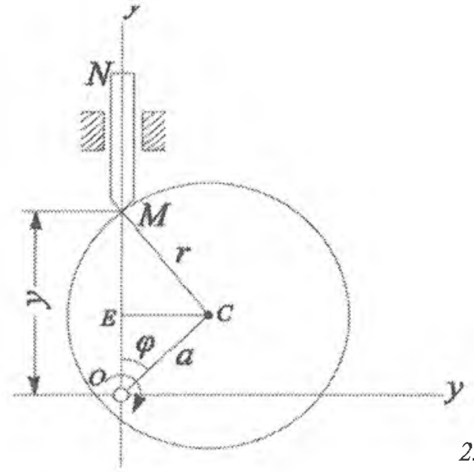
6. Jismning ilgarilanma harakat tenglamalari yoki jism tezligining o'qlardagi proyeksiyalariga ko'ra uning tezlanishining o'qlardagi proyeksiyalari aniqlanadi.

7. Jism tezlanishining o'qlardagi proyeksiyalariga ko'ra uning miqdori va yo'nalishi aniqlanadi.

18-§. Qattiq jismning ilgarilanma harakatiga doir masalalar

1-masala. Diametri $d = 2r$ bo'lgan ekssentrik O nuqta atrofida aylanadi, bunda φ burchak $\varphi = \frac{\pi}{2}t$ qonunga muvofiq o'zgaradi. Ekssentrik geometrik markazi bo'lgan C va O nuqtalar orasidagi masofa $OC = a = \frac{r}{3}$. Vertikal yo'nalishda harakatlanuvchi MN sterjen M nuqtasining to'g'ri chiziqli harakat tenglamasi tuzilsin *(2.5-rasm)* hamda $t_1 = 3\ s$ vaqt oni uchun mazkur nuqtaning tezligi va tezlanishi aniqlansin.

Yechish: masala shartiga ko'ra MN sterjen O nuqtadan o'tuvchi vertikal chiziq bo'ylab to'g'ri chiziqli harakatda bo'ladi, ya'ni MN sterjen ilgarilanma harakat sodir etadi. Shuning uchun sterjenning M nuqtasi ham O nuqtasidan o'tuvchi to'g'ri chiziq bo'ylab harakatlanadi.



2.5-rasm

Mazkur to‘g‘ri chiziq bo‘ylab *Oy* koordinata o‘qini o‘tkazamiz; koordinata boshi sifatida *O* nuqta olinadi; *Ox* o‘qi gorizontal holda yo‘naltiriladi. Rasmdan $OM = y$;

Bu masofa vaqtga bog‘liq holda o‘zgaradi. Bu holni aniqlash uchun *OM* masofani φ burchak orqali ifodalash lozim.

Buning uchun *C* nuqtadan *OMS* uchburchakning *OM* tomoniga *CE* balandlikni o‘takazamiz.

Natijada, $OM = OE + EM$ bo‘ladi.

Lekin, $OE = a\cos\varphi$, $EC = a\sin\varphi$,

$$EM = \sqrt{(MC)^2 - (EC)^2} = \sqrt{r^2 - a^2\sin^2\varphi}.$$

Shuning uchun

$$y = OM = a\cos\varphi + \sqrt{r^2 - a^2\sin^2\varphi}.$$

Hosil bo‘lgan ifodaga φ burchak qiymatini qo‘yib, $r = 3a$ ekanligini e’tiborga olsak, *M* nuqtaning harakat tenglamasi uchun quyidagi ifoda kelib chiqadi:

$$y = a\left(\cos\frac{\pi}{2}t + \sqrt{9 - \sin^2\frac{\pi}{2}t}\right).$$

M nuqtaning tezligini quyidagicha aniqlaymiz:

$$v = \frac{dy}{dt} = \frac{d}{dt}\left[a\left(\cos\frac{\pi}{2}t + \sqrt{9 - \sin^2\frac{\pi}{2}t}\right)\right] =$$

$$= -\frac{\pi}{2}a\sin\frac{\pi}{2}t \cdot \left(1 + \frac{\cos\frac{\pi}{2}t}{\sqrt{9 - \sin^2\frac{\pi}{2}t}}\right) =$$

$$= -\frac{\pi a\sin\frac{\pi}{2}t}{2\sqrt{9 - \sin^2\frac{\pi}{2}t)}} \cdot \left(\cos\frac{\pi}{2}t + \sqrt{9 - \sin^2\frac{\pi}{2}t}\right),$$

yoki

$$v = -\frac{\pi}{2} \cdot \frac{\sin\frac{\pi}{2}t}{\sqrt{9 - \sin^2\frac{\pi}{2}t}} \cdot y$$

M nuqtaning tezlanishini aniqlaymiz:

$$a = \frac{dv}{dt} = \frac{d^2y}{dt^2} = -\frac{\pi}{2}\left[y\frac{d}{dt}\left(\frac{\sin\frac{\pi}{2}t}{\sqrt{9 - \sin^2\frac{\pi}{2}t}}\right) + \frac{\sin\frac{\pi}{2}t}{\sqrt{9 - \sin^2\frac{\pi}{2}t}}\frac{dy}{dt}\right].$$

Agar $\frac{dy}{dt} = v$ va $\frac{d}{dt}\left(\frac{\sin\frac{\pi}{2}t}{\sqrt{9 - \sin^2\frac{\pi}{2}t}}\right) = \frac{9\pi\cos\frac{\pi}{2}t}{2\left(9 - \sin^2\frac{\pi}{2}t\right)^{\frac{3}{2}}},$

ekanligini e'tiborga olsak,

$$a=-\frac{\pi}{2}\left[y\frac{9\cos\frac{\pi}{2}t}{2\left(9-\sin^2\frac{\pi}{2}t\right)^{\frac{3}{2}}}+v\frac{\sin\frac{\pi}{2}t}{\sqrt{9-\sin^2\frac{\pi}{2}t}}\right].$$

Shunday qilib,

$$a=-\frac{\pi^2}{4}y\frac{\left[9\cos\frac{\pi}{2}t-\sin^2\frac{\pi}{2}t\sqrt{9-\sin^2\frac{\pi}{2}t}\right]}{\left(9-\sin^2\frac{\pi}{2}t\right)}.$$

M nuqtaning tezligi va tezlanishi t_1 =3 *s* da quyidagi qiymatlarga ega bo'ladi:

$$v_1=\frac{\pi}{2}a\ sm/s,$$

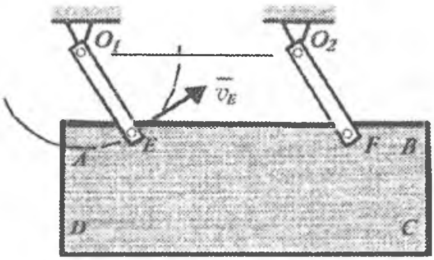
$$a_1=\frac{\pi^2}{8\sqrt{2}}a\ sm/s.$$

$v_1>0$ va $a_1>0$ bo'lganligi uchun, *M* nuqtaning tezligi va tezlanishi *O* nuqtadan vertikal holda yuqoriga yo'nalgan bo'ladi, ya'ni *M* nuqta tezlanuvchan harakatda bo'ladi.

2-masala. O'lchamlari $l_{AB}=3l$ va $l_{Bc}=30\ sm=l$ bo'lgan to'g'ri burchakli plastina o_1 va o_2 nuqtalardagi sharnirlarga sterjenlar yordamida biriktirilgan, plastinka *E* nuqtasining tezligi v_E = 0,6 *m/s*. Boshlang'ich paytda sterjenlar vertikal holatda bo'lgan plastina *ABC* va *D* nuqtalarining $t=t_1=0,5$ *s* dagi tezliklari va tezlanishlari aniqlansin *(2.6-rasm)*. $O_1O_2=EF=2l$, $O_1E=O_2F=l$.

Yechish:

masalada *ABCD* plastina ilgarilanma harakat sodir etiladi, chunki u O_1E va O_2F strjenlarning aylanma harakatlari natijasida har doim o'zining boshlang'ich holatiga parallel qolgan holda harakatlanadi.



2.6-rasm

Boshlang'ich paytda sterjenlar vertikal holatda bo'lgan. Plastinkaning $t = t_1$ vaqt momentidagi vaziyati E nuqtaning burchak tezligi orqali quyidagicha aniqlanadi:

$$\omega_E = \frac{v_E}{l} = \frac{0{,}6}{0{,}3} = 2\ rad/s.$$

Demak, O_1E va O_2F sterjenlar tekis aylanma harakatda bo'ladi.

Sterjenlarning boshlang'ich holatiga nisbatan (vertikal holati) og'ish burchagi quyidagicha topiladi:

$$\theta = \omega_E \cdot l$$

Bunday $t = t$ vaqt momenti uchun sterjenlarning boshlang'ich holatidan og'ish burchagi quyidagiga teng bo'ladi:

$$\theta = \theta(t_1) = 2 \cdot 0{,}5 = 1\ rad = 57°32.$$

Demak, t–t_1 vaqt momentiga boshlang'ich holati vertikal bo'lgan sterjenlar θ = 57°32 burchakka burilar ekan.

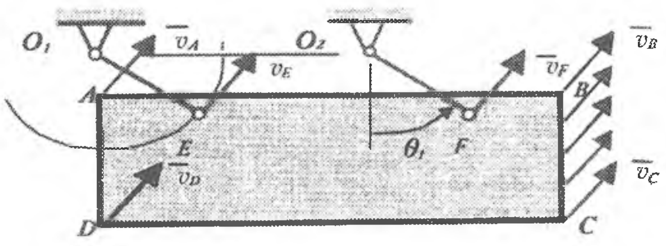
Plastinka ilgarilanma harakatda bo'lishi tufayli, uning barcha nuqtalarning tezligi E nuqtaning tezligiga teng bo'ladi va $\vec{v}_E$ bilan bir xil yo'nalishda bo'ladi:

$$v_A = v_B = v_C = v_D = v_E = v_F = 0{,}6\ m/s.$$

Plastinka nuqtalari tezliklarining taqsimoti *(2.7-rasm)*da ko'rsatilgan.

Plastina E nuqtasining tezlanishini aniqlaymiz. E nuqtaning tezlanishi urinma va normal tashkil etuvchilardan iborat bo'ladi:

$$\vec{a}_E = \vec{a}_{E\tau} + \vec{a}_{En}.$$



2.7-rasm

Bunda

$$a_{E_\tau} = v_E' = 0,$$

$$a_{En} = \frac{v_E^2}{0_1 E} = \frac{(0,6)^2}{0,3} = 1,2\ m/s^2.$$

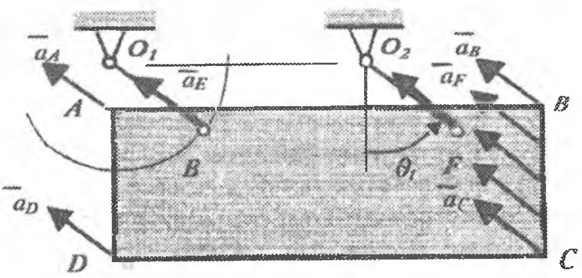
Demak, E nuqtaning tezlanishi u tekis aylanma harakatda bo'lishi sababli, normal markazga intilma tezlanishdan iborat bo'lar ekan:

$$a_E = a_{En} = 1,2\ m/s^2.$$

Bu tezlanish E nuqta chizadigan aylana radiusi bo'ylab, aylana markazi tomon yo'naladi. Plastina ilgarilanma harakatda bo'lishi tufayli, uning barcha nuqtalarining tezlanishlari ham o'zaro teng bo'ladi va $a_{\bar{E}}$ yo'nalishi bilan bir xil bo'ladi:

$$a_A = a_B = a_C = a_D = a_E = a_F = 1,2\ m/s^2.$$

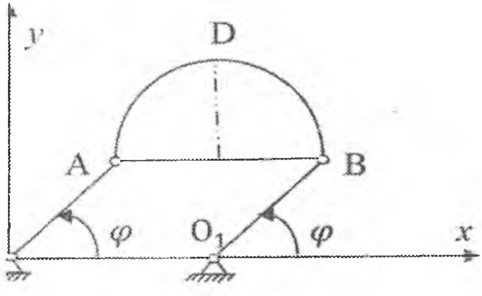
Plastina nuqtalari tezlanishlarining taqsimoti *(2.8-rasm)*da ko'rsatilgan.



2.8-rasm

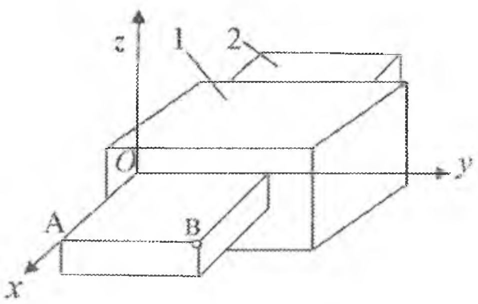
19-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan masalalar

1-masala. Uzunliklari $OA=O_1B=0,16$ *m* bo'lgan ikki krivoshiplarning harakat qonuni $\varphi=\pi t$ bo'lib, yarim aylana shaklidagi *ABD* jismni ilgarilanma harakatga keltiradi. Agar $AB=0,25$ *m* bo'lsa, $t=2s$ da jismning *D* nuqtasi trayektoriyasining egrilik radiusini toping *(2.9-rasm).*



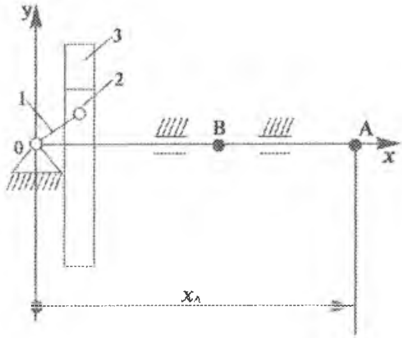
2.9-rasm

2-masala. 1 g'ilof ichida 2 polzun harakat qiladi. Agar polzunning ilgarilanma harakat qonuni $x_A=0,1$ cost, $y_A=0,1$, $z_A=0$ bo'lsa, $t=\pi$(sek) paytda *B* nuqtaning tezligini aniqlang. Bunda masofa $AB=0,3$ *m (2.10-rasm).*



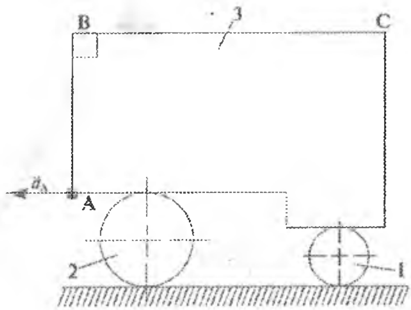
2.10-rasm

3-masala. Krivoship 1 va polzun 2 yordamida ilgarilanma harakatga keluvchi 3 kulisali mexanizm $x_A=0,4-0,1\sin t^2$ qonun asosida siljisa, $t=2$ *s* dagi *B* nuqtaning tezligini aniqlang *(2.11-rasm).*



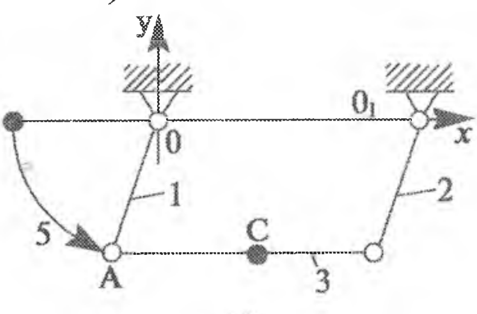
2.11-rasm

4-masala. Ikkita 1 va 2 silindrik o'qlarga o'rnatilgan 3 jism ilgarilanma harakat qiladi. Agar masofalar $BC=2AB=1$ *m* bo'lib, jismning *A* nuqtasi 2 m/s^2 tezlanishga ega bo'lsa, *C* nuqtasining tezlanishini hisoblang *(2.12-rasm)*.



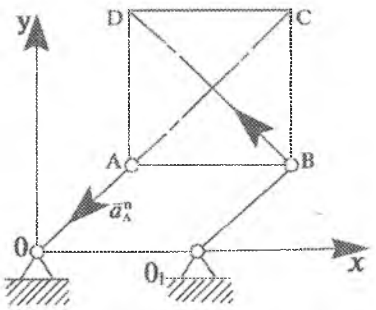
2.12-rasm

5-masala. Bir xil uzunlikdagi $OA=O_1B=0,2$ *m* 1 va 2 krivoshiplarga o'rnatilgan 3 sterjen *Oxy* tekisligida ilgarilanma harakat qiladi. Uning *A* nuqtasining harakat qonuni $s=0,2\pi t$ bo'lsa, $t=0$ paytdagi sterjen o'rtasidagi *C* nuqtaning tezlanishini aniqlang. Bunda masofa $AB=0,36$ *m* *(2.13-rasm)*.



2.13-rasm

6-masala. *ABCD* kvadrat plastina *Oxy* tekisligida ilgarilanma harakat qiladi. Agar uning *A* nuqtasi $a_a^n = 4\ m/s^2$ normal tezlanishga va *B* nuqtasi $a_B^\tau = 3\ m/s^2$ urinma tezlanishga ega bo'lsa, *C* nuqtasining tezlanishini toping *(2.14-rasm).*



2.14-rasm

20-§. Qattiq jismning qo'zg'almas o'q atrofidagi aylanma harakati

Qattiq jismning harakatida ikki nuqtasi doimo qo'zg'almasdan qolsa, uning bunday harakati qo'zg'almas o'q atrofidagi aylanma harakat, qo'zg'almas nuqtalardan o'tuvchi o'q esa aylanish o'qi deyiladi.

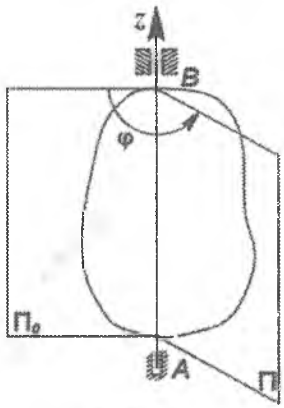
Qattiq jismning qo'zg'almas o'q atrofidagi aylanma harakatida uning aylanish o'qida yotuvchi barcha nuqtalari qo'zg'almas bo'ladi. Aylanish o'qida yotmaydigan boshqa barcha nuqtalar aylanish o'qiga perpendikular tekisliklarda yotuvchi, markazi aylanish o'qida bo'lgan aylanalar bo'ylab harakatlanadi.

Qattiq jismning aylanma harakatini o'rganish uchun aylanish o'qi orqali o'tuvchi qo'zg'almas Π_0 va jismga mahkam biriktirilgan, u bilan birga harakatlanadigan Π tekisliklarni o'tkazamiz.

Jism aylanish o'qi A_Z atrofida harakatlanganda Π tekislik Π_0 tekislikka nisbatan φ burchakka buriladi. Bu burchak aylanish burchagi deyiladi va Π tekislik jism bilan mahkam biriktirilganligidan jismning holati φ burchak bilan aniqlanadi.

Jism A_Z o'q atrofida aylanganda uning aylanish burchagi φ vaqtning uzluksiz, bir qiymatli funksiyasi sifatida o'zgaradi:

$$\varphi = f(t). \tag{2.6}$$



2.15-rasm

Bu tenglama jismning qo'zg'almas o'q atrofida aylanma harakatining kinematik tenglamasi deyiladi. Aylanish burchagi radianlarda o'lchanadi.

Qo'zg'almas o'q atrofida aylanma harakatda bo'lgan jismning asosiy kinematik xarakteristikalari uning burchak tezligi va burchak tezlanishi hisoblanadi.

21-§. Qo'zg'almas o'q atrofida aylanma harakatda bo'lgan jismning burchak tezligi. Tekis aylanma harakat

Burchak tezlik aylanma harakatda bo'lgan jism aylanish burchagining o'zgarishini ifodalovchi kattalik bo'lib, u aylanish burchagidan vaqt bo'yicha olingan birinchi tartibli hosilaga teng:

$$\omega = \varphi' = \frac{d\varphi}{dt}. \tag{2.7}$$

Burchak tezlik φ burchakning o'zgarish qonuniga mos ravishda musbat yoki manfiy qiymatga ega bo'lishi mumkin.

Agar $\varphi' = \frac{d\varphi}{dt} > 0$ bo'lsa, aylanma harakat aylanish o'qining musbat yo'nalishidan qaraganda soat milining aylanishiga teskari yo'nalishda yuz beradi; $\varphi' = \frac{d\varphi}{dt} < 0$ bo'lsa, jism soat milining aylanish yo'nalishida aylanma harakatda bo'ladi.

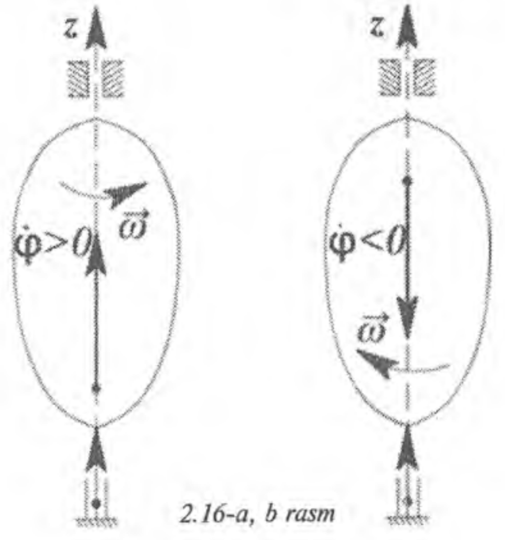
Burchak tezlik vektori aylanish o'qi bo'ylab yo'naladi va uning musbat yo'nalishidan qaraganda, aylanish soat mili harakatiga teskari yo'nalishda ko'rinadi *(2.16-rasm)*. Burchak tezlik vektori aylanish o'qining ixtiyoriy nuqtasiga qo'yiladi. Shuning uchun ham u erkin vektor hisoblanadi. Burchak tezlik vektorining moduli

$$\omega = \left|\frac{d\varphi}{dt}\right| \qquad (2.8)$$

formula yordamida aniqlanadi.

Burchak tezlik SI birliklar sistemasida rad/s yoki 1/s da o'l-chanadi.

Jism harakati davomida $\omega = \omega_0 = \text{const}$ bo'lsa, u tekis aylanma harakatda bo'ladi.



2.16-a, b rasm

Bu holda $\frac{d\varphi}{at} = \omega_0 = \text{const}$, shuning uchun

$$d\varphi = \omega_0 dt. \qquad (2.9)$$

Vaqt 0 dan t gacha o'zgarganda aylanish burchagi φ_0 dan φ gacha o'zgarishini e'tiborga olib, (2.9)ni integrallasak,

$$\varphi = \varphi_0 + \omega_0 t \qquad (2.10)$$

bo'ladi.

(2.10) ifoda jismning tekis aylanma harakat tenglamasini ifodalaydi.

Texnikada tekis aylanma harakatda burchak tezlik ko'pincha bir minutdagi aylanishlar soni bilan o'lchanadi.

Jism bir marta to'la aylanganda $\varphi = 2\pi$ bo'ladi. Agar jism bir minutda n marta aylansa, tekis aylanma harakatning burchak tezligi quyidagiga teng bo'ladi:

$$\omega = \frac{2\pi n}{60} = \frac{\pi n}{30}\ rad/s. \qquad (2.11)$$

22-§. Qo'zg'almas o'q atrofida aylanma harakatda bo'lgan jismning burchak tezlanishi. Tekis o'zgaruvchan aylanma harakat

Burchak tezlanishi aylanma harakatda bo'lgan jism burchak tezligining o'zgarishini ifodalovchi kattalik bo'lib, u burchak tezligidan vaqt bo'yicha olingan birinchi tartibli hosilaga yoki aylanish o'qi atrofidagi aylanish burchagidan vaqt bo'yicha olingan ikkinchi tartibli hosilaga teng bo'ladi:

$$\varepsilon = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2}. \qquad (2.12)$$

Burchak tezlanish ham burchak tezlik kabi vektor kattalik hisoblanadi.

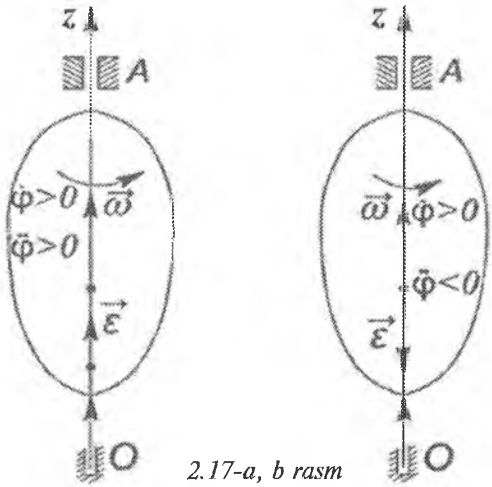
Agar $\frac{d\varphi}{dt}$ va $\frac{d^2\varphi}{dt^2}$ bir xil ishorali bo'lsa, ya'ni aylanma harakat tezlanuvchan bo'lsa, burchak tezlik va burchak tezlanish vektorlari aylanish o'qi bo'ylab bir tomonga *(2.17-a rasm)*, turli ishorali bo'lsa, ya'ni aylanma harakat sekinlanuvchan bo'lsa, qarama-qarshi tomonlarga yo'naladi *(2.17-b rasm)*. Burchak tezlanish vektorining moduli

$$\varepsilon = \left|\frac{d^2\varphi}{dt^2}\right| = \left|\frac{d\omega}{dt}\right| \tag{2.13}$$

formula yordamida aniqlanadi. Burchak tezlanish SI birliklar sistemasida *rad/s*^{2} yoki $1/s^2$ larda o'lchanadi.

Agar aylanma harakat davomida $\frac{d\varphi}{dt} > 0$ bo'lsa, φ orta boradi va bunday harakat ***tezlanuvchan aylanma harakat*** deyiladi; $\frac{d\varphi}{dt} < 0$ bo'lsa, ω kamaya boradi va bunday harakat ***sekinlanuvchan aylanma harakat*** deyiladi.

Agar aylanma harakat davomida $\varepsilon = \varepsilon_0 = \text{const}$ bo'lsa, jismning harakati tekis o'zgaruvchan aylanma harakat bo'ladi.



2.17-a, b rasm

Bunday holda

$$\frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2} = \varepsilon = \varepsilon_0 = \text{const}. \tag{2.14}$$

Vaqt 0 dan t gacha o‘zgarganda, burchak tezlik ω_0 dan ω gacha o‘zgarishini e’tiborga olib, (2.14)ni integrallasak,

$$\omega = \omega_0 + \varepsilon t \tag{2.15}$$

bo‘ladi. (2.15) tenglik yordamida tekis o‘zgaruvchan aylanma harakat burchak tezligi aniqlanadi.

Tekis o‘zgaruvchan aylanma harakat tenglamasini ifodalash uchun (2.15)ni quyidagicha yozamiz:

$$\frac{d\varphi}{dt} = \omega_0 + \varepsilon t. \tag{2.16}$$

Bundan

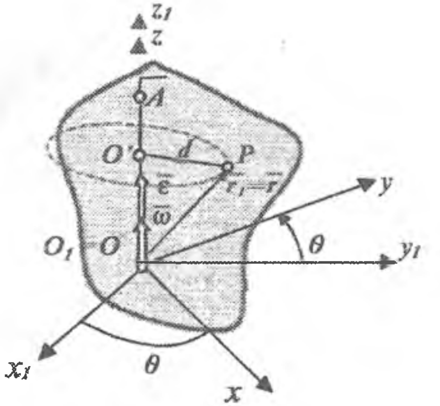
$$d\varphi = (\omega_0 + \varepsilon t)dt. \tag{2.17}$$

(2.10)ni e’tiborga olib, (2.17)ni integrallasak,

$$\varphi = \varphi_0 + \omega_0 t + \frac{\varepsilon t^2}{2} \tag{2.18}$$

ko‘rinishdagi tekis o‘zgaruvchan aylanma harakat tenglamasi kelib chiqadi.

Qattiq jismning qo‘zg‘almas o‘q atrofida aylanma harakatini quyidagi rasm orqali soddaroq holda tushuntirish mumkin *(2.18-rasm)*.



2.18-rasm

Rasmda θ burchak orqali jismning O_z aylanish o‘qi atrofidagi aylanma harakatida burilish burchagi ko‘rsatilgan (*Oxyz* – koordinata o‘qlari sistemasi qo‘zg‘aluvchan sistema, u jism bilan bog‘langan).

Chizmada $\vec{r}_i$ vektor orqali jism ixtiyoriy nuqtasining radius vektori ko'rsatilgan. Shuning uchun,

$$\vec{r}_i = \vec{r}$$

bo'lib, jismning aylanma harakat burchak tezligi va burchak tezlanishlari quyidagicha aniqlanadi:

$$\omega = \omega_z = \theta'$$

$$\varepsilon = \frac{d\omega}{dt} = \frac{d\omega_z}{dt} = \theta''.$$

23-§. Qo'zg'almas o'q atrofida aylanuvchi jism nuqtasining chiziqli tezligi

Qo'zg'almas O_z o'qi atrofida ω burchak tezlik bilan aylanuvchi qattiq jismning aylanish o'qidan *R* masofada joylashgan *M* nuqtasining tezligini aniqlaymiz *(2.19-rasm)*. Biror *t* vaqtda mazkur nuqta *M* holatda bo'lib, *dt* vaqt oralig'ida jism $d\varphi$ burchakka aylansin. Bunda *M* nuqta aylanish o'qiga perpendikular tekislikda aylana bo'ylab harakatlanib, $ds = Rd\varphi$ yoyni bosib o'tadi. *M* nuqta tezligining algebraik qiymati quyidagi formulaga muvofiq aniqlanadi:

$$v_\tau = \frac{ds}{dt} = R\frac{d\varphi}{dt} = R\omega. \qquad (2.19)$$

Tezlikning moduli esa

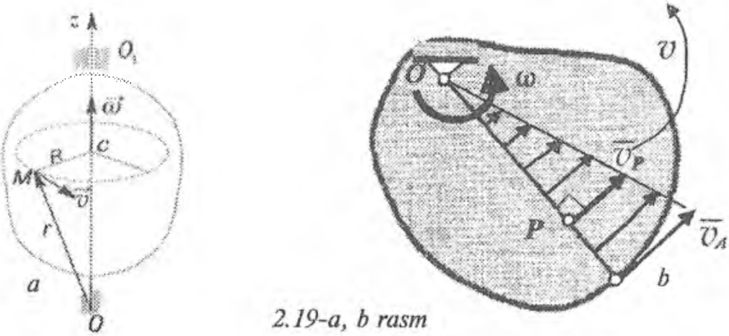
$$v = \left|\frac{ds}{dt}\right| = R\left|\frac{d\varphi}{dt}\right|$$

formula bilan aniqlanadi.

(2.19) formula bilan aniqlanadigan tezlik qo'zg'almas o'q atrofida aylanuvchi jism nuqtasining chiziqli tezligi deyiladi.

Shunday qilib, qo'zg'almas o'q atrofida aylanma harakatda bo'lgan jism ixtiyoriy nuqtasining chiziqli tezligi miqdor jihatdan jism burchak tezligining mazkur nuqtadan aylanish o'qigacha bo'lgan masofaga ko'paytmasiga teng bo'ladi.

Jism barcha nuqtalarining burchak tezliklari berilgan onda bir xil qiymatga ega bo'lgani uchun (2.19)dan qo'zg'almas o'q atrofida aylanma harakatda bo'lgan jism nuqtalarining chiziqli tezliklari mazkur nuqtalardan aylanish o'qigacha bo'lgan masofaga to'g'ri proporsional holda o'zgarishi ma'lum bo'ladi *(2.19-b rasm).*



2.19-a, b rasm

Nuqtaning chiziqli tezligi vektori $\vec{v}$ nuqta chizgan aylanaga harakat yo'nalishi bo'yicha o'tkazilgan urinma bo'ylab yo'naladi.

Chiziqli tezlik vektori burchak tezlik vektori bilan mazkur nuqtaning aylanish o'qidagi *O* nuqtaga nisbatan radius-vektorining vektor ko'paytmasiga teng bo'ladi:

$$\vec{v} = \vec{\omega} \times \vec{r}. \qquad (2.20)$$

Chunki mazkur vektor ko'paytmaning moduli

$$|\vec{\omega} \times \vec{r}| = \omega \cdot r \ \sin(\vec{\omega}\hat{\ }\vec{r}) = \omega \cdot R$$

tezlikning moduliga teng bo'ladi. $\vec{\omega} \times \vec{r}$ vektori, $\vec{\omega}$ va $\vec{r}$ yotgan tekislikka perpendikular holda jismning aylanish yo'nalishi bo'yicha yo'naladi, ya'ni $\vec{\omega} \times \vec{r}$ ning yo'nalishi $\vec{v}$ yo'nalishi bilan bir xil bo'ladi.

Aylanma harakatdagi jism ixtiyoriy *P* nuqtasining tezligini quyidagicha aniqlash ham mumkin *(2.19-a rasm)*:

$$\vec{v}_M = \vec{\omega} \times \vec{r} = \omega\vec{k} \times \vec{r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \omega \\ x & y & z \end{vmatrix} = -\omega y\vec{i} + \omega x\vec{j}.$$

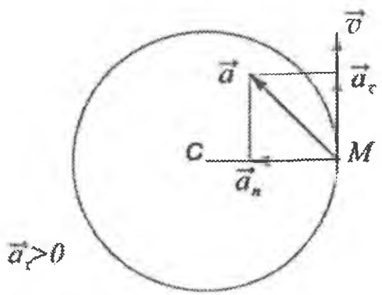
M nuqta tezligining moduli esa quyidagicha aniqlanadi:

$$v_M = \omega\sqrt{x^2 + y^2} = \omega \cdot R.$$

2.17-b rasmda aylanma harakatda bo'lgan jism nuqtalari tezliklarining taqsimoti ko'rsatilgan.

24-§. Qo'zg'almas o'q atrofida aylanuvchi jism nuqtasining tezlanishi

Qo'zg'almas o'q atrofida aylanma harakatdagi jism nuqtalari aylanish o'qiga perpendikular tekislikda aylanalar bo'ylab harakatlanishi tufayli M nuqtaning tezlanishi urinma va normal tezlanishlardan tashkil topadi *(2.20-rasm):*



2.20-rasm

$$\vec{a} = \vec{a}_\tau + \vec{a}_n. \qquad (2.21)$$

Agar ko'rilayotgan holda $\rho = R$ va $v = R\omega$ ekanligini e'tiborga olsak,

$$a_\tau = \frac{dv_\tau}{dt} = \frac{d}{dt}(R\omega) = R\varepsilon, \qquad (2.22)$$

$$a_n = \frac{v^2}{\rho} = \frac{R^2\omega^2}{R} = \omega^2 R \qquad (2.23)$$

bo'ladi.

Urinma tezlanish $\vec{a}_\tau$ aylanma harakat tezlanuvchan bo'lganda, trayektoriyaga o'tkazilgan urinma bo'ylab harakat yo'nalishida, sekinlanuvchan aylanma harakatda esa unga teskari yo'naladi. Normal tezlanish $\vec{a}_\tau$ doimo bosh normal bo'yicha aylanish o'qi tomon yo'naladi *(2.21-a rasm)*. Ba'zan $\vec{a}_\tau$ *aylanma tezlanish*, $\vec{a}_n$ *esa markazga intilma tezlanish* deb ham yuritiladi.

Agar $\vec{\omega}$ va $\vec{\varepsilon}$ vektorlar bir xil ishorali bo'lsa, aylanish o'qi bo'ylab bir tomonga *(2.21-a rasm)*, turli ishorali bo'lsa, qarama-qarshi tomonga yo'naladi *(2.21-b rasm)*.

M nuqta tezlanishining moduli:

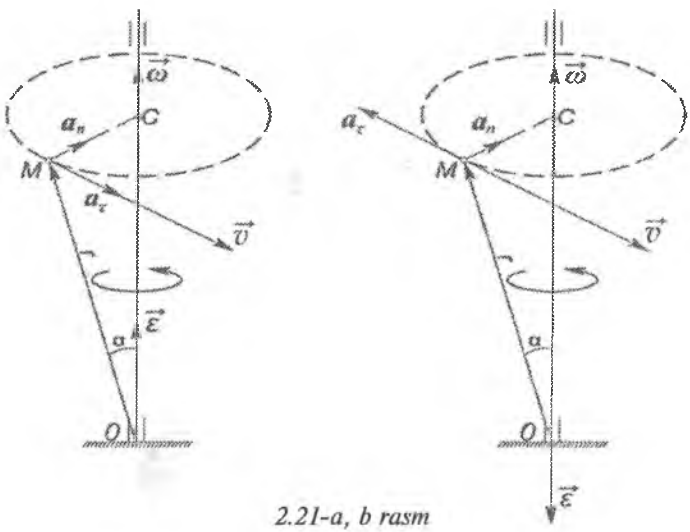
$$a_M = \sqrt{a_\tau^2 + a_n^2} = R\sqrt{\varepsilon^2 + \omega^4} \qquad (2.24)$$

formula orqali aniqlanadi.

Aylanma harakatdagi qattiq jism ixtiyoriy *M* nuqtasining tezlanishini quyidagicha aniqlash ham mumkin:

$$\vec{a}_M = \varepsilon\vec{k}\cdot\vec{r} + \omega\vec{k}\cdot(\omega\vec{k}\cdot\vec{r}) = (-\varepsilon y - \omega^2 x)\vec{i} + (\varepsilon x - \omega^2 y)\vec{j}\,.$$

Tezlanish moduli esa quyidagicha aniqlanadi:



2.21-a, b rasm

$$a_M = \sqrt{(x^2 + y^2)(\varepsilon^4 + \omega^4)} = d\cdot\sqrt{\varepsilon^2 + \omega^4},$$

bunda $d = R$.

M nuqta tezlanishining yo‘nalishi bosh normal bilan $\vec{a}$ tezlanish vektori orasidagi μ burchak orqali aniqlanadi *(2.20-rasm):*

$$\operatorname{tg}\mu = \frac{|a_\tau|}{a_n} = \frac{\varepsilon}{\omega^2}. \qquad (2.25)$$

Aylanma harakatdagi jismning barcha nuqtalari uchun ω va ε lar bir xil bo‘lganidan, jism nuqtalarining tezlanishi aylanish o‘qidan mazkur nuqtalargacha bo‘lgan masofalarga proporsional ravishda o‘zgaradi.

Berilgan onda jismning barcha nuqtalari uchun μ burchak ham bir xil bo‘ladi.

Urinma va normal tezlanishlarning vektorli ifodalarini aniqlash uchun (2.20) dan vaqt bo‘yicha hosila olamiz:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}. \qquad (2.26)$$

Bunda

$$\frac{d\vec{\omega}}{dt} = \vec{\varepsilon}, \quad \frac{d\vec{r}}{dt} = \vec{v}. \qquad (2.27)$$

Shuning uchun

$$\vec{a} = \vec{\varepsilon} \times \vec{r} + \omega \times \vec{v}. \qquad (2.28)$$

Bu formulada

$$\vec{\varepsilon} \times \vec{r} = \vec{a}_\tau \qquad (2.29)$$

urinma tezlanish vektorini ifodalaydi.

Ko‘rinib turibdiki, qo‘zg‘almas o‘q atrofida aylanma harakatdagi jism ixtiyoriy nuqtasining urinma tezlanishi jismning burchak tezlanishi vektori bilan mazkur nuqtaning aylanish o‘qidagi ixtiyoriy O nuqtaga nisbatan radius vektorining vektorli ko‘paytmasiga teng bo‘lar ekan.

(2.28) formulada

$$\vec{a}_n = \vec{\omega} \times \vec{v} \qquad (2.30)$$

normal tezlanish vektorini ifodalaydi.

Demak, qo‘zg‘almas o‘q atrofida aylanma harakatdagi jism ixtiyoriy nuqtasining normal yoki markazga intilma tezlanishi jismning burchak tezlik vektori bilan mazkur nuqta chiziqli tezligining vektorli ko'paytmasiga teng bo'lar ekan.

Takrorlash uchun savollar:

1. Qattiq jismning qanday harakatiga ilgarinlanma harakat deyiladi va bu harakatning asosiy xususiyatlari?

2. Qattiq jismning qanday harakatiga qo'zg'almas o'q atrofidagi aylanma harakat deyiladi?

3. Aylanma harakat tenglamasi qanday ifodalanadi?

4. Aylanma harakat qilayotgan qattiq jismning burchak tezlik va burchak tezlanish modullari qanday formula bilan aniqlanadi?

5. Qo'zg'almas o'q atrofidagi aylanma harakat qilayotgan qattiq jism burchak tezlik va burchak tezlanish vektorlari qanday yo'nalgan bo'ladi?

6. Aylanma harakat qilayotgan nuqtaning chiziqli tezligi qanday formula orqali ifodalanadi?

7. Aylanma harakat qilayotgan nuqtaning chiziqli tezlanishi qanday formula orqali ifodalanadi?

8. Eyler formulasi qanday ko'rinishda bo'ladi?

9. Aylanma harakat qilayotgan nuqtaning tezlik vektori qanday ifodalanadi?

10. Aylanma harakat qilayotgan nuqtaning tezlanish vektori qanday ifodalanadi?

25-§. Qattiq jismning qo'zg'almas o'q atrofida aylanma harakatiga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Qattiq jismning qo'zg'almas o'q atrofida aylanishiga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1. Koordinatalar sistemasi tanlab olinadi, bunda koordinata o'qlaridan birini (z o'qini) aylanish o'qi bo'ylab yo'naltirish maqsadga muvofiq bo'ladi.

2. Qattiq jismning aylanma harakati tenglamasi tuziladi.

3. Qattiq jismning aylanish burchagidan vaqt bo'yicha birinchi tartibli hosila hisoblab, burchak tezlikning aylanish o'qidagi proyeksiyasi aniqlanadi.

4. Qattiq jismning aylanish burchagidan vaqt bo'yicha ikkinchi tartibli hosila hisoblab, burchak tezlanishning aylanish o'qidagi proyeksiyasi aniqlanadi.

5. Aylanma harakat burchak tezligini bilgan holda, jism nuqtasining chiziqli tezligi va normal tezlanishi aniqlanadi.

6. Aylanma harakat burchak tezlanishini bilgan holda, jism nuqtasining urinma tezlanishi aniqlanadi.

7. Aniqlangan normal va urinma tezlanishlar orqali jism nuqtasining to'la tezlanishi aniqlanadi.

Agar masalada qattiq jismning burchak tezlanishi yoki burchak tezligi berilgan bo'lib, aylanma harakat tenglamasini qattiq jism nuqtalarining tezligi va tezlanishini aniqlash talab etilsa, masalani quyidagi tartibda yechish maqsadga muvofiq bo'ladi.

1. Qattiq jism burchak tezlanishining aylanish o'qidagi proyeksiyasini ifodalovchi tenglamani integrallab, burchak tezlikning aylanish o'qidagi proyeksiyasini aniqlaymiz. Bundan integrallash doimiylari — o'zgarmaslari boshlang'ich kattaliklar orqali aniqlanadi.

2. Burchak tezlikning aylanish o'qidagi proyeksiyasini ifodalovchi tenglamani integrallab, jismning aylanma harakat tenglamasini aniqlaymiz. Bunda ham integrallash o'zgarmaslari boshlang'ich kattaliklar orqali aniqlanadi.

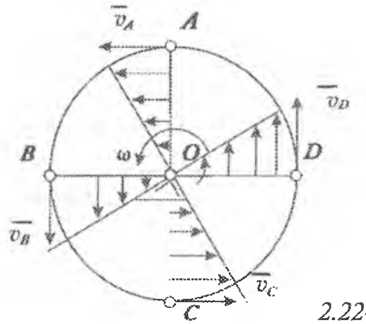
3. Burchak tezlikning aylanish o'qidagi proyeksiyasi ifodasidan foydalanib, jism nuqtalarining tezligini va normal tezlanishini aniqlaymiz.

4. Burchak tezlanishning aylanish o'qidagi proyeksiyasi ifodasidan foydalanib, jism nuqtalarining urinma tezlanishlarini aniqlaymiz.

5. Jism nuqtalarining normal va urinma tezlanishlarini bilgan holda uning to'la tezlanishi aniqlanadi.

26-§. Qattiq jismning qo'zg'almas o'q atrofidagi aylanma harakatiga doir masalalar

1-masala. Radius R=40 *sm* bo'lgan disk qo'zg'almas *O* nuqta atrofida o'zgarmas $\omega = 0{,}5$ *rad/s* burchak tezlik bilan aylanadi *(2.22-rasm).* Disk gorizontal va vertikal diametrlari uchlaridagi nuqtalarning tezligi va tezlanishi aniqlansin va mazkur diametrlar nuqtalari tezliklarining taqsimoti ko'rsatilsin.



2.22-rasm

Yechish: disk qo'zg'almas O nuqtadan o'tuvchi o'q atrofida qo'zgarmas $\omega = 0{,}5$ burchak tezlik bilan aylanma harakatda bo'lishi sababli disk gorizontal va vertikal diametrlari uchlaridagi nuqtalarning tezliklari quydagicha aniqlanadi:

$$v_A = OA \cdot \omega = 40 \cdot 0{,}5 = 20\ sm/s;$$

$$v_B = OB \cdot \omega = 40 \cdot 0{,}5 = 20\ sm/s;$$

$$v_C = OC \cdot \omega = 40 \cdot 0{,}5 = 20\ sm/s;$$

$$v_D = OD \cdot \omega = 40 \cdot 0{,}5 = 20\ sm.$$

Mazkur tezliklar nuqtalar radiuslariga perpendikular holda ω yo'nalishi tomon yo'naladi:

$$v_{\bar{A}} \perp_{\bar{O}A};\ v_{\bar{B}} \perp_{\bar{O}B};\ v_{\bar{C}} \perp_{\bar{O}C};\ v_{\bar{D}} \perp_{\bar{O}D}.$$

Diskning gorizontal va vertikal diametridagi nuqtalar tezliklarining taqsimoti *2.22-rasmda* ko'rsatilgan.

Diskning gorizontal va vertikal diametrlari uchlaridagi nuqtalar tezlanishlarining normal va urinma tashkil etuvchilarini aniqlaymiz.

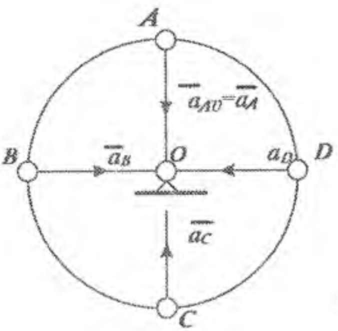
A nuqtaning tezlanishi:

$$a_{A\tau} = OA \cdot \varepsilon = OA \cdot \omega' = 0,$$

$$a_{An} = OA \cdot \omega^2 = 40 \cdot (0{,}5)^2 = 10\ sm/s^2.$$

Nuqta normal tezlanishi nuqta radiusi bo'ylab, disk markazi tomon yo'naladi.

Disk *B, C, D* nuqtalarining tezlanishlari ham miqdor jihatdan *A* nuqtasining tezlanishiga teng bo'lib, nuqtalar radiuslari bo'ylab, disk markazi tomon yo'naladi *(2.23-rasm)*.

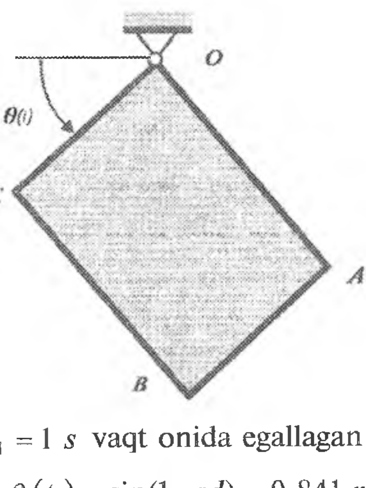


2.23-rasm

2-masala. Ko'rsatilgan *OABC* plastina chizma tekisligida *O* nuqta atrofida aylanadi. Agar plastinaning aylanma harakat tenglamasi

$$\theta(t) = \sin t(rad)$$

bo'lib, *OA*=40 *sm*, *AB*=30 *sm* bo'lsa, plastina *A*, *B* va *C* nuqtalarining tezligi, tezlanishi aniqlansin. Plastina *OA* va *AB* tomonlari nuqtalarining $t_1 = 1\ s$ vaqt onidagi tezliklarining taqsimoti ham ko'rsatilsin *(2.24-rasm).*



2.24-rasm

Yechimi:

1. Plastinaning $t_1 = 1\ s$ vaqt onida egallagan o'rnini aniqlaymiz.

$$\theta_1 = \theta(t_1) = \sin(1\ rad) = 0,841\ rad.$$

Bundan

$$\theta_1 = 48°23'.$$

Plastina nuqtalarining tezliklarini aniqlash uchun dastlab uning burchak tezligini aniqlaymiz:

$$\omega_1 = \frac{d\theta_1}{dt} = \cos t_1 = 0{,}54\frac{1}{s}.$$

Burchak tezlik burilish burchak o'sish tomoniga qarab yo'naladi (ularning ishoralari bir xil).

Plastina A nuqtasining tezligini aniqlaymiz:

$$\vec{v}_A = \omega_1 \cdot OA = 0{,}54 \cdot 40 = 21{,}6\ sm/s.$$

$\vec{v}_A$ tezlik vektori aylanish markazi O nuqtadan o'tkazilgan OA radiusga perpendikular holda ω_1 tomon yo'naladi. Plastina B nuqtasining tezligini aniqlaymiz:

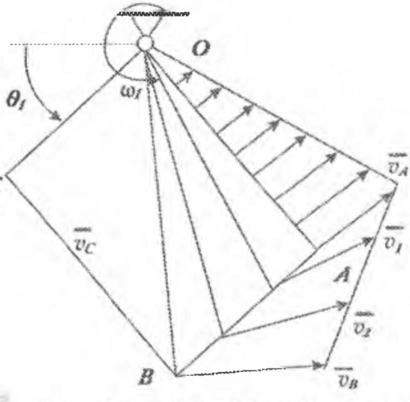
$$v_B = v_1 \cdot OB = v_1 \cdot \sqrt{(OA)^2 + (AB)^2} = 0{,}54 \cdot 50 = 27 sm/s.$$

$\vec{v}_B$ tezlik vektori aylanish markazi O nuqtadan o'tkazilgan OB radiusga perpendikular holda ω_1 tomon yo'naladi.

Plastina C nuqtasining tezligi quyidagiga teng bo'ladi:

$$v_c = \omega_1 \cdot OC = 0{,}54 \cdot 30 = 16{,}2 sm/s.$$

$\vec{v}_C$ tezlik vektori aylanish markazi O nuqtadan o'tkazilgan OC radiusga perpendikular holda ω_1 tomon yo'naladi.



2.25-rasm

Plastina OA tomoni nuqtalari tezliklarining taqsimoti *2.25-rasmda* ko'rsatilgan:

Plastina OA tomoni nuqtalarining tezligi aylanish markazigacha bo'lgan masofalarga to'g'ri proporsinal holda o'sib boradi.

Plastina AB tomoni nuqtalari tezliklarining taqsimoti ham *2.25-rasmda* ko'rsatilgan.

Plastina nuqtalari tezliklarining miqdorlari nuqtalardan aylanish markazigacha bo'lgan masofalarga to'g'ri proporsional bo'ladi. Mazkur vektorlar uchlari $\vec{v}_B$ va $\vec{v}_A$ vektorlar uchlarini birlashtiruvchi to'g'ri chiziq kesmasida yotadi.

Plastina nuqtalarining tezlanishlarini aniqlash uchun plastinaning aylanma harakat burchak tezlanishini aniqlaymiz:

$$\varepsilon_1 = \frac{d\omega_1}{dt} = -\sin t_1 = -0{,}841\frac{1}{s}.$$

Burchak tezlanishining «manfiy» ishorasi plastinaning aylanma harakati tekis sekinlashuvlar ekanligidan dalolat beradi. ε_1 va ω_1 yo'nalishlari qarama-qarshi bo'ladi *(2.25, 2.26-rasmlar).*

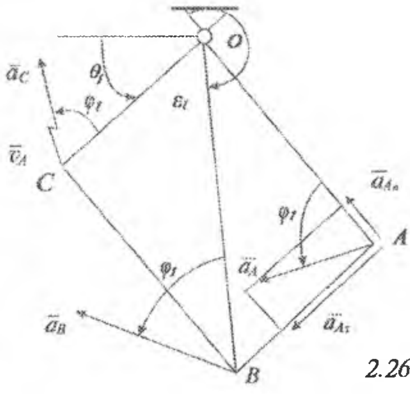
Plastina A nuqtaning urinma tezlanishi quyidagiga teng bo'ladi:

$$a_{A\tau} = OA \cdot \varepsilon_1 = 40 \cdot 0{,}841 = 33{,}64\ sm/s^2.$$

Plastina A nuqtasining normal tezlanishi esa quyidagiga teng bo'ladi:

$$a_{An} = OA \cdot \omega_1^2 = 40 \cdot (0{,}54)^2 = 11{,}64\ sm/s^2.$$

$\vec{a}_{A\tau}$ vektor OA radiusga perpendikular holda ε_1 tomon, $\vec{a}_{An}$ vektor esa A nuqtadan OA radius bo'ylab aylanish markazi tomon yo'naladi.



2.26-rasm

A nuqta tezlanishining moduli quyidagiga teng:

$$a_A = \sqrt{(a_{A\tau})^2 + (a_{An})^2} = \sqrt{(33,64)^2 + (11,64)^2} = 35,59 sm/s^2.$$

$\vec{a}_A$ ning yo'nalishi quyidagi formula asosida aniqlanadi:

$$\mathrm{tg}\varphi_1 = \frac{a_{A\tau}}{a_{An}} = \frac{33,64}{11,64} = 2,89,$$

$$\varphi_1 = \mathrm{arctg}2,89 = 70°91!$$

Plastina B nuqtasi tezlanishining miqdori va yo'nalishi quyidagicha topiladi:

$$a_B = OB \cdot \sqrt{\varepsilon_1^2 + \omega_1^4} = 50\sqrt{(0,84)^2 + (0,54)^2} = 50 \cdot 0,89 = 44,5\ sm/s^2;$$

$$\mathrm{tg}\varphi_1 = \frac{\varepsilon_1}{\omega_1^2} = \frac{0,841}{(0,54)^2} = 2,89.$$

Plastina C nuqtasi tezlanishi miqdorini va yo'nalishi quyidagicha aniqlanadi:

$$a_C = OC \cdot \sqrt{\varepsilon_1^2 + \omega_1^4} = 30 \cdot 0,89 = 26,7\ sm/s^2$$

$\vec{a}_C$ vektorning plastina OC tomoni bilan hosil qilgan burchagi ham φ_1 ga teng bo'ladi.

Plastina B va C nuqtalarning tezlanishlari ham 2.26-rasmda ko'rsatilgan.

3-masala. Radiusi R = 2 m bo'lgan maxovik tinch holatdan boshlab tekis tezlanish bilan aylanadi, gardishida yotuvchi nuqtalar t = 10 s dan keyin $v = 100\ m/s$ chiziqli tezlikka ega bo'ladi. G'ildirak gardishida yotgan nuqtaning t_1 = 15 s bo'lgan vaqtdagi tezligi, urinma va normal tezlanishlari topilsin.

Yechish:

Maxovik tinch holatdan boshlab, tekis tezlanish bilan aylanadi. Shuning uchun ω_0 = 0. t = 10 s vaqt oni uchun maxovikning burchak tezligini aniqlaymiz:

$$v = \omega \cdot R.$$

Bundan

$$\omega = \frac{v}{R} = \frac{100\ m/s}{2\ m} = 50\frac{rad}{s}.$$

Maxovikning shu ondagi burchak tezlanishi esa quyidagicha aniqlanadi:

$$\omega = \varepsilon t.$$

Bundan

$$\varepsilon = \frac{\omega}{t} = \frac{50\ rad/s}{10\ s} = 5\frac{rad}{s^2} = \text{const}.$$

Maxovik gardishida yotgan nuqtaning $t_1 = 15\ s$ dagi tezligi quyidagiga teng bo'ladi:

$$v_1 = \omega_1 \cdot R.$$

Bunda

$$\omega_1 = \varepsilon t_1 = 5 \cdot 15 = 75\ rad/s.$$

Shuning uchun

$$v_1 = 150\ m/s.$$

Maxovik gardishida yotgan nuqtaning urinma va normal tezlanishlarini aniqlaymiz:

$$a_\tau = \varepsilon R = 5 \cdot 2 = 10\ m/s^2,$$

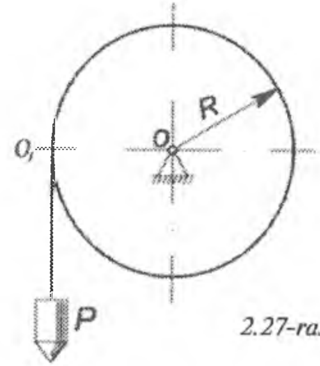
$$a_n = v_1^2 / R = 11250\ m/s^2.$$

4-masala. Radiusi $R = 10\ sm$ bo'lgan val unga ipda osilgan P tosh bilan aylantiriladi. Toshning harakati $x = 100t^2$ tenglama bilan ifodalanadi, bunda x – toshdan qo'zg'almas OO_1 gorizontalgacha bo'lgan, santimetrlar hisobida ifodalangan masofa, t vaqt (sekundlar hisobida). t paytida valning burchak tezligi va burchak tezlanishi, shuningdek, val sirtidagi M nuqtaning tezligi va to'la tezlanishi aniqlansin *(2.27-rasm)*.

Yechish:

toshning tezligi uning harakat tenglamasidan vaqt bo'yicha olingan birinchi tartibli hosilaga teng:

$$v = x' = (100t^2)' = 200t.$$



2.27-rasm

Tosh osilgan ipni cho‘zilmaydi deb faraz qilsak, val O_1 nuqtasining chiziqli tezligi tosh tezligiga teng bo‘ladi $v = v_{O_1}$.

Shuning uchun valning burchak tezligini quyidagicha aniqlash mumkin:

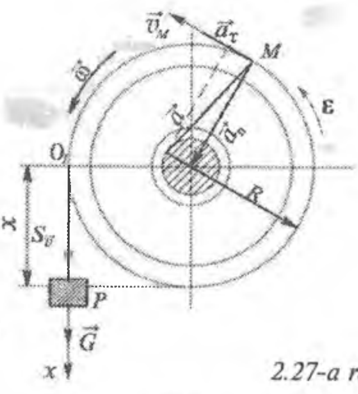
$$v = v_{O_1} = \omega \cdot R.$$

Bundan

$$\omega = \frac{v_{O_1}}{R} = v/R \;\; rad/s = \frac{200t}{10} = 20t \; rad/s.$$

Valning burchak tezlanishi uning burchak tezligidan vaqt bo‘yicha hisoblangan birinchi tartibli hosilaga teng:

$$\varepsilon = \frac{d\omega}{dt} = (20t)' = 20 \; rad/s^2.$$



2.27-a rasm

ω va ε lar yo‘nalishlari $\vec{v}$ yo‘nalishi orqali aniqlanadi, $\vec{v}$ esa toshning harakati tomon yo‘naladi *(2.27-a rasm).*

Val sirtidagi nuqtaning to‘la tezlanishi quyidagicha aniqlanadi:

$$a = \sqrt{a_\tau^2 + a_n^2}.$$

Bunda

$$a_\tau = \varepsilon \cdot R = 20 \cdot 10 = 200 \ sm/s^2,$$

$$a_n = \omega^2 R = 400t^2 \cdot 10 = 4000t^2 \ sm/s^2.$$

Shuning uchun

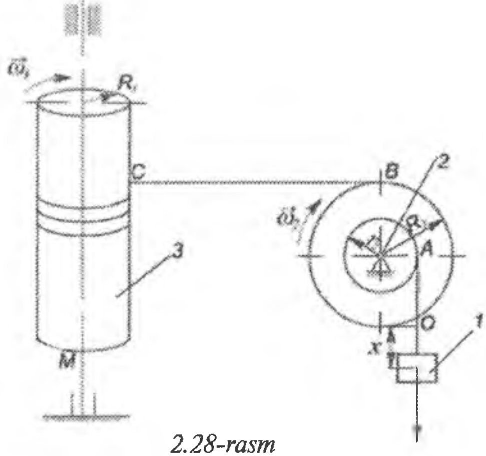
$$a = \sqrt{(200)^2 + (4000t^2)} = 200\sqrt{1 + (4000t^4)} \ sm/s^2.$$

27-§. Jismlarning ilgarilanma va aylanma harakatlarining mexanizmlarda qo‘llanilishiga doir masalalar

1-masala. 1-jismning harakat tenglamasi $x = 10 + 30t^2$ ga ko‘ra, uning vertikal o‘q bo‘ylab, $s = 0{,}3$ *m* yo‘lni o‘tgan vaqt momentida, 3 jism *M* nuqtasining tezligi, urinma, normal va to‘la tezlanishi topilsin *(2.28-rasm).*

Shkiv va silindr o‘lchamlari quyidagicha:

$$R_2 = 30 \ sm, \ r_2 = 20 \ sm, \ R_3 = 30 \ sm.$$



2.28-rasm

Yechish:

1-jismning $s = 0{,}3\ m = 30\ sm$ yo'lni o'tish vaqti τ ni topamiz:

$$s = x_{t=\tau} - x_{t=0} = 10 + 30\tau^2 - 10 = 30\tau^2.$$

Bundan

$$t = \sqrt{s/30} = \sqrt{s/30} = 1\ s.$$

1-jism tezligini aniqlash uchun uning harakat tenglamasidan vaqt bo'yicha birinchi tartibli hosila hisoblaymiz:

$$v_1 = \frac{dx}{dt} = (10 + 30t^2)' = 60t.$$

Agar birinchi jism osilgan hamda 2- va 3-jismlarni biriktiruvchi tasmalarni cho'zilmaydi deb olsak, $\vec{v}_A = \vec{v}_1$ bo'ladi. U vaqtda 2-jismning burchak tezligi

$$\omega_2 = \frac{v_A}{r_2} = \frac{v_1}{r_2} = \frac{60t}{20} = 3t$$

bo'ladi.

Jism B nuqtasining tezligi:

$$v_B = \omega_2 \cdot R_2 = 3t \cdot 30 = 90t.$$

BC tasmani ham cho'zilmaydi deb olsak,

$$v_B = v_c = 90t$$

bo'ladi.

Bu holatda 3-jismning burchak tezligi quyidagi formula bilan aniqlanadi:

$$\omega_3 = \frac{v_c}{R_3} = \frac{90t}{30} = 3t\frac{1}{s}.$$

3-jismning burchak tezlanishi esa uning burchak tezligidan vaqt bo'yicha hisoblangan birinchi tartibli hosilaga teng:

$$\varepsilon_3 = \frac{d\omega_3}{at} = (3t)' = 3\frac{1}{s^2}.$$

M va C nuqtalar silindrning sirtida, ya'ni aylanish o'qidan bir xil masofada yotganligi uchun ularning tezliklari, urinma, normal va to'la tezlanishlari o'zaro teng bo'ladi. Shuning uchun:

$$v_M = v_c = \omega_3 \cdot R_3 = 3t \cdot 30 = 90t,$$

$$a_\tau = \varepsilon_3 R_3 = 3 \cdot 30 = 90,$$

$$a_n = \omega_3^2 \cdot R_3 = 9t^2 \cdot 30 = 270t^2,$$

$$a = \sqrt{a_\tau^2 + a_n^2} = \sqrt{90^2 + (270t^2)^2} = \sqrt{8100 + 72900t^4}.$$

$t = \tau = 1$ sekundda:

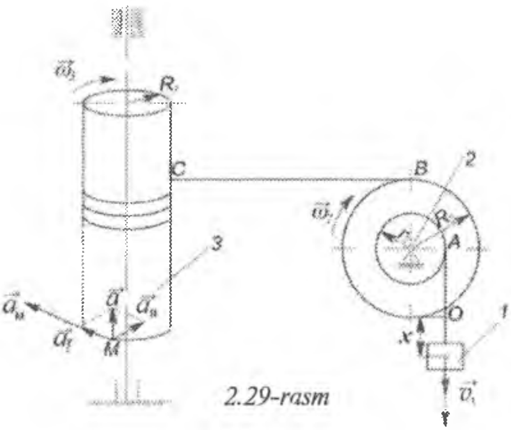
$$v_M = 90 \cdot 1 = 90\ sm/s,$$

$$a_\tau = 90\ sm/s^2,$$

$$a_n = 270 \cdot 1 = 270\ sm/s,$$

$$a = \sqrt{8100 + 72900} = 284{,}6\ sm/s^2.$$

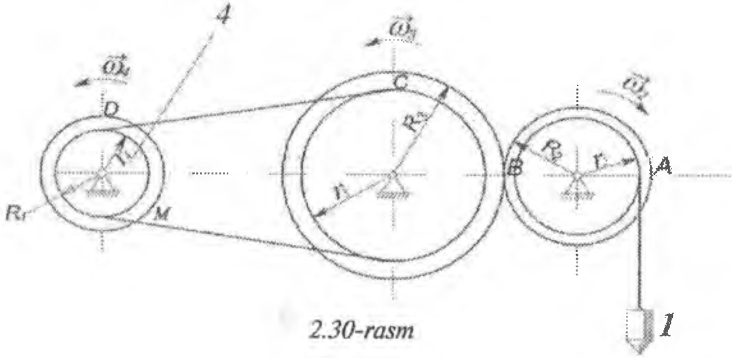
M nuqtaning tezligi, urinma, normal va to'la tezlanishi (*2.29-rasmda* ko'rsatilgan).



2.29-rasm

2-masala. 1-jismning harakat tenglamasi $x = 5 + 8t^2$ ga ko'ra, uning vertikal o'q bo'ylab, $s = 0{,}32$ m yo'lni bosib o'tgan vaqt momentida, 4-jism M nuqtasining tezligi, urinma, normal va to'la tezlanishlari topilsin. 2-, 3-, 4-jismlar radiuslari $r_2 = 16\ sm$,

$R_2 = 20$ *sm*, $r_3 = 20$ *sm*, $R_3 = 25$ *sm*, $r_4 = 8$ *sm/s*, $R_4 = 12$ *sm* ga teng *(2.30-rasm)*.



2.30-rasm

Yechish:

1-jismning $s = 0{,}32$ *m* yo'lni bosib o'tish vaqti τ ni aniqlaymiz:

$$s = x_{t=\tau} - x_{t=0} = 5 + 8t^2 - 5 = 8t^2.$$

Bundan

$$\tau = \sqrt{S/8} = \sqrt{32/8} = 2\ s.$$

Birinchi jism tezligini topamiz. Buning uchun uning harakat tenglamasidan vaqt bo'yicha birinchi tartibli hosila hisoblaymiz:

$$v_1 = \frac{dx}{at} = 16t.$$

Yuk osilgan arqonni cho'zilmaydi deb hisoblasak, 2-jismning burchak tezligi quyidagicha aniqlanadi:

$$\omega_2 = \frac{v_A}{r_2} = \frac{v_1}{r_2} = \frac{16t}{16} = t,$$

bunda $v_A = v_1$ ekanligi e'tiborga olindi.

2-jism *B* nuqtasining tezligi esa $v_B = \omega_2 \cdot R_2 = 20t$ bo'ladi.

B nuqtani 2- va 3-jismlar uchun umumiy deb olib, 3-jismning burchak tezligini topamiz:

$$\omega_3 = \frac{v_B}{R_3} = \frac{20t}{25} = 0{,}8t.$$

3-jism C nuqtasining tezligi esa quyidagiga teng bo'ladi:

$$v_c = \omega_3 \cdot r_3 = 0.8t \cdot 20 = 16t.$$

Agar 3- va 4-jismlarni biriktiruvchi tasmani cho'zilmaydi deb hisoblasak,

$$v_c = v_D$$

bo'ladi.

Shuning uchun 4-jismning burchak tezligi

$$\omega_4 = \frac{v_D}{r_4} = \frac{16t}{8} = 2t$$

bo'ladi.

4-jismning burchak tezlanishini aniqlash uchun burchak tezligidan vaqt bo'yicha birinchi tartibli hosila hisoblaymiz:

$$\varepsilon_4 = \frac{d\omega_4}{at} = (2t)' = 2.$$

4-jism M nuqtasining tezligi, urinma, normal va to'la tezlanishlari quyidagicha aniqlanadi:

$$v_M = \omega_4 \cdot R_4 = 2t \cdot 12 = 24t,$$

$$a_\tau = \varepsilon_4 \cdot R_4 = 2 \cdot 12 = 24,$$

$$a_n = \omega_4^2 \cdot R_4 = 4t^2 \cdot 12 = 48t^2,$$

$$a = \sqrt{a_\tau^2 + a_n^2} = \sqrt{576 + 2304t^2}.$$

$t = \tau = 2$ sekundda:

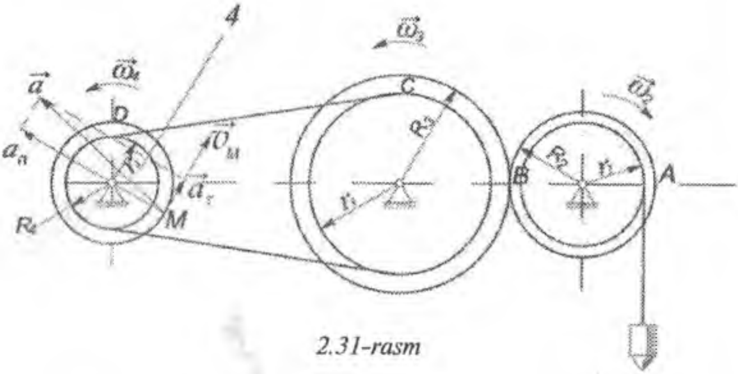
$$v_m = 24 \cdot 2 = 48\ sm/s^2,$$

$$a_\tau = 24\ sm/s^2,$$

$$a_n = 48 \cdot 4 = 192\ sm/s^2,$$

$$a = \sqrt{576 + 36864} = 193{,}4\ sm/s^2.$$

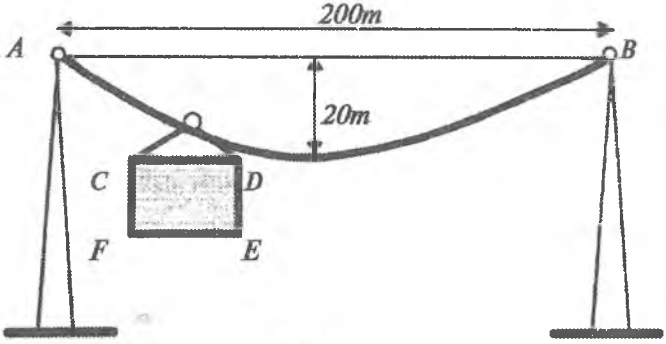
Tezlik va tezlanishlar uchun masshtab tanlab, ularni chizmada ko'rsatamiz: M nuqtaning tezlik vektori nuqtadan trayektoriyaga o'tkazilgan urinma bo'ylab, to'la tezlanish vektori esa urinma va normal tezlanishlarga qurilgan paralellogramning diagonali bo'ylab yo'naladi *(2.31-rasm)*.



2.31-rasm

28-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo. Lift kabinasi parabola qismi bo'ylab A nuqtadan B nuqtaga tortilgan arqonga bog'langan holda harakatlanmoqda. Kabina harakati A nuqtadan gorizontal $v_{gor} = 1\ m/s$ o'zgarmas tezlik bilan boshlangan. Kabina AB arqon o'rtasida bo'lgan paytda $t_1 = 3$ vaqt momenti uchun uning $CDFE$ nuqtalarining tezligi va tezlanishi aniqlansin *(2.32-rasm)*.



2.32-rasm

2-muammo. Kvadrat plastina chizma tekisligida o‘zgarmas $\varepsilon = 1\ rad/s^2$ burchak tezlanish bilan aylanma harakat qilmoqda. Agar $t = 0$ vaqt onida plastina *OA* tomoni gorizontal holatni egallagan bo‘lib, *B* nuqtaning tezligi $v_{BO} = 1\ m/s$ ga teng bo‘lsa, kvadrat plastina uchlarining tezligi va tezlanishi aniqlansin va $t = 1\ s$ vaqt oni uchun kvadrat plastina diagonallarida yotuvchi nuqtalar tezliklarining taqsimoti ko‘rsatilsin. Kvadrat plastina tomoni $l = 0{,}5\ m$ ga teng.

3-muammo. Jism qo‘zg‘almas o‘q atrofida $\varepsilon = 5\ rad/s^2$ burchak tezlanish bilan aylanadi. Boshlang‘ich paytda, $t_0 = 0$ da jismning burchak tezligi $\omega_0 = 0$ bo‘lsa, $t = 2\ s$ da uning aylanish o‘qidan $r = 0{,}2\ m$ masofadagi nuqtasining tezligini aniqlang.

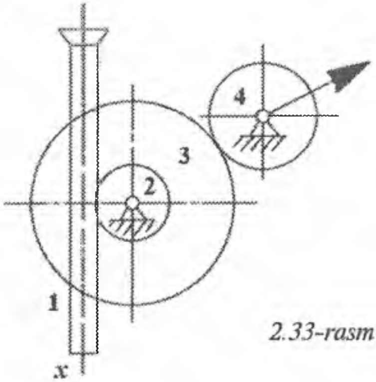
4-muammo. Qo‘zg‘almas o‘q atrofida aylanayotgan jismning aylanish o‘qidan $r = 0{,}2\ m$ masofadagi nuqtasining tezligi $v = 4t^2$ qonun bo‘yicha o‘zgarsa, $t = 2\ s$ dagi jismning burchak tezlanishini toping.

5-muammo. Jismning burchak tezligi $\omega = 1 + t$ qonun bo‘yicha o‘zgarsa, $t = 1\ s$ paytda uning aylanish o‘qidan $r = 0{,}2\ m$ masofadagi nuqtasining tezlanishini toping.

6-muammo. Strelkani indikator mexanizmida harakat o‘lchov shtiftining (1) reykasidan (2) shesterniyaga uzatiladi; (2) shesterniyaning o‘qiga (3) tishli g‘ildirak o‘rnatilgan, (3) g‘ildirak esa strelka biriktirilgan (4) shesternya bilan tishlashadi. Agar shtiftning harakati $x = a\sin kt$ tenglama bilan berilgan bo‘lsa va tishli g‘ildiraklarning radiuslari tegishlicha r_2, r_3 va r_4 bo‘lsa, strelkaning burchak tezligi aniqlansin *(2.33-rasm).*

7-muammo. Jismning burchak tezligi $\omega = -8t$ qonun bo‘yicha o‘zgarsa, $t = 3s$ da uning burilish burchagi φ ni toping. Boshlang‘ich paytda, $t = 0$ da, $\varphi_0 = 5$ rad.

8-muammo. Maxovikning aylanish chastotasi $t_1 = 10\ s$ da 3 marta kamayib, 30 *ayl/min* ga teng bo‘lsa, aylanishni tekis sekunlanuvchan deb, uning burchak tezlanishini aniqlang.



2.33-rasm

9-muammo. Agar jismning burchak tezligi $\omega = 2 - 8t^2$ qonun bo‘yicha o‘zgarsa, u to‘xtaguncha qancha vaqt o‘tadi?

10-muammo. Jism $\varphi = 2\pi\cos\pi t^2$ qonun asosida qo‘zg‘almas o‘q atrofida aylansa, $t = 2$ *s* dan keyin uning burilish burchagini toping.

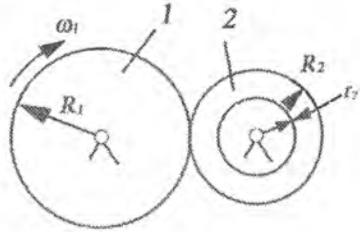
11-muammo. Soat mexanizmining balansiri $\omega = \pi\sin4\pi t$ burchak tezlik bilan harakatlansa, $t = 0{,}125$ *s* paytdagi uning aylanish o‘qidan $h = 6$ *mm* masofadagi nuqtasining tezligini toping (*sm/s*).

12-muammo. Jism qo‘zg‘almas o‘q atrofida $\varphi = 2t^2$ qonun bo‘yicha aylanadi. Jismning aylanish o‘qidan $r = 0{,}2$ *m* masofada joylashgan nuqtasi normal tezlanishining $t = 2$ *s* paytdagi qiymatini aniqlang.

13-muammo. Jism $\omega = 2t^2$ burchak tezlik bilan aylanayotgan bo‘lsa, uning aylanish o‘qidan $r = 0{,}2$ *m* masofada joylashgan nuqtasi urinma tezlanishining $t = 2$ *s* dagi qiymatini aniqlang.

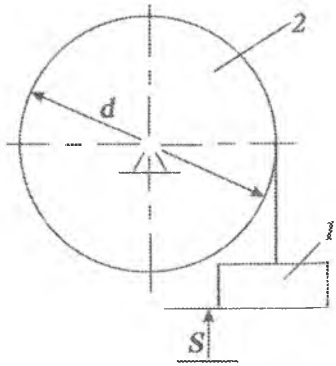
14-muammo. Elektrodvigatelning rotori berilgan paytda $\omega = 3\pi$ burchak tezlik va $\varepsilon = 8\pi$ burchak tezlanish bilan aylanayotgan bo‘lsa, rotorning aylanish o‘qidan 0,04 *m* masofadagi nuqtasining to‘la tezlanishini toping.

15-muammo. Radiuslari $R_1 = 1$ *m*, $R_2 = 0{,}8$ *m* va $r_2 = 0{,}4$ *m* bo‘lgan pog‘onali g‘ildiraklarga (3) yuk osilgan. Agar (1) g‘ildirakning burchak tezligi $\omega_1 = 2t^2$ berilgan bo‘lsa, $t = 2$ *s* da (3) yukning tezlanishini aniqlang *(2.34-rasm).*



2.34-rasm

16-muammo. Diametri $d = 50$ sm li 2 baraban, 1 yukni $s = 7+5t^2$ (*sm*) qonun bo'yicha yuqoriga tortadi. $t = 3$ *s* paytda 2 barabanning burchak tezligini aniqlang *(2.35-rasm).*



2.35-rasm

29-§. Talabalar mustaqil bajarishi uchun ko'p variantli keyslar (hisob chizma ishlari uchun)

Ilgarilanma va aylanma harakatlarda qattiq jism nuqtalarining tezliklari va tezlanishlarini aniqlash.

1-yukning harakati

$$x = c_2t^2 + c_1t + c_0$$

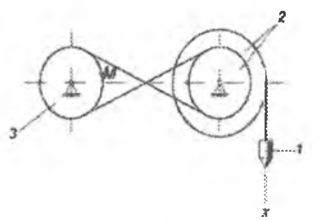
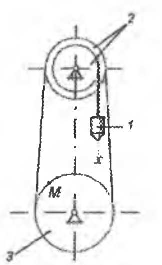
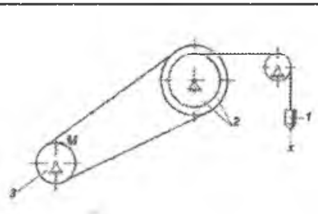
tenglama bilan tavsiflanishi kerak. Bu yerda t – vaqt, c; c_0; c_1; c_2 – doimiylar.

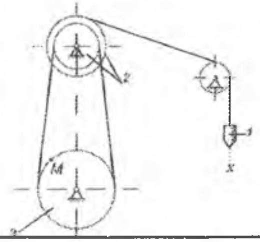
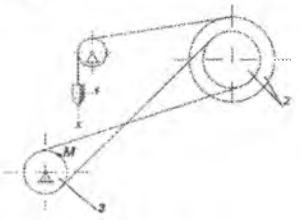
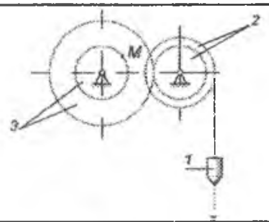
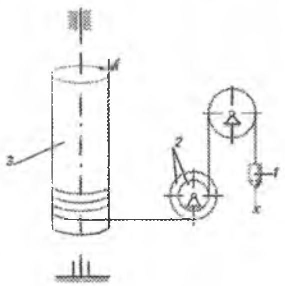
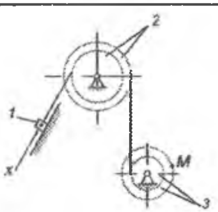
Vaqtning boshlang'ich onida ($t=0$) yukning koordinatasi x_0, tezligi esa v_0 bo'lishi kerak.

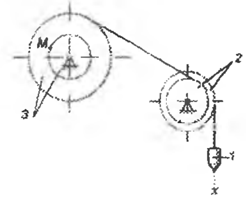
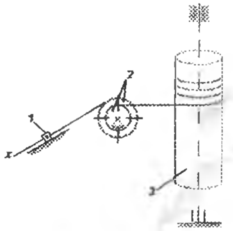
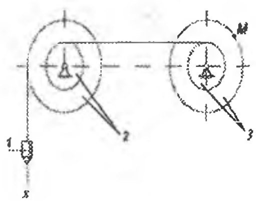
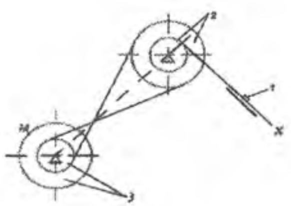
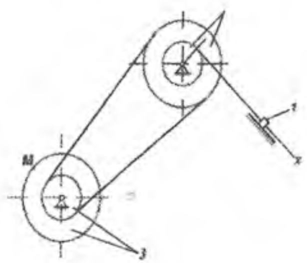
Undan tashqari, $t=t_2$ vaqt onida yukning koordinatasi x_2 ga teng bo'lishi lozim.

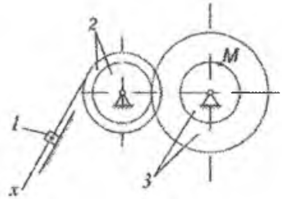
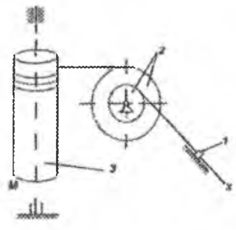
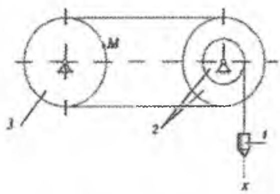
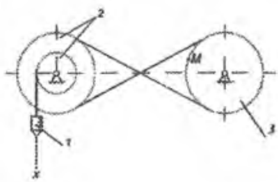
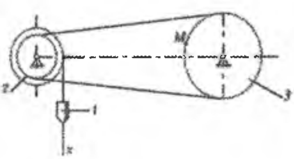
c_0, c_1, c_2 koeffitsiyentlar shunday aniqlansinki, bunda yuk 1 ning talab qilgan harakati amalga oshsin. Shuningdek, $t = t_1$ vaqt onida yukning hamda mexanizm g'ildiraklaridan birining M nuqtasining tezligi va tezlanishi aniqlansin.

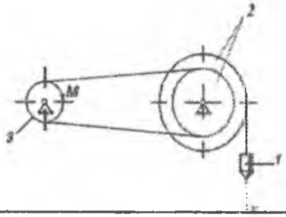
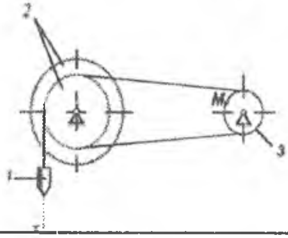
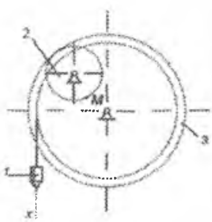
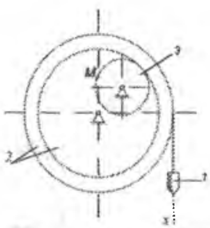
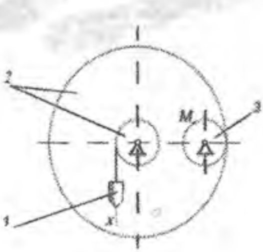
Mexanizmlarning sxemalari, hisoblash uchun kerakli ma'lumotlar jadvalda keltirilgan.

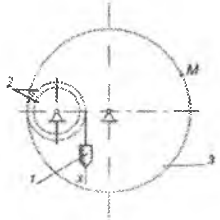
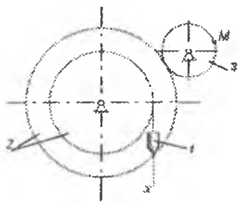
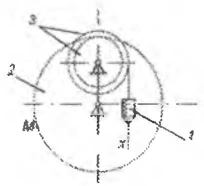
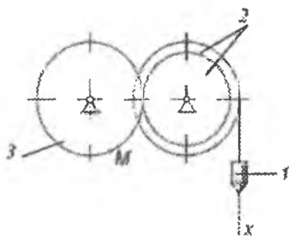
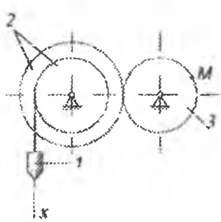
Variant raqamlari	**Mexanizmlarning sxemalari**	**Radiuslar, sm**	**1 yukning koordinatalari va tezliklari**	**Hisob uchun vaqt onlari**
1	2	3	4	5
1.		$R_2=20$ *sm* $r_2=15$ *sm* $R_3=15$ *sm*	$x_0=4$ *sm* $v_0=6$ *sm/s* $x_2=220$ *sm*	$t_2=4$ *s* $t_1=3$ *s*
2.		$R_2=20$ *sm* $r_2=15$ *sm* $R_3=25$ *sm*	$x_0=8$ *sm* $v_0=4$ *sm/s* $x_2=44$ *sm*	$t_2=2$ *s* $t_1=1$ *s*
3.		$R_2=25$ *sm* $r_2=15$ *sm* $R_3=10$ *sm*	$x_0=3$ *sm* $v_0=12$ *sm/s* $x_2=211$ *sm*	$t_2=4$ *s* $t_1=1$ *s*

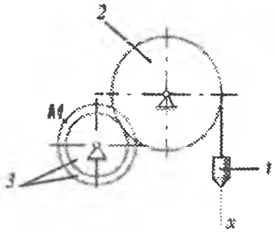
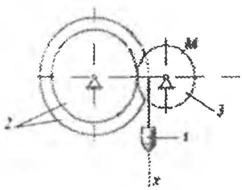
4.		R_2=20 *sm* r_2=15 *sm* R_3=25 *sm*	x_0=5 *sm* v_0= 10 *sm/s* x_2=505 *sm*	t_2=5 *s* t_1=3 *s*
5.		R_2=30 *sm* r_2=20 *sm* R_3=15 *sm*	x_0=10 *sm* v_0= 8 *sm/s* x_2=277 *sm*	t_2=3 *s* t_1=1 *s*
6.		R_2=25 *sm* r_2=15 *sm* R_3=35 *sm* r_3=15 *sm*	x_0=6 *sm* v_0 = 5 *sm/s* x_2=356 *sm*	t_2=5 *s* t_1=2 *s*
7.		R_2=40 *sm* r_2=25 *sm* R_3=50 *sm*	x_0=7 *sm* v_0= 6 *sm/s* x_2=103 *sm*	t_2=2 *s* t_1=1 *s*
8.		R_2=50 *sm* r_2=40 *sm* R_3=40 *sm* r_3=20 *sm*	x_0=5 *sm* v_0= 9 *sm/s* x_2=194 *sm*	t_2=3 *s* t_1=2 *s*

9.		R_2=40 *sm* r_2=20 *sm* R_3=50 *sm* r_3=25 *sm*	x_0=9 *sm* v_0 = 8 *sm/s* x_2=105 *sm*	t_2=4 *s* t_1=2 *s*
10.		R_2=60 *sm* r_2=25 *sm* R_3=70 *sm*	x_0=8 *sm* v_0= 4 *sm/s* x_2=119 *sm*	t_2=3 *s* t_1=2 *s*
11.		R_2=40 *sm* r_2=20 *sm* R_3=40 *sm* r_3=20 *sm*	x_0=6 *sm* v_0= 14 *sm/s* x_2=862 *sm*	t_2=4 *s* t_1=2 *s*
12.		R_2=50 *sm* r_2=20 *sm* R_3=50 *sm* r_3=20 *sm*	x_0=5 *sm* v_0 = 6 *sm/s* x_2=193 *sm*	t_2=2 *s* t_1=1 *s*
13.		R_2=50 *sm* r_2=25 *sm* R_3=50 *sm* r_3=20 *sm*	x_0=8 *sm* v_0 = 5 *sm/s* x_2=347 *sm*	t_2=3 *s* t_1=2 *s*

14.		R_2=30 *sm* r_2=22 *sm* R_3=60 *sm* r_3=30 *sm*	x_0–4 *sm* v_0= 6 *sm/s* x_2=32 *sm*	t_2=2 *s* t_1=1 *s*
15.		R_2=40 sm r_2=20 sm R_3=25 sm	x_0=5 *sm* v_0 =7 *sm/s* x_2=128 *sm*	t_2=2 *s* t_1=1 *s*
16.		R_2=30 *sm* r_2=20 *sm* R_3=30 *sm*	x_0=5 *sm* v_0 =2 *sm/s* x_2=189 *sm*	t_2=4 *s* t_1=2 *s*
17.		R_2=30 *sm* r_2=20 *sm* R_3=30 *sm*	x_0–6 *sm* v_0 =3 *sm/s* x_2=80 *sm*	t_2=2 *s* t_1=1 *s*
18.		R_2=40 *sm* r_2=25 *sm* R_3=50 *sm*	x_0=7 *sm* v_0=0 *sm/s* x_2=557 *sm*	t_2=5 *s* t_1=2 *s*

19.		$R_2=40$ *sm* $r_2=30$ *sm* $R_3=20$ *sm*	$x_0=5$ *sm* $v_0=10$ *sm/s* $x_2=179$ *sm*	$t_2=3$ *s* $t_1=2$ *s*
20.		$R_2=50$ *sm* $r_2=30$ *sm* $R_3=25$ *sm*	$x_0=9$ *sm* $v_0=8$ *sm/s* $x_2=65$ *sm*	$t_2=2$ *s* $t_1=1$ *s*
21.		$R_2=30$ *sm* $r_2=80$ *sm* $R_3=70$ *sm*	$x_0=5$ *sm* $v_0=3$ *sm/s* $x_2=129$ *sm*	$t_2=4$ *s* $t_1=3$ *s*
22.		$R_2=40$ *sm* $r_2=30$ *sm* $R_3=15$ *sm*	$x_0=10$ *sm* $v_0=7$ *sm/s* $x_2=48$ *sm*	$t_2=2$ *s* $t_1=1$ *s*
23.		$R_2=40$ *sm* $r_2=15$ *sm* $R_3=15$ *sm*	$x_0=5$ *sm* $v_0=2$ *sm/s* $x_2=111$ *sm*	$t_2=3$ *s* $t_1=2$ *s*

24.		R_2=60 *sm* r_2=45 *sm* R_3=130 *sm*	x_0=8 *sm* v_0=16 *sm/s* x_2=124 *sm*	t_2=4 *s* t_1=3 *s*
25.		R_2=120 *sm* r_2=72 *sm* R_3=36 *sm*	x_0=7 *sm* v_0=16 *sm/s* x_2=215 *sm*	t_2=4 *s* t_1=2 *s*
26.		R_3=80 *sm* r_3=45 *sm* R_2=30 *sm*	x_0=3 *sm* v_0=15 *sm/s* x_2=102 *sm*	t_2=3 *s* t_1=2 *s*
27.		R_2=58 *sm* r_2=45 *sm* R_3=60 *sm*	x_0=4 *sm* v_0 =4 *sm/s* x_2=172 *sm*	t_2=4 *s* t_1=3 *s*
28.		R_2=120 *sm* r_2=72 *sm* R_3=90 *sm*	x_0=8 *sm* v_0 =6 *sm/s* x_2=40 *sm*	t_2=4 *s* t_1=2 *s*

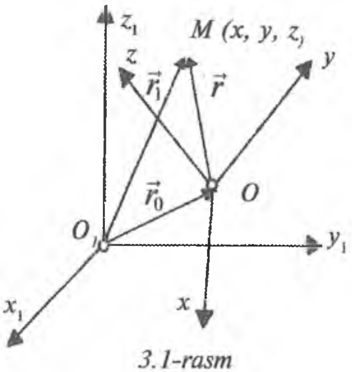
29.		R_2=100 *sm* r_3=75 *sm* R_3=60 *sm*	x_0=5 *sm* v_0=10 *sm/s* x_2=41 *sm*	t_2=2 *s* t_1=1 *s*
30.		R_2=60 *sm* r_2=45 *sm* R_3=36 *sm*	x_0−2 *sm* v_0=12 *sm/s* x_2=173 *sm*	t_2=3 *s* t_1=2 *s*

III BOB
NUQTANING MURAKKAB HARAKATI

30-§. Nuqtaning nisbiy, ko'chirma va absolut harakatlari

Nuqta bir vaqtning o'zida ikki yoki undan ortiq harakatda ishtirok etsa, bunday harakat murakkab harakat deyiladi.

Nuqtaning murakkab harakatini o'rganish uchun qo'zg'almas O_1x_1, y_1, z_1 va unga nisbatan ixtiyoriy ravishda harakatlanadigan $Oxyz$ koordinatalar sistemasini tanlab olamiz *(3.1-rasm)*. *M* nuqtaning qo'zg'aluvchi *Oxyz* koordinatalar sistemasiga nisbatan harakati nisbiy harakat deyiladi.



3.1-rasm

Nuqtaning bunday harakatdagi tezlik va tezlanishi mos ravishda nisbiy tezlik va nisbiy tezlanish deyiladi hamda $\vec{v}_n$ va $\vec{a}_n$ bilan belgilanadi.

M nuqtaning qo'zg'aluvchi koordinatalar sistemasi bilan birgalikda qo'zg'almas koordinatalar sistemasiga nisbatan harakati ko'chirma harakat deyiladi. Qo'zg'aluvchi koordinatalar sistemasining berilgan onda M nuqta bilan ustma-ust tushuvchi nuqtasining tezligi va tezlanishi ko'chirma tezlik va ko'chirma tezlanish deyiladi hamda $\vec{v}_k$ *va* $\vec{a}_k$ *bilan belgilanadi.*

M nuqtaning qo'zg'almas koordinatalar sistemasiga nisbatan harakati ***absolut harakat*** deyiladi.

Nuqtaning ***absolut harakati*** o'z navbatida nisbiy va ko'chirma harakatlardan tashkil topgani tufayli nuqtaning absolut harakatini murakkab deb atash mumkin. Absolut harakatdagi nuqtaning tezlik va tezlanishi mos ravishda absolut tezlik va absolut tezlanish deyiladi hamda $\vec{v}_a$ va $\vec{a}_a$ bilan belgilanadi.

Nuqtaning nisbiy va ko'chirma harakatini bilgan holda uning absolut harakatini, binobarin, absolut harakat tezligi va tezlanishini aniqlash nuqta murakkab harakati kinematikasining asosiy masalasi hisoblanadi.

31-§. Murakkab harakatdagi nuqtaning tezliklarini qo'shish haqidagi teorema

Faraz qilaylik, *M* nuqta qo'zg'almas O_1x_1, y_1, z_1 koordinatalar sistemasiga nisbatan murakkab harakatda bo'lsin *(3.1-rasm)*.

Nuqtaning qo'zg'almas va qo'zg'aluvchan koordinatalar sistemasiga nisbatan holatini aniqlovchi radius-vektorlarni $\vec{r}_1$ va $\vec{r}$ deb, qo'zg'aluvchan sistemani qo'zg'almas sistemaga nisbatan holatini aniqlovchi radius-vektorni $\vec{r}_0$ deb belgilasak, *3.1-rasmdan:*

$$\vec{r}_1 = \vec{r}_0 + \vec{r} = \vec{r}_0 + (\vec{i}x + \vec{j}y + \vec{k}z). \qquad (3.1)$$

Nuqtaning tezligi uning holatini aniqlovchi radius-vektordan vaqt bo'yicha hisoblangan birinchi tartibli hosilaga teng:

$$\vec{v} = \frac{d\vec{r}_1}{dt} = \frac{d\vec{r}_0}{dt} + \frac{d\vec{r}}{dt}. \qquad (3.2)$$

$\vec{r}_1$ — vektor *M* nuqtaning qo'zg'almas koordinatalar sistemasiga nisbatan holatini aniqlovchi radius-vektor bo'lgani uchun, $\frac{d\vec{r}_1}{dt}$ hosila nuqtaning absolut tezligi $\vec{v}_a$ni, $\vec{r}$ — vektor *M* nuqtaning qo'zg'aluvchi koordinatalar sistemasiga nisbatan holatini aniqlovchi radius-vektor bo‘lgani uchun $\frac{d\vec{r}}{dt}$ hosila nuqtaning nisbiy harakat tezligi $\vec{v}_n$ ni, $\vec{r}_0$ — vektor qo‘zg‘aluvchi koordinatalar sistemasining qo‘zg‘almas sistemaga nisbatan holatini aniqlovchi radius-vektor bo‘lgani uchun $\frac{d\vec{r}_0}{dt}$ hosila nuqtaning ko‘chirma harakat tezligi $\vec{v}_k$ ni ifodalaydi.

Shuning uchun (3.2)dan:

$$\vec{v}_a = \vec{v}_n + \vec{v}_k. \tag{3.3}$$

Shuni ta’kidlash lozimki, qo‘zg‘aluvchi $Oxyz$ koordinatalar sistemasi O nuqtadan o‘tuvchi OR oniy o‘q atrofida ω_k burchak tezlik bilan aylanma harakatda bo‘lishi mumkin. Shuning uchun $Oxyz$ koordinatalar sistemasini qo‘zg‘almas $O_1x_1y_1z_1$ koordinatalar sistemasiga nisbatan erkin jism kabi harakatlanishini e`tiborga olsak, M nuqtaning ko‘chirma tezligi quyidagicha yoziladi:

$$\vec{v}_k = \vec{v}_0 + \vec{\omega}_k \times \vec{\varepsilon}. \tag{3.4}$$

Agar qo‘zg‘aluvchi $Oxyz$ koordinatalar sistemasi qo‘zg‘almas O_1x_1, y_1, z_1 koordinatalar sistemasiga nisbatan ilgarilanma harakatda bo‘lsa $\omega_k = 0$ bo‘lib, $\vec{v}_k = \vec{v}_0$ bo‘ladi.

(3.4)ni e’tiborga olsak, umumiy holda, nuqtaning absolut tezligi quyidagicha ifodalanadi:

$$\vec{v}_a = \vec{v}_k + \vec{v}_n = \vec{v}_0 + \vec{\omega}_k \times \vec{r} + \frac{d\vec{r}}{at} \tag{3.5}$$

yoki

$$\vec{v}_a = \frac{d\vec{r}_0}{at} + \left(\frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}\right) + \left(x\frac{d\vec{i}}{dt} + y\frac{d\vec{j}}{dt} + z\frac{d\vec{k}}{dt}\right)$$

Binobarin, murakkab harakatdagi nuqtaning absolut tezligi nisbiy va ko‘chirma harakat tezliklarining geometrik yig‘indisiga teng.

(3.5) tenglama murakkab harakatdagi nuqtaning tezliklarini qo‘shish haqidagi teoremani ifodalaydi.

Quyidagi hollarda nuqtaning ko'chirma tezligini hisoblashni ko'rib chiqamiz.

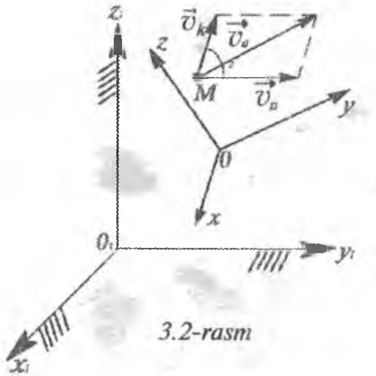
1. Agar ko'chirma harakat ilgarilanma harakatdan iborat bo'lsa, qo'zg'aluvchan sanoq sistemasi o'qlari o'ziga parallel ravishda ko'chadi. Bu holda $\omega_k = 0$ bo'lib, (3.4)ga ko'ra ko'chirma tezlik uchun ushbu munosabat o'rinli bo'ladi.

$$\vec{v}_k = \vec{v}_0 .$$

2. Agar *M* nuqtaning harakati davomida qo'zg'aluvchi koordinatalar sistemasining boshi qo'zg'almasdan qolsa, $\vec{v}_0 = 0$ bo'lib, *M* nuqtaning ko'chirma harakat tezligi xuddi sferik harakatdagi jism nuqtasining tezligi kabi aniqlanadi:

$$\vec{v}_k = \omega \times \vec{r} .$$

Absolut tezlik vektori nisbiy va ko'chirma harakat tezliklariga qurilgan parallelogrammning diagonali bo'ylab yo'nalgan bo'lib, moduli quyidagi formula asosida aniqlanadi *(3.2-rasm)*:



3.2-rasm

$$v = \sqrt{v_n^2 + v_k^2 + 2v_n v_k \cos\alpha} .$$

Bunda:

a) Agar $\alpha = 90°$, ya'ni $\vec{v}_n \perp \vec{v}_k$ bo'lsa, absolut tezlik moduli

$$v_a = \sqrt{v_n^2 + v_k^2}$$

formula yordamida hisoblanadi.

b) Agar $\alpha = 0°$, ya'ni $\vec{v}_n$ va $\vec{v}_k$ bir to'g'ri chiziq bo'ylab bir tomonga yo'nalsa, absolut tezlik moduli

$$v_a = \sqrt{v_n^2 + v_k^2 + 2v_n v_k} = (v_n + v_k)$$

formula orqali aniqlanadi.

d) Agar $\alpha = 180°$ bo'lsa, ya'ni $\vec{v}_n$ bilan bir to'g'ri chiziq bo'ylab qarama-qarshi tomonga yo'nalsa, absolut tezlik moduli

$$v_a = \sqrt{v_n^2 + v_k^2 - 2v_n v_k} = (v_n - v_k)$$

formuladan aniqlanadi.

Absolut tezlik modulini proyeksiyalash usuli yordamida ham aniqlash mumkin. Buning uchun koordinata o'qlari o'tkaziladi va (3.5) tenglik koordinata o'qlariga proyeksiyalanadi:

$$v_{ax} = v_{nx} + v_{kx},$$

$$v_{ay} = v_{ny} + v_{ky}.$$

Absolut tezlik moduli va yo'nalishi quyidagi formulalar asosida aniqlanadi:

$$v_a = \sqrt{v_{ax}^2 + v_{ay}^2},$$

$$\cos(\vec{v}_a \wedge x) = \frac{v_{ax}}{v_a}, \quad \cos(\vec{v}_a \wedge y) = \frac{v_{ay}}{v_a}.$$

Shuni ta'kidlash lozimki, nuqtaning nisbiy tezligini aniqlash uchun ko'chirma harakat xayolan to'xtatiladi.

Ko'chirma harakat tezligini aniqlash uchun nisbiy harakat xayolan to'xtatiladi va berilgan onda qo'zg'aluvchan sanoq sistemasinning *M* nuqta bilan ustma-ust tushuvchi nuqtasining tezligi aniqlanadi.

32-§. Murakkab harakatdagi nuqtaning tezlanishlarini qo'shish haqidagi Koriolis teoremasi

Agar qo'zg'aluvchan *Oxyz* koordinatalar sistemasi qo'zg'almas koordinatalar sistemasiga nisbatan ilgarilanma harakatda bo'lmasa, nuqtaning absolut tezlinishi quyidagi teorema yordamida aniqlanadi.

Teorema. *Ko'chirma harakat ilgarilanma bo'lmagan murakkab harakatda nuqtaning absolut tezlanishi uning nisbiy, ko'chirma va Koriolis tezlanishlarining geometrik yig'indisi teng bo'ladi.*

Teoremani isbotlash uchun nuqtaning absolut tezligi ifodasidan foydalanamiz:

$$\vec{v}_a = \frac{d\vec{r}_o}{dt} + \left(\frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}\right) + \left(x\frac{d\vec{i}}{dt} + y\frac{d\vec{j}}{dt} + z\frac{d\vec{k}}{dt}\right) \qquad (3.6)$$

Nuqtaning absolut tezlanishi uning absolut tezligidan vaqt bo'yicha olingan birinchi tartibli hosilaga teng:

$$\vec{a}_a = \frac{d\vec{v}_a}{dt} = \frac{d^2\vec{r}_o}{dt^2} + \left(\frac{d^2\vec{i}}{dt^2}x + \frac{d^2\vec{j}}{dt^2}y + \frac{d^2\vec{k}}{dt^2}z\right) + \left(\vec{i}\frac{d^2x}{dt^2} + \vec{j}\frac{d^2y}{dt^2} + \vec{k}\frac{d^2z}{dt^2}\right) +$$

$$+2\left(\frac{d\vec{i}}{dt}\frac{dx}{dt} + \frac{d\vec{j}}{dt}\frac{dy}{dt} + \frac{d\vec{k}}{dt}\frac{dz}{dt}\right) \qquad (3.7)$$

Bu ifodada

$$\frac{d^2\vec{r}_o}{dt^2} = \vec{a}_o \quad - 0 \text{ qutbning tezlanishi,} \qquad (3.8)$$

$\frac{d^2x}{dt^2}\vec{i} + \frac{d^2y}{dt^2}\vec{j} + \frac{d^2z}{dt^2}\vec{k} = \vec{a}_n$ – M nuqtaning nisbiy tezlanishini. Yuqorida yozilgan ifodalarni e'tiborga olsak, (3.7)ni quyidagicha yozish mumkin:

$$\vec{a}_a = \vec{a}_o + \vec{\varepsilon}_k \times \vec{r} + \vec{\omega}_k \times (\vec{\omega}_k \times \vec{r}) + \vec{a}_n + 2(\vec{\omega}_k \times \vec{v}_n). \qquad (3.9)$$

M nuqtaning ko'chirma tezlanishi $Oxyz$ qo'zg'aluvchi koordinatalar sistemasining shu nuqta bilan ustma-ust tushgan nuqtasining $O_1x_1y_1z_1$ sanoq sistemasiga nisbatan tezlanishiga teng bo'ladi. Ko'rilayotgan holda $Oxyz$ koordinatalar sistemasi xuddi erkin jism kabi harakatlangani uchun $\vec{a}_k$ ko'chirma harakat tezlanishi O qutbning tezlanishi $\vec{a}_o$ hamda qutb atrofidagi aylanma harakat tezlanishi

$\vec{a}^{\varepsilon} = \vec{\varepsilon} \times \vec{r}$ va oniy o'qqa intilma tezlanish $\vec{a}^{\omega} = \vec{\omega}_k \times (\vec{\omega}_k \times \vec{r})$ dan tashkil topadi:

$$\vec{a}_k = \vec{a}_o + \vec{\varepsilon}_k \times \vec{r} + \vec{\omega}_k \times (\vec{\omega}_k \times \vec{r}). \qquad (3.10)$$

(3.9)dagi $2(\vec{\omega}_k \times \vec{v}_n) = \vec{a}_c$ – Koriolis tezlanishi deyiladi (3.11).

Binobarin, nuqtaning absolut tezlanishi quyidagi tenglikdan aniqlanadi:

$$\vec{a}_a = \vec{a}_n + \vec{a}_k + a_c. \qquad (3.12)$$

(3.12) tenglik ko'chirma harakati ilgarilanma harakat bo'lmagan nuqtaning tezlanishlarini qo'shish haqidagi Koriolis (1792–1843) teoremasini ifodalaydi.

Agar ko'chirma harakat ilgarilanma harakatdan iborat bo'lsa, u holda $\vec{\omega}_k = 0$, $\vec{\varepsilon}_k = 0$ bo'ladi. Shu sababli qo'zg'aluvchi koordinatalar sistemasi bilan bog'langan barcha nuqtalarning tezlanishlari o'zaro teng bo'lib, qutbning tezlanishi $\vec{a}_o$ bilan belgilanadi:

$$\vec{a}_k = a_o.$$

Bunday holda Koriolis tezlanishi ham nolga teng bo'ladi:

$$\vec{a}_c = 2(\vec{\omega}_k \times \vec{v}_n) = 0.$$

Shu sababli nuqtaning absolut tezlanishi quyidagi tenglik orqali ifodalanadi:

$$\vec{a}_a = \vec{a}_n + \vec{a}_k. \qquad (3.13)$$

(3.13) formula ko'chirma harakati ilgarilanma harakatdan iborat bo'lgan nuqta uchun tezlanishlarni qo'shish haqidagi quyidagi teoremani ifodalaydi:

Ko'chirma harakati ilgarilanma harakatdan iborat bo'lgan nuqtaning absolut tezlanishi uning nisbiy va ko'chirma tezlanishlarining geometrik yig'indisiga teng bo'ladi. Bu holda absolut tezlanishning moduli quyidagicha hisoblanadi:

$$a_a = \sqrt{a_n^2 + a_k^2 + 2a_k a_n \cos(\vec{a}_k \hat{\ } a_n)}.$$

33-§. Koriolis tezlanishi

Nisbiy tezlikning yo'nalishini ko'chirma harakatda, ko'chirma tezlik miqdori va yo'nalishini nisbiy harakatda o'zgarishlarini xarakterlovchi kattalik Koriolis tezlanishi deyiladi.

Koriolis tezlanishi murakkab harakatdagi nuqtaning ko'chirma harakat burchak tezligi vektori bilan nisbiy harakat tezligi vektor ko'paytmasining ikkilanganiga teng:

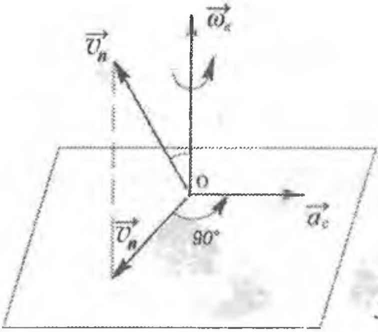
$$\vec{a}_c = 2(\vec{\omega}_k \times \vec{v}_n)\,. \qquad (3.14)$$

Agar $\vec{\omega}_k$ bilan $\vec{v}_n$ orasidagi burchakni α bilan belgilasak, Koriolis tezlanishining moduli

$$a_c = 2\omega_k v_n \sin\alpha \qquad (3.15)$$

formuladan aniqlanadi.

Koriolis tezlanishining yo'nalishini aniqlash uchun nuqtaning nisbiy tezligini ko'chirma harakat aylanish o'qiga perpendikular tekislikka proyeksiyalab, bu proyeksiyani mazkur tekislikda ko'chirma harakat aylanishi yo'nalishida 90° burchakka burish kerak (3.4-rasm).



3.4-rasm

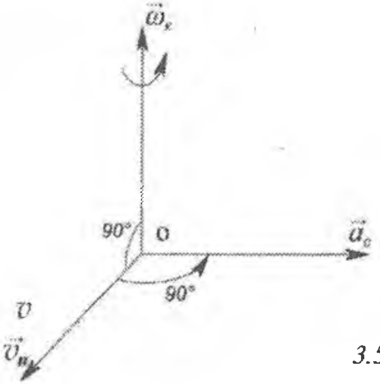
Bu usul Jukovskiy qoidasi deyiladi.

Agar $\vec{\omega}_k \perp \vec{v}_n$ bo'lsa *(3.4-rasm)*, $\sin\alpha = 1$ bo'ladi.

U holda

$$a_c = 2\omega_k \cdot v_n\,. \qquad (3.16)$$

Koriolis tezlanishining yo'nalishini $\vec{\omega}_k \times \vec{v}_n$ vektor ko'paytma qoidasiga muvofiq aniqlash ham mumkin. Qoidaga ko'ra, Koriolis tezlanishi $\vec{\omega}_k$ va $\vec{v}_n$ vektorlar joylashgan tekislikka perpendikular holda shunday tomonga qarab yo'nalgan bo'ladiki, u tomondan qaraganda $\vec{\omega}_k$ vektorni $\vec{v}_n$ vektor bilan kichik burchak orqali ustma-ust tushirish uchun qilinadigan aylanma harakat soat mili harakatiga qarama-qarshi yo'nalishda ro'y beradi *(3.5-rasm)*.



3.5-rasm

Koriolis tezlanishi quyidagi hollarda nolga teng bo'ladi:

a) agar $\omega_k = 0$, ya'ni ko'chirma harakat ilgarilanma harakatdan iborat bo'lsa;

b) agar $v_n = 0$, ya'ni nisbiy harakat tezligi biror onda nolga teng bo'lsa;

c) agar $\alpha = 0$ yoki $\alpha = 180°$, ya'ni nisbiy harakat ko'chirma harakat aylanish o'qiga parallel ravishda sodir bo'lsa yoki berilgan onda nisbiy harakat tezligi vektori mazkur o'qqa parallel bo'lsa.

Takrorlash uchun savollar:

1. *Nuqtaning qanday harakatiga nisbiy harakat deyiladi?*
2. *Nuqtaning qanday harakatiga ko'chirma harakat deyiladi?*
3. *Nuqtaning qanday harakatiga absolut harakat deyiladi?*
4. *Nuqtaning nisbiy tezligi qanday topiladi?*
5. *Nuqtaning ko'chirma tezligi qanday topiladi?*
6. *Nuqtaning absolut tezligi qanday topiladi?*
7. *Tezliklarni qo'shish haqidagi teoremani aytib bering.*
8. *Absolut tezlik modulini topish formulasini yozib bering.*
9. *Nuqtaning nisbiy tezlanishi qanday aniqlanadi.*

10. Nuqtaning ko'chirma tezlanishi qanday aniqlanadi?
11. Nuqtaning absolut tezlanishi qanday aniqlanadi?
12. Koriolis tezlanishining yuzaga kelish shartlarini aytib bering.
13. Koriolis tezlanishi qanday yo'naladi.
14. Koriolis tezlanishining moduli qanday ifodalanadi?
15. Koriolis tezlanishining nolga teng bo'lishi shartlarini aytib bering.

34-§. Nuqtaning nisbiy va absolut harakatlarida uning trayektoriyasi va harakat tenglamalarini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Mazkur mavzuga doir masalalarni ikki asosiy turga ajratish mumkin:

1) nuqtaning nisbiy va ko'chirma harakatlarini bilgan holda absolut harakat trayektoriyasi va uning tenglamalarini tuzish talab etiladi.

2) nuqtaning absolut va ko'chirma harakatlarini bilgan holda nisbiy harakat trayektoriyasi va uning tenglamalarini tuzish talab etiladi.

Birinchi turdagi masalalarni yechishda nuqtaning nisbiy va ko'chirma harakatlarini qo'shish lozim.

Ikkinchi turdagi masalalarni yechishda nuqtaning berilgan absolut harakatini masala shartida ma'lum bo'lgan ko'chirma va noma'lum nisbiy harakatlarga ajratish talab etiladi.

Masalalarni yechishda, dastavval qo'zg'almas va qo'g'aluvchan sanoq sistemalari tanlab olinadi va qo'zg'aluvchan sanoq sistemasida bog'langan jism harakati, ya'ni ko'chirma harakat o'rganiladi. Natijada, nuqtaning absolut va nisbiy harakatlarining xususiyatlarini oson aniqlash imkoni tug'iladi.

Birinchi turdagi masalalarni quyidagi tartibda yechish maqsadga muvofiq bo'ladi:

1)nuqtaning absolut harakati ko'chirma va nisbiy harakatlarga ajratiladi;

2) shartli ravishda qo'zg'almas deb qabul qilingan absolut va harakatdagi jism bilan bog'langan nisbiy sanoq sistemalari tanlab olinadi;

3) nuqta nisbiy harakatining tenglamalari tuziladi;

4) nuqta absolut harakatining parametrik tenglamalari tuziladi;

5) trayektoriyaning parametrik tenglamalaridan vaqtni qisqartirib, absolut harakat trayektoriyasining tenglamasi tuziladi.

Ikkinchi turga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1) nuqtaning masala shartidan ma'lum bo'lgan absolut harakati ko'chirma va nisbiy harakatlarga ajratiladi;

2) shartli ravishda qo'zg'almas deb qabul qilingan absolut va harakatdagi jism bilan bog'langan nisbiy sanoq sistemalari tanlab olindi;

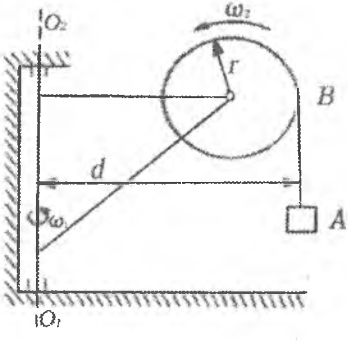
3) nuqta absolut harakatining tenglamalari tuziladi;

4) nuqta nisbiy harakatining parametrik formadagi tenglamalari tuziladi;

5) nisbiy harakatning parametrik tenglamalaridan parametr-vaqtni qisqartirib, koordinatalar ko'rinishidagi nisbiy harakat tenglamalari tuziladi.

35-§. Nuqta absolut harakatining tenglamalari va trayektoriyasini aniqlashga doir masalalar

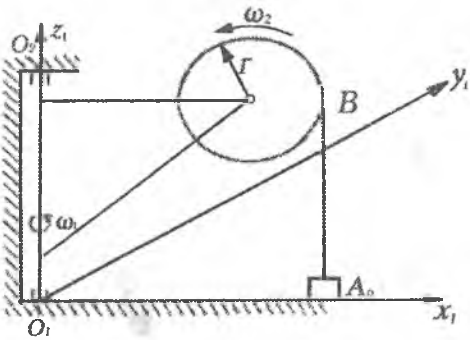
1-masala. Aylanuvchi kranning O_1O_2 o'q atrofida ω_1 o'zgarmas burchak tezlik bilan aylanishida A yuk B barabanga o'ralgan kanat yordamida yuqoriga ko'tariladi. r radiusli B baraban ω_2 o'zgarmas burchak tezlik bilan aylanadi. Agar kranning qulochi d ga teng bo'lsa, yukning absolut harakati trayektoriyasi aniqlansin *(3.6-rasm)*.



3.6-rasm

Yechish: aylanuvchi kranning O_1O_2 o'q atrofida ω_1 o'zgarmas burchak tezlik bilan aylanishi ko'chirma harakat hisoblanadi. *A* nuqtaning *B* barabanga o'ralgan kanat yordamida yuqoriga ko'tarilishi nisbiy harakat hisoblanadi.

Aylanuvchi kran asosi bilan bog'langan $O_1x_1y_1z_1$ koordinata o'qlari sistemasi qo'zg'almas sanoq sistemasini tashkil etadi. Aylanuvchi kran bilan bog'lashgan va u bilan birga aylanuvchi O_2xyz koordinata o'qlari sistemasi qo'zg'aluvchan sanoq sistemasini tashkil etadi *(3.7-rasm).*

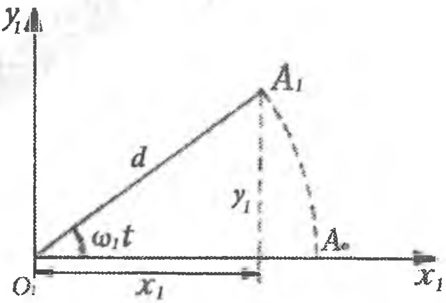


3.7-rasm

Bunda x_1 o'q O_1O_2 o'q va yukning boshlang'ich holatidan o'tadi, z_1 o'q esa kran aylanish o'qi bo'ylab yo'naladi.

A yukning holati uning absolut harakatida quyidagi koordinatalar orqali aniqlanadi *(3.8-rasm):*

$$\begin{cases} x_1 = d\cos\omega_1 t, \\ y_1 = d\sin\omega_1 t, \\ z_1 = r\omega_2 t. \end{cases}$$



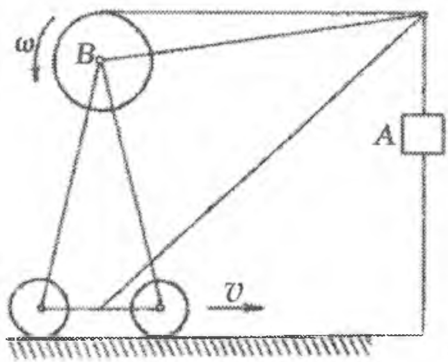
3.8-rasm

Hosil bo'lgan tenglamalarni A yukning absolut harakat trayektoriyasining parametrik ko'rinishidagi tenglamalari sifatida qarash mumkin.

Koordinatalar formasidagi trayektoriya tenglamasini tuzish uchun yuqoridagi tenglamalardan parametr-vaqtni qisqartiramiz. Natijada, A yuk absolut harakati trayektoriyasining tenglamalari hosil bo'ladi:

$$x_1 = d\cos\frac{\omega_1 z_1}{\omega_2 r},\ y_1 = d\sin\frac{\omega_1 z_1}{\omega_2 r}.$$

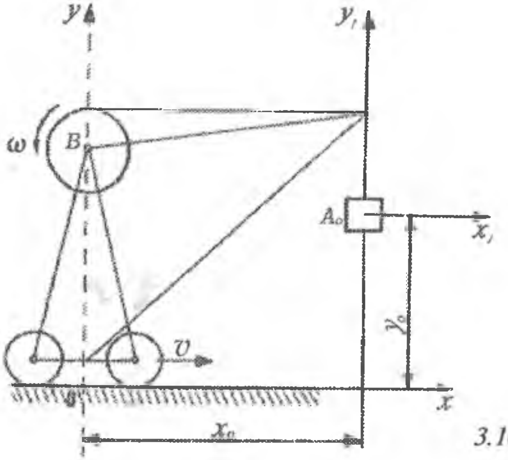
2-masala. Yukni ko'tarish va kranni siljitish mexanizmlarining ishlarini birlashtirishda A yuk gorizontal va vertikal yo'nalishlarda siljiydi. r=0,5 *m* radiusli B barabanga o'ralgan kanat vositasida A yuk ushlab turiladi. B baraban ishga tushirilishida $\omega = 2\pi\ rad/s$ burchak tezlik bilan aylanadi. Kran gorizontal yo'nalishda $\upsilon = 0{,}5\ m/s$ doimiy tezlik bilan siljiydi. Agar yukning boshlang'ich koordinatalari x_0=10 *m*, y_0=6 *m* bo'lsa, uning absolut trayektoriyasi aniqlansin *(3.9-rasm)*.



3.9-rasm

Yechish: kranning gorizontal yo'nalishda $\upsilon = 0{,}5\ m/s$ doimiy tezlik bilan siljishi ko'chirma harakat deyiladi. A nuqtaning vertikal yo'nalishda siljishi nisbiy harakat sifatida qaraladi. Yer bilan bog'langan *Oxy* koordinata o'qlari sistemasi shartli ravishda qo'zg'almas sanoq sistemasi sifatida qaraladi. Gorizontal yo'nalishda

$v = 0{,}5\ m/s$ doimiy tezlik bilan siljuvchi kran bilan bog'langan Ax_1y_1 koordinata o'qlari sistemasi qo'zg'luvchan sanoq sistemasini tashkil etadi, bunda qo'zg'luvchan sanoq sistemasining boshi A yukning boshlang'ich holati bilan ustma-ust tushadi *(3.10-rasm)*.



3.10-rasm

A yukning qo'zg'almas – absolut sanoq sistemasidagi holati quyidagi koordinatalar orqali aniqlanadi:

$$\begin{cases} x = x_0 + vt, \\ y = y_0 + \omega \cdot rt. \end{cases}$$

Hosil bo'lgan tenglamalar sistemasini A yuk absolut harakati trayektoriyasining parametrik tenglamalari sifatida qarash mumkin. A yuk absolut harakati trayektoriyasining koordinatalar formasidagi tenglamalarini tuzish uchun yuqoridagi tenglamalardan parametr-vaqtni qisqartiramiz.

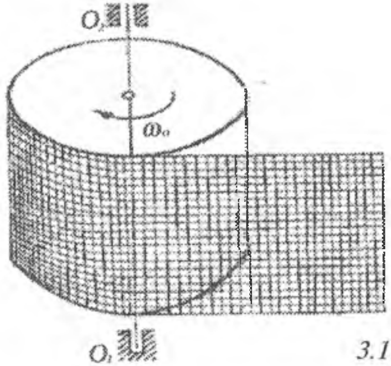
Natijada, A yuk absolut harakati trayektoriyasining tenglamalari hosil bo'ladi:

$$t = \frac{x - x_0}{v},$$

$$y = y_0 + \omega r\left(\frac{x - x_0}{v}\right) = 6 + 2\pi \cdot 0{,}5\left(\frac{x - 10}{0{,}5}\right) = (6{,}28x - 56{,}8)m.$$

36-§. Nuqta nisbiy harakatining tenglamalari va trayektoriyasini aniqlashga doir masalalar

1-masala. Yozib oluvchi moslamaning barabani ω_0 burchak tezlik bilan bir tekis aylanadi. Barabanning radiusi r. O'ziyozar, vertikal yo'nalishda $y = a \sin \omega_1 t$ qonun bilan harakatlanuvchi detall bilan birlashtirilgan. Qog'oz lentada pero yozib olgan egri chiziqning tenglamasi topilsin *(3.11-rasm).*



3.11-rasm

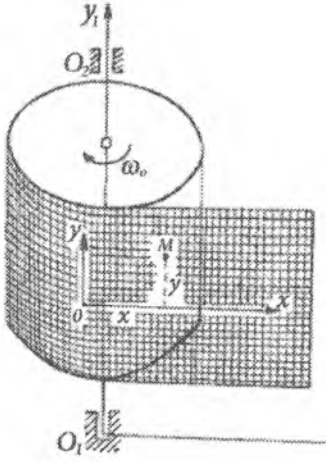
Yechish. Yozib oluvchi moslama uchun barabanning ω_0 burchak tezlik bilan aylanishi ko'chirma harakat hisoblanadi. O'ziyozar apparat perosining harakati nisbiy harakat sifatida qaraladi. Yer bilan bog'langan va shartli ravishda qo'zg'almas deb qabul qilingan $O_1x_1y_1$ koordinata o'qlari sistemasi qo'zg'almas – absolut sanoq sistemasini tashkil etadi. Aylanuvchi baraban bilan bog'langan Oxy koordinata o'qlari sistemasi qo'zg'aluvchan sanoq sistemasi sifatida qaraladi *(3.12-rasm).*

Faraz qilaylik, t vaqt onida o'ziyozar apparat perosi M holatda bo'lsin. Pero M holatining koordinatalari quyidagicha aniqlanadi:

$$x = vt = \omega_0 rt,$$

$$y = a\sin\omega_1 t.$$

Yozilgan tenglamalar o'ziyozar apparat perosi nisbiy harakatining parametrik tenglamalarini ifodalaydi. Pero nisbiy harakatining koordinatalar ko'rinishidagi tenglamasini yozish uchun yuqoridagi



3.12-rasm

tenglamalardan parametr-vaqtni qisqartiramiz. Natijada, pero nisbiy harakatining quyidagi ko'rinishdagi tenglamasiga ega bo'lamiz:

$$t = \frac{x}{\omega_0 r};\ y = a\sin\frac{\omega_1 x}{\omega_0 r}.$$

2-masala. Krivoship shatunli mexanizmda uzunligi *OA=r* bo'lgan krivoship *O* nuqtadan chizma tekisligiga perpendikular holda o'tuvchi o'q atrofida o'zgarmas ω_0 burchak tezlik bilan aylanadi, bunda $\varphi = \omega_0 t$.

Shatun uzunligi *AB*=l, *B* polzun *O* nuqtadan o'tuvchi gorizontal chiziq bo'ylab harakatlanadi. *B* polzunni moddiy nuqta sifatida qarab, uning nisbiy harakati tenglamasi tuzilsin *(3.13-rasm)*.

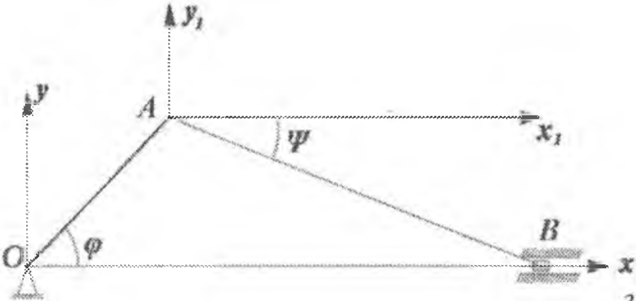
3.13-rasm

Yechish: *AB* shatun murakkab harakatini ikki sodda harakatlarga ajratamiz:

a) *O* nuqta atrofida ω_0 burchak tezlik bilan yuz beruvchi ko'chirma harakat;

b) A nuqta atrofida notekis yuz beruvchi aylanma harakat – nisbiy harakat.

Qo‘zg‘almas sanoq sistemasi sifatida Yer bilan bog‘langan O nuqtadan o‘tuvchi Oxy koordinata o‘qlari sistemasini tanlaymiz. Qo‘zg‘luvchan sanoq sistemasi sifatida Krivoship va shatun birlashadigan A nuqtadan o‘tuvchi Ax_1y_1 koordinata o‘qlari sistemasi olinadi *(3.14-rasm)*.



3.14-rasm

AB shatunning mazkur sanoq sistemasidagi holati $\Psi = \angle x_1AB = \angle ABO$ burchak orqali aniqlanadi. Ψ burchak qiymati sinuslar teoremasi orqali aniqlanadi:

$$\frac{r}{\sin\Psi} = \frac{l}{\sin\varphi},$$

bundan

$$\sin\Psi = \frac{r\sin\varphi}{l} = \frac{r\sin\omega_0 t}{l}$$

yoki

$$\Psi = \arcsin\left(\frac{r}{l}\sin\omega_0 t\right)\cdot rad.$$

Hosil bo‘lgan tenglama AB shatun nisbiy harakatining tenglamasini ifodalaydi.

B polzun nisbiy harakati tenglamalarini tuzish uchun uning nisbiy koordinatalari x_1y_1 larni yuqorida aniqlangan Ψ burchak qiymati orqali ifodalashimiz lozim:

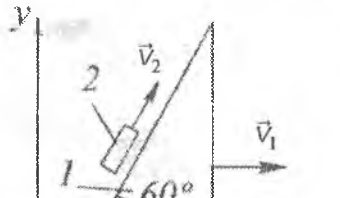
$$x_1 = l\cos\Psi = l\sqrt{1-\frac{r^2}{l^2}\sin^2\omega_0 t},$$

$$y_1 = -l\sin\Psi = -r\sin\varphi = -r\sin\omega_0 t(m).$$

37-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo. Platforma gorizontal yo'l bo'ylab 1 *m/s* tezlik bilan tekis harakatlanadi. Platforma ichidagi moddiy nuqta ham shu yo'nalish bo'yicha unga nisbatan s=0,5t qonun asosida siljisa, boshlang'ich paytda t=0 va x=0 deb, t=4s paytdagi nuqtaning x koordinatasini hisoblang.

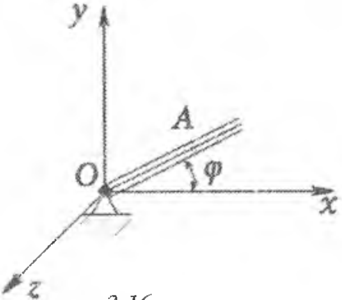
2-muammo. 1-jism o'zgarmas $v_1 = 2\ m/s$ tezlik bilan gorizontal tekislik bo'ylab harakat qiladi. Uning ustida esa 2-jism o'zgarmas $v_2 = 4\ m/s$ tezlik bilan yuqoriga ko'tarilmoqda. Agar boshlang'ich paytda t=0 s da x_2=0 bo'lsa, t=0,5s paytdagi 2-jismning x_2 koordinatasini aniqlang. 2-jism moddiy nuqta deb qaralsin *(3.15-rasm)*.



3.15-rasm

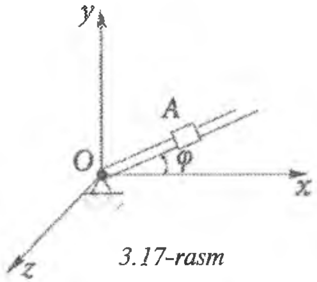
3-muammo. O_z o'qi atrofida $\varphi = 4t$ qonun bo'yicha aylanayotgan naycha ichidagi A sharcha $OA = 5t^2$ tenglama asosida harakat qilsa, t=0,25 s paytdagi A nuqtaning x_A koordinatasini toping *(3.16-rasm)*.

4-muammo. O_z o'qi atrofida φ=2t qonun bo'yicha aylanayotgan sterjen bo'ylab A polzun OA=3t^2 tenglama asosida harakat qilsa,



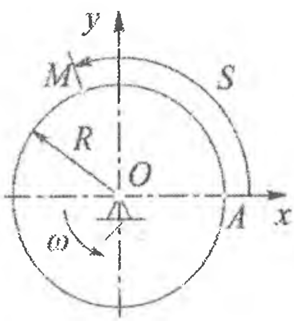
3.16-rasm

polzunning o'lchamlarini hisobga olmay, t=0,5s paytdagi uning y_A koordinatasini hisoblang *(3.17-rasm)*.



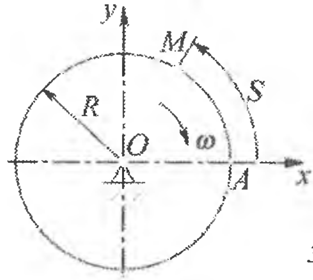
3.17-rasm

5-muammo. Radiusi R=0,5 m bo'lgan disk o'zgarmas burchak tezlik ω =2 *rad/s* bilan aylanadi. M nuqta esa diskning gardishi bo'ylab s =2t^2 tenglama asosida harakat qiladi. Agar boshlang'ich paytda M nuqta Ox o'qida bo'lgan bo'lsa, t =0,5 s paytdagi nuqtaning yoy koordinatasi s ni aniqlang *(3.18-rasm)*.



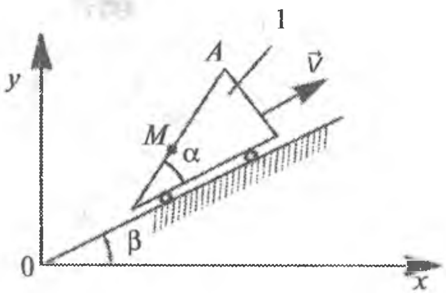
3.18-rasm

6-muammo. Radiusi R=0,5 *m* bo‘lgan disk o‘zgarmas burchak tezlik ω =2 *rad/s* bilan aylanadi. *M* nuqta esa diskning gardishi bo‘ylab $s=2t^2$ qonun asosida harakat qiladi. Agar boshlang‘ich paytda *M* nuqta *Ox* o‘qida bo‘lgan bo‘lsa, t=1*s* paytda nuqtaning yoy koordinatasi *s* ni aniqlang *(3.19-rasm).*



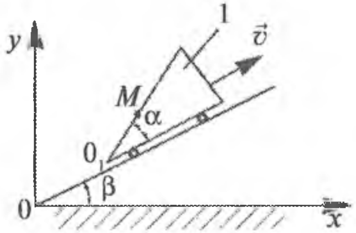
3.19-rasm

7-muammo. 1-jism o‘zgarmas $v_1 = 2\ m/s$ tezlik bilan qiya tekislik bo‘ylab yuqoriga ko‘tarilmoqda. Uning ustidagi *M* nuqta jismga nisbatan AM=0,5 t qonun bo‘yicha harakat qilsa, boshlang‘ich paytda, t=0 da, x_M=0 deb olib, t=2 *s* dagi nuqtaning x_M koordinatasini hisoblang. Bunda $\alpha = \beta = 30°$ deb oling *(3.20-rasm).*



3.20-rasm

8-muammo. 1-jism o‘zgarmas $v_1 = 2\ m/s$ tezlik bilan qiya tekislik bo‘ylab harakat qiladi. Uning ustida esa 2-jism o‘zgarmas $v_2 = 4\ m/s$ tezlik bilan yuqoriga ko‘tarilmoqda. Agar boshlang‘ich paytda, t=0 *s* da x_0=0 bo‘lsa, t=0,5 *s* paytdagi 2-jismning x_2 koordinatasini aniqlang. 2-jism moddiy nuqta deb qaralsin *(3.21-rasm).* Bunda $\alpha = \beta = 45°$ deb olinsin.



3.21-rasm

38-§. Nuqtaning nisbiy, ko‘chirma va absolut tezligini aniqlashga doir masalalarni yechish uchun uslubiy ko‘rsatmalar

Nuqtaning murakkab harakatida absolut tezligini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1) masala shartiga ko‘ra nuqtaning nisbiy, ko‘chirma va absolut harakatlari aniqlanadi;

2) qo‘zg‘almas va qo‘zg‘aluvchan sanoq sistemalari tanlab olinadi;

3) ko‘chirma harakat xayolan to‘xtatiladi va nuqtaning nisbiy harakat tezligi aniqlanadi;

4) nisbiy harakat xayolan to‘xtatiladi va nuqtaning ko‘chirma harakat tezligi aniqlanadi;

5) murakkab harakatda nuqtaning tezliklarini qo‘shish haqidagi teoremadan foydalanib, nuqtaning absolut tezligi aniqlanadi.

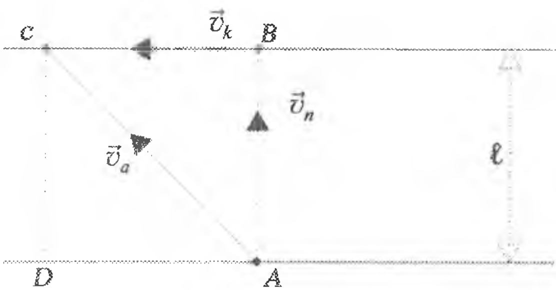
39-§. Murakkab harakatda nuqtaning nisbiy, ko‘chirma va absolut tezligini aniqlashga doir masalalar

1-masala. Daryo qirg‘oqlari parallel; qayiq *A* nuqtadan chiqib, qirg‘oqlarga tik kurs oldi va jo‘naganidan 10 daqiqa o‘tib, narigi qirg‘oqqa borib yetdi. Bunda u *A* nuqtadan daryoning oqimi bo‘ylab hisoblaganda 120 *m* pastdagi *C* nuqtaga keldi. Qayiq *A* nuqtadan chiqib, qirg‘oqqa tik bo‘lgan *AB* to‘g‘ri chiziqqa nisbatan qandaydir burchak ostida va oqimga qarshi kurs olishi kerak; bu holda qayiq narigi qirg‘oqqa, 12,5 daqiqada yetadi. Daryo kengligi *l*, qayiqning suvga nisbatan nisbiy tezligi *u* va daryo oqimining tezligi *v* aniqlansin.

Yechish: masalada qayiqning daryo oqimiga nisbatan harakati nisbiy harakat deyiladi. Daryoning qirg'oqqa nisbatan harakati (qirg'oq qo'zg'almas sanaladi) ko'chirma harakat sifatida qaraladi.

Qayiqning qirg'oqqa nisbatan harakati absolut harakat hisoblanadi.

a). Qayiqning *A* nuqtadan qirg'oqqa perpendikular yo'nalishdagi harakatini o'rganamiz. Bunda qayiq qarama-qarshi qirg'oqning *C* nuqtasiga borib yetadi *(3.22-rasm)*.



3.22-rasm

Bunday harakat t_1=10 minutda amalga oshadi.

Masalada qayiqning daryo oqimiga nisbatan harakatidagi tezligi nisbiy tezlik hisoblanadi va u $\vec{v}_n$ vektor orqali belgilanadi. Daryoning qayiq turgan nuqtasining qirg'oqqa nisbatan oqimi tezligi ko'chirma tezlik hisoblanadi va u $\vec{v}_k$ vektor orqali belgilanadi. *3.22-rasmdan*

$$v_k = \frac{AD}{t_1} = \frac{120}{10} = 12 \ m/min.$$

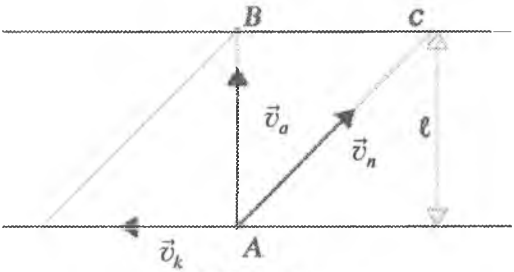
Bunda daryo kengligi quyidagi formula asosida aniqlanadi:

$$l = v_n \cdot t_1.$$

Bundan

$$v_n = \frac{l}{t_1}. \qquad (3.17)$$

b). Qayiqning *AB* tog'ri chiziqqa ma'lum burchak ostida daryo oqimiga qarshi yo'nalishdagi harakatini o'rganamiz.



3.23-rasm

Bunda qayiq qarama-qarshi qirg'oqqa t_2=12,5 minut vaqt o'tgach yetadi. *3.23-rasmdan:*

$$l = v_a \cdot t_2.$$

Bundan

$$v_a = \frac{l}{t_2}. \qquad (3.18)$$

ABC uchburchakdan

$$v_n^2 = v_a^2 + v_k^2. \qquad (3.19)$$

(3.17) va (3.18) ifodalarni (3.19) ifodaga qo'ysak:

$$\frac{l^2}{t_1^2} = \frac{l^2}{t_2^2} + v_k^2$$

yoki

$$\frac{l^2}{10^2} = \frac{l^2}{(12,5)^2} + 144.$$

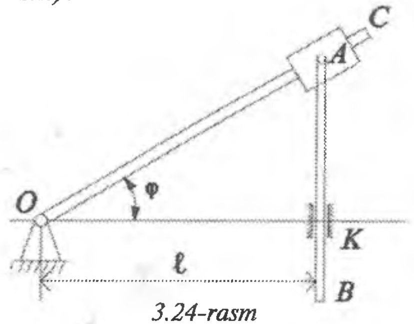
Hosil bo'lgan ifodadan daryo kengligi aniqlanadi:

$$l = \sqrt{\frac{144}{0,0036}} = 200\ m.$$

Bunday holda qayiqning daryo oqimiga nisbatan tezligi quyidagicha aniqlanadi:

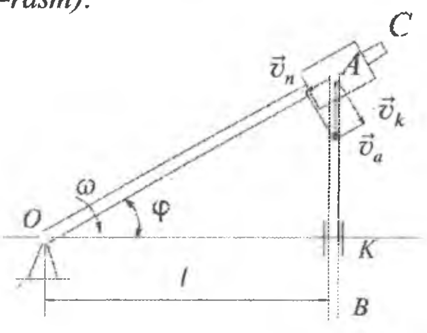
$$\vec{v}_n = \frac{l}{t_1} = \frac{200}{10} = 20\ m/s.$$

2-masala. Kulisali mexanizmda OC krivoshipning rasm tekisligiga perpendikular bo'lgan O o'q atrofida tebranishi natijasida, A polzun OC krivoship bo'ylab surilib, vertikal k yo'naltiruvchilarda harakatlanuvchi AB sterjenni harakatga keltiradi. Masofa $OK=l$ A polzunning OC krivoshipga nisbatan harakatidagi tezligi, krivoshipning burchak tezligi ω aylanish burchagi φ funksiayasi sifatida aniqlansin *(3.24-rasm).*



3.24-rasm

Yechish: masalada A polzun uchun OC krivoshipning chizma tekisligiga perpendikular holda O nuqtadan o'tuvchi o'q atrofidagi tebranishi ko'chirma harakat hisoblanadi. Polzunning OC krivoshipga nisbatan harakati esa nisbiy harakat deb qaraladi. OC krivoshipning qaralayotgan vaqt momentida A polzun bilan ustma-ust tushuvchi nuqtaning tezligi A polzun uchun ko'chirma tezlik hisoblanadi *(3.25-rasm).*



3.25-rasm

Shuning uchun

$$v_k = \omega \cdot OA.$$

$\vec{v}_k$ vektor OC krivoshipga perpendikular holda krivoshipning A nuqtasidan uning aylanishi tomon yo'naladi.

3.25-rasmdan

$$OA = \frac{l}{\cos\varphi}.$$

Shuning uchun

$$v_k = \frac{\omega l}{\cos\varphi}. \tag{3.20}$$

A polzunning nisbiy tezligi $\vec{v}_n$ OC krivoship bo'ylab yo'naladi.

Mexanizmning A nuqtasida tezliklar parallelogrammini chizamiz, tezliklar parallelogrammidan

$$v_n = v_k \cdot \mathrm{tg}\varphi. \tag{3.21}$$

(1)ni (2)ga qo'ysak, A polzunning nisbiy tezligi uchun quyidagi ifodaga ega bo'lamiz:

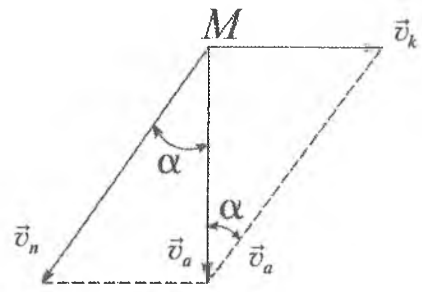
$$v_n = \frac{\omega l}{\cos\varphi} \cdot \mathrm{tg}\varphi.$$

3-masala. Gorizontal yo'lda 72 *km/soat* tezlik bilan borayotgan avtomobildagi passajir kabinaning yon oynasiga tushgan yomg'ir tomchisining vertikalga nisbatan 40° ga teng burchakka og'gan trayektoriyasini kuzatadi. Vertikal tushayotgan yomg'ir tomchisining absolut tezligi aniqlansin. Tomchi bilan oyna orasidagi ishqalanish hisobga olinmasin.

Yechish: avtomobilning gorizontal yo'ldagi harakati passajir uchun ko'chirma harakat hisoblanadi. Yomg'ir tomchisining avtomobil oynasida vertikalga nisbatan 40° burchakka og'gan trayektoriyasi nisbiy harakatni ifodalaydi. Vertikal tushayotgan yomg'ir tomchisining harakati absolut harakat hisoblanadi. Murakkab harakatda tezliklarni qo'shish teoremasiga asosan yomg'ir tomchisining absolut tezligi uning nisbiy va ko'chirma tezliklarining geometrik yig'indisiga teng bo'ladi:

$$\vec{v}_a = \vec{v}_n + \vec{v}_k.$$

Ko'chirma, nisbiy va absolut tezliklarning yo'nalishini bilgan holda tezliklar parallelogrammini chizamiz *(3.26-rasm).*



3.26-rasm

Chizilgan parallelogrammdan

$$\frac{v_k}{v_a} = \text{tg}40°.$$

Agar

$$v_k = 72\frac{km}{soat} = 20\ m/s$$

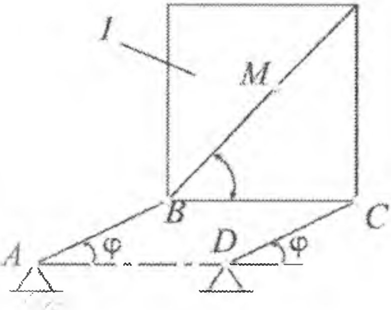
ekanligini e'tiborga olsak,

$$v_a = \frac{v_k}{\text{tg}40^0} = \frac{20\ m/s}{0{,}839} = 23{,}8\ m/s.$$

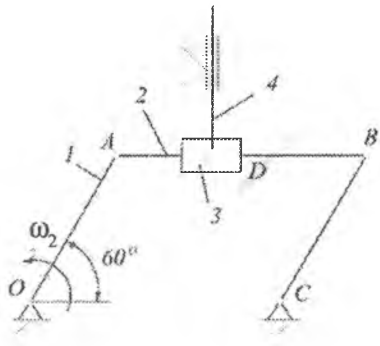
40-§. Murakkab harakatda nuqtaning nisbiy, ko'chirma va absolut tezligini aniqlashga doir mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo. O'zaro teng krivoshiplar *(AB=DC=0,5 m)*, $\varphi = 0{,}25\pi t$ qonun bo'yicha aylanadi. Krivoshiplarga o'rnatilgan kvadrat plastina diagonali bo'ylab harakatlanayotgan *M* nuqtaning tenglamasi $BM=0{,}1\ t^2$ bo'lsa, *t*=1 *s* paytdagi *M* nuqtaning absolut tezligini aniqlang *(3.27-rasm).*

2-muammo. *OABC* sharnirli parallelogrammning (2) shatuni bo'ylab (3) halqasimon polzun (vtulka) harakat qiladi. O'z o'rnida (3) polzun (4) sterjenni harakatga keltiradi. Mexanizmning berilgan holati uchun 1 krivoship *A* nuqtasining tezligini 2 *m/s* deb olib, (4) sterjenning tezligini toping *(3.28-rasm).*

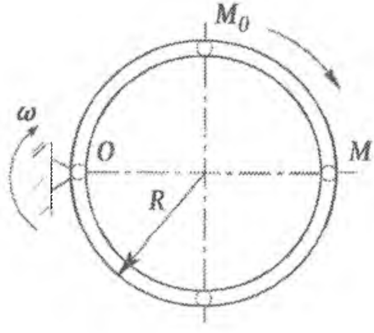


3.27-rasm



3.28-rasm

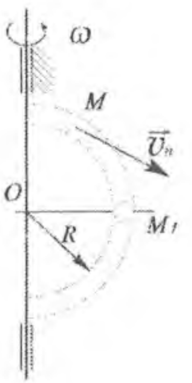
3-muammo. Radiusi $R=0,1$ *m* bo'lgan halqa shakl tekisligida *O* nuqta atrofida o'zgarmas $\omega=4$ *rad/s* burchak tezlik bilan aylanadi. Halqadagi *M* shar esa $M_0M=0,1t$ qonun bo'yicha nisbiy harakat qilsa, ko'rsatilgan holat uchun *M* sharning absolut tezligini toping *(3.29-rasm).*



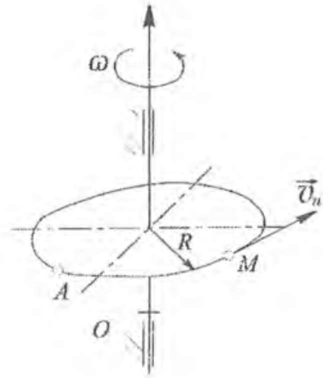
3.29-rasm

4-muammo. Radiusi $R=1$ *m* bo'lgan yarim doira shaklidagi naycha $\omega=3$ *rad/s* burchak tezlik bilan aylanadi. Naycha ichidagi *M* sharcha o'zgarmas nisbiy tezlik $v_n=3\ m/s$ bilan harakatlansa, *M* sharchaning M_1 holatga kelgan paytdagi absolut tezligini aniqlang *(3.30-rasm).*

5-muammo. Radiusi $R=1$ *m* bo'lgan disk O_z o'qi atrofida $\varphi=4\sin 3t$ qonun bilan aylanadi. *M* nuqta esa diskning gardishi bo'ylab $AM=0,66\sin 6t+4$ tenglama bo'yicha harakatlanadi. Vaqtning $t=0,35$ *s* paytda *M* nuqtaning absolut tezligini toping *(3.31-rasm).*



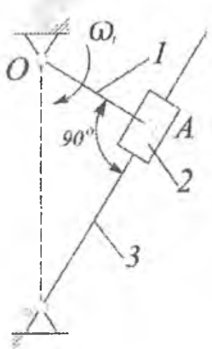
3.30-rasm



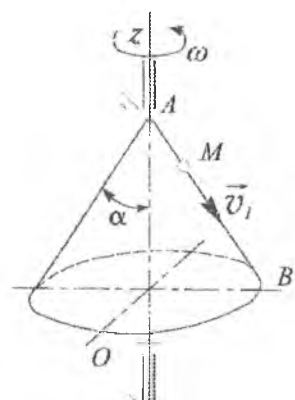
3.31-rasm

6-muammo. Uzunligi OA=0,1 *m* bo'lgan 1 krivoship O o'q atrofida ω_1 =5 *rad/s* burchak tezlik bilan aylanadi. Shaklda ko'rsatilgan holat uchun (2) polzunning (3) kulisaga nisbatan tezligini aniqlang *(3.32-rasm).*

7-muammo. Konussimon jism O_z o'qi atrofida ω =3 *rad/s* burchak tezlik bilan aylanadi. Uning yasovchisi bo'ylab M nuqta o'zgarmas $v_1 = 4\ m/s$ tezlikka ega bo'lgan holda A dan B ga qarab harakatlanadi. Agar $\alpha = 30°$ bo'lsa, M nuqta AM=2 *m* yo'l bosgan paytdagi uning absolut tezligini toping *(3.33-rasm).*

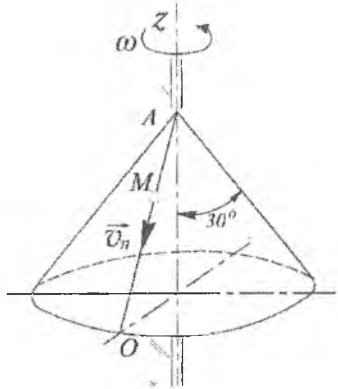


3.32-rasm



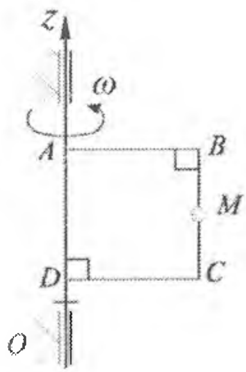
3.33-rasm

8-muammo. Konussimon jism $\varphi=4\sin 0{,}4t$ qonun bo‘yicha O_z o‘qi atrofida aylanadi. Uning yasovchisi bo‘ylab $AM=2\,t$ tenglama asosida harakatlanayotgan M nuqtaning $t=2\ s$ paytdagi ko‘chirma tezligi miqdorini hisoblang *(3.34-rasm)*.



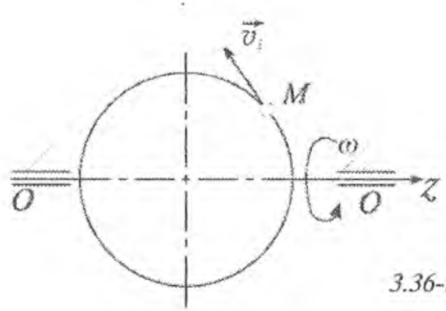
3.34-rasm

9-muammo. To‘g‘ri to‘rtburchak shaklidagi $ABCD$ plastina O_z o‘qi atrofida $\omega=4t$ burchak tezlik bilan aylanadi. Uning BC tomoni bo‘ylab M nuqta o‘zgarmas 9 m/s tezlik bilan B dan C da tomon harakatlandi. $T=3\ s$ da nuqtaning absolut tezlik miqdorini toping. Bunda $AB=1\ m$ deb oling *(3.35-rasm)*.



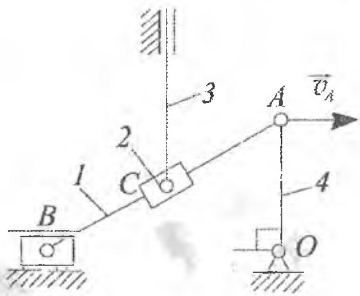
3.35-rasm

10-muammo. Disk O_z o‘qi atrofida aylanadi. Uning gardishi bo‘ylab M nuqta o‘zgarmas nisbiy tezlik $v_1=9\ m/s$ bilan harakatlanadi. M nuqtaning absolut tezligi 15 m/s bo‘lgan paytda uning ko‘chirma tezligini toping *(3.36-rasm)*.



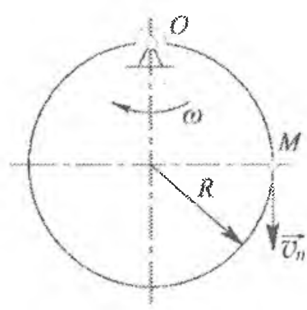
3.36-rasm

11-muammo. Krivoship-polzunli mexanizmning 1 shatuniga 2 halqasimon polzun (vtulka)o'rnatilgan bo'lib, u o'z navbatida 3 sterjenni harakatga keltiradi. Agar o'lchamlar $OA=0,5AB$ bo'lsa, mexanizmning berilgan holati uchun krivoship A nuqtasining tezligini $v_A = 3\ m/s$ hisoblab, 3 sterjenning tezligini toping *(3.37-rasm).*



3.37-rasm

12-muammo. Radiusi $R=0,1\ m$ bo'lgan disk O nuqta atrofida $\varphi = 0,4t$ qonun asosida aylanadi. Diskning gardishi bo'ylab M nuqta $OM=0,3t$ tenglama bilan harakatlansa, uning absolut tezligini aniqlang *(3.38-rasm).*



3.38-rasm

41-§. Murakkab harakatda ko'chirma harakat ilgarilanma harakat bo'lgan hol uchun nuqtaning absolut tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Murakkab harakatda ko'chirma harakat ilgarilanma harakatdan iborat bo'lsa, nuqtaning absolut tezlanishi nisbiy va ko'chirma tezlanishlarining geometrik yig'indisidan iborat bo'ladi:

$$\vec{a}_a = \vec{a}_n + \vec{a}_k \qquad (3.22)$$

yoki

$$\vec{a}_a = \vec{a}_n^n + a_n^\tau + \vec{a}_k^n + \vec{a}_k^\tau . \qquad (3.23)$$

Bu ifodada:

$\vec{a}_n^n$ va $\vec{a}_n^\tau$ – nuqtaning nisbiy harakatida markazga intilma va aylanma tezlanishlar.

$\vec{a}_k^k$ va $\vec{a}_k^\tau$ – ko'chirma harakatda nuqtaning normal va urinma tezlanishlari.

Agar murakkab harakatda nuqtaning nisbiy va ko'chirma harakatlari to'g'ri chiziqli harakatlardan iborat bo'lsa, nuqtaning nisbiy markazga intilma va ko'chirma normal tezlanishlari nolga teng bo'ladi.

Agar murakkab harakatda nuqtaning nisbiy va ko'chirma harakatlari egri chiziqli tekis harakatlardan iborat bo'lsa, nuqtaning nisbiy aylanma va ko'chirma urinma tezlanishlari nolga teng bo'ladi.

Mavzuga doir masalalarni ikki usulda yechish tavsiya etiladi: geometrik va analitik usullar.

a) masalalarni geometrik usulda yechishda tanlangan masshtabda tezlanishlar parallelogrammi yoki ko'p burchagi chiziladi.

b) masalalarni analitik usulda yechishda proyeksiyalar metodidan foydalanish tavsiya etiladi.

Buning uchun koordinata o'qlari o'tkaziladi va (3.23) tenglamani chap va o'ng tomonlari tanlab olingan koordinata o'qlariga proyeksiyalanadi:

$$(a_a)_x = (a_n^n)_x + (a_n^\tau)_x + (a_k^n)_x + (a_k^\tau)_x ,$$

$$(a_a)_y = (a_n^n)_y + (a_n^\tau)_y + (a_k^n)_y + (a_k^\tau)_y,$$

$$(a_a)_z = (a_n^n)_z + (a_n^\tau)_z + (a_k^n)_z + (a_k^\tau)_z.$$

Bunda absolut tezlanishning moduli

$$a_a = \sqrt{(a_a)_x^2 + (a_a)_y^2 + (a_a)_z^2}$$

formula yordamida, yo'nalishi esa

$$\cos(\vec{a}_a \wedge x) = \frac{(a_a)_x}{a_a},$$

$$\cos(\vec{a}_a \wedge y) = \frac{(a_a)_y}{a_a},$$

$$\cos(\vec{a}_a \wedge z) = \frac{(a_a)_z}{a_a}$$

formulalar asosida aniqlanadi.

Mavzuga doir masalalarni quyidagi tartibda yechish maqsadga muvofiq bo'ladi:

1) masala shartidan nuqtaning nisbiy, ko'chirma va absolut harakatlari aniqlab olinadi;

2) qo'zg'almas va qo'zg'aluvchan koordinata o'qlari sistemasi tanlab olinadi;

3) ko'chirma harakat xayolan to'xtatilib, nuqtaning nisbiy tezligi va nisbiy tezlanishi aniqlab olinadi;

4) nisbiy harakat xayolan to'xtatilib, nuqtaning ko'chirma harakat tezligi va tezlanishi aniqlab olinadi;

5) masalani geometrik usulda yechishda tezlanishlar parallelogrammi yoki ko'p burchagi chiziladi va ulardan noma'lum tezlanish aniqlanadi;

6) masalani analitik usulda yechishda proyeksiyalar usulidan foydalanish tavsiya etiladi, ya'ni absolut tezlanishning o'qlardagi proyeksiyalari aniqlanadi;

7) absolut tezlanishning o'qlardagi proyeksiyalariga ko'ra, uning moduli va yo'nalishi topiladi.

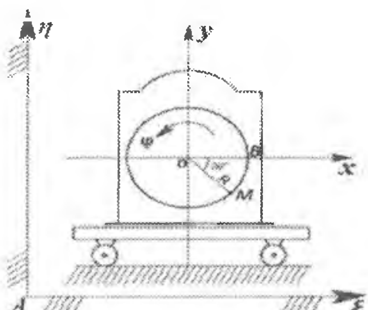
42-§. Ko'chirma harakat ilgarilanma harakatdan iborat bo'lganda nuqtaning absolut tezlanishini aniqlashga doir masalalar

1-masala.

O'ng tomonga gorizontal yo'nalishda $x_k = t^3 + 4t$ m qonunga muvofiq harakat qiluvchi aravachaga elektr motori o'rnatilgan. Uning rotori harakatga keltirish vaqtida $\varphi = t^2$ tenglamaga muvofiq aylanadi, bunda φ burchak radianlarda o'lchanadi. Rotor gardishidagi *M* nuqtaning $t = 1$ *s* bo'lgandagi absolut tezligi va absolut tezlanishi aniqlansin. Rotorning radiusi 0,2 *m* ga teng. Shu paytda *M* nuqta rasmda ko'rsatilgan holda turadi *(3.39-a rasm).*

Yechish:

rasmda ko'rsatilgan $A\xi\eta$ o'qlar sistemasi qo'zg'almas sanoq sistemasini, aravacha bilan bog'langan va u bilan birga harakatlanuvchi *Oxy* o'qlar sistemasi qo'zg'aluvchan sanoq sistemasini tashkil etadi.



3.39-a rasm

Rotor gardishidagi *M* nuqtaning motor korpusi aravachaga bog'langan *Oxy* sanoq sistemasiga nisbatan harakati nisbiy, rotorning qo'zg'aluvchan *O, x, y* sanoq sistemasi bilan birgalikda qo'zg'almas $A\xi\eta$ sanoq sistemasiga nisbatan harakati *M* nuqta uchun ko'chirma va *M* nuqtaning bevosita qo'zg'almas $A\xi\eta$ sanoq sistemasiga nisbatan harakati murakkab harakat hisoblanadi.

M nuqtaning absolut tezligini nuqtaning murakkab harakatida tezliklarni qo'shish teoremasiga asosan aniqlaymiz.

Teoremaga ko'ra:

$$\vec{v}_a = \vec{v}_n + \vec{v}_k. \qquad (3.24)$$

Nisbiy tezlikning moduli

$$v_n = R \cdot \omega_n, \qquad (3.25)$$

bu yerda R — rotorning radiusi,

ω_n — rotor burchak tezligining moduli:

$$\omega_n = [\tilde{\omega}_n], \ \tilde{\omega}_n = \frac{d\varphi_n}{dt} = 2t.$$

$t = 1$ sekundda

$$\tilde{\omega}_n = 2 \ rad/s, \ \omega_n = 2 \ rad/s.$$

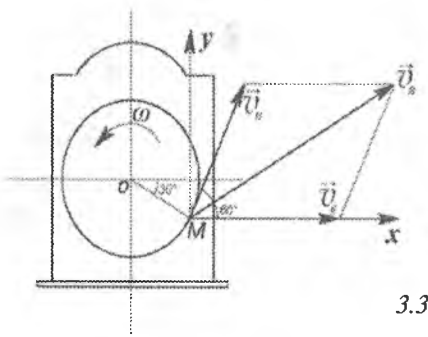
$\tilde{\omega}_n$ kattalikning oldidagi musbat ishora rotorning aylanishi φ burchakning o'sish tomoniga qarab ro'y berishini ko'rsatadi.

Nisbiy tezlikning moduli (3.25) formula asosida aniqlanadi:

$$v_n = 0{,}2 \cdot 2 = 0{,}4 \ m/s.$$

$\vec{v}_n$ vektor, M nuqta nisbiy harakatda chizgan aylanaga urinma bo'ylab, rotorning aylanish tomoniga qarab yo'naladi *(3.39-b rasm)*.

M nuqtaning ko'chirma tezligi qaralayotgan vaqt momentida motor korpusi aravachaning M nuqta bilan ustma-ust tushuvchi nuqtasining tezligiga teng bo'ladi:



3.39-b rasm

$$v_k = |\tilde{x}_k'| = |3t^2 + 4|. \qquad (3.26)$$

$t = 1$ sekundda

$$v_k = \tilde{x}_k' = 7 \ sm/s, \ x_k' = 7 \ sm/s.$$

Demak, $v_k = 7\ sm/s$.

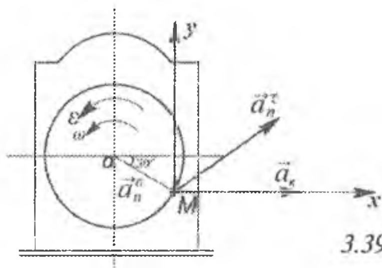
$\vec{v}_k$ vektor, $\dot{x}_k'$ kattalik oldidagi ishora musbat bo'lganligi uchun, x ning o'sish tomoniga, ya'ni aravachaning harakat yo'nalishi tomon yo'naladi *(3.39-a rasm).*

M nuqtaning absolut tezligi uning nisbiy va ko'chirma harakat tezliklaridan ko'rilgan parallelogrammning diagonali orqali ifodalanadi *(3.39-c rasm).*

Uning moduli:

$$v_a = \sqrt{v_n^2 + v_k^2 + 2v_n v_k \cos 60°} = 7{,}21\ sm/s.$$

M nuqtaning absolut tezlanishini nuqtaning murakkab harakatida tezlanishlarni qo'shish teoremasidan aniqlaymiz.



3.39-c rasm

Ko'chirma harakat ilgarilanma harakat bo'lganligi uchun

$$\vec{a}_a = \vec{a}_n + \vec{a}_k \qquad (3.27)$$

yoki yoyilgan ko'rinishda

$$\vec{a}_a = \vec{a}_n^\tau + \vec{a}_n^n + \vec{a}_k. \qquad (3.28)$$

Nisbiy urinma tezlanishning moduli:

$$a_n^\tau = R\varepsilon_n, \qquad (3.29)$$

bu yerda $\varepsilon_n = |\vec{\varepsilon}_n|$ — rotor burchak tezlanishining moduli.

$$\tilde{\varepsilon}_n = \frac{d^2\varphi_n}{at^2} = 2\frac{rad}{s^2},\ \varepsilon_n = \frac{2\ rad}{s^2},$$

$\vec{\varepsilon}_n$ va $\vec{\omega}_n$ larning ishoralari bir xil. Demak, $\vec{a}_n^\tau$ va $\vec{v}_n$ vektorlar bir xil yo'nalishga ega bo'ladi *(3.39-b, c rasmlar).*

Nisbiy urinma tezlanish moduli:

$$a_n^\tau = 0{,}2 \cdot 2 = 0{,}4 \ sm/s^2.$$

Nisbiy normal tezlanishning moduli:

$$a_n^n = R\omega_n^2 = 0{,}2 \cdot 4 = 0{,}8 \ sm/s^2.$$

$\vec{a}_n^n$ vektor rotor M nuqtasining nisbiy harakatda chizgan aylanasining markazi O nuqta tomon yo'naladi *(3.39-c rasm).*

M nuqtaning ko'chirma tezlanishi qaralayotgan vaqt momentida motor korpusi — aravachaning M nuqtasi bilan ustma-ust tushuvchi nuqtasining tezlanishiga teng bo'ladi:

$$ak = |\ddot{x}_k''| = |6t|.$$

$t = 1$ sekundda

$$\ddot{x}_k'' = 6sm/s^2, \ x_k'' = 6 \ sm/s^2.$$

Demak, $a_k = 6 \ sm/s^2$.

$\bar{x}'$ va $\bar{x}''$ kattaliklarning ishoralari bir xil bo'lganligi uchun $\vec{v}_k$ va $\vec{a}_k$ vektorlarning yo'nalishlari ustma-ust tushadi *(3.39-a, c rasmlar).*

M nuqta absolut tezlanishining modulini proyeksiyalash usuli yordamida topamiz:

$$(a_a)_x = a_k - a_n^n \cos 30° + a_n^\tau \cos 60° = 5{,}52 \ sm/s^2,$$

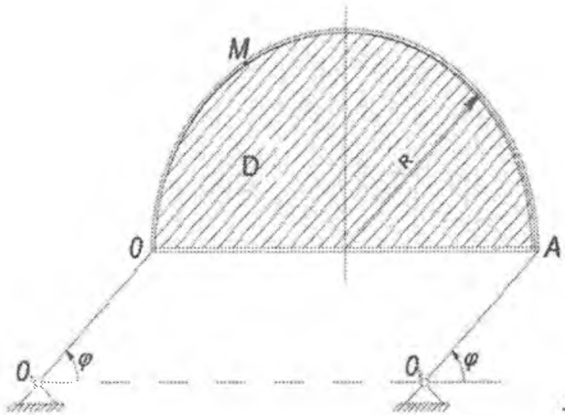
$$(a_a)_y = a_n^n \cos 60° + a_n^\tau \cos 30° = 0{,}74 sm/s^2,$$

$$a_a = \sqrt{(a_a)_x^2 + (a_a)_y^2} = 5{,}6 \ sm/s^2.$$

Hisob natijalari jadvalda ko'rsatilgan.

$\vec{\omega}_n$, rad/s^2	Tezlik, sm/s			$\bar{\varepsilon}_n$ rad/s^2	Tezlanish, sm/s²					
	v_n	v_k	v_a		a_n^τ	a_n^n	a_k	$(a_a)_x$	$(a_a)_y$	a_a
2	0,4	7	7,21	2	0,4	0,8	6	5,52	0,74	5,6

2-masala. M nuqta D jismga nisbatan $OM = s_n = 6\pi t^2$ tenglama bo'yicha harakatlanadi. D jism O_1OAO_2 sharnirli to'rt zvenolikka mahkamlangan. To‘rt zvenolikning O_1O va O_2A sterjenlari O_1 va O_2 nuqtalar atrofida $\varphi = \frac{\pi t^3}{6}$ qonunga muvofiq aylanadi. M nuqtaning $t = t_1$ vaqt onidagi absolut tezligi va absolut tezlanishi aniqlansin *(3.40-a rasm)*.



3.40-a rasm

Masalada:

$$OM = s_n = 6\pi t^2 (sm),$$

$$\varphi = \frac{\pi t^3}{6}(rad),$$

$$t_1 = 1\ s,$$

$$R = 18\ sm,$$

$$O_1O = O_2A = 20\ sm.$$

Yechimi: masalada to‘rt zvenolikning O_1O va O_2A sterjenlari O_1 va O_2 sharnirlar atrofida aylanadi, OA sterjen esa ilgarilanma harakatda bo‘ladi. Yarim doira ham OA sterjenga mahkamlanganligi tufayli ilgarilanma harakatda bo‘ladi. M nuqta uchun D yarim doiraning harakati ko‘chirma harakat hisoblanadi.

Shuning uchun masalada ko‘chirma harakat ilgarilanma harakat bo‘ladi. M nuqtaning D jismga nisbatan harakati esa nisbiy harakat hisoblanadi.

Berilgan vaqt momentida *M* nuqtaning *D* jismdagi o‘rni $\alpha = \frac{S_n}{R}$ burchak orqali aniqlanadi:

$t_1 = 1\ s$ da

$$\alpha = \frac{6\pi t_1^2}{18} = \frac{\pi}{3} = 60°.$$

D jismning tekislikdagi holati φ burchak orqali aniqlanadi:

$t_1 = 1\ s$ da

$$\varphi = \frac{\pi t_1^3}{6} = \frac{\pi}{6} = 30°.$$

Nuqtaning murakkab harakatida tezliklarni qo‘shish haqidagi teoremaga asosan *M* nuqtaning absolut tezligi uning nisbiy va ko‘chirma harakat tezliklarining geometrik yig‘indisiga teng bo‘ladi:

$$\vec{v}_a = \vec{v}_n + \vec{v}_k.$$

Nisbiy tezlikning miqdorini aniqlaymiz:

$$v_n = s_n' = (6\pi t^2) = 12\pi t,$$

$t_1 = 1\ s$ da

$$v_n = 12 \cdot \pi \cdot 1 = 37{,}68\ sm/s.$$

Ilgarilanma harakatdagi jismning barcha nuqtalari bir xil trayektoriya bo‘ylab harakatlanadi va har onda miqdor va yo‘nalishlari bir xil bo‘lgan tezlik va tezlanishga ega bo‘ladi. Shuning uchun *M* nuqtaning ko‘chirma tezligi *O* nuqtaning ko‘chirma tezligiga teng bo‘ladi:

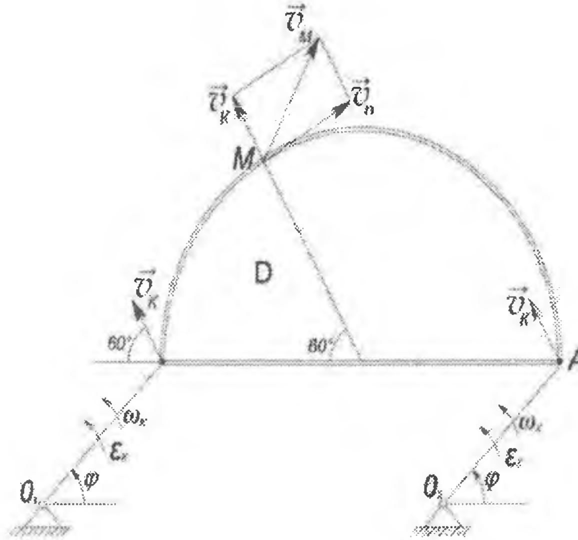
$$v_K = v_O = \omega \cdot 0_1 0_1.$$

Bunda:

$$\omega_K = \varphi' = \left(\frac{\pi t^3}{6}\right)' = \frac{\pi t^2}{2},$$

$t_1 = 1\ s$ da

$$\omega_k = 1{,}57\ rad/s.$$



3.40-b rasm

Binobarin,

$$v_k = \omega \cdot O_1O = 1{,}57 \cdot 20 = 3{,}14 \ sm/s.$$

$\vec{v}_n = \vec{v}_K$ vektorlar o'zaro perpendikular yo'nalgan *(3.40-b rasm)*. Shuning uchun *M* nuqta absolut tezligining miqdori quyidagicha aniqlanadi:

$$v_M = v_a = \sqrt{v_n^2 + v_K^2} = \sqrt{(37{,}68)^2 + (3{,}14)^2} = 49{,}05 \ sm/s.$$

M nuqtaning absolut tezlanishini aniqlaymiz. Ko'chirma harakat ilgarilanma harakat bo'lganligi uchun kariolis tezlanishi $\vec{a}_c = 2(\vec{\omega}_k \times \vec{v}_n) = 0.$

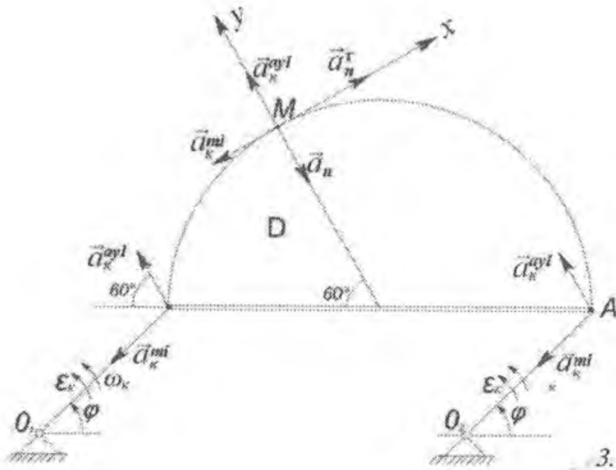
Shuning uchun

$$\vec{a}_a = \vec{a}_n + \vec{a}_k = \vec{a}_n^n + \vec{a}_n^\tau + \vec{a}_k^{mi} + \vec{a}_k^{ayl}.$$

M nuqta nisbiy tezlanishlarining miqdorlari:

$$a_n^n = \frac{v_n^2}{R} = \frac{(37{,}68)^2}{18} = 78{,}88 \ sm/s^2,$$

$$a_n^\tau = \frac{dv_n}{dt} = 12\pi = 37{,}68 \ sm/s^2.$$



3.40-c rasm

M nuqtaning ko'chirma tezlanishlarining miqdorlari:

$$a_k^{mi} = \omega^2 \cdot O_1O = (1{,}57)^2 \cdot 20 = 49{,}30 \ sm/s^2;$$
$$a_k^{ayl} = \varepsilon \cdot O_1O.$$

Bunda: $\varepsilon = \varepsilon_k = \dfrac{d\omega_k}{dt}$.

$t = 1 \ s$ da

$$\varepsilon_k = \frac{d\omega_k}{dt} = \pi t = 3{,}14 \ rad/s^2.$$

Shuning uchun

$$a_k^{ayl} = 3{,}14 \cdot 20 = 62{,}8 \ sm/s^2.$$

M nuqtaning nisbiy va ko'chirma tezlanishlari *3.40-c rasmda* ko'rsatilgan. *M* nuqtaning absolut tezlanishining miqdorini proyeksiyalash usulidan foydalanib aniqlaymiz:

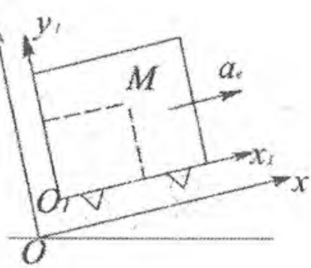
$$(a_a)_x = a_n^{\tau} - a_k^{mi} = 37{,}68 - 49{,}30 = -11{,}62,$$

$$(a_a)_y = -a_n^n + a_k^{ayl} = -78{,}88 + 62{,}8 = -16{,}08,$$

$$a = \sqrt{(a_M)_X^2 = (a_M)_Y^2} = 19{,}84 \ sm/s^2$$

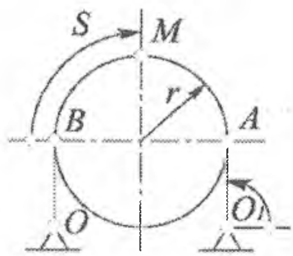
43-§. Talabalarga mustaqil yechish uchun tavsiya etiladigan muammolar

1-muammo. Arava qiya tekislikda $a_k = 2m/s^2$ tezlanish bilan harakat qiladi. Aravadagi *M* nuqta esa shakl tekisligida $x_1=3t^2$ va $y_1=4t^2$ tenglamalar bo'yicha harakatlanadi. Nuqtaning absolut tezlanishini toping *(3.41-rasm).*



3.41-rasm

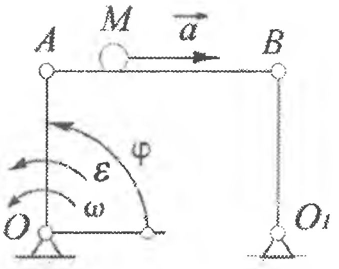
2-muammo. O_1A zveno $\varphi=2t$ qonun bilan aylanib, radiusi $r=0{,}5$m li diskni harakatga keltiradi. Diskning gardishi bo'ylab esa *M* nuqta $s=2rt$ tenglama asosida aylanadi. Nuqtaning $t=0{,}25p$ paytdagi absolut tezlanishi miqdorini toping *(3.42-rasm).*



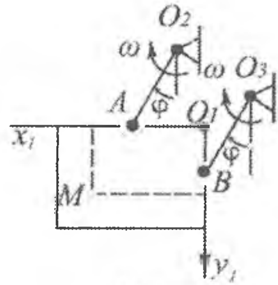
3.42-rasm

3-muammo. Uzunligi *OA*=0,1 *m* bo'lgan sterjen ω = 4 *rad/s* burchak tezlik va ε = 0,4 *rad/s²* burchak tezlanish bilan aylanib, $OABO_1$ sharnirli parallelogrammni harakatga keltiradi. *M* nuqta *AB* sterjen bo'ylab *a* =0,4 *m/s²* tezlanish bilan harakat qiladi. *M* nuqtaning absolut tezlanish modulini $\varphi=(0{,}5\ \pi)$ holat uchun aniqlang *(3.43-rasm).*

4-muammo. To'rtburchak shakldagi plastina uzunliklari $AO_2=BO_3=1$ *m* bo'lgan krivoshiplar yordamida harakatga keltiriladi.



3.43-rasm

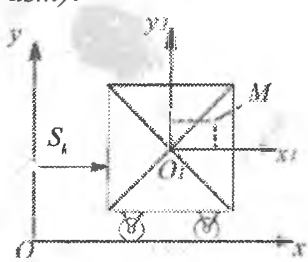


3.44-rasm

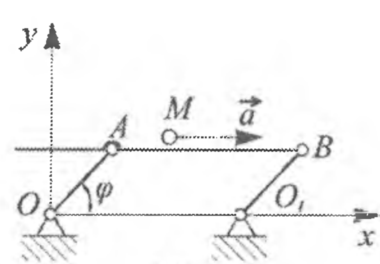
M nuqta esa plastina bo'ylab $x_1 = 0{,}2t^3$ va $y_1 = 0{,}3t^2$ tenglama asosida harakatlanadi. Agar krivoshiplar o'zgarmas $\omega = 2\pi$ burchak tezlik bilan aylansa, $t = 1$ *s* paytda $\varphi = 30°$ holat uchun M nuqtaning absolut tezlanishini hisoblang *(3.44-rasm).*

5-muammo. Arava gorizontal yo'lda $s_k = 0{,}5t^3$ qonun bilan harakatlanadi. Aravadagi M nuqta esa vertikal shakl tekisligida $x_1 = 0{,}3\,t$ va $y_1 = 0{,}1\,t^2$ tenglamalar asosida harakat qiladi. $t = 1$ *s* paytdagi nuqtaning absolut tezlanishini toping *(3.45-rasm).*

6-muammo. Uzunligi $OA = 2$ *m* bo'lgan sterjen $\varphi = t$ qonun bilan aylanib, $OABO_1$ sharnirli parallelogrammni harakatga keltiradi. M nuqta AB sterjen bo'ylab esa a =cost tezlanish bilan harakat qilsa, M nuqtaning $t = \pi$ paytdagi absolut tezlanish miqdorini toping *(3.46-rasm).*

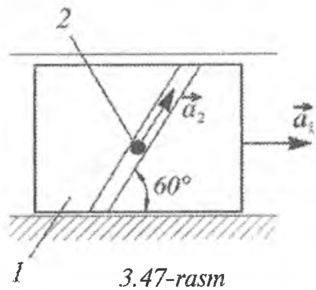


3.45-rasm



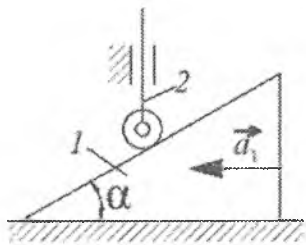
3.46-rasm

7-muammo. Polzun gorizontal yo'l bo'ylab o'zgarmas $a_1 = 4$ m/s^2 tezlanish bilan harakat qiladi. M nuqta polzunga nisbatan $a_2 = 3$ m/s^2 tezlanish bilan harakat qilsa, M nuqtaning absolut tezlanishini toping *(3.47-rasm).*



3.47-rasm

8-muammo. Uchburchak prizma 1 gorizontal tekislik bo'ylab a_1=0,6 m/s^2 tezlanish bilan harakatlanadi. Uchiga g'ildirak o'rnatilgan 2 sterjen esa vertikal yuqoriga siljiydi. Agar α=30° bo'lsa, 2 sterjenning tezlanishini aniqlang *(3.48-rasm).*

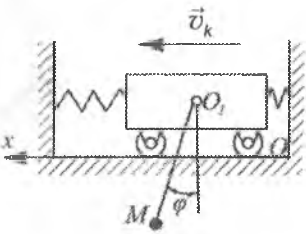


3.48-rasm

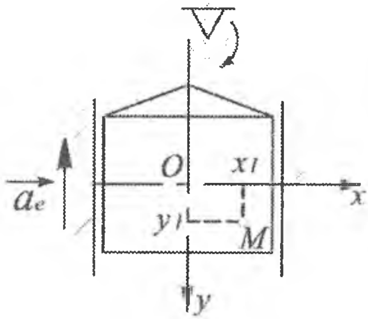
9-muammo. *M* nuqtaning nisbiy harakat tenglamasi $x_n = 3t$ va $y_n = 2\sin t$, ko'chirma harakat tenglamasi esa $x_k = 2t$ va $y_k = 2\cos t$ bo'lsa, uning t=0 paytdagi absolut tezlanishini hisoblang. Bunda absolut va nisbiy koordinata o'qlarini o'zaro parallel deb oling.

10-muammo. Arava gorizontal yo'lda $v_k = \sin(\pi/3)t$ tezlik bilan harakatlanadi. Uning markaziga mahkamlangan uzunligi O_1M=1 *m* li mayatnik φ=0,5πt qonun bo'yicha tebranadi. Vaqtning t=0,5 *s* da *M* nuqtaning absolut tezlanishini toping *(3.49-rasm).*

11-muammo. Lift kabinasi a_k=5 m/s^2 o'zgarmas tezlanish bilan yuqoriga ko'tariladi. Uning ichida shakl tekisligi bo'ylab *M* nuqta x_1=0,5t^2 va y_1=0,3t^2 qonun bo'yicha harakat qiladi. Nuqtaning absolut tezlanishini toping *(3.50-rasm).*



3.49-rasm



3.50-rasm

44-§. Koriolis tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko‘rsatmalar

Kariolis tezlanishini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1. Masala shartiga ko‘ra murakkab harakatdagi nuqtaning nisbiy harakati tenglamasi aniqlanadi.
2. Nuqtaning nisbiy harakat tenglamasiga ko‘ra uning nisbiy tezligining miqdori va yo‘nalishi aniqlanadi.
3. Masala shartiga ko‘ra ko‘chirma harakat tenglamasi aniqlanadi.
4. Ko‘chirma harakat tenglamasiga ko‘ra nuqta ko‘chirma harakatining burchak tezligining miqdori va yo‘nalish aniqlanadi.
5. Nuqtaning aniqlangan nisbiy tezligi va ko‘chirma harakati burchak tezligining miqdori va yo‘nalishiga asosan uning Koriolis tezlanishi aniqlanadi.

45-§. Murakkab harakatda nuqtaning Koriolis tezlanishini aniqlashga doir masalalar

1-masala. Eni 500 m bo‘lgan daryo janubdan shimolga qarab 1,5 *m/s* tezlik bilan oqadi. 60° shimoliy kenglikda *M* suv zarrasining a_c Koriolis tezlanishi aniqlansin. Keyin suv daryoning qaysi qirg‘og‘ida ekanligi va qancha baland ekanligi aniqlansin; suv sathi, Koriolis tezlanishiga teng va unga qarama-qarshi yo‘nalgan vektor bilan og'irlik kuchining tezlanishi **g** vektorning vektor yig'indisiga teng bo'lgan vektor yo'nalishiga perpendikular.

Yechish: daryo suv zarrasining Koriolis tezlanishi quyidagi formula asosida aniqlanadi:

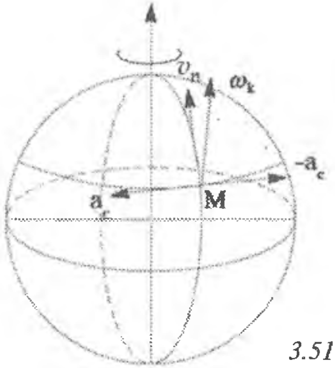
$$\vec{a}_c = 2(\vec{\omega}_k \times {}^\wedge \vec{v}_n).$$

Uning moduli esa

$$a_c = 2\omega_k v_n \sin(\vec{\omega}_k \, ^\wedge \, v_n)$$

formula yordamida hisoblanadi.

Agar $v_n = 1{,}5\ m/s$ $\omega_k = \dfrac{2\pi}{24 \cdot 60 \cdot 60} = 0{,}000073\dfrac{1}{s}$; $\sin(\vec{\omega}_k \, ^\wedge \, \vec{v}_n) = $ $= \sin 60° = 0{,}87$ ekanligini e'tiborga olsak, daryo suv zarrasining Koriolis tezlanishi quyidagi miqdorga teng bo'ladi:



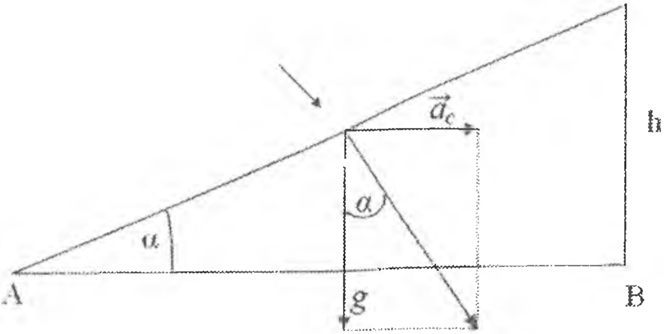
3.51-rasm

$$a_c = 2 \cdot 0{,}000073 \cdot 1{,}5 \cdot 0{,}87 = 1{,}89 \cdot 10^{-4}\, m/s^2.$$

Yerning shimoliy yarim sharida, Yer aylanishi tufayli, Yer sirtida harakatlanayotgan har qanday jism o'ng tomonga og'adi. Binobarin, suv sathi daryoning o'ng qirg'og'ida baland bo'ladi.

Daryo suvining o'ng qirg'og'i qancha baland bo'lishini aniqlash uchun suv sathi, Koriolis tezlanishiga teng va unga qarama-qarshi yo'nalgan vektor bilan og'irlik kuchining tezlanishi $\vec{g}$ vektorning geometrik yig'indisiga teng bo'lgan vektor yo'nalishiga perpendikular bo'lishini e'tiborga olamiz.

Koriolis tezlanishining yo'nalishi *3.51-rasmda* ko'rsatilgan.



3.52-rasm

3.52-rasmdan $h = AB \cdot \text{tg}\alpha$; ikkinchi tomondan $\text{tg}\alpha = \dfrac{a_c}{g}$.

Shuning uchun

$$h = 500 \cdot \text{tg}\alpha = 500 \cdot \frac{1{,}89 \cdot 10^{-4}}{9{,}81} = 0{,}0096\ m.$$

Demak, suv sathi o'ng qirg'oqda h=0,0096 m baland bo'lar ekan.

2-masala. Meridian bo'yicha harakatlanuvchi elektrovoz ekvatorni kesib o'tayotgan paytda uning g'ildiragidagi M_1, M_2, M_3 va M_4 nuqtalarning Koriolis tezlanishlari aniqlansin. Elektrovoz g'ildiragi markazining tezligi $v_0 = 40\ m/s$.

Yechish: elektrovoz g'ildiragi Yerning meridiani bo'ylab harakatlanib, ekvatorni kesib o'tadi. Elektrovoz g'ildiragining rasmdagi holatida M_1 nuqta g'ildirak nuqtalarining Yer sirtidagi nisbiy harakati tezliklarining oniy markazi hisoblanadi *(3.53-rasm).*

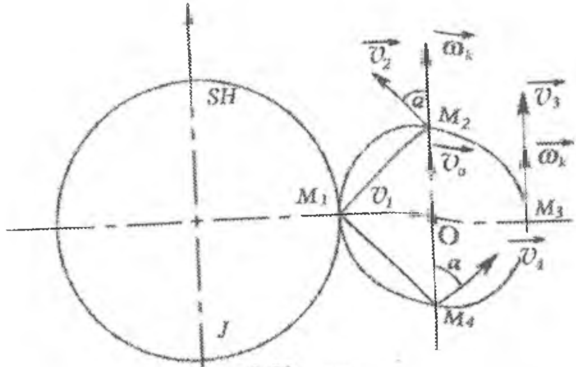
Rasmdan $M_1M_2 = M_1M_4 = r\sqrt{2}$, bunda r – elektrovoz g'ildiragining radiusi.

Elektovoz g'ildiragining oniy burchak tezligini aniqlaymiz:

$$\omega = \frac{v_0}{r} = \frac{v_2}{r\sqrt{2}} = \frac{v_4}{r\sqrt{2}}.$$

Bundan $v_2 = v_4 = v_0\sqrt{2}\ m/s$,

v_2 va v_4 – g'ildirak 2 va 4 nuqtalarining nisbiy tezliklari.



3.53-rasm

Yerning aylanishi elektrovoz uchun ko'chirma harakat hisoblanadi.

Uning burchak tezligi

$$\omega_k = \frac{2\pi}{24 \cdot 60 \cdot 60} = 0{,}000073\frac{1}{s}.$$

Ma'lumki, elektrovoz g'ildiragi nuqtalarining Koriolis tezlanishi quyidagi formula asosida aniqlanadi:

$$a_c = 2 \cdot \omega_k \cdot v_n \sin\alpha.$$

Bu ifodada $\alpha - \vec{\omega}_k$ va $\vec{v}_n$ vektorlar orasidagi burchak.

Elektrovoz g'ildiragi nuqtalarning Koriolis tezlanishlarini aniqlaymiz:

$$a_{M_1} = 2 \cdot \omega_k \cdot v_1 \sin\alpha = 0, \text{ chunki } v_1 = 0;$$

$$a_{M_2} = 2 \cdot \omega_k \cdot v_2 \sin 45 = 2 \cdot 0{,}000073 \cdot 40 \cdot \sqrt{2} \cdot 0{,}71 = = 5{,}81 \cdot 10^{-3} m/s^2;$$

$$a_{M_3} = 2 \cdot \omega_k \cdot v_3 \sin 0 = 0, \text{ chunki } \sin 0^\circ = 0;$$

$$a_{M_4} = a_{M_2} = 5{,}81 \cdot 10^{-3} m/s^2.$$

3-masala. Shimoliy kenglik paralleli bo'ylab o'tkazilgan temir yo'lda teplovoz g'arbdan sharqqa qarab $v_n = 20\ m/s$ tezlik bilan harakat qiladi. Teplovozning Koriolis tezlanishi a_c topilsin.

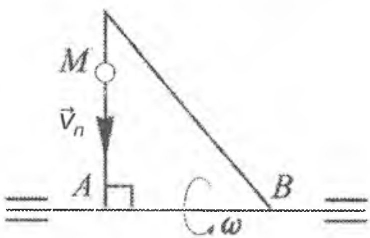
Yechish: masala shartiga ko'ra, shimoliy kenglik paralleli bo'ylab o'tkazilgan temir yo'lda teplovoz g'arbdan sharqqa qarab $v_n = 20\ m/s$ tezlik bilan harakat qiladi. Yerning aylanishi teplovoz uchun ko'chirma harakat hisoblanadi. Uning burchak tezlik vektori $\vec{\omega}_k$ yerning aylanish o'qi bo'ylab yuqoriga yo'nalgan. Shuning uchun $\vec{\omega}_k \perp \vec{v}_n$. Natijada, teplovozning Koriolis tezlanishi quyidagiga teng bo'ladi:

$$a_c = 2 \cdot \omega_k \cdot v_n = 2 \cdot 0{,}000073 \cdot 20 = 2{,}91 \cdot 10^{-3}\, m/s.$$

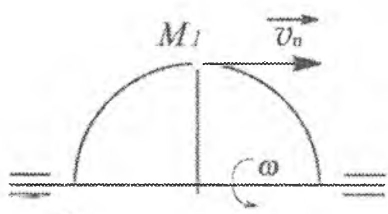
46-§. Talabalarga mustaqil yechish uchun tavsiya etiladigan muammolar

1-muammo. Uchburchak shaklidagi jism AB tomoni atrofida ω = 8 *rad/s* burchak tezlik bilan aylanadi. M nuqta esa uchburchakning AB ga perpendikular tomoni bo'ylab $v_n = 4\ m/s$ nisbiy tezlik bilan harakat qiladi. M nuqtaning Koriolis tezlanishini toping *(3.54-rasm).*

2-muammo. Yarim doira shaklidagi jism ω = 4 *rad/s* burchak tezlik bilan aylanadi. M nuqta esa uning yoyi bo'ylab $\vec{v}_n$ tezlik bilan harakat qilsa, M nuqtaning Koriolis tezlanishini toping *(3.55-rasm).*

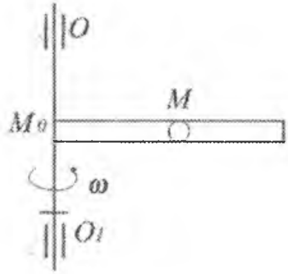


3.54-rasm



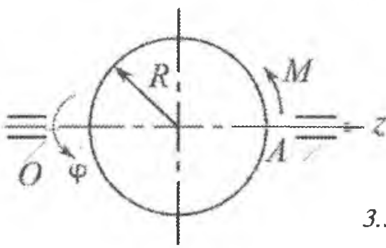
3.55-rasm

3-muammo. Naycha $O\ O_1$ o'q atrofida ω = 1,5 *rad/s* burchak tezlik bilan ayanadi. Uning ichida M nuqta $M_0M = 4t$ qonun bo'yicha harakat qilsa, nuqtaning Koriolis tezlanishini aniqlang *(3.56-rasm).*



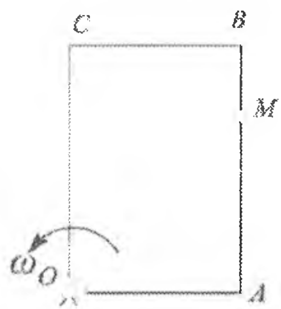
3.56-rasm

4-muammo. Radiusi R=0,4 *m* bo‘lgan disk O_z o‘qi atrofida $\varphi = 4\sin 0,25\pi t$ qonun bo‘yicha aylanadi. Uning gardishi bo‘ylab *M* nuqta $AM = 0,25\pi R t^2$ tenglama bilan harakatlansa, t=1 *s* paytda nuqtaning Koriolis tezlanishini toping *(3.57-rasm)*.



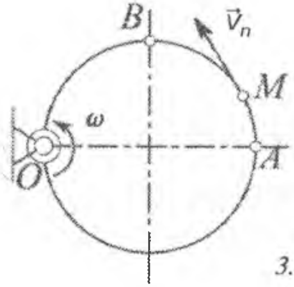
3.57-rasm

5-muammo. To‘rtburchak shaklidagi plastina shakl tekisligida *O* nuqta atrofida aylanadi. *M* nuqta *AB* qirrasi bo‘ylab *AM*=3sin (π/3)*t* qonun bo‘yicha harakatlanadi. Agar *t*=2 *sek* da *M* nuqtaning Koriolis tezlanishi $4\pi(m/s)$ bo‘lsa, plastinaning ω_k ko‘chirma burchak tezligini toping *(3.58-rasm)*.



3.58-rasm

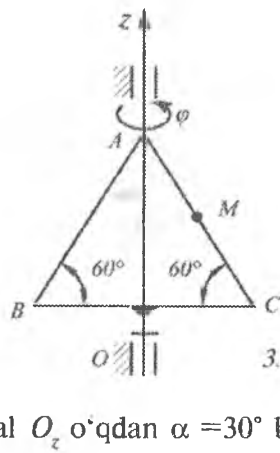
6-muammo. Shakl tekisligida $\omega = 2$ *rad/s* burchak tezlik bilan aylanuvchi diskning gardishi bo'ylab *M* nuqta $v_n = 0,2\ m/s$ nisbiy tezlik bilan harakat qiladi. *M* nuqta *A* holatdan *B* holatga o'tgan bo'lsa, uning Koriolis tezlanishining miqdori o'zgaradimi? *(3.59-rasm).*



3.59-rasm

7-muammo. Teng tomonli uchburchak shaklidagi *ABC* jism O_z o'qi atrofida $\varphi = 5t^2$ qonun bo'yicha aylanadi.

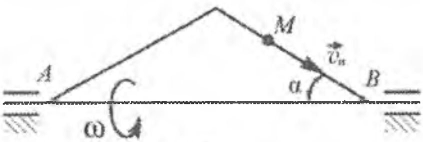
Agar *M* nuqta $AM=4t^2$ tenglamaga asosan harakatlansa, t=0,5 *s* paytdagi nuqtaning Koriolis tezlanishini toping *(3.60-rasm).*



3.60-rasm

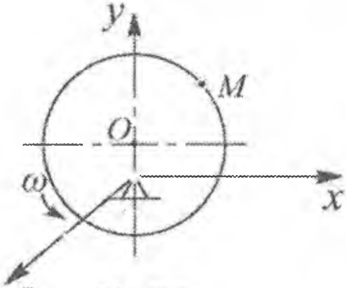
8-muammo. Vertikal O_z o'qdan $\alpha = 30°$ burchak ostidagi sterjen, shu o'q atrofida ω_k=4 *rad / s* burchak tezlik bilan aylanadi. *M* nuqta esa sterjen bo'ylab, koordinata boshidan $v_n = 2\ m/s$ tezlik bilan yuqoriga ko'tariladi. Serjen O_{yz} tekisligida joylashgan paytda *M* nuqtaning Koriolis tezlanishining *Ox* o'qidagi proyeksiyasini toping.

9-muammo. Uchburchak shaklidagi jism *AB* tomoni atrofida $\omega = 4\ rad/s$ burchak tezlik bilan aylanadi. *M* nuqta esa uchburchakning tomoni bo'ylab $v_n = 2\ m/s$ tezlik bilan harakatlanadi. Agar $\alpha = 30°$ bo'lsa, *M* nuqtaning Koriolis tezlanishini aniqlang *(3.61-rasm).*



3.61-rasm

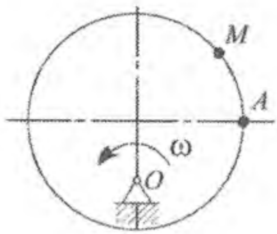
10-muammo. Ekssentrikka ega bo'lgan disk O_z o'qi atrofida tinch holatdan tekis tezlanuvchan ω =4 *rad/s* burchak tezlik bilan aylanadi. Uning gardishi bo'ylab *M* nuqta 0,1 *m/s* tezlik bilan harakatlansa, *t*=3 *s* paytdagi nuqtaning Koriolis tezlanishini aniqlang *(3.62-rasm).*



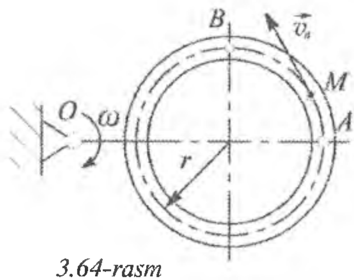
3.62-rasm

11-muammo. Ekssentrikka ega bo'lgan disk shakl tekisligida tekis aylanadi. Uning gardishi bo'ylab *M* nuqta $AM = 4t^2$ qonun bilan harakat qiladi. *t*=1 *s* paytda *M* nuqta Koriolis tezlanishi 24 m/s^2 ga teng bo'lishi uchun disk qanday o'zgarmas burchak tezlikka ega bo'ladi? *(3.63-rasm).*

12-muammo. Radiusi *r*=0,5 m bo'lgan halqa shakl tekisligida O_z o'q atrofida $\omega = const$ burchak tezlik bilan aylanadi. *M* nuqta esa halqa bo'ylab v_n = const nisbiy tezlik bilan harakat qiladi. Agar nuqta *A* holatdan *B* holatga o'tsa, uning Koriolis tezlanishining miqdori o'zgaradimi? *(3.64-rasm).*



3.63-rasm



3.64-rasm

47-§. Ko'chirma harakat ilgarilanma harakat bo'lmagan holda nuqtaning absolut tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Nuqtaning murakkab harakatida uning absolut tezlanishini aniqlashga doir masalalarni yechishda ko'chirma harakatning ko'rinishi muhim ahamiyat kasb etadi.

Agar nuqtaning murakkab harakatida ko'chirma harakat ilgarilanma harakat bo'lmasa, ya'ni qo'zg'aluvchi koordinatalar sistemasining berilgan ondagi burchak tezligi ma'lum bo'lsa, nuqtaning absolut tezlanishi uning nisbiy, ko'chirma va Koriolis tezlanishlarining geometrik yig'indisidan iborat bo'ladi:

$$\vec{a}_a = \vec{a}_n + \vec{a}_k + \vec{a}_c = \vec{a}_n + \vec{a}_k + 2(\vec{\omega}_k \times {}^\wedge v_n).$$

Murakkab harakatda ko'chirma harakat ilgarilanma harakat bo'lmagan holda nuqtaning absolut tezlanishini aniqlashda quyidagi tartibga rioya etish tavsiya etiladi:

1) masala shartiga ko'ra nuqtaning nisbiy, ko'chirma va absolut harakatlari aniqlab olinadi;

2) qo'zg'almas va qo'zg'aluvchan sanoq sistemalari tanlab olinadi;

3) ko'chirma harakat xayolan to'xtatilib, nuqtaning nisbiy tezlanishi aniqlanadi;

4) nisbiy harakat xayolan to'xtatilib, nuqtaning ko'chirma tezlanishi aniqlanadi;

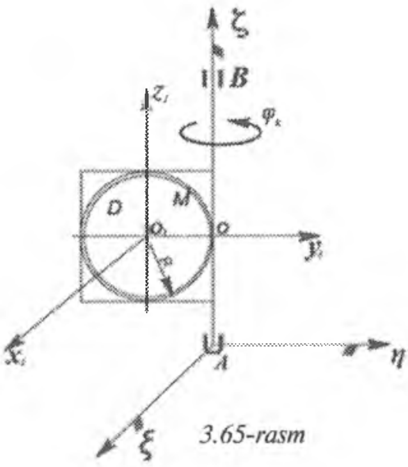
5) nuqtaning nisbiy tezligi va ko'chirma harakatning burchak tezliklarini bilgan holda, nuqtaning Koriolis tezlanishi aniqlanadi;

6) Koriolis teoremasiga asosan nuqtaning absolut tezlanishi aniqlanadi.

48-§. Ko'chirma harakat ilgarilanma harakat bo'lmagan hol uchun nuqtaning absolut tezlanishini aniqlashga doir masalalar

1-masala.

To'g'ri burchakli ramka AB qo'zg'almas o'q atrofida $\varphi_k = 3t - 0,5t^3$ rad qonun bo'yicha aylanadi. M nuqta to'g'ri burchakli ramkaga nisbatan unda chizilgan radiusi R=40 sm bo'lgan aylana bo'ylab O nuqtadan $OM = s_n = 40\pi \cos\frac{\pi t}{3}\, sm$ qonun bo'yicha harakatlanadi. M nuqtaning $t = 1$ sekunddagi absolut tezligi va absolut tezlanishi topilsin *(3.65-rasm)*.



3.65-rasm

Yechish:

1. M nuqtaning absolut tezligini aniqlash.

Berilgan vaqt onida chizma tekisligi to'g'ri burchakli ramkaning tekisligi bilan ustma-ust tushadi deb faraz qilinadi.

Shaklda ko'rsatilgan $A\xi\eta\zeta$ o'qlar sistemasi qo'zg'almas sanoq sistemasini, to'g'ri burchakli ramka bilan bog'langan va u bilan birga aylanuvchi $O_1x_1y_1z_1$ o'qlar sistemasi qo'zg'aluvchan sanoq sistemasini tashkil etadi.

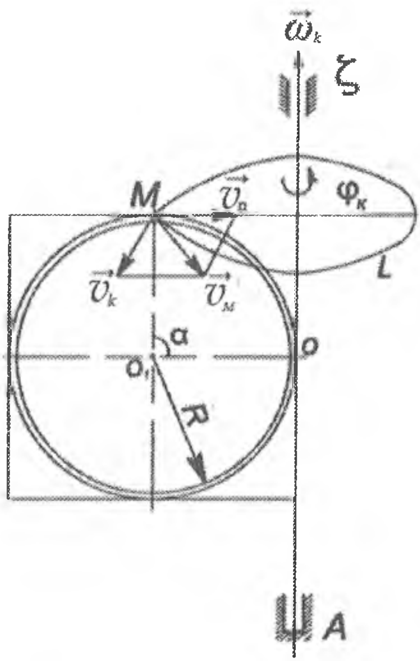
M nuqtaning to'g'ri burchakli ramka bilan bog'langan $O_1x_1y_1z_1$ sanoq sistemasiga nisbatan harakati nisbiy, to'g'ri burchakli ramkaning va u bilan bog‘langan $O_1x_1y_1z_1$ sanoq sistemasining qo‘zg‘almas $A\xi\eta\zeta$ sanoq sistemasiga nisbatan harakati ko‘chirma va nuqtaning qo‘zg‘almas $A\xi\eta\zeta$ sanoq sistemasiga nisbatan harakati absolut harakat hisoblanadi.

M nuqtaning to‘g‘ri burchakli ramkada chizilgan aylanadagi holatini uning aylana bo‘ylab harakat qonunidan foydalanib, quyidagi α burchak orqali aniqlaymiz:

$$\alpha = \frac{s_n}{R} = \frac{40\pi\cos\frac{\pi t}{3}}{40};$$

t=1 s da $\alpha = 90°$.

M nuqtaning absolut tezligini nuqataning murakkab harakatida tezliklarni qo‘shish haqidagi teoremaga asosan, nisbiy va ko‘chirma tezliklarning geometrik yig‘indisi kabi aniqlaymiz *(3.66-rasm):*



3.66-rasm

$$\vec{v}_a = \vec{v}_n + \vec{v}_k. \qquad (3.30)$$

Nisbiy tezlikning moduli:

$$v_n = |\tilde{v}_n|, \qquad (3.31)$$

bu yerda

$$\tilde{v}_n = \frac{ds_n}{at} = -\frac{40\pi^2}{3}\sin\frac{\pi t}{3}.$$

$t=1$ sekundda

$$\tilde{v}_n = \frac{-40 \cdot (3,14)^2}{3} \cdot 0,86 = -113,06\ sm/s,$$

$$v_n = 113,06\ sm/s.$$

$\tilde{v}_n$ kattalikning oldidagi manfiy ishora *M* nuqtaning nisbiy tezligi s_n ning kamayish tomoniga qarab aylanaga urinma holda yo'nalishini bildiradi *(3.66-rasm)*.

M nuqtaning ko'chirma tezligini aniqlaymiz.

Ko'chirma tezlikning moduli:

$$v_k = R_k\omega_k. \qquad (3.32)$$

Bu ifodada R_k to'g'ri burchakli ramkaning qaralayotgan vaqt onida *M* nuqta bilan ustma-ust tushuvchi nuqtasi tomonidan, $A\xi$ o'q atrofida chizadigan *L* aylanasining radiusi, $R_k = R = 40$ *sm.*

ω_k – to'g'ri burchakli ramka burchak tezligining moduli:

$$\omega_k = |\tilde{\omega}_k|, \quad \tilde{\omega}_k = \frac{d\varphi_k}{at} = 3 - 1,5t^2.$$

$t=1$ sekundda,

$$\tilde{\omega}_k = 1,5\ rad/s, \quad \omega_k = 1,5\ rad/s.$$

$\vec{\omega}_k$ kattalikning musbat ishorasi to'g'ri burchakli ramkaning $A\xi$ o'q atrofidagi aylanishi ω_k burchakning o'sish tomoniga ro'y berishini ko'rsatadi. $\vec{\omega}_k$ ko'chirma tezlikning moduli (3.33) formula bo'yicha hisoblanadi:

$$v_k = 40 \cdot 1,5 = 60\ sm/s.$$

$\vec{v}_k$ vektor L aylanaga urinma bo'ylab, to'g'ri burchakli ramkaning aylanish tomoniga qarab yo'nalgan.

$\vec{v}_k$ va $\vec{v}_n$ vektorlar o'zaro perpendikular bo'lgani uchun M nuqta absolut tezligining moduli *(3.66-rasm)*:

$$v_a = \sqrt{v_n^2 + v_k^2} = 128\ sm/s.$$

2. M nuqtaning absolut tezlanishini aniqlaymiz.

M nuqtaning absolut tezlanishini nuqtaning murakkab harakatida tezlanishlarni qo'shish teoremasidan aniqlaymiz. Masalada, ko'chirma harakat ilgarilanma bo'lmagan murakkab harakat bo'lganligi uchun absolut tezlanish nisbiy, ko'chirma va Koriolis tezlanishlarining geometrik yig'indisiga teng:

$$\vec{a}_a = \vec{a}_M = \vec{a}_n + \vec{a}_k + \vec{a}_c \tag{3.34}$$

yoki yoyilgan ko'rinishda

$$\vec{a}_a = \vec{a}_M = \vec{a}_n^\tau + \vec{a}_n^n + \vec{a}_k^{ayl} + \vec{a}_k^{mi} + \vec{a}_c. \tag{3.35}$$

Nisbiy urinma tezlanishning moduli:

$$a_n^\tau = \left|\vec{a}_n^\tau\right|, \tag{3.36}$$

bu ifodada

$$\tilde{a}_n^\tau = \frac{d^2 s_n}{dt^2} = -\frac{40\pi^2}{9}\cos\frac{\pi t}{3}.$$

t=1 sekundda

$$\tilde{a}_n^\tau = -21,91\ sm/s^2,$$

$$a_n^\tau = 21,91\ sm/s^2.$$

$\vec{a}_n^\tau$ ning manfiy ishorasi $\vec{a}_n^\tau$ vektorning s_n ning kamayish tomoniga qarab yo'nalganligini ko'rsatadi. $\vec{a}_n^\tau$ va $\vec{v}_n$ ishoralari bir xil. Demak, $\vec{a}_n^\tau$ va $\vec{v}_n$ vektorlari bir xil yo'nalishga ega.

Nisbiy normal tezlanish:

$$a_n^n = \frac{v_n^2}{R} = 319,56\ sm/s^2.$$

$\vec{a}_n^n$ vektor M nuqtadan O_1 nuqta tomon yo'nalgan *(3.67-rasm).* Ko'chirma aylanma tezlanishning moduli *(3.67-rasm):*

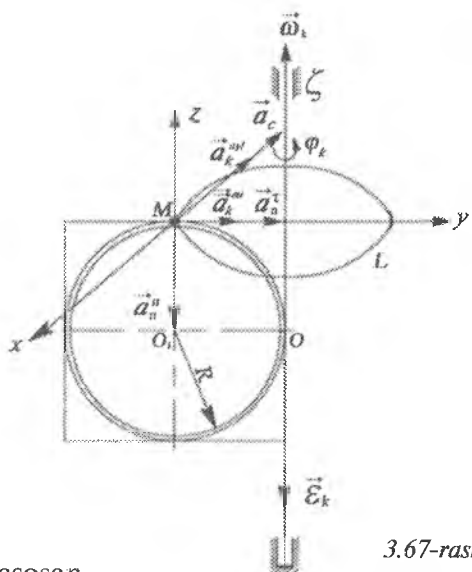
$$a_k^{ayl} = R_k \varepsilon_k. \qquad (3.37)$$

Bu ifodada $\varepsilon_k = |\tilde{\varepsilon}_k|$ — to'g'ri burchakli ramkaning burchak tezlanishining algebraik qiymati:

$$\tilde{\varepsilon}_k = \frac{d^2\varphi_k}{dt^2} = -3t. \qquad (3.38)$$

t=1 sekundda,

$\tilde{\varepsilon}_k = -3\ rad/s^2$, $\varepsilon_k = 3 rad/s^2$. $\tilde{\varepsilon}_k$ va $\tilde{\omega}_k$ larning ishoralari har xil. Demak, to'g'ri burchakli ramkaning aylanishi sekinlanuvchan, $\vec{\omega}_k$ va $\vec{\varepsilon}_k$ vektorlarning yo'nalishlari qarama-qarshi bo'ladi *(3.67-rasm).*



3.67-rasm

(3.37)ga asosan,

$$a_k^{ayl} = 40 \cdot 3 = 120\ sm/s^2.$$

$\vec{a}_k^{ayl}$ va $\vec{v}_k$ vektorlar qarama-qarshi tomonlarga yo'nalgan *(3.66-3.67-rasmlar).*

Ko‘chirma markazga intilma-normal tezlanishning moduli

$$a_k^{mi} = R_k \omega_k^2 = 40 \cdot (1,5)^2 = 90\ sm/s^2. \qquad (3.39)$$

$\vec{a}_k^{\,mi}$ vektor L aylananing markazi tomon yo‘nalgan.

Koriolis tezlanishining moduli

$$a_c = 2\omega_k v_n \sin(\vec{\omega}_k \vec{v}_n), \qquad (3.40)$$

bu ifodada

$$\sin(\vec{\omega}_k \hat{\ } \vec{v}_n) = \sin 90° = 1 \cdot$$

ω_k va v_n larning yuqorida topilgan qiymatlarini hisobga olgan holda a_c uchun quyidagi natijaga ega bo‘lamiz:

$$a_c = 2 \cdot 1,5 \cdot 113,06 = 339,18\ sm/s^2.$$

$\vec{a}_c$ vektor $(\vec{\omega}_k \times \vec{v}_n)$ vektor ko‘paytma qoidasiga muvofiq yo‘nalgan *(3.67-rasm).*

M nuqta absolut tezlanishining modulini proyeksiyalash usuli orqali aniqlaymiz:

$$(a_a)_a = a_{Mx} = -a_k^{ayl} - a_c = 459,18\ \text{sm}/s^2;$$

$$(a_a)_y = a_{My} = a_n^{\tau} + a_k^{mi} = 111,91\ sm/s^2;$$

$$(a_a)_z = a_{Mz} = -a_n^{n} = -319,56\ sm/s^2;$$

$$a_M = \sqrt{a_{Mx}^2 + a_{My}^2 + a_{Mz}^2} = 570,5\ sm/s^2.$$

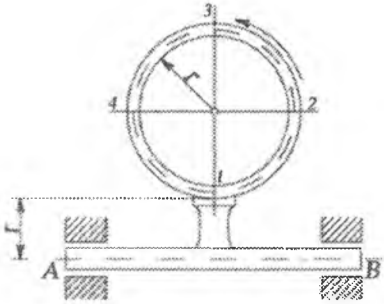
$\bar{\omega}_k$, rad/s.	Tezlik, sm/s.			$\bar{\varepsilon}_k$, rad/s^2	Tezlanish, sm/s²								
	v_k	v_n	v_M		a_k^{mi}	a_k^{ayl}	a_n^{τ}	a_n^{n}	a_c	a_{mi}	a_{my}	a_{mz}	a_m
115	60	113,06	128	–3	90	120	21,91	319,56	339,18	459,18	111,91	319,56	570,5

M nuqta absolut tezlanishining tashkil etuvchilarining yo‘nalishlari *3.67-rasmda* ko‘rsatilgan, hisoblash natijalari jadvalda keltirilgan.

2-masala.

Radius *r* bo‘lgan kovak halqa *AB* val bilan mahkam biriktirilgan, bunda valning o‘qi halqa o‘qining tekisligida joylashgan. Halqa

...masida ko'rsatilgan strelka yo'nalishida o'zgarmas, u nisbiy tezlik bilan harakat qiluvchi suyuqlik bilan to'ldirilgan.



3.68-a rasm

Agar aylanish o'qi bo'yicha *A* dan *B* ga qaralsa, *AB* val soat strelkasi aylanadigan tomonga aylanadi. Valning ω burchak tezligi o'zgarmas. 1, 2, 3 va 4-nuqtalardagi suyuqlik zarralarining absolut tezlanishlari miqdorlari aniqlansin *(3.68-a rasm).*

Yechish: masalada suyuqlik zarralarining halqa ichidagi harakati nisbiy harakat, halqaning esa *AB* val bilan birgalikda soat strelkasi aylanadigan tomonga aylanishi ko'chirma harakat hisoblanadi.

Nuqtaning murakkab harakatida tezlanishlarni qo'shish teoremasiga asosan 1, 2, 3 va 4-nuqtalardagi suyuqlik zarralarining absolut tezlanishlari quyidagi formula asosida aniqlanadi:

$$\vec{a} = \vec{a}_n + \vec{a}_k + \vec{a}_c = \vec{a}_n^{mi} + \vec{a}_n^{\tau} + \vec{a}_k^{mi} + \vec{a}_k^{\tau} + \vec{a}_c.$$

Masala shartiga ko'ra:

$$v_n = \text{const}; \quad \omega_k = \text{const}.$$

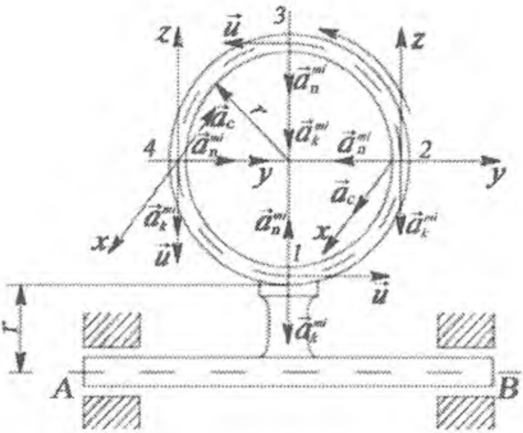
Shuning uchun barcha nuqtalarda $a_n^{\tau} = 0, \quad a_k^{\tau} = 0.$

1-nuqtada suyuqlik zarralarining absolut tezlanishining miqdorini aniqlaymiz *(3.68-b rasm):*

$$a_n^{mi} = \frac{u^2}{r}; \quad a_k^{mi} = \omega^2 \cdot r; \quad a_c = 2\omega_k v_n \cdot \sin(\vec{\omega}_n \hat{,} v_n) = 0.$$

Shuning uchun

$$a_1 = \left| a_k^{mi} - a_n^{mi} \right| = \left| \omega^2 r - \frac{u^2}{r} \right|.$$



3.68-b rasm

2-nuqtada suyuqlik zarralarining absolut tezlanishining miqdorini aniqlaymiz *(3.68-b rasm)*:

$$a_n^{mi} = \frac{u^2}{r}; \quad a_k^{mi} = \omega^2 \cdot 2r;$$

$$a_c = 2 \cdot \omega_k v_n \sin(\vec{\omega}_k \hat{} \vec{v}_n) = 2 \cdot \omega_k \cdot v_n = 2\omega \cdot u.$$

Bularni e'tiborga olsak,

$$a_2 = \sqrt{(a_c)^2 + (-a_n^{mi})^2 + (-a_k^{mi})^2} = \sqrt{4\omega^2 u^2 + \frac{u^4}{r^2} + 4\omega^4 r^2} =$$

$$= \sqrt{\frac{4\omega^2 u^2 r^2 + u^4 + 4\omega^4 r^4}{r^2}} = \frac{1}{r}\sqrt{(u^2 + 2\omega^2 r^2)^2} =$$

$$= \frac{1}{r}(u^2 + 2\omega^2 r^2) = \frac{u^2}{r} + 2\omega^2 r.$$

3-nuqtada suyuqlik zarralarining absolut tezlanishining miqdori quyidagiga teng bo'ladi *(3.68-b rasm)*:

$$a_n^{mi} = \frac{u^2}{r}; \quad a_k^{mi} = \omega^2 \cdot 3r;$$

$$a_c = 2 \cdot \omega_k v_n \sin(\vec{\omega}_k \hat{} \vec{v}_n) = 0.$$

Shuning uchun

$$a_3 = a_n^{mi} + a_k^{mi} = \frac{u^2}{r} + 3\omega^2 r.$$

4-nuqtada suyuqlik zarralarining absolut tezlanishining miqdorini aniqlaymiz. *(3.68-b rasm):*

$$a_n^{mi} = \frac{u^2}{r}; \quad a_k^{mi} = \omega^2 \cdot 2r;$$

$$a_c = 2 \cdot \omega_k v_n \sin\left(\vec{\omega}_k \hat{\ } \vec{v}_n\right) = 2\omega_k v_n = 2\omega u.$$

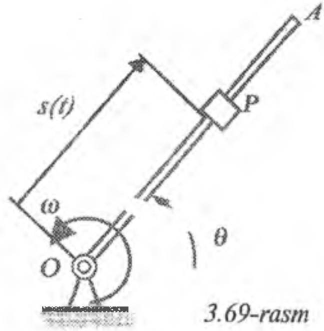
Shuning uchun

$$a_4 = \sqrt{(-a_c)^2 + (-a_n^{mi})^2 + (-a_k^{mi})^2} =$$

$$= \sqrt{(-2\omega u)^2 + \left(\frac{u^2}{r}\right)^2 + (2\omega^2 r)^2} = \frac{u^2}{r} + 2\omega^2 r.$$

3-masala. Uzunligi $l = 1$ m bo'lgan sterjen chizma tekisligiga perpendikular holda O nuqtadan o'tuvchi o'q atrofida $\omega = \frac{\pi t}{3} \cdot \frac{1}{s}$ burchak tezlik bilan aylanmoqda. Shu vaqtning o'zida sterjen bo'ylab P polzun $S(t) = OP = 12{,}5t^2$ qonunga muvofiq harakatlanadi *(3.69-rasm)*.

Sterjen harakati gorizontal holatdan boshlanadi deb faraz qilib, P polzunning, u sterjenning yarmida bo'lgan holatida, absolut tezligi va absolut tezlanishi aniqlansin.



3.69-rasm

Yechish: sterjenning *O* nuqtadan o'tuvchi o'q atrofidagi aylanma harakati *P* polzun uchun ko'chirma harakat, *P* polzunning sterjen bo'ylab *O* nuqtadan sterjenga nisbatan harakati nisbiy harakat hisoblanadi. Agar t_1 vaqt onida polzun sterjenning yarmida bo'ladi deb faraz qilsak,

$$S(t_1)=\frac{l}{2};\quad 12{,}5t_1^2=50;\ t_1^2=4s^2$$

bo'ladi.

Bundan t_1=2 *s.*

Shu vaqt onida sterjenning boshlang'ich holati (gorizontal holat) bilan tashkil etgan burchak quyidagicha aniqlanadi:

$$\omega=\frac{d\theta}{dt};\quad d\theta=\omega dt;$$

$$\theta(t)=\int\omega dt+C$$

yoki

$$\theta(t)=\frac{\pi t^2}{6}+C.$$

Bu ifodada *C* — integrallash doimiysi.

Agar boshlang'ich paytda θ =0 ekanligini e'tiborga olsak, *C*=0 bo'ladi. Natijada,

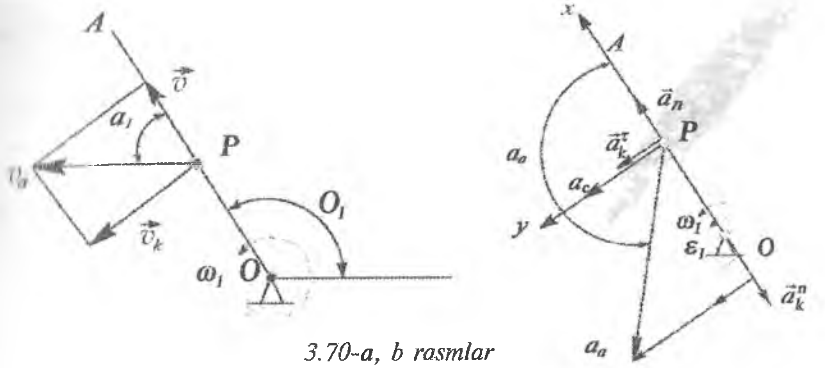
$$\theta(t)=\frac{\pi t^2}{6}.$$

t_1 = 2 *s* da θ_1 = $2\pi/3$. (*3.70-a rasm*).

Polzunning nisbiy tezlik va nisbiy tezlanishini aniqlash uchun sterjenning aylanma harakatini — ko'chirma harakatini xayolan to'xtatamiz. Natijada, *P* polzunning nisbiy tezligi va nisbiy tezlanishi uchun quyidagi kattaliklarga ega bo'lamiz:

$$v_n=\frac{dS(t_1)}{dt}=25t_1=50\ sm/s,$$

$$a_n=\frac{d^2S(t_1)}{dt^2}=\frac{dv_n}{dt}=25\ sm/s^2.$$



3.70-a, b rasmlar

P polzun ko‘chirma harakat tezligi va tezlanishini aniqlash uchun *P* polzunning nisbiy harakatini xayolan to‘xtatamiz. Natijada, *P* polzun ko‘chirma tezligi va ko‘chirma tezlanishlari uchun quyidagi ifodalarga ega bo‘lamiz:

$$v_k = OP(t_1) \cdot \omega(t_1) = \frac{\ell}{2} \cdot \frac{\pi t_1}{3} = 104{,}7 \ sm/s \, .$$

$\vec{v}_k$ vektor *OP* kesmaga perpendikular holda ω_1 yo‘nalishida yo‘naladi.

P polzunning ko‘chirma tezlanishi uning urinma va normal tezlanishlaridan tashkil topadi:

$$\vec{a}_k = \vec{a}_k^\tau + \vec{a}_k^n .$$

P polzun ko‘chirma urinma tezlanishi:

$$a_k^\tau = OP(t_1) \cdot \varepsilon(t_1) = OP(t_1) \cdot \frac{d\omega(t_1)}{dt} = \frac{100}{2} \cdot \frac{\pi}{3} = 52{,}34 \ sm/s^2 .$$

P polzun ko‘chirma normal tezlanishi:

$$a_k^n = OP(t_1) \cdot \omega^2(t_1) = \frac{100}{2} \cdot \frac{\pi^2 t_1^2}{9} = 219{,}1 \ sm/s^2 .$$

P polzunning Koriolis tezlanishini aniqlaymiz:

$$\vec{\omega}_{1k} = \omega_1(t_1) \cdot \vec{k} = 2{,}09\vec{k},$$

$$\vec{v}_n = v_n \vec{i} = 50\vec{i} \, .$$

Bulardan foydalansak, *P* polzun Koriolis tezlanishini quyidagiga teng bo'ladi:

$$\vec{a}_c = 2\vec{\omega}_1(t_1)\times v_n(t_1) = 2\cdot 2{,}09\vec{k}\times 50\vec{i} = 209{,}4\vec{j}.$$

Yuqorida hisoblangan kattaliklarning qiymatlarini e'tiborga olsak, *P* polzunning absolut tezligi quyidagiga teng bo'ladi *(3.70-a rasm)*:

$$v_a = \sqrt{v_n^2 + v_k^2} = \sqrt{(1004{,}7)^2 + (50)^2} = 106{,}02\ sm/s^2.$$

Absolut tezlikning yo'nalishini quyidagicha topiladi:

$$\operatorname{tg}\alpha_1 = \frac{v_k}{v_n} = \frac{104{,}7}{50} = 2{,}09,$$

$$\alpha_1 = \operatorname{arc\,tg}\alpha_1 = 64°47.$$

P polzunning absolut tezlanishini aniqlaymiz:

$$\vec{a}_n = \vec{a}_k + \vec{a}_n + \vec{a}_c.$$

Absolut tezlanishi moduli va yo'nalishini aniqlash uchun proyeksiyalar usulidan foydalanilamiz:

$$(a_a)_x = a_n - a_k^n = 25 - 219{,}1 = -194{,}1\ sm/s^2,$$

$$(a_a)_y = a_{k\tau} + a_c = 52{,}34 + 209{,}4 = 261{,}74\ sm/s^2.$$

Natijada, *P* polzun absolut tezlanishi quyidagiga teng bo'ladi:

$$a_a = \sqrt{a^2{}_{ax} + a^2{}_{ay}} = \sqrt{(-194{,}1)^2 + (261{,}74)^2} = 325{,}82\ sm/s^2$$

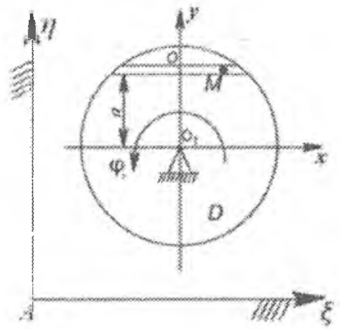
Absolut tezlanish yo'nalishi esa uning P_x o'qi bilan hosil qilgan burchak orqali aniqlanadi:

$$\cos\alpha_2 = \frac{(a_a)_x}{a_a} = \frac{-194{,}1}{325{,}82} = -0{,}595$$

$$\alpha_2 = \arccos\alpha_2 = 180° - 53°44' = 126°56'$$

Absolut tezlik vektori, absolut tezlanish vektori va ularning yo'nalishlari *3.70-a, b rasmlarda* ko'rsatilgan.

4-masala. M nuqta D jismga (doiraga) nisbatan $OM = s_n = 20\sin\frac{\pi t}{6}$ qonun bo'yicha harakat qiladi. D jism (doira) chizma tekisligiga perpendikular o'q atrofida $\varphi_k = 2t - 0,5t^2$ qonun bo'yicha aylanadi. M nuqtaning $t = 1$ sekunddagi absolut tezligi va absolut tezlanishi topilsin *(3.71-rasm).*



3.71-rasm

Yechish:

chizmada ko'rsatilgan Axh o'qlar sistemasi qo'zg'almas sanoq sistemasini, doira bilan bog'langan va u bilan birga aylanuvchi O_1xy sanoq sistemasi qo'zg'aluvchan sanoq sistemasini tashkil etadi.

M nuqtaning qo'zg'aluvchan O_1xy sanoq sistemasiga nisbatan harakati nisbiy, D jismning qo'zg'aluvchan O_1xy sanoq sistemasi bilan birgalikda qo'zg'almas sanoq sistemasiga nisbatan harakati M nuqta uchun ko'chirma va M nuqtaning bevosita qo'zg'almas Axh sanoq sistemasiga nisbatan harakati absolut harakat hisoblanadi.

Nuqtaning $t = 1$ sekunddagi holatini uning OM to'g'ri chiziq bo'ylab harakat qonunidan foydalanib topamiz va chizmada ko'rsatamiz.

$$t = 1 \text{ sekundda } OM = s_n = 20\sin\frac{\pi t}{6} = 10 \text{ } sm.$$

M nuqtaning absolut tezligini nuqtaning murakkab harakatida tezliklarni qo‘shish haqidagi teoremaga asosan nisbiy va ko‘chirma tezliklarning geometrik yig‘indisi kabi topamiz:

$$\vec{v}_M = \vec{v}_n + \vec{v}_k. \tag{3.41}$$

Nisbiy tezlikning moduli

$$v_n = |\tilde{v}_n|, \tag{3.42}$$

bu ifodada,

$$\tilde{v}_n = \frac{dS_n}{at} = \frac{20\pi}{6}\cos\frac{\pi t}{6}.$$

t=1 sekundda,

$$\tilde{v}_n = 9\ sm/s, \quad v_n = 9\ sm/s.$$

$\tilde{v}_n$ oldidagi musbat ishora v_n vektorning s_n ning o‘sish tomoniga qarab yo‘nalganligini ko‘rsatadi *(3.71-a rasm).*

Ko‘chirma tezlikning moduli

$$v_k = R_k\omega_k, \tag{3.43}$$

bu ifodada R_k – doiraning berilgan onda M nuqta bilan ustma-ust tushuvchi nuqtasi tomonidan chiziladigan L aylananing radiusi, ω_k – doira burchak tezligining moduli.

$$R_k = O_1M = \sqrt{a^2 + (OM)^2} = 22,36\ sm, \tag{3.44}$$

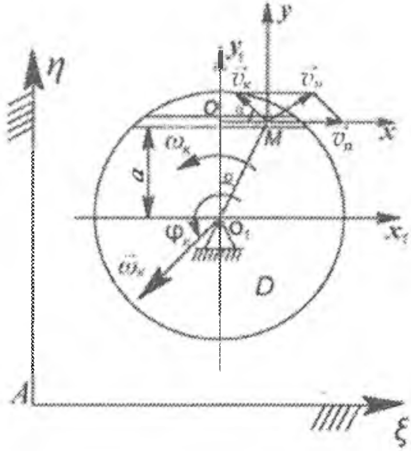
$$\omega_k = |\tilde{\omega}_k|, \quad \tilde{\omega}_k = \frac{d\varphi_k}{at} = 2 - t. \tag{3.45}$$

t=1 sekundda,

$$\tilde{\omega}_k = 1\frac{rad}{s}, \quad \omega_k = 1\ rad/s.$$

(L aylana rasmda ko‘rsatilmagan).

$\tilde{\omega}_k$ kattalikning oldidagi musbat ishora doiraning O_1 nuqta atrofida aylanishi φ_k burchakning o‘sish tomoniga qarab ro‘y berishini ko‘rsatadi. Shuning uchun $\vec{\omega}_k$ vektor chizma tekisligiga perpendikular holda O_1 nuqtadan o‘tkazilgan aylanish o‘qi bo‘ylab tepaga



3.71-a rasm

qarab yo'nalgan *(3.71-a rasm)*. Ko'chirma tezlikning moduli (3.43) formula bo'yicha topiladi:

$$v_k = 22,36 \ sm/s.$$

$\vec{v}_k$ vektor L aylanaga urinma bo'ylab, doiraning aylanish tomoniga qarab yo'nalgan.

M nuqtaning absolut tezligi uning nisbiy va ko'chirma harakat tezliklaridan qurilgan parallelogrammning diagonali orqali ifodalanadi. Uning modulini proyeksiyalash usuli orqali aniqlaymiz: *(3.71-a rasm)*

$$(v_a)_x = v_{Mx} = v_n - v_k \cos\alpha; \quad \cos\alpha = 0,89.$$

$$(v_a)_y = v_{My} = v_k \sin a; \quad \sin a = 0,45.$$

Shuning uchun,

$$v_{Mx} = -10,9 \ sm/s;$$

$$v_{My} = 10,06 \ sm/s;$$

$$v_a = v_M = \sqrt{v_{Mx}^2 + v_{My}^2} = 14,83 \ sm/s.$$

M nuqtaning absolut tezlanishi nuqtaning murakkab harakatida tezlanishlarni qo'shish teoremasidan aniqlanadi. Masalada ko'chirma harakat ilgarilanma bo'lmagan murakkab harakat bo'lganligi uchun absolut tezlanish nisbiy, ko'chirma va Koriolis tezlanishlarining geometrik yig'indisiga teng:

$$\vec{a}_a = \vec{a}_M = \vec{a}_n + \vec{a}_k + \vec{a}_c, \qquad (3.46)$$

yoki

$$\vec{a}_M = \vec{a}_n^\tau + \vec{a}_n^n + \vec{a}_k^{ayl} + \vec{a}_k^{mi} + \vec{a}_c. \qquad (3.47)$$

Nisbiy urinma tezlanishning moduli

$$a_n^\tau = \left|\vec{a}_n^\tau\right|, \qquad (3.48)$$

bu ifodada

$$\vec{a}_n^t = \frac{d^2 S_n}{dt^2} = -\frac{20 p^2}{36} \sin \frac{pt}{6}.$$

t=1 sekundda

$$\vec{a}_n^\tau = -2,74\ sm/s^2,\ a_n^\tau = 2,74\ sm/s^2.$$

$\vec{a}_n^\tau$ ning manfiy ishorasi $\vec{a}_n^\tau$ vektorning s_n ning kamayish tomoniga qarab yo'nalganligini ko'rsatadi. $\vec{a}_n^\tau$ va $\vec{v}_n$ larning ishoralari har xil. Demak, $\vec{a}_n^\tau$ va $\vec{v}_n$ vektorlar qarama-qarshi tomonlarga yo'nalgan bo'ladi.

M nuqtaning nisbiy harakatdagi normal tezlanish

$$a_n^n = \frac{v_n^2}{\rho} = 0,$$

chunki nisbiy harakat trayektoriyasi to'g'ri chiziq ($\rho = \infty$).

Ko'chirma aylanma-urinma tezlanishning moduli

$$a_k^{ayl} = R_k \varepsilon_k, \qquad (3.49)$$

bu ifodada: $\varepsilon_k = \left|\vec{\varepsilon}_k\right|$ — doira burchak tezlanishining moduli

$$\vec{\varepsilon}_k = \frac{d^2\varphi_k}{at^2} = -1\ rad/s^2,\ \varepsilon_k = 1\ rad/s^2.$$

$\vec{\varepsilon}_k$ va $\vec{\omega}_k$ larning ishoralari har xil. Demak, doira sekinlanuvchan aylanma harakatda bo'lar ekan. $\vec{\omega}_k$ va $\vec{\varepsilon}_k$ vektorlar qarama-qarshi tomonlarga yo'naladi *(3.71-a, b rasmlar).*

(3.49)ga asosan,

$$a_k^{ayl} = 22,36\ sm/s^2.$$

$\vec{a}_k^{ayl}$ vektor $\vec{v}_k$ vektorga qarama-qarshi holda yo'nalgan.

M nuqtaning ko'chirma harakat markazga intilma tezlanishning moduli:

$$a_k^{mi} = R_k\omega_k^2 = 22,36\ sm/s^2. \qquad (3.50)$$

$\vec{a}_k^{mi}$ vektor L aylananing markaziga (O_1 nuqta tamon) yo'nalgan.

Koriolis tezlanishining moduli

$$a_c = 2\omega_k\vartheta_n \sin(\vec{\omega}_k \wedge \vec{\vartheta}_n).$$

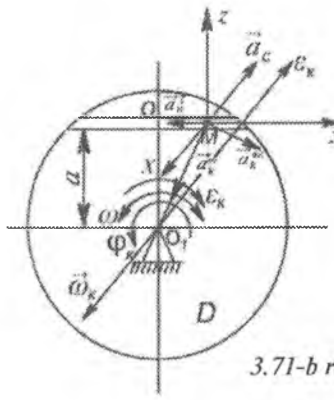
Masalada

$$\sin\sin(\omega_k \hat{} \vartheta_n) = \sin 90° = 1.$$

ω_k va ϑ_n larning yuqorida topilgan qiymatlarini hisobga olsak.

$$a_c = 18\ sm/s^2.$$

$\vec{a}_c$ vektor $(\vec{\omega}_k \mathrm{x} \vec{\vartheta}_n)$ vektor ko'paytma qoidasiga muvofiq yo'naladi *(3.71-b rasm)*.



3.71-b rasm

M nuqtaning absolut tezlanishining modulini proyeksiyalash usuli yordamida aniqlaymiz:

$$(a_a)_x = a_{Mx} = -a_c = -18\ sm/s^2;$$

$$(a_a)_y = a_{My} = -a_n^{\tau} - a_k^{mi}\sin\alpha + a_k^{ayl}\cos\alpha = 7,1\ sm/s^2;$$

$$a_{Mz} = -a_k^{mi} \cos a - a_k^{ayl} \sin \alpha = 30,5 \ sm/s^2;$$

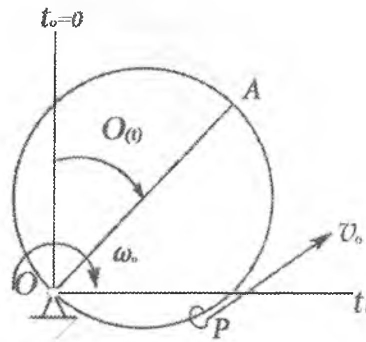
$$a_a = a_M = \sqrt{a_{Mx}^2 + a_{My}^2 + a_{Mz}^2} = 36,12 \ sm/s^2.$$

Hisoblash natijalari quyidagi jadvalda keltirilgan.

ω_k, ga rad/s.	Tezlik, sm/s.			ε_k, rad/s²	Tezlanish, sm/s²								
	v_k	v_n	v_M		a_k^{mi}	a_k^{ayl}	a_n^{τ}	a_n^{n}	a_c	a_{Mx}	a_{My}	a_{Mz}	a_M
1	22,36	9	14,83	−1	22,36	22,36	2,74	0	18	−18	7,1	30,5	36,12

49-§. Talabalarga mustaqil o'rganish uchun tavsiya etiladigan muammolar

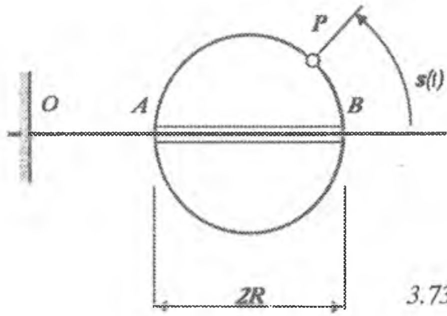
1-muammo. Radiusi R=30 sm bo'lgan doira chizma tekisligiga O nuqtadan o'tuvchi o'q atrofida $\omega_0 = 1 \ rad/s$ o'zgarmas tezlik bilan aynalmoqda. Shu vaqtning o'zida P nuqta O nuqtadan $v_0 = 25 \ sm/s$ tezlik bilan doira gardishi bo'ylab harakatlana boshlaydi. Boshlang'ich paytda vertikal holatda bo'lgan doira diametrik gorizontal holatini egallaganda P nuqtaning absolut tezligi va absolut tezlanishi aniqlanadi *(3.72-rasm)*.



3.72-rasm

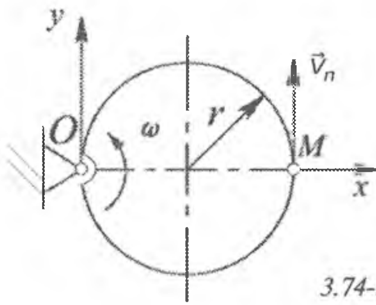
2-muammo. Disk uning diametridan o'tuvchi gorizontal to'g'ri chiziq bo'ylab $OA = at^2$ qonunga muvofiq ilgarilanma harakatlanmoqda. Shu vaqtning o'zida disk gardishi bo'ylab P nuqta $BP = S(t) = \frac{R\pi t^2}{12}$ qonunga muvofiq harakatlanadi.

Agar $a=10$ sm/s^2, $R=25$ sm bo'lsa, $t_1=2$ s vaqt oni uchun P nuqtaning absolut tezligi va absolut tezlanishi aniqlansin *(3.73-rasm)*.



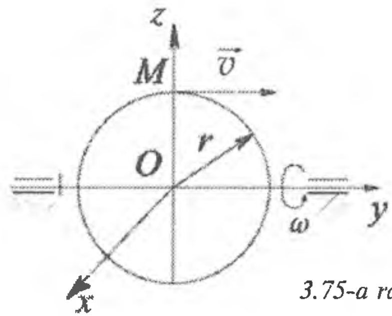
3.73-rasm

3-muammo. Radiusi $r=0,5$ m bo'lgan halqa shakl tekisligida o'zgarmas $\omega = 4\ rad/s$ burchak tezlik bilan aylanadi. M nuqta esa halqa bo'ylab o'zgarmas $v = 2\ m/s$ tezlik bilan harakat qiladi. Ko'rsatilgan holat uchun M nuqtaning absolut tezlanishini toping *(3.74-rasm)*.



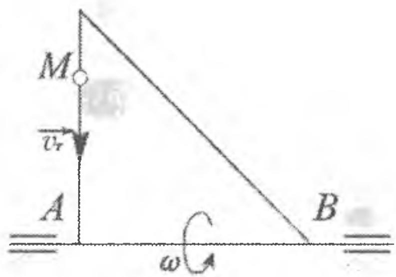
3.74-rasm

4-muammo. *Oy* o'qi atrofida o'zgarmas $\omega=4$ *rad/s* burchak tezlik bilan aylanayotgan halqa bo'ylab M nuqta o'zgarmas $v=2\ m/s$ tezlik bilan harakat qiladi. Agar halqaning radiusi $r=0,5$ m bo'lsa, ko'rsatilgan holat uchun M nuqtaning absolut tezlanishini toping *(3.75-a rasm)*.



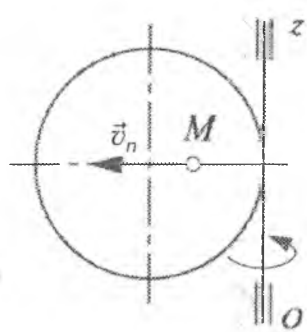
3.75-a rasm

5-muammo. Uchburchak shaklidagi jism *AB* tomoni atrofida ω burchak tezlik bilan aylanadi. *M* nuqta esa uning tomoni bo'ylab $v_n = 3t^2$ nisbiy tezlik bilan harakatlanadi. Nuqtaning $t = 2$ s paytdagi nisbiy tezlanishini aniqlang *(3.75-b rasm).*



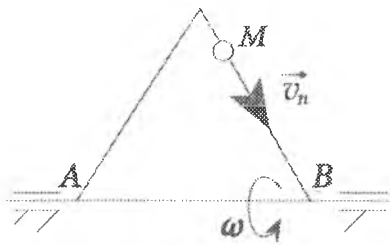
3.75-b rasm

6-muammo. Disk O_z o'qi atrofida aylanadi. *M* nuqta esa nisbiy tezlik bilan diskning diametri bo'ylab harakatlanadi. Nuqtaning $t = 1$ *s* paytdagi nisbiy tezlanishini toping *(3.76-rasm).*



3.76-rasm

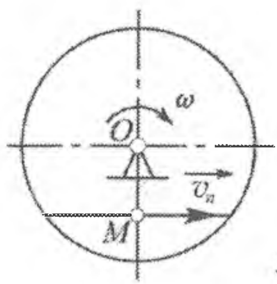
7-muammo. Uchburchak shaklidagi jism *AB* tomoni atrofida ω burchak tezlik bilan aylanadi. *M* nuqta esa uchburchakning tomoni bo'ylab $v_n = 2\sin 4t$ nisbiy tezlik bilan harakatlanadi. Nuqtaning $t = \pi/8$ *s* paytdagi nisbiy tezlanishini hisoblang *(3.77-rasm).*



3.77-rasm

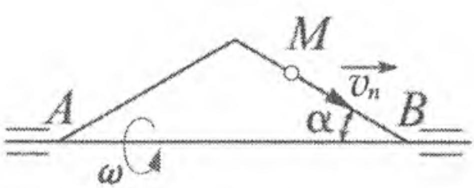
8-muammo. Disk shakl tekisligida *O* o'qi atrofida ω =0,5 *rad/s* burchak tezlik bilan aylanadi. Uning vatari bo'ylab *M* nuqta $v_n = 0{,}5t$ nisbiy tezlik bilan harakat qiladi.

Agar $t=2$ *s* paytda *OM*=0,02 *m* bo'lsa, *M* nuqtaning absolut tezlanishini toping *(3.78-rasm).*



3.78-rasm

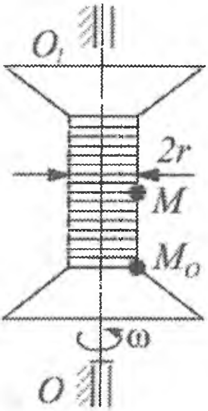
9-muammo. Uchburchak shaklidagi jism *AB* tomoni atrofida o'zgarmas ω =4 *rad/s* burchak tezlik bilan aylanadi. Uning bir tomoni bo'ylab *M* nuqta v_n nisbiy tezlikka ega bo'lsa, *MB*=0,5 *m*



3.80-rasm

bo'lgan paytda *M* nuqtaning ko'chirma tezlanishini aniqlang. Bunda α =30° *(3.80-rasm)*.

10-muammo. Radiusi r=0,02 *m* bo'lgan g'altak OO_1 o'q atrofida ω =2 *rad/s* burchak tezlik bilan aylanadi. *M* nuqta esa g'altakning chekkasi bo'ylab $MoM = 0{,}04t^2$ qonun bo'yicha harakatlanadi. *M* nuqtaning absolut tezlanishini hisoblang *(3.81-rasm)*.



3.81-rasm

50-§. Talabalar tomonidan mustaqil bajariladigan hisob-chizma ishlari variantlari

Nuqtaning absolut tezligi va absolut tezlanishini aniqlash

M nuqta *D* jismga nisbatan harakat qiladi. *M* nuqta nisbiy harakatining va D jism harakatining berilgan tenglamalariga ko'ra *M* nuqtaning $t=t_1$ vaqt onidagi absolut tezligi va absolut tezlanishi aniqlansin.

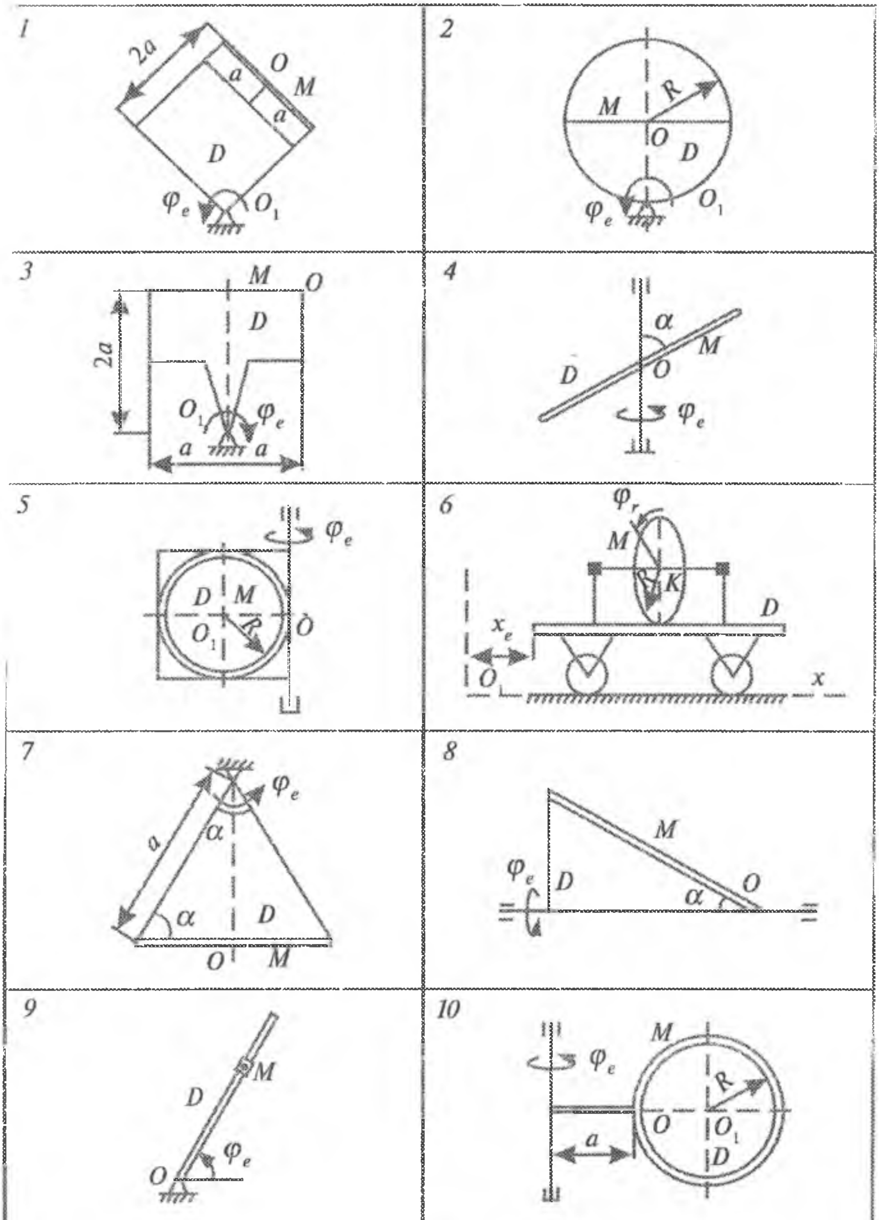
Mexanizimlarning sxemalari va hisoblash uchun ma'lumotlar jadvalda keltirilgan.

Variant raqami	M nuqtaning nisbiy harakat tenglamasi	Jismning harakat tenglamasi		t_1,c	R,sm	a, sm	α, grad	Qo'shimcha ma'lumotlar
		$\varphi_e=\varphi_e(t)$, rad	$x_e=x_e(t)$, sm					
1.	$18\sin(\pi t/4)$	$2t^3-t^2$	–	2/3	–	25	–	
2.	$20\sin\pi t$	$0,4t^2+t$	–	5/3	20	–	–	
3.	$6t^3$	$2t+0,5t^2$	–	2	–	30	–	
4.	$10\sin(\pi t/6)$	$0,6t^2$	–	1	–	–	60	
5.	$40\cos(\pi t/6)$	$3t-0,5t^3$	–	2	30	–	–	
6.	–	–	$3t+0,27t^3$	10/3	15	–	–	$\varphi_n=0,15\pi t^3$
7.	$20\cos 2\pi t$	$0,5t^2$	–	3/8	–	40	60	
8	$6(t+0,5t^2)$	t^3-5t	–	2	–	–	30	
9	$10(1+\sin 2\pi t)$	$4t+1,6t^2$	–	1/8	–	–	–	
10	$20\cos(\pi t/4)$	$1,2t-t^2$	–	4/3	20	20	–	
11	$25\sin(\pi t/3)$	$2t^2-0,5t$	–	4	–	25	–	
12	$15\pi t^3/8$	$5t-4t^2$	–	2	30	30	–	
13	$120\pi t^2$	$8t^2-3t$	–	1/3	40	–	–	
14	$3+14\sin\pi t$	$4t-2t^2$	–	2/3	–	–	30	
15	$5\sqrt{2}$	$0,2t^3+t$	–	2	–	60	45	
16	$20\sin\pi t$	$t-0,5t^2$	–	1/3	–	20	–	

17	$8t^3+2t$	$0,5t^2$	–	1	–		–	
18	$10t+t^3$	$8t-t^2$	–	2	–	–	60	
19	$6t+4t^3$	$t+3t^2$	–	2	40	–	–	
20	$30\pi\cos w(\pi t/6)$	$6t+t^2$	–	3	60	–	–	

Eslatma: $\varphi_e = \varphi_e(t) \rightarrow \varphi_k = \varphi_k(t)\ (rad)$

$x_e = x_e(t) \rightarrow x_k = x_k(t)\ (sm)$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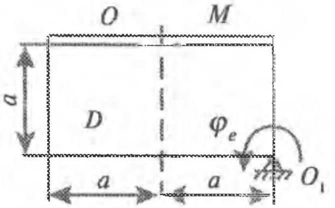

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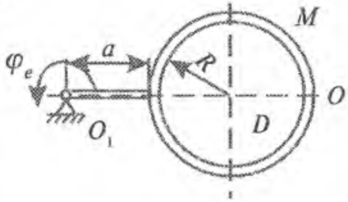

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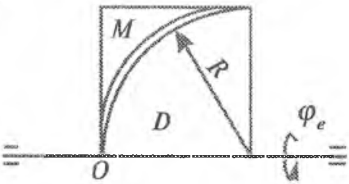

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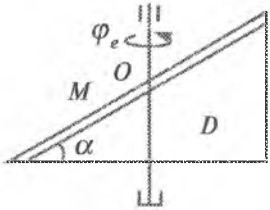

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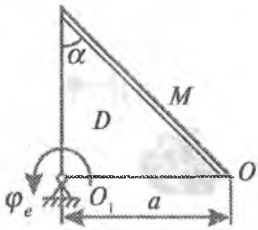

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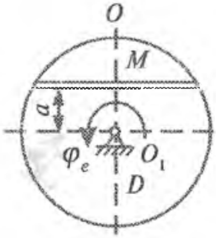

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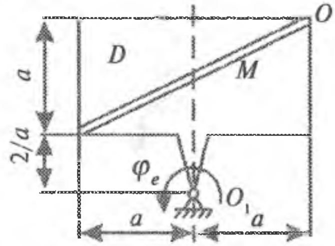

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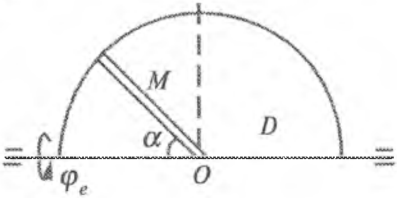

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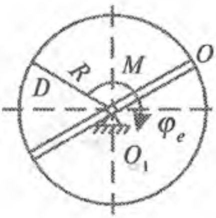

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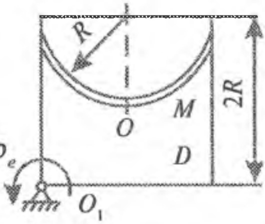


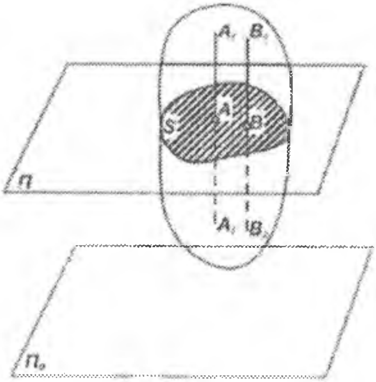
IV BOB
QATTIQ JISMNING TEKISLIKKA PARALLEL HARAKATI

Agar jismning barcha nuqtalari berilgan qo'zg'almas tekislikka parallel tekisliklarda harakatlansa, uning bunday harakati tekislikka parallel harakat deyiladi.

Jismning tekislikka parallel harakatiga misol tariqasida to'g'ri chiziqli relsda g'ildirakning dumalashini, bir tekislikda harakatlanuvchi mashina va mexanizm qismlarining harakatini va hokazolarini keltirish mumkin.

51-§. Qattiq jismning tekislikka parallel harakatining xususiyatlari. Tekis shaklning harakat tekisligidagi ko'chishi

Jismning tekislikka parallel harakatini o'rganish uchun uni qo'zg'almas Π_0 tekislikka parallel bo'lgan Π tekislik bilan fikran kesamiz. Kesish natijasida hosil bo'lgan kesimni S bilan belgilab, uni tekis shakl deb ataymiz. Jismning tekislikka parallel harakati ta'rifiga ko'ra, jismning harakati davomida bu tekis shakl doimo qo'zg'almas Π_0 tekislikka parallel bo'lgan Π tekislikda harakatlanadi. Tekislikka parallel harakatdagi jismda Π tekislikka perpendikular qilib olingan A_1A_2 kesma o'ziga parallel holda ko'chadi, ya'ni, kesma ilgarilanma harakatda bo'ladi. Shu sababli jismning bu kesmada yotgan barcha nuqtalarining harakatini o'rganish o'rniga, ulardan birining, masalan, S tekis shakl A nuqtasining harakatini o'rganish yetarli bo'ladi. Π tekislikka perpendikular B_1B_2 kesmaning harakatini o'rganishda ham xuddi shunday xulosaga kelish mumkin. Shunday qilib, qattiq jismning tekislikka parallel harakatini o'rganish uchun Π_0 qo'zg'almas tekislikka parallel bo'lgan S tekis shaklning Π_0 tekislikdagi harakatini o'rganish kifoya bo'lar ekan.



4.1-rasm

Tekis shakl harakatlanadigan П tekislik tekis shaklning harakat tekisligi deyiladi *(4.1-rasm).*

52-§. Tekis shaklning harakat tekisligida ko'chishini qutb bilan birgalikdagi ilgarilanma harakat va qutb atrofidagi aylanma harakatlarga ajratish

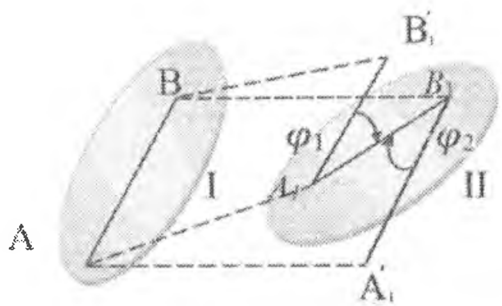
Tekis shakl harakatini undagi kinematik holati aniq bo'lgan nuqta harakatiga bog'lab o'rganish qulay bo'ladi. Bunday nuqta qutb deb ataladi.

Tekis shaklning harakat tekisligidagi har qanday ko'chishi quyidagi teorema orqali ifodalanadi: *tekis shaklning harakat tekisligidagi har qanday ko'chishi qutb bilan birgalikdagi ilgarilanma ko'chish hamda qutb atrofidagi aylanma ko'chishdan tashkil topadi.*

Teoremani isbotlash uchun tekis shaklning harakat tekisligidagi ixtiyoriy ikki holatini olamiz. Tekis shaklning I holati AB kesmaning tekislikdagi o'rni bilan, II holati esa A_1B_1 kesmaning tekislikdagi o'rni bilan aniqlansin *(4.2-rasm).*

Tekis shaklning harakat tekisligidagi ko'chishini ilgarilanma va aylanma harakatlardan tashkil topgan deb qarash mumkin. Quyidagi 2 variantni qarab chiqamiz.

1-variant. Qutb sifatida A nuqtani tanlab, tekis shaklga shunday ilgarilanma ko'chish beramizki, natijada A nuqta A_1 nuqta bilan



4.2-rasm

ustma-ust tushsin. Bunda tekis shaklning ilgarilanma ko'chishi $\overrightarrow{AA_1}$ vektor bilan aniqlanadi. B nuqta esa B'_1 holatga o'tadi. Tekis shaklni A_1 nuqtadan tekis shakl tekisligiga perpendikular holda o'tuvchi o'q atrofida shunday φ_1 burchakka aylantiramizki, natijada B'_1 nuqta B_1 nuqta bilan ustma-ust tushsin. Bunday harakatlar natijasida tekis shakl II holatni egallaydi.

2-variant. Qutb sifatida B nuqtani tanlab, tekis shaklga shunday ilgarilanma ko'chish beramizki, natijada B nuqta B_1 nuqta bilan ustma-ust tushsin.

Bunda tekis shaklning ilgarilanma ko'chishi $\overrightarrow{BB_1}$ vektor bilan aniqlanadi. A nuqta esa A'_1 holatga o'tadi, tekis shaklni B_1 nuqtadan tekis shaklga perpendikular holda o'tuvchi o'q atrofida φ_2 burchakka aylantirsak, A'_1 nuqta A_1 holatga o'tadi hamda amalga oshirilgan ilgarilanma va aylanma harakatlar natijasida tekis shakl II holatni egallaydi.

4.2-rasmdan ko'ramizki, $AA_1 \neq BB_1$, ya'ni tekis shaklning ilgarilanma harakati qutbni tanlashga bog'liq bo'lar ekan. $A_1B'_1 // A'_1B_1$ va A_1B_1 umumiy bo'lgani uchun, $\varphi_1 = \varphi_2$ hamda aylanish yo'nalishi bir xil bo'ladi.

Mazkur holatlar, tekis shaklni qutb atrofida aylanishi qutbni tanlashga bog'liq bo'lmasligini ko'rsatadi. Shunday qilib, teorema isbotlandi.

53-§. Tekis shaklning harakat tenglamasi

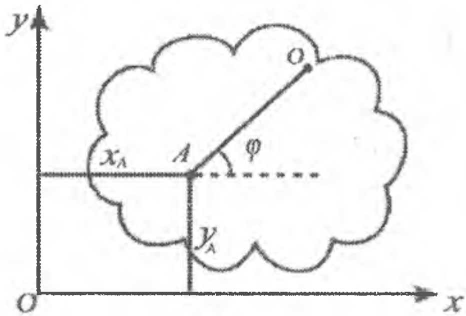
Yuqorida isbotlangan teoremaga asosan, tekis shaklning o'z tekisligidagi harakatini qutb bilan birgalikdagi ilgarilanma harakat va qutbdan harakat tekisligiga perpendikular ravishda o'tuvchi o'q atrofidagi aylanma harakatlardan tashkil topgan deb qarash mumkin.

Binobarin, tekis shaklning harakat tenglamasi ham qutb bilan birgalikdagi ilgarilanma harakat va qutb atrofidagi aylanma harakat tenglamalaridan iborat bo'ladi.

Tekis shaklda biror *A* nuqtani qutb sifatida qabul qilib, uning qo'zg'almas *Oxy* koordinatalar sistemasidagi koordinatalarini *x*, *y* orqali belgilasak,

$$x_A = f_1(t),\quad y_A = f_2(t) \qquad (4.1)$$

tenglamalar tekis shaklning ilgarilanma harakatini ifodalaydi *(4.3-rasm)*.



4.3-rasm

Tekis shaklda olingan ixtiyoriy *AO* kesmaning *x* o'qi bilan tashkil qilgan burchagini φ orqali belgilasak, mazkur burchak vaqt o'tishi bilan o'zgarishi tufayli

$$\varphi = f_3(t) \qquad (4.2)$$

tenglama tekis shaklning aylanma harakat tenglamasini ifodalaydi.

Natijada, tekis shaklning o‘z tekisligidagi harakati

$$\left.\begin{aligned} x_A &= f_1(t), \\ y_A &= f_2(t), \\ \varphi &= f_3(t) \end{aligned}\right\} \tag{4.3}$$

tenglamalar bilan ifodalanishi aniq bo‘ladi. (4.3.) tenglamalar tekis shaklning harakat tenglamalarini ifodalaydi.

Xususiy holda, tekis shaklning harakatida φ=const bo‘lsa, tekis shakl ilgarilanma harakatda bo‘ladi.

Agar tekis shaklning harakati davomida x_A, y_A koordinatalar o‘zgarmas qiymatga ega bo‘lib, φ burchak o‘zgarsa, tekis shakl bunday holda aylanma harakatda bo‘ladi.

54-§. Tekis shaklning burchak tezligi va burchak tezlanishi

Tekis shaklning qutb atrofida aylanishida uning barcha nuqtalari har onda bir xil burchak tezlik va bir xil burchak tezlanishga ega bo‘ladi.

Tekis shaklning aylanish burchagidan vaqt bo‘yicha olingan hosila tekis shaklning burchak tezligi deyiladi:

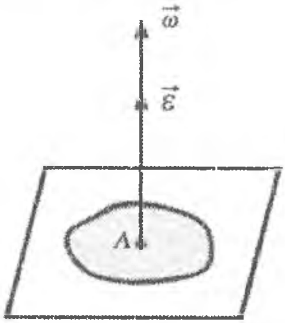
$$\omega = \frac{d\varphi}{at}. \tag{4.4}$$

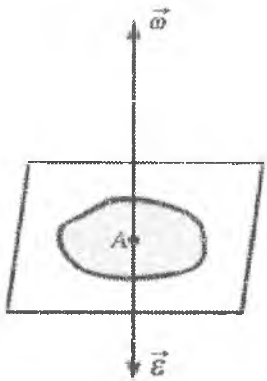
Tekis shaklning burchak tezligidan vaqt bo‘yicha olingan birinchi tartibli hosila yoki tekis shakl aylanish burchagidan vaqt bo‘yicha olingan ikkinchi tartibli hosila tekis shaklning burchak tezlanishi deyiladi:

$$\varepsilon = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2}. \tag{4.5}$$

Tekis shaklning burchak tezligi va burchak tezlanishi qutbning tanlab olinishiga bog‘liq bo‘lmaydi, chunki tekis shaklning qutb atrofida aylanish burchagi qutbni tanlashga bog‘liq bo‘lmaydi.

Burchak tezlik $\vec{\omega}$ va burchak tezlanish $\vec{\varepsilon}$ vektorlari tekis shakl tekisligiga A qutb orqali perpendikular holda o'tgan o'qda yotadi. Agar tekis shaklning qutb atrofidagi aylanma harakati tezlanuvchan bo'lsa, $\vec{\omega}$ va $\vec{\varepsilon}$ lar bir tomonga, sekinlanuvchan bo'lsa, qarama-qarshi tomonga yo'naladi. *(4.4-a, 4.4-b-rasmlar)*





4.4-a, 4.4-b-rasmlar

Takrorlash uchun savollar:

1. *Qattiq jismning tekislikka parallel harakatini ta'riflang.*
2. *Qanday tekislik shaklning harakat tekisligi deyiladi?*
3. *Qanday nuqta qutb sifatida tanlanadi?*
4. *Tekis shaklning harakat tekisligidagi har qanday ko'chishi qanday harakatlardan tashkil topadi?*
5. *Tekis shaklning tekislikka parallel harakati tenglamalarini yozing.*
6. *Agar tekis shaklning harakat tekisligida* φ = sonst *bo'lsa, tekis shakl qanday harakatda bo'ladi?*
7. *Tekis shaklning burchak tezligini ta'riflang.*
8. *Tekis shaklning burchak tezlanishini ta'riflang.*
9. *Tekis shakl qutb atrofida tezlanuvchan aylanma harakatda bo'lish shartini ta'riflang.*
10. *Tekis shakl qutb atrofida sekinlashuvchan aylanma harakatda bo'lish shartini ta'riflang.*

55-§. Tekis shaklning harakat tenglamalari, tekis shakl nuqtasining harakat tenglamalari, tekis shakl burchak tezligi va burchak tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

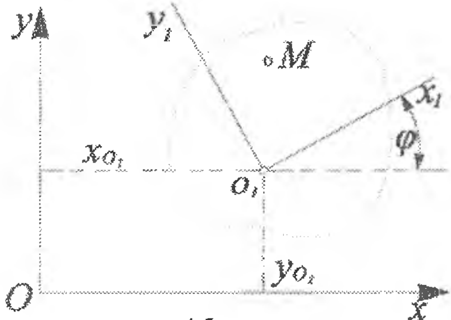
Tekis shaklning harakat tekisligida har qanday ko'chishi qutb bilan birgalikdagi ilgarilanma harakati hamda qutbdan harakat tekisligiga perpendikular ravishda o'tuvchi o'q atrofidagi aylanma harakatidan tashkil topadi.

Tekis shaklning harakat tekisligida qo'zg'alamas O nuqtani tanlab, Oxy qo'zg'almas sanoq sistemasini o'tkazamiz.

Qo'zg'aluvchan sanoq sistemasi sifatida markazi qutb deb tanlangan O_1 nuqtada bo'lgan va tekis shakl bilan bog'langan $O_1x_1y_1$ o'qlar sistemasi olinadi.

Bunday holda tekis shaklning harakat tenglamasi quyidagi ko'rinishda yoziladi:

$$x_{0_1} = f_1(t), \ y_{0_1} = f_2(t), \ \varphi = f_3(t). \qquad (4.6)$$



4.5-rasm

Yozilgan ifodalarda x_{01}, y_{01} — qutb sifatida tanlangan O_1 nuqtaning koordinatalari, φ — qo'zg'aluvchan o'qlar sistemasining qo'zg'almas o'qlar sistemasiga nisbatan burilish burchagi *(4.5-rasm)*.

(4.6) tenglamalar sistemasi tekis shaklning ixtiyoriy vaqt onidagi holatini aniqlashga imkon beradi.

Tekis shaklda olingan ixtiyoriy M nuqtaning harakat tenglamalari quyidagi ko'rinishda yoziladi:

$$\begin{cases} x = x_{0_1} + x_1 \cos\varphi - y_1 \sin\varphi, \\ y = y_{0_1} + x_1 \sin\varphi - y_1 \cos\varphi. \end{cases} \quad (4.7)$$

Bu tenglamalarda: x,y – M nuqtaning qo‘zg‘almas o‘qlar sistemasidagi koordinatalari; x_{o_1}, y_{o_1} qutb – O_1 nuqtaning koordinatalari, x_1, y_1 M nuqtaning tekis shakl bilan bog‘langan qo‘zg‘aluvchan o‘qlar sistemasidagi koordinatalari; φ – qo‘zg‘aluvchan o‘qlar sistemasining burilish burchagi.

Shuni ta’kidlash lozimki x_1, y_1 koordinatalar tekis shaklning harakati davomida doimo o‘zgarmas miqdor sifatida saqlanadi. qolgan barcha kattaliklar esa vaqtning funksiyalari hisoblanadi va (4.7) ifodalar orqali aniqlanadi.

Tekis shaklning harakat tenglamalarini, tekis shakl nuqtasining harakat tenglamasi va trayektoriyasini, burchak tezligi va burchak tezlanishini aniqlashda quyidagi tartibda amal qilish tavsiya etiladi:

1) qo‘zg‘almas va qo‘zg‘aluvchan o‘qlar sistemalari tanlab olinadi;

2) tekis shaklning harakat tenglamalari yoziladi;

3) tekis shakl nuqtasining harakat tenglamalari tuziladi;

4) tuzilgan harakat tenglamalaridan vaqtni qisqartirib, nuqta trayektoriyasining tenglamasi tuziladi;

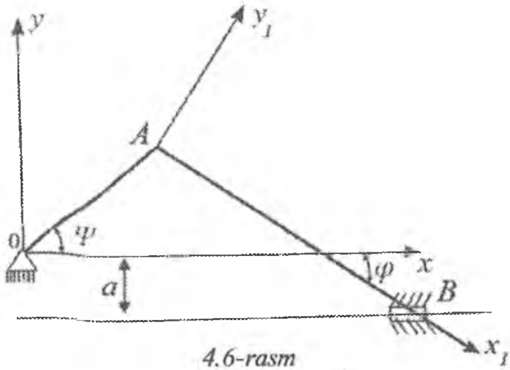
5) tekis shakl burilish burchagidan birinchi tartibli hosila olinib, burchak tezlik aniqlanadi;

6) tekis shakl burchak tezligidan birinchi tartibli hosila hisoblanib, burchak tezlanish aniqlanadi.

56-§. Tekis shaklning harakat tenglamalari, burchak tezligi va burchak tezlanishini aniqlashga doir masalalar

1-masala. Krivoship – polzunli mexanizmda krivoshipning aylanish markazi *B* polzun trayektoriyasidan *a* masofa uzoqlikda joylashgan.

Krivoship *O* nuqta atrofida $\Psi = kt$ qonunga muvofiq aylanadi, bunda *k* – doimiy koeffisiyent. Krivoship uzunligi $OA = r$, shatun uzunligi $AB = l$ ekanligini e’tiborga olib *(4.6-rasm), AB* shatun harakat tenglamasini aniqlang.



4.6-rasm

Yechilishi. Qo'zg'almas sanoq sistemasi sifatida markazi *O* nuqtada bo'lgan, *Ox* o'qi gorizontal holda o'ng tomon, *Oy* o'qi vertikal yuqoriga yo'nalgan o'qlar sistemasini tanlaymiz. Qo'zg'aluvchan sanoq sistemasi sifatida markazi *A* nuqtada bo'lgan, Ax_1 o'qi *AB* shatun bo'ylab, Ay_1 o'qi unga perpendikular holda yo'nalgan o'qlar sistemasi olinadi.

Shatun *A* nuqtasini qutb sifatida tanlaymiz. Qutbning harakat tenglamasi quyidagi ko'rinishda yoziladi:

$$x_A = OA\cos\Psi = r\cos kt,$$

$$y_A = OA\sin\Psi = r\sin kt.$$

Qutb *A* nuqtaning aylanma harakatini ifodalovchi tenglamani tuzish uchun *AB* kesmani *Oy* o'qiga proyeksiyalaymiz.

Agar Ax_1 va *Ox* o'qlar orasidagi burchakni φ orqali belgilasak, *4.6-rasmdan*

$$AB\sin\varphi = OA\sin\Psi + a.$$

Agar *AB*=l, *OA*=*r*, Ψ=*kt* ekanligini e'tiborga olsak,

$$\sin\varphi = \frac{r}{l}\sin kt + \frac{a}{l}.$$

Bu ifodadan *AB* shatun harakat tenglamalaridan uchinchisi quyidagi ko'rinishda yozilishi ma'lum bo'ladi:

$$\varphi = \arcsin\left(\frac{r}{l}\sin kt + \frac{a}{l}\right).$$

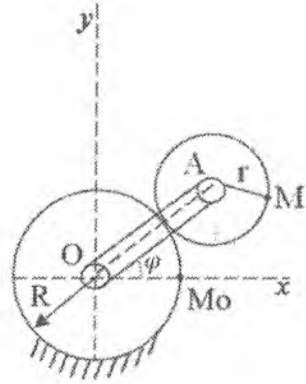
Natijada, *AB* shatun harakat tenglamalari quyidagi ko'rinishda yozilishi ma'lum bo'ladi:

$$x = r\cos kt,$$

$$y = r\sin kt,$$

$$\varphi = \arcsin\left(\frac{r}{l}\sin + \frac{a}{l}\right).$$

2-masala. *R* radiusli qo'zg'almas tishli g'ildirak bo'ylab dumalovchi *r* radiusli tishli g'ildirak *OA* krivoship bilan harakatga keltiriladi; krivoship qo'zg'almas tishli g'ildirakning *O* o'qi atrofida ε_0 burchak tezlanish bilan tekis tezlanuvchan aylanma harakat qiladi. Agar $t = 0$ da krivoshipning burchak tezligi $\omega_0 = 0$ va boshlang'ich aylanish burchagi $\varphi_0 = 0$ bo'lsa, qo'zg'aluvchan tishli g'ildirakning harakat tenglamalari tuzilsin; uning *A* markazi qutb deb qabul qilinsin *(4.7-rasm).*



4.7-rasm

Yechish:

qutb sifatida *A* nuqtani tanlab, uning koordinatalarini aniqlaymiz:

$$x_A = (R+r)\cos\varphi,$$

$$y_A = (R+r)\sin\varphi.$$

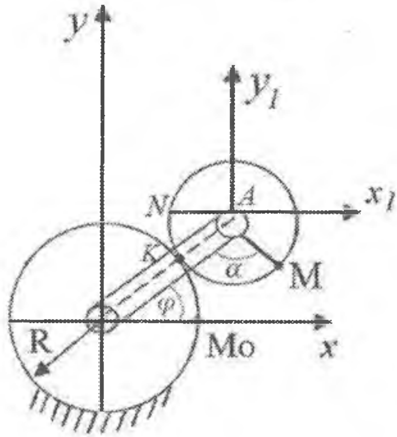
Qo'zg'aluvchan g'ildirak qo'zg'almas tishli g'ildirakning *O* o'qi atrofida ε_0 burchak tezlanish bilan aylanma harakat qiladi. Agar $t = 0$ da krivoshipning burchak tezligi $\omega_0 = 0$ bo'lsa, krivoshipning aylanish burchagi quyidagicha aniqlanadi:

$$\varphi = \frac{(\varepsilon_o t^2)}{2}.$$

Shuning uchun

$$x_A = (R + r)\cos\frac{(\varepsilon_o t^2)}{2},$$

$$y_A = (R + r)\sin\frac{(\varepsilon_o t^2)}{2}.$$



4.7-a rasm

Qo‘zg‘aluvchan g‘ildirakning harakat tenglamalaridan uchinchisini aniqlash uchun qo‘zg‘aluvchan g‘ildirakning qutb – *A* nuqta atrofida aylanish burchagini aniqlaymiz. Krivoshipning boshlang‘ich aylanish burchagi $\varphi_0=0$ bo‘lganda *M* nuqta M_O nuqta bilan ustma-ust tushadi.

G‘ildiraklar tegib turgan *K* nuqta hamda *N* nuqta ham boshlang‘ich paytda M_O nuqta bilan ustma-ust tushadi.

Shuning uchun qo‘zg‘aluvchan g‘ildirakning aylanish burchagi

$$\angle MAN = \beta = \alpha + \varphi. \qquad (4.8)$$

4.7-a rasmdan

$$\overset{\frown}{KM_0} = \overset{\frown}{KM}$$

yoki

$$R\varphi = r\alpha.$$

Bundan

$$\alpha = \frac{R\varphi}{r} \quad (4.9)$$

(4.9)ni (4.8)ga qo'ysak,

$$\beta = (1 + \frac{R}{r})\varphi$$

yoki

$$\beta = \left(1 + \frac{R}{r}\right)\frac{\varepsilon_0 t^2}{2}. \quad (4.10)$$

Shunday qilib, qo'zg'aluvchan g'ildirakning harakat tenglamalari quyidagi ko'rinishda yozilar ekan:

$$x_A = (R + r)\cos\frac{\varepsilon_o t^2}{2},$$

$$y_A = (R + r)\sin\frac{\varepsilon_o t^2}{2},$$

$$\beta = (1 + \frac{R}{r})\frac{\varepsilon_o t^2}{2}.$$

3-masala. *R* radiusli qo'zg'almas tishli g'ildirakning ichida dumalovchi r radiusli tishli g'ildirak *OA* krivoship bilan harakatga keltiriladi; krivoship qo'zg'almas tishli g'ildirakning *O* o'qi atrofida o'zgarmas ω_0 burchak tezlik bilan aylanadi. t=0 bo'lganda φ_0=0. *A* markazni qutb deb qabul qilib, qo'zg'aluvchan g'ildirakning harakat tenglamalari tuzilsin *(4.8-rasm).*

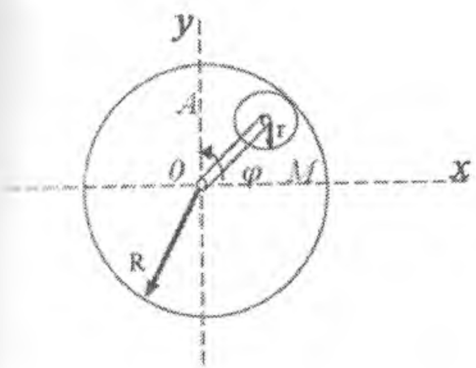
Yechish: qo'zg'aluvchan g'ildirakning harakat tenglamalarini tuzish uchun dastlab qutb deb qabul qilingan *A* nuqtaning koordinatalarini aniqlaymiz.

OAB uchburchakdan *(4.8-a rasm).*

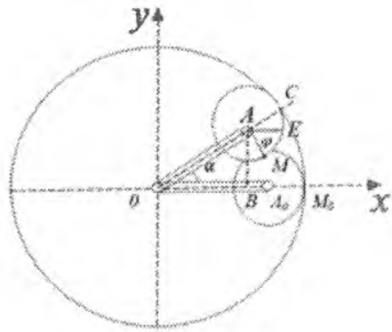
$$x_A = OB = OA\cos\alpha,$$

$y_A = AB = OA\sin\alpha$. Lekin $OA = OC - AC = R - r,$

$$\alpha = \omega t.$$



4.8-rasm



4.8 a-rasm

Shuning uchun

$$x_A=(R-r)\cos\omega t,$$

$$y_A=(R-r)\sin\omega t.$$

Qo'zg'aluvchan g'ildirak harakat tenglamalaridan uchinchisini tuzish uchun qo'zg'aluvchan g'ildirakning qutb A nuqta atrofida aylanish burchagi φ ni aniqlaymiz. Buning uchun boshlang'ich holati A_oM_o bo'lgan AM radiusning harakatini o'rganamiz.

4.8-a rasmda AE kesma x o'qiga parallel, binobarin, $\varphi = \angle EAM$.

Qo'zg'aluvchan g'ildirak qo'zg'almas tishli g'ildirak ichida sirpanmasdan aylanishi tufayli

$$\overset{\frown}{CM_0}=\overset{\frown}{CM}.$$

Lekin

$$\overset{\frown}{CM_0}=\overset{\frown}{R\alpha}.$$

Shuning uchun $r\ (\varphi+\alpha)=R\alpha$.

Bundan

$$\varphi=\left(\frac{R}{r}-1\right)\omega t.$$

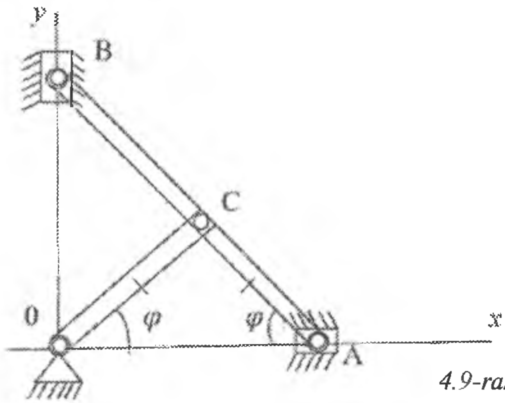
Shunday qilib, qo'zg'aluvchan g'ildirakning quyidagi ko'rinishdagi harakat tenglamalariga ega bo'lamiz:

$$x_A = (R - r)\cos\omega t,$$

$$y_A = (R - r)\sin\omega t,$$

$$\varphi = \left(\frac{R}{r} - 1\right)\omega t .$$

4-masala. Ellipsograf lineykasi *O* o'q atrofida ω_0 o'zgarmas burchak tezlik bilan aylanuvchi *OC* krivoship yordamida harakatga keltiriladi. *A* polzunni qutb deb qabul qilib, ellipsograf lineykasi tekislikka parallel harakatining tenglamasi topilsin. Masalada: *OC=BC=AC=r,* boshlang'ich paytda *AB* lineyka gorizontal joylashgan *(4.9-rasm).*



4.9-rasm

Yechish: ellipsograf lineykasining harakati davomida A nuqta *Ox* o'qi bo'ylab harakatlanadi. Shuning uchun y_A=0. Agar *AB* lineykani *Ox* o'qi bilan tashkil qiladigan burchagini φ orqali belgilasak, *AOC* teng yonli uchburchakdan:

$$AO = 2\cdot OC\cdot\cos\varphi$$

yoki

$$x_A = 2\cdot OC\cdot\cos\varphi.$$

AB lineyka harakat tenglamalaridan uchinchisini tuzish uchun φ burchakni aniqlaymiz. Krivoship o'zgarmas burchak tezlik bilan aylanishi tufayli

$$\varphi = \omega_0 t.$$

Shunday qilib, *AB* lineykaning harakat tenglamalarini quyidagicha ifodalaymiz:

$$x_A = 2r\cos\omega_{0t},$$
$$y_A = 0,$$
$$\varphi = \omega_0 t.$$

5-masala. $OABO_1$ antiparallelogrammning 00_1 katta zvenosiga qo'yilgan *OA* krivoship ω burchak tezlik bilan tekis aylanadi. Agar $OA=O_1B=a$ va $OO_1=AB=b$ (bunda $a<b$) bo'lsa, *A* nuqtani qutb deb olib, *AB* zvenoning harakat tenglamalari tuzilsin; boshlang'ich paytda *OA* krivoship OO_1 bo'ylab yo'nalgan *(4.10-rasm).*

Yechish: masala shartidan

$$OO_1 = OA\cos wt + AB\ \cos j - BO_1\cos(wt + j)$$

yoki

$$b = a\ \cos\omega t + b\ \cos\varphi - a\cos(\omega t + \varphi)\,. \qquad (4.11)$$

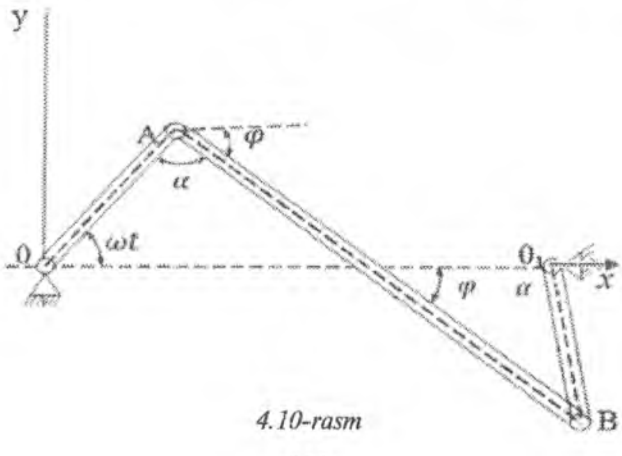
Tenglamani o'zgartirib yozamiz:

$$(b - a\ \cos\omega t)\cdot(1 - \cos\varphi) = a\sin\omega t \cdot \sin\varphi. \qquad (4.12)$$

4.10-rasmdan $\alpha = 180 - (\omega t + \varphi).$ (4.13)

(4.12)ni soddalashtiramiz:

$$(b - a\ \cos\omega t)\cdot 2\sin^2\frac{\varphi}{2} = a\sin\omega t \cdot 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}. \qquad (4.14)$$



4.10-rasm

Hosil bo'lgan tenglamani yechsak:

$$\operatorname{tg}\frac{\varphi}{2}=\frac{a\sin\omega t}{b-a\cos\omega t} \tag{4.15}$$

bo'ladi. Bundan *AB* zveno harakat tenglamalaridan biri — aylanma harakat tenglamasi kelib chiqadi:

$$\varphi=2\operatorname{arc\,tg}\frac{a\sin\omega t}{b-a\cos\omega t}. \tag{4.16}$$

Masala shartida A nuqta qutb sifatida qabul qilingan. *A* nuqtaning koordinatalarini aniqlaymiz:

$$x=a\cos\omega t,$$

$$y=a\sin\omega t. \tag{4.17}$$

(4.17) tenglamalar mexanizm *AB* zvenosining ilgarilanma harakat tenglamalarini ifodalaydi.

Shunday qilib, antiparallelogrammning *AB* zvenosining harakat tenglamalari quyidagi ko'rinishda yozilar ekan.

$$x_A=a\cos\omega t,$$

$$y_A=a\sin\omega t,$$

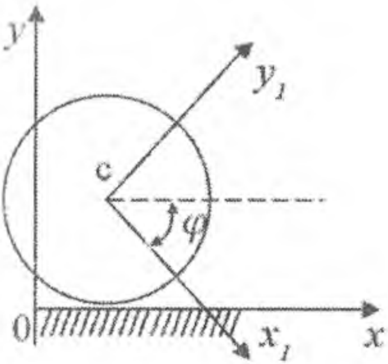
$$\varphi=2\operatorname{arc\,tg}\frac{a\sin\omega t}{b-a\cos\omega t}.$$

57-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

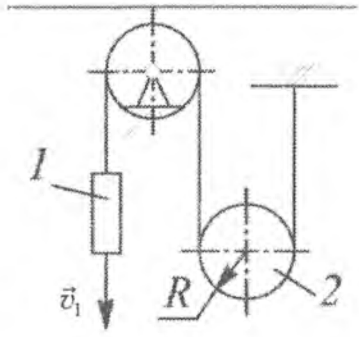
1-muammo. Radiusi *R* bo'lgan g'ildirak gorizontal to'g'ri chiziq bo'ylab sirpanmasdan g'ildiraydi. G'ildirak markazi *C* ning tezligi o'zgarmas va v ga teng. G'ildirak bilan bog'langan y_1 o'q boshlang'ich paytda vertikal bo'lib, qo'zg'almas *y* o'q shu paytda g'ildirakning *C* markazi orqali o'tadi.

G'ildirakning harakat tenglamalari aniqlansin. *C* nuqta qutb deb olinsin *(4.11-rasm).*

2-masala. Agar 1 yukning tezligi $v_1=0{,}5$ *m/s* bo'lsa, radiusi $R=0{,}1$ *m* bo'lgan qo'zg'aluvchan 2 blokning burchak tezligi qancha bo'ladi? *(4.12-rasm).*



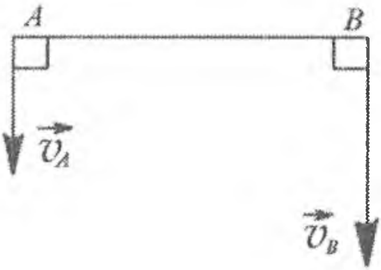
4.11-rasm



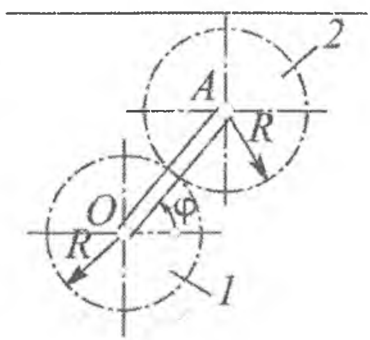
4.12-rasm

3-muammo. Uzunligi 80 *sm* bo'lgan *AB* sterjen shakl tekisligida harakat qilib, *A* va *B* nuqtalari v_A=0,2 m/s va v_B=0,6 m/s tezlikka ega bo'lsa, sterjenning burchak tezligini aniqlang *(4.13-rasm).*

4-muammo. *OA* krivoship φ=0,5t^2 qonun bo'yicha aylansa, 2-g'ildirakning burchak tezlanishini aniqlang *(4.14-rasm).*

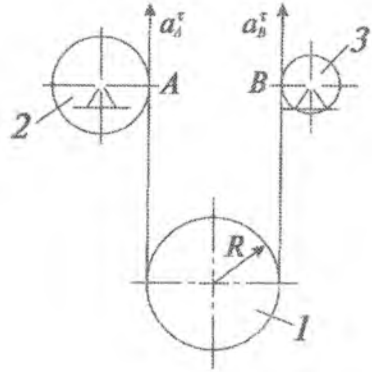


4.13-rasm



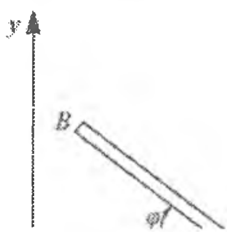
4.14-rasm

5-muammo. Qo'zg'almas 2- va 3-bloklarning *A* va *B* nuqtalari $a_A^\tau = 5m/s^2$ va $a_B^\tau = 10m/s^2$ tangensial tezlanishga ega bo'lsa, radiusi *R*=0,5 m li 1 qo'zg'aluvchan blokning burchak tezlanishini toping *(4.15-rasm).*



4.15-rasm

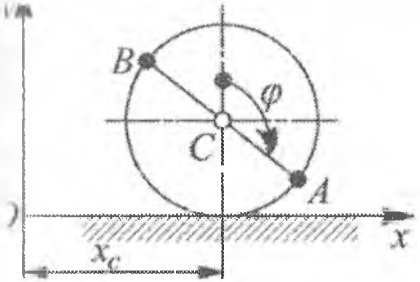
6-muammo. *AB* sterjen $x_A=2+t$, $y_A=0$, $\varphi=0{,}25\pi t$ tenglamalar asosida harakat qiladi. Agar $AB=3$ m bo'lsa, $t_1=1s$ paytda *B* nuqtaning x_B abssissasini hisoblang *(4.16-rasm).*



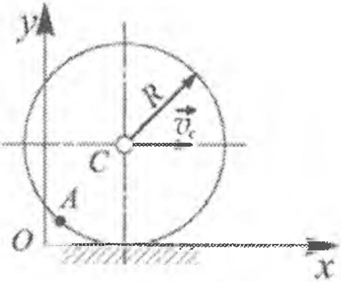
4.16-rasm

7-muammo. To'g'ri chiziqli yo'lda dumalayotgan g'ildirakning markazi $x_C=0{,}3\,t^2$, $y_C=0{,}15$ *m* qonun bo'yicha harakat qiladi. Agar boshlang'ich paytda *AB* to'g'ri chiziq *Oy* o'qi bilan ustma-ust tushgan bo'lsa, $t_1=1$ *s* paytda B nuqtaning ordinatasi y_B ni toping *(4.17-rasm).*

8-muammo. Radiusi $R=0{,}2$ m bo'lgan g'ildirak zarba ta'sirida dumalaydi. Uning markazi *C* o'zgarmas $v_c=0{,}1$ m/s tezlikka ega. Agar boshlang'ich paytda, $t_0=0$ da, g'ildirakning *A* nuqtasi koordinata boshi bilan ustma-ust tushsa, $t_1=1$ *s* paytda *A* nuqtaning abssissasini aniqlang *(4.18-rasm).*



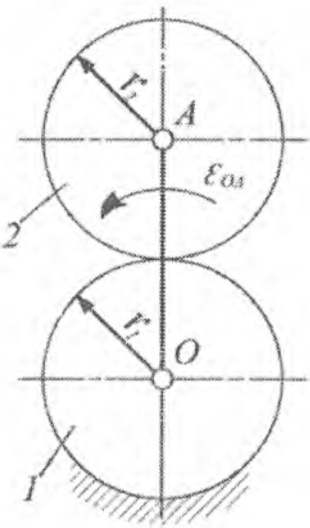
4.17-rasm



4.18-rasm

9-muammo. Radiusi R=10 sm bo'lgan g'ildirak to'g'ri chiziqli yo'lda dumalaydi. Uning markazi C o'zgarmasi=2π sm/s^2 tezlanishga ega. Agar harakat boshlangan paytda $v_c(0)=0$ bo'lsa, $t_1=10$ *s* da g'idirak necha marta dumalashga ulgiradi? *(4.18-rasm).*

10-muammo. Radiuslari teng $r_1=r_2=$ 10 sm ikkita shesternani bog'lab turuvchi *OA* krivoship tinch holatdan $\varepsilon_{OA}=0{,}1\pi$ burchak tezlanish bilan tekis aylana boshlaydi. Ikki shesterna 10 *s* ichida necha marta aylanadi? *(4.19-rasm).*



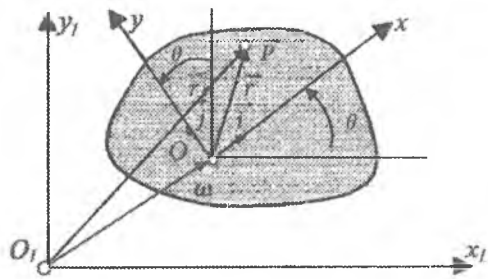
4.19-rasm

58-§. Tekis shakl nuqtasining tezligini qutb usulida aniqlash

Tekis shakl nuqtalarining tezliklari orasidagi bog'lanish quyidagi teorema yordamida ifodalanadi.

Teorema. Tekis shakl ixtiyoriy *P* nuqtasining tezligi qutb sifatida olingan *O* nuqtaning tezligi bilan mazkur nuqtaning qutb atrofidagi aylanma harakatidagi chiziqli tezligining geometrik yig'indisidan iborat bo'ladi *(4.20-rasm).*

Xorijiy o'quv adabiyotlarida mazkur teorema quyidagicha isbotlanadi.



4.20-rasm

Agar tekis shaklning o'z tekisligidagi Ox va O_1x_1 o'qlar orasidagi burchakni $\theta(t)$ orqali belgilasak, qo'zg'aluvchan Oxy o'qlar sistemasining birlik vektorlari quyidagicha ifodalanadi:

$$\vec{i} = \cos\theta\vec{i}_1 + \sin\theta\vec{j}_1,$$

$$\vec{j} = \sin\theta\vec{i}_1 + \cos\theta\vec{j}_1.$$

Tekis shaklning qutb atrofidagi aylanma harakati qutbdan shakl tekisligiga perpendikular bo'lgan O_z o'q atrofida yuzaga kelishini e'tiborga olsak, aylanma harakatning burchak tezligini quyidagicha aniqlash mumkin:

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = \dot{\theta}\vec{k}_1.$$

Bunday holda qutb sifatida olingan *O* nuqtaning tezligi quyidagi formula asosida yoziladi:

$$\vec{v}_0 = v_{Ox_1}\cdot\vec{i}_1 + v_{Oy_1}\cdot\vec{j}_1 = v_{Ox}\cdot\vec{i} + v_{Oy}\cdot\vec{j}.$$

Natijada, tekis shaklda olingan ixtiyoriy
teoremaga asosan, quyidagicha ifodalanadi:

$$\vec{v}_P = \vec{v}_o + \vec{\omega}\times\vec{r} = v_{Ox}\cdot\vec{i} + v_{Oy}\cdot\vec{j} + \omega k\ddot{y}\big((x$$

$$= \vec{i}\,((v_{Ox} - \omega y)) + \vec{j}\,((v_{Oy} + \omega x))$$

Bu ifodadan P nuqta tezligining qo'zg'luvchan Ox va
proeksiayalari quyidagi ifodalar orqali aniqlanishi ma'l

$$v_{Px} = v_{Ox} - \omega y,$$

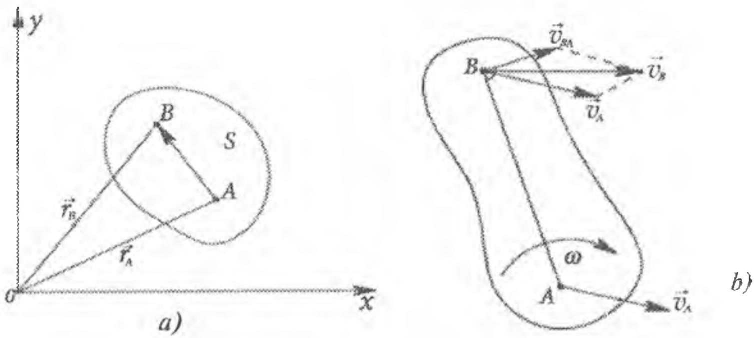
$$v_{Py} = v_{Oy} + \omega x.$$

Tekis shakl nuqtasining tezligini (4.18) formula asosida aniqlash qutb usulida aniqlash deyiladi.

Hozirgi kunda amalda bo'lgan darslik va o'quv qo'llanmalarda yuqorida bayon etilgan teorema quyidagi ko'rinishda isbotlanadi:

Tekis shakl ixtiyoriy B nuqtasining tezligi qutb sifatida olingan A nuqta tezligi bilan mazkur nuqtaning qutb atrofida aylana bo'ylab harakatidagi chiziqli tezligining geometrik yig'indisiga teng bo'ladi.

Tekis shakl harakatini qo'zg'almas Oxy koordinatalar sistemasiga nisbatan o'rganamiz. Agar A nuqta qutb sifatida olinsa, A va B nuqtalar radius vektorlari quyidagicha bog'lanadi *(4.21-a rasm).*



4.21-a, b rasm

$$\vec{r}_B = \vec{r}_A + \overrightarrow{AB}\,. \qquad (4.6)$$

Tezlik ta'rifiga ko'ra:

$$\vec{v}_B = \frac{d\vec{r}_B}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d(\overrightarrow{AB})}{dt}. \qquad (4.19)$$

Bunda

$$\frac{d\vec{r}_A}{dt} = \vec{v}_A, \qquad \frac{d\overrightarrow{AB}}{dt} = \vec{v}_{BA} = \vec{\omega}\times\overrightarrow{AB}. \qquad (4.20)$$

Shuning uchun

$$\vec{v}_B = \vec{v}_A + \vec{v}_{BA} = \vec{v}_A + \vec{\omega}\times\overrightarrow{AB}. \qquad (4.21)$$

Tekis shakl biror nuqtasining tezligi va tekis shakl aylanma harakatining burchak tezligi berilganda, tekis shakl boshqa biror nuqtasining tezligini (4.21) formula vositasida aniqlash, uni qutb usulida aniqlash deyiladi (4.21-b rasm).

59-§. Tekis shakl ikki nuqtasi tezliklarining proyeksiyalariga oid teorema

Teorema. Tekis shaklning ikkita nuqtasi tezliklarining shu nuqtalardan o'tuvchi o'qdagi proyeksiyalari o'zaro teng bo'ladi.

(4.21) dan

$$\vec{v}_B = \vec{v}_A + \vec{v}_{BA}. \qquad (4.22)$$

Teoremaga ko'ra, (4.22)ning har ikki tomonini Ax o'qiga proyeksiyalaymiz:

$$(\vec{v}_B)_x = (\vec{v}_A)_x + (\vec{v}_{BA})_x. \qquad (4.23)$$

Lekin

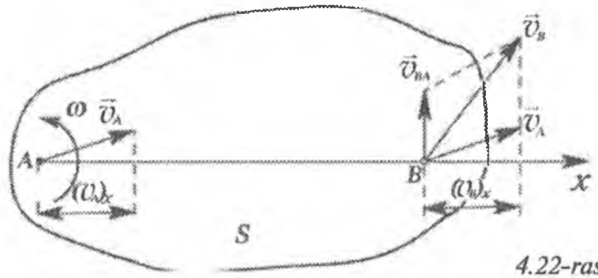
$$(v_{BA})_x = 0, \text{ chunki } \vec{v}_{BA} \perp Ax.$$

Shuning uchun *4.22-rasmdan*:

$$(v_B)_x = (v_A)_x. \qquad (4.24)$$

Mazkur teoremani isbotlashda 1-teoremadan foydalandik.

Tekis shaklning biror A nuqtasining tezligi, boshqa B nuqtasining tezligi yo'nalishi ma'lum bo'lganda, B nuqtaning tezligi miqdorini mazkur teoremadan aniqlash qulay bo'ladi.

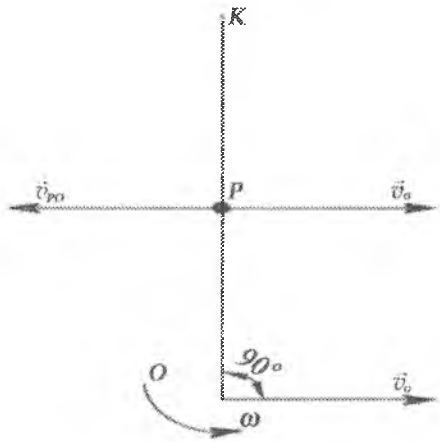


4.22-rasm

60-§. Tekis shakl nuqtalari tezliklarning oniy markazi

Tekis shaklning berilgan onda tezligi nolga teng bo'lgan nuqtasi tezliklar oniy markazi yoki aylanish oniy markazi deyiladi.

Agar tekis shaklning burchak tezligi noldan farqli bo'lsa, albatta, tezliklar oniy markazi mavjud bo'ladi *(4.23-rasm).*



4.23-rasm

Tekis shakl biror *O* nuqtasining tezligi $\vec{v}_o$ va shu *O* nuqta atrofidagi aylanma harakatning burchak tezligi ω berilgan bo'lsin. *O* nuqtani qutb deb olib, tekis shaklning aylanish yo'nalishida $\vec{v}_o$ga perpendikular *OK* chiziqni o'tkazamiz. *OK* chiziqda $OP = \frac{v_0}{\omega}$ tenglikka mos keluvchi P nuqtani olib, (4.22) formulaga asosan uning tezligini topamiz.

$$\vec{v}_P = \vec{v}_O + \vec{v}_{PO}. \tag{4.25}$$

Bunda

$$v_{PO} = \omega \cdot OP,\ \ OP = \frac{v_O}{\omega}$$

bo'lgani uchun

$$v_{PO} = \omega \cdot \frac{v_O}{\omega} = v_O,\ \ \vec{v}_{PO} = -\vec{v}_O.$$

U holda (4.25) dan $\vec{v}_p = 0$ bo'ladi. Demak, P nuqta tekis shakl nuqtalari tezliklarining oniy markazi ekan.

Berilgan onda tekis shakl nuqtalari tezliklarining oniy markazini qutb deb olsak, (4.22) formulaga asosan, tekis shakl *A, B, C* nuqtalarining tezliklari quyidagicha aniqlanadi:

$$\vec{v}_A = \vec{v}_P + \vec{v}_{AP};\ \ \vec{v}_B = \vec{v}_P + \vec{v}_{BP};\ \ \vec{v}_C = \vec{v}_P + \vec{v}_{CP}. \tag{4.26}$$

Lekin $\vec{v}_p = 0$.

Shuning uchun

$$\vec{v}_A = \vec{v}_{AP};\ \ \vec{v}_B = \vec{v}_{BP};\ \ \vec{v}_C = \vec{v}_{CP}. \tag{4.27}$$

Bunda

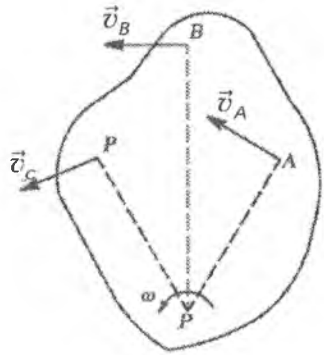
$$v_A = \omega \cdot AP,\ \ v_B = \omega \cdot BP,\ \ v_C = \omega \cdot CP \tag{4.28}$$

va

$$\vec{v}_A \perp \overrightarrow{AP}, \vec{v}_B \perp \overrightarrow{BP}, \vec{v}_C \perp \overrightarrow{CP}.$$

Demak, biror onda tezliklarining oniy markazi ma'lum bo'lgan tekis shakl nuqtalarining tezliklarini oniy markaz atrofida aylanma harakatdagi nuqtalarining tezliklari kabi topish mumkin ekan. Agar tezliklar oniy markazi tekis shakl konturidan tashqarida yotsa, tezliklar oniy markazi uchun tekis shaklga biriktirilgan tekislikning nuqtasi olinadi. *(4.24-rasm)*

(4.28)dan tekis shakl nuqtalarining ayni paytdagi tezliklari orasidagi quyidagi munosabatni aniqlash mumkin:



4.24-rasm

$$\frac{v_A}{AP} = \frac{v_B}{BP} = \frac{v_C}{CP}. \qquad (4.29)$$

Demak, tekis shakl nuqtalarining tezliklari, shu nuqtalardan tezliklar oniy markazigacha bo‘lgan masofalarga to‘g‘ri proporsional bo‘lar ekan *(4.24-rasm)*.

61-§. Ba’zi hollarda tezliklarning oniy markazini aniqlash

1) Tekis shakl biror *A* nuqtasining tezligi $\vec{v}_A$ va *B* nuqtasining tezligi yo‘nalishi ma’lum bo‘lsin. Bunday holda tekis shakl nuqtalari tezliklarining oniy markazi *A* va *B* nuqtalar tezliklariga o‘tkazilgan perpendikularlarning kesishgan nuqtasida bo‘ladi *(4.25-rasm)*.

A nuqta tezligining moduli ma’lum bo‘lgani uchun, (4.28) dan tekis shaklning burchak tezligini aniqlaymiz:

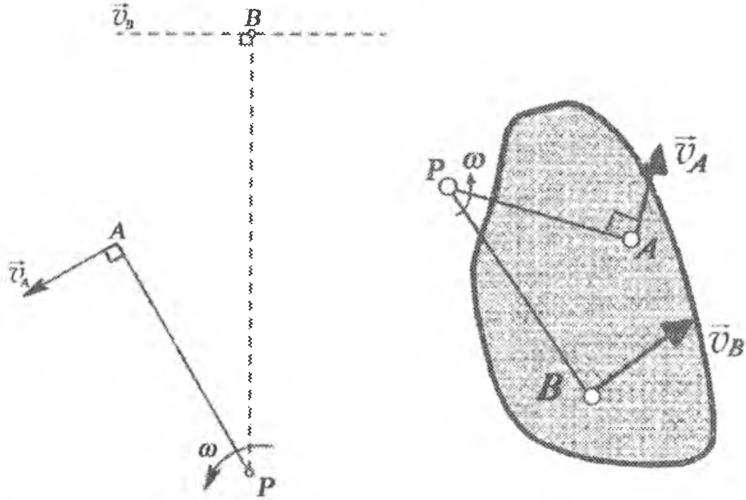
$$\omega = \frac{v_A}{AP}, \qquad (4.30)$$

AP masofa chizmadan aniqlanadi.

U paytda *B* nuqta tezligining miqdori quyidagiga teng bo‘ladi:

$$v_B = \omega \cdot BP.$$

2) Tekis shakl *A* va *B* nuqtalarining tezliklari parallel va *AB* kesmaga perpendikular bo‘lsa, tezliklarning oniy markazini aniqlash uchun tezliklar moduli ham ma’lum bo‘lishi kerak.

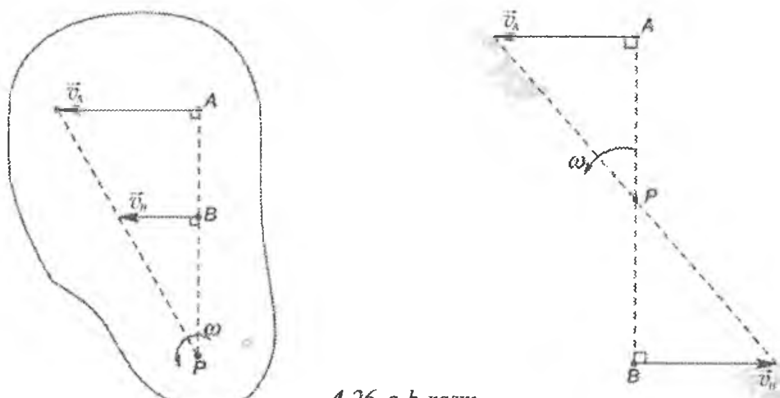


4.25-rasm

(4.29) ga ko'ra:

$$\frac{v_B}{v_A} = \frac{BP}{AP} \tag{4.31}$$

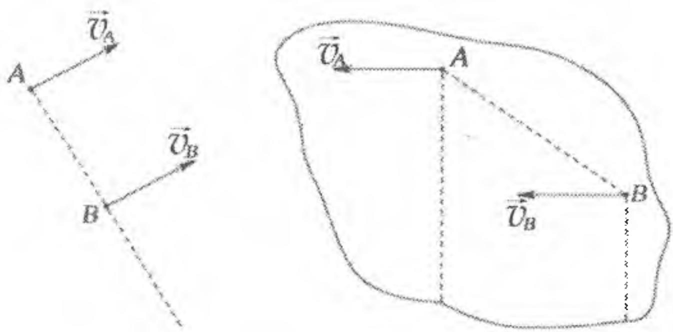
Shuning uchun ham, *A* va *B* nuqtalar tezliklarining uchlari oniy markaz orqali o'tuvchi chiziqda yotadi. Shu chiziqning *AB* chiziq bilan kesishgan nuqtasi tezliklar oniy markazini ifodalaydi *(4.26-a, b rasmlar).*



4.26-a,b rasm

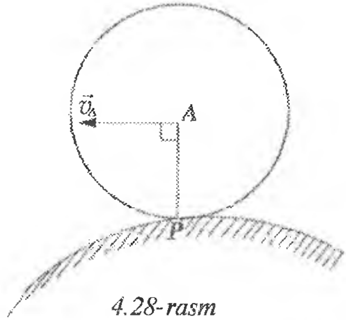
Agar tekis shakl *A* va *B* nuqtalarining tezliklari o'zaro teng va parallel yo'nalgan bo'lsa, u holda tezliklar oniy markazi cheksizlikda bo'ladi $(AP = \infty)$.

Tekis shakl burchak tezligi bunday holda nolga teng bo'lib, u berilgan onda ilgarilanma harakatda bo'ladi *(4.27-a, b rasmlar)*:



4.27-a, b rasm

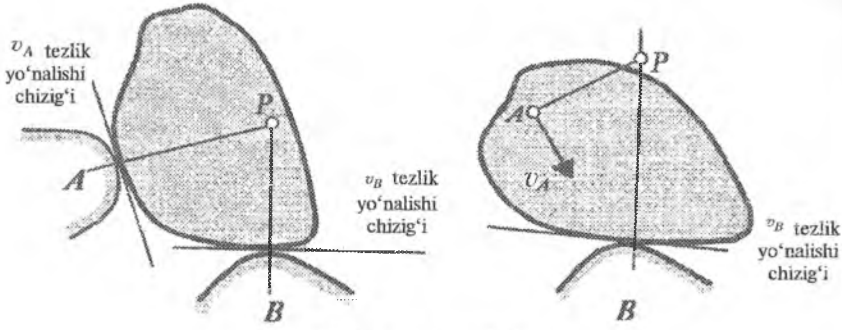
3) Tekis shakl konturi biror qo'zg'almas chiziq ustida sirpanmasdan dumalasa, tekis shakl konturining qo'zg'almas chiziqqa tegib turgan nuqtasining tezligi nolga teng bo'ladi. Shuning uchun oniy markaz shu urinish nuqtasida yotadi *(4.28-rasm)*.



4.28-rasm

4) Tekis shakl konturi *A* va *B* qo'zg'almas *(4.29-rasm)* yoki *B* qo'zg'almas *(4.29-b rasm)* chizma ustida sirpanmasdan dumalasa.

Shakl tezliklarining oniy markazi A va B nuqtalari tezliklarining yo'nalishlariga o'tkazilgan perpendikularlarning kesishgan nuqtasida bo'ladi *(4.29-a, b rasmlar).*



(4.29-a, b rasmlar)

62-§. Tekislikka parallel harakatdagi jism nuqtalarining tezliklarini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Umuman, tekis shakl nuqtalarining tezliklarini quyidagi 3 usulda aniqlash mumkin:

1. Analitik usul.
2. Grafik usul.
3. Grafoanalitik usul.

Mazkur o'quv qo'llanmada tekis shakl nuqtalarining tezliklarini aniqlashning grafoanalitik usuli bilan tanishamiz.

Grafoanalitik usulning o'zi ham ikki yo'ldan iborat.

a) Tekis shakl nuqtalarining tezliklarini qutb usulida aniqlash.

Bu usulda tekis shakl nuqtalarining tezliklari quyidagicha aniqlanadi.

1. Tezligi ma'lum yoki masala shartiga ko'ra aniqlanishi mumkin bo'lgan tekis shakl nuqtasi qutb sifatida tanlanadi.

2. Tekis shaklda tezligining yo'nalishi ma'lum bo'lgan boshqa nuqta aniqlanadi.

3. Bu nuqtaning tezligi tekis shakl nuqtalarining tezliklari haqidagi teorema asosida hisoblanadi.

4. Tekis shaklning shu vaqt onidagi burchak tezligi aniqlanadi.

5. Tekis shakl burchak tezligini bilgan holda, yuqorida bayon etilgan tekis shakl nuqtalarining tezliklari haqidagi teoremadan, tekis shaklning so'ralgan nuqtasining tezligi aniqlanadi.

b) Tekis shakl nuqtalarining tezliklarini tezliklarning oniy markazi orqali aniqlash.

Bu usulda tekis shakl nuqtalarining tezliklari quyidagicha aniqlanadi:

1. Tekis shakl nuqtalari tezliklarining oniy markazi aniqlanadi.

2. Tekis shaklning tezligi ma'lum bo'lgan nuqtasining oniy radiusi aniqlanadi va tezlik modulini oniy radiusga bo'lib, tekis shaklning burchak tezligi topiladi.

3. Tekis shaklning burchak tezligini bilgan holda, so'ralgan nuqtaning tezligi aniqlanadi.

Takrorlash uchun savollar:

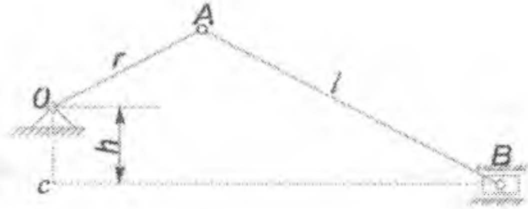
1. *Tekis shakl nuqtalarining tezliklari orasidagi bog'lanishni ta'riflang.*
2. *Tekis shakl nuqtasining tezligini qutb usulida aniqlash deb qanday usulga aytiladi?*
3. *Tekis shakl ikki nuqtasi tezliklarining proyeksiyalariga oid teoremani ta'riflang.*
4. *Tezliklar oniy markazi deb qanday nuqtaga aytiladi?*
5. *Tekis shakl nuqtalari tezliklarining oniy markazini aniqlash hollarini ko'rsating.*
6. *Agar tekis shakl A va B nuqtalarining tezliklari teng va parallel yo'nalgan bo'lsa, tezliklar oniy markazi qayerda joylashadi?*

63-§. Tekislikka parallel harakatdagi jism nuqtalarining tezliklarini aniqlashga doir masalalar

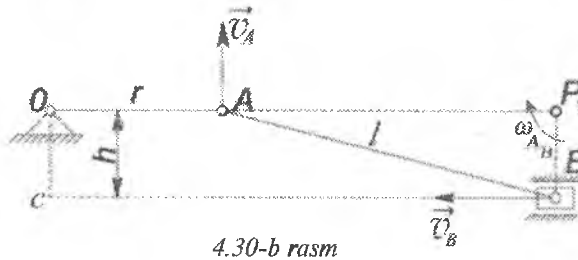
1-masala. *O* val $\omega=1{,}5$ *rad/s* burchak tezlik bilan aylanuvchi krivoshipning ikkita gorizontal va ikkita vertikal holatida markaziy bo'lmagan krivoship mexanizmi *B* polzuni tezligining qancha bo'lishi topilsin; $OA=40$ *sm*, $AB=200$ *sm*, $OC=20$ *sm (4.30-a rasm).*

Yechish:

1. Krivoshipning birinchi gorizontal holatida *B* polzunning tezligi qancha bo'lishini aniqlaymiz *(4.30-b rasm).*



4.30-a rasm



4.30-b rasm

Krivoship A nuqtasining tezligi:

$$v_A = \omega \cdot OA = 1{,}5 \cdot 40 = 60\,sm/s;\ \vec{v}_A \perp \vec{OA}\,.$$

B polzun tezligini aniqlash uchun AB shatun nuqtalari tezliklarining oniy markazini aniqlaymiz. Buning uchun $\vec{v}_A$ va $\vec{v}_B$ yo'nalishlariga perpendikular chiziqlar o'tkazib, ularning kesishish nuqtasi P_1 ni topamiz. P_1 nuqta AB shatun nuqtalari tezliklarining oniy markazini ifodalaydi. P_1 nuqta qutb sifatida qabul qilinsa, krivoship A nuqtasining tezligi quyidagicha yoziladi:

$$v_A = \omega_{AB} \cdot AP_1.$$

Bundan

$$\omega_{AB} = \frac{v_A}{AP_1}.$$

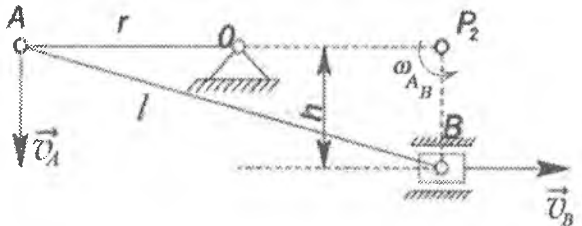
Bunday holda

$$v_B = \omega_{AB} \cdot BP_1.$$

Agar $BP_1 = h$ ekanligini e'tiborga olsak:

$$v_B = \frac{v_A}{AP_1} \cdot h = \frac{60}{\sqrt{\ell^2 - n^2}} \cdot h = 6{,}03\ sm/s\,.$$

2. Xuddi shunday mulohazalar asosida, krivoshipning ikkinchi gorizontal holatida B polzunning tezligini aniqlaymiz *(4.30-c rasm).*

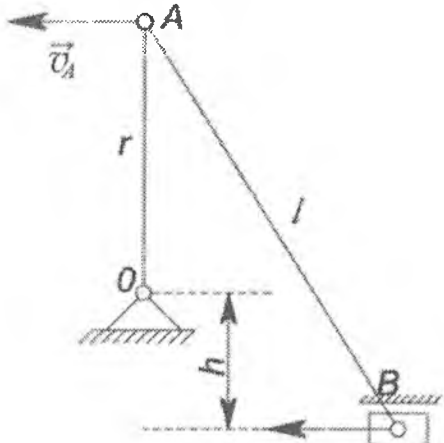


4.30-c rasm

4.30-c rasmdan ko'rinib turibdiki, krivoshipning ikkinchi gorizontal holatida ham B polzun tezligining miqdori 1 gorizontal holatidagi tezligiga teng bo'lar ekan:

$$v_B = 6{,}03\, sm/s.$$

3. Yuqorida keltirilgan mulohazalar asosida, krivoshipning birinchi vertikal holatida B polzun tezligining miqdorini aniqlaymiz *(4.30-d rasm).*



4.30-d rasm

Krivoshipning bunday vertikal holatida B polzun tezligining miqdori krivoship A nuqtasining tezligiga teng bo'ladi:

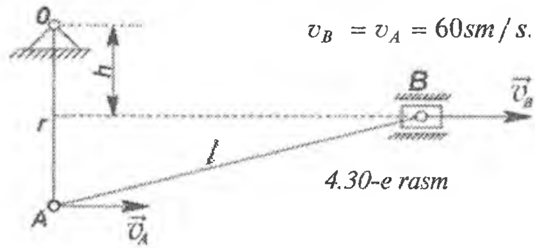
$$v_B = v_A = 60 sm/s.$$

Chunki krivoshipning bunday holatida *AB* polzun oniy ilga-rilanma harakatda bo'ladi:

$$AP_3 = \infty, \ \omega_{AB} = \frac{v_A}{AP_3} = 0.$$

4. Xuddi shunday hol krivoshipning ikkinchi vertikal holatida ham yuzaga keladi *(4.30-e rasm)*. Shuning uchun bu holatda ham:

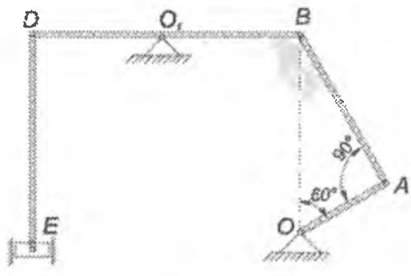
$$AP_3 = \infty, \quad \omega_{AB} = 0, \ v_{B4} = v_{B3} = 60 \ sm/s.$$



$$v_B = v_A = 60 sm/s.$$

4.30-e rasm

2-masala. *OA* krivoship 2 *rad/s* burchak tezlik bilan bir tekis aylanadi. Agar *OA*=20 *sm*, $O_1B = O_1D$ bo'lsa, rasmda ko'rsatilgan holat uchun nasosning uzatmali mexanizmi *E* porshenining tezligi aniqlansin *(4.31-a rasm)*.

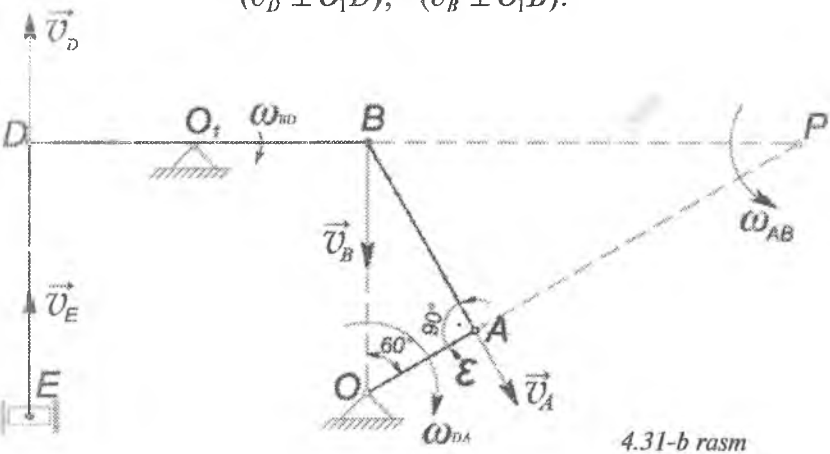


4.31-a rasm

Yechish: *A* nuqtaning tezligi $\vec{v}_A$ *OA* krivoshipga perpendikular holda ω_{OA} tomon yo'naladi ($\vec{v}_A \perp \overline{OA}$) *(4.31-b rasm)*.

BD krivoship O_1 nuqta atrofida aylanishi tufayli *D* nuqtaning tezligi v_D O_1D kesmaga perpendikular holda, *B* nuqtaning tezligi $\vec{v}_B$ esa O_1B kesmaga perpendikular holda ω_{BD} tomon yo'naladi:

$$(\vec{v}_D \perp O_1\bar{D}), \quad (\vec{v}_B \perp O_1\bar{B}).$$



4.31-b rasm

Shuning uchun:

$$\omega_{BD} = \frac{v_B}{O_1B} = \frac{v_D}{O_1D}.$$

Lekin $O_1B = O_1D$ bo'lganligi uchun, $v_B = v_D$.
Rasmda *BD* krivoship gorizontal holatda bo'lganligi uchun

$$\vec{v}_D // \vec{v}_E; \ \vec{v}_D = \vec{v}_E.$$

Binobarin,

$$v_E = v_B.$$

Demak, *E* nuqtaning tezligini aniqlash uchun *B* nuqtaning tezligini aniqlash yetarli ekan.

B nuqtaning tezligini aniqlash uchun berilgan mexanizm *AB* qismining harakatini o'rganamiz. *AB* qism nuqtalari tezliklarining oniy markazi $\vec{v}_A$ va $\vec{v}_B$ vektorlarga o'tkazilgan perpendikularlarning kesishish nuqtasi *P* hisoblanadi *(4.31-b rasm).*

Shuning uchun

$$\omega_{AB} = \frac{v_A}{AP} = \frac{v_B}{BP}.$$

Bundan

$$v_B = \frac{v_A}{AP} \cdot BP.$$

Agar

$$v_A = \omega_{OA} \cdot OA = 2 \cdot 20 = 40\, sm/s\ ;$$

$$OB = \frac{AO}{\cos 60} = 2 \cdot AO = 40 sm;$$

$$OP = \frac{OB}{\cos 60} = 2 \cdot OB = 80\ sm;$$

$$AP = OP - OA = 80 - 20 = 60\ sm;$$

$$BP = OB \operatorname{tg} 60^\circ = 40\sqrt{3} = 69,3\, sm$$

ekanligini e'tiborga olsak,

$$v_B = \frac{v_A}{AP} \cdot BP = \frac{40}{60} \cdot 69,3 = 46,2\ sm/s\ .$$

Demak,

$$v_E = v_B = 46,2\ sm/s\ .$$

3-masala. Sharnirli $OABO_1$ parallelogrammning AB sterjeni o'rtasida D nuqtaga K polzunni ilgarilanma-qaytma harakatga keltiruvchi DE sterjen sharnir yordamida birlashtirilgan. Agar OA=O_1B=$2DE$=20 *sm* bo'lsa, mexanizmning rasmda tasvirlangan holati uchun K polzunning tezligi va DE sterjenning burchak tezligi aniqlansin; OA zvenoning berilgan paytdagi burchak tezligi 1 *rad/s.*

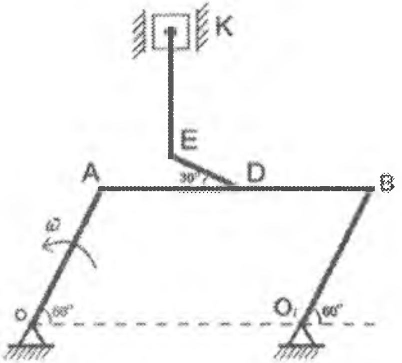
Yechish: chizmada sharnirli parallelogrammning A va B nuqtalari tezliklarining yo'nalishlarini ko'rsatamiz *(4.33-rasm)*:

Sharnirli parallelogrammda

$$OA = O_1B.$$

Shuning uchun sharnirli parallelogrammning harakati davomida OA va O_1B krivoshiplarning burilish burchaklari o'zaro teng bo'ladi. Keltirilgan mulohazalar sharnirli parallelogrammda AB sterjen tekislikka parallel harakatda bo'lishini e'tirof etadi.

Bu hol $v_A = v_B$ bo'lishini taqozo etadi.



4.32-rasm

D nuqta ham *AB* sterjinga taalluqli. Shuning uchun

$$\vec{v}_D = \vec{v}_A = \vec{v}_B\,;$$

$$\vec{v}_D \parallel \vec{v}_A \parallel \vec{v}_B\,.$$

Agar $\angle AED = 30°$ ekanliginni e'tiborga olsak, *D* nuqtaning tezligi $\vec{v}_D$ *DE* shatun bo'ylab yo'nalishi ma'lum bo'ladi. Mexanizmda *K* polzun vertikal yo'nalishda harakatlanadi. *K* va *E* nuqtalar *KE* sterjenga taalluqli. *KE* sterjin vertikal yo'nalishda ilgarilanma-qaytalanma harakatda bo'lishi tufayli *K* nuqtaning tezligi *E* nuqta tezligiga teng bo'ladi.

$$\vec{v}_K = \vec{v}_E\,.$$

KE sterjen *E* nuqtasining tezligini aniqlash uchun *DE* shatunning harakatini o'rganamiz. *DE* shatun shakl tekisligida harakatlanadi.

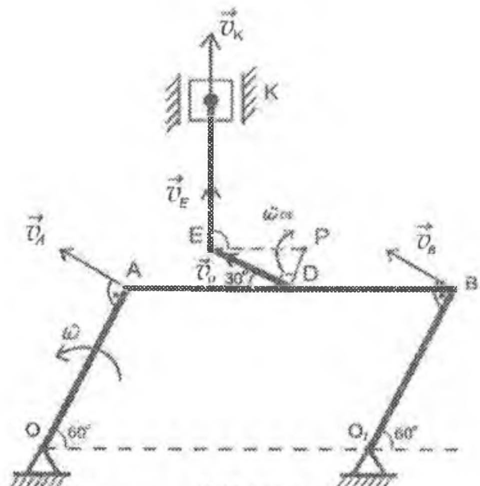
DE shatun nuqtalari tezliklarining vektorlariga perpendikular chiziqlar tushiramiz. Ularning kesishish nuqtasi tezliklarning oniy markazini ifodalaydi *(4.33-rasm).*

Bunday holda DE shatun burchak tezligi quyidagicha aniqlanadi:

$$\omega_{DE} = \frac{v_D}{DP} = \frac{v_E}{EP}\,.$$

Bunda

$$v_D = v_A = \omega_{OA} \cdot OA = 1 \cdot 20 = 20\,\frac{rad}{s}\,.$$



4.33-rasm

$$DP = DE\ \mathrm{tg}30° = \frac{10\sqrt{3}}{3} = 5{,}8\ sm.$$

$$EP = \frac{DE}{\cos 30°} = \frac{10 \cdot 2 \cdot 10}{\frac{\sqrt{3}}{2}\sqrt{3}} = 11{,}5\ sm.$$

Aniqlangan kattaliklarni hisobga olsak,

$$\omega_{DE} = \frac{v_D}{DP} = \frac{20}{5{,}8} = 3{,}5\frac{1}{s}.$$

4-masala. 1-qo'zg'aluvchi va 2-qo'zg'almas bloklar cho'zilmaydigan ip bilan bog'langan. Ipning uchiga biriktirilgan *K* yuk $x_k = 2t^2$ m qonun bilan vertikal bo'ylab pastga tushadi. $t = 1$ *s* bo'lgan paytda rasmda tasvirlangan holat uchun harakatlanuvchi blok gardishida yotuvchi *C, D, B* va *E* nuqtalarning tezliklari topilsin; qo'zg'aluvchi 1-blok radiusi 0,2 *m* ga teng, $CD \perp BE$.

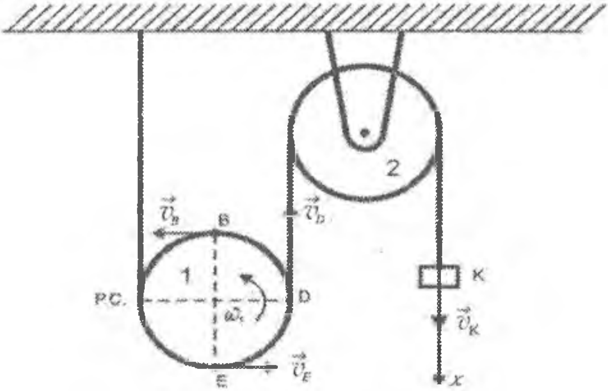
Shuningdek, 1-blokning burchak tezligini ham toping *(4.34-rasm).*

Yechish: *K* yuk tezligini aniqlaymiz:

$$v_K = \frac{dx_K}{dt} = (2t^2)^1 = 4t,$$

$$t_1 = 1\ s \text{ da} \quad v_K = 4 \cdot 1 = 4\ m/s.$$

$\vec{v}_K$ K nuqtaga qo'yilgan bo'lib, vertikal pastga yo'nalgan.



4.34-rasm

Chizmadan harakatlanuvchan 1-blok D nuqtasining tezligi K yuk tezligiga teng bo'lishi ma'lum:

$$v_D = v_K = 4\ m/s.$$

1-blok uchun C nuqta tezliklar oniy markazini ifodalaydi. Shuning uchun

$$v_C = 0.$$

Bunday holda 1-blok burchak tezligi:

$$\omega_1 = \frac{v_D}{2R_1} = \frac{4}{2 \cdot 0{,}2} = 10\frac{1}{s}.$$

ω_1 ning yo'nalishi $\vec{v}_D$ yo'nalishi orqali aniqalanadi.

1-qo'zg'aluvchi blok B va E nuqtalarining tezliklari mazkur nuqatalarning tezliklar oniy markazi C nuqta atrofidagi aylanma harakat tezligi kabi topiladi:

$$v_B = \omega \cdot CB,$$

$$v_E = \omega \cdot CE.$$

Bunda

$$CB = R_1\sqrt{2} = 0,2 \cdot 1,41 = 0,28\ m,$$

$$CE = R_1\sqrt{2} = 0,2 \cdot 1,41 = 0,28\ m$$

ekanligini e'tiborga olsak,

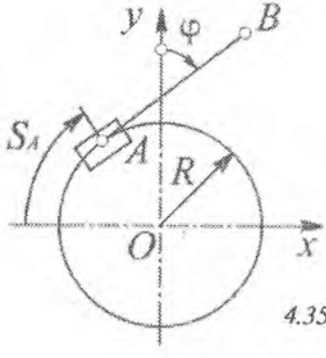
$$\upsilon_B = 10 \cdot 0,28 = 2,8 m/s,$$

$$\upsilon_E = 10 \cdot 0,28 = 2,8 m/s$$

bo'ladi. $\vec{\upsilon}_B$ va $\vec{\upsilon}_E$ lar yo'nalishlari 4.34-rasmda ko'rsatilgan.

64-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo. *AB* sterjenning A nuqtasi radiusi R=1 m bo'lgan aylana bo'ylab S_A=1,05t qonun bo'yicha harakat qiladi. Bir vaqtning o'zida sterjen φ =t qonun bilan aylanadi. Agar sterjenning uzunligi

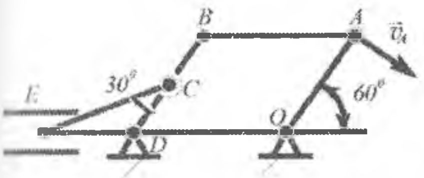


4.35-rasm

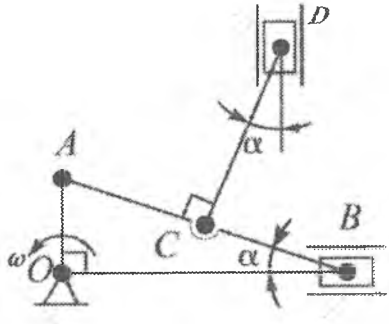
AB=1 m bo'lsa, t_1=1 s paytda uning *B* nuqtasi tezligining *Oy* o'qiga proyeksiyasini aniqlang *(4.35-rasm).*

2-muammo. Sharnirli parallelogramm *OABD* ga *CE* shatun biriktirilgan bo'lib, uning uchida *E* polzun harakatlanadi. Agar *A* nuqtaning tezligi 0,4 m/s va parallelogrammning o'lchamlari *OA*=*BD*=20 sm, *BC*=*BD*/2 bo'lsa, *E* polzunning tezligini toping *(4.36-rasm).*

3-muammo. Uzunligi 0,2 m bo'lgan *OA* krivoship $\omega = 8\ rad/s$ burchak tezlik bilan tekis aylanadi. *AB* shatunning *C* nuqtasiga *CD*



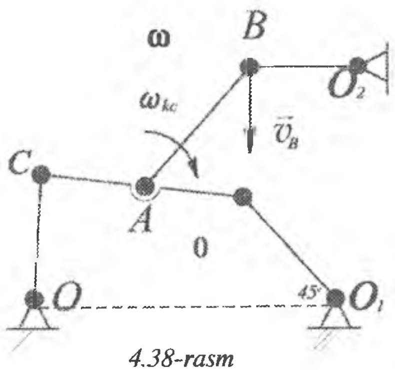
4.36-rasm



4.37-rasm

shatun biriktirilgan. Berilgan holat, α=20° uchun, *D* polzunning tezligini aniqlang *(4.37-rasm).*

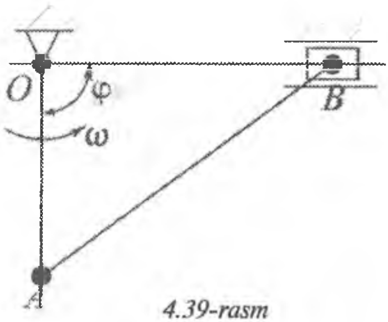
4-muammo. Berilgan paytda *B* nuqtaning tezligi 20 *m/s* va *AB* zvenoning burchak tezligi 10 *rad/s* bo‘lsa, *B* nuqtadan *AB* zvenoning tezliklar oniy markazigacha bo‘lgan masofani aniqlang *(4.38-rasm).*



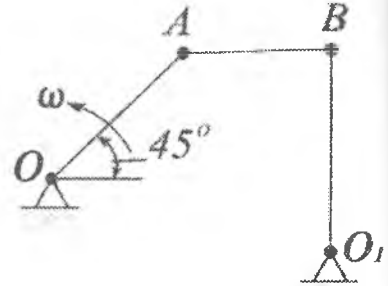
4.38-rasm

5-muammo. Mexanizmning *OA* krivoshipi tekis aylanma harakat qilib, *OB* yo‘nalishga φ = 90° burchak tashkil qilgan paytda *B* polzundan *AB* zveno tezliklari oniy markazigacha bo‘lgan masofani toping *(4.39-rasm).*

6-muammo. Uzunligi *AB*=0,6 *m* bo‘lgan mexanizmning krivoshipi ω =10 *rad/s* burchak tezlik bilan aylansa, shaklda ko‘rsatilgan holat uchun *A* nuqtada *AB* sterjenning tezliklar oniy markazigacha bo‘lgan masofani toping *(4.40-rasm).*

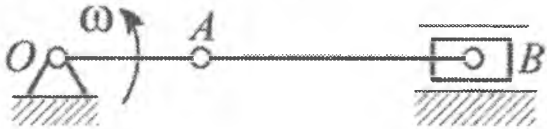


4.39-rasm



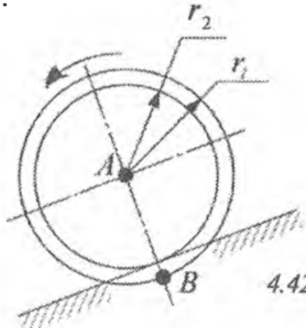
4.40-rasm

7-muammo. Mexanizmning *OA* krivoshipi o'zgarmas burchak tezlik ω bilan aylanadi. Agar krivoshipning uzunligi OA=80 *mm*, shatunning uzunligi esa AB=160 *mm* bo'lsa, *A* nuqtadan *AB* zvenoning tezliklar oniy markazigacha bo'lgan masofani toping *(4.41-rasm).*



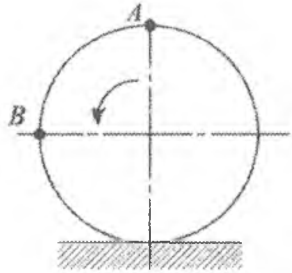
4.41-rasm

8-muammo. Tekislik bo'ylab sirpanmasdan dumalayotgan pog'onali g'ildirakning radiuslari r_1=0,6 *m* va r_2=0,5 *m* bo'lib, *A* nuqtasining tezligi v_A=2 *m/s* bo'lsa, *B* nuqtasining tezligini aniqlang *(4.42-rasm).*



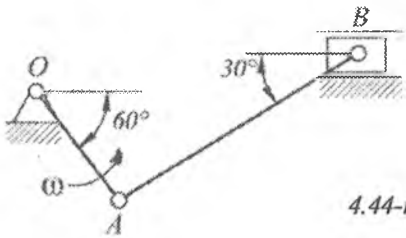
4.42-rasm

9-muammo. Tekislik bo'ylab sirpanmasdan dumalayotgan g'ildirakning *A* nuqtasi 2 *m/s* tezlikka ega bo'lsa, *B* nuqtasining tezligini toping *(4.43-rasm).*



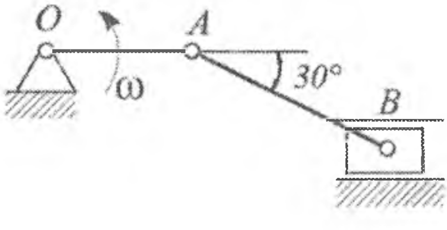
4.43-rasm

10-muammo. Krivoship-polzunli mexanizim shatunining uzunligi *AB*=1 *m* bo'lgan, *A* nuqtasining tezligi $v_A = 3m/s$ bo'lsa, ko'rsatilgan holat uchun *AB* shatunning burchak tezligini aniqlang *(4.44-rasm).*



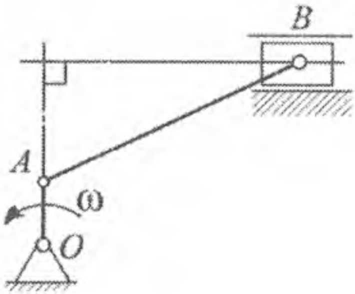
4.44-rasm

11-muammo. Krivoship-polzunli mexanizim shatunining uzunligi *AB*=3 *m* bo'lgan, *A* nuqtasining tezligi $v_A = 3m/s$ bo'lsa, ko'rsatilgan holat uchun *AB* shatunning burchak tezligini toping *(4.45-rasm).*



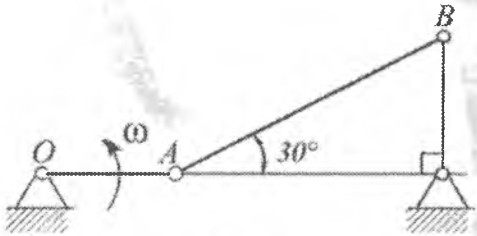
4.45-rasm

12-muammo. Krivoship-polzunli mexanizm krivoshipining uzunligi *OA*=0,1 *m* bo'lib, polzunning tezligi v_B=2 *m/s* bo'lsa, *OA* krivoshipning burchak tezligini aniqlang *(4.46-rasm).*



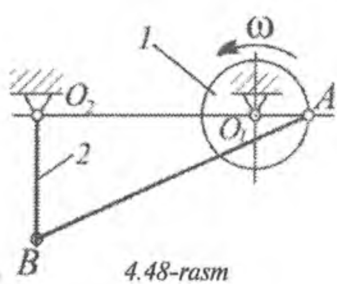
4.46-rasm

13-muammo. Berilgan mexanizm *AB* zvenosining uzunligi 0,2 *m* bo'lib, *A* nuqtasining tezligi 1 *m/s* bo'lsa, ko'rsatilgan holat uchun *AB* zvenosining burchak tezligini hisoblang *(4.47-rasm).*



4.47-rasm

14-muammo. Mexanizm radiusi r=0,1 *m* bo'lgan 1 shkiv, uzunligi 0,25 *m* bo'lgan 2 sterjen va *AB* richagdan iborat. Agar 1 shkiv 120 *ayl/min* chastota bilan aylansa, ko'rsatilgan holat uchun *AB* richakning burchak tezligini toping. Aylanish o'qlari orasidagi masofa O_1O_2=0,45 *m* ga teng *(4.48-rasm).*



4.48-rasm

65-§. Tekis shakl nuqtasining tezlanishi

Tekis shakl nuqtalarining tezlanishlari orasidagi bog'lanish quyidagi teorema yordamida aniqlanadi:

Teorema. *Tekis shakl ixtiyoriy nuqtasining tezlanishi qutbning tezlanishi bilan mazkur nuqtaning qutb atrofida aylanishidagi tezlanishining geometrik yig'indisiga teng.*

Ma'lumki, A nuqtani qutb deb olsak, tekis shakl ixtiyoriy nuqtasining tezligi (4.21) formula orqali aniqlanar edi:

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \overrightarrow{\mathrm{AB}} .$$

B nuqtaning tezlanishini aniqlash uchun (4.21)dan vaqt bo'yicha hosila olamiz:

$$\vec{a}_B = \frac{d\vec{v}_B}{dt} = \frac{d\vec{v}_A}{dt} + \frac{d\vec{\omega}}{dt} \times \overrightarrow{AB} + \vec{\omega} \times \frac{d\overrightarrow{AB}}{dt} . \qquad (4.32)$$

Bu ifodada

$$\frac{d\vec{v}_A}{at} = \vec{a}_A, \frac{d\vec{\omega}}{at} = \vec{\varepsilon}, \quad \frac{d\overrightarrow{AB}}{at} = \vec{v}_{BA} = \vec{\omega} \times \overrightarrow{AB} .$$

Shuning uchun

$$\vec{a}_B = \vec{a}_A + \vec{\varepsilon} \times \overrightarrow{AB} + \vec{\omega} \times \vec{v}_{BA} . \qquad (4.33)$$

(4.33)da $\vec{a}_A - A$ nuqtaning tezlanishi; $\varepsilon \times \overrightarrow{AB} = \vec{a}_{BA}^{\tau} - B$ nuqtaning A qutb atrofida aylanishidagi aylanma tezlanishi; $\vec{\omega} \times \vec{v}_{BA} = \vec{a}_{BA}^{n} B$ nuqtaning A qutb atrofida aylanishidagi markazga intilma tezlanishi.

Shuning uchun

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}^{\tau} + \vec{a}_{BA}^{n} .$$

Agar

$$\vec{a}_{BA}^{\tau} + \vec{a}_{BA}^{n} = \vec{a}_{BA}$$

ekanligini e'tiborga olsak,

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} \qquad (4.34)$$

ifodaga ega bo'lamiz.

$$a_{BA}^{\tau} = AB \cdot \varepsilon; \quad a_{BA}^{n} = AB \cdot \omega^2$$

ekanligini e'tiborga olsak,

$\vec{a}_{BA}$ ning moduli quyidagicha aniqlanadi:

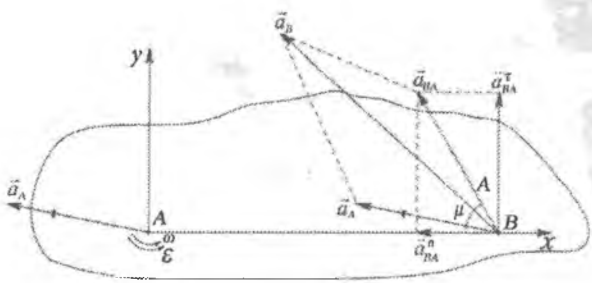
$$a_{BA} = \sqrt{(a_{BA}^{\tau})^2 + (a_{BA}^{n})^2} = AB\sqrt{\varepsilon^2 + \omega^4}\,. \qquad (4.35)$$

B ning yo'nalishi esa quyidagicha aniqlanadi:

$$tg\mu = \frac{|\varepsilon|}{\omega^2}. \qquad (4.36)$$

B nuqtaning *A* qutb atrofida aylanishi tezlanuvchan bo'lganda B nuqtaning tezlanishi *4.49-a rasmda*, sekinlanuvchan bo'lganda *4.49-b rasmda* ko'rsatilgan.

Masala yechishda, *B* nuqtaning tezlanishini proyeksiyalash usulida aniqlash qulay bo'ladi. Buning uchun o'qlardan birini, masalan, *x* o'qni aylanish radiusi bo'ylab, ikkinchisini esa, unga perpendikular ravishda o'tkazish maqsadga muvofiq bo'ladi.



4.49-a rasm

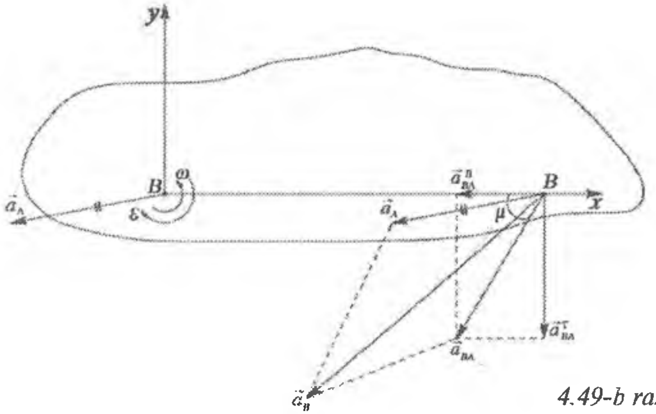
Xorijiy o'quv adabiyotlarida mazkur teorema quyidagicha bayon etiladi.

Tekis shakl ixtiyoriy *P* nuqtasining tezlanishini, qutb sifatida *O* nuqtani tanlab olib, quyidagi formula yordamida ham aniqlash mumkin:

$$\vec{a}_p = \vec{a}_o + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}). \qquad (4.37)$$

Agar

$$\vec{a}_o = a_{o_x}\vec{i} + a_{o_y}\vec{j}\,;$$



4.49-b rasm

$$\vec{\varepsilon} = \varepsilon \vec{k};$$

$$\vec{\omega} = \omega \vec{k};$$

$$\vec{r} = x \times \vec{i} + y\vec{j}$$

ekanligini e'tiborga olsak, (4.37) ifoda quyidagi ko'rinishda yoziladi:

$$\vec{a}_p = \vec{i}(a_{o_x} - \varepsilon y - \omega^2 x) + \vec{j}(a_{o_y} + \varepsilon x + \omega^2 y).$$

Mazkur formula yordamida tekis shakl ixtiyoriy nuqtasining tezlanishini aniqlash mumkin.

66-§. Tezlanishlarning oniy markazi va undan foydalanib tekis shakl nuqtalarining tezlanishlarini aniqlash

Tekis shaklning berilgan ondagi tezlanishi nolga teng bo'lgan nuqtasi (yoki tekis shaklga bog'langan va u bilan birga harakatlanuvchi tekislikning nuqtasi) tezlanishlarning oniy markazi deyiladi.

Tezlanishlarning oniy markazini aniqlash uchun tekis shakl biror A nuqtasining tezlanishi $\vec{a}_A$ va tekis shaklning burchak tezligi ω hamda burchak tezlanishi ε berilgan bo'lishi kerak.

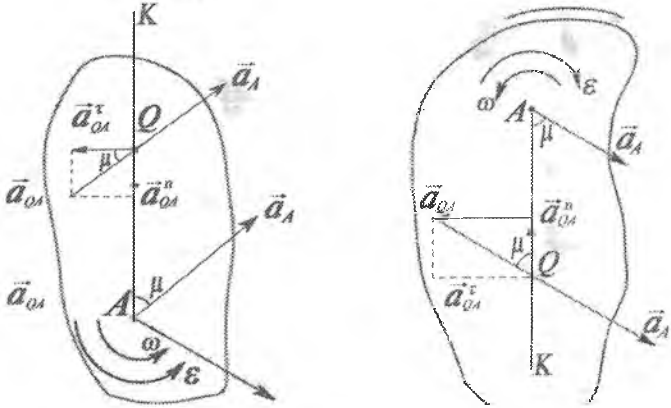
Dastlab,

$$\text{tg}\,\mu = \frac{|\varepsilon|}{\omega^2}$$

formula orqali μ burchak aniqlanadi. So'ngra, tezlanuvchan aylanma harakatda $\vec{a}_A$ vektorga harakat yo'nalishida, sekinlanuvchan aylanma harakatda, harakat yo'nalishiga teskari yo'nalishda burchak ostida to'g'ri chiziq o'tkazib, A nuqtadan

$$AQ = \frac{a_A}{\sqrt{\varepsilon^2 + \omega^4}} \tag{4.38}$$

masofa uzoqlikda yotuvchi Q nuqta aniqlanadi *(4.50-a, b rasmlar)*. Mazkur nuqta tekis shakl nuqtalari tezlanishlarining oniy markazini ifodalaydi.



4.50-a rasm *4.50-b rasm*

Haqiqatan:

$$\vec{a}_Q = \vec{a}_A + \vec{a}_{QA}; \; a_{QA} = QA\sqrt{\varepsilon^2 + \omega^4} = \vec{a}_A.$$

$\vec{a}_{QA}$ miqdor jihatdan $\vec{a}_A$ ga teng, yo'nalishi esa, $\vec{a}_A$ ga qarama-qarshi. Shu sababli,

$$\vec{a}_Q = \vec{a}_A + \vec{a}_{QA} = 0.$$

Agar tekis shakl nuqtalari tezlanishlarining oniy markazini qutb deb olsak, tekis shakl ixtiyoriy B nuqtasining tezlanishi (4.34) va (4.35) formulalarga asosan

$$\vec{a}_B = \vec{a}_{BQ} \tag{4.39}$$

va

$$a_B = BQ\sqrt{\varepsilon^2 + \omega^4} \qquad (4.40)$$

ifodalar orqali aniqlanadi.

(4.40)dan ko'rinib turibdiki, tekis shakl nuqtalarining berilgan ondagi tezlanishlari, mazkur nuqtalardan tezlanishlarning oniy markazigacha bo'lgan masofalarga mutanosib bo'lar ekan:

$$\frac{a_B}{BQ} = \frac{a_A}{AQ} = \frac{a_C}{CQ} = \ldots = \sqrt{\varepsilon^2 + \omega^4}. \qquad (4.41)$$

Takrorlash uchun savollar:

1. *Tekis shakl nuqtalarining tezlanishlari orasidagi bog'lanish qanday ta'riflanadi?*
2. *Tekis shakl ixtiyoriy nuqtasining tezlanishini aniqlash uchun tekis shakl harakatini xarakterlovchi qanday kinematik xarakteristikalar ma'lum bo'lishi lozim?*
3. *Tezlanishlarning oniy markazi deb qanday nuqtaga aytiladi?*
4. *Tekis shakl nuqtalarining berilgan ondagi tezlanishlari va bu nuqtalardan tezlanishlar oniy markazigacha bo'lgan masofalar orasidagi qanday munosabat mavjud?*
5. *Tekis shakl nuqtalari tezlanishlarining oniy markazini aniqlash uchun qanday kattaliklar ma'lum bo'lishi lozim?*
6. *Tekis shakl nuqtalarining berilgan ondagi tezlanishlari va bu nuqtalardan tezlanishlarning oniy markazigacha bo'lgan masofalar orasida qanday bog'lanish mavjud?*

67-§. Tekislikka parallel harakatda bo'lgan jism nuqtalarining tezlanishlarini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Tekislikka parallel harakatda bo'lgan jism nuqtalarining tezlanishlarini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1. Tekis shakl nuqtalari tezliklarining oniy markazini aniqlash usullaridan foydalanib, berilgan masalada, tekis shakl nuqtalari tezliklarining oniy markazi aniqlanadi.

2. Tekis shakl nuqtalari tezliklarining oniy markazini bilgan holda shaklning burchak tezligi aniqlanadi.

3. Tekis shaklning burchak tezligini bilgan holda, tekis shakl ikkinchi nuqtasining birinchi nuqta atrofidagi aylanma harakatidagi markazga intilma tezlanishi topiladi.

4. Ikkinchi nuqtaga uning tezlanishini tashkil etuvchi tezlanishlar vektorlari qo'yiladi. Agar birinchi nuqta A, ikkinchi nuqta B bo'lsa:

$$\vec{a}_B = \vec{a} + \vec{a}_{BA} = \vec{a}_A^{\tau} + \vec{a}_A^{n} + \vec{a}_{BA}^{\tau} + \vec{a}_{BA}^{n}.$$

5. Koordinata o'qlarini o'tkazib, yuqoridagi vektor tenglikning har ikki tomoni koordinata o'qlariga proyeksiyalanadi.

6. Hosil bo'lgan proyeksiyalar tenglamalaridan noma'lum $\vec{a}_{BA}^{\tau}$ va $\vec{a}_B$ lar aniqlanadi.

7. Proyeksiyalar tenglamalaridan topilgan a_{BA}^{τ} tezlanish modulini bilgan holda, tekis shakl burchak tezlanishi aniqlanadi:

$$a_{BA}^{\tau} = \varepsilon \cdot AB,$$

bundan

$$\varepsilon = \frac{a_{BA}^{\tau}}{AB}.$$

8. Tekis shakl burchak tezligi va burchak tezlanishini bilgan holda, tekis shakl nuqtalarining tezlanishlari haqidagi teorema yordamida, so'ralgan ixtiyoriy nuqtaning tezlanishi aniqlanadi.

Izoh: tekis shaklda, $\vec{a}_B$ va $\vec{a}_{BA}^{\tau}$ larning modullarini B nuqtada tanlangan masshtabda chizilgan, tomonlari tashkil etuvchi tezlanishlar, yopuvchi tomoni esa nuqtaning tezlanishi bo'lgan ko'p burchakdan grafik usulda aniqlash ham mumkin.

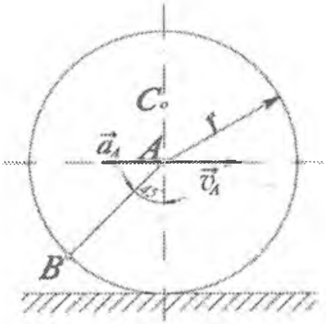
68-§. Tekislikka parallel harakatda bo'lgan jism nuqtalarining tezlanishlarini aniqlashga doir masalalar

1-masala. Radiusi r=30 *sm* bo'lgan g'ildirak yo'lning to'g'ri chiziqli gorizontal uchastkasida sirpanmay dumalaydi. Bu paytda g'ildirak markazining tezligi $v_A = 50\ m/s$, tezlanishi a_A=30 m/s^2, AC=10 *sm*.

G'ildirak B va C nuqtalarining tezligi va tezlanishi aniqlansin *(4.51-a rasm).*

Yechish:

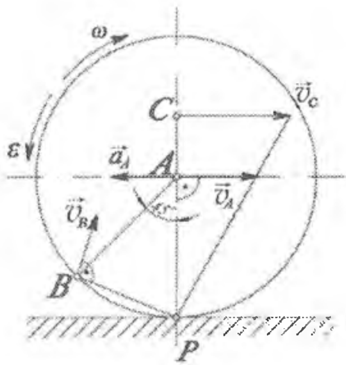
1. Nuqtalarning tezliklarini va g'ildirak burchak tezligini aniqlash.



4.51-a rasm

Masala shartida g'ildirak markazi A nuqtaning tezligi $\vec{v}_A$ berilgan.

G'ildirakning qo'zg'almas chiziqqa tegib turgan nuqtasining tezligi nolga teng bo'lishi sababli, g'ildirak nuqtalari tezliklarining oniy markazi shu urinish nuqtasida yotadi *(4.51-b rasm).*



4.51-b rasm

Berilgan onda g'ildirak nuqtalari tezliklarining oniy markazi P nuqtani qutb deb olsak, g'ildirak nuqtalarining shu ondagi tezliklarini oniy markaz atrofida aylanma harakatdagi jism nuqtalarining tezliklari kabi aniqlash mumkin bo'ladi:

$$v_A = w \cdot PA,$$
$$v_B = w \cdot PB,$$
$$v_C = w \cdot PC$$

yoki

$$\frac{v_A}{PA} = \frac{v_B}{PB} = \frac{v_C}{PC}.$$

Masala shartiga ko‘ra:

$PA = r = 30$ *sm*. PB va PC masofalarni *4.51-b rasmdan* aniqlaymiz:

$$PB = \sqrt{r^2 + r^2 - 2r^2 \cos 45^\circ} = 22{,}8 \ sm.$$

Shuning uchun

$$v_B = \frac{v_A \cdot PB}{PA} = 38{,}1 \ sm/s,$$
$$v_C = \frac{v_A \cdot PC}{PA} = 66{,}7 \ sm/s.$$

G‘ildirak nuqtalarining tezliklarini g‘ildirakning burchak tezligini aniqlash orqali ham topish mumkin:

$$v_A = \omega \cdot PA.$$

Bundan

$$\omega = \frac{v_A}{PA} = \frac{50}{30} = 1{,}67 \ rad/s.$$

Burchak tezlikning yo‘nalishi $\vec{v}_A$ yo‘nalishi orqali aniqlanadi *(4.51-b rasm)*.

Bunday holda, g‘ildirak B va C nuqtalarining tezligi quyidagilarga teng bo‘ladi:

$$v_B = \omega \cdot PB = 1{,}67 \cdot 22{,}8 = 38{,}1 \ sm/s,$$
$$v_C = \omega \cdot PC = 1{,}67 \cdot 40 = 66{,}7 \ sm/s.$$

II. G‘ildirak nuqtalarning tezlanishlari va g‘ildirak burchak tezlanishini aniqlash.

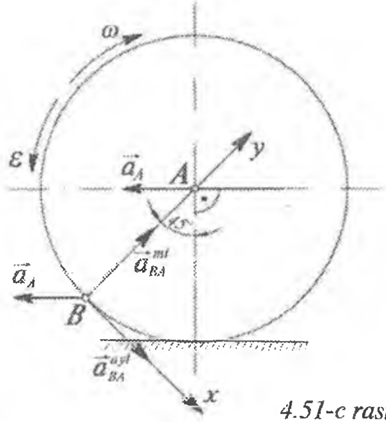
Masala shartida A nuqtaning tezlanishi a_A berilgan.

Tekis shakl nuqtalarining tezlanishlari haqidagi teoremaga asosan:

$$\vec{a}_B=\vec{a}_A+\vec{a}_{BA}^{n}+\vec{a}_{BA}^{\tau}=\vec{a}_A+\vec{a}_{BA}^{mi}+\vec{a}_{BA}^{ayl}.$$

Bunda A nuqta qutb sifatida qabul qilindi.

G'ildirakning A qutb atrofida aylanma harakatida B nuqtasining markazga intilma normal tezlanishi quyidagicha aniqlanadi:



4.51-c rasm

$$a_{BA}^{mi}=a_{BA}^{n}=\omega^2\cdot AB=83{,}7\ sm/s^2.$$

$\vec{a}_{BA}^{mi}=\vec{a}_{BA}^{n}$ vektor B nuqtadan A nuqtaga tomon yo'naladi.

G'ildirakning A qubt atrofida aylanma harakatida B nuqtasining aylanma urinma tezlanishi quyidagi formula asosida aniqlanadi:

$$a_B^{ayl}=a_{BA}^{\tau}=\varepsilon\cdot AB.$$

Bu ifodada ε — g'ildirakning burchak tezlanishi. Burchak tezlanish ta'rifiga ko'ra g'ildirakning burchak tezlanishini aniqlaymiz:

$$\varepsilon=\frac{d\omega}{dt}=\frac{d}{dt}\left(\frac{v_A}{PA}\right)=\frac{1}{PA}\frac{dv_A}{dt}=\frac{a_A}{PA}=1\ rad/s.$$

Burchak tezlanishining yo'nalishi $\vec{a}_A$ vektor yo'nalishi orqali aniqlanadi.

B nuqtaning A nuqta atrofida aylanma harakatida aylanma-urinma tezlanishining miqdorini aniqlaymiz:

$$a_{BA}^{ayl} = a_{BA}^{\tau} = \varepsilon \cdot AB = 30\ sm/s^2,$$

$\vec{a}_{BA}^{ayl} = \vec{a}_{BA}^{\tau}$ vektor g'ildirakning B nuqtasiga ε yo'nalishida o'tkazilgan urinma bo'ylab yo'naladi *(4.51 c-rasm).*

B nuqta tezlanishining modulini proyeksiyalash yo'li bilan aniqlaymiz. Koordinata o'qlarini *4.51-c rasmdagidek* o'tkazsak:

$$(a_B)x = a_{BA}^{ayl} - a_A \cos 45° = 30 - 30 \cdot 0,71 = 8,7\ sm/s^2;$$

$$(a_B)_y = a_{BA}^{mi} - a_A \cos 45° = 83,7 - 30 \cdot 0,71 = 62,4\ sm/s^2.$$

Bular orqali B nuqta tezlanishining moduli quyidagicha aniqlanadi:

$$a_B = \sqrt{(a_B)_x^2 + (a_B)_y^2} = \sqrt{75,69 + 3893,76} = 63\ sm/s^2.$$

G'ildirak C nuqtasining tezlanishini aniqlaymiz *(4.51-d rasm).*

Tekis shakl nuqtalarining tezlanishlari haqidagi teoremaga asosan:

$$\vec{a}_c = \vec{a}_A + \vec{a}_{CA},$$

yoki

$$\vec{a}_c = \vec{a}_A + \vec{a}_{CA}^n + \vec{a}_{CA}^{\tau} = \vec{a}_A + \vec{a}_{CA}^{mi} + \vec{a}_{CA}^{ayl}$$

G'ildirakning A qutb atrofida aylanma harakatida C nuqtasining markazga intilma tezlanishi quyidagicha aniqlanadi:

$$\vec{a}_{CA}^{mi} = \omega^2 \cdot CA = 16,7 \frac{sm}{s^2}.$$

Mazkur tezlanish C nuqtadan A nuqta tomon yo'naladi.

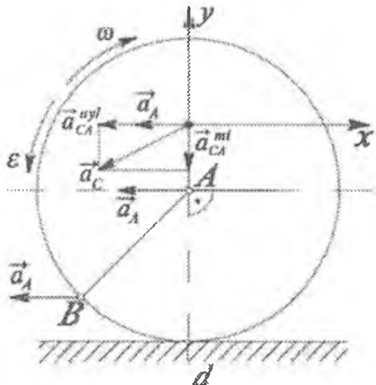
G'ildirakning A qutb atrofida aylanma harakatida C nuqtasining aylanma-urinma tezlanishi quyidagicha hisoblanadi:

$$\vec{a}_{CA}^{ayl} = \vec{a}_{CA}^{t} = \varepsilon \cdot CA = 10\ sm/s^2.$$

Mazkur tezlanish C nuqtada $\vec{a}_{CA}^{mi}$, $\vec{a}_{CA}^{ayl}$ ga perpendikular holda ε tomon yo'naladi.

$\vec{a}_{CA}^{mi}$, $\vec{a}_{CA}^{ayl}$ vektorlar *4.51-d rasmda* ko'rsatilgan.

C nuqtaning tezlanishini ham proyeksiyalash yo'li bilan aniqlanadi.



4.51-d rasm

Koordinata o'qlarini *4.51-d rasmdagidek* o'tkazsak,

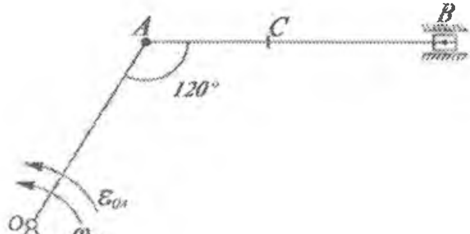
$$(a_c)_x = -a_A - a_{CA}^{ayl} = -30 - 10 = -40\ sm/s^2,$$

$$(a_c)_y = -a_{CA}^{mi} = -16{,}7\ sm/s^2.$$

Bular orqali $\vec{a}_c$ moduli aniqlanadi:

$$a_c = \sqrt{(a_C)_x^2 + (a_C)_y^2} = 43{,}34\ sm/s^2.$$

2-masala. Mexanizmning berilgan holati uchun *A, B, C* nuqtalarining tezliklari va tezlanishlari hamda shu nuqtalar tegishli bo'lgan zvenoning burchak tezligi va burchak tezlanishi topilsin *(4.52-rasm).*



4.52-rasm

Masalada quyidagi kattaliklar berilgan:

$$OA = 40\ sm,\ AB = 80\ sm,\ AC = 30\ sm,$$

$$\omega_{OA} = 2\ rad/s,\ \varepsilon_{OA} = 6\ rad/s^2.$$

Yechish:

1. Nuqtalarning tezliklarini va *AB* zvenoning burchak tezligini aniqlash.

Mexanizmning berilgan harakatida *OA* krivoship *A* panjasi tezligining modulini hisoblaymiz:

$$v_A = \omega_{OA} \cdot OA = 2 \cdot 40 = 80 \ sm/s. \qquad (4.42)$$

A nuqtaning tezligi $\vec{v}_A$ *OA* krivoshipga perpendikular holda, ω_{OA} yo'nalishi bo'yicha yo'naladi *(4.52-a rasm).*

B polzunning tezligi gorizontal holda *B* nuqtadan *A* nuqta tomon yo'nalgan. Uning modulini aniqlash uchun *AB* zveno nuqtalari tezliklarining oniy markazidan foydalanamiz. *AB* zveno-shatun nuqtalari tezliklarining oniy markazi P_{AB} *A* va *B* nuqtalardan, ularning $\vec{v}_A$ va $\vec{v}_B$ tezliklariga o'tkazilgan perpendikulalarning kesishgan nuqtasida yotadi.

Bunday holda *A* nuqtaning tezlikini *AB* zveno nuqtalari tezliklarining oniy markazi orqali quyidagicha ifodalash mumkin:

$$v_A = \omega_{AB} \cdot AP_{AB}.$$

Bunday holda *AB* shatun burchak tezligi quyidagi ifodadan aniqlanadi.

$$\omega_{AB} = \frac{v_A}{AP_{AB}}. \qquad (4.43)$$

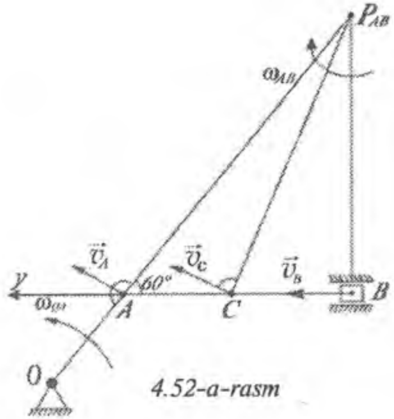
ω_{AB} ning yo'nalishi $\vec{v}_A$ vektor yo'nalishi orqali aniqlanadi *(4.52-a rasm).*

Shatun *B* va *C* nuqtalari tezliklarining modullarini mazkur nuqtalarning oniy markaz P_{AB} nuqta atrofidagi aylanma harakat tezliklari kabi aniqlaymiz. Shuning uchun

$$v_B = \omega_{AB} \cdot BP_{AB}, \ v_C = \omega_{AB} \cdot CP_{AB}. \qquad (4.44)$$

AP_{AB}, BP_{AB}, CP_{AB} masofalar chizmadagi ABP_{AB} va ACP_{AB} uchburchaklardan topiladi *(4.52-a rasm).*

$$AP_{AB} = \frac{AB}{\cos 60^\circ} = 160 \ sm, \ BP_{AB} = AP_{AB} \cdot \sin 60^\circ = 137{,}6 \ sm;$$



4.52-a-rasm

$$CP_{AB} = \sqrt{(BC)^2 + (BP_{AB})^2} = \sqrt{2500 + 18933{,}8} = 146{,}4\ sm.$$

Yuqoridagilarni e'tiborga olsak:

$$\omega_{AB} = 0{,}5\ rad/s,\ v_B = 68{,}8\ sm/s,\ v_C = 73{,}2\ sm/s.$$

$\vec{v}_C$ vektor CP_{AB} kesmaga perpendikular holda, ω_{AB} yo'nalishi tomon yo'nalgan *(4.52-a rasm).*

Bajarilgan hisoblashlarning to'g'riligiga ishonch hosil qilish uchun nuqtaning tezligini, tekis shakl ikki nuqtasi tezliklarining bu nuqtalardan o'tuvchi o'qdagi proyeksiyalarining o'zaro tengligi haqidagi teoremadan foydalanib aniqlaymiz.

Buning uchun y o'qini shatun bo'ylab B nuqtadan A nuqta tomon yo'naltiramiz. Teoremaga asosan:

$$v_A \cos(\vec{v}_A \hat{\ } y) = v_B \cos(\vec{v}_B \hat{\ } y). \qquad (4.45)$$

4.52-a rasmdan:

$$v_A \cos 30° = v_B,$$

chunki

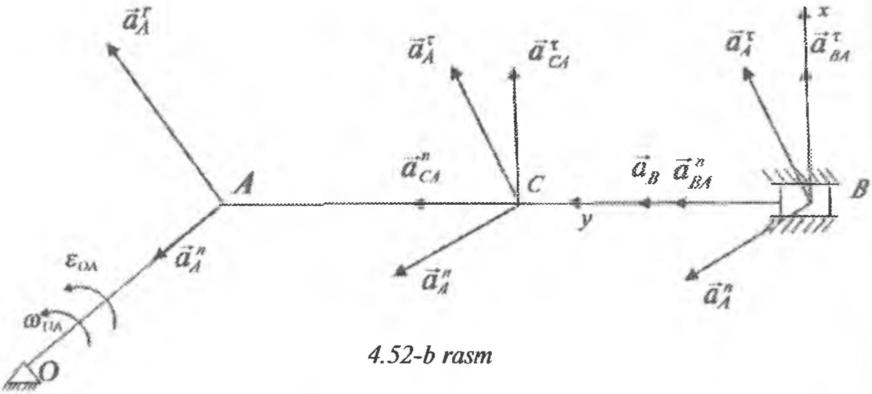
$$\cos(\vec{v}_B \hat{\ } y) = 1\,.$$

Demak, $v_B = 68{,}8\ sm/s$, hisoblashlar to'g'ri bajarilgan.

C nuqtaning avval topilgan tezligi v_C ham shu teorema yordamida tekshirilishi mumkin.

2. Nuqtalarning tezlanishlari va *AB* zvenoning burchak tezlanishini aniqlash.

A nuqta *O* nuqta atrofida aylana bo'ylab harakatlanishi tufayli uning tezlanishi aylanma-urinma va markazga intilma normal tezlanishlardan tashkil topadi *(4.52-b rasm).*



4.52-b rasm

$$\vec{a}_A = \vec{a}_A^\tau + \vec{a}_A^n. \qquad (4.46)$$

Bunda:

$$a_A^\tau = \varepsilon_{OA} \cdot OA = 240\ sm/s^2;\ a_A^n = \omega_{OA}^2 \cdot OA = 160\ sm/s^2.$$

$$a_A = \sqrt{(a_A^\tau)^2 + (a_A^n)^2} = 288{,}4 sm/s^2.$$

$\vec{a}_A^\tau$ vektor *OA* krivoshipga perpendikular holda, ε_{OA} yo'nalishi bo'yicha yo'naladi. $\vec{a}_A^n$ vektor *A* nuqtadan *O* nuqta tomon yo'naladi.

Tekis shakl nuqtalarining tezlanishlari haqidagi teoremaga asosan:

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}$$

yoki

$$\vec{a}_B = \vec{a}_A^\tau + \vec{a}_A^n + \vec{a}_{BA}^\tau + \vec{a}_{BA}^n. \qquad (4.47)$$

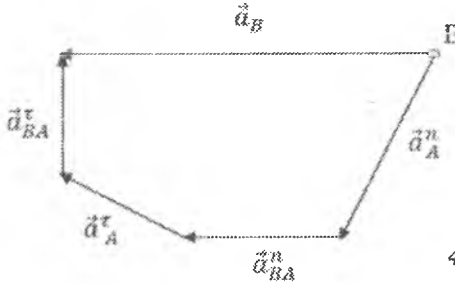
Bunda tezlanishi $\vec{a}_A$ ma'lum bo'lgan *A* nuqta qutb deb olindi.

AB shatunning *A* qutb atrofidagi aylanma harakatida *B* nuqtaning markazga intilma tezlanishi quyidagiga teng bo'ladi:

$$a_{BA}^n = \omega_{AB}^2 \cdot AB = 20\ m/s^2. \qquad (4.48)$$

$\vec{a}^n_{BA}$ vektor *B* nuqtadan *A* nuqta tomon yo'naladi.

B nuqtaning tezlanishi $\vec{a}_B$ va *B* nuqtaning *A* qutb atrofidagi aylanma harakatidagi aylanma tezlanishi $\vec{a}^\tau_{BA}$ larning faqat yo'nalish chiziqlari ma'lum: $\vec{a}_B$ gorizontal, $\vec{a}^\tau_{BA}$ esa *AB* shatunga perpendikular yo'nalgan. Ularning ko'rsatilgan yo'nalish chiziqlari bo'ylab qaysi tomonlarga yo'nalishlarini ixtiyoriy tanlab olamiz *(4.52-b rasm).*



4.52-c rasm

Mazkur tezlanishlarning modullari (4.47) vektor tenglikning koordinata o'qlariga proyeksiyalari tenglamalaridan aniqlanadi. Javobning ishorasiga qarab, vektorning haqiqiy yo'nalishini, hisoblashda qabul qilinganiga mos kelishi yoki kelmasligi orqali aniqlanadi. *x* va *y* o'qlarining yo'nalishlarini *4.52-b rasmda* ko'rsatilgandek o'tkazib, quyidagilarni hosil qilamiz:

$$0=-a^n_A\cos30°+a^\tau_A\cos60°+a^\tau_{BA}, \tag{4.49}$$

$$a_B = a^n_{BA} + a^n_A\cos60° + a^\tau_A\cos30°. \tag{4.50}$$

(4.49)dan:

$$a^\tau_{BA} = a^n_A\cos30° - a^\tau_A\cos60° = 17,6\ sm/s^2.$$

(4.50)dan

$$a_B = 306,4\ sm/s^2.$$

Javoblarning ishoralari musbat. Shuning uchun $\vec{a}^\tau_{BA}$ va $\vec{a}_B$ vektorlarning haqiqiy yo'nalishlari, hisoblashda qabul qilingan yo'nalishlarga mos kelar ekan.

AB shatunning burchak tezlanishini quyidagicha aniqlanadi:

Ma'lumki,

$$a_{BA}^{\tau} = \varepsilon_{AB} \cdot AB.$$

Bundan

$$\varepsilon_{AB} = \frac{a_{BA}^{\tau}}{BA} = 0,22\ rad/s^2. \qquad (4.51)$$

ε_{AB} ning yo'nalishi $\vec{a}_{BA}^{\tau}$ vektor yo'nalishi orqali aniqlanadi.

$\vec{a}_B$ va $\vec{a}_{BA}^{\tau}$ larning modullarini grafik usulda *B* nuqtada tezlanishlar ko'p burchagini chizish orqali ham aniqlash mumkin. Buning uchun (4.47)ga asosan *B* nuqtadan boshlab, tanlangan masshtabda ketma-ket $\vec{a}_A^n$, $\vec{a}_{BA}^n$ va $\vec{a}_A^{\tau}$ $\vec{a}_{BA}^{\tau}$ vektorlarni qo'yamiz *(4.52-c rasm)*. $\vec{a}_{BA}^{\tau}$ vektorning oxiri orqali *AB* shatunga perpendikular holda o'tkazilgan to'g'ri chiziqni tezlanishning yo'nalish chizig'i bilan kesishguncha davom ettiramiz. Mazkur to'g'ri chiziq uzunligi tanlangan masshtabda $\vec{a}_{BA}^{\tau}$ ning modulini ifodalaydi. $\vec{a}_B$ vektorning moduli tezlanishlar ko'p burchagining yopuvchi tomoni kabi aniqlanadi.

Shuning uchun ko'pburchakning yopuvchi tomonining uzunligi tanlangan masshtabda $\vec{a}_B$ modulini ifodalaydi *(4.52-c rasm)*.

C nuqtaning tezlanishini aniqlaymiz:

$$\vec{a}_C = \vec{a}_A + \vec{a}_{CA} = \vec{a}_A^{\tau} + \vec{a}_A^n + \vec{a}_{CA}^{\tau} + \vec{a}_{CA}^n. \qquad (4.52)$$

AB shatunning *A* nuqta atrofidagi aylanma harakatida *C* nuqtaning aylanma va markazga intilma tezlanishlari quyidagilarga teng bo'ladi:

$$a_{CA}^{\tau} = \varepsilon_{AB} \cdot AC = 6,6\ sm/s^2,$$

$$a_{CA}^{n} = \omega_{AB}^2 \cdot AC = 7,5\ sm/s^2.$$

$\vec{a}_{CA}^n$ vektor *C* nuqtadan *A* nuqta tomon yo'naladi. $\vec{a}_{CA}^{\tau}$ vektor esa $\vec{a}_{CA}^n$ vektorga perpendikular holda, ε_{AB} burchak tezlanishining yo'nalishi tomon yo'naladi.

$\vec{a}_C$ ning modulini proyeksiyalash usuli bilan aniqlaymiz.

Buning uchun (4.52)ni x va y o‘qlarga proyeksiyalaymiz *(4.52-b rasm):*

$$(a_C)_x = +a_A^\tau \cos 60° - a_A^n \cos 30° + a_{CA}^\tau = -11\ sm/s^2,$$

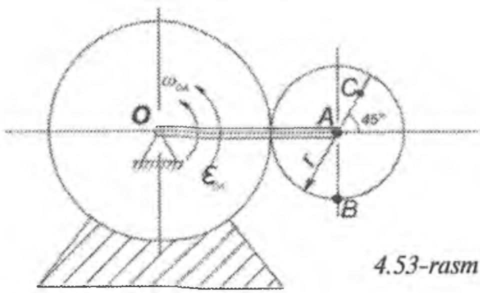
$$(a_C)_y = a_A^\tau \cos 30° + a_A^n \cos 60° + a_{CA}^n = 293{,}9\ sm/s^2.$$

Natijada,

$$a_C = \sqrt{(a_C)_x^2 + (a_C)_y^2} = 294{,}1\ sm/s^2.$$

3-masala. Radius $r = 20$ sm bo‘lgan tishli g‘ildirak radiusi $R = 40$ sm bo‘lgan qo‘zg‘almas tishli g‘ildirakning O o‘qi atrofida aylanuvchi OA krivoship bilan harakatga keltiriladi; krivoship shu paytda ω=2*rad/s* burchak tezligiga ega bo‘lib, ε =2*rad/s²* burchak tezlanish bilan aylanadi.

Kivoship A nuqtasining va qo‘zg‘aluvchi g‘ildirakning B va C nuqtalarining tezliklari va tezlanishlari hamda qo‘zg‘aluvchi g‘ildirakning burchak tezligi va burchak tezlanishi aniqlansin, AC=15 *sm (4.53-rasm).*



4.53-rasm

Yechish:

1. Nuqtalarning tezliklarini va qo‘zg‘aluvchi g‘ildirak burchak tezligini aniqlash.

Mexanizmning berilgan holatida OA krivoship A panjasi tezligining modulini aniqlaymiz:

$$v_A = \omega_{OA} \cdot OA = 120\ sm/s. \tag{4.53}$$

A nuqtaning tezligi $\vec{v}_A$ OA krivoshipga perpendikular holda w_{OA} yo‘nalishi tomon yo‘naladi.

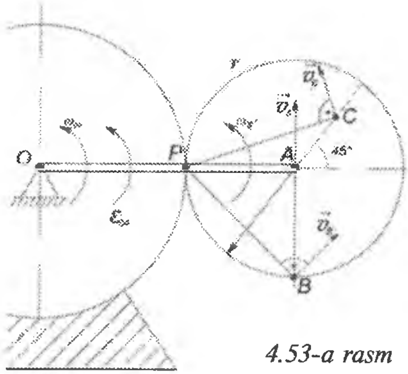
Qo‘zg‘aluvchi g‘ildirak nuqtalari tezliklarining oniy markazi P nuqta bo‘lganligi uchun *(4.53-a rasmga qarang)*

$$v_A = \omega_{g'} \cdot r.$$

Bundan

$$\omega_{g'} = \frac{v_A}{r} = 6 \ rad/s. \qquad (4.54)$$

Qo‘zg‘aluvchi g‘ildirak burchak tezligi $\omega_{g'}$ ning yo‘nalishi $\vec{v}_A$ vektor yo‘nalishi orqali aniqlanadi *(4.53-a rasm).*



4.53-a rasm

Qo‘zg‘aluvchi g‘ildirak B va C nuqtalari tezliklarining modullari quyidagi ifodalar orqali aniqlanadi:

$$v_B = \omega_{g'} \cdot BP, \qquad (4.55)$$

$$v_C = \omega_{g'} \cdot CP. \qquad (4.56)$$

4.53-a rasmdan

$$BP = r\sqrt{2} = 28,2 \ sm,$$

$$CP = \sqrt{r^2 + (AC)^2 + \ 2 \cdot r \cdot AC \cos 45^\circ} = 32,4 \ sm.$$

Yuqoridagilarni e’tiborga olsak:

$$v_B = 169,2 \ sm/s, \ v_C = 194 \ sm/s.$$

$\vec{v}_B$ vektor BP kesmaga, $\vec{v}_C$ vektor CP kesmaga perpendikular holda $\omega_{g'}$ yo‘nalishi tomon yo‘naladi *(4.53-a rasm).*

2. Nuqtalarning tezlanishlari va qo'zg'aluvchi g'ildirak burchak tezlanishini aniqlash.

A nuqta *O* nuqta atrofida aylanma harakatda bo'lishi tufayli, uning tezlanishi aylanma-urima va markazga intilma normal tezlanishlardan tashkil topadi *(4.53-b rasm)*:

$$\vec{a}_A = \vec{a}_A^{\tau} + \vec{a}_A^{n}. \quad (1.5)$$

Bunda:

$$a_A^{\tau} = \varepsilon_{OA} \cdot OA = 120 \ sm/s^2,$$

$$a_A^{n} = \omega_{OA}^2 \cdot OA = 240 \ sm/s^2.$$

A nuqta tezlanishining moduli quyidagi ifodadan aniqlanadi:

$$a_A = \sqrt{(a_A^{\tau})^2 + (a_A^{n})^2} = 268{,}3 \ sm/s^2.$$

$\vec{a}_A^{n}$ vektor *A* nuqtadan *O* nuqta tomon yo'naladi. $\vec{a}_A^{\tau}$ vektor $\vec{a}_A^{n}$ vektorga perpendikular holda, ε_{OA} yo'nalishi tomon yo'naladi *(4.53-b rasm)*.

Qo'zg'aluvchi g'ildirak *B* va *C* nuqtalarining tezlanishlarini tekis shakl nuqtalarining tezlanishlari haqidagi teoremadan foydalanib, aniqlaymiz:

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}$$

yoki

$$\vec{a}_B = \vec{a}_A^{\tau} + \vec{a}_A^{n} + \vec{a}_{BA}^{\tau} + \vec{a}_{BA}^{n}. \quad (4.57)$$

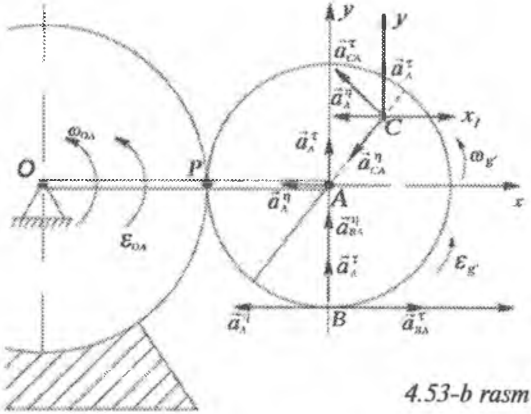
Bunda tezlanishi $\vec{a}_A$ ma'lum bo'lgan A nuqta, ***qutb*** deb olinadi.

$\vec{a}_{BA}^{n}$ vektor *B* nuqtadan *A* nuqta tomon yo'naladi, uning moduli quyidagi formuladan topiladi:

$$a_{BA}^{n} = \omega_{g'}^2 \cdot BA = 720 \ sm/s^2. \quad (4.58)$$

$\vec{a}_{BA}^{\tau}$ vektorni aniqlash uchun qo'zg'aluvchi g'ildirak burchak tezlanishini aniqlash lozim.

Burchak tezlanish ta'rifiga ko'ra,



4.53-b rasm

$$\varepsilon_{g'} = \frac{d\omega_{g'}}{at} = \frac{1}{r}\left(\frac{dv_A}{at}\right) = \frac{a_A^{\tau}}{r} = 6\, rad/s^2. \qquad (4.59)$$

$\varepsilon_{g'}$ ishorasi $\omega_{g'}$ ishorasi bilan bir xil bo'lganligi uchun ular bir xil yo'nalishga ega bo'ladilar.

$\vec{a}_{BA}^{\tau}$ vektor $\vec{a}_{BA}^{n}$ vektorga perpendikular holda $\varepsilon_{g'}$ yo'nalishi tomon yo'naladi, uning moduli quyidagi formuladan topiladi:

$$a_{BA}^{\tau} = \varepsilon_{g'} \cdot AB = 360\ sm/s^2. \qquad (4.60)$$

$\vec{a}_B$ vektor modulini aniqlash uchun $\vec{a}_A^{\tau}$, $\vec{a}_A^{n}$, $\vec{a}_{BA}^{\tau}$, $\vec{a}_{BA}^{n}$ vektorlarni B nuqtaga qo'yamiz va proyeksiyalash usulidan foydalanamiz. x o'qini B nuqtadan gorizontal, y o'qini esa vertikal yo'naltiramiz. (4.57)ni x va y o'qlariga proyeksiyalasak:

4.53-b rasm

$$(a_B)_x = -a_A^n + a_{BA}^{\tau} = 120\ sm/s^2;$$

$$(a_B)_y = a_A^{\tau} + a_{BA}^{n} = 840\ sm/s^2.$$

Natijada,

$$a_B = \sqrt{(a_B)_x^2 + (a_B)_y^2} = 848{,}5\ sm/s^2.$$

C nuqtaning tezlanishi B nuqtaning tezlanishi kabi topiladi *(4.53-rasm)*:

$$\vec{a}_C = \vec{a}_A + \vec{a}_{CA} \qquad (4.61)$$

yoki

$$\vec{a}_C = \vec{a}_A^\tau + \vec{a}_A^n + \vec{a}_{CA}^\tau + \vec{a}_{CA}^n. \qquad (4.62)$$

Bunda

$$a_{CA}^\tau = \varepsilon_{g'} \cdot AC = 90 \; sm/s^2,$$

$$a_{CA}^n = \omega_{g'}^2 \cdot AC = 540 \; sm/s^2.$$

$\vec{a}_C$ vektor modulini ham proyeksiyalash usulidan foydalanib aniqlaymiz. Buning uchun C nuqtaga $\vec{a}_A^\tau$, $\vec{a}_A^n$, $\vec{a}_{CA}^\tau$, $\vec{a}_{CA}^n$ vektorlarni qo'yamiz. x o'qini C nuqtadan gorizontal, y o'qini esa vertikal yo'naltiramiz.

(4.62)ni x va y o'qlariga proyeksiyalasak:

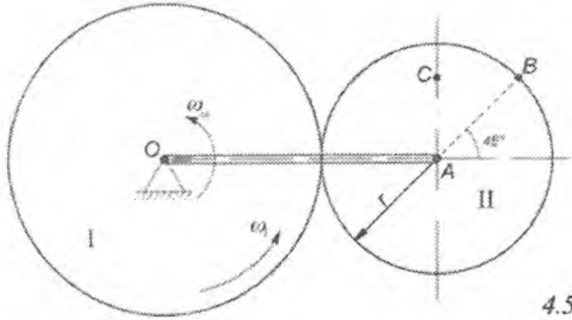
$$(a_C)_x^\tau = -a_A^n - a_{CA}^n \cos 45° - a_{CA}^\tau \cos 45° = -687{,}3 \; sm/s^2$$

$$(a_C)_y = +a_A^\tau - a_{CA}^n \cos 45° + a_{CA}^\tau \cos 45° = -199{,}5 \; sm/s^2.$$

Natijada,

$$a_C = \sqrt{(a_C)_x^2 + (a_C)_y^2} = \sqrt{512181{,}54} = 715{,}6 \; sm/s^2. \qquad (4.63)$$

4-masala. Uzunligi 60 *sm* bo'lgan *OA* krivoship chizma tekisligiga perpendikular bo'lgan qo'zg'almas *Ox* o'q atrofida $\omega_{OA} = 4 rad/s$ burchak tezlik bilan aylanadi. Xuddi shu *Ox* o'qqa I g'ildirak o'tqazilgan, krivoship *A* nuqtasiga esa radiusi $r = 20$ *sm* bo'lgan, I g'ildirakka tashqari tomonidan ilashgan, II g'ildirak o'rnatilgan.



4.54-rasm

I g'ildirakning burchak tezligi $\omega_I = 15\ rad/s$. $AC = 10\ sm$ *(4.54-rasm)*. Ikkinchi g'ildirak *A, B, C* nuqtalarining tezliklari va tezlanishlari aniqlansin hamda ikkinchi g'ildirakning burchak tezligi topilsin.

Yechish:

A) II g'ildirak burchak tezligi va *A, B, C* nuqtalarining tezliklarini aniqlash.

I va II g'ildiraklar tashqari tomonidan ilashganligi uchun, II g'ildirakning burchak tezligi ω_{II} ni Villis formulasidan aniqlaymiz *(4.54-a rasm)*:

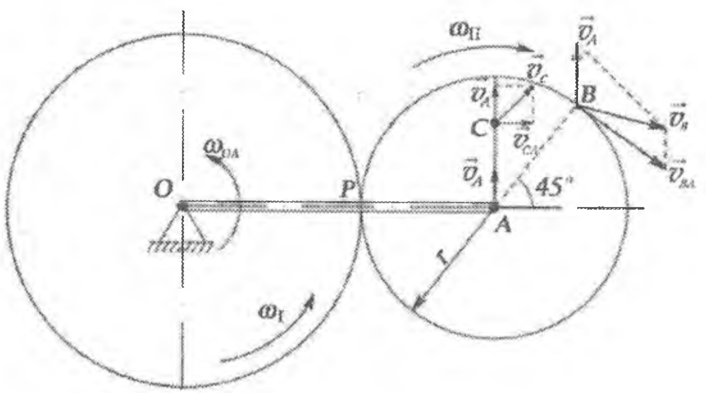
$$\frac{\omega_I - \omega_{OA}}{\omega_{II} - \omega_{OA}} = -\frac{r}{OA - r}, \qquad (4.64)$$

bundan $\omega_{II} = 8\ rad/s$.

ω_I va ω_{II} larning yo'nalishlari chizmada ko'rsatilgan *(4.54-a rasm).*

Mexanizmning berilgan holatida krivoship *A* panjasi tezligining modulini aniqlaymiz:

$$\upsilon_A = \omega_{OA} \cdot OA = 4 \cdot 60 = 240\ sm/s. \qquad (4.65)$$



4.54-a rasm

A nuqtaning tezligi $\vec{\upsilon}_A$ *OA* krivoshipga perpendikular holda ω_{OA} yo'nalishi tomon yo'naladi.

II g'ildirak B va C nuqtalarining tezliklarini aniqlash uchun tekis shakl nuqtalarining tezliklari haqidagi teoremadan foydalanamiz. Buning uchun qutb sifatida tezligi $\vec{v}_A$ ma'lum bo'lgan A nuqtani tanlaymiz. Teoremaga asosan, B nuqtaning tezligi quyidagicha aniqlanadi:

$$\vec{v}_B = \vec{v}_A + \vec{v}_{BA}.$$

Bunda

$$v_{BA} = \omega_{II} \cdot BA = 160\ sm/s,$$

shuning uchun

$$v_B = \sqrt{v_A^2 + v_{BA}^2 + 2v_A v_{BA} \cos 45} = 288\ sm/s. \quad (4.66)$$

II g'ildirak C nuqtasining tezligi ham B nuqtaning tezligi kabi topiladi:

$$\vec{v}_C = \vec{v}_A + \vec{v}_{CA},$$

bunda

$$v_{CA} = \omega_{II} \cdot CA = 80\ sm/s.$$

$\vec{v}_A$ va $\vec{v}_{CA}$ lar o'zaro perpendikular bo'lganligi uchun

$$v_C = \sqrt{v_A^2 + v_{CA}^2} = 252{,}8\ sm/s. \quad (4.67)$$

$\vec{v}_A$ va $\vec{v}_C$ vektorlar $\vec{v}_A$ va $\vec{v}_{BA}$ hamda $\vec{v}_A$ va $\vec{v}_{CA}$ vektorlardan qurilgan parallelogrammlar diagonallari orqali ifodalanadi *(4.54-a rasm)*.

B) II g'ildirak A, B, C nuqtalarining tezlanishlarini aniqlash.

A nuqta O nuqta atrofida aylana bo'ylab harakatlanishi tufayli, uning tezlanishi aylanma-urinma va markazga intilma normal tezlanishlardan tashkil topadi *(4.54-b rasm)*:

$$\vec{a}_A = \vec{a}_A^\tau + \vec{a}_A^n, \quad (4.68)$$

bunda

$$a_A^\tau = \varepsilon_{OA} \cdot OA = 0,$$

chunki

$$\varepsilon_{OA} = \frac{d\omega_{OA}}{at} = 0.$$

$$a_A^n = \omega_{OA}^2 \cdot OA = 960\ sm/s^2.$$

Natijada,

$$a_A = a_A^n = 960\ sm/s^2. \qquad (4.69)$$

A nuqtaning tezlanishi $\vec{a}_A$ *A* nuqtaning *O* nuqta atrofidagi aylanma harakati markazga intilma tezlanishiga teng bo'ladi *(4.54-b rasm).*

II g'ildirak *B* va *C* nuqtalarining tezlanishlarini aniqlash uchun tekis shakl nuqtalarining tezlanishlari haqidagi teoremadan foydalanamiz. Qutb sifatida tezlanishi $\vec{a}_A$ ma'lum bo'lgan *A* nuqtani tanlaymiz.

Bunda, teoremaga ko'ra, *B* nuqtaning tezlanishi quyidagicha ifodalanadi:

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}$$

yoki

$$\vec{a}_B = \vec{a}_A^\tau + \vec{a}_A^n + \vec{a}_{BA}^\tau + \vec{a}_{BA}^n. \qquad (4.70)$$

Bunda

$$a_{BA}^\tau = 0,$$

chunki

$$\varepsilon_{II} = \frac{d\omega_{II}}{at} = 0.$$

$\vec{a}_{BA}^n$ vektor *B* nuqtadan *A* nuqta tomon yo'naladi. Uning moduli:

$$a_{BA}^n = \omega_{II}^2 \cdot BA = 1280\ sm/s^2.$$

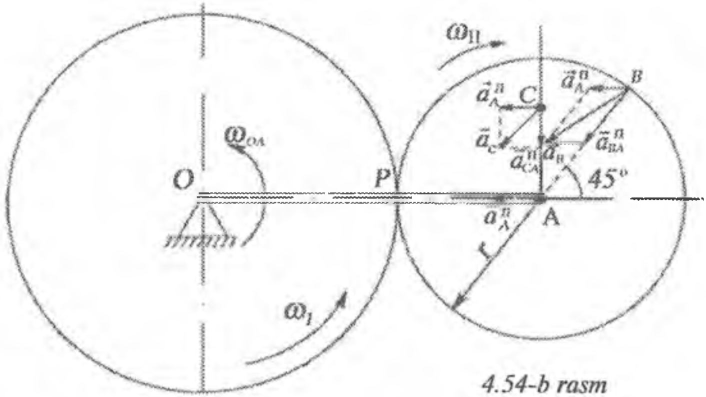
$\vec{a}_B$ vektor $\vec{a}_A^n$ va $\vec{a}_{BA}^n$ tezlanishlardan qurilgan parallelogrammning diagonali orqali ifodalanadi *(4.54-b rasm).*

Uning moduli:

$$a_B = \sqrt{(a_A^n)^2 + (a_{BA}^n)^2 + 2a_A^n a_{BA}^n \cos 45^\circ} = 2074\ sm/s^2. \quad (4.71)$$

C nuqtaning tezlanishi B nuqta tezlanishi kabi topiladi:

$$\vec{a}_C = \vec{a}_A + \vec{a}_{CA} \qquad (4.72)$$



4.54-b rasm

yoki

$$\vec{a}_C = \vec{a}_A^{\tau} + \vec{a}_A^n + \vec{a}_{CA}^{\tau} + \vec{a}_{CA}^n. \qquad (4.73)$$

Bunda

$$a_{CA}^{\tau} = 0,$$

chunki

$$\varepsilon_{II} = O;$$

$$a_{CA}^n = \omega_{II}^2 \cdot CA = 640\ sm/s^2.$$

$\vec{a}_{CA}^n$ vektor C nuqtadan A nuqta tomon yo'naladi.

$\vec{a}_C$ vektor $\vec{a}_A^n$ va $\vec{a}_{CA}^n$ vektorlardan qurilgan parallelogramm diagonali orqali ifodalanadi *(4.54-b rasm).*

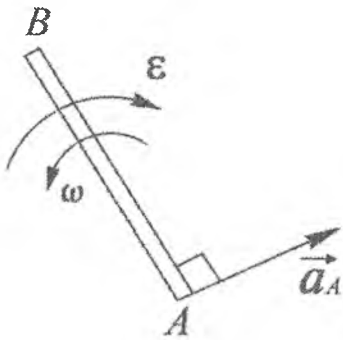
$\vec{a}_A^n$ va $\vec{a}_{CA}^n$ o'zaro perpendikular bo'lganliklari uchun $\vec{a}_C$ ning moduli quyidagicha aniqlanadi:

$$a_C = \sqrt{(a_A^n)^2 + (a_{CA}^n)^2} = 1153\ sm/s^2. \qquad (4.74)$$

69-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

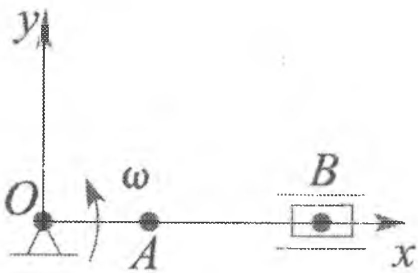
1-muammo. Radiusi 0,5 *m* bo'lgan g'ildirak tekislik bo'ylab $s=2t$ qonun bo'yicha harakat qiladi. G'ildirakning tekislikka tegib turgan nuqtasining tezlanishini aniqlang.

2-muammo. Uzunligi AB=1 m bo'lgan sterjen tekislik bo'ylab harakatlanadi. Agar uning burchak tezligi $\omega = 2\ rad/s$, burchak tezlanishi $\varepsilon = 2\ rad/s^2$ va A nuqtasining tezlanishi $a_A = 1\ m/s^2$ bo'lsa, B nuqtasining tezlanishini hisoblang *(4.55-rasm).*



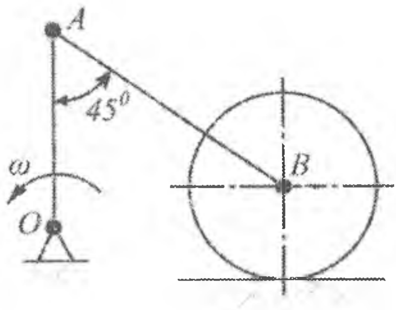
4.55-rasm

3-muammo. Krivoship-polzunli mexanizmning OA krivoshipi o'zgarmas $\omega = 10\ rad/s$ burchak tezlik bilan aylanadi. Shaklda ko'rsatilgan holat uchun AB shatunning burchak tezlanishini toping *(4.56-rasm).*



4.56-rasm

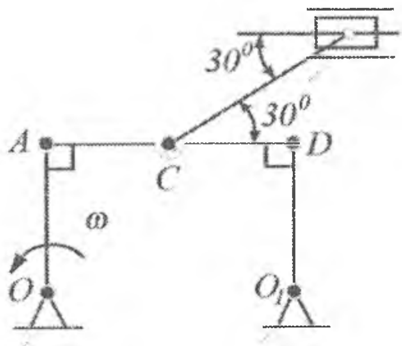
4-muammo. Krivoship-shatunli mexanizmning o'lchamlari OA=0,3 m va AB=0,45 m bo'lib, OA krivoship o'zgarmas burchak tezlik $\omega = 10\ rad/s$ bilan aylanadi. AB shatunning burchak tezlanishini hisoblang *(4.57-rasm).*



4.57-rasm

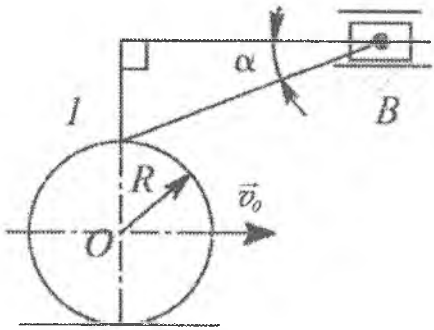
5-muammo. Sharnirli parallelogrammning *OA* krivoshipi o'zgarmas burchak tezlik $\omega = 0,4\ rad/s$ bilan aylanadi.

Agar mexanizmning o'lchamlari *OA*=20 *sm*, *CD*=30 *sm* bo'lsa, ko'rsatilgan holat uchun *CD* shatunning burchak tezlanishini toping *(4.58-rasm).*



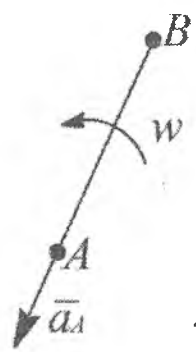
4.58-rasm

6-muammo. Mexanizmning polzuni B radius *R*=50 sm li 1-g'ildirakka sharnir yordamida bog'langan bo'lib, uning markazi o'zgarmas $v_0 = 5\ m/s$ tezlik bilan harakatlansa, *B* polzunning tezlanishini aniqlang. Bunda $\alpha = 30°$ *(4.59-rasm).*



4.59-rasm

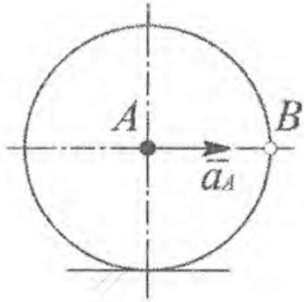
7-muammo. Uzunligi 2 *m* bo'lgan *AB* sterjen tekis parallel harakat qiladi. Agar sterjenning burchak tezligi $\omega = 1$ *rad / s*, burchak tezlanishi ε=0 va *A* nuqtasining tezlanishi 1 *m/s*^{2} bo'lsa, *B* nuqtasining tezlanishini toping *(4.60-rasm).*



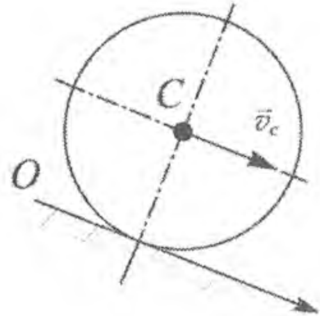
4.60-rasm

8-muammo. G'ildirak sirpanmasdan dumalaydi. Uning *B* nuqtasining tezlanishini *A* nuqtaning tezligi va tezlanishi: $v_A = 0$ va $a_A = 2$ *m/s*^{2} holat uchun hisoblang *(4.61-rasm).*

9-muammo. Sirpanmasdan dumalayotgan g'ildirakning markazi o'zgarmas tezlikka ega. G'ildirak tezliklar oniy markazi bo'lib hisoblangan nuqtaning tezlanish vektori *Ox* o'q bilan qanday burchak hosil qiladi. *4.62-rasm* (*Ox* o'q gorizontal holda joylashgan).



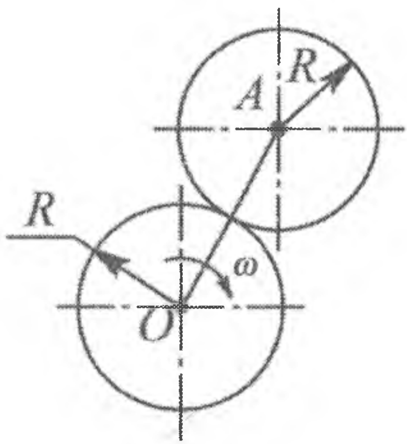
4.61-rasm



4.62-rasm

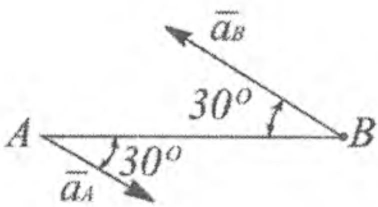
10-muammo. Planetar mexanizmining krivoshipi *OA* o'zgarmas burchak tezlik $\omega = 1$ *rad/s* bilan aylanadi.

Agar g'ildiraklarning radiuslari $R = 0{,}1$ bo'lsa, qo'zg'aluvchi g'ildirak tezliklar oniy markazi bo'lgan nuqtaning tezlanishini toping *(4.63-rasm)*.



4.63-rasm

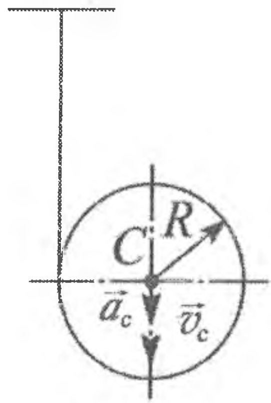
11-muammo. Uzunligi $AB = 40$ *sm* bo'lgan sterjen shakl tekisligida harakat qiladi. Biror vaqtdan keyin uning *A* va *B* nuqtalari $a_A = 2$ m/s^2 va $a_B = 6$ m/s^2 tezlanishlarga ega bo'lsa, sterjenning burchak tezlanishini toping *(4.64-rasm)*.



4.64-rasm

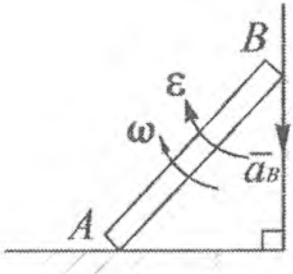
12-muammo. Radiusi R=0,066 *m* bo'lgan ip o'raglan slindr pastga tushadi.

Agar berilgan paytda uning markazi $v_c = 0,66\ m/s$ tezlikka va a_c = 6,6 m/s^2 tezlanishga ega bo'lsa, *C* markazdan tezlanishlar oniy markazigacha bo'lgan masofani toping *(4.65-rasm).*



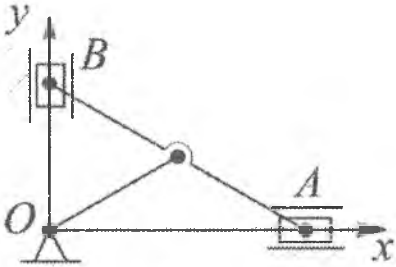
4.65-rasm

13-muammo. Uzun *AB* jism bir uchi bilan devorga, ikkinchi uchi bilan yerga tiralib, pastga tushadi. Agar berilgan paytda jismning burchak tezligi ω = 0,6 *rad/s* va burchak tezlanishi ε = 0,36 *rad/s* ga teng bo'lsa, *B* nuqtaning tezlanish vektori $\vec{a}_B$ va *B* nuqtadan jism tezlanishlar oniy markazi — *Q*, ya'ni *BQ* kesma orasidagi burchakning radian qiymatini toping *(4.66-rasm).*



4.66-rasm

14-muammo. Ellipsograf chizg'ichining A polzuni $a_A = 4\ m/s^2$ tezlanishiga ega bo'lib, tezlanishlar oniy markazi Q dan A va B nuqtalargacha bo'lgan masofalar $AQ = 33\ m$ $BQ = 53\ sm$ bo'lsa, B polzunning tezlanishini hisoblang *(4.67-rasm).*

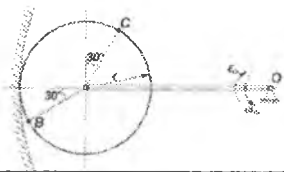
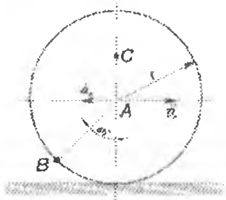
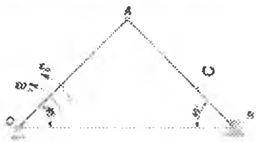
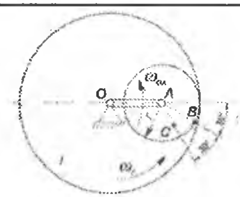
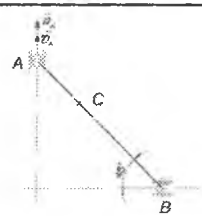
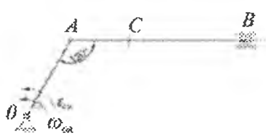


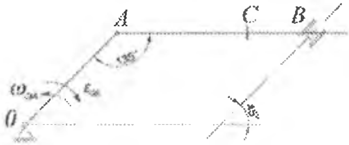
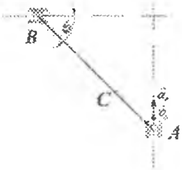
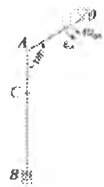
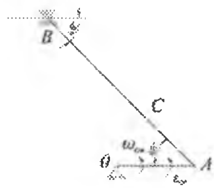
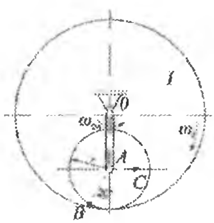
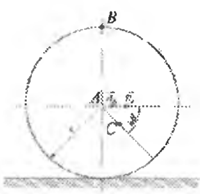
4.67-rasm

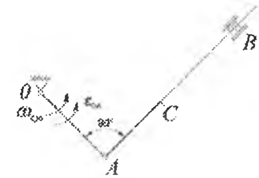
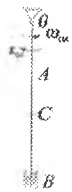
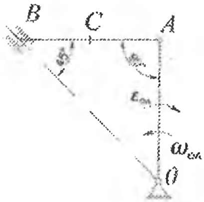
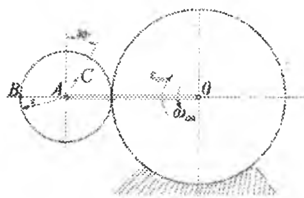
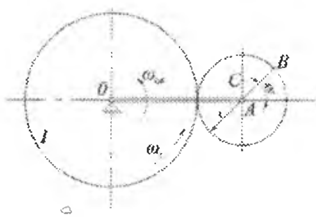
70-§. Mustaqil yechish uchun talablarga tavsiya etiladigan masalalar

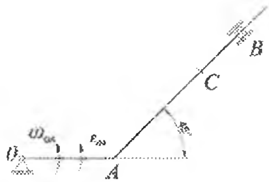
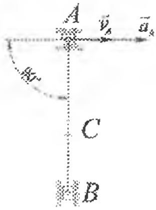
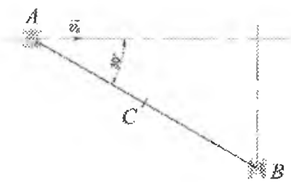
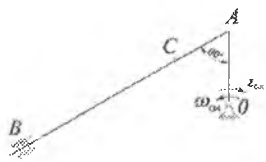
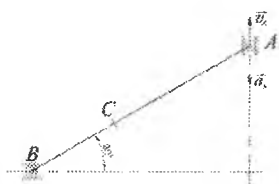
Mexanizmning berilgan holati uchun B va C nuqtalarning tezliklari va tezlanishlari hamda shu nuqtalar tegishli bo'lgan zvenoning burchak tezligi va burchak tezlanishi topilsin.

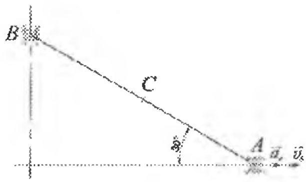
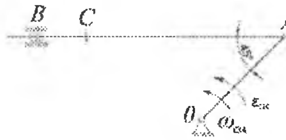
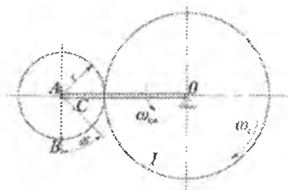
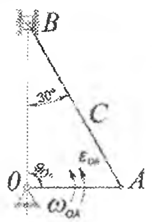
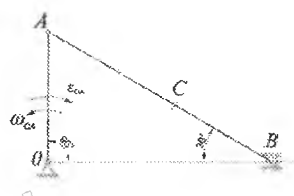
Mexanizmlarning sxemalari va hisoblash uchun kerakli ma'lumotlar quyidagi jadvalda keltirilgan.

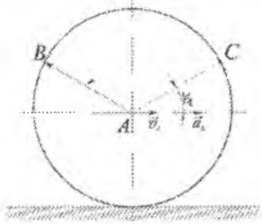
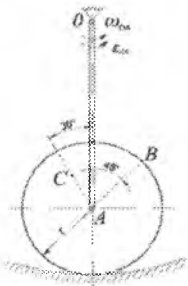
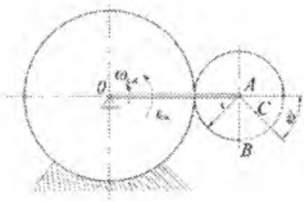
Variant raqam-lar	Mexanizmlarning sxemalari	Hisoblash uchun kerak ma'lumotlar
1.		$OA=60$ sm, $r=20$ sm, $\omega_{OA}=2\ rad/s$, $\varepsilon_{OA}=4\ rad/s^2$.
2.		$r=45$ sm, $AC=15$ sm, $v_A=100\ sm/s$, $a_A=50$ sm/s².
3.		$OA=20$ sm, $AB=20$ sm, $AC=10$ sm, $\omega_{OA}=2\ rad/s$, $\varepsilon_{OA}=6\ rad/s^2$.
4.		$OA=30$ sm, $r=20$ sm, $AC=15$ sm, $\omega_{OA}=2\ rad/s$, $w_1=2{,}5 rad/s$, $\varepsilon_{OA}=0 rad/s^2$.
5.		$AB=30$ sm, $AC=10$ sm, $v_A=10 sm/s$, $a_A=15$sm/s².
6.		OA=30 sm, $AB=60$ sm, $AC=20$ sm, $w_{OA}=2\ rad/s$, $\varepsilon_{OA}=6 rad/s^2$.

7.		OA=40 sm, AB=60 sm, AC=40 sm, $w_{OA}=3\ rad/s$, $\varepsilon_{OA}=8 rad/s^2$.
8.		AB=60 sm, AC=20 sm, $\vec{v}_A = 5 sm/s$, a_A=10 sm/s^2.
9.		OA=30 sm, AB=40 sm, AC=15 sm, $\omega_{OA}=3\ rad/s$, $\varepsilon_{OA}=3\ rad/s^2$.
10.		OA=30 sm, AB=80 sm, AC=25 sm, $\omega_{OA}=1\ rad/s$, $\varepsilon_{OA}=2\ rad/s^2$.
11.		OA=20 sm, AB=15 sm, AC=10 sm, $\omega_1=1{,}2\ rad/s$, $\varepsilon_{OA}=0$, $\omega_{OA}=2\ rad/s$.
12.		r=20 sm, AC=10 sm, $v_A = 60 sm/s$, a_A=30 sm/s^2.

13.		OA=30 sm, AB=60 sm, AC=25 sm, $w_{OA}=1\ rad/s$, $\varepsilon_{OA}=1\ rad/s^2$.
14.		OA=20 sm, AB=40 sm, AC=15 sm, $w_{OA}=4\ rad/s$, $\varepsilon_{OA}=6\ rad/s^2$.
15.		OA=40 sm, AC=20 sm, $\omega_{OA}=4\ rad/s$, $\varepsilon_{OA}=8\ rad/s^2$.
16.		OA=50 sm, r=20 sm, AC=10 sm, $\omega_{OA}=1\ rad/s$, $\varepsilon_{OA}=8\ rad/s^2$.
17.		OA=60 sm, r=25 sm, $\omega_{OA}=1\ rad/s$, AC=10 sm, $\omega_1=12 rad/s$, $\varepsilon_{OA}=0$.

18.		OA=30 sm, AB=60 sm, AC=40 sm, $\omega_{OA}=1\ rad/s$, $\varepsilon_{OA}=2\ rad/s^2$.
19.		AB=40 sm, AC=25 sm, $v_A=20\ sm/s$, a_A=20 sm/s^2.
20.		AB=40 sm, AC=20 sm, $v_A=10\ sm/s$, a_A=0.
21.		OA=30 sm, AB=60 sm, AC=20 sm, $\omega_{OA}=2\ rad/s$, $\varepsilon_{OA}=2\ rad/s^2$.
22.		AB=50 sm, AC=30 sm, $v_A=20\ sm/s$, $a_A=10 sm/s^2$.

23.		AB=50 sm, AC=30 sm, $v_A = 20\ sm/s$, a_A=20sm/s^2.
24.		OA=30 sm, AB=60 sm, AC=40 sm, ω_{OA}=4 rad/s, ε_{OA}=10 rad/s^2.
25.		OA=60 sm, r=15 sm, AC=6 sm, ω_{OA}=1 rad/s, ω_1=1 rad/s, ε_{OA}=0.
26.		OA=25 sm, AC=20 sm, ω_{OA}=1 rad/s, ε_{OA}=1 rad/s^2.
27.		OA=40 sm, AC=50 sm, ω_{OA}=4 rad/s, ε_{OA}=8 rad/s^2.

28.		r=50 sm, $v_A = 50sm/s$, a_A=100 sm/s^2.
29.		OA=40 sm, r=20 sm, AC=10 sm, ω_{OA}=3 rad/s, ε_{OA}=2 rad/s^2.
30.		OA=40 sm, r=15 sm, AC=8 sm, w_{OA}=1 rad/s, ε_{OA}=1 rad/s^2.

Eslatma. ω_{OA}, ε_{OA} – OA krivoship mexanizmning berilgan vaziyatidagi burchak tezligi va burchak tezlanishi; ω_1 – 1-g'ildirakning burchak tezligi (doimiy); v_A – va a_A – A nuqtaning tezligi va tezlanishi. G'ildiraklar sirpanishsiz aylanadi.

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MUNDARIJA

II BOB.
QATTIQ JISMNING ILGARILANMA VA QO‘ZG‘ALMAS O‘Q ATROFIDA AYLANMA HARAKATI

III BOB.
NUQTANING MURAKKAB HARAKATI

IV BOB.
QATTIQ JISMNING TEKISLIKKA PARALLEL HARAKATI

KO‘CHARBOY KENJAYEV

NAZARIY MEXANIKA

Misol va masalalarda

II qism

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