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# MAXSUS YO'L BILAN YECHILADIGAN ALGEBRAIK MASALALAR

*OLIY O'QUV YURTLARIGA KIRUVCHILAR HAMDA FANLAR OLIMPIADASIGA TAYYORGARLIK KO'RAYOTGANLAR UCHUN*

$$f_1^{\,2}(x)+f_2^{\,2}(x)+\ldots+f_n^{\,2}(x)=0$$

$$\begin{cases} f_1(x)=0 \\ f_2(x)=0 \\ \dots\dots\dots \\ f_n(x)=0 \end{cases}$$

$$\left(\frac{1}{16}\right)^x=\log_{\frac{1}{16}} x$$

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**Akademik litsey, kasb-hunar kollejlari va umumiy o'rta maktab o'quvchilari uchun o'quv qo'llanma**

*"O'zbekiston milliy ensiklopediyasi"*
*Davlat ilmiy nashriyoti*
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# So'z boshi

Hurmatli o'quvchim! Hozirgi kunda Respublikamizning oliy o'quv yurtlariga qabul imtihonini test uslubida o'tkazish an'anaga aylandi. Test sinovlarini muvaffaqiyatli topshirishlaring uchun avvalo Davlat test markazi tomonidan chop etilgan „Axborotnoma"larda berilgan test savollarining yechimlarini o'rganib chiqishlaringiz kerak bo'ladi. Ayrim testlar murakkabroq tuzilgan bo'lib, ularni yechish o'quvchidan alohida tayyorgarlik ko'rinishni talab qiladi. E'tiboringizda taqdim etilayotgan ushbu kitobda yechilishi maxsus yo'lni talab qiladigan ayrim masalalar jamlangan. Ularni yechish uchun nazariy ma'lumotlar ham berilgan. Har bir mavzudan keyin mustaqil yechish uchun masalalar keltirilgan.

Ushbu kitob umumiy o'rta ta'lim maktablarining yuqori sinf o'quvchilari, o'rta maxsus-kasb hunar ta'limi o'quvchilari va o'qituvchilariga mo'ljallangan. Qo'llanmadan hamkasblarim o'zlarining repetitorlik faoliyatlarida, to'garak mashg'ulotlarida, o'quvchilarni fan olimpiyadalariga tayyorlashda, dars jarayonida yoki takrorlash uchun berilgan dars soatlarida, fan oyliklarida, o'quvchi esa mustaqil ravishda oliy o'quv yurtlariga kirish va fan olimpiyadalariga tayyorgarlik ko'rishda foydalanishlari mumkin.

O'quv qo'llanmaning mundarijasidan shu narsa ko'rinib turibdiki, unda darsliklarda e'tibor berilmagan yoki yetarli yoritilmagan mavzular keltirilgan. Masalan, sonning butun va kasr qismlari, funksiyaning davrlarini topish, funksiyaning eng katta va eng kichik qiymatlarini topish, Diofant tenglamalari yechishda keng o'rin berilgan.

Ushbu o'quv qo'llanmani o'qib chiqib o'zlarini maslahatlarini bergan Ter.DU ning dotsenti P.Ismoilovga, kompyuterda o'quv qo'llanmani tergan shogirtlarim Akbar va Abbos Xolmamatovlarga CH. Mamatmurodovga, hamda kitobni chiqarishda amaliy yordam bergan shogirdim A.Sultonov, Boysun tuman bandlikka ko`maklashish markazi direktori Z.Norovga o'z minnatdorchiligimni bildiraman.

Aziz o'quvchim! Davlat test markazi tomonidan e'lon qilingan testlarni mukammalroq o'rganib, Sizni oliy o'quv yurtiga kirishingizga va matematikadan fan olimpiyadasida sovrinli o'rinlarni olishingizga muvoffaqiyatlar tilayman.

Ushbu kitob haqida qimmatli vaqtini ayamay o'z fikr va mulohazalarini bildirgan o'quvchilarimga oldindan minnatdorchiligimni izhor qilaman.

Muallif

## 1-§. Sonning butun va kasr qismlari

**1-ta'rif.** $x$ sonining butun qismi deb, $x$ dan katta bo'lmagan eng katta butun songa aytiladi. $x$ sonining butun qismi $[x]$ simvoli bilan belgilanadi.

Masalan: $[3,2]=3$, $[\pi]=3$, $[-3,2]=-4$, $[-\pi]=-4$, $[-8]=-8$

$f(x)=[x]$ funksiyani qaraylik. Bu funksiya quyidagi xossalarga ega:

1. Funksiyani aniqlanish sohasi barcha haqiqiy sonlar, qiymatlar sohasi butun sonlar to'plamidan iborat.
2. Ixtiyoriy $x$ haqiqiy son uchun $[x]\le x<[x]+1$ tengsizlik o'rinli.
3. Istalgan $x_1$, $x_2$ haqiqiy sonlar uchun $[x_1+x_2]\ge[x_1]+[x_2]$ bo'ladi.
4. Ixtiyoriy $x$ haqiqiy va $n$ butun sonlar uchun $[x+n]=[x]+n$ tenglik to'g'ri.
5. Agar $[x]=[y]$ bo'lsa, $|x-y|<1$ bo'ladi.
6. Agar $n$ natural son bo'lsa, u holda $\left[\frac{[x]}{n}\right]=\left[\frac{x}{n}\right]$ bo'ladi.

**2- ta'rif.** $x$ sonining kasr qismi deb, $x$ bilan uning butun qismi orasidagi ayirmaga aytiladi. $x$ sonining kasr qismi $\{x\}$ simvoli bilan belgilanadi. U holda $\{x\}=x-[x]$.

Masalan: $\{3,2\}=3,2-[3,2]=3,2-3=0,2$

$\{-3,2\}=-3,2-[-3,2]=-3,2+4=0,8$

Istalgan $x$ sonini uning butun va kasr qismlari yig'indisi, ya'ni $x=[x]+\{x\}$ ko'rinishida tasvirlash mumkin.

$g(x)=\{x\}$ funksiyani qaraylik. Bu funksiya quyidagi xossalarga ega:

1. Funksiyani aniqlanish sohasi barcha haqiqiy sonlar to'plami, qiymatlar sohasi $[0;1)$ oraliqdan iborat.
2. Ixtiyoriy $x$ haqiqiy son uchun $\{[x]\}=0$ va $[\{x\}]=0$ bo'ladi.
3. $g(x)=\{x\}$ funksiya asosiy davri 1 ga teng davriy funksiyadir, ya'ni $\{x\}=\{x+1\}$ tenglik istalgan $x$ haqiqiy son uchun bajariladi.

1(00-2-3). Ifodani hisoblang. $[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+...+[\sqrt{9}]+[\sqrt{10}]$ bunda $[a]$- $a$ sonining butun qismi. A)15 B)19 C)18 D)17 E)21

**Yechish.** Bizga ma'lumki: $\sqrt{1}=1$, $1<\sqrt{2}<2$, $1<\sqrt{3}<2$, $\sqrt{4}=2$, $2<\sqrt{5}<3$, $2<\sqrt{6}<3$, $2<\sqrt{7}<3$, $2<\sqrt{8}<3$, $\sqrt{9}=3$. U holda $[\sqrt{1}]=1$, $[\sqrt{2}]=1$, $[\sqrt{3}]=1$, $[\sqrt{4}]=2$, $[\sqrt{5}]=2$, $[\sqrt{6}]=2$, $[\sqrt{7}]=2$, $[\sqrt{8}]=2$, $[\sqrt{9}]=3$, $[\sqrt{10}]=3$. Bularni hisobga olsak $[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+...+[\sqrt{10}]=1+1+1+2+2+2+2+3+3=15$

**Javob.** 15 (A)

2(99-8-30). Yig'indini hisoblang. Bunda $[a]$ yozuv $a$ sonining butun qismini bildiradi. $[\lg 28]+[\lg 0{,}026]$

A) 0 B) 1 C) -1 D) -2 E) 2

**Yechish.** $1<\lg 28<2$, $\lg 0{,}026=\lg(26\cdot 10^{-3})=\lg 26-3$, $1<\lg 26<2$ bo'lganligi sababli $[\lg 28]=1$, $[\lg 0{,}026]=-2$ bo'ladi. U holda $[\lg 28]+[\lg 0{,}026]=1-2=-1$

**Javob.** -1 (C)

3(97-5-18). Tenglamani yeching. $[x^2]=9$

A)3 B)-3 C)$(-\sqrt{10};-3)\cup(3;\sqrt{10})$ D)$\left[-\sqrt{10};-3\right]\cup\left[3;\sqrt{10}\right]$ E)$\left(-\sqrt{10};-3\right]\cup\left[3;\sqrt{10}\right)$

**Yechish.** $x^2=t$ belgilash kiritaylik. U holda $[t]=9$ bo'ladi. $9\le t<10$ tengsizlikni qanoatlantiruvchi $t$ sonlar $[t]=9$ tenglamani yechimi bo'ladi. Belgilashni hisobga olsak $9\le x^2<10$ bo'ladi. Bu tengsizlikni yechamiz. $y=x^2$ funksiyaning $[0;\infty)$ oraliqda o'suvchiligidan $\sqrt{9}\le\sqrt{x^2}<\sqrt{10}$, yoki $3\le|x|<\sqrt{10}$ bo'ladi. Bu tengsizlikni yechishda ikki holni qaraymiz.

**1-hol.** $x>0$ bo'lsa, $|x|=x$ bo'ladi. U holda berilgan tenglamani $3\le x<\sqrt{10}$ yechimini topamiz.

**2-hol.** $x<0$ bo'lsa, $|x|=-x$ bo'ladi. Buni hisobga olsak, $3\le -x<\sqrt{10}$ yoki $-\sqrt{10}<x\le -3$ bo'ladi. Yuqoridagi ikki holni birlashtirib, berilgan tenglamani yechimini hosil qilamiz.

**Javob.** $\left(-\sqrt{10};-3\right]\cup\left[3;\sqrt{10}\right)$ (E).

**4-misol.** $[\lg x]\cdot\{\lg x\}=\lg x$ tenglamani yeching.

**Yechish.** $[\lg x]=n$, $n\in Z$ va $\{\lg x\}=\alpha$, $0\le\alpha<1$ bo'lsin. U holda $\lg x=n+\alpha$ bo'ladi. Bularni tenglamaga qo'ysak, $n\alpha=n+\alpha$ ga ega bo'lamiz. Bundan $n=\dfrac{\alpha}{\alpha-1}$ yoki $\alpha=\dfrac{n}{n-1}$ bo'ladi. $0\le\alpha<1$ ekanligidan $n\le 0$ ekanligi kelib chiqadi. Demak, $\lg x=\dfrac{n^2}{n-1}$. Bundan $x=10^{\frac{n^2}{n-1}}$

**Javob.** $x=10^{\frac{n^2}{n-1}}$, $n\le 0$, $n$-butun son.

**5-misol.** $\left[\dfrac{5+6x}{8}\right]=\dfrac{15x-7}{5}$ tenglamani yeching.

**Yechish.** $\dfrac{15x-7}{5}=t$ deb belgilasak, $x=\dfrac{5t+7}{15}$ bo'lib, bularni hisobga olsak, tenglama $\left[\dfrac{10t+39}{40}\right]=t$ ko'rinishni oladi. Butun qismning ta'rifiga asosan $0\le\dfrac{10t+39}{40}-t<1$ bo'ladi. Bu tengsizlikni yechib,

$-\frac{1}{30} < t \le 1.3$ ni hosil qilamiz. $t$ sonining butun son ekanligidan oxirgi tengsizlikdan $t=0$ yoki $t=1$ ekanligi kelib chiqadi. $t=0$ bo'lsa $x_1=\frac{7}{15}$, $t=1$ bo'lsa $x_2=\frac{4}{5}$ bo'ladi.

**Javob.** $x_1=\frac{7}{15}$; $x_2=\frac{4}{5}$

**6-misol.** $x^2-2\{x\}-5=0$ tenglamani yeching.

**Yechish.** Tenglamadan $2\{x\}=x^2-5$ kelib chiqadi. $0\le\{x\}<1$ dan $5\le x^2<7$ yoki $\sqrt{5}\le|x|<\sqrt{7}$ bo'ladi. Bu yerdan $[x]=2$ yoki $[x]=-3$ bo'ladi. Endi $\{x\}=x-[x]$ ekanligini hisobga olsak, berilgan tenglama quyidagi ikkita tenglamaga ajraladi.

1) $x^2-2(x-2)-5=0$ yoki 2) $x^2-2(x+3)-5=0$.

Birinchi tenglamani yechib $x=1\pm\sqrt{2}$ ni hosil qilamiz. $x=1+\sqrt{2}$ ildiz $[x]=2$ shartni qanoatlantiradi. Ikkinchi tenglamani yechib $[x]=-3$ shartni qanoatlantiruvchi $x=1-2\sqrt{3}$ ildizini hosil qilamiz.

**Javob.** $x=1+\sqrt{2}$, $x=1-2\sqrt{3}$

**7-misol.** $[-x^2+3x]=\left[x^2+\frac{1}{2}\right]$ tenglamani yeching.

**Yechish.** $x^2+\frac{1}{2}>0$ ekanligi ma'lum. Shuning uchun $\left[x^2+\frac{1}{2}\right]=n\ge0$.

Ikkinchi tomondan $-x^2+3x\ge3$ yoki $x^2-3x+3\le0$ tengsizlik yechimga ega emas. Shuning uchun $n$ soni 0;1;2 qiymatlar qabul qilishi mumkin.

1) $n=0$ bo'lsin. $[-x^2+3x]=0$; $\begin{cases}0\le -x^2+3x\le 1\\ 0\le x^2+\frac{1}{2}<1\end{cases}$ bundan $0\le x<\frac{1}{2}(3-\sqrt{5})$

2) $n=1$ bo'lsin. $[-x^2+3x]=1$; $\begin{cases}1\le -x^2+3x\le 2\\ 1\le x^2+\frac{1}{2}<2\end{cases}$ bundan $\frac{\sqrt{2}}{2}\le x<1$

3) $n=2$ bo'lsin. $[-x^2+3x]=2$; $\begin{cases}2\le -x^2+3x\le 3\\ 2\le x^2+\frac{1}{2}<3\end{cases}$ bundan $\frac{\sqrt{6}}{2}\le x<\frac{\sqrt{10}}{2}$

Sonning butun qismi qo'llaniladigan quyidagi Lejandr teoremasini keltiraylik.

**Teorema.** $p$ tub soni $n!$ sonining tub ko'paytuvchilariga yoyilmasida quyidagi $\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\ldots$ daraja ko'rsatgichi bilan qatnashadi.

( yetarli katta $k$ lar uchun $\left[\frac{n}{p^k}\right]=0$)

8 **misol.** $1\cdot2\cdot3\cdot...\cdot150$ soni nechta nol bilan tugaydi?

**Yechish.** $1\cdot2\cdot3\cdot...\cdot150$ ko'paytma nechta nol bilan tugallanishi *150*! ni tub ko'paytuvchilarga ajratganda *2* va *5* ko'paytuvchilar necha marta qatnashishiga bog'liq (chunki $2\cdot5=10$ ). *150*! da *5* ko'paytuvchi *2* ko'paytuvchiga nisbatan kamroq qatnashadi. *5* ko'paytuvchini necha marta qatnashishini hisoblash yetarli.

$$\alpha=\left[\frac{150}{5}\right]+\left[\frac{150}{25}\right]+\left[\frac{150}{125}\right]=30+6+1=37$$

**Javob.** *150*! soni *37* ta nol bilan tugallanadi.

9(00-6-1). *10* dan boshlab *75* dan katta bo'lmagan barcha natural sonlarni ko'paytirish natijasida hosil bo'ladigan sonning oxirida nechta nol qatnashadi? A)15 B)16 C)17 D)18 E)14

**Yechish.** $\left[\frac{75}{5}\right]+\left[\frac{75}{25}\right]+\left[\frac{75}{125}\right]-\left[\frac{5}{5}\right]=15+3+0-1=17$

**Javob.** 17 (C)

10(02-12-22). *20* dan katta bo'lmagan barcha natural sonlarning ko'paytmasi $n(n\in N)$ ning qanday eng katta qiymatida $2^n$ ga qoldiqsiz bo'linadi?

A)10 B)18 C)20 D)16 E)14

**Yechish.** $1\cdot2\cdot3\cdot...\cdot20$ ko'paytmada 2 tub ko'paytuvchi necha marta qatnashishini hisoblaymiz. Lejandr teoremasiga asosan

$$\left[\frac{20}{2}\right]+\left[\frac{20}{4}\right]+\left[\frac{20}{8}\right]+\left[\frac{20}{16}\right]=10+5+2+1=18$$

**Javob.** 18 (B)

## Mustaqil yechish uchun masalalar

1(97-9-18). $[x^2]=36$ tenglamani yeching.

A)6 B)-6 C)$\left(-\sqrt{37};-6\right]\cup\left[6;\sqrt{37}\right)$ D)$\left(-37;-6\right)\cup\left(6;\sqrt{37}\right)$

E)$[-37;-6)\cup[6;37]$

12(00-1-21). *1* dan *50* gacha bo'lgan sonlarning ko'paytmasi nechta nol bilan tugaydi? *A)14 B)10 C)13 D)11 E)12*

13. $[x]^2-6[x]+8=0$ tenglamani yeching.

14. $x^2-[x]=3$ tenglamani yeching.

15. $\left[\frac{8x+19}{7}\right]=\frac{16(x+1)}{11}$ tenglamani yeching.

16. $[x+[x]]=2$ tenglamani yeching.

17. $2^{[x]}=1+2x$ tenglamani yeching.

18(99-7-3). $\sqrt{50}$ ning butun qismini toping. A)8 B)7 C)6 D)9 E)5

19(02-10-40). $-\frac{21}{6}+2,(2)$ ning butun qismini toping. A)-2 B)-1 C)0 D)1 E)2

20(97-4-4). $k$ raqamining qanday qiymatlarida $\sqrt{30+k}$ ning butun qismi 5 bo'ladi?
A)6;7;8;9 B)0;1;2 C)1;2;3 D)5;6 E) 0;1;2;3;4;5

21(97-9-64). $n$ raqamining qanday qiymatlarida $\sqrt{49+n}$ ning butun qismi 7 bo'ladi?
A)0;1;2 B)0;1 C)3;4;5 D) hech qanday qiymatida E) barcha qiymatlarida

## 2-§.Ko'phadlar to'g'risida kerakli ma'lumotlar

$n$-darajali $P(x)=a_0x^n+a_1x^{n-1}+...+a_{n-1}x+a_n$ (1) ko'phad berilgan bo'lsin. Bu ko'phad uchun quyidagi fikrlar o'rinli.

1) (1) ko'phadning barcha koeffitsientlari yig'indisi, uning $x=1$ nuqtadagi qiymatiga, ya'ni $P(1)=a_0+a_1+...+a_{n-1}+a_n$ ga teng;

2) (1) ko'phadda $x$ ning toq darajalari oldidagi koeffitsientlarining yig'indisi $\frac{P(1)-P(-1)}{2}$ ga teng;

3) (1) ko'phadda $x$ ning juft darajalari oldidagi koeffitsientlarining yig'indisi $\frac{P(1)+P(-1)}{2}$ ga teng;

4) $g(x)=x^n+a_1x^{n-1}+...+a_{n-1}x+a_n$ ko'phad $x-a$ ga bo'linsa, qoldiq bu ko'phadning $x=a$ bo'lgandagi qiymati $g(a)$ ga teng bo'ladi;

5) $n$- darajali ko'phad $n$ tadan ortiq bo'lmagan haqiqiy ildizlarga ega bo'ladi(ildizlar karralisini hisobga olganda);

6) toq darajali ko'phad hech bo'lmaganda bitta haqiqiy ildizga ega bo'ladi;

7) agar $P_n(x)=Q_m(x)\cdot K_l(x)$ tenglik o'rinli bo'lsa, u holda $P_n(x)$ ko'phadning har bir ildizi $Q_m(x)$ yoki $K_l(x)$ ko'phadlarning birining ildizi bo'ladi. $Q_m(x)$

va $K_i(x)$ ko'phadlarning har bir ildizlari $P_n(x)$ ko'phadning ildizlari bo'ladi;

8) agar $a$ soni $P_n(x)$ ko'phadning ildizi bo'lsa, u holda $P_n(x)=(x-a)\cdot Q_{n-1}(x)$ tenglik o'rinli bo'ladi;

9) $\frac{p}{q}$ ($p$-butun son, $q$-natural son) qisqarmas ratsional son $P_n(x)$ ko'phadning ildizi bo'lishi uchun $p$ soni ozod had $a_n$ ning, $q$ soni esa bosh koeffitsient $a_0$ ning bo'luvchisi bo'lishi zarur va yetarli;

10) agar $P_n(x)$ da bosh koeffitsient $a_0=1$ bo'lsa, va ko'phadning koeffitsientlari butun sonlardan iborat bo'lsa, u holda bu ko'phadning ratsional ildizlari butun sondan iborat bo'ladi va ozod had $a_n$ ning bo'luvchisidan iborat bo'ladi;

11) (Viyet teoremasi). Agar $x_1$, $x_2$, $x_3$, ..., $x_n$ sonlar (1) ko'phadning haqiqiy ildizlari bo'lsa, u holda quyidagi tengliklar o'rinli bo'ladi:

$$\begin{cases} x_1+x_2+\ldots+x_n=-\frac{a_1}{a_0} \\ x_1x_2+x_1x_3+\ldots+x_{n-1}x_n=\frac{a_2}{a_0} \\ x_1x_2x_3+\ldots+x_{n-2}x_{n-1}x_n=-\frac{a_3}{a_0} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_1x_2\cdot\ldots\cdot x_n=(-1)^n\cdot\frac{a_n}{a} \end{cases}$$

1(98-6-19). Agar $(x-1)^2\cdot(x+1)^3+3x-1$ ifoda standart shaklidagi ko'phad ko'rinishida yozilsa, koeffitsientlarining yig'indisi nechaga teng bo'ladi?

A)10 B)4 C)2 D)3 E)1

**Yechish.** 1) ga asosan: $x=1$ bo'lsa, $(1-1)^2(1+1)^3+3\cdot1-1=2$

**Javob.** 2 (C)

2(98-11-68). Agar $(x^3-x+1)^3+x$ ifoda standart shaklidagi ko'phad ko'rinishida yozilsa, $x$ ning toq darajalari oldidagi koeffitsientlarining yig'indisi nechaga teng bo'ladi?

A)1 B)7 C)4 D)5 E)3

**Yechish.** 2) ga asosan: $\frac{P(1)-P(-1)}{2}=\frac{(1^3-1+1)^3+1-((-1)^3+1+1)^3+1}{2}=\frac{2+0}{2}=1$

**Javob.** 1 (A)

3(03-3-26). $f(x)=(x^3+2x^2-1)^2-3x^2$ ko'phadning juft darajali koeffitsientlarining yig'indisini toping. A)-6 B)-2 C)3 D)-3 E)-1

**Yechish.** 3) ga asosan: $f(1)=(1^3+2\cdot1^2-1)^2-3\cdot1^2=1$,

$f(-1)=((-1)^3+2\cdot(-1)^2-1)^2-3\cdot(-1)^2=-3$. U holda $\frac{f(1)+f(-1)}{2}=\frac{1-3}{2}=-1$

**Javob.** – 1 (E)

4(02-8-4). $x^{2001}+3x^{2000}+3x+13$ ko'phadni $x+3$ ga bo'lganda qoldiq necha bo'ladi? A)4 B)3 C)5 D)2 E)1

**Yechish.** 4) ga asosan berilgan ko'phadni $x=-3$ nuqtadagi qiymatini hisoblash yetarli. $(-3)^{2001}+3\cdot(-3)^{2000}+3\cdot(-3)+13=(-3)^{2001}-(-3)^{2001}-9+13=4$

**Javob.** 4 (A)

5(02-8-5). $x^6+x^4-3x^2+5$ ko'phadni $x^2-\sqrt{3}$ ga bo'lgandagi qoldiqni toping. A)8 B)7 C)6 D)9 E)5

**Yechish.** $x^2-\sqrt{3}=(x-\sqrt[4]{3})(x+\sqrt[4]{3})$ bo'lganligi sababli (4) ga asosan berilgan ko'phadni $x=\sqrt[4]{3}$ nuqtadagi qiymatini topish yetarli.

$(\sqrt[4]{3})^6+(\sqrt[4]{3})^4-3\cdot(\sqrt[4]{3})^2+5=3\sqrt{3}+3-3\sqrt{3}+5=8$

**Javob.** 8 (A)

6(02-9-16). Agar $a+b+c=12$; $ab+bc+ac=-15$ bo'lsa, $a^2+b^2+c^2$ ning qiymatini toping. A)84 B)114 C)144 D)174 E)204

**Yechish.** $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ac)$ formulaga asosan:

$a^2+b^2+c^2=(a+b+c)^2-2(ab+bc+ac)=12^2-2(-15)=144+30=174$

**Javob.** 174 (D)

7(02-10-18). $a^4+4b^4$ni ratsional ko'paytuvchilarga ajrating.

A)$(a^2-2ab+2b^2)(a^2+2ab+2b^2)$ B)$(a^2-2b^2)^2$ C) $(a^2+2b^2)^2$

D)$(a^2-2b^2)(a^2+2b^2)$ E)$(a^2+b^2)(a^2-4b^2)$

**Yechish.**

$a^4+4b^4=((a^2)^2+4a^2b^2+(2b^2)^2)-(2ab)^2=(a^2+2b^2)^2-(2ab)^2=(a^2+2b^2-2ab)(a^2+2b^2+2ab)$

**Javob.** $(a^2-2ab+2b^2)(a^2+2ab+2b^2)$ (A)

## Mustaqil yechish uchun masalalar

8(03-5-14). $x^3-3x^2-4x+12$ ko'phad quyidagilarning qaysi biriga bo'linmaydi?

A)$x+3$ B)$x-3$ C)$x+2$ D)$x-2$ E)$x^2-x-6$

9(00-5-25). Ko'paytuvchilarga ajrating. $a^3+9a^2+27a+19$

A) $(a+1)(a^2-3a+19)$ B)$(a+1)(a^2+3a+19)$ C) $(a+1)(a^2+8a+19)$

D) $(a-1)(a^2+3a+19)$ E) $(a-1)(a^2+8a+19)$

**10**(00-10-77). Ko'phadni ko'paytuvchilarga ajrating.
$(x-y)^3-(z-y)^3+(z-x)^3$
A) $3(x-y)(y-z)(x-z)$ B) $-3(x-y)(z-y)(x-z)$ C) $3(y-x)(y-z)(z-x)$
D) $-3(x-y)(z-y)(z-x)$ E) ko'paytuvchiga ajralmaydi
**11**(97-5-16). Ko'paytuvchilarga ajrating. $x^4+x^2+1$
A) $(x^2+x+1)(x^2+x-1)$ B) $(x^2+x+1)(x^2-x+1)$ C) $(x^2+x+1)(x^2-x-1)$
D) $(x^2+x+1)(-x^2+x-1)$ E)ko'paytuvchilarga ajratib bo'lmaydi
**12**(99-9-9). Ushbu $x^{12}-1$ ifodani ko'paytuvchilarga ajratganda, nechta ratsional ko'paytuvchilardan iborat bo'ladi? A)4 B)5 C)6 D)8 E)7
**13**(99-1-14). Agar $f\left(\frac{ax-b}{bx-a}\right)=x^{50}+x^{49}+x^{48}+...+x^2+x+1$ $(|a|\neq|b|)$ bo'lsa, $f(1)$ ni toping. A)1 B)2 C)51 D)4 E)5

## 3-§. Yuqori darajali tenglamalarni yechish

$ax^3+bx^2+cx+d=0$ ko'rinishdagi kub tenglamalar uchun quyidagi Viyet teoremasi o'rinli.

**Teorema**. Agar $x_1,x_2,x_3$ sonlar uchinchi darajali tenglamaning ildizlari bo'lsa, u holda quyidagi tengliklar o'rinli bo'ladi.

$$x_1+x_2+x_3=-\frac{b}{a},\quad x_1x_2+x_1x_3+x_2x_3=\frac{c}{a},\quad x_1x_2x_3=-\frac{d}{a}$$

uchinchi darajali tenglamalarni umumiy yechish formulasi oliy ta'limda o'rganilib, quyida berilgan misollarni kub tenglamani chap qismini ko'paytuvchilarga ajratish usuli bilan yechamiz.

**1**(99-10-6). Ushbu $x^3-px^2-qx+4=0$ tenglamaning ildizlaridan biri *1* ga teng.Shu tenglama barcha koeffitsientlarini yig'indisini toping.
A)-1 B)0 C)1 D)1,5 E)2
**Yechish.** $f(x)=$ $x^3-px^2-qx+4$ ko'phadning barcha koeffitsientlari yig'indisi uning $x=1$ dagi qiymatiga teng. Haqiqatdan: $f(1)=1\cdot1^3-p\cdot1^2-q\cdot1+4=1-p-q+4$. $x=1$ soni $f(x)$ ko'phadning ildizi bo'lganligi sababli $f(1)=0$ bo'ladi. Demak, $1-p-q+4=0$.
**Javob.** *0* (B)
**2**(97-1-12). Tenglamaning ildizlari yig'indisini toping. $x^3+2x^2-9x-18=0$
A)9 B)-2 C)6 D)-18 E)2

Yechish. 1-usul. Tenglamani chap qismini ko'paytuvchilarga ajrataylik. [illegible] $=0$, $x_1=-2$ $x_2=3$, $x_3=-3$ U holda $x_1+x_2+x_3=-2$ bo'ladi.

**Javob.** -2 (B)

**2-usul.** Viyet teoremasiga asosan bu tenglamaning ildizlarining yig'indisi qarama-qarshi ishora bilan olingan $x^2$ oldidagi koeffitsientga teng bo'ladi. Bundan $x^2$ oldidagi koeffitsientning qarama-qarshisi *-2*.

**Javob.** -2 (B)

3(97-6-12). Tenglamaning ildizlari ko'paytmasini toping. $x^3-3x^2-4x+12=0$

A)6 B)4 C)12 D)-12 E)24

**Yechish.** **1-usul.** Tenglamaning chap qismini ko'paytuvchilarga ajrataylik. $x^2(x-3)-4(x-3)=0,$ $(x-3)(x-2)(x+2)=0,$ bundan $x_1=3$ $x_2=2$ $x_3=-2$ ildizlarni topamiz. U holda $x_1x_2x_3=3\cdot2\cdot(-2)=-12$

**Javob.** *-12* (D)

**2-usul.** Viyet teoremasiga asosan uchinchi darajali keltirilgan tenglamaning ildizlari ko'paytmasi qarama-qarshi ishora bilan olingan ozod hadga teng. Bu misolda ozod hadning qarama-qarshisi *-12* ga teng.

**Javob.** *-12* (D)

4(02-11-24). $x^3-7x-6=0$ tenglamaning barcha haqiqiy ildizlari o'rta geometrigini toping. A) $\sqrt{6}$ B) $\sqrt[3]{6}$ C) $-\sqrt[3]{6}$ D) $2\sqrt{2}$ E)-2 .

**Yechish. 1-usul.** Tenglamaning chap qismini ko'paytuvchilarga ajrataylik.

$(x^3+x^2)-(x^2+x)-6(x+1)=0,$ $x^2(x+1)-x(x+1)-6(x+1)=0,$ $(x+1)(x^2-x-6)=0,$ $(x+1)(x+2)(x-3)=0.$ Bundan $x_1=-1$ $x_2=-2$ $x_3=3$ ildizlarni topamiz. U holda $\sqrt[3]{x_1x_2x_3}=\sqrt[3]{6}$

**Javob.** $\sqrt[3]{6}$ (B)

**2-usul.** Viyet teoremasiga asosan uchinchi darajali keltirilgan tenglamaning ildizlari ko'paytmasi qarama-qarshi ishora bilan olingan ozod hadga teng. Bu misolda ozod hadning qarama-qarshi *6* ga teng.U holda ildizlarning o'rta geometrigi $\sqrt[3]{x_1x_2x_3}=\sqrt[3]{6}$ ga teng.

**Javob.** $\sqrt[3]{6}$ (B)

5(03-3-22). $x^3-13x+12=0$ tenglamaning haqiqiy ildizlarining o'rta arifmetigini toping. A) $2\frac{2}{3}$ B) $1\frac{1}{3}$ C)0 D) $-\frac{1}{2}$ E) $-1\frac{1}{3}$

**Yechish. 1-usul.** Tenglamani chap qismini ko'paytuvchilarga ajrataylik.

$(x^3-x^2)+(x^2-x)-(12x-12)=0,$ $x^2(x-1)+x(x-1)-12(x-1)=0,$

$(x-1)(x^2+x-12)=0,$ $(x-1)(x+4)(x-3)=0.$ Bundan $x_1=1$ $x_2=-4$ $x_3=3$

ildizlarni topamiz. U holda ildizlarning o'rta arifmetigi $\frac{x_1+x_2+x_3}{3}=\frac{0}{3}=0$ ga teng bo'ladi.
**Javob.** *0* (C)
**2-usul.** Viyet teoremasiga asosan bu tenglamaning ildizlarining yig'indisi qarama-qarshi ishora bilan olingan $x^2$ oldidagi koeffitsientga teng bo'ladi. Bundan $x^2$ oldidagi koeffitsient 0 ga teng bo'lganligi sababli $\frac{x_1+x_2+x_3}{3}=\frac{0}{3}=0$ bo'ladi.
**Javob.** 0 (C)
6(98-10-45). Quydagi mulohazalardan qaysi biri to'g'ri.
A) $6x^4+3x^3+8=0$ tenglamaning ildizi $x=3$ bo'lishi mumkin.
B) $3x^6+4x=-9$ tenglama musbat ildizga ega.
C) $12x^3+7x=2$ tenglama manfiy ildizga ega.
D) $x^2-2x-8=0$ tenglamaning ildizlari qarama-qarshi ishorali.
E) $p\neq 0$ da $x^2-px+p^2=0$ tenglama ikkita musbat ildizga ega.
**Yechish.** Mulohazalarni birma-bir ko'rib chiqaylik:
A) $6x^4+3x^3+8=0$ tenglama $x=3$ ildizga ega bo'lsa, $6x^4+3x^3+8>0$ bo'ladi. Demak bu mulohaza noto'g'ri.
B) $3x^6+4x=-9$ tenglama musbat ildizga ega bo'lsa, u holda $3x^6+4x>0$ bo'ladi. Musbat son manfiy songa teng bo'lmaganligi sababli bu mulohaza noto'g'ri C) $12x^3+7x=2$ tenglama manfiy ildizga ega bo'lsa, $12x^3+7x<0$ bo'ladi. Manfiy son musbat songa teng bo'lmaganligi sababli bu mulohaza ham noto'g'ri.
D) $x^2-2x-8=0$ tenglama Viyet teoremasiga asosan ikkita qarama-qarshi $x_1=-2,\quad x_2=4$ ildizlarga ega. Demak, bu mulohaza tog'ri.
Tekshirishni shu yerda to'xtatamiz.
**Javob.** (D)
7(98-12-18). $a$ ning qanday qiymatida $\frac{a^3}{a^2-1}$ kasrning qiymati $\frac{27}{8}$ ga teng bo'ladi. A)3 B)2 C)27 D)8 E)9
**Yechish. 1-usul.** Test javoblarini ko'zdan kechirib, $\frac{27}{8}=\frac{3^3}{3^2-1}$ ekanligini hisobga olsak, $\frac{a^3}{a^2-1}=\frac{27}{8}=\frac{3^3}{3^2-1}$ $a=3$ ekanligi kelib chiqadi.
**2-usul.** $\frac{a^3}{a^2-1}=\frac{27}{8}$ tenglamani yechamiz. Bundan $8a^3-27a^2+27=0$. Tenglamaning chap qismini ko'paytuvchilarga ajrataylik:
$(8a^3-24a^2)+(-3a^2+9a)-(9a-27)=0,$
$8a^2(a-3)-3a(a-3)-9(a-3)=0,\quad (a-3)(8a^2-3a-9)=0\quad a=3$

**Javob.** *a=3* (A)

**8(99-2-15).** Ushbu $x^4=3x^2-2x$ tenglamaning eng katta va eng kichik ildizlari yig'indisini toping. A)3 B)-3 C)1 D)-1 E)-2

**Yechish.** $x^4-3x^2+2x=0$, $x(x^3-3x+2)=0$, $x=0$ yoki $x^3-3x+2=0$. Tanlash yordami bilan bu tenglamaning *x=1* ildizini ozod hadning bo'luvchisi sifatida topamiz. U holda tenglamaning chap qismi ko'paytuvchiga ajraladi. $(x^3-x^2)+(x^2-x)+(-2x+2)=0$,

$x^2(x-1)+x(x-1)-2(x-1)=0$, $(x-1)(x^2+x-2)=0$, $x=1$ yoki

$x^2+x-2=0$, $x_2=-2$, $x_3=1$. Tenglamaning eng katta ildizi *1*, eng kichik ildizi -2. U holda *1-2=-1*

**Javob.** *-1* (D)

**9(00-10-49).** *m* ning qanday qiymatida $x(x+a)(x+b)(x+a+b)+4m^2$ ifoda to'la kvadrat bo'ladi? *A)* $\frac{a^2b^2}{4}$ *B)* $\pm\frac{ab}{4}$ *C)* $\pm\frac{a+b}{4}$ *D)* $\frac{ab^2}{4}$ *E)* bunday qiymatlar mavjud emas.

**Yechish.** Ifodani shakl almashtiraylik:

$x(x+a)(x+b)(x+a+b)+4m^2=$

$(x(x+a+b))((x+a)(x+b))+4m^2=(x^2+(a+b)x)(x^2+(a+b)x+ab)+4m^2;$

$x^2+(a+b)x=t$ belgilash kiritaylik. U holda ifoda $t(t+ab)+4m^2=t^2+abt+4m^2$ ko'rinishni oladi. Bu kvadrat uchhad to'la kvadrat bo'lishi uchun uning diskriminanti *0* ga teng bo'lishi kerak. $D=(ab)^2-16m^2=0$ bundan $m=\pm\frac{ab}{4}$.

**Javob.** $m=\pm\frac{ab}{4}$ (B)

## Mustaqil yechish uchun masalalar

**10(97-11-12).** Tenglamaning ildizlari ko'paytmasini toping.
$x^3+5x^2-4x-20=0$
A)-10 B)20 C)-4 D)-20 E)16

**11(00-8-12).** Tenglama ildizlarining yig'indisini toping.
$x^3+3x^2-4x-12=0$
A)-3 B)-7 C)4 D)12 E)0

**12(02-11-22).** $x^3-3x^2-2x+6=0$ tenglamaning ildizlari ko'paytmasini toping. A)3 B)-6 C)6 D)-3 E)1

**13(03-3-21).** $x^3-5x^2-2x+10=0$ tenglamaning ildizlari ko'paytmasini toping.
A)10 B)-10 C)20 D)5 E)-5

**14-misol.** Tenglamani yeching. $(x+2)(x+4)(x-6)(x-8)=2925$

**15-misol.** Tenglamani yeching.

1) $x^3-3x^2-4x+12=0$; 2) $x^5+x^4-6x^3-14x^2-11x-3=0$; 3) $x^4-3x^3-2x^2-6x-8=0$

**16-misol.** $2x^3+mx^2+nx+12=0$ tenglama $x_1=1$, $x_2=-2$ ildizlarga ega. Shu tenglamaning uchinchi ildizini toping.

**17-misol.** $m$ ning qanday qiymatida $x(x+5a)(x+2b)(x+5a+2b)+25m^2$ ifoda to'la kvadrat bo'ladi?

*A)* $\pm\frac{ab}{5}$ *B)* bunday qiymatlar mavjud emas. *C)* to'g'ri javob keltirilmagan *D)* $\frac{a^2b^2}{25}$ *E)* $\pm ab$

**18-misol.** $m$ ning qanday qiymatida $x(x+a)(x+2b)(x+a+2b)+m^2$ ifoda to'la kvadrat bo'ladi?

*A)* $a^2b^2$ *B)* bunday qiymatlar mavjud emas. *C)* to'g'ri javob keltirilmagan *D)* $\pm ab$ *E)* $\pm\frac{ab}{2}$

**19(99-7-43).** $27^x+12^x-2\cdot 8^x=0$ **tenglamaning ildizini uch baravarini toping.**
A) -6 B) 3 C) -3 D) 6 E) 0

## 4-§. $f_1^2(x)+f_2^2(x)+\ldots+f_n^2(x)=0$ ko'rinishdagi tenglamalarni yechish

$f_1^2(x)+f_2^2(x)+\ldots+f_n^2(x)=0$ ko'rinishdagi tenglama quyidagi tenglamalar sistemasiga teng kuchli:

$$\begin{cases} f_1(x)=0 \\ f_2(x)=0 \\ \ldots\ldots\ldots\ldots \\ f_n(x)=0 \end{cases}$$

1(97-12-10). Agar $(a-|b|)^2+(a-2)^2=0$ bo'lsa, $2a-3b$ ning qiymatini toping.
A)-2 B)10 C)2 va 10 D)-2 va 10 E)-10

**Yechish.** Berilgan tenglik quyidagi tenglamalar sistemaga teng kuchli

$$\begin{cases} a-|b|=0 \\ a-2=0 \end{cases}$$

Bu tenglamalar sistemasini yechamiz.

$$\begin{cases} |b|=a \\ a=2, \end{cases} \quad \begin{cases} |b|=2 \\ a=2, \end{cases} \quad \begin{cases} b=\pm 2 \\ a=2. \end{cases}$$

1) agar $a=2,\ b=2$ bo'lsa, $2a-3b=2\cdot 2-3\cdot 2=-2$.
2) agar $a=2,\ b=-2$ bo'lsa, $2a-3b=2\cdot 2-3\cdot(-2)=10$

**Javob.** -2 va 10 (D)

2(98-11-61). Agar $x$ va $y$ sonlari $x^2+y^2+(y-1)^2=2xy$ tenglikni qanoatlantirsa, $x+y$ qanchaga teng bo'ladi? A)4 B)1 C)3 D)2 E)5

**Yechish.** Berilgan tenglikni $x^2-2xy+y^2+(y-1)^2=0$ yoki $(x-y)^2+(y-1)^2=0$ ko'rinishda yozamiz. Bu tenglik quyidagi tenglamalar sistemasiga teng kuchli.

$\begin{cases} x-y=0 \\ y-1=0. \end{cases}$ Bu sistemani yechamiz. $\begin{cases} x=y \\ y=1 \end{cases} \Leftrightarrow \begin{cases} x=1 \\ y=1. \end{cases}$

$x=1$ va $y=1$ bo'lsa $x+y=1+1=2$ bo'ladi.

**Javob.** 2 (D)

3(98-12-80). Agar $x^2+y^2+2(2x-3y)+|z-xy|+13=0$ bo'lsa, $x+y+z$ ni toping.

A)8 B)11 C)-5 D)-7 E)aniqlab bo'lmaydi.

**Yechish.** Berilgan tenglikni quyidagi ko'rinishda yozamiz.

$(x^2+4x+4)+(y^2-6y+9)+|z-xy|=0,\quad (x+2)^2+(y-3)^2+|z-xy|=0.$

Nomanfiy sonlarning yig'indisini 0 ga teng bo'lish shartiga asosan:

$\begin{cases} x+2=0 \\ y-3=0 \\ |z-xy|=0, \end{cases} \begin{cases} x=-2 \\ y=3 \\ z=xy, \end{cases} \begin{cases} x=-2 \\ y=3 \\ z=-6 \end{cases}$ U holda $x+y+z=-2+3-6=-5$ bo'ladi.

**Javob.** -5 (C)

4(99-5-10). $m,n,k\in N$ $m^2+2n^2-2nk=25,\ 2mn-k^2=25$ bo'lsa, $\frac{(m+n)^2}{2k}$ ni hisoblang. A)1 B)2 C)5 D)10 E)15

**Yechish.** Masala shartidagi tengliklarni hadma-had ayirib shakl almashtiraylik. $m^2+2n^2-2nk-2mn+k^2=0,\ (m-n)^2+(n-k)^2=0.$ Bundan $\begin{cases} m-n=0 \\ n-k=0, \end{cases}$ $m=n=k$. Buni masala shartidagi birinchi tenglikga hisobga olsak: $m^2+2m^2-2m^2=25,\ m^2=25,\ m=5$. U holda

$\frac{(m+n)^2}{2k}=\frac{(5+5)^2}{2\cdot 5}=\frac{100}{10}=10$ bo'ladi.

**Javob.** 10 (D)

5(99-5-33). Agar $8(x^4+y^4)-4(x^2+y^2)+1=0$ bo'lsa, $|x|+|y|$ ning qiymatini toping. A)1 B)$\frac{1}{2}$ C)$\frac{1}{4}$ D)2 E)$\frac{1}{16}$

**Yechish.** Berilgan tenglikni ikkala qismini 2ga ko'paytiraylik. $16(x^4+y^4)-8(x^2+y^2)+2=0$, $(4x^2-1)^2+(4y^2-1)^2=0$. Bundan

$\begin{cases} 4x^2-1=0 \\ 4y^2-1=0, \end{cases}$ $\begin{cases} x=\pm\frac{1}{2} \\ y=\pm\frac{1}{2} \end{cases}$ Bularni hisobga olsak: $|x|+|y|=\frac{1}{2}+\frac{1}{2}=1$

**Javob.** 1 (A)

6(99-9-8). Agar $n-m=(a-2)^2$, $p-n=(b-3)^2$ va $m-p=(c-4)^2$ bo'lsa, $a+b+c$ yig'indi nechaga teng? *A)8 B)10 C)10 D)7 E)9*

**Yechish.** Berilgan tengliklarni hadma-had qo'shaylik. $n-m+p-n+m-p=$

$=(a-2)^2+(b-3)^2+(c-4)^2$, $(a-2)^2+(b-3)^2+(c-4)^2=0$. Bundan $\begin{cases} a-2=0 \\ b-3=0 \\ c-4=0, \end{cases}$

$\begin{cases} a=2 \\ b=3 \\ c=4. \end{cases}$ U holda $a+b+c=2+3+4=9$.

**Javob.** 9 (E)

7(99-10-8). Agar $m-n=(2x+y)^2$ $n-m=(4x-y-12)^2$ bo'lsa, $xy$ ni toping. *A)-6 B)6 C)-8 D)8 E)9*

**Yechish.** Berilgan tengliklarni hadma-had qo'shaylik. $m-n+n-m=(2x+y)^2+(4x-y-12)^2$, Yoki $(2x+y)^2+(4x-y-12)^2=0$.Bundan

$\begin{cases} 2x+y=0 \\ 4x-y-12=0, \end{cases}$ Bu sistemani qo'shish usuli bilan yechaylik $6x=12$, $x=2$.

$y=-2\cdot 2=-4$. U holda $xy=2\cdot(-4)=-8$.

**Javob.** *-8 (C)*

8(99-10-18). Nuqtaning koordinatalari $x^2-4x+y^2-6y+13=0$ tenglamani qanoatlantiradi. Nechta nuqta shu tenglamani qanoatlantiradi. *A)2 B)3 C)1 D)4 E)*Birorta ham nuqta qanoatlantirmaydi.

Yechish. Berilgan tenglamani chap qismidan ikkihadni kvadratini ajrataylik. $(x^2-4x+4)+(y^2-6y+9)=0$, $(x-2)^2+(y-3)^2=0$, $\begin{cases} x-2=0 \\ y-3=0, \end{cases}$ $\begin{cases} x=2 \\ y=3. \end{cases}$

*(2;3)*

**Javob.** *1 (C)*

9(00-2-7). Agar $4a^2+9b^2+16c^2-4a-6b-8c+3=0$ bo'lsa, $abc$ ko'paytmaga teskari sonni toping. *A)*$\frac{1}{24}$ *B)12 C)48 D)24 E)* $\frac{1}{12}$

**Yechish.** Tenglikni chap qismini shakl almashtiraylik
$(4a^2-4a+1)+(9b^2-6b+1)+(16c^2-8c+1)=0$. $(2a-1)^2+(3b-1)^2+(4c-1)^2=0$.

Bundan $\begin{cases} 2a-1=0 \\ 3b-1=0 \\ 4c-1=0, \end{cases}$ $\begin{cases} a=\frac{1}{2} \\ b=\frac{1}{3} \\ c=\frac{1}{4} \end{cases}$ U holda $abc=\frac{1}{2\cdot 3\cdot 4}=\frac{1}{24}$. $\frac{1}{abc}=24$

**Javob.** *24* *(D)*

**10**(00-6-14). Tenglamalar sistemasi nechta yechimga ega?

$$\begin{cases} y=x^2+7x+11 \\ y=y^2+3x+15 \end{cases}$$

*A)4 B)3 C)2 D)1 E)∅*

**Yechish.** Tenglamalar sistemasidagi tenglamalarni hadma-had qo'shib, shakl almashtiralik $2y=x^2+y^2+10x+26$,

$(x^2+2\cdot 5\cdot x+25)+(y^2-2y+1)=0$,

$(x+5)^2+(y-1)^2=0$, $\begin{cases} x+5=0 \\ y-1=0, \end{cases}$ $\begin{cases} x=-5 \\ y=1, \end{cases}$ $(-5;1)$

**Javob.** *1* ta yechimga ega *(D)*

**11**(00-9-39). $9(x^4+y^4)-6(x^2+y^2)+2=0$ ekanligini bilgan holda, $x^2+y^2$ ning qiymatini hisoblang. *A)*$\frac{1}{3}$ *B)1* *C)*$\frac{2}{3}$ *D)*$\frac{4}{3}$ *E)3*

**Yechish.** Masaladagi tenglikni shakl almashtiraylik.

$(9x^4-6x^2+1)+(9x^4-6y^2+1)=0$, $(3x^2-1)^2+(3y^2-1)^2=0$, $\begin{cases} 3x^2-1=0 \\ 3y^2-1=0, \end{cases}$

$\begin{cases} x^2=\frac{1}{3} \\ y^2=\frac{1}{3}, \end{cases}$ U holda $x^2+y^2=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

**Javob.** $\frac{2}{3}$ *(C)*

**12**(00-10-80). Ushbu $(|x_1|-1)^2+(|x_2|-2)^2+....+(|x_n|-n)^2+...=0$ tenglikni qanoatlantiradigan $(x_n)$ arifmetik progressiyalar nechta?

*A)2 B)1 C)n D)2n E)n-1*

**Yechish.** Berilgan tenglik quyidagi tenglamalar sistemasiga teng kuchli

$$\begin{cases}|x_1|-1=0\\|x_2|-2=0\\\dots\\|x_n|-n=0,\end{cases}\quad\begin{cases}x_1=\pm1\\x_2=\pm2\\\dots\\x_n=\pm n\end{cases}\quad\begin{cases}x_1=1\\x_2=2\\\dots\\x_n=n\end{cases}\ yoki\ \begin{cases}x_1=-1\\x_2=-2\\\dots\\x_n=-n\end{cases}$$

Bizga ma'lumki, natural sonlar va manfiy butun sonlar ketma-ketligi arifmetik progressiya tashkil etadi.

**Javob.** *2 (A)*

13(98-4-20). Agar $x$ va $z$ orasida $x^2+z^2+x+z+\frac{1}{2}=0$ munosabat o'rinli bo'lsa, $xz$ ning qiymati qanchaga teng bo'ladi ?
*A)0,25 B)0,4 C)0,5 D)1 E)-0,8*

**Yechish.** Berilgan tenglikni chap qismini shakl almashtirib ikkihadlar kvadratlarining yig'indisi shaklida yozaylik.

$(x^2+2\cdot\frac{1}{2}x+\frac{1}{4})+(z^2+2\cdot\frac{1}{2}z+\frac{1}{4})=0,\quad (x+\frac{1}{2})^2+(z+\frac{1}{2})^2=0,\begin{cases}x=-0,5\\z=-0,5\end{cases}$ U holda $xz=0,25$.

**Javob.** *0,25 (A)*

14(99 -5-42). Agar $x,y,z$ $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ va $\sqrt{2-tgx-ctgx}+\sqrt[4]{\sin y-1}+\sqrt[6]{\cos 2z-1}=0$ bo'lsa, $\frac{3y}{2x+5z}$ ning qiymatini toping. *A)*$\frac{1}{2}$ *B)1 C)2 D)3 E)*$\frac{3}{7}$

**Yechish.** Juft darajali ildizlarning qiymati nomanfiy bo'la olmaydi. Berilgan tenglik o'rinli bo'lishi uchun ildizlarning har birining qiymati nolga teng bo'lishi kerak.

$$\begin{cases}2-tgx-ctgx=0\\\sin y-1=0\\\cos 2z-1=0,\end{cases}\quad\begin{cases}tgx+ctgx=2\\\sin y=1\\\cos 2z=1,\end{cases}\quad\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$$ kesmaga tegishli

$tgx+ctgx=2$ tenglamaning ildizi $x=\frac{\pi}{4}$, $siny=1$ tenglamani ildizi $y=\frac{\pi}{2}$, $cos2z=1$ tenglamaning ildizi $z=0$. U holda

$$\frac{3y}{2x+5z}=\frac{3\cdot\frac{\pi}{2}}{2\cdot\frac{\pi}{4}+5\cdot0}=\frac{3\pi}{2}\cdot\frac{4}{2\pi}=3.$$

**Javob.** *3 (D)*

15(02-9-8). Agar $16a^2+9b^2+4c^2+3=8a+6b+4c$ bo'lsa, $a+b+c$ ga teskari sonni toping. *A)*$-1\frac{1}{12}$ *B)*$\frac{12}{13}$ *C)*$\frac{12}{11}$ *D)*$-\frac{11}{12}$ *E)*$-\frac{12}{13}$

**Yechish.** Berilgan tenglikni shakl almashtiraylik.

$(16a^2-8a+1)+(9b^2-6b+1)+(4c^2-4c+1)=0, (4a-1)^2+(3b-1)^2+(2c-1)^2=0$

$\begin{cases}4a-1=0\\3b-1=0\\2c-1=0\end{cases}$ $\begin{cases}a=\frac{1}{4}\\b=\frac{1}{3}\\c=\frac{1}{2}\end{cases}$ U holda, $a+b+c=\frac{1}{4}+\frac{1}{3}+\frac{1}{2}=\frac{3+4+6}{12}=\frac{13}{12}$,

$\frac{1}{a+b+c}=\frac{12}{13}$

**Javob.** $\frac{12}{13}$ *(B)*

16(01-9-44). Tenglamani yeching

$\log^2{}_7(x^2+5x-13)+\log^2{}_{1/7}(x^2-8x+13)=0$

*A)3 B)2 C)5 D)1 E)∅*

**Yechish.** Berilgan tenglama quyidagi tenglamalar sistemasiga teng kuchli $\begin{cases}\log_7(x^2+5x-13)=0\\ \log_{1/7}(x^2-8x+13)=0,\end{cases} \Leftrightarrow \begin{cases}x^2+5x-13=1\\x^2-8x+13=1\end{cases}$

$\Leftrightarrow \begin{cases}x^2+5x-14=0\\x^2-8x+12=0\end{cases} \Leftrightarrow \begin{cases}x_1=-7, & x_2=2\\x_1=2, & x_2=6\end{cases} \quad x=2$

**Javob.** 2 (B)

17(03-5-42). $\cos^2\left(\frac{\pi x}{6}\right)+\sqrt{2x^2-5x-3}=0$ tenglamani yeching.

*A)3 B)$\frac{3}{2}$ C)$-\frac{1}{2}$ D)-3 E)$\frac{1}{2}$*

**Yechish.** Nomanfiy sonlarning yig'indisini nolga teng bo'lish shartiga asosan: $\begin{cases}\cos\frac{\pi x}{6}=0\\2x^2-5x-3=0.\end{cases}$ $2x^2-5x-3=0$ kvadrat tenglamaning ildizlari $x_1=3,\ x_2=-\frac{1}{2}$

Agar $x=3$ bo'lsa $\cos\frac{\pi\cdot 3}{6}=\cos\frac{\pi}{2}=0$ bo'ladi, ya'ni bu ildiz birinchi tenglamani qanoatlantirdi. $x=3$ berilgan tenglamani ildizi bo'ladi. Agar $x=-\frac{1}{2}$ bo'lsa, $\cos\left(\frac{\pi}{6}\cdot\left(-\frac{1}{2}\right)\right)=\cos\frac{\pi}{12}\neq 0$ bo'ladi, bu ildiz birinchi tenglamani qanoatlantirmaganligi sababli bu ildiz tenglamani ildizi bo'lmaydi

**Javob.** $x=3$ *(A)*

**18(99-4-54).** Ushbu $1+tg^4x=cos^2 2x$ tenglamaning $[-2\pi;2\pi]$ kesmada nechta ildizi bor? *A)6 B)5 C)4 D)2 E)1*

**Yechish.** Tenglamani soddalashtiraylik. $1-cos^2 2x+tg^4x=0$,

$sin^2 2x+tg^4x=0$, $\begin{cases} \sin 2x=0 \\ tgx=0 \\ \cos x\neq 0, \end{cases}$ $\begin{cases} 2\sin x\cos x=0 \\ tgx=0 \\ \cos x\neq 0, \end{cases}$ $\begin{cases} \sin x=0 \\ tgx=0 \\ \cos x\neq 0, \end{cases}$ $tgx=0$

tenglamani ildizlari $x=\pi n,\ n\in Z$. Bu ildizlardan $[-2\pi;2\pi]$ kesmaga tegishli ildizlari: $x=-2\pi,\ x=-\pi,\ x=0,\ x=\pi,\ x=2\pi$

**Javob.** 5 ta *(B)*

**19(99-5-2).** Agar $A^2+B^2+C^2=AB+AC+BC$ *bo'lsa,* $\frac{A+B}{C}+\frac{B+C}{A}$ ning qiymati nechaga teng bo'ladi? *A)* aniqlab bo'lmaydi *B)1 C)2 D)3 E)4*

**Yechish.** Berilgan tenglikni ikkala qismini 2 ga ko'paytirib, shakl almashtiraylik. $(A^2-2AB+B^2)+(A^2-2AC+C^2)+(B^2-2BC+C^2)=0$,

$(A-B)^2+(A-C)^2+(B-C)^2=0$, $\begin{cases} A-B=0 \\ A-C=0 \\ B-C=0, \end{cases}$ bundan $A=B=C$.

Agar $A=B=C$ bo'lsa, $\frac{A+B}{C}+\frac{B+C}{A}=\frac{A+A}{A}+\frac{A+A}{A}=2+2=4$.

**Javob.** *4 (E)*

**20(00-9-11).** Agar $A(A-B)+B(B-C)+C(C-A)=0$ va $ABC\neq 0$ bo'lsa $\frac{A^3+B^3+C^3}{A(B+C)^2+B(A+C)^2+C(A+B)^2}$ ning qiymati nechaga teng bo'ladi?

*A)0,25 B)0,5 C)0,75 D)1 E)1,25*

**Yechish.** Berilgan tenglikni ikkala qismini ikkiga ko'paytirib shakl almashtiraylik. $(A-B)^2+(A-C)^2+(B-C)^2=0$, $\begin{cases} A-B=0 \\ A-C=0 \\ B-C=0, \end{cases}$ $\begin{cases} A=B \\ A=C \\ B=C, \end{cases}$

bundan $A=B=C$. U holda

$$\frac{A^3+B^3+C^3}{A(B+C)^2+B(A+C)^2+C(A+B)^2}=\frac{A^3+A^3+A^3}{A(A+A)^2+A(A+A)^2+A(A+A)^2}=\frac{3A^3}{12A^3}=0,25.$$

**Javob.** *0,25 (A)*

## Mustaqil yechish uchun masalalar

**21.** Tenglamalar sistemasini yeching.

$$a)\begin{cases}1+x_1^2=2x_2\\1+x_2^2=2x_3\\1+x_3^2=2x_4\\1+x_4^2=2x_1\end{cases}\qquad b)\begin{cases}x^4+y^4+z^4=3\\x^5+y^5+z^5=3\\x^6+y^6+z^6=3\end{cases}$$

Javob: $a) x_1=x_2=x_3=x_4=1$

$b) x=y=z=1$

**22.** $x$ va $y$ ning qanday qiymatlarida tenglik to'g'ri?

$x^2+5y^2+4xy+2y+1=0$ Javob: $x=2$, $y=-1$

**23.** $x$ va $y$ ning qanday qiymatlarida tenglik to'g'ri?

$x^2-2\sqrt{2}x+y-2\sqrt{y}+3=0$ Javob: $x=\sqrt{2}, y=1$

**24.** $x^2-5x-4\sqrt{x}+13=0$ tenglama haqiqiy ildizga ega emasligini isbotlang.

**25.** $5x^2+5y^2+8xy+2y-2x+2=0$ tenglikni qanoatlantiruvchi barcha $x,y$ sonlar juftini toping. Javob: $x=1$, $y=-1$

**26.** $4x^2+9y^2+16z^2-4x-6y-8z+3=0$ tenglik qanday $x, y, z$ haqiqiy sonlar uchun bajariladi?

**27.** $\log_2^2(x+y)+\log_2^2(xy)+1=2\log_2(x+y)$ tenglamani yeching. Javob: $x=y=1$.

**28.** $\sin^2 x+\sin^2 y=\sin x\sin y+\sin x+\sin y-1$ tenglamani yeching.

**29.** Tenglamalar sistemasini yeching.

$$1)\begin{cases}x^2-yz=y-x\\y^2-xz=z-y\\z^2-xy=x-z\end{cases}\quad 2)\begin{cases}x^2+2y+1=0\\y^2+2z+1=0\\z^2+2x+1=0\end{cases}\quad 3)\begin{cases}4x^2+4y+1=0\\4y^2+4z+1=0\\4z^2+4x+1=0\end{cases}$$

$$4)\begin{cases}x+y+z=3\\2xy-2y-z^2=4\end{cases}\quad 5)\begin{cases}x^2+y^2=2y\\y^2+1=2xy\end{cases}$$

## 5-§. Tenglama va tengsizliklarni yechishda funksiyaning chegaralanganlik xossasidan foydalanish

**1-teorema.** Agar haqiqiy sonlarning biror M to'plamida $f(x)\le a, g(x)\le b$ tengsizliklar o'rinli bo'lsa, u holda $f(x)+g(x)=a+b$ (1) tenglama M to'plamda $\begin{cases}f(x)=a\\g(x)=b\end{cases}$ (2)tenglamalar sistemasiga teng kuchli bo'ladi.

**2-teorema.** Agar haqiqiy sonlarning biror M to'plamida $f(x)\ge a$, $g(x)\le a$ tengsizliklar o'rinli bo'lsa, u holda M to'plamda $f(x)=g(x)$ tenglama $\begin{cases}f(x)=a\\g(x)=a\end{cases}$ tenglamalar sistemasiga teng kuchli bo'ladi.

**3-teorema.** Agar haqiqiy sonlarning biror M to'plamida $|f(x)|\ge a$, $|g(x)|\ge b$, (yoki $|f(x)|\le a$, $|g(x)|\le b$) bo'lsa, u holda M to'plamda $f(x)\cdot g(x)=ab$ tenglama tenglamalarning quyidagi sistemasining birlashmasiga teng kuchli:

$$\left[\begin{array}{l}\begin{cases} f(x)=a, \\ g(x)=b \end{cases} \\ \begin{cases} f(x)=-a \\ g(x)=-b \end{cases}\end{array}\right.$$

1(00-5-42). Tenglamani yeching. *sin5x-3cos2x=4*

*A)* $-\frac{\pi}{2}+2\pi n,\ n\in Z$ *B)* $\frac{\pi}{2}+2\pi n,\ n\in Z$ *C)* $\pi+\pi n, n\in Z$ *D)* $\frac{\pi}{2}+2\pi n,\ n\in Z$ *E)* $2\pi n, n\in Z$

**Yechish.** $\sin 5x-3\cos 2x=1+3,\ \sin 5x\le 1,\ -3\cos 2x\le 3$. U holda 1-teoremaga asosan tenglama $\begin{cases}\sin 5x=1\\ -3\cos 2x=3\end{cases}$ tenglamalar sistemasiga teng kuchli. Bundan ikkinchi tenglamani yechaylik. $\cos 2x=-1$, $2x=\pi+2\pi n, n\in Z$, $x=\frac{\pi}{2}+\pi n, n\in Z$. Bu ildizlarni sistemaning birinchi tenglamasini qanoatlantirishini tekshiramiz.

$$\sin 5(\frac{\pi}{2}+\pi n)=\sin(\frac{5\pi}{2}+5\pi n)=\sin(\frac{\pi}{2}+2\pi+4\pi n+\pi n)=\sin(\frac{\pi}{2}+\pi n)=\cos\pi n=$$

$$=\begin{cases}1, & n=2k \quad bo'lsa\\ -1, & n=2k-1 \quad bo'lsa\end{cases}$$

Demak, $x=\frac{\pi}{2}+\pi n, n\in Z$ ildizlardan $n=2k$ bo'lsa,hosil bo'lgan ildizlar 1-tenglamani qanoatlantiradi.

**Javob.** $x=\frac{\pi}{2}+2\pi n, n\in Z$

2(00-6-55). Ushbu *cosxcos2xcos4x=1* tenglama [ $-2\pi;2\pi$. ] kesmada nechta ildizga ega? *A)1 B)2 C)3 D)4 E)* ∅

**Yechish.** Kosinuslar ko'paytmasini yig'indiga almashtiraylik.

$$\cos x\cos 2x\cos 4x=$$

$$=\frac{1}{2}(\cos(x+2x)+\cos(x-2x))\cos 4x=\frac{1}{2}(\cos 3x+\cos x)\cos 4x=\frac{1}{2}(\cos 3x\cos 4x+\cos x\cos 4x)=$$

$$=\frac{1}{2}(\frac{1}{2}(\cos 7x+\cos x)+\frac{1}{2}(\cos 5x+\cos 3x))$$

Demak tenglama quyidagi ko'rinishni oladi: $\cos x+\cos 3x+\cos 5x+\cos 7x=4$.

$\cos x\le 1,\ \cos 3x\le 1,\ \cos 5x\le 1,\ \cos 7x\le 1$ bo'lganligi sababli $\cos x+\cos 3x+\cos 5x+\cos 7x=1+1+1+1$ tenglama 1-teoremaga asosan :

$\begin{cases}\cos x=1\\ \cos 3x=1\\ \cos 5x=1\\ \cos 7x=1\end{cases}$ tenglamalar sistemasiga teng kuchli bo'ladi.

$\cos x=1$ tenglamani ildizlari $x=2\pi n, n\in Z$ sistemaning barcha tenglamalarini qanoatlantirgani sababli, bu ildizlar sistemani yechimi bo'ladi.Bu yechimlardan $[-2\pi;2\pi]$ kesmaga tegishli ildizlari $x_0=0,\ x_1=2\pi,\ x_2=-2\pi$ bo'ladi.

**Javob.** 3 ta (C)

3(00-9-37). Ushbu $cos\left(\frac{\sqrt{3}\pi}{12}x\right)=13+4\sqrt{3}x+x^2$ tenglama $[-2\pi;2\pi]$ kesmada nechta ildizga ega? *A)∅ B)1 C)2 D)3 E)4*

**Yechish.** $\cos\frac{\sqrt{3}}{12}\pi x\le 1,\ x^2+4\sqrt{3}x+13=(x+2\sqrt{3})^2+1\ge 1$ bo'lganligi sababli, 2-teoremaga asosan bu tenglama $\begin{cases}\cos(\frac{\sqrt{3}\pi}{12}x)=1,\\(x+2\sqrt{3})^2+1=1\end{cases}$ tenglamalar sistemasiga teng kuchli. Bu sistemaning ikkinchi tenglamasini yechaylik: $(x+2\sqrt{3})^2=0$, bundan $x=-2\sqrt{3}$ ildiz topiladi. Topilgan bu ildiz sistemaning birinchi tenglamasini qanoatlantirishini tekshiramiz.

$\cos(\frac{\sqrt{3}\pi}{12}(-2\sqrt{3}))=\cos\frac{\pi}{2}=0\ne 1$. Demak $x=-2\sqrt{3}$ ildiz tenglamalar sistemasining birinchi tenglamasini qanoatlantirmaganligi sababli bu sistema va tenglama yechimga ega emas.

**Javob.** ∅ (A)

4(00-9-46). $\alpha\in\left(0;\frac{\pi}{2}\right)$ va $\beta,\gamma\in[0;\pi]$ miqdorlar $2cos\gamma+3\sin 2\beta+\frac{4}{tg^2\alpha+ctg^2\alpha}=7$ tenglikni qanoatlantiradi. $\frac{3\alpha-\gamma}{5\gamma+6\beta}$ ning qiymatini hisoblang.

*A)* $\frac{3}{8}$ *B)* $\frac{1}{4}$ *C)* $\frac{2}{5}$ *D)* $\frac{1}{2}$ *E)* $\frac{4}{11}$

**Yechish.** Bizga ma'lumki $2\cos\gamma\le 2;\ 3\sin 2\beta\le 3;$

$\frac{4}{tg^2\alpha+ctg^2\alpha}=\frac{4}{(tg\alpha-ctg\alpha)^2+2}\le\frac{4}{2}=2$. Demak 1-teoremaga asosan

$2\cos\gamma+3\sin 2\beta+\frac{4}{tg^2\alpha+ctg^2\alpha}=2+3+2$ tenglik $2\cos\gamma=2,\ 3\sin 2\beta=3,$

$\frac{4}{tg^2\alpha+ctg^2\alpha}=2$ bo'lsa bajariladi. $\cos\gamma=1$ dan $\gamma=0$, $\sin 2\beta=1$ dan $2\beta=\frac{\pi}{2}$,

$\beta=\frac{\pi}{4}$. $\frac{4}{tg^2\alpha+ctg^2\alpha}=2$ dan $tg^2\alpha+ctg^2\alpha=2,\ tg^2\alpha+\frac{1}{tg^2\alpha}=2,\ (tg^2\alpha-1)^2=0,$

$tg^2\alpha=1,\ tg\alpha=1,\ \alpha=\frac{\pi}{4}$. U holda: $\frac{3\alpha-\gamma}{5\gamma+6\beta}=\frac{3\cdot\frac{\pi}{4}-0}{5\cdot 0+6\cdot\frac{\pi}{4}}=\frac{1}{2}$.

**Javob.** $\frac{1}{2}$ (D)

5(98-4-22). $k$ ning qanday qiymatida $|\ln(x+15)|=-(x+k)^2$ tenglama yechimga ega bo'ladi? *A)-15 B)14 C)15 D)10 E)-e*

**Yechish.** $f(x)=|\ln(x+15)|\ge 0$. $g(x)=-(x+k)^2\le 0$ bo'lganligi sababli berilgan tenglama $\begin{cases}|\ln(x+15)|=0\\ -(x+k)^2=0\end{cases}$ tenglamalar sistemasiga teng kuchli.Bundan

$\begin{cases}\ln(x+15)=0\\ k=-k,\end{cases}$ $\begin{cases}x=-15+1\\ x=-k,\end{cases}$ $k=14$

Javob. k=14 (B)

6(99-5-51). Ushbu $7^{x^2+|x|}=5^{-x^4}$ munosabat $x$ ning nechta qiymatida o'rinli? *A)∅ B)1 C)2 D)3 E)4*

**Yechish.** $x^2+|x|\ge 0$ va $y=7^t$ funksiyani o'suvchiligidan $7^{x^2+|x|}\ge 7^0=1$. $-x^4\le 0$ va $y=5^t$ ni o'suvchiligidan $5^{-x^4}\le 5^0=1$ bo'ladi. 2- teoremaga asosan berilgan tenglama $\begin{cases}7^{x^2+|x|}=1\\ 5^{-x^4}=1\end{cases}$ tenglamalar sistemasiga teng kuchli. Bu sistemani yechamiz. Ikkinchi tenglamadan $5^{-x^4}=5^0$,
$-x^4=0,\quad x=0$. $x=0$ ildiz sistemaning birinchi tenglamasini qanoatlantiradi.

**Javob.** $x=0$. 1 ta (B)

7(99-4-53). Tengsizlikni yeching $cos^2(x+1)\cdot\log_4(3-2x-x^2)\ge 1$
*A)*$[-1;0)$ *B)*$[-2;-1]$ *C)-2;-1 D)-1 E)(-3;0)∪(0;1)*

**Yechish.** $0\le\cos^2(x+1)\le 1$ va $y=\log_4 t$ funksiyani o'suvchiligidan $\log_4(3-2x-x^2)=\log_4(4-(x+1)^2)\le\log_4 4=1$ bo'ladi. Ikkita 1 dan katta bo'lmagan sonni ko'paytmasi hech vaqt 1 dan katta bo'lmaydi.Demak, yuqoridagi tengsizlikda faqat tenglik bajariladi. Shuning uchun berilgan tengsizlik quyidagi sistemaga teng kuchli. $\begin{cases}\cos^2(x+1)=1,\\ \log_4(4-(x+1)^2)=1.\end{cases}$ Bundan
$\log_4(4-(x+1)^2)=1,\ 4-(x+1)^2=4,\quad (x+1)^2=0,\quad x=-1$ ildizni topamiz. Bu topilgan ildiz $\cos^2(x+1)=1$ tenglamani qanoatlantirganligi sababli sistemani yechimi bo'ladi.

Javob: $x=-1$ (D)

8(00-9-24). Tenglamani ildizi nechta? $log_3x+log_x3=2cos(6\pi x^2)$
*A)∅ B)1 C)2 D)3 E)4*

**Yechish.** $0<x<1$ va $x>1$ hollarni alohida qaraymiz:

**1-hol.** $0<x<1$ bo'lsin . U holda $\log_3 x<0$ bo'ladi.

$\log_3 x+\log_x 3=\log_3 x+\frac{1}{\log_3 x}\le -2$ va $2cos(6\pi x^2)\ge -2$ tengsizliklar o'rinli bo'lganligi sababli berilgan tenglama quyidagi sistemaga teng kuchli.
$\begin{cases}\log_3 x+\log_x 3=-2\\ 2\cos(6\pi x^2)=-2\end{cases}$ Sistemani 1-tenglamasini yechamiz. $\log_3 x+\frac{1}{\log_3 x}=$

$=-2$, $\log_3^2 x+2\log_3 x+1=0$, $\log_3 x=-1$, $x=\frac{1}{3}$. Bu ildiz $0<x<1$ shartni qanoatlantiradi, ammo sistemaning 2-tenglamasini qanoatlantirmaganligi : $\cos(6\pi\cdot\frac{1}{9})=\cos\frac{2\pi}{3}\neq-1$ sababli bu holda tenglama ildizga ega emas.

**2-hol.** $x>1$ bo'lsa $\log_3 x+\log_x 3=\log_3 x+\frac{1}{\log_3 x}\geq 2$ $2\cos(6\pi x^2)\leq 2$ bo'lganligi sababli berilgan tenglama quyidagi tenglamalar sistemasiga teng kuchli

$$\begin{cases}\log_3 x+\log_x 3=2\\ 2\cos(6\pi x^2)=2\end{cases}$$

Sistemaning birinchi tenglamasidan $x=3$ ildizni topamiz. Bu ildiz sistemaning 2-tenglamasini qanoatlantiradi. Chunki $2\cos(6\pi\cdot 9)=2\cos(0+27\cdot 2\pi)=2\cos 0=2$

**Javob.** 1 ta (B)

9(01-2-67) Tenglamani nechta ildizi bor?

$\sqrt{3x^2+6x+7}+\sqrt{5x^2+10x+14}=4-2x-x^2$

*A)*cheksiz ko'p *B)1 C)2 D)3 E)4*

**Yechish.** Tenglamani chap qismini shakl almashtirib, $y=\sqrt{x}$ funksiyani o'suvchiligidan foydalanamiz.

$\sqrt{3x^2+6x+7}+\sqrt{5x^2+10x+14}=\sqrt{3(x+1)^2+4}+\sqrt{5(x+1)^2+9}\geq\sqrt{4}+\sqrt{9}=2+3=5$

Tenglamani o'ng qismini shakl almashtiramiz $4-2x-x^2=5-(x+1)^2\leq 5$.

U holda berilgan tenglama quyidagi tenglamalar sistemasiga teng kuchli.

$$\begin{cases}\sqrt{3(x+1)^2+4}+\sqrt{5(x+1)^2+9}=5\\ 5-(x+1)^2=5\end{cases}$$

2-tenglamadan $x=-1$ ildizni topamiz. Bu ildiz sistemaning 1-tenglamasini qanoatlantiradi.

**Javob.** 1 ta (B)

**10**(01-8-34). Ushbu $3\text{-}4x\text{-}4x^2=2^{4x^2+4x+3}$ tenglamaning ildizlari yig'indisini toping. *A)2 B)-0,5 C)6 D)4,5 E)-6,5*

**Yechish.** Tenglamani chap va o'ng qismini shakl almashtiraylik: $2^{4x^2+4x+3}=2^{(2x+1)^2+2}\geq 2^2=4$ , $3-4x-4x^2=4-(2x+1)^2\leq 4$ U holda 2-teoremaga asosan berilgan tenglama quyidagi tenglamalar sistemasiga teng kuchli:

$$\begin{cases}4-(2x+1)^2=4\\ 2^{(2x+1)^2+2}=2^2\end{cases}$$

Bundan: $\begin{cases}x=-\frac{1}{2}\\ x=-\frac{1}{2}\end{cases}$ $x=-\frac{1}{2}$ ildizni topamiz.

**Javob.** $x=-0{,}5$ (B)

**11**(01-8-55). Tenglamani yeching $tg^4 x-\cos^2 2x=\cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}-\frac{7}{8}$

*A)* $\frac{\pi k}{2},k\in Z$ *B)* $\pi k,k\in Z$ *C)* $2\pi k,k\in Z$ *D)* $\frac{\pi}{4}+\pi k,k\in Z$ *E)* $\varnothing$

**Yechish.** Tenglamaning o'ng qismidagi kosinuslar ko'paytmasini alohida hisoblab, ikkilangan burchak sinusi va keltirish formulasidan foydalanamiz.

$$\cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}=\frac{1}{2\sin\frac{\pi}{7}}\cdot\sin\frac{2\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}=\frac{1}{4\sin\frac{\pi}{7}}\cdot\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}=$$

$$=\frac{1}{8\sin\frac{\pi}{7}}\cdot\sin\frac{8\pi}{7}=\frac{1}{8\sin\frac{\pi}{7}}\cdot\sin(\pi+\frac{\pi}{7})=-\frac{1}{8\sin\frac{\pi}{7}}\cdot\sin\frac{\pi}{7}=-\frac{1}{8}.$$ U holda tenglama

quyidagi ko'rinishni oladi. $tg^4x-\cos^2 2x=-1,\ 1+tg^4x=\cos^2 2x$

$1+tg^4x\ge 1;\quad \cos^2 2x\le 1$ ekanligini hisobga olsak berilgan tenglama

quyidagi sistemaga teng kuchli $\begin{cases}1+tg^4x=1\\ \cos^2 2x=1\end{cases}$ Birinchi tenglamadan $tgx=0$,

$x=\pi n, n\in Z$, bu ildizlar sistemaning 2-tenglamasini qanoatlantirganligi sababli, u berilgan tenglamani ham yechimi bo'ladi.

**Javob.** $x=\pi n, n\in Z$ *(B).*

**12(03-12-60).** $\sin^3 x+\cos^3 x+\sin^2 x=2$ tenglama $[-2\pi;2\pi]$ kesmada nechta ildizga ega? *A) ildizi yo'q B)1 ta C)2 ta D)3 ta E)4 ta*

**Yechish.** Tenglamani shakl almashtiraylik. $\sin^3 x+\cos^3 x=1+(1-\sin^2 x)$,

$\sin^3 x+\cos^3 x=1+\cos^2 x,\quad \sin^3 x+\cos^2 x(\cos x-1)=1+0,\quad \sin^3 x\le 1, \cos^2(\cos x-1)\le 0$

ekanligidan berilgan tenglama quyidagi tenglamalar sistemasiga teng kuchli

$\begin{cases}\sin^3 x=1\\ \cos^2 x(\cos x-1)=0\end{cases}$ Bu sistema quyidagi 2 ta tenglamalar sistemasining

birlashmasiga teng kuchli.

a) $\begin{cases}\sin x=1\\ \cos^2 x=0\end{cases}$ b) $\begin{cases}\sin x=1\\ \cos x-1=0\end{cases}$

a) sistemani yechaylik $x=\frac{\pi}{2}+\pi n, n\in Z$ Bu ildizlardan n=0 da $x=\frac{\pi}{2}$,

ildiz *n=-1* da $x=-1{,}5\pi$ berilgan oraliqqa tegishli. *b)* sistema yechimga ega emas, chunki *sinx=1* va *cosx=1* tengliklar bir vaqtda bajarilmaydi.

**Javob.** 2 ta *(C)*

13(99-4-56). Tengsizlikni yeching. $\cos 4\cos x\ge\sqrt{\frac{\cos x}{1+ctg^2 x}}$

$A)\left(\pi n;\frac{\pi}{2}+\pi n\right], n\in Z$ $B)\left[0;\frac{\pi}{2}\right]$ $C)\frac{\pi}{2}+\pi n, n\in Z$ $D)\pi n,\ n\in Z$ $E)\left[-\frac{\pi}{2}+2\pi n;\frac{\pi}{2}+2\pi n\right]$ $n\in Z$

**Yechish.** Tengsizlikning o'ng qismi $\sqrt{\frac{\cos x}{1+ctg^2x}} \geq 0$ dan kasrning maxraji $1+ctg^2x>0$ bo'lganligi sababli kasrning surati $cosx \geq 0$ bo'ladi. U holda tengsizlikning chap qismi $cos4cosx$ ni qaraylik. $\pi < 4 < \frac{3\pi}{2}$ bo'lganligi sababli $cos4<0$ bo'ladi. $cosx \geq 0$ dan tengsizlikning chap qismi $cos4cosx \leq 0$

bo'ladi. Yuqoridagilardan berilgan tengsizlik $cosx=0$ bo'lgandagina bajarilishi kelib chiqadi. Bundan $x=\frac{\pi}{2}+\pi n, n \in Z$ ildizlarni topamiz.

**Javob.** $x=\frac{\pi}{2}+\pi n, n \in Z$ (C)

14(99-5-16). Tenglamaning ildizlari nechta? $cos\left(\lg\left(2-3^{x^2}\right)\right)=3^{x^2}$

*A)* $\varnothing$ *B)*cheksiz ko'p *C)1* *D)2* *E)3*

**Yechish.** $-1 \leq \cos t \leq 1$ bo'lganligi sababli tenglamaning chap qismining eng katta qiymati 1 ga teng. $y=3^t$ fungsiya o'suvchi ekanligidan $3^{x^2} \geq 3^0 = 1$

bo'lganligi sababli tenglamaning o'ng qismini eng kichik qiymati 1 ga teng. Tenglik bajarilishi uchun $\begin{cases} \cos(\lg(2-3^{x^2}))=1 \\ 3^{x^2}=1 \end{cases}$ bo'lishi kerak. Ikkinchi tenglamadan $3^{x^2}=3^0$, $x^2=0$, $x=0$ ildizini topamiz. $x=0$ ildiz sistemaning 1- tenglamasini ham qanoatlantiradi. Shuning uchun tenglama yagona ildizga ega.

Javob: 1 ta (C).

**15-misol.** $tgx+ctgx=\sqrt{2}(\sin x+\cos x)$ tenglamani yeching.

A) $\frac{\pi}{4}+\pi k,\ k \in Z$ B) $-\frac{\pi}{4}+2\pi k,\ k \in Z$ C) $\frac{\pi}{4}+2\pi k,\ k \in Z$ D) $\frac{\pi}{4}$ E) $(-1)^k\frac{\pi}{4}+\pi k,\ k \in Z$

**Yechish.** Berilgan tenglamada shakl almashtiraylik. $\cos(x-\frac{\pi}{4})\cdot\sin 2x=1$.

3-teoremani tadbiq etamiz.

$$\left[\begin{array}{l} \begin{cases} \cos(x-\frac{\pi}{4})=1 \\ \sin 2x=1 \end{cases} \\ \begin{cases} \cos(x-\frac{\pi}{4})=-1 \\ \sin 2x=-1 \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} x=\frac{\pi}{4}+2\pi k,\ k \in Z \\ x=\frac{\pi}{4}+\pi n,\ n \in Z \end{cases} \\ \begin{cases} x=\frac{5\pi}{4}+2\pi k,\ k \in Z \\ x=-\frac{\pi}{4}+\pi n,\ n \in Z \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{4}+2\pi k,\ k \in Z \\ \varnothing \end{array}\right.$$

**Javob.** $x=\frac{\pi}{4}+2\pi k,\ k \in Z$

## Mustaqil yechish uchun masalalar

16(99-5-27). Tenglama $[-\pi;2\pi]$ oraliqda nechta ildizga ega? $5\sin 2x+8\cos x=13$ *A)*$\varnothing$ *B) 1 C) 2 D) 3 E) 4*

17(01-2-31) Tengsizlikni yeching $\cos^2(x+1)\cdot\lg(9-2x-x^2)\geq 1$

*A)*$(-\infty;-1]$ *B)* $\{-1\}$ *C)*$[-1;0)$ *D)*$(0;\infty)$ *E)*$[-1;1)$

18(01-10-52). Ushbu $2|x-3|+x-1+2\sin\frac{\pi x}{2}=0$ tenglama nechta ildizga ega?

*A)*$\varnothing$ *B)1 C)2 D)4 E)* cheksiz ko'p.

19(03-9-15). $\sqrt{25-x^2}+\sqrt{9-x^2}=9x^4+8$ tenglamaning ildizlari quyida keltirilgan oraliqlarning qaysi biriga tegishli? *A)*$[-3;-1]$ *B)*$(-2;0)$ *C)*$[0;2]$ *D)*$(0;2)$ *E)*$(1;3)$

20(02-1-6). $2\cos\frac{x}{20}=2^x+2^{-x}$ tenglama nechta yechimga ega?

*A)1 B)2 C)* cheksiz ko'p *D)*$\varnothing$ *E)5*

21(01-12-22). Ushbu $\cos^2\frac{x}{2}-\sin^2\left(\frac{\sqrt{3}x}{2}\right)=1$ tenglamaning $[-\pi;\pi]$ oraliqda nechta yechimi bor? *A)1 B)2 C)3 D)*yechimi yo'q *E)4*

22(03-2-19). $6x-x^2-5=2^{x^2-6x+11}$ tenglamaning ildizlari yig'indisini toping.

*A)-5 B)-3 C)6 D)4 E)3*

23. $\log_2(x^2+4)-\log_2 x=4x-x^2-2$ tenglamani yeching.

24. $\log_2(4-\sin 3x)\leq\cos\frac{12x}{5}$ tengsizlikni yeching.

25(01-10-36). Ushbu $3\sin 2x+5\sin 4x=8$ tenglama $[-2\pi;2\pi]$ kesmada nechta ildizga ega? A) $\varnothing$ B) 1 C) 2 D) 3 E) 4

26(02-6-41). $3\sin 5x+4\cos 5x=6$ tenglama $[-\pi;2\pi]$ kesmada nechta ildizga ega?

A) $\varnothing$ B) 1 C) 2 D) 3 E) 4

27. $\log_{\sqrt{6}}(x^2+9)-\log_{\sqrt{6}} x=6x-x^2-7$ tenglamani yeching.

28. $3x^2-2x^3=\log_2(x^2+1)-\log_2 x$ tenglamani yeching.

29. Tengsizlikni yeching.

a) $2+\sin x>\frac{1}{1+x^2}$ b) $2-\cos x>\frac{1}{1+x^2}$

30. Tenglamani yeching. $2^{|x|}=\cos x^2$

## 6-§. Tenglama va tengsizliklarni yechishda funksiyaning o'suvchi va kamayuvchiligidan foydalanish

**1-teorema.** Agar haqiqiy sonlarning biror M to'plamida $f(x)$ monoton funksiya bo'lsa, u holda M to'plamda $f(x)=a$ tenglama bittadan ortiq ildizga ega emas, (ya'ni $f(x)=a$ tenglamaning ildizi yoki yo'q, yoki yagona bo'ladi).

**2-teorema.** $f(x)=g(x)$ tenglamaning har ikkala qismi biror M to'plamda aniqlangan bo'lsin. $f(x)$ funksiya o'suvchi, $g(x)$ funksiya kamayuvchi funksiya bo'lsin (yoki aksincha). U holda $f(x)=g(x)$ tenglama bittadan ortiq ildizga ega emas.

**3-teorema.** Agar haqiqiy sonlarning biror M to'plamida $f(x)$ o'suvchi funksiya bo'lsa, u holda M to'plamda $f(x)>c$ tengsizlikni yechish uchun $f(x)=c$ tenglamani yechib, uning $x_0$ ildizini topsak, u holda tengsizlikni yechimini $x>x_0$ ko'rinishda bo'ladi. Bunda $x_0$ ildiz $f(x)$ funksiyaning aniqlanish sohasiga tegishli bo'lishi lozim.

**4-teorema.** Agar $f(x)$ va $g(x)$ funksiyalar qat'iy o'suvchi va o'zaro teskari funksiyalar bo'lsa, u holda $f(x)=g(x)$ tenglama $f(x)=x$ yoki $g(x)=x$ tenglamalar teng kuchli bo'ladi.

1(02-2-58). $5^x+7^x=12^x$ tenglama nechta ildizga ega? *A)1 B)2 C)3 D)*cheksiz ko'p *E)* yechimi yo'q

**Yechish.** Berilgan tenglamaning ildizi $x=1$ ni tanlash yordamida osongina topish mumkin. Endi tenglamaning boshqa ildizi yo'qligini isbotlaymiz. Tenglamani ikkala qismini $12^x$ ga bo'lamiz. U holda $\left(\frac{5}{12}\right)^x+\left(\frac{7}{12}\right)^x=1$ tenglama hosil bo'ladi. Tenglamaning chap qismidagi ikkita kamayuvchi funksiyaning yig'indisi kamayuvchi funksiya bo'ladi. 1-teoremaga asosan bu tenglamaning boshqa ildizi yo'q.

**Javob.** 1 ta (A)

2(03-1-13). $\sqrt{5^x}+\sqrt{12^x}=\sqrt{13^x}$ tenglama nechta ildizga ega?
*A)∅ B)1 C)2 D)3 E)4*

**Yechish.** Berilgan tenglamaning $x=4$ ildizini tanlash yordamida topamiz. Uning boshqa ildizi yo'qligini isbotlaymiz. Tenglamaning ikkala qismini $\sqrt{13^x}$ ga bo'lamiz. U holda
$\left(\sqrt{\frac{5}{13}}\right)^x+\left(\sqrt{\frac{12}{13}}\right)^x=1$ tenglama hosil bo'ladi. Bu tenglamaning chap qismi

$f(x)=\left(\sqrt{\frac{5}{13}}\right)^x+\left(\sqrt{\frac{12}{13}}\right)^x$ kamayuvchi funksiyadan iborat. 1-teoremaga asosan, berilgan tenglama $x=4$ yagona ildizga ega.

**Javob.** 1 ta (B)

3.Tenglamani yeching. $\left(\sqrt{2+\sqrt{2}}\right)^x+\left(\sqrt{2-\sqrt{2}}\right)^x=2^x$

**Yechish.** Tenglamaning $x=2$ ildizini tanlash yordamida osongina topish mumkin. Tenglamaning boshqa ildizi yo'qligini isbotlash uchun uning ikkala qismini $2^x$ ga bo'laylik. U holda $\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)^x+\left(\frac{\sqrt{2-\sqrt{2}}}{2}\right)^x=1$ tenglama hosil bo'ladi. Tenglamaning chap qismida $y_1=\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)^x$ va $y_2=\left(\frac{\sqrt{2-\sqrt{2}}}{2}\right)^x$ kamayuvchi funksiyalar yig'indisi kamayuvchi bo'lganligi sababli 1-teoremaga asosan bu tenglama boshqa ildizga ega emas.

4. Tengsizlikni yeching. $x^5+x^4+2\sqrt{x}>4.$

**Yechish.** Tengsizlikni aniqlanish sohasi $x\ge 0$ dan iborat. $x^5+x^4+2\sqrt{x}=4$ tenglamaning chap qismidagi $f(x)=$ $x^5+x^4+2\sqrt{x}$ funksiya aniqlanish sohasida uchta o'suvchi fungsiyaning yig'indisi sifatida o'suvchi funksiyadan iborat. $x^5+x^4+2\sqrt{x}=4$ tenglamaning ildizi $x_0=1$ ni osongina topish mumkin. 1-teoremaga asosan bu tenglama boshqa ildizga ega emas.Shuning uchun 3-teoremaga asosan berilgan tengsizlikni yechimi $x>x_0$ yoki $x>1$ bo'ladi.

**Javob.** $x>1$

5.Tengsizlikni yeching. $\sqrt{x-1}+2^x+\log_2 x\ge 2$

**Yechish.** Tengsizlikning aniqlanish sohasini topaylik: $\begin{cases}x-1\ge 0\\ x>0\end{cases}$ $\begin{cases}x\ge 1\\ x>0\end{cases}$ $\Rightarrow$ $x\ge 1$. $x\ge 1$ bo'lganda tengsizlikning chap qismi f(x) $=\sqrt{x-1}+2^x+\log_2 x$ o'suvchi funksiyadan iborat. $\sqrt{x-1}+2^x+\log_2 x=2$ tenglamaning ildizi $x_0=1$ ni tanlash yordamida osongina topamiz. 1-teoremaga asosan bu tenglama boshqa ildizga ega emas. U holda 3-teoremaga asosan berilgan tengsizlik $x\ge 1$ yechimga ega.

**Javob.** $x\ge 1$

6.Tenglamani yeching. $\log_2 x=3-x$

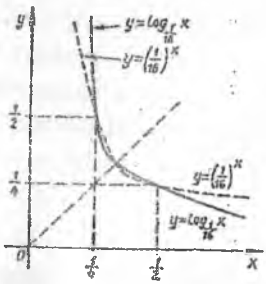
**Yechish.** Tenglamani aniqlanish sohasi $x>0$ dan iborat. U holda $f(x)=$ $\log_2 x$ funksiya o'suvchi, $g(x)=3-x$ chiziqli funksiya sifatida kamayuvchi, chunki

$k = l < 0$, bo'ladi. Bu tenglamaning $x = 2$ ildizini tanlash yordamida topamiz. 2-teoremaga asosan bu tenglama yagona $x = 2$ ildizga ega.
**Javob.** $x = 2$

7. Tenglamani yeching. $\left(\frac{1}{16}\right)^x = \log_{\frac{1}{16}} x$

**Yechish.** Tenglamaning chap va o'ng qismida kamayuvchi funksiya turganligi sababli yuqoridagi teoremalarni bu tenglama uchun qo'llay olmaymiz. Grafik yordamiga bu tenglama uchta ildizga ega ekanligini aniqlaymiz. Bu ildizlardan $x_1 = \frac{1}{2}$, $x_2 = \frac{1}{4}$ lar grafik yordamida aniq topiladi.Bu tenglamaning uchinchi ildizining aniq ko'rinishi elementar funksiyalar orqali ifodalanmaydi.Bu ildiz $x = \log_{\frac{1}{16}} x$ va $(\frac{1}{16})^x = x$ tenglamalarning umumiy ildizidan iborat.

8(03-5-24). $2^x = x^3$ tenglama nechta haqiqiy ildizga ega.
A)2 B)1 C)3 D)∅ E)aniqlab bo'lmaydi.
**Yechish.** $y_1 = 2^x$ va $y_2 = x^3$ funksiyalar R da o'suvchi funksiyalardir.
Bu funksiyalarning grafiklarini bitta koordinata tekisligida yasab,



$1 < x < 2$ oraliqda bitta ildizi borligini aniqlaymiz. $x = 9$ da $y_1(9) = 2^9 = 512$, $y_2(9) = 9^3 = 729$. $x = 10$ da $y_1(10) = 2^{10} = 1024$, $y_2(10) = 10^3 = 1000$
Hisoblashlardan ko'rinadiki bu tenglama $9 < x < 10$ oraliqda ikkinchi ildizga ham ega ekan.
**Javob.**2 ta (A)
**9-misol.** $\sqrt[3]{x-9} = (x-3)^3 + 6$ tenglamani yeching.
**Yechish.** Tenglamani $\sqrt[3]{x-9} + 3 = (x-3)^3 + 9$ ko'rinishida yozaylik. U holda $y = (x-3)^3 + 9$ va $y = \sqrt[3]{x-9} + 3$ funksiyalarning har biri o'suvchi funksiyalardan iborat. $y = (x-3)^3 + 9$ funksiyaga teskari funsiyani topaylik. Buning uchun berilgan funksiyani $x$ ga nisbatan yechaylik va $x$ va $y$ ni o'rinlarini almashtiraylik. $y = (x-3)^3 + 9$,

$x)=\left(\sqrt{\frac{5}{13}}\right)^x+\left(\sqrt{\frac{12}{13}}\right)^x$ kamayuvchi funksiyadan iborat. 1-teoremaga asosan, erilgan tenglama $x=4$ yagona ildizga ega.
**Javob.** 1 ta (B)

3.Tenglamani yeching. $\left(\sqrt{2+\sqrt{2}}\right)^x+\left(\sqrt{2-\sqrt{2}}\right)^x=2^x$

**Yechish.** Tenglamaning $x=2$ ildizini tanlash yordamida osongina topish mumkin. Tenglamaning boshqa ildizi yo'qligini isbotlash uchun uning ikkala qismini $2^x$ ga bo'laylik. U holda $\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)^x+\left(\frac{\sqrt{2-\sqrt{2}}}{2}\right)^x=1$ tenglama hosil bo'ladi. Tenglamaning chap qismida $y_1=\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)^x$ va $y_2=\left(\frac{\sqrt{2-\sqrt{2}}}{2}\right)^x$ kamayuvchi funksiyalar yig'indisi kamayuvchi bo'lganligi sababli 1-teoremaga asosan bu tenglama boshqa ildizga ega emas.

4. Tengsizlikni yeching. $x^5+x^4+2\sqrt{x}>4$.

**Yechish.** Tengsizlikni aniqlanish sohasi $x\geq 0$ dan iborat. $x^5+x^4+2\sqrt{x}=4$ tenglamaning chap qismidagi $f(x)=$ $x^5+x^4+2\sqrt{x}$ funksiya aniqlanish sohasida uchta o'suvchi fungsiyaning yig'indisi sifatida o'suvchi funksiyadan iborat. $x^5+x^4+2\sqrt{x}=4$ tenglamaning ildizi $x_0=1$ ni osongina topish mumkin. 1-teoremaga asosan bu tenglama boshqa ildizga ega emas.Shuning uchun 3-teoremaga asosan berilgan tengsizlikni yechimi $x>x_0$ yoki $x>1$ bo'ladi.

**Javob.** $x>1$

5.Tengsizlikni yeching. $\sqrt{x-1}+2^x+\log_2 x\geq 2$

**Yechish.** Tengsizlikning aniqlanish sohasini topaylik: $\begin{cases}x-1\geq 0\\x>0\end{cases}$ $\begin{cases}x\geq 1\\x>0\end{cases}$ $\Rightarrow x\geq 1$. $x\geq 1$ bo'lganda tengsizlikning chap qismi f(x) $=\sqrt{x-1}+2^x+\log_2 x$ o'suvchi funksiyadan iborat. $\sqrt{x-1}+2^x+\log_2 x=2$ tenglamaning ildizi $x_0=1$ ni tanlash yordamida osongina topamiz. 1-teoremaga asosan bu tenglama boshqa ildizga ega emas. U holda 3-teoremaga asosan berilgan tengsizlik $x\geq 1$ yechimga ega.

**Javob.** $x\geq 1$

6.Tenglamani yeching. $\log_2 x=3-x$

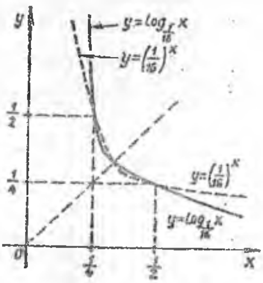
**Yechish.** Tenglamani aniqlanish sohasi $x>0$ dan iborat. U holda $f(x)=$ $\log_2 x$ funksiya o'suvchi, $g(x)=3-x$ chiziqli funksiya sifatida kamayuvchi, chunki

$k=-1<0$, bo'ladi. Bu tenglamaning $x=2$ ildizini tanlash yordamida topamiz. 2-teoremaga asosan bu tenglama yagona $x=2$ ildizga ega.
**Javob.** $x=2$

7. Tenglamani yeching. $\left(\frac{1}{16}\right)^x = \log_{\frac{1}{16}} x$

**Yechish.** Tenglamaning chap va o'ng qismida kamayuvchi funksiya turganligi sababli yuqoridagi teoremalarni bu tenglama uchun qo'llay olmaymiz. Grafik yordamiga bu tenglama uchta ildizga ega ekanligini aniqlaymiz. Bu ildizlardan $x_1=\frac{1}{2}$, $x_2=\frac{1}{4}$. lar grafik yordamida aniq topiladi.Bu tenglamaning uchinchi ildizining aniq ko'rinishi elementar funksiyalar orqali ifodalanmaydi.Bu ildiz $x=\log_{\frac{1}{16}} x$ va $(\frac{1}{16})^x = x$ tenglamalarning umumiy ildizidan iborat.

8(03-5-24). $2^x = x^3$ tenglama nechta haqiqiy ildizga ega.
A)2 B)1 C)3 D)∅ E)aniqlab bo'lmaydi.
**Yechish.** $y_1=2^x$ va $y_2=x^3$ funksiyalar R da o'suvchi funksiyalardir.
Bu funksiyalarning grafiklarini bitta koordinata tekisligida yasab,




$1<x<2$ oraliqda bitta ildizi borligini aniqlaymiz. $x=9$ da $y_1(9)=2^9=512$, $y_2(9)=9^3=729$. $x=10$ da $y_1(10)=2^{10}=1024$, $y_2(10)=10^3=1000$ Hisoblashlardan ko'rinadiki bu tenglama $9<x<10$ oraliqda ikkinchi ildizga ham ega ekan.
**Javob.**2 ta (A)
**9-misol.** $\sqrt[3]{x-9}=(x-3)^3+6$ tenglamani yeching.
**Yechish.** Tenglamani $\sqrt[3]{x-9}+3=(x-3)^3+9$ ko'rinishida yozaylik. U holda $y=(x-3)^3+9$ va $y=\sqrt[3]{x-9}+3$ funksiyalarning har biri o'suvchi funksiyalardan iborat. $y=(x-3)^3+9$ funksiyaga teskari funsiyani topaylik. Buning uchun berilgan funksiyani $x$ ga nisbatan yechaylik va $x$ va $y$ ni o'rinlarini almashtiraylik. $y=(x-3)^3+9$,

$x-3=\sqrt[3]{y-9},\ x=\sqrt[3]{y-9}+3,\ y=\sqrt[3]{x-9}+3$. Demak, $y=(x-3)^3+9$ va $y=\sqrt[3]{x-9}+3$ funksiyalar o'zaro teskari funsiyalar ekan. U holda 4-teoremaga asosan berilgan tenglama $(x-3)^3+9=x$ tenglamaga teng kuchli. Bu tenglamani yechaylik.

$x^3-9x^2+26x-18=0,\ (x-1)(x^2-8x+18)=0,\ x=1,\ x^2-8x+18=(x+4)^2+2>0$.

**Javob.** $x=1$

## Mustaqil yechish uchun misollar

**10.** $3^x+4^x=25$ tenglamani yeching.

**11.** $2^x+3^x=5\cdot6^x$ tenglamani yeching.

**12.** $x^{10}+\sqrt{x-1}\geq1$ tengsizlikni yeching.

**13.** $\sqrt{x+3}+\sqrt{3x-2}<7$ tengsizlikni yeching.

**14.** $\log_2 x+3\log_2(x+3)=6$ tenglamani yeching.

**15.** $4^x+(x-13)\cdot2^x-2x+22=0$ tenglamani yeching.

**16.** $8^x+3\cdot2^{x+1}<7$ tengsizlikni yeching.

**17.** $27^x-7\cdot\sqrt[3]{7\cdot3^x+6}=6$ tenglamani yeching.

**18.** $x^2+x-\frac{1}{4}=\frac{1}{2}(\sqrt{4x+2}-1)$ tenglamani yeching.

**19(02-6-35).** Ushbu $3^{x^2}+\log_2 x^3=84$ tenglama nechta ildizga ega?

A) $\varnothing$ B) 1 C) 2 D) 3 E) 4

**20(01-10-32).** Ushbu $2^{x^2}+\log_3 x^3=515$ tenglama nechta ildizga ega?

A) $\varnothing$ B) 1 C) 2 D) 3 E) 4

## 7-§. Tenglama va tengsizliklarni yechishda aniqlanish sohasidan foydalanish

Ba'zi hollarda, tenglama yoki tengsizliklarda qatnashayotgan funksiyalarning aniqlanish sohasini bilish, tenglama yoki tengsizliklarning yechimi mavjud emasligini bilishga, yoki yechimini topishga yordam beradi.

1(02-4-9). $\sqrt{2-x^2}\cdot\sqrt{x^2-4}=0$ tenglamani ildizlari sonini toping.

A)0 B)1 C)2 D)3 E)4

**Yechish.** Tenglamani aniqlanish sohasini topaylik.

$$\begin{cases}2-x^2\ge 0\\ x^2-4\ge 0\end{cases}\Leftrightarrow\begin{cases}x^2\le 2\\ x^2\ge 4\end{cases}\Leftrightarrow\begin{cases}|x|\le\sqrt{2}\\ |x|\ge 2\end{cases}\quad\varnothing$$

Tenglamaning aniqlanish sohasi bo'sh to'plam bo'lganligi sababli, tenglama yechimga ega emas.

**Javob.** 0 (A)

**2-misol.** $\sqrt{x-2}+\sqrt{2x-1}=\sqrt{3-2x}$ tenglamani yeching.

**Yechish.** Tenglamani aniqlanish sohasini topaylik.

$$\begin{cases}x-2\ge 0\\ 2x-1\ge 0\\ 3-2x\ge 0\end{cases}\Leftrightarrow\begin{cases}x\ge 2\\ x\ge 0{,}5\\ x\le 1{,}5\end{cases}\quad\varnothing$$

Tenglamaning aniqlanish sohasi bo'sh to'plam bo'lganligi sababli, tenglama ildizga ega emas.

**Javob.** $\varnothing$

**3-misol.** $\sqrt{1-x}+\sqrt{1+x^2}+\sqrt{x-1}=\sqrt{2}$ tenglamani yeching.

**Yechish.** Tenglamani aniqlanish sohasini topaylik.

$$\begin{cases}1-x\ge 0\\ x-1\ge 0\end{cases}\Leftrightarrow\begin{cases}x\le 1\\ x\ge 1\end{cases}\Rightarrow x=1$$

Tenglamaning aniqlanish sohasi faqat bitta $x=1$ nuqtadan iborat. $x=1$ ni berilgan tenglamani qanoatlantirishini tekshiramiz. $x=1$ bo'lsa $\sqrt{1-1}+\sqrt{1+1^2}+\sqrt{1-1}=\sqrt{2}$, $\sqrt{2}=\sqrt{2}$ tenglik to'g'ri. Demak, tenglama faqat $x=1$ ildizga ega.

**Javob.** 1

4(01-2-20). Tengsizlikni yeching. $\sqrt{3x-8}<-2$

A) $x<4$ B) $x\in\varnothing$ C) $x>\frac{8}{3}$ D) $x>4$ E) $\left[\frac{8}{3};4\right]$

**Yechish.** Tengsizlikni chap va o'ng qismini taqqoslaylik. Bizga ma'lumki, aniqlanish sohasida $\sqrt{3x-8}\ge 0$ bo'ladi. Tengsizlikning chap qismi o'zining aniqlanish sohasida nomanfiy, o'ng qismi manfiy.

Nomanfiy sonlar manfiy sonlardan kichik bo'lmaydi. Shuning uchun bu tengsizlik yechimga ega emas.

**Javob:** $\varnothing$

5(03-11-14). $\sqrt{\frac{8-x}{x-18}} > -1$ tengsizlikning butun yechimlari yig'indisini toping.

A)125 B)130 C)143 D)136 E)124

**Yechish.** Tengsizlikning chap qismi nomanfiy, o'ng qismi esa manfiydir. Nomanfiy sonlar manfiy sonlardan doim katta bo'ladi. Shuning uchun berilgan tengsizlik aniqlanish sohasidagi barcha $x$ uchun o'rinli. Aniqlanish sohasini topaylik. $\frac{8-x}{x-18} \geq 0$, $\frac{x-8}{x-18} \leq 0$ $x \in [8;18)$.

Bu oraliqqa tegishli butun sonlar yig'indisi $8+9+10+\ldots+17=\frac{8+17}{2}\cdot 10 = 25\cdot 5 = 125$

**Javob.** 125 (A)

6(03-1-51). $\cos x \geq \left(-\frac{\pi}{2}x\right)$ tengsizlikni yeching.

A)yechimga ega emas B)$[-\pi+2\pi n;\pi+2\pi n]$ $n\in Z$

C)$\left[-\frac{\pi}{2}+2\pi n;\frac{\pi}{2}+2\pi n\right]$ $n\in Z$

D)$\left[-\frac{\pi}{2}+\pi n;\frac{\pi}{2}+\pi n\right]$ $n\in Z$ E)$(-\infty;\infty)$

**Yechish.** $\left(-\frac{\pi}{2}x\right) = -\frac{\pi}{2} \approx -1{,}5$ va $-1 \leq \cos x \leq 1$ bo'lganligi sababli tengsizlikning chap qismi o'zining eng kichik qiymatini qabul qilganda ham, tengsizlik to'g'ri bo'ladi. Shuning uchun berilgan tengsizlikning yechimi uning aniqlanish sohasi R dan iborat.

**Javob.** $(-\infty;\infty)$ (E)

**7-misol.** $x+2^x+\sqrt{x-1} \geq 2+\sqrt{x}$ tengsizlikni yeching.

**Yechish.** Tengsizlikning aniqlanish sohasini topaylik.

$$\begin{cases} x-1\geq 0 \\ x\geq 0 \end{cases} \Leftrightarrow \begin{cases} x\geq 1 \\ x\geq 0 \end{cases} \Leftrightarrow x\geq 1 \quad [1;\infty)$$

$x\geq 1$ bo'lsa, berilgan tengsizlik to'g'ri mulohazadan iborat bo'ladi. Haqiqatdan $x\geq 1$ bo'lsa, $x\geq\sqrt{x}$, $2^x\geq 2$ va $\sqrt{x-1}\geq 0$ bo'ladi.

**Javob:** $[1;\infty)$

**8-misol.** $\lg x < \sqrt[4]{1-x^2}$ tengsizlikni yeching.

**Yechish.** Tengsizlikning aniqlanish sohasi $x>0$ va $1-x^2\geq 0$ shartlarni qanoatlantiruvchi sonlardan iborat, ya'ni $0<x\leq 1$. Ravshanki, $x=1$ tengsizlik yechimi bo'la olmaydi. $(0,1)$ oraliqdan olingan har bir $x$

uchun $\lg x<0$ va tengsizlikning o'ng tomoni musbat. Demak, $(0;1)$ oraliq tengsizlik yechimi bo'ladi.

**Javob:** $0<x\le 1$

9(01-12-21). Tengsizlikni yeching. $\arcsin x<\sqrt{x^2-1}$

A) $\{1\}$ B) $\{-1\}$ C) $\{-1;1\}$ D) $\left(0;\frac{\pi}{2}\right]$ E) $\left[-\frac{\pi}{2};0\right)$

**Yechish.** Tengsizlikning aniqlanish sohasini topaylik.

$$\begin{cases}-1\le x\le 1\\ x^2-1\ge 0\end{cases}\Leftrightarrow\begin{cases}-1\le x\le 1\\ |x|\ge 1\end{cases}\quad x=-1;x=1$$

$x=-1$ bo'lsa, $\arcsin(-1)<\sqrt{(-1)^2-1}$, $-\frac{\pi}{2}<0$ to'g'ri tengsizlik. Demak $x=-1$ tengsizlikning yechimi. $x=1$ bo'lsa, $\arcsin(1)<\sqrt{1^2-1}$, $\frac{\pi}{2}<0$ noto'g'ri tengsizlik. $x=1$ tengsizlikning yechimi emas.

**Javob.** $\{-1\}$ (B)

**10**(00-10-65). Tengsizlikni yeching. $x^2-4x\arccos(x^2-4x+5)<0$

*A)* $\{2\}$ *B)* $(1;5)$ *C)* $(-2;3)$ *D)* $(\arccos 1;10)$ *E)* yechimi yo'q

**Yechish.** $\arccos a$, $|a|\le 1$ bo'lsa ma'noga ega. $x^2-4x+5=(x-2)^2+1\ge 1$. Demak, $\arccos a$, $(x-2)^2+1=1$ bo'lsa ma'noga ega. Bundan tengsizlikning aniqlanish sohasi $x=2$ kelib chiqadi. Agar $x=2$ bo'lsa, $2^2-4\cdot 2\cdot\arccos 1<0$, $4<0$ noto'g'ri tengsizlik hosil bo'ladi. Demak, $x=2$ tengsizlikni yechimi bo'lmaydi.

**Javob.** yechimi yo'q (E)

## Mustaqil yechish uchun masalalar

**11-misol.** $\sqrt{4-x}=\lg(x-4)$ tenglamani yeching.

**Javob.** ildizi yo'q

**12-misol.** $2-\sqrt{4-x^2}=\sqrt[4]{x^4-16}+x$ tenglamani yeching.

**Javob.** $x=2$

13(03-1-8). $\sqrt{\frac{2-3x}{x+4}}>-2$ tengsizlikni eng kichik butun yechimini toping.

A)0 B)-1 C)-2 D)-3 E)-5

**Javob.** -3 (D)

14(03-1-18). $\sin x<1+\frac{x^2}{4}$ tengsizlikni yeching.

A)∅ B) $\left(-\frac{\pi}{2}+2\pi n;\frac{\pi}{2}+2\pi n\right)$, $n\in Z$ C) $[-\pi;\pi]$ D) $\left[-\frac{\pi}{6}+2\pi n;\frac{\pi}{6}+2\pi n\right]$ $n\in Z$

E) $(-\infty;\infty)$

**Javob.** $(-\infty;\infty)$ (E)

15(03-12-16). $\sqrt{\frac{2x-3}{5x+7}}\geq -2$ tengsizlikni yeching.

A) $(-\infty;-1,2]\cup[2,5;\infty)$ B) $(-\infty;-1,4]\cup[1,5;\infty)$ C) $[1,5;4]$ D) $(-\infty;-1,4)\cup[1,5;\infty)$

E) $(-\infty;-1,4)\cup[2,5;\infty)$

**Javob.** $(-\infty;-1,4)\cup[1,5;\infty)$ (D)

**16-misol.** $\sqrt{1-x^2}+\sqrt{x^4-1}<2^x-\frac{2}{1+x^2}$ tengsizlikni yeching.

**Javob.** $x=1$

17(02-4-44). $\sqrt{1+x}\leq \arccos(x+2)$ tengsizlikning eng katta butun yechimini toping. *A)-2 B)-1 C)0 D)1 E)2*

**Javob.** -1 (B)

**18-misol.** $\left(\frac{\sqrt{x}+\sqrt{-x}}{2}\right)^2=\sin\frac{x}{2}$ tenglamani yeching. **Javob.** $x=0$

## 8-§. $f(f(x))=x$ ko'rinishdagi tenglamalarni yechish

**Teorema.** Agar $y=f(x)$ funksiya monoton o'suvchi funksiya bo'lsa, u holda $f(x)=x$ (1) va $f(f(x))=x$ (2) tenglamalar teng kuchli bo'ladi.

Bu teorema umumiy hol uchun ham uchun o'rinli. Agar $y=f(x)$ monoton o'suvchi funksiya bo'lsa, u holda istalgan k natural son uchun $\underbrace{f(f(...f(x)))...)}_{k}=x$ va $f(x)=x$ tenglamalar teng kuchli bo'ladi.

**1-misol.** $\sqrt[3]{x}+1=2(2x-1)^3$ tenglamani yeching.

**Yechish.** Tenglamaning shaklini quyidagicha o'zgartiraylik:

$$\frac{\sqrt[3]{x}+1}{2}=(2x-1)^3$$

$\sqrt[3]{\frac{\sqrt[3]{x}+1}{2}}=2x-1$, $\sqrt[3]{\frac{\sqrt[3]{x}+1}{2}}+1=2x$, $\frac{1}{2}\left(\sqrt[3]{\frac{\sqrt[3]{x}+1}{2}}+1\right)=x$, $f(x)=\frac{1}{2}(\sqrt[3]{x}+1)$ desak,

$f(f(x))=\frac{1}{2}(\sqrt[3]{f(x)}+1)=\frac{1}{2}\left(\sqrt[3]{\frac{1}{2}(\sqrt[3]{x}+1)}+1\right)=x$ bo'ladi. U holda teoremaga asosan

$f(f(x))=x$ va $f(x)=x$ tenglamalar teng kuchli, chunki $f(x)=\frac{1}{2}(\sqrt[3]{x}+1)$

funksiya o'suvchidir. $\frac{1}{2}\left(\sqrt[3]{x}+1\right)=x$ tenglamani yechamiz. $\left(\sqrt[3]{x}+1\right)=2x$ $\sqrt[3]{x}=y$ deb belgilash kiritsak, $2y^3-y-1=0$ tenglama hosil bo'ladi. Bu tenglamani yechaylik. $2y^3-2y^2+y^2-1=0$, $2y^2(y-1)+(y-1)=0$, $(y-1)(2y^2+1)=0$, $y=1$, $\sqrt[3]{x}=1$, $x=1$

**Javob.** $x=1$

**2-misol.** $\ln(1+\ln x)=x-1$ tenglamani yeching.

**Yechish.** $\ln(1+\ln x)+1=x$, $f(x)=1+\ln x$ deb olsak, $f(f(x))=\ln(1+\ln x)+1$ bo'ladi. $f(x)=1+\ln x$ funksiya $x>0$ bo'lganda o'suvchi funksiyadan iborat. U holda teoremaga asosan: $f(f(x))=x$ va $f(x)=x$ tenglamalar teng kuchli bo'ladi. Bundan $1+\ln x=x$ tenglama hosil bo'ladi. Bu tenglamaning ildizi $x=1$ ni tanlash yoki grafik yordamida aniqlaymiz.

**Javob.** $x=1$

**3-misol.** $\begin{cases} y^3-3x+2=0 \\ z^3-3y+2=0 \\ x^3-3z+2=0 \end{cases}$ tenglamalar sistemasini yeching.

**Yechish.** $\begin{cases} y^3=3x-2 \\ z^3=3y-2 \\ x^3=3z-2 \end{cases}$ $\begin{cases} y=\sqrt[3]{3x-2} \\ z=\sqrt[3]{3y-2} \\ x=\sqrt[3]{3z-2} \end{cases}$ $f(x)=\sqrt[3]{3x-2}$ funksiyani kiritsak, bu funksiya o'suvchidir. U holda tenglamalar sistemasi quyidagi ko'rinishni oladi.

$\begin{cases} y=f(x) \\ z=f(y) \\ x=f(z) \end{cases}$ bundan ketma-ket o'rniga qo'yish yo'li bilan $x=f(f(f(x)))$ tenglama hosil bo'ladi. Bu esa teoremaga asosan $f(x)=x$ tenglamaga teng kuchli. $\sqrt[3]{3x-2}=x$ tenglamani yechamiz. Bundan $x^3-3x+2=0$, $x^3-x-2x+2=0$, $x(x^2-1)-2(x-1)=0$,

$(x-1)(x^2+x-2)=0$, $x-1=0$, $x=1$. $x^2+x-2=0$, $x_2=-2$, $x_3=1$.

Demak tenglamalar sistemasining yechimi $x=1$, $y=1$, $z=1$ yoki $x=-2$, $y=-2$ $z=-2$ bo'ladi.

**Javob.** (1;1;1), (-2;-2;-2)

## Mustaqil yechish uchun misollar

4. $\sqrt{1+\sqrt{x}}=x-1$ tenglamani yeching. **Javob.** $x=\frac{3+\sqrt{5}}{2}$

5. $x^3+1=2\sqrt[3]{2x-1}$ tenglamani yeching. **Javob.** $x_1=1$, $x_{2,3}=\frac{-1\pm\sqrt{5}}{2}$

6. $\begin{cases} x^3+2x^2+2x=y, \\ y^3+2y^2+2y=z, \\ z^3+2z^2+2z=x \end{cases}$ tenglamalar sistemasini yeching. **Javob.** (0;0;0), (-1;-1;-1)

7. $\sqrt{a+\sqrt{a+x}}=x$ tenglamani yeching.Bunda $a$ ixtiyoriy son.

8. Agar $f(x)$ funksiya uchun biror $a$ soni topilib quyidagi shartlar bajarilsa

$$\begin{cases} f(x)\ge x, agar \quad x<a \quad bo'lsa \\ f(a)=a \\ 0\le f(x)\le x, barcha \quad x>a \quad bo'lsa \end{cases}$$

u holda $f(f(x))=x$ va $f(x)=x$ tenglamalar teng kuchli ekanligini isbotlang.

Bundan foydalanib quyidagi tenglamalar sistemalarini yeching.

a) $\begin{cases} y^3-6x^2+12x-8=0 \\ z^3-6y^2+12y-8=0 \\ x^3-6z^2+12z-8=0 \end{cases}$ **Javob.** $x=y=z$

b) $\begin{cases} x-5y=2y^2+2 \\ y-5z=2z^2+2 \\ z-5x=2x^2+2 \end{cases}$ v) $\begin{cases} x=\dfrac{2y^2}{1+y^2} \\ y=\dfrac{2z^2}{1+z^2} \\ z=\dfrac{2x^2}{1+x^2} \end{cases}$

## 9-§. Diofant tenglamalarini yechish

Butun koefisientli algebraik tenglamalar yoki algebraik tenglamalar sistemalari Diofant tenglamalari deyiladi va bunda ularning butun yoki ratsional yechimlarini topish ko`zda tutiladi.

Bunday tenglamalarda o`zgaruvchilar soni ikkitadan kam bo`lmasligi lozim . Odatda , Diofant tenglamalarining yechimi ko`p bo`ladi , shu sababli ,ularni aniqmas tenglamar ham deb atashadi. Aniqmas tenglamalar III asrda yashagan yunon matematigi Diofant sharafiga shunday nomlangan. Uning „Arifmetika" kitobida ko`pgina masalalar jamlangan.

Bunday tenglamalarni yechishning umumiy usuli yo`q. Tenglamalarni butun sonlarda yechish juda qiziqarli masala. Qadim zamonlardan boshlab tayin Diofant tenglamalarini yechishning ko`pgina usullari yig`ilib qolgan, biroq ularni tekshirishning umumiy usullari faqat bizning asrimizda paydo bo`ldi, $x^3+y^3+z^3=30$ tenglamani hozirgacha yechimlari topilmagan.

$x^3+y^3+z^3=3$ tenglamani to`rtta butun (1; 1;1), (4; 4; -5), (4; -5; 4), (-5; 4; 4) yechimlari topilgan . Ammo boshqa yechimlari bor yoki yo`qligi hal qilinmagan.

$x^3+y^3+z^3=2$ tenglama cheksiz ko`p: $(x;y;z)=(1+6\alpha^3;1-6\alpha^3;-6\alpha^2)$, bu yerda $\alpha$ -butun son, yechimlari topilgan. Ammo bu tenglamaning barcha yechimlari shu formula bilan topiladimi? Hozirgacha hal qilinmagan.

P. Fermaning katta teoremasi nomini olgan $x^n+y^n=z^n$ ($n>2$) tenglamani butun yechimlari mavjud emasligi ısbotlash 350 yildan keyin 1998 yil AQSH Priston Universiteti professori , angliyalik matematik Endryu Uaylisga nasib etdi.

Asrimizning 20-yillarida angliyalik matematik E. I. Mordell uchinchi darajali Diofant tenglamalari odatda, chekli sondagi butun sonli yechimga ega bo`lishi lozim, degan farazni o`rtaga tashladi. Bu farazni gollandiyalik G. Faltings 1983 yilda isbot qildi.
1970 yili leningradlik (hozirgi sank-peterburglik) matematik Y. B. Matiyasevich Diofant tenglamalarini yechishnig umumiy usuli yo`qligini isbotladi.

Sonlar nazariyasi bilan shug`ullanuvchi har bir matematik $x^4+y^4+z^4=t^4$ tenglamani butun sonlarda yechimi mavjudmi?, - degan savolga javob topa olmayaptilar.

1914 yilda A. Verebyusov bu tenglamani yechilmasligini isbotladi. Ammo keyinchalik uni isbotida xatolik mavjudligi aniqlandi. Eyler ham o`z zamonasida bu tenglamaning yechilmasligini aytgan. 1945 yilda bu tenglamani $|t|<10^8$ ni qanoatlantiruvchi butun sonlar uchun yechimi mavjud ekanligini isbotladi, lekin bu to`la yechim emasdir.

Tenglamalarni butun sonlarda yechish matematikaning chiroyli bo`limlaridan biri. Birorta ham yirik matematik Diofant tenglamalari nazariyasini chetlab o`tgan emas . Ferma, Eyler, Logranj, Dirixle, Gauss, Chebishev va Rimanlar bu qiziq sohada o`chmas iz qoldirganlar.

Diofant tenglamalarini ba`zilarini yechish uchun quydagi foydali maslahatlardan foydalanish lozimdir.
1.Agar tenglamaning chap qismi o`zgaruvchilarning natural qiymatlarida butun qiymatlar qabul qiluvchi ko`paytiruvchilarga

ajratilsa , tenglamaning o`ng qismi esa butun sondan iborat bo`lsa u holda berilgan tenglamani unga teng kuchli bo`lgan tenglamalar sistemasining birlashmasiga almashtirish mumkin.

2.Agar o`zgaruvchilarning istalgan natural qiymatlarida tenglamaning chap va o`ng qismlarida quydagi shartlar bajariladigan butun sonlar hosil bo`lsa , u holda bunday tenglamalar natural sonlar to`plamida yechimga ega bo`lmaydi:

a) tenglamaning chap va o`ng qismlarini bir xil natural sonlarga bo`lganda har xil qoldiqlar hosil bo`lsa;

b) tenglamaning chap va o`ng qismlari har xil raqamlar bilan tugallansa ;

v) tenglamaning bir tomoni to`la kvadrat ( kub va hokazo ) bo`lib , boshqa tomoni esa bunday xususiyatga ega bo`lmasa .

3. $a^2+b^2=c^2$ tenglamaning barcha butun yechimlari

$a=m^2+n^2$ $b=2mn$ $c=m^2-n^2$ formula bilan topiladi.

Biz quyida bunday tenglamalarning yechimini ayrim usullari to`g`risida to`ztalib o`tamiz.

## 1. Ko`paytuvchilarga ajratish

**1-misol.** $x^2-y^2=105$ tenglamani butun sonlarda yeching.

**Yechish.** Tenglamaning chap qismini ko`paytuvchilarga ajrataylik. $(x+y)(x-y)=105$.

Tenglamani natural sonlarda yechish yetarli , chunki $x$ va $y$ yechimi bo`lsa , $-x$ va $-y$ ham yechimi bo`ladi.

$x-y<x+y$ bo`lganligi sababli $105=1\cdot105=5\cdot21=3\cdot35=7\cdot15$ 4 ta usul bilan ko`paytuvchilarga ajaratiladi .

U holda berilgan tenglama 4 ta sistemaga teng kuchli

a) $\begin{cases}x-y=1\\x+y=105\end{cases}$ $\begin{cases}x-y=1\\2x=106\end{cases}$ $\begin{cases}y=x-1=53-1=52\\x=53\end{cases}$ (53; 52)

b) $\begin{cases}x-y=5\\x+y=21\end{cases}$ $\begin{cases}y=x-5\\2x=26\end{cases}$ $\begin{cases}y=13-5=8\\x=13\end{cases}$ (13; 8)

v) $\begin{cases}x-y=3\\x+y=35\end{cases}$ $\begin{cases}y=x-3\\2x=38\end{cases}$ $\begin{cases}y=19-3=16\\x=19\end{cases}$ (19; 16)

g) $\begin{cases} x-y=7 \\ x+y=15 \end{cases}$ $\begin{cases} y=x-7 \\ 2x=22 \end{cases}$ $\begin{cases} y=11-7 \\ x=11 \end{cases}$ (11; 4)

Demak , berilgan tenglama 8 juft butun yechimga ega ekan:
(53; 52), (13; 8), (19; 16), (11; 4),
(-53; -52), (-13; -8), (-19; -16), (-11; -4) .

**2-misol.** $xy-2x+3y=16$ tenglamaning butun yechimlarini toping.
Yechish. Berilgan tenglama quyidagi tenglamalarga teng kuchli :

$x(y-2)+3y-6=10$

$x(y-2)+3(y-2)=10$

$(x+3)(y-2)=10$

Oxirgi tenglamadan $x+3$ va $y-2$ sonlar 10 sonining bo`luvchilari ekanligi kelib chiqadi. 10 soni 8ta : $\pm1$; $\pm2$; $\pm5$; $\pm10$ bo`luvchilarga ega.

$$(x+3)(y-2)=\pm1\cdot(\pm10)=\pm2\cdot(\pm5)=\pm5\cdot(\pm2)=\pm10\cdot(\pm1)$$

U holda 8 ta quydagi tenglamalar sistemasi hosil bo`ladi.

1) $\begin{cases} x+3=1 \\ y-2=10 \end{cases}$ 2) $\begin{cases} x+3=-1 \\ y-2=-10 \end{cases}$ 3) $\begin{cases} x+3=5 \\ y-2=2 \end{cases}$ 4) $\begin{cases} x+3=-5 \\ y-2=-2 \end{cases}$

5) $\begin{cases} x+3=2 \\ y-2=5 \end{cases}$ 6) $\begin{cases} x+3=-2 \\ y-2=-5 \end{cases}$ 7) $\begin{cases} x+3=10 \\ y-2=1 \end{cases}$ 8) $\begin{cases} x+3=-10 \\ y-2=-1 \end{cases}$

Bu tenglamalar sistemasini yechib 8 juft:
(-2; 12), (-4; -8), (-1; 7), (-5; -3), (2; 4), (-8, 0), (7; 3), (-13; 1) butun yechimlarni topamiz.

**3-misol.** $x^3+91=y^3$ tenglamani butun yechimlarini toping.
**Yechish.** $y^3-x^3=91$, $(y-x)(y^2+xy+x^2)=91$

$y^2+xy+x^2=(y+\frac{x}{2})^2+\frac{3}{4}x^2\geq0$ bo`lganligi sababli quyidagi 4 hol ro`y beradi.

$91=1\cdot91=7\cdot13=13\cdot7=91\cdot1$

Natijada berilgan tenglama butun sonlar to`plamida quyidagi 4 ta sistemaga teng kuchli:

a) $\begin{cases} y-x=1 \\ y^2+xy+x^2=91 \end{cases}$ dan $x=5$ , $y=6$ yoki $x=-6$ , $y=-5$

b) $\begin{cases} y-x=7 \\ y^2+xy+x^2=13 \end{cases}$ dan $x=-3$ , $y=4$ yoki $x=-4$ , $y=3$

v) $\begin{cases} y-x=13 \\ y^2+xy+x^2=7 \end{cases}$ bu sistema haqiqiy yechimga ega emas

g) $\begin{cases} y-x=91 \\ y^2+xy+x^2=1 \end{cases}$ bu sistema haqiqiy yechimga ega emas

**Javob.** (5; 6), (-6; -5), (-3; 4), (-4; 3)

**4-misol.** (99-8-13). Nechta $(x;y)$ butun sonlar jufti $(x+1)(y-2)=2$ tenglikni qanoatlantiradi? A)4 B)2 C)1 D)3 E)5

**Yechish.** 2 soni butun sonlar to'plamida 4 usul bilan ko'paytuvchilarga ajraladi.

$(x+1)(y-2)=2\cdot 1=1\cdot 2=(-2)\cdot(-1)=(-1)\cdot(-2)$

Demak, berilgan tenglamani qanoatlantiruvchi 4 juft butun sonlar mavjud.

Agar sizga bu butun sonlar kerak bo'lsa, ularni mustaqil toping.

**5-misol.** $15x^3+2x-1-4x^2y(x^2y-x+1)=0$ tenglamani butun yechimlarini toping

**Yechish.** Tenglamani $x(15x^2+2-4xy(x^2y-x+1))=1$ ko'rinishda yozaylik. Bundan, $x$ soni 1 ning bo'luvchilari ekanligi kelib chiqadi. Bundan $x=\pm1$ kelib chiqadi.

Agar $x=1$ bo'lsa, tenglamadan $y^2=4 \Rightarrow y=\pm2$

Agar $x=-1$ bo'lsa, $2y(y+2)=-9$ tenglama hosil bo'ladi. Oxirgi tenglikning chap qismi juft, o'ng qismi toq bo'lganligi sababli, uni yechimi yo'q.

**Javob.** (1;-2),(1;2)

**6-misol.** To'g'ri burchakli uchburchakning katetlaridan biri 13 sm gat eng. Qolgan ikki tomonini butun son bilan ifodalanishi ma'lum bo'lsa, ularni toping.

**Yechish.** $a=13$ $b$ - katetlar, $c$ - gipotenuza bo'lsin . U holda Pifagor teoremasiga asosan $c^2=13^2+b^2$ $c^2-b^2=196$; $c>b$ bo`lganligi uchun

$(c-b)(c+b)=1\cdot196=13\cdot13$

$\begin{cases} c-b=1 \\ c+b=196 \end{cases} \Rightarrow c=85, \quad b=84$

$\begin{cases} c-b=13 \\ c+b=13 \end{cases} \Rightarrow c=13, \quad b=0$

Javob: $b=84sm, \quad c=85sm$

7(02-6-4). $xy^2-xy-y^2+y=94$ tenglamaning natural yechimlari juftini toping.

A) (48; 2) B) (48; 3) C) (49; 1) D) (49; 2) E) (48; 1)

**Yechish.** Tenglamani chap qismini ko`paytuvchilarga ajrataylik. $xy(y-1)-y(y-1)=(xy-y)(y-1)=(x-1)y(y-1)$ o`ng tomoni $94=47\cdot2\cdot1$ bo`lganda yechimga ega. $x=48$ va $y=2$

**Javob.** (48; 2)

8(02-6-14). Nechta natural $(x;y)$ sonlar jufti $x^2-y^2=53$ tenglikni qanoatlabtiradi.

A) ∅  B) 1  C) 2  D) 3  E) 4

**Yechish.** $(x-y)(x+y)=1\cdot53$  $x-y<x+y$

$\begin{cases} x-y=1 \\ x+y=53, \end{cases}$ $\begin{cases} x=27 \\ y=26. \end{cases}$ 1 ta

**Javob.** (B)

## 2.Tanlash usuli

**9-misol.** $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$ tenglamani natural sonlar to`plamida yeching.

**Yechish.** $x,y,z$ – o`zgaruvchilar tenglamada simmetrik holda qatnashganligi sababli $x\le y\le z$ deb olish mumkin. Boshqa yechimlar o`zgaruvchilarning o`rinlarini almashtirish yordamida hosil qilinadi. Quyidagi hollarni qaraymiz:

a) $x=1$ bo`lsa, tenglama yechimga ega emas, chunki $\frac{1}{y}+\frac{1}{z}\neq0$

b) $x=2$ bo`lsin, u holda $\frac{1}{y}+\frac{1}{z}=\frac{1}{2}$ $(y-2)\cdot(z-2)=4$ tenglama hosil bo`ladi. $0\le y-2\le z-2$ bo`lganligi sababli 2 ta yechim hosil bo`ladi. $(y-2)\cdot(z-2)=1\cdot4=2\cdot2$

$\begin{cases} y-2=1 \\ z-2=4, \end{cases}$ $\begin{cases} y=3 \\ z=6 \end{cases}$

$\begin{cases} y-2=2 \\ z-2=2, \end{cases}$ $\begin{cases} y=4 \\ z=4 \end{cases}$

Bu holda (2;3;6), (2;4;4) yechimlar hosil bo`ladi.

v) $x=3$ bo`lsin. Shakl almashtirishdan so`ng, $\frac{1}{y}+\frac{1}{z}=\frac{2}{3}$ tenglama hosil bo`ladi.

Agar $y\ge4$ bo`lsa, $z\ge4$ bo`ladi. Bundan $\frac{1}{y}+\frac{1}{z}\le\frac{1}{4}+\frac{1}{4}=\frac{1}{2}<\frac{2}{3}$ hosil bo`ladi. Bunday bo`lishi mumkin emas. Bu holda yagona (3; 3; 3) yechim topiladi.

g) $x\ge4$ bo`lsin . U holda $y\ge4$ va $z\ge4$ bo`ladi. Tenglamadan esa $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\le\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}\le1$ kelib chiqadi . Demak $x\le y\le z$ bo`lsa ,

tenglama 3 ta (2; 3; 6), (2; 4; 4), (3; 3; 3) yechimga ega. Dastlabki farazni bekor qilsak,
8 juft: (4; 2; 4), (4; 4; 2), (2; 6; 3), (3; 2; 6), (3; 6; 2), (6; 2; 3), (6; 3; 2) yechim hosil bo`ladi.

## 3.Teskarisidan faraz qilish usuli bilan isbotlash

**10-misol.** $x^2-y^2=1982$ tenglik to`g`ri bo`ladigan $x$ va $y$ butun sonlar mavjud emasligini isbotlang.

**Yechish.** Teskarisini faraz qilaylik va $x^2-y^2=1982$ tenglamani qanoatlantiruvchi $x$, $y$ butun sonlar mavjud bo`lsin deylik. Bizga ma`lumki, $x-y$ va $x+y$ sonlar bir vaqtda juft yoki bir vaqtda toq bo`ladi. Birinchi holda $x^2-y^2$ son 4 ga bo`linadi. U holda tenglikning o`ng qismi 1982 soni 4 ga bo`linmaydi. Bu qarama-qarshilik farazimizni noto`g`ri ekanligini bildiradi. Ikkinchi hol bu yerda bo`lishi mumkin emasligi ko`rinib turibdi.

**11-misol.** $15x^2=9+7y^2$ tenglamani qanoatlantiruvchi $x$ va $y$ butun sonlar mavjud emasligini isbotlang.

**Yechish.** Tenglamani qanoatlantiruvchi $x$, $y$ butun sonlar mavjud bo`lsin deylik. U holda tenglaning chap qismi 3 ga bo`linganligi sababli, o`ng qismi ham 3 ga bo`linishi kelib chiqadi; $y^2$ to`la kvadrat bo`lganligi sababli $y$ 9 ga bo`linadi, ya`ni $y^2=9k^2$. U holda $5x^2=3\cdot(1+7k^2)$ bo`ladi. Bundan $x^2$ ni 3 ga bo`linishi kelib chiqadi, ya`ni $x^2=9p^2$ ekanligi kelib chiqadi. Bundan $15p^2=1+7k^2$. Tenglamaning o`ng qismi 3 ga bo`linmaydi.

Chunki $k=3n$ bo`lsa, $7k^2+1=3\cdot21\cdot n^2+1$

$k=3n+1$ bo`lsa, $7k^2+1=3\cdot(7\cdot3\cdot n^2+7\cdot2n+2)+2$

$k=3n+2$ bo`lsa, $7k^2+1=3\cdot(21\cdot n^2+28n+9)+2$

Demak $7k^2+1$ soni 3 ga bo`linmas ekan.

Ammo $15p^2=1+7k^2$ tenglamani chap qismi 3 ga bo`linadi. Bunday bo`lishi mumkin emas. Demak, bizning farazimiz noto`g`ri. Tenglamani qanoatlantiruvchi $x$ va $y$ butun sonlar mavjud emas.

**12-misol.** $x^5+3x^4y-5x^3y^2-15x^2y^3+4xy^4+12y^5=33$ tenglamani butun sonlar to`plamida yeching.

**Yechish.** Tenglamani chap qismini ko`paytuvchilarga ajrataylik.

$x^5+3x^4y-5x^3y^2-15x^2y^3+4xy^4+12y^5=x^4(x+3y)-5x^2y^2(x+3y)+4y^4(x+3y)=$

$(x+3y)(x^4-5x^2y^2+4y^4)=(x+3y)(x^2-y^2)(x^2-4y^2)=(x+3y)(x+y)(x-y)(x+2y)(x-2y)$

Faraz qilaylik bu tenglama qandaydir $(x_0;y_0)$ yechimga ega bo`lsin. U holda $y_0 \neq 0$ bo`ladi, chunki $y_0=0$ bo`lsa, $x_0^5=33$ bo`lib bu tenglik butun sonlar to`plamida yechimga ega emas. Agar $y_0 \neq 0$ bo`lsa, $x_0+3y_0$, $x_0+y_0$, $x_0-y_0$, $x_0+2y_0$, $x_0-2y_0$ sonlar juft-jufti bilan har xil sonlar bo`ladi. Demak 33 sonini 5 ta har xil ko`paytuvchilarning ko`paytmasi shaklida yozish mumkin. Bu esa bo`lishi mumkin emas, chunki $33=(-3)\cdot 1\cdot(-1)\cdot 11=(-1)\cdot 1\cdot 3\cdot(-11)$ 2 xil usul bilan 4 ta ko`paytuvchilarga ajraladi. Demak, farazimiz no`to`g`ri., berilgan tenglama yechimga ega emas.

**13-misol.** $x^2+y^2=3\cdot(z^2+u^2)$ tenglamani qanoatlantiruvchi natural $x,y,z,u$ sonlar mavjud emasligini isbotlang.

**Yechish.** Teskarisini faraz qilaylik. Berilgan tenglamani qanoatlantiruvchi natural $x,y,z$ va $u$ sonlar mavjud bo`lsin. Bulardan $x^2+y^2$ miqdor eng kichik bo`lganini qaraylik. Bu sonlar $a,b,c,d$ bo`lsin. $a^2+b^2=3(c^2+d^2)$ dan $a^2+b^2$ son 3 ga karrali ekanligi ko`rinib turibdi.

$a^2+b^2$ 3 ga, $a$ va $b$ sonlar 3 ga bo`linganda bo`linadi.

$a=3m$, $b=3n$ bo`lsin. U holda $a^2+b^2=9m^2+9n^2=3\cdot(c^2+d^2)$ dan $c^2+d^2=3\cdot(m^2+n^2)$. Oxirgi tenglikdan berilgan tenglamani qanoatlantiruvchi $c,d,m,n$-sonlarini topdik. Bu to`rtlik uchun $c^2+d^2<a^2+b^2$ o`rinli. Bu esa, $a,b,c,d$-sonlarni tanlanishga zidd. Demak bizning farazimiz noto`g`ri ekan.

## 4.Yagonalik usuli

**14-misol.** $(x_1^2+1)\cdot(x_2^2+2^2)\cdot\ldots\cdot(x_n^2+n^2)=2^n\cdot n!x_1\cdot\ldots\cdot x_n$ tenglamani natural sonlarda yeching, bunda $n\in Z$.

**Yechish.** Koshi tengsizligiga asosan quyidagini hosil qilamiz.

$$x_1^2+1\ge 2x_1,$$
$$x_2^2+2^2\ge 2\cdot 2x_2,$$
$$\ldots$$
$$x_n^2+n^2\ge 2n\cdot x_n$$

Bu tengsizliklarga "=" belgisi $x_1=1$, $x_2=2,\ldots,x_n=n$ bo`lganda bajariladi. Tengsizliklarni hadmahad ko`paytirib $(x_1^2+1)\cdot(x_2^2+2^2)\cdot\ldots\cdot(x_n^2+n^2)\ge 2^n\cdot n!x_1\cdot\ldots\cdot x_n$ hosil qilamiz. Bundan $x_1=1$, $x_2=2,\ldots,x_n=n$ ekanligi kelib chiqadi.

**Javob.** (1; 2;...n), $n\in N$



## 5. Xususiy holdan umumiy holga o`tish

**15-misol.** Tenglamani butun sonlar to`plamida yeching.

$\sqrt{x+\sqrt{x+\ldots\sqrt{x}}}=z$, ($y$ ta ildiz)

**Yechish.** $x=0$, $z=0$, $y$-istalgan natural son berilgan tenglamaning yechimi ekanligi ko`rinib turibdi.

1) $y=1$ bo`lsin. U holda $\sqrt{x}=z$ bo`ladi. Bundan $x=z^2$, $(x\ge 0)$. Bu tenglamani yechimi $x=t^2$, $z=t$, bunda $t\in Z_0$, bo`ladi. Bu holda yechim $x=t^2$, $z=t$, $y$-istalga natural son bo`ladi.

2) $y=2$ bo`lsin. $\sqrt{x+\sqrt{x}}=z$ (1) bo`ladi. Bundan $x+\sqrt{x}=z^2$ (2), $\sqrt{x}=z^2-x$. Oxirgi tenglamadan $\sqrt{x}$ni natural son ekanligini aniqlasak, $\sqrt{x}=t$, $(t\in N)$. $x=t^2$ni (2) tenglamaga qo`ysak, $t\cdot(t+1)=z^2$ bo`ladi. $t^2<t(t+1)<(t+1)^2$ ekanligidan $t^2<z^2<(t+1)^2$ kelib chiqadi. Bundan ikkita ketma-ket natural son orasida natural son mavjud bo`lmaydi.

Bu holda ham $x=0$, $z=0$, $y$- istalgan natural son tenglamani yechimi bo`ladi.

3) $y=3$ bo`lsin. $\sqrt{x+\sqrt{x+\sqrt{x}}}=z$ (3). Bundan ketma-ket hosil qilamiz. $\sqrt{x+\sqrt{x}}=z^2-x$ yoki $\sqrt{x+\sqrt{x}}=z_1$, bunda $z_1=z^2-x$. Demak, bunda (1) tenglamaga o`xshash tenglama hosil bo`ldi. Bu tenglama ham $x=0$; $z=0$; $y_1$- istalgan natural son yechimga ega.

Xuddi shunday, keyingi bosqichlarni tahlil qilib borib, tenglamani $x=0$; $z=0$; $y=n$, $n\in N$ yechimini topamiz.

**16-misol.** $1!+2!+3!+\ldots+x!=y^2$ tenglamani butun yechimlarini toping.

**Yechish.** 1) $x=1$ bo`lsin. U holda $1!=y^2$, $y^2=1$, $y=\pm 1$. Bu holda (1; 1), (1; -1) yechim bo`ladi.

2) $x=2$ bo`lsin. $1!+2!=y^2$ tenglama hosil bo`ladi. $y^2=3$ tenglama butun yechimga ega emas.

3) $x=3$ bo`lsin. $y^2=1!+2!+3!=1+2+6=9$ $y=\pm 3$

Bu holda yechim (3; 3), (3; -3).

4) $x=4$ bo`lsin. $y^2=1!+2!+3!+4!=1+2+6+24=33$ $y^2=33$ bu tenglama butun yechimga ega emas.

5) agar $x\ge 4$ bo`lsa, tenglamaning chap qismi 3 raqami bilan tugallanadi. To`la kvadrat 3 raqami bilan tugallanmasligi sababli, bu holda tenglama butun yechimga ega emas.

**Javob.** (1; 1), (1; -1), (3; 3), (3; -3).

## 6. Zanjirli kasrdan foydalanish

**Teorema.** Har bir $\alpha$ haqiqiy songa qiymati shu $\alpha$ dan iborat bo`lgan yagona zanjir kasr mos keladi. Agar $\alpha \in Q$ bo`lsa, zanjir kasr chekli bo`ladi, agar $\alpha$ iratsional bo`lsa zanjir kasr cheksiz bo`ladi va aksincha.

**17-misol.** $\frac{17}{11}$ ni zanjir kasrga yoying.

**Yechish.** $\frac{17}{11}=1+\frac{6}{11}=1+\frac{1}{\frac{11}{6}}=1+\frac{1}{1+\frac{5}{6}}=1+\frac{1}{1+\frac{1}{\frac{6}{5}}}=1+\frac{1}{1+\frac{1}{1+\frac{1}{5}}}$

**18-misol.** $\sqrt{3}$ ni zanjir kasrga yoying.

Yechish. $[\sqrt{3}]=1$, $\sqrt{3}=1+\frac{1}{x}$, $x>0$

$x=\frac{1}{\sqrt{3}-1}=\frac{\sqrt{3}+1}{2}$, $[x]=1$, $x=1+\frac{1}{y}$, $y>1$

$y=\frac{1}{x-1}=\sqrt{3}+1$; $[y]=2$, $y=2+\frac{1}{z}$, $(z>1)$

$z=\frac{1}{y-2}=\frac{1}{\sqrt{3}-1}=x$; demak,

$$\sqrt{3}=1+\frac{1}{x}=1+\frac{1}{1+\frac{1}{y}}=1+\frac{1}{1+\frac{1}{2+\frac{1}{z}}}=1+\frac{1}{1+\frac{1}{2+\frac{1}{x}}}=1+\frac{1}{1+\frac{1}{2+\frac{1}{1+\frac{1}{2+\frac{1}{1+\dots}}}}}$$

19(97-5-15). Tenglamani natural sonlardagi yechimida $z$ nimaga teng.

$$x+\frac{1}{y+\frac{1}{z}}=\frac{10}{7}$$

A) 3 B) 4 C) 1 D) 2 E) 7

**Yechish. 1-usul.** Tenglamani quyidagicha almashtiramiz. $x+\frac{z}{1+yz}=1+\frac{3}{7}$.

U holda $x$- biror sonning butun qismi, $\frac{z}{1+yz}$ –uning kasr qismi bo'ladi, shuning uchun $x=1$ va $\frac{z}{1+yz}=\frac{3}{7}$ bo'ladi. Bundan : $\frac{1+yz}{z}=\frac{7}{3}$ yoki $y+\frac{1}{z}=2+\frac{1}{3}$. Bundan $y=2$, $z=3$

**Javob.** $z=3$ (A)

**2-usul.** $\frac{10}{7}$ sonini zanjirli kasr ko`rinishida yozaylik.

$\frac{10}{7}=\frac{7+3}{7}=1+\frac{3}{7}=1+\frac{1}{\frac{7}{3}}=1+\frac{1}{2+\frac{1}{3}}$, $\frac{10}{7}$ soni yagona usulda zanjirli kasrga yoyiladi . $x+\frac{1}{y+\frac{1}{z}}=1+\frac{1}{2+\frac{1}{3}}$

Oxirgi tenglamadan $z=3$ ekanligi kelib chiqadi.

**20-misol.** $55\cdot(x^3y^3+x^2+y^2)=229(xy^3+1)$ tenglamani natural sonlar to`plamida yeching.

**Yechish.** Tenglamani shaklini almashtiraylik.

$$\frac{x^3y^3+x^2+y^2}{xy^3+1}=\frac{229}{55},\quad \frac{x^2(xy^3+1)+y^2}{xy^3+1}=4+\frac{9}{55},\quad x^2+\frac{y^2}{xy^3+1}=4+\frac{1}{\frac{55}{9}},$$

$x^2+\frac{1}{xy+\frac{1}{y^2}}=4+\frac{1}{6+\frac{1}{9}}$ . Oxirgi tenglikdan $x^2=4;\quad xy=6;\quad y^2=9$

Bundan $\begin{cases}x^2=4\\ xy=6\\ y^2=9\end{cases}$ $\begin{cases}x=2\\ y=3\end{cases}$

**Javob.** $x=2;\quad y=3$

## 7. Parametrlash usuli

Diofant tenglamalarini yechimlarini cheksiz ko`p ekanligini isbotlashda parametrlash usuli kuchli metod hisoblanadi. Bu metodning mohiyati shundan iboratki , $x,y,...,z$ o`zgaruvchilarni $\alpha,\beta,...,\gamma$ parametrlarga bog`liq bo`lgan $x=A(\alpha,\beta,...,\gamma),\quad y=B(\alpha,\beta,...,\gamma),...,\quad z=C(\alpha,\beta,...,\gamma)$ butun koefitsientli $A,B,...,C-$ ko`phadlar orqali ifodalanadi.

Parametrlash usuli -original usul bo`lib, juda katta topqirlikni talab etadi.

**21-misol.** $x^3+y^3+z^3=2$ tenglamani yeching.

**Yechish.** $x=a+b,\quad y=a-b$ deb olaylik . $a,b\in Z$ u holda

$(a+b)^3+(a-b)^3+z^3=2,$

$a^3+3a^2b+3ab^2+b^3+a^3-3a^2b+3ab^2-b^3+z^3=2,\quad 2a^3+6ab^2+z^3=2$ .

$a=1$ bo`lsin , $6b^2=-z^3$, bunda $z=6c$, $(c\in Z)$, ya`ni $b^2=-36bc^3$. Bu holda $c=-\alpha^2$, $(\alpha\in Z)$. U holda , $b=6\alpha^3$. Natijada, $(1+6\alpha^3)^3+(1-6\alpha^3)^3+(-6\alpha^2)^3=2$.
$x=A(\alpha)=1+6\alpha^3$, $y=B(\alpha)=1-6\alpha^3$, $z=C(\alpha)=-6\alpha^2$, $\alpha\in Z$.
$\alpha$ ni o`rniga istalgan butun sonlarni qo`ysak , butun yechimlarni hosil qilamiz.

**22-misol.** $x^4+y^6+z^{12}=t^4$ tenglamani yeching.

**Yechish.** $x=a-b$, $t=a+b$, bunda $a,b\in Z$, deb olaylik. U holda, $(a-b)^4+y^6+z^{12}=(a+b)^4$ yoki $a^4-4a^3b+6a^2b^2-4ab^3+b^4+y^6+z^{12}=$
$=a^4+4a^3b+6a^2b^2+4ab^3+b^4$, $y^6+z^{12}=8a^3b+8ab^3$. $y^6=8a^3b$, $z^{12}=8ab^3$ deb olaylik. U holda $a=c^3$, $b=d^3$, $(c,d\in Z)$ va $y^2=2c^3d$, $z^4=2cd^3$)
Bundan $2cd$ ni to`la kvadrat ekanligi ko`rinadi. $c=\alpha^4$, $d=2\beta^4$ deb olsak, $(\alpha,\beta\in Z)$
$(\alpha^{12}-8\beta^{12})^4+(2\alpha^6\beta^2)^6+(2\alpha\beta^3)^{12}=(\alpha^{12}+8\beta^{12})^4$ tenglik hosil bo`ladi.

**Javob.** $x=\alpha^{12}-8\beta^{12}$, $y=2\alpha^6\beta^2$, $z=2\alpha\beta^3$, $t=\alpha^{12}+8\beta^{12}$

**23-misol.** $\begin{cases}x^3+y^3+z^3=2\\x+y+z=2\end{cases}$ tenglamalar sistemasini butun sonlarda yeching.

**Yechish.** Tenglamalar sistemasidan $x,y$ va $z$ sonlardan biri 0 ga , qolgan ikkitasi 1 ga teng ekanligi ko`rinib turibdi.Buni isbotlaymiz.
$x=1+\alpha$, $y=1+\beta$ deb olaylik $\alpha,\beta\in Z$. U holda ikkinchi tenglamadan $z=2-(x+y)=2-(1+\alpha+1+\beta)=-(\alpha+\beta)$ bo`ladi. Bularni sistemaning birinchi tenglamasiga qo`ysak $(\alpha+1)^3+(\beta+1)^3+(-(\alpha+\beta))^3=2$
$\alpha^3+3\alpha^2+3\alpha+1+\beta^3+3\beta^2+3\beta+1-\alpha^3-3\alpha^2\beta-3\alpha\beta^2-\beta^3=2$,
$3\alpha^2+3\beta^2+3\alpha+3\beta=3\alpha^2\beta+3\alpha\beta^2$ yoki $\alpha^2+\beta^2+\alpha+\beta=\alpha\beta(\alpha+\beta)$ yoki $\alpha+\beta=(\alpha+\beta+2)(\alpha+\beta-\alpha\beta)$. Oxirgi tenglikdan $\alpha+\beta$ soni $\alpha+\beta+2$ ga bo`linishi kelib chiqadi. Bunda quyidagi 4 hol ro`y beradi.
1) $\alpha+\beta=0$ ; 2) $\alpha+\beta=-1$; 3) $\alpha+\beta=-3$; 4) $\alpha+\beta=-4$
1) $\alpha+\beta=0$ bo`lsa, oxirgi tenglikdan $0=(0+2)(0-\alpha\beta)$. $\alpha\beta=0$ demak $\alpha=0$, $\beta=0$. U holda $x=1+\alpha=1+0=1$, $y=1+\beta=1+0=1$, $z=-(\alpha+\beta)=-(0+0)=0$.
Demak, $x=1$ , $y=1$, $z=0$.
2) $\alpha+\beta=-1$ bo`lsa, $-1=(-1+2)(-1-\alpha\beta)$, $-1=-1-\alpha\beta$, $\alpha\beta=0$, $\alpha=0$, $\beta=0$. Bu holda ham $x=1$ , $y=1$, $z=0$ bo'ladi.
3) $\alpha+\beta=-3$ bo'lsin. U holda $-3=(-3+2)(-3-\alpha\beta)$ bundan $\alpha\beta=-6$ bo'ladi.
a) $\alpha=1$, $\beta=-6$ b) $\alpha=-6$, $\beta=1$ v) $\alpha=2$, $\beta=-3$ g) $\alpha=-3$, $\beta=2$
Bu hollarda topilgan yechimlar sistemani qanoatlantirmaydi.
4) $\alpha+\beta=-4$ bo'lsa, 3-holdek bo'ladi. Bu holda ham yechim mavjud emas.

## 8. Taqqoslamalardan foydalanish

Ayrim Diofant tenglamalarini yechish uchun taqqoslamalar qo`llaniladi. Agar $a-b$ soni $k$ soniga bo`linsa, $a$ va $b$ sonlar $k$ modul bo`yicha taqqoslama deyiladi va $a \equiv b(\mathrm{mod}\, k)$ deb yoziladi.

$a$ va $b$ sonlar $k$ modul bo`yicha taqqoslama bo`lishi uchun ularning har birini $k$ ga bo`lganda bir xil qoldiq qolishi lozim, chunki $a=k\cdot q+n$, $b=k\cdot m+n$ bo`lsa $a-b=k\cdot(q-m)$ bo`ladi.

**24-misol.** $x^3+117y^3=5$ tenglamani yeching.

**Yechish.** Bu tenglamani "9 modul" bo`yicha qaraymiz.

Har qanday sonni kubini 9 ga bo`lganda, yoki bu son 9ga bo`linadi, yoki 9 ga bo`lganda 1 yoki 8 qoldiq qoladi. $x^3=0(\mathrm{mod}\,9)$ yoki $x^3=1(\mathrm{mod}\,9)$, $x^3=8(\mathrm{mod}\,9)$. 117 soni 9 ga bo`linadi. Bu holda $117y^3 \equiv 0(\mathrm{mod}\,9)$. Shunday qilib: $x^3+117y^3 \equiv 0(\mathrm{mod}\,9)$ yoki $x^3+117y^3 \equiv 1(\mathrm{mod}\,9)$ yoki $x^3+117y^3 \equiv 8(\mathrm{mod}\,9)$. Shuning uchun $x^3+117y^3=5$ tenglama butun yechimlarga ega emas.

**25-misol.** $1988\cdot x^{1989}+1989\cdot y^{1990}=1991$ tenglamaning qanoatlantiruvchi $x$ va $y$ butun sonlar mavjudmi?

**Yechish.** tenglamadan $y$ ning toq son ekanligi ko'rinib turibdi (nima uchun? )

Tenglamaning chap qismini 4 ga bo'lganda 1 qoldiq qoladi. O'ng qismini 4 ga bo'lganda 3 qoldiq qoladi. Demak, tenglamani qanoatlantiruvchi butun $x$ va $y$ sonlari mavjud emas ekan.

Bu taqqoslama tilida quyidagicha yoziladi:

$1988 \equiv 0(\mathrm{mod}\,4)$, $1988x^{1989} \equiv 0(\mathrm{mod}\,4)$, $1989 \equiv 1(\mathrm{mod}\,4)$, $1989y^{1990} \equiv 1(\mathrm{mod}\,4)$

$1991 \equiv 3(\mathrm{mod}\,4)$, $1988x^{1989}+1989y^{1990} \equiv 1(\mathrm{mod}\,4)$

## 9. $ax+by=c$ ko'rinishidagi tenglama

**26-misol.** $8x-5y=19$ tenglamani butun yechimlarini toping.

**Yechish.** Bu misolni yechish orqali $ax-by=c$ tenglamalarni yechish usuli bilan tanishtiramiz.

$x=3$, $y=1$ berilgan tenglamani qanoatlantiradi, ya'ni uning xususiy yechimi bo'ladi, chunki $8\cdot 3-5\cdot 1=19$ tenglik to'g'ri .Bu yechimni tanlash orqali topdik . $8x-5y=19$ dan $8\cdot 3-5\cdot 1=19$ ni ayiraylik. $8(x-3)-5(y-1)=0$ yoki $8\cdot(x-3)=5\cdot(y-1)$. Oxirgi tenglikdan, $x-3$ butun son bo`lishi uchun $y-1$ soni 8 ga bo`linishi, ya`ni $y-1=8n$, $n\in Z$. Oxiri tenglamaga $y-1$ ni o`rniga $8n$ ni qo`yamiz. U holda $x-3=5n$, $n\in Z$. Demak, berilgan tenglamaning butun yechimlari cheksiz ko`p bo`lib bu yechimlar umumiy holda $x=5n+3$, $y=8n+1$, $n\in Z$ ko`rinishda yozish mumkin.

Ikki o`zgaruvchili chiziqli tenglamani butun sonlarda yechish uchun quydagi teoremani bilish foydalidir.

**Agar EKUB*(a;b)=1* bo`lsa, $ax+by=1$ tenglama hech bo`lmaganda bitta yechimga ega bo`ladi.**

**Agar EKUB $(a;b)=d>1$ bo`lib, $c$ son $d$ ga bo`linmasa, u holda $ax+by=c$ tenglama butun yechimga ega bo`lmaydi.**

**Agar $ax+by=c$ tenglamada EKUB $(a;b)=d>1$ va $c$ son $d$ ga bo`linsa, u holda bu tenglamaning yechimlari $a_1x+b_1y=c_1$,( bunda EKUB $(a_1;b_1=1)$) tenglamaning yechimlari bilan ustma-ust tushadi.**

**Agar $ax+by=c$ tenglamada EKUB*(a;b)=1* bo`lsa, uning yechimlari quydagi formula bilan topiladi:**

**$x=x_0c+bt$, $y=y_0c-at$, bunda $x_0, y_0$ ushbu $ax+by=1$ tenglamaning butun yechimi va $t$ -ixtiyoriy butun son.**

Yuqorida keltirilgan teoremalar $ax+by=c$ tenglamaning , bunda EKUB(a;b)=1, butun sonlar to'plamida yechish uchun quyidagi qoidaga amal qilish lozimligini bildiradi.

**Qoida.** 1) $ax+by=1$ tenglamaning yechimlaridan birini topish uchun 1 sonini $a$ va $b$ sonlarining chiziqli kombinatsiyalari ko'rinishida tasvirlash lozim.

2) $ax+by=c$ tenglamaning barcha butun yechimlarini $x=x_0c+bt$, $y=y_0c-at$, ko'rinishida yozish lozim bo'ladi. Bunda $x_0, y_0$ - $ax+by=1$ ning butun yechimlari, $t$ -ixtiyoriy butun son.

**27-misol.** $15x+37y=1$ tenglamaning butun yechimlaridan birini toping.

**Yechish.** 1) $37=15\cdot2+2$, $15=7\cdot2+1$.

2) $1=15-7\cdot2=15-(37-15\cdot2)\cdot2=15\cdot5+37\cdot(-2)$. Demak, $x_0=5$, $y_0=-2$

**28-misol.** $18x+20y=1$ tenglamaning butun yechimlarini toping.

**Yechish.** EKUB(18;20)=2. 9 soni 2 ga bo'linmaydi. Shuning uchun tenglamaning butun yechimi yo'q.

**28-misol.** $37x-25y=3$ tenglamaning butun yechimlarini toping.

**Yechish.** 1). $37x-25y=1$ tenglamaning butun yechimlarini topamiz.

$$256=37\cdot6+34;$$
$$37=34\cdot1+3;$$
$$34=3\cdot11+1$$

$$1=34-3\cdot11=256-37\cdot6-11(37-256+37\cdot6)=256\cdot12-37\cdot83=37\cdot(-83)-256\cdot(-12)$$

$x_0=-83,\quad y_0=-12$

2). Berilgan tenglamaning barcha yechimlari:

$$x=-83\cdot3-256t=-249-256t$$
$$y=-12\cdot3-37t=-36-37t$$

**30-masala.** 1 so'm pulni 25 dona 1 tiyinlik, 3 tiyinlik va 5 tiyinlik tangalar bilan maydalash mumkinmi?

**Yechish.** Faraz qilaylik maydalash mumkin bo'lsin.

| Tangalar | Tangalar soni | Pul |
|---|---|---|
| 1 tiyin | $x$ ta | $x$ tiyin |
| 3 tiyin | $y$ ta | $3y$ tiyin |
| 5 tiyin | $z$ ta | $5z$ tiyin |
| Jami | 25 ta | 1 so'm=100 tiyin |

U holda $\begin{cases} x+y+z=25 \\ x+3y+5z=100 \end{cases}$ tenglamalar sistemasi hosil bo'ladi. Uni yechaylik. Birinchi tenglamadan $z=25-x-y$ bo'ladi. Buni ikkinchi tenglamaga qo'yib soddalashtirsak, $2\cdot(2x+y)=25$ tenglama hosil bo'ladi. Bu tenglikning chap qismi juft son, ammo o'ng qismi toq sondir. Bunday bo'lishi mumkin emas. Demak, farazimiz notog'ri ekan.

**Javob.** mumkin emas.

## 10. $x^2-dy^2=1$ ko'rinishidagi tenglama (bunda $d$ to`la kvadrat emas)

$x^2-dy^2=1$ ko'rinishidagi tenglamalar Pell(1620-1685y.) tenglamalari deyiladi. $x_0=1,\ y_0=0$ yechim ko'rinib turibdi. $ax+by=c$ tenglamani xususiy yechimlarini bilgan holda uning umumiy yechimlarini topdik. Xuddi shunday usulni $x^2-dy^2=1$ tenglamaga qo`llash mumkin. Buning uchun quyidagi misollarga murojaat qilaylik.

**31-misol.** $x^2-2y^2=1$ tenglamaning butun yechimlarini toping.

**Yechish.** Agar $(x_0;y_0)$ sonlar jufti berilgan tenglamaning yechimlari bo`lsa, $(-x;y)$, $(x;-y)$, $(-x;-y)$ sonlar jufti ham uning yechimi bo'ladi. Shuning uchun berilgan tenglamaning natural yechimlarini topish yetarli ekan.

$x=3$, $y=2$ sonlar berilgan tenglamani qanoatlantiradi. Uni tanlash usuli bilan topdik.

$(x+y\sqrt{2})(x-y\sqrt{2})=1$ $x=3$, $y=2$ sonlar juftini tenglamaga qo`ysak: $(3+2\sqrt{2})\cdot(3-2\sqrt{2})=1$ (1). Agar oxirgi tenglikni ikkala qismini kvadratga oshirsak berilgan tenglamani ikkinchi yechimi topiladi. Haqiqatdan, $(3+2\sqrt{2})^2(3-2\sqrt{2})^2=1^2$, $(17+12\sqrt{2})(17-12\sqrt{2})=1$ bundan $x=17$, $y=12$ sonlar jufti tenglamani yechimi ekanligi ko`rinadi. Bu usulni qo`llash, $a+\sqrt{2}b$ sonlarni qo`shish, ayirish, ko`paytirish va bo`lish yana shu ko`rinishdagi sonlar ekanligidan kelib chiqadi.

(1) tenglikni ikkala qismini kubga ko`tarsak, yangi yechimini hosil qilamiz. Haqiqatdan, $(3+2\sqrt{2})^3\cdot(3-2\sqrt{2})^3=1$, $(99+70\sqrt{2})\cdot(99-70\sqrt{2})=1$. $x=99$, $y=70$ sonlar jufti yechim bo`ladi. Bu usulni davom ettirib yangi yechimlarini topamiz. Demak $(3+2\sqrt{2})^n=x_n+y_n\cdot\sqrt{2}$ bo`ladi. Bunda $x_n, y_n$ sonlar jufti berilgan tenglamani yechimi bo`ladi.

$(3+2\sqrt{2})^n=x_n+y_n\cdot\sqrt{2}$, $(3-2\sqrt{2})^n=x_n-y_n\cdot\sqrt{2}$. $x_n=\frac{1}{2}((3+2\sqrt{2})^n+(3-2\sqrt{2})^n)$

$y_n=\frac{1}{2}((3+2\sqrt{2})^n-(3-2\sqrt{2})^n)$

32-misol(Geron masalasi). Tomonlari ketma- ket natural sonlardan , yuzasi esa butun sondan iborat uchburchakning tomonlariini toping.

**Yechish.** $x-1;x;x+1-$ uchburchak tomonlari bo`lsin. U holda Geron formulasiga asosan, $p=\frac{3x}{2}$; $p-a=\frac{x}{2}+1$, $p-b=\frac{x}{2}$, $p-c=\frac{x}{2}-1$.

$S=\sqrt{\frac{3x}{2}\cdot(\frac{x}{2}+1)\cdot\frac{x}{2}\cdot(\frac{x}{2}-1)}$ , $S=\frac{x}{2}\sqrt{3\cdot(\frac{x^2}{4}-1)}$ , $\frac{x}{2}=m-$butun son bo`lishi lozim, aks holda $S-$butun son bo`lmaydi. U holda $S=m\cdot\sqrt{3\cdot(m^2-1)}$

bundan $m^2-1=3n$ bo'lishi lozim, chunki $\sqrt{3(m^2-1)}-$ butun son bo`lishi kerak. $S=3mn$. Demak, bu masalani yechish uchun $m^2-3n^2=1$ tenglama hosil bo`ldi. Tanlash yordamida oxirgi tenglamani $m=2$, $n=1$ xususiy yechimlarini topamiz. $(m+n\sqrt{3})(m-n\sqrt{3})=1$, yoki topilgan yechimga qo`ysak, $(2+\sqrt{3})(2-\sqrt{3})=1$. Oldingi misolga asosan $x_p=2m_p=(2+\sqrt{3})^p+(2-\sqrt{3})^p$

$p=1$ bo`lsa, $x_1=4$, $S=6$

$p=2$ bo`lsa, $x_2=14$, $S=84$

$p=3$ bo`lsa, $x_3=52$, $S=1170$

$p=4$ bo`lsa, $x_4=194$, $S=16296$ va hokazo

Bunday tenglamalarni yechish uchun quyidagi teorema o`rinli.

**Teorema.** $x^2-dy^2=1$, bunda $d>0$, $d$–to`la kvadrat emas, tenglamaning har qanday yechimi $x_n=\frac{1}{2}((x_0+y_0\sqrt{d})^n+(x_0-y_0\sqrt{n})^n)$

$y_n=\frac{1}{2\sqrt{7}}((x_0+y_0\sqrt{A})^n-(x_0-y_0\sqrt{A})^n)$ ko`rinishga ega, bunda $(x_0;y_0)$–berilgan tenglamaning eng kichik yechimi.

**Eslatma.** $x^2-dy^2=1$ tenglamaning eng kichik yechimini topish uchun umumiy metod bor.

(Qarang Kvant № 5;6 1983 yil)

## 11. Boshqa usullarni qo'llash bilan tenglamalarni yechish

33(99-5-41) . Ushbu $y=\log_3(x^2-8x+7)$ funksiya grafigining ikkala koordinatasi ham butun sonlardan iborat nechta nuqta bor?

A) ∅  B) 1  C) 2  D) 3  E) 4

**Yechish.** Tenglamani $x^2-8x+7=3^y$ ko`rinishda yozaylik . $x^2-8x+7>0$ bo`lishi lozim. $x^2-8x+(7-3^y)=0$. $x$ga nisbatan kvadrat tenglama sifatida yechaylik. $x_{1,2}=4\pm\sqrt{16-(7-3^y)}=4\pm\sqrt{9+3^y}$ . Agar $9+3^y$ to`la kvadrat bo`lsa $x$ butun son bo`ladi, ya`ni $9+3^y=a^2$ bo`lsin. Bundan $3^y=(a+3)(a-3)$ oxirgi tenglik o`rinli bo`lishi uchun $\begin{cases}a-3=3^k\\a+3=3^n\end{cases}$ bo`lishi lozim. Bunda $k<n$ . $3^k-3^n=-6$ yoki $3^n=6+3^k$ . 3 ning darajalari orasida bir-biriga 6 ga farq qiluvchisi faqat $n=2$, $k=1$ bo`lganda bo`ladi. $n=2$ bo`lsa $a+3=3^n$ dan $a=6$. $k=1$ bo`lsa $a-3=3^k$ dan $a=6$ kelib chiqadi. $a=6$ bo'lsa, $y=3$ bo'ladi. $x_1=-2$, $x_2=10$.

**Javob.** 2 ta (C)

34(03-5-9). $mn^3=18$ va $m^2k=20$ bo`lib, $m,n$ va $k$ natural son bo`lsa, $n$ ni toping. A) 3  B) 2  C) 5  D) 4  E) 6

**Yechish.** $\begin{cases}mn^2=18\\m^2k=20\end{cases}$ dan $k\cdot n^2m^3=18\cdot20$, $k\cdot n^2m^3=5\cdot3^2\cdot2^3$ dan $m=2$, $n=3$, $k=5$.

**Javob.** $n=3$ (A)

**35**(99-4-22) . Tenglamaning nechta butun yechimi bor? $(x+2)^2=-\frac{3}{x}$

A) 4 B) 2 C) 1 D) 3 E) ildizi yo`q.

**Yechish.** Tenglamaning chap qismi to`la kvadrat. Demak, $(x+2)^2\geq 0$ bo`ladi. O`ng tomoni nomanfiy bo`lishi uchun $x<0$ dan va $x$ soni 3 ning bo`luvchisi bo`lishi lozim. Bu esa ikki holda bo`ladi.

**1-hol.** $x=-1$. Agar $x=-1$ bo`lsa, o'ng tomoni to`la kvadrat emas.

**2-hol.** $x=-3$. Agar $x=-3$ bo`lsa o`ng tomoni to`la kvadrat bo`ladi.

**Javob.** 1 ta (C)

**36-misol.** $x^2+y^3+z^5=t^7$ tenglamani natural sonlar to`plamida cheksiz ko`p yechimga ega ekanligini isbotlang.

**Yechish.** $3^a+3^a+3^a=3^{a+1}$ bo'lganligi sababli , $a$ ning qiymatini 2,3 va 5 ga karrali qilib shunday tanlaymizki, $a+1$ soni 7 ga bo'linsin. U holda tenglamaning quyidagi yechimlarini hosil qilamiz. $x=3^{\frac{a}{2}}$, $y=3^{\frac{a}{3}}$, $z=3^{\frac{a}{5}}$, $t=3^{\frac{a+1}{7}}$ . Bu xossaga 90 soni egadir. Bu songa $2\cdot 3\cdot 5\cdot 7$ soniga bo'linadigan sonni qo'shsak ham, hosil bo'lgan son tenglamani qanoatlantiradi. Shuning uchun $a=210n+90,\ n\in N$ .

**Javob.** $x=3^{105n+45}$, $y=3^{70n+30}$, $z=3^{42n+18}$, $t=3^{30n+13}$ , $n\in N$

## Mustaqil yechish uchun misollar

Aniqmas tenglamalarni yeching:

**37.** $xy+3x-5y=-3$

**38.** $x^2-656xy-657y^2=1983$

Tenglamalarning natural yechimlarini toping:

**39.** $x+y+z=xyz$

**40.** $\frac{1}{x^2}+\frac{1}{xy}+\frac{1}{y^2}=1$

**41.** Shunday $(x;y;z)$ sonlarini topingki, ulardan istalgan ikkitasini 1 bilan yig`indisi uchinchi songa teng bo`lsin.

**42.** Agar $p$-tub son bo`lsa, $(p>2)$, u holda $\frac{2}{p}$ sonini yagona usulda $\frac{2}{p}=\frac{1}{x}+\frac{1}{y}$ ko`rinishida yozish mumkinligini isbotlang. Bunda $x$ va $y$- har xil butun sonlar.

Tenglamani butun yechimlarini toping.

**43.** $27x-40y=1$

**44.** $13x-15y=7$

45. $42x+34y=5$

46(97-9-15). Tenglamani natural sonlardagi yechimida $y$ nimaga teng.

$$x+\frac{1}{y+\frac{1}{z}}=\frac{17}{15}$$

A) 4 B) 3 C) 2 D) 7 E) 16

47. $2x^2-xy-y^2+2x+7y=84$ tenglamani butun nomanfiy sonlar to'plamida yeching.

**Javob.** (13;20),(6,0)

48. $17x+19y=300$ tenglamani natural sonlar to'plamida yeching.

**Javob.** $x=2, \quad y=14$

47. Tenglamani butun manfiy bo'lmagan yechimlarini toping.

$31(xyzt+xy+xt+zt+1)=40(yzt+y+t)$

48. Butun musbat sonlarda tenglamani yeching. $x^4+2x^7y-x^{14}-y^2=7$

49. $(x-y)^3+(y-z)^3+(z-x)^3=30$ tenglama butun sonlar to'plamida yechimga ega emasligini isbotlang.

## 10-§. Eng katta va eng kichik qiymatni topishga doir misollar

Ekstremal masalalarni yechishning turli usullari mavjud. Bunga, avvalo, hosilaladan foydalanib yechish usulini misol qilib keltirish mumkin. Tengsizliklardan foydalanib, ekstremal masalalarni ham hal qilish mumkin. Ekstremal masalalarni yechishda tengsizliklar usulini mohiyati quyidagicha bayon etilishi mumkin.

Ixtiyoriy musbat $x_1$, $x_2$, ..., $x_n$ sonlar berilgan bo'lsin. Ular yordamida ushbu $A_n=\frac{x_1+x_2+...+x_n}{n}$, $G_n=\sqrt[n]{x_1x_2\cdot...\cdot x_n}$ kattaliklarni tuzamiz, bunda $A_n$ – berilgan sonlarning o'rta arifmetigi, $G_n$ – esa ularning o'rta geometrigi deyiladi.

Mazkur kattaliklar orasida $A_n \geq G_n$ (1) munosabat mavjud bo'lib, unda tenglik belgisi $x_1=x_2=...=x_n$ bo'lganda va faqat shu holdagina erishiladi.

1) ushbu $x_1+x_2+...+x_n$ kattalik o'zgarmas bo'lsin, uni $a$ bilan belgilaylik. U holda (1) tengsizlikdan ko'rinadiki, $G_n$ ning eng katta qiymati faqat $A_n=\frac{a}{n}$ dan iborat bo'lishi mumkin. Bunga esa

$x_1 = x_2 = ... = x_n$ bo'lgandagina erishiladi. Demak, $n$ ta musbat son yig'indisi o'zgarmas son bo'lsa, shu sonlar ko'paytmasidan chiqarilgan $n$-darajali ildiz o'zining eng katta qiymatiga ular o'zaro teng bo'l gandagina erishadi va bu eng katta qiymat $A_n = \frac{a}{n}$ ning o'zi bo'ladi. Shunday qilib, $\max G_n = \frac{a}{n}$;

2) endi $x_1 x_2 \cdot ... \cdot x_n$ kattalik o'zgarmas deylik, uni $b$ bilan belgilaymiz. U holda (1)tengsizlikdan ravshanki, $A_n$ning eng kichik qiymati $\sqrt[n]{b}$ bo'lishi mumkin. Bunga esa $x_1 = x_2 = ... = x_n$ bo'lgandagina erishiladi. Demak, $n$ ta musbat son ko'paytmasi o'zgarmas bo'lsa, shu sonlarning o'rta arifmetigi o'zining eng kichik qiymatiga ular o'zaro teng bo'lgandagina erishiladi va bu eng kichik qiymat $G_n = \sqrt[n]{b}$ ning o'zi bo'ladi. Shunday qilib, $\min A_n = \sqrt[n]{b}$.

**1**(99-5-12). Ifodaning eng kichik qiymatini toping. $5a^8 + 10a^{-4}b^{-4} + 5b^8$

*A)10 B)20 C)100 D)25 E)50.*

**Yechish. 1-usul.** Ikki sonning o'rta arifmetigi va o'rta geometrigi haqidagi tengsizlikni ikki marta qo'llaylik.

$$5a^8 + 10a^{-4}b^{-4} + 5b^8 = 5(a^8 + b^8) + 10a^{-4}b^{-4} \geq 5 \cdot 2\sqrt{a^8 b^8} + \frac{10}{a^4 b^4} = 10(a^4 b^4 + \frac{1}{a^4 b^4}) \geq$$

$\geq 10 \cdot 2\sqrt{a^4 b^4 \cdot \frac{1}{a^4 b^4}} = 20$. Demak, berilgan ifodaning eng kichik qiymati 20 ekan.

**Javob.** 20 (B)

**2-usul.** $A_4 \geq B_4$ tengsizlikka asosan:

$$5a^8 + 10a^{-4}b^{-4} + 5b^8 = 5(a^8 + \frac{1}{a^4 b^4} + \frac{1}{a^4 b^4} + b^8) \geq$$

$\geq 5 \cdot 4\sqrt[4]{a^8 \cdot b^8 \cdot \frac{1}{a^4 b^4} \cdot \frac{1}{a^4 b^4}} = 20$. Demak, berilgan ifodaning eng kichik qiymati 20 ekan.

**Javob.** 20 (B)

**2**(99-8-22). Ko'phadning eng kichik qiymatini toping. $x^2 - 2x + 2y^2 + 8y + 9$

*A)0 B)8 C)1 D)9 E)-1*

**Yechish. 1-usul.** Ko'phaddan ikki hadning kvadratini ajratishga harakat qilaylik: $x^2 - 2x + 2y^2 + 8y + 9 = (x^2 - 2x + 1) + 2(y^2 + 4y + 4) = (x-1)^2 + 2(y+2)^2 \geq 0$ bo'ladi. Oxirgi natijadan berilgan ko'phadning eng kichik qiymati 0 bo'lib, bu esa $x = 1$ va $y = -2$ bo'lganda bo'lishligi kelib chiqadi.

**Javob.** 0 (A)

**2-usul.** Ko'phadni $x$ ga nisbatan kvadrat uchhad sifatida qaraylik, $x^2-2x+(2y^2+8y+9)$. Bu kvadrat uchhadning grafigi parabola bo'lib, $a=1>0$ bolganligi sababli tarmoqlari yuqoriga qaralgan. Parabola o'zining eng kichik qiymatiga uchining ordinatasiga erishadi. $x_0=-\frac{b}{2a}=-\frac{-2}{2\cdot 1}=1$, $x=1$ bo'lsa $1-2\cdot 1+2y^2+8y+9=2y^2+8y+8=2(y+2)^2\geq 0$

Oxirgi natijadan berilgan ko'phadning eng kichik qiymati 0 bo'lib, bu esa $x=1$ va $y=-2$ bo'lganda bo'lishligi kelib chiqadi.

**Javob.** 0 (A)

3(00-1-17). Ushbu $2x^2+2xy+2y^2+2x-2y+3$ ko'phad eng kichik qiymatga erishganda, $xy$ ning qiymati qanday bo'ladi?

*A)1 B)-2 C)2 D)1.5 E)-1*

**Yechish.** Berilgan ko'phaddan ikkihadning kvadratini ajrataylik. $2x^2+2xy+2y^2+2x-2y+3=(x^2+2x+1)+(x^2+2xy+y^2)+(y^2-2y+1)+1=$

$=(x+1)^2+(x+y)^2+(y-1)^2+1\geq 1$. Oxirgi natijadan ko'rinadiki, ko'phadning eng kichik qiymati 1 bo'lib, bu $x=-1$ va $y=1$ bo'lganda hosil bo'ladi. U holda $x\cdot y=-1\cdot 1=-1$

**Javob.** -1 (E)

4(97-7-63). 30 ta gugurt cho'pidan ularni sindirmay eng katta yuzali to'g'ri to'rtburchak yasalgan. Shu to'g'ri to'rtburchakning yuzini toping.

*A)64 B)62 C)56 D)52 E)49*

**Yechish.** To'g'ri to'rtburchakning tomonlari $a$ va $b$ bo'lsin. To'g'ri to'rtburchakning perimetri $P=2(a+b)=30$ dan $a+b=15$ kelib chiqadi. U holda uning yuzi $S=ab$ bo'ladi. Gugurt cho'plarini sindirmay degani masalani natural sonlar to'plamida yechimini top degani. $a=7$, $b=8$ yoki $a=8$, $b=7$ bo'lsa, $S=ab=56$ eng katta qiymatga erishadi.

**Javob.** 56 (C)

5(99-8-29) Agar $x^2+(\frac{x}{x-1})^2=8$ bo'lsa, $\frac{x^2}{x-1}$ ifodaning eng katta qiymatini toping. *A)4 B)8 C)2 D)16 E)* $\frac{1}{4}$

**Yechish. 1-usul.** Berilgan tenglikni shakl almashtirib quyidagi ko'rinishda yozaylik.

$x^2+2x\cdot\frac{x}{x-1}+\left(\frac{x}{x-1}\right)^2-2x\cdot\frac{x}{x-1}=8,\quad \left(x+\frac{x}{x-1}\right)^2-2x\cdot\frac{x}{x-1}-8=0,\quad \left(\frac{x^2}{x-1}\right)^2-2\cdot\frac{x^2}{x-1}-8=0$

$\frac{x^2}{x-1}=t$ deb belgilash kiritsak, $t^2-2t-8=0$ kvadrat tenglama hosil bo'ladi.

Viyet teoremasiga asosan $t_1 = -2$. $t_2 = 4$. U holda $\frac{x^2}{x-1}$ ning eng katta qiymati 4 ekanligi kelib chiqadi.

**Javob.** 4 (A)

**2-usul.** $A_2 \geq G_2$ tengsizlikni qo'llaymiz. $x^2 + (\frac{x}{x-1})^2 \geq 2\sqrt{x^2 \cdot (\frac{x}{x-1})^2} = 2\left|\frac{x^2}{x-1}\right|$

yoki $2\left|\frac{x^2}{x-1}\right| \leq 8$, $\left|\frac{x^2}{x-1}\right| \leq 4$. Oxirgi natijadan $\frac{x^2}{x-1}$ ning eng katta qiymati 4 ekanligi kelib chiqadi.

6(00-3-20). Ifodalarni taqqoslang. $p=a^2+b^2+c^2$, $q=ab+ac+bc$

*A)p<q B) p=q C) p>q D) p≤q E) p≥q*

**Yechish.** $p$ va $q$ ifodalarni 2 ga ko'paytirib, hosil bo'lgan ifodalarni ayiraylik. U holda

$2p-2q = 2a^2+2b^2+2c^2-2ab-2ac-2bc = (a-b)^2+(a-c)^2+(b-c)^2 \geq 0$, $2p-2q \geq 0$, $p \geq q$

**Javob.** $p \geq q$ (E)

7(98-4-18). Ushbu $tg^{100}x + ctg^{100}x$ yig'indining eng kichik qiymatini toping.

*A)4 B)0 C)2 D)1 E)100*

**Yechish.** Berilgan ifodadan ikkihadning kvadratini ajrataylik. $tg^{100}x + ctg^{100}x = (tg^{50}x - ctg^{50}x)^2 + 2 \geq 2$. Oxirgi natijadan berilgan ifodaning eng kichik qiymati 2 ekanligi ko'rinib turibdi.

**Javob.** 2 (C)

8(98-6-15). $x$, $x \in [0;\pi]$ ning qanday qiymatlarida $sin^2 x + \cos x$ funksiya o'zining eng katta qiymatiga erishadi? *A)0 B)$\frac{\pi}{3}$ C)$\frac{\pi}{6}$ D)$\frac{\pi}{4}$ E)$\frac{\pi}{12}$*

**Yechish.** Berilgan funksiyaning shaklini o'zgartiraylik. $y = sin^2 x + \cos x = 1 - \cos^2 x + \cos x = \frac{5}{4} - \left(\cos x - \frac{1}{2}\right)^2 \leq \frac{5}{4}$. Demak, bu funksiya o'zining eng katta qiymati $\frac{5}{4}$ ni $\cos x - \frac{1}{2} = 0$ bo'lganda erishadi. Bundan $\cos x = \frac{1}{2}$, $x = \frac{\pi}{3}$

**Javob.** $\frac{\pi}{3}$ (B)

9(99-10-28). Ifodaning eng kichik qiymatini toping.

$(1+\cos^2 2\alpha)(1+tg^2\alpha)+4\sin^2\alpha$

*A)2,5 B)1,5 C)2 D)3 E)3,5*

**Yechish.** Berilgan ifodani kosinus va sinusning darajasini pasaytirish formulasidan foydalanib soddalashtiraylik.

$$(1+\cos^2 2\alpha)(1+tg^2\alpha)+4\sin^2\alpha=\frac{1+\cos^2 2\alpha}{\cos^2\alpha}+4\cdot\frac{1-\cos 2\alpha}{2}=$$

$$=\frac{1+\cos^2 2\alpha}{\frac{1+\cos 2\alpha}{2}}+2(1-\cos 2\alpha)=\frac{2\cdot(1+\cos^2 2\alpha)}{1+\cos 2\alpha}+2\cdot(1-\cos 2\alpha)=\frac{4}{1+\cos 2\alpha}.$$

Oxirgi natijadan kasrning maxraji eng katta qiymatga erishganda ifoda eng kichik qiymatga erishishi ko'rinib turibdi.

$$\max(1+\cos 2\alpha)=1+1=2,\qquad \min=\frac{4}{2}=2$$

**Javob.** 2 (C)

10(02-3-27). $a_n=-3n^2+18n+1$ $(n\in N)$ formula bilan berilgan ketma-ketlikning nechanchi hadi eng katta qiymatga ega bo'ladi?

*A)3 B)2 C)6 D)8 E)5*

**Yechish.** Berilgan ketma-ketlikdan ikkihadning kvadratini ajrataylik. $a_n=-3n^2+18n+1=-3\cdot(n-3)^2+28$. Agar $-3(n-3)^2=0$ bo'lsa, berilgan ketma-ketlik o'zining eng katta qiymati 28 ga erishadi. Bu esa n=3 bo'lganda bo'ladi.

**Javob.** 3 (A)

11(02-7-21). $2^x+2^{\frac{2-x-y}{2}}+2^y$ ning eng kichik qiymatini aniqlang

*A)4 B)2 C)3 D)5 E)6*

**Yechish.** $A_2\ge G_2$ tengsizlikni ikki marta qo'llaylik. U holda

$$2^x+2^{\frac{2-x-y}{2}}+2^y=(2^x+2^y)+\frac{2}{\sqrt{2^{x+y}}}\ge 2\cdot\sqrt{2^{x+y}}+\frac{2}{\sqrt{2^{x+y}}}=2\left(\sqrt{2^{x+y}}+\frac{1}{\sqrt{2^{x+y}}}\right)\ge 2\cdot 2\sqrt{\sqrt{2^{x+y}}\cdot\frac{1}{\sqrt{2^{x+y}}}}=$$

=4. Demak, berilgan ifodaning eng kichik qiymati 4 ekan.

**Javob.** 4 (A)

12(02-3-55). Agar $5\le x\le y\le z\le t\le 320$ bo'lsa, $\frac{x}{y}+\frac{z}{t}$ ifodaning eng kichik qiymatini toping. A) 0,25 B) 0,5 C) 1,6 D) 0,16 E) topib bo'lmaydi

Yechish: $\frac{x}{y}+\frac{z}{t}$ ifoda o'zining eng kichik qiymatiga $y=z$ bo'lsa erishadi. $x$ ga eng kichik qiymat $t$ eng katta qiymat qabul qilganda bu hol ro'y beradi. $A_2\ge G_2$ tengsizlikka asosan $\frac{x}{y}+\frac{y}{t}\ge 2\sqrt{\frac{x}{y}\cdot\frac{y}{t}}=2\sqrt{\frac{x}{t}}$. Oxirgi natijada $x=5,\ t=320$ bo'lsa, ifodaning eng kichik qiymati $2\sqrt{\frac{x}{t}}=2\cdot\sqrt{\frac{5}{320}}=2\cdot\frac{1}{8}=0{,}25$ bo'ladi.

**Javob.** 0,25 (A)

13(00-5-67). $a$ ning qanday eng kichik qiymatida istalgan $ABC$ uchburchak uchun $cosA+cosB+cosC \le a$ tengsizlik hamisha o'rinli bo'ladi?

*A)1 B)2 C)$\frac{3}{2}$ D)$\frac{5}{2}$ E)3*

**Yechish.** $A+B+C=180^\circ$; $\frac{A+B}{2}=90^\circ-\frac{C}{2}$, $\cos\frac{A+B}{2}=\cos(90^\circ-\frac{C}{2})=\sin\frac{C}{2}$, $0\le\cos\frac{A-B}{2}\le 1$ bo'lganligi uchun $\cos\frac{A+B}{2}\cos\frac{A-B}{2}\le\sin\frac{C}{2}$ bo'ladi. U holda $cosA+cosB+cosC=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}+\cos C\le 2\sin\frac{C}{2}+\cos C$ bo'ladi. $2\sin\frac{C}{2}+\cos C$ ifodani qaraymiz. $2\sin\frac{C}{2}+\cos C=2\sin\frac{C}{2}+1-2\sin^2\frac{C}{2}$. $x=\sin\frac{C}{2}$ belgilash olaylik. U holda $2\sin\frac{C}{2}+1-2\sin^2\frac{C}{2}=-2x^2+2x+1$ bo'ladi. Endi $-2x^2+2x+1\le a$ tengsizlikni qaraymiz. $-2x^2+2x+1-a\le 0$ tengsizlikda $D\le 0$, $a<0$ bo'lsa barcha $x$ uchun bajariladi. $D=4-4\cdot(-2)\cdot(1-a)=12-8a$, $12-8a\le 0$ tengsizlikni qanoatlantiruvchi eng kichik $a$ ni topamiz. $8a\ge 12$, $a\ge\frac{3}{2}$. Oxirgi tengsizlikni qanoatlantiruvchi eng kichik $a$ soni $a=\frac{3}{2}$

**Javob.** $\frac{3}{2}$ (C)

14. $\frac{x^4+x^2+5}{(x^2+1)^2}$ ifodaning eng kichik qiymatini toping.

**Yechish.** Ifodani shakl almashtiraylik. $y=\frac{x^4+x^2+5}{(x^2+1)^2}=\frac{(x^2+1)^2-(x^2+1)+5}{(x^2+1)^2}=$ $=1-\frac{1}{x^2+1}+\frac{5}{(x^2+1)^2}$; $\frac{1}{1+x^2}=u$ deb belgilaylik. U holda holda $y=5u^2-u+1$ bo'ladi. Bu funksiya $u=-\frac{b}{2a}=-\frac{-1}{2\cdot 5}=\frac{1}{10}$ nuqtada eng kichik qiymatga erishadi. $y_{min}=\frac{19}{20}$. $\frac{1}{1+x^2}=\frac{1}{10}$ dan $x_1=-3$ va $x_2=3$ lar topiladi.

**Javob.** $y_{min}=\frac{19}{20}$

15. $f(x)=\frac{\sqrt[3]{(x^2+1)^2(x^2+3)}}{3x^2+4}$ funksiyaning eng katta qiymatini toping.

**Yechish.** $f(x)$ ni quyidagi ko'rinishda yozib olaylik. $\sqrt[3]{50}f(x)=\sqrt[3]{\frac{5x^2+5}{3x^2+4}\cdot\frac{5x^2+5}{3x^2+4}\cdot\frac{2(x^2+3)}{3x^2+4}}$. Ildiz ostidagi har bir ko'paytuvchi musbat bo'lganligi sababli $\sqrt[3]{abc}\le\frac{a+b+c}{3}$ tengsizlikni qo'llaymiz.

$\sqrt[3]{50}f(x)\le\frac{12x^2+16}{3(3x^2+4)}=\frac{4(3x^2+4)}{3(3x^2+4)}=\frac{4}{3}$. Demak, $\sqrt[3]{50}f(x)\le\frac{4}{3}$. Oxirgi tengsizlikda „=" belgisi $5x^2+5=2x^2+6$ bo'lsa erishadi. Bundan $x=\pm\frac{1}{\sqrt[3]{3}}$ ni topamiz.

**Javob.** $\max f(x)=\frac{4}{3\cdot\sqrt[3]{50}}$

## Mustaqil yechish uchun misollar

**16**(97-9-56). 18 ta go'gurt cho'pidan ularni sindirmay eng katta yuzali to'g'ri to'rtburchak yasalgan.Shu to'g'ri to'rtburchakning yuzini toping.
*A)16 B)20 C)24 D)28 E)30*

**17**(02-9-17). $2a^2-2ab+b^2-2a+2$ ning eng kichik qiymatini toping.
*A)-2 B)1 C)2 D)4 E)8*

**18**(00-10-53). Agar $16\le x\le y\le z\le t\le 100$ bo'lsa, $\frac{x}{y}+\frac{z}{t}$ ifodaning eng kichik qiymatini toping. A) 0,9 B) 200 C)0,8 D) 0,2 E) topib bo'lmaydi
**Javob.** (C)

**19**(02-1-15). Agar $25\le x\le y\le z\le t\le 64$ bo'lsa, $\frac{x}{y}+\frac{z}{t}$ ifodaning eng kichik qiymatini toping. A) 1,25 B) 1,6 C) $\frac{25}{32}$ D) 0,2 E) topib bo'lmaydi
**Javob.** (A)

**20**(02-2-15). Agar $7\le x\le y\le z\le t\le 112$ bo'lsa, $\frac{x}{y}+\frac{z}{t}$ ifodaning eng kichik qiymatini toping. A) 0,5 B) 0,2 C) 0,7 D) 0,8 E) topib bo'lmaydi
**Javob.** (A)

**21**(02-2-62). Agar $16\le x\le y\le z\le t\le 121$ bo'lsa, $\frac{x}{y}+\frac{z}{t}$ ifodaning eng kichik qiymatini toping. A) $\frac{8}{11}$ B) $\frac{11}{8}$ C) $\frac{4}{11}$ D) $\frac{2}{11}$ E) topib bo'lmaydi
**Javob.** (A)

**22**(02-1-7). Agar $9\le x\le y\le z\le t\le 81$ bo'lsa, $\frac{x}{y}+\frac{z}{t}$ ifodaning eng kichik qiymatini toping. A) $\frac{2}{3}$ B) $\frac{3}{2}$ C) $\frac{1}{5}$ D) $\frac{1}{3}$ E) topib bo'lmaydi

**23**(02-3-23). Agar $8\le x\le y\le z\le t\le 200$ bo'lsa, $\frac{x}{y}+\frac{z}{t}$ ifodaning eng kichik qiymatini toping. A)0,4 B)0,9 C)0,7 D)0,2 E) topib bo'lmaydi

## 11-§. O'zaro bog'liq bo'lmagan miqdorlarning eng katta va eng kichik qiymatlarini topish

Agar $x$ va $y$ lar, bir-biri bilan o'zaro bog'liq bo'lmagan chekli o'zgaruvchi miqdorlar bo'lsa, u holda quyidagi tasdiqlar o'rinli ekanligini anglash qiyin emas:

1) $x+y$ yig'indi o'zining eng katta qiymatiga, $x$ ham, $y$ ham eng katta qiymatga erishganda erishadi ;

2) $x+y$ yig'indi o'zining eng kichik qiymatiga, $x$ ham, $y$ ham eng kichik qiymatga erishganda erishadi ;

3) $x$-$y$ ayirma o'zining eng katta qiymatiga , $x$ eng katta qiymat va $y$ eng kichik qiymatga erishganda erishadi;

4) $x$-$y$ ayirma o'zining eng kichik qiymatiga, $x$ eng kichik qiymat va $y$ eng katta qiymatga erishganda erishadi;

5) $\frac{x}{y}$ nisbat, $y>0$ bo'lganda o'zining eng katta qiymatiga, $x$ eng katta qiymat va $y$ eng kichik qiymatga erishganda erishadi;

6) $\frac{x}{y}$ nisbat, $y>0$ bo'lganda o'zining eng kichik qiymatiga, $x$ eng kichik qiymat va $y$ eng katta qiymatga erishganda erishadi;

7) $z^2+m$ ifodaning eng kichik qiymati $m$ ga teng va bu qiymatga $z=0$ bo'lganda erishadi;

8) $z^2-m$ ifodaning eng kichik qiymati $-m$ ga teng va bu qiymatga $z=0$ bo'lganda erishadi;

9) $\pm m-z^2$ ifodaning eng katta qiymati $\pm m$ ga teng va bu qiymatga $z=0$ bo'lganda erishadi, bu yerda $m$ qandaydir o'zgarmas qiymat.

1(98-11-64). Agar $|a|\le 1$ , $|b|\le 1$ bo'lsa, *arccosa-4arcsinb* ifodaning eng katta qiymati qanchaga teng bo'ladi? *A)2π B)1 C)3π D)5π E)4π*

**Yechish.** Bizga ma'lumki, $0\le \arccos a\le \pi$ va $-\frac{\pi}{2}\le \arcsin b\le \frac{\pi}{2}$ tengsizliklar o'rinli. Berilgan ifoda $\arccos a$ eng katta qiymat, $\arcsin b$ eng kichik qiymat qabul qilganda eng katta qiymatni qabul qiladi. Bu qiymat $\max=\pi-4\cdot(-\frac{\pi}{2})=\pi+2\pi=3\pi$ ga teng.

**Javob.** $3\pi$ (C)

2(98-12-77). Ushbu $\frac{10}{x^2+8x+41}+\cos 5y$ ifodaning eng katta qiymati nechaga teng bo'lishi mumkin? *A)1,8 B)1,5 C)1,4 D)2 E)2,15*

**Yechish.** Berilgan ifodadagi birinchi qo'shiluvchining maxraji eng kichik qiymat va ikkinchi qo'shiluvchi eng katta qiymat qabul qilganada eng katta qiymat qabul qiladi. $y_1 = x^2 + 8x + 41$ kvadrad funksiyaning eng kichik qiymati $y_0 = -\frac{b^2 - 4ac}{4a} = -\frac{64 - 4 \cdot 41}{4} = 25$ ga, $\cos 5y$ ning eng katta qiymati 1 ga teng. U holda berilgan ifodaning eng katta qiymati $\frac{10}{25} + 1 = 1,4$ ga teng.

**Javob.** 1,4 (C)

3(99-5-30). Ifodaning eng katta qiymatini toping. $\frac{8\cos 2\alpha - 5\cos 3\beta}{7 + 2\cos 4\gamma}$

*A)2,2 B)2,3 C)2,4 D)2,5 E)2,6*

**Yechish.** Berilgan kasrning surati eng katta, maxraji eng kichik qiymat qabul qilganda ifoda o'zining eng katta qiymatini qabul qiladi. $\max(8\cos 2\alpha - 5\cos 3\beta) = 8 \cdot 1 - 5 \cdot (-1) = 13$

$\min(7 + 2\cos 4\alpha) = 7 + 2 \cdot (-1) = 5$. U holda berilgan ifodaning eng katta qiymati $\frac{13}{5} = 2,6$

**Javob.** 2,6 (E)

4(00-9-35). Ifodaning eng katta qiymatini toping. $\frac{5}{tg^2\alpha + ctg^2\alpha} + \frac{5\sin 2\alpha - \cos\gamma}{5 + \cos 3t}$

*A)5 B)2 C)3 D)6 E)4*

**Yechish.** Birinchi kasrning maxraji eng kichik qiymat, ikkinchi kasrning surati eng katta, maxraji eng kichik qiymat qabul qilganda ifoda eng katta qiymatni qabul qiladi. Bu qiymatlarni topamiz.

$tg^2\alpha + ctg^2\alpha = (tg\alpha - ctg\alpha)^2 + 2 \ge 2$, $\max(5\sin 2\alpha - \cos\gamma) =$

$= 5 \cdot 1 - (-1) = 6$, $\min(5 + \cos 3t) = 5 - 1 = 4$. U holda berilgan ifodaning eng katta qiymati $\frac{5}{2} + \frac{6}{4} = 2,5 + 1,5 = 4$.

**Javob.** 4 (E)

5(01-10-38). Ushbu $\frac{3\sin\alpha + 2}{5 + \cos\beta} + \frac{7}{tg^2\gamma + ctg^2\gamma}$ ifodaning eng katta qiymatini toping. *A)6,25 B)4,75 C)3,45 D)2,75 E)* aniqlab bo'lmaydi

**Yechish.** Birinchi kasrning surati eng katta, maxraji esa eng kichik va ikkinchi kasrning maxraji eng kichik qiymat qabul qilganda ifoda eng katta qiymat qabul qiladi.

$\max = \frac{3 \cdot 1 + 2}{5 - 1} + \frac{7}{2} = 1,25 + 3,5 = 4,75$

**Javob.** 4,75 (B)

6(02-6-39). $\frac{2\sin\alpha-1}{5-2\sin\beta}+\frac{tg^2\gamma+ctg^2\gamma}{2}$ ning eng kichik qiymatini toping.

*A)0* *B)1* *C)-1* *D)*$\frac{4}{7}$ *E)* aniqlab bo'lmaydi

**Yechish.** Berilgan ifodada birinchi kasrning surati eng katta, maxraji eng kichik va ikkinchi kasrning surati eng kichik qiymat qabul qilsa, ifoda eng kichik qiymat qabul qiladi. $\min=\frac{2\cdot(-1)-1}{5+2}+\frac{1+1}{2}=-\frac{3}{7}+1=\frac{4}{7}$

**Javob.** $\frac{4}{7}$ (D)

## Mustaqil yechish uchun misollar

**7-misol.** $2\arccos x+5\arcsin y$ ifodaning eng kichik qiymatini toping.

**8-misol.** $\frac{20}{x^2+2x+1}+2\sin 10y$ ifodaning eng katta qiymatini toping.

**9-misol.** Ifodaning eng katta qiymatini toping. $\frac{20\sin 2x+5\cos 10y}{\cos 8z-3}$.

**10-misol.** $\frac{8\cos x+3}{2+\sin y}+\frac{12}{tg^2z+ctg^2z}$ ifodaning eng katta qiymatini toping.

11(2007-var.106). $\frac{3\sin\alpha+2}{5+\cos\beta}+\frac{3}{tg^2\gamma+ctg^2\gamma}$ ifodaning eng katta qiymatini toping.

A) 4,75 B) 6,25 C) 2,75 D) 3,45

## 12-§. Parametrli tenglamalarni yechish

**1.** $Ax=B$ tenglamada 3 hol ro'y beradi.

a) $A\neq 0$ bo'lsa, tenglama bitta ildizga ega: $x=\frac{B}{A}$;

b) $\begin{cases}A=0\\B=0\end{cases}$ bo'lsa, tenglama cheksiz ko'p yechimga ega bo'ladi: $x\in R$;

v) $\begin{cases}A=0\\B\neq 0\end{cases}$ bo'lsa, tenglama ildizga ega bo'lmaydi: $\varnothing$

**2.** Keltirilgan kvadrat tenglamaning ildizlarining joylashuvi to'g'risida.

**1)** $x^2+px+q=0$ kvadrat tenglama ikkita musbat ildizga ega bo'lsa,

$\begin{cases}p^2-4q\geq 0\\p<0\\q>0\end{cases}$ bo'ladi.

2) $x^2+px+q=0$ kvadrat tenglama ikkita manfiy ildizga ega bo'lsa, $\begin{cases} p^2-4q\ge 0 \\ p>0 \\ q>0 \end{cases}$ bo'ladi.

3) $x^2+px+q=0$ kvadrat tenglama har biri biror $c$ sonidan katta bo'lgan ikkita ildizga ega bo'lsa, u holda $\begin{cases} p^2-4q\ge 0 \\ -\frac{p}{2}>c \\ c^2+pc+q>0 \end{cases}$ bo'ladi.

4) $x^2+px+q=0$ kvadrat tenglama har biri biror $c$ sonidan kichik bo'lgan ikkita ildizga ega bo'lsa, u holda $\begin{cases} p^2-4q\ge 0 \\ -\frac{p}{2}<c \\ c^2+pc+q>0 \end{cases}$ bo'ladi.

5) $x^2+px+q=0$ kvadrat tenglama biri biror $c$ sonidan kichik bo'lgan ikkinchisi $c$ sonidan katta bo'lgan ikkita ildizga ega bo'lsa, u holda $c^2+pc+q<0$ bo'ladi.

**1-misol.** $x^2-2(a-1)x+(2a+1)=0$ tenglama ikkita musbat ildizga ega bo'ladigan $a$ ning barcha qiymatlarini toping.

**Yechish.** 1) ga asosan : $\begin{cases} (-2(a-1))^2-4(2a+1)\ge 0 \\ -2(a-1)<0 \\ 2a+1>0 \end{cases}$ bo'lishi kerak. Bu sistemani yechamiz. $\begin{cases} a(a-4)\ge 0 \\ a-1>0 \\ 2a>-1 \end{cases}$ $\begin{cases} a\le 0, \quad a\ge 4 \\ a>1 \\ a>-\frac{1}{2} \end{cases}$ $\Rightarrow a\ge 4$

**Javob.** $a\ge 4$

**2-misol.** $2x^2-2(2a+1)x+a(a-1)=0$ tenglama $x_1<a<x_2$ shartni qanoatlantiruvchi ikkita $x_1$ va $x_2$ ildizga ega bo'ladigan $a$ ning barcha qiymatlarini toping.

**Yechish.** 5) ga asosan $a$ ning bunday qiymatlari $2a^2-2(2a+1)a+a(a-1)<0$ tengsizlikni qanoatlantirishi kerak. Bundan $-a^2-3a<0$, $a(a+3)>0$. $a<-3$ yoki $a>0$.

**Javob.** $a<-3$ yoki $a>0$.

**3-misol.** (03-9-5) $x^2-2ax+a^2-1=0$ tenglamaning ikkala ildizi $-2$ va $4$ orasida joylashgan bo'lsa, $a$ ning qiymati qaysi oraliqqa o'zgaradi.

*A) (-3;3) B) (-1;5) C)(-3;-1)∪(3;5) D)(-1;3) E) (0;3)*

**Yechish.** $\begin{cases} D \geq 0 \\ x_0 = -\dfrac{b}{2a} > -2 \\ x_0 = -\dfrac{b}{2a} < 4 \\ (-2)^2 - 2a(-2) + a^2 - 1 > 0 \\ 4^2 - 2a \cdot 4 + a^2 - 1 > 0 \end{cases}$ $\begin{cases} 4a^2 - 4(a^2 - 1) \geq 0 \\ a > -2 \\ a < 4 \\ a^2 + 4a + 3 > 0 \\ a^2 - 8a + 15 > 0 \end{cases}$ $\begin{cases} -2 < a < 4 \\ (-\infty; -3) \cup (-1; \infty) \\ (-\infty; 3) \cup (5; \infty) \end{cases}$.

Oxirgi sistemadan $a \in (-1;3)$ kelib chiqadi.

**Javob.** $a \in (-1;3)$ (D)

4(99-3-12). $n$ ning qanday qiymatlarida $4x^2 - 3nx + 36 = 0$ tenglama ikkita manfiy ildizga ega? *A)*$|n| \geq 8$ *B)*$n \leq -8$ *C)n<8* *D)n<-8* *E)n>8*

**Yechish.** $\begin{cases} D > 0 \\ p > 0 \\ q > 0 \end{cases}$ bo'lsa, kvadrat tenglama ikkita manfiy ildizga ega.

$\begin{cases} 9n^2 - 4 \cdot 4 \cdot 36 > 0 \\ -\dfrac{3n}{4} > 0 \\ \dfrac{36}{4} > 0 \end{cases}$ $\begin{cases} n^2 - 16 \cdot 4 > 0 \\ n < 0, \end{cases}$ $\begin{cases} |n| > 8 \\ n < 0, \end{cases}$ $-n > 8,$ $n < -8$

**Javob.** $n < -8$ (D)

5(03-2-1). $a$ ning qanday haqiqiy qiymatlarida $x^4 + a = x^2 + a^2$ tenglama uchta turli haqiqiy ildizlarga ega bo'ladi?

*A)(0;4)* *B)2* *C)0 va 1* *D)*$[0;1]$ *E)0*

**Yechish.** Tenglamani quyidagi ko'rinishda yozib olaylik. $x^4 - x^2 + (a - a^2) = 0$ $x^2 = t$ belgilash kiritsak, $t^2 - t + (a - a^2) = 0$ kvadrat tenglama hosil bo'ladi. Agar bu kvadrat tenglamaning ilgizidan biri 0, ikkinchisi musbat bo'lsa, bikvadrat tenglama uchta haqiqiy ildizga ega bo'ladi. Bu esa kvadrat tenglamaning ozod hadi 0 ga teng bo'lganda bo'ladi. $a - a^2 = 0$ dan $a = 0$ yoki $a = 1$ hosil bo'ladi.

**Javob.** $a = 0$ yoki $a = 1$ (C)

6(02-1-22). $m$ ning qanday qiymatlarida $x^2 + mx + 8 = 0$ va $x^2 + x + m = 0$ tenglamalar umumiy yechimga ega bo'ladi?

*A)-6* *B)-7* *C)9* *D)-5* *E)5*

**Yechish.** $\begin{cases} x^2 + mx + 8 = 0 \\ x^2 + x + m = 0 \end{cases}$ $\begin{cases} x^2 + (-x - x^2)x + 8 = 0 \\ m = -x - x^2 \end{cases}$ $\begin{cases} x = 2 \\ m = -2 - 4 = -6 \end{cases}$

**Javob.** $m = -6$

7(99-5-29). Tenglama ildizga ega bo'ladigan $a$ ning barcha qiymatlarini ko'rsating. $\sin^4 x+\cos^4 x=a\sin x\cdot\cos x$ *A)* $[1;\infty)$ *B)* $[-1;1]$ *C)* $[1;5]$ *D)* $(-\infty;-1]\cup[1;\infty)$ *E)* $[-3;-1]\cup[1;3]$

**Yechish.** **1-usul.** Berilgan tenglamani $(\sin^2 x-\cos^2 x)^2+2\sin^2 x\cos^2 x=a\sin x\cos x$ yoki $2\cos^2 2x+\sin^2 2x=a\sin 2x$ ko'rinishida yozamiz. Bu tenglamani $\cos^2 2x+1=a\sin 2x$ ko'rinishida yozish mumkin. Tenglamani chap tomoni 1 dan kichik emas, demak, $a\sin 2x\geq 1$ bo'lishi kerak. Bundan $a>0$ va $a<0$ hollarni alohida qaraymiz.

**1-hol.** $a>0$ bo'lsin. U holda $\begin{cases}\sin 2x\geq\dfrac{1}{a}\\ \sin 2x\geq 0\end{cases}\Rightarrow a\in[1;\infty)$

**2-hol.** $a<0$ bo'lsin. U holda $\begin{cases}\sin 2x\leq\dfrac{1}{a}\\ \sin 2x\leq 0\end{cases}\Rightarrow a\in(-\infty;-1]$

**Javob.** $(-\infty;-1]\cup[1;\infty)$ (D)

**2-usul.** Tenglamani $(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x=a\sin x\cos x$ yoki $2-\sin^2 2x=a\sin 2x$ ko'rinishda yozib, $\sin^2 2x+a\sin 2x-2=0$ tenglamaga kelamiz. Bundan $\sin 2x=\dfrac{-a\pm\sqrt{a^2+8}}{2}$. $\sin 2x$ ning chegaralanganligini hisobga olsak, quyidagi tengsizliklarni hosil qilamiz:

$-1\leq\dfrac{-a-\sqrt{a^2+8}}{2}\leq 1$, $-1\leq\dfrac{-a+\sqrt{a^2+8}}{2}\leq 1$. Bu tengsizliklarni alohida-alohida yechalik. $\begin{cases}-a-\sqrt{a^2+8}\leq 2\\ -a-\sqrt{a^2+8}\geq -2\end{cases}\Leftrightarrow\begin{cases}\sqrt{a^2+8}\geq -a-2\\ \sqrt{a^2+8}\leq 2-a\end{cases}\Leftrightarrow$

$$\Leftrightarrow\begin{cases}a^2+8\geq a^2+4a+4\\ -a-2\geq 0\\ a^2+8\leq 4-4a+a^2\\ 2-a\geq 0\end{cases}\cup\begin{cases}a^2+8\geq a^2+4a+4\\ -a-2\leq 0\\ a^2+8\leq 4-4a+a^2\\ 2-a\geq 0\end{cases}\Rightarrow\begin{cases}a\leq 1\\ a\leq -2\\ a\leq -1\\ a\leq 2\end{cases}\cup\begin{cases}a\leq 1\\ a\geq -2\\ a\leq -1\\ a\leq 2\end{cases}\Leftrightarrow a\leq -2\cup -2\leq a\leq -1$$

Demak, $a\in(-\infty;-1]$. Xuddi shunday ikkinchi tengsizlikni yechib, $a\in[1;\infty)$ bo'lishini topamiz.

**Javob.** $(-\infty;-1]\cup[1;\infty)$ (D)

## Mustaqil yechish uchun misollar

8(99-9-6). $t$ ning qanday qiymatlarida $x^2+(t-2)x+0{,}25=0$ tenglamaning ikkita ildizi ham manfiy bo'ladi? *A)* $t<2$ *B)* $t<1$ *C)* $t>2$ *D)* $t\leq 1$ *E)* $t>3$

9(02-3-78). $cos^2 x + 6\sin x = 4a^2 - 2$ tenglama $a$ ning qanday qiymatlarida yechimga ega bo'ladi ? *A)* $[-\sqrt{2};\sqrt{2}]$ *B)* $[0;\sqrt{2}]$ *C)*$[0;2)$ *D)(- 2;2)* *E)*$[1;0)$

10(00-3-11). $k$ ning qanday qiymatida $k(k+6)x = k+7(x+1)$ tenglama yechimga ega bo'lmaydi? A) 1 va 7 B) 1 C) 7 D) 1 va -7 E)-7

11(98-3-14). Ushbu $(k^2 - 4k + 2)x = k - x - 3$ yoki $(k+2)x - 1 = k + x$ tenglamalardan biri cheksiz ko'p yechimga ega bo'ladigan $k$ ning nechta qiymati mavjud.

A)0 B)1 C) 2 D) 3 E) cheksiz ko'p

13(00-8-14). $k$ ning qanday qiymatlarida $x^2 + 2(k-9)x + k^2 + 3k + 4$ ifodani to'la kvadrat shaklida tasvirlab bo'lmaydi. A) $\frac{11}{3}$ B) 3 C) 4 D) $\frac{5}{7}$ E) $\frac{7}{9}$

13(98-6-21). Ushbu $x^2 + ax - 2 = 0$ *va* $x^3 + ax^2 - 2 = 0$ tenglamalar umumiy ildizga ega bo'lsa $a$ ni toping.

*A)1* *B)2* *C)1,5* *D)3* *E)-1*

14(03-12-61). $a$ parametrning qanday qiymatlarida $\sin^6 x + \cos^6 x = a$ tenglama yechimga ega?

A)$[0;1]$ B)$[0,5;1]$ C)$[0,25;0,5]$ D)$[0,25;1]$ E)$[0,25;0,75]$

15(98-12-75). Ushbu $1 + a\cos x = (a+1)^2$ tenglama hech bo'lmaganda bitta yechimga ega bo'ladigan $a$ ning nechta butun qiymati mavjud?

A)4 B)3 C)5 D)2 E)1

16(00-4-1). $a$ ning qanday qiymatlarida $\sin^4 x + \cos^4 x = a$ tenglama yechimga ega?

A)$[0,5;1]$ B)$[0;0,5]$ C)$[0,5;\infty)$ D)$(-\infty;1]$ E)$[0;1]$

17(00-9-36). Ushbu $a(\sin^6 x + \cos^6 x) = \sin^4 x + \cos^4 x$ tenglama ildizga ega bo'ladigan $a$ ning barcha qiymatlarini ko'rsating.

A)$[-1;1]$ B)$[0;1]$ C)$[1;2]$ D)$[1;1,5]$ E)$[1;2,5]$

18(03-7-73). $a$ parametrning qanday qiymatlarida $7\sin x - 5\cos x = a$ tenglama yechimga ega bo'ladi.

A)$-1 \le a \le 1$ B)$-\sqrt{24} \le a \le \sqrt{24}$ C)$0 \le a \le 1$ D)$2 \le a \le 12$ E)$-\sqrt{74} \le a \le \sqrt{74}$

## 13-§.Davriy funksiyalar

**Ta'rif.** Agar shunday $T \neq 0$ son mavjud bo'lsaki, $y = f(x)$ funksiyaning aniqlanish sohasidagi istalgan $x$ uchun $f(x-T) = f(x) = f(x+T)$ tenglik bajarilsa, $f(x)$ funksiya davriy funksiya deyiladi. T son $f(x)$ funksiyaning davri deyiladi.

Bizga ma'lumki, $y = \sin x,\ y = \cos x,\ y = tgx,\ y = ctgx$ funksiyalar uchun $\sin(x - 2\pi) = \sin(x + 2\pi) = \sin x,\ \ \cos(x - 2\pi) = \cos(x + 2\pi) = \cos x,$ $tg(x - \pi) = tg(x + \pi) = tgx,\ \ ctg(x - \pi) = ctg(x + \pi) = ctgx$ tengliklar bajarilganligi sababli $y = \sin x,\ y = \cos x$ funksiyalar $T = 2\pi$ davrli, $y = tgx,\ y = ctgx$ funksiyalar $T = \pi$ davrli funksiyalardir.

**Teorema.** Agar T son $y = f(x)$ funksiyaning davri bo'lsa, $n \neq 0$ butun son uchun $n \cdot T$ sonlar ham shu funksiyaning davri bo'ladi.

Bu teoremadan ko'rinadiki, berilgan funksiyaning davrlari cheksiz ko'p bo'ladi. Masalan, $y = \sin x$ funksiyani davri $T = 2\pi$ bo'lganligi sababli bu funksiyaning davri cheksiz bo'lib, $\pm 2\pi;\ \ \pm 4\pi;\ \ \pm 6\pi...\ \ \pm 2\pi n...$ sonlar ham uning davri bo'ladi. Bizga maktab darsliklaridan $y = \sin x,\ y = \cos x,\ y = tgx,\ y = ctgx$ funksiyalar uchun quyidagi tengliklar bajarilishi ma'lum: $\sin(x + 2\pi n) = \sin x,\ \cos(x + 2\pi n) = \cos x,\ tg(x + \pi n) = tgx,$ $ctg(x + \pi n) = ctgx,\ n \in Z$. Bu esa $y = \sin x,\ y = \cos x$ funksiyalarning davrlari $2\pi n$, $y = tgx,\ y = ctgx$ funksiyalarning davrlari $\pi n$ ekanligini bildiradi.

Agar bu davrlar ichida eng kichik musbat davri mavjud bo'lsa, u holda bu davr berilgan funksiyaning asosiy davri yoki eng kichik davri deb ataladi. Shuning uchun berilgan funksiyani eng kichik musbat davrini topish yetarli. Lekin, davriy funksiyalar eng kichik musbat davrga ega bo'lmasligi ham mumkin.

Masalan,

1) $f(x) = C = const$ funksiya davriy funksiya, chunki $T \neq 0$ son uchun $f(x+T) = C,\ \ f(x) = C\ \ f(x+T) = f(x)$ tenglik bajarilmoqda. Musbat sonlar orasida eng kichigi mavjud emas, shuning uchun uning eng kichik musbat davrini topib bo'lmaydi.

2) Dirixle funksiyasi deb ataluvchi

$$f(x) = \begin{cases} 1, & agar\ \ x\ \ ratsional\ \ son\ \ bo'lsa, \\ 0, & agar\ \ x\ \ irratsional\ \ son\ \ bo'lsa. \end{cases}$$

funksiya davriy funksiya bo'lib, asosiy davrga ega emasdir. Haqiqatdan ixtiyoriy T ratsional son bu funksiya uchun davr bo'ladi.

$$f(x+T)=\begin{cases}1, & agar \quad x+T \quad ratsional \quad son \quad bo'lsa,\\ 0, & agar \quad x+T \quad irratsional \quad son \quad bo'lsa.\end{cases}$$

Bizga ma'lumki, $x$ va $T$ ratsional son bo'lsa, $x+T$ ratsional son bo'ladi. $x$ irratsional son bo'lsa, $x+T$ irratsional son bo'ladi. Demak, $f(x)=f(x+T)$ tenglik bajarilmoqda, ya'ni ixtiyoriy ratsional son bu funksiyaning davri bo'adi. Lekin musbat ratsional sonlar ichida eng kichigi yo'qdir. Shuning uchun Dirixle funksiyasi eng kichik musbat davrga ega emas.

Maktab darsligida berilgan funksiyani davriy ekanligini isbotlash uchun o'quvchi ta'rifni bajarilishini osongina tekshira oladi. Agar davri berilmagan bo'lsa, ko'pincha uning davri tanlash yo'li bilan aniqlanib, keyin ta'rif tekshirib ko'riladi. Biz quyida tajribadan kelib chiqib ayrim funksiyalarning davrini topishni o'rgatamiz.

Agar davriy funksiyaning eng kichik musbat davri topilsa, uning grafigini yasash osonlashadi. Uzunligi davrga teng istalgan oraliqda uning grafigini chizib, bu grafikni OX o'qi bo'ylab o'ngga va chapga istalgancha parallel ko'chirish yetarli.

**Teorema.** Agar T soni $y=f(x)$ funksiyaning asosiy davri bo'lsa, u holda $y=Af(kx+b)$, (bunda $A\neq 0$, $k\neq 0$, $b$ sonlar) funksiyaning asosiy davri $\frac{T}{|k|}$ bo'ladi.

Bu teoremadan ko'rinadiki, davriy funksiyaning grafigini

1) OX va OY o'qlari bo'ylab parallel ko'chirish,

2) OX va OY oqi bo'ylab cho'zish yoki qisish,

asosiy davrini o'zgartirmaydi. Bu teoremadan foydalanish ayrim misollarni yechishni osonlashtiradi.

**1-misol.** $y=2\sin(2x+3)$ funksiyaning eng kichik musbat davrini toping.

**Yechish:** $y=\sin x$ ni davri $T=2\pi$ bo'lganligi sababli, $y=2\sin(2x+3)$ ni davri $T_1=\frac{2\pi}{2}=\pi$ bo'ladi.

**2-misol.** $f(x)=\frac{1}{3}tg(-\frac{1}{2}x-3)$ funksiyani asosiy davrini toping.

**Yechish:** $T=\frac{\pi}{\left|-\frac{1}{2}\right|}=2\pi$

Yuqoridagi teoremadan :

**1)** $y=A\sin(kx+b)$ va $y=A\cos(kx+b)$ lar uchun eng kichik musbat davr $T=\frac{2\pi}{|k|}$

3) $y = Atg(kx+b)$ va $y = Actg(kx+b)$ lar uchun eng kichik musbat davr $T = \frac{\pi}{|k|}$

4) $y = A\{kx+b\}$ funksiya uchun eng kichik musbat davr $T = \frac{1}{|k|}$ kelib chiqadi.

Agar $g(x)$ funksiya T davrli funksiya bo'lsa, u holda $F(x) = f(g(x))$ funksiya ham T davrli bo'ladi.

**Eslatma.** $y = f(g(x))$ murakkab funksiyada $u = g(x)$ davriy funksiya bo'lib, uning eng kichik musbat davri T bo'lsa, u holda T soni $y = f(g(x))$ murakkab funksiyaning davri bo'ladi. Lekin umuman olganda uning asosiy davri T dan kichik bo'lishi mumkin.

Masalan, $y = \sin^2 x$ funksiya $y = u^2$ va $u = \sin x$ funksiyalardan tuzilgan murakkab funksiyadir. $u = \sin x$ asosiy davri $2\pi$ bo'lib, $y = \sin^2 x$ asosiy davri $y = \sin^2 x = \frac{1-\cos 2x}{2}$

tenglikdan $T = \frac{2\pi}{2} = \pi$ ekanligi kelib chiqadi.

Agar $g(x)$ funksiya davriy funksiya bo'lib, $f(x)$ esa istalgan funksiya bo'lganda juda ko'p davriy funksiyalarni $f(g(x))$ formula bilan tuzish mumkin.

Masalan, $\sin^3 2x$; $\sqrt{tg\frac{x}{2}}$; $\log_2 \cos(x-4)$; $\arcsin(\cos x)$; $\ln\sqrt{4-tg^2(x+\frac{\pi}{3})}$ - funksiyalar davriy funksiyalardir.

Berilgan funksiyani davriy emasligini isbotlash ba'zan qiyinchiliklarga olib keladi. Bu qiyinchiliklarni bartaraf qilish uchun quyidagi mantiqan to'g'ri fikrdan foydalanamiz.

Agar barcha davriy funksiyalar biror xossaga ega bo'lsa, berilgan bu funksiya ushbu xossaga ega bo'lmasa, u holda bu funksiya davriy funksiya bo'lmaydi. Shuning uchun davriy funksiyalarning xossalarni bilish bunga yordam beradi.

## 1. Davriy funksiyalarning xossalari

1-xossa. Agar $x_0$ nuqta T davrli davriy funksiyaning aniqlanish sohasiga tegishli bo'lsa, u holda uning aniqlanish sohasiga barcha $x_0 + nT$, (bunda $n \in Z$) nuqtalar ham tegishli bo'ladi.

**Natija.** Davriy funksiyalarning aniqlanish sohasi juda katta musbat va juda kichik manfiy sonlarni o'z ichiga oladi.

Bundan $y=\log_a x$ funksiyani davriy emasligi kelib chiqadi, chunki uning aniqlanish sohasi $(0;\infty)$ dan iborat bo'lib, manfiy sonlarni o'z ichiga olmaydi. $y=\arccos x$ funksiya davriy emas, chunki uning aniqlanish sohasi $[-1;1]$ dan iborat bo'lib, 1 dan katta va -1 dan kichik sonlarni ichiga olmaydi.

**2-xossa.** Davriy funksiya o'zining har bir qiymatini argumentning cheksiz ko'p qiymatlarida qabul qiladi. Bu argumentning qiymatlari orasida juda katta musbat va juda kichik manfiy sonlar mavjuddir.

**Natija.** Davriy funksiya o'zining aniqlanish sohasida o'suvchi yoki kamayuvchi bo'la olmaydi.

Masalan, $y=a^x$ ko'rsatkichli funksiya, $y=kx+b$ *(bunda* $k\neq 0$) chiziqli funksiya davriy funksiya bo'la olmaydi.

2-xossani quyidagi ko'rinishda ham foydalansa bo'ladi. Agar $y=f(x)$ davriy funksiya bo'lsa, u holda istalgan $a$ haqiqiy soni uchun $f(x)=a$ tenglama yo ildizga ega bo'lmaydi, yoki cheksiz ko'p ildizga ega bo'ladi.

Masalan,

1) $y=2x-\cos x$ funksiya davriy funksiya emasdir, chunki uning hosilasi $y'=2+\sin x>0$ bo'lganligi sababli u o'suvchi funksiyadir.

2) $y=\dfrac{3x^2-x+2}{x^2+x+1}$ funksiya davriy emasdir, chunki $\dfrac{3x^2-x+2}{x^2+x+1}=a$ tenglama ikkitadan ortiq ildizga ega emas.

**3-xossa.** Agar $y=f(x)$ funksiya T davrli davriy funksiya bo'lsa, u holda $f(x+T)=f(x)$,

bunda T – noma'lum son, $x$- parametr, tenglama aniqlanish sohasidagi barcha $x$ uchun hech bo'lmaganda bitta 0 dan farqli $T=T_0$ yechimga ega bo'ladi.

3-xossadan $y=f(x)$ funksiyani davriy emasligini isbotlash uchun argumentning shunday $x=a$ va $x=b$ qiymatlarini ko'rsatish yetarliki, T ga nisbatan $f(a+T)=f(a)$ va $f(b+T)=f(b)$ tenglamalar umumiy 0 dan farqli yechimga ega bo'lmasligi lozimdir.

Masalan, $y=\{x\}+\sin x$ funksiya davriy funksiyami?

Yechish: Faraz qilaylik bu funksiya T davrli davriy funksiya bo'lsin. U holda istalgan $x\in Z$ uchun $\{x+T\}+\sin(x+T)=\{x\}+\sin x$ tenglik o'rinli bo'ladi.Xususiy holda $x=0$ bo'lsa, $\{0+T\}+\sin(0+T)=\{0\}+\sin 0$ yoki $\{T\}+\sin T=0$ tenglama hosil bo'ladi. $x=-T$ bo'lsa, $\{-T\}-\sin T=0$ tenglama hosil bo'ladi. $\begin{cases}\{T\}+\sin T=0\\ \{-T\}-\sin T=0\end{cases}$

Bu tenglamalarni qo'shsak, $\{T\}+\{-T\}=0$

Bizga ma'lumki, $x$ ning kasr qismi manfiy bo'la olmaydi. Shuning uchun oxirgi tenglik $\{T\}=\{-T\}=0$ bo'lsa o'rinli bo'ladi. Bundan $T=0$ kelib chiqadi. Shuning uchun sistema yagona T=0 umumiy ildizga ega ekan. Bundan $y=f(x)$ funksiya nodavriy funksiya ekanligi kelib chiqadi.

**4-xossa.** Agar T davrli $y=f(x)$ funksiya aniqlanish sohasidagi uzunligi T ga teng bo'lgan $[a;a+T]$ kesmalar chegaralangan bo'lsa, ya'ni $|f(x)|\le M$ tengsizlik bajarilsa, u holda bu funksiya aniqlanish sohasidagi barcha $x$ uchun chegaralangan bo'ladi. Bundan uzluksiz funksiya aniqlanish sohasidagi barcha nuqtalarda chegaralanmagan bo'lsa, u davriy funksiya bo'lmasligi kelib chiqadi.

Masalan, $y=x^2\cos x$ funksiyaning davriy emasligini isbotlang.

**Yechish.** Bu funksiya barcha $x\in R$ da uzluksiz. Bu funksiyaning chegaralanmaganligini ko'rsatamiz. Ixtiyoriy M>0 soni uchun $x_0=(2+2[M])\pi$ sonini olsak, u holda $\left|x_0^2\cos x_0\right|=\pi^2(2+2[M])^2\cos(2\pi+2[M]\pi)=\pi^2(2+2[M])^2>M$. Demak, $y=f(x)$ funksiya chegaralanmagan ekan. Shuning uchun u davriy bo'la olmaydi.

**5-xossa.** Davriy funksiya o'zining aniqlanish sohasida chekli miqdordagi uzilish nuqtasiga ega bo'la olmaydi.

Masalan, $f(x)=10^{\frac{1}{x(x+4)}}$ funksiya ikkita uzilish nuqtasi $x=0$ va $x=-4$ ga ega. Shuning uchun bu davriy bo'la olmaydi.

**6-xossa.** $f_1(x)$ va $f_2(x)$ funksiyalar sonlar to'g'ri chizig'ining barcha nuqtalarida aniqlangan davriy funksiyalar bo'lsin. Agar $T_1>0$ soni $f_1(x)$ funksiyaning, $T_2>0$ soni $f_2(x)$ funksiyaning davri bo'lib, $\frac{T_1}{T_2}$ son ratsional sondan iborat bo'lsa, $f_1(x)+f_2(x)$ funksiya davriy funksiya bo'ladi.

Misol. $f(x)=\cos3\pi x+\sin2\pi x$ funksiyani davriy funksiya ekanligini isbotlang.

**Yechish.** $y_1=\cos3\pi x$ funksiyaning davri $T_1=\frac{2\pi}{3\pi}=\frac{2}{3}$, $y_2=\sin2\pi x$ funksiyaning davri $T_2=\frac{2\pi}{2\pi}=1$, $\frac{T_1}{T_2}=\frac{2}{3}$ ratsional son bo'lganligi sababli 6-xossaga asosan $f(x)$ funksiya davriy funksiya bo'ladi. Uning davri $T=3\cdot T_1=2\cdot T_2=2$ soni bo'ladi.

**7-xossa.** Agar $f(x)$ davriy funksiya differensiyalanuvchi funksiya bo'lsa, uning hosilasi ham shu davrli davriy funksiya bo'ladi.

## 2.Misollar yechish namunalari

**1(96-9-48).** Ushbu $y = tg\frac{x}{3} - 3\sin\frac{x}{2} + 3\cos\frac{2}{3}x$ funksiyaning eng kichik davrini toping.

A) $4\pi$ B) $6\pi$ C) $3\pi$ D) $12\pi$ E) $15\pi$

**Yechish.** $y_1 = tg\frac{x}{3}$ uchun $T_1 = \frac{\pi}{\frac{1}{3}} = 3\pi$. $y_2 = \sin\frac{x}{2}$ uchun $T_2 = \frac{2\pi}{\frac{1}{2}} = 4\pi$.

$y_3 = 3\cos\frac{2}{3}x$ uchun $T_3 = \frac{2\pi}{\frac{2}{3}} = 3\pi$.

U holda T=EKUK($T_1, T_2 T_3$)=EKUK($3\pi, 4\pi, 3\pi$)=$\pi$EKUK(3,4,3)= $12\pi$ son uning eng kichik davri bo'ladi.

**Javob.** $12\pi$ (D)

**2(99-3-31).** Funksiyaning eng kichik musbat davrini toping.

$y = 2\sin\frac{\pi x}{3} + 3\cos\frac{\pi x}{4} - tg\frac{\pi x}{2}$

A) 12 B) $12\pi$ C) $2\pi$ D) $24\pi$ E) 24

**Yechish.** $y_1 = 2\sin\frac{\pi x}{3}$ uchun $T_1 = \frac{2\pi}{\frac{\pi}{3}} = 6$. $y_2 = 3\cos\frac{\pi x}{3}$ uchun $T_2 = \frac{2\pi}{\frac{\pi}{4}} = 8$.

$y_3 = tg\frac{\pi x}{3}$ uchun $T_3 = \frac{\pi}{\frac{\pi}{2}} = 2$. Berilgan funksiyaning eng kichik musbat davri

T=EKUK(6,8,2)=24

**Javob.** 24 (E)

**3(03-10-43).** $y = \sin^6 x + \cos^6 x$ funksiyaning eng kichik musbat davrini aniqlang.

A) $2\pi$ B) $\pi$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{4}$ E) $\frac{\pi}{3}$

**Yechish.** Funksiyani soddalashtiraylik.

$y = \sin^6 x + \cos^6 x = (\sin^2 x + \cos^2 x)(\sin^4 x + \sin^2 x\cos^2 x + \cos^4 x) = (\sin^2 x + \cos^2 x)^2 - \sin^2 x\cos^2 x$

$= 1 - \frac{1}{4}\sin^2 2x = 1 - \frac{1}{4}\cdot\frac{1-\cos 4x}{2} = \frac{1}{2} + \frac{1}{8}\cos 4x$. Funksiyaning eng kichik davri

T=$\frac{2\pi}{4} = \frac{\pi}{2}$

**Javob.** $\frac{\pi}{2}$

**Misol.** $f(x)=\sqrt[4]{\log_5 \cos\frac{2\pi x}{\sqrt{13}}}$ funksiyani davriy funksiya ekanligin isbotlang va uning davrlaridan birin toping.

**Yechish.** Funksiyani aniqlanish sohasini topaylik.

$\log_5 \cos\frac{2\pi x}{\sqrt{13}} \geq 0 \Leftrightarrow \cos\frac{2\pi x}{\sqrt{13}} \geq 1 \Leftrightarrow \cos\frac{2\pi x}{\sqrt{13}} = 1 \Leftrightarrow x=\sqrt{13}n, \quad n\in Z$.

Shunday qilib, $f(x)$ funksiyaning aniqlanish sohasi $D=\{\sqrt{13}n | n\in Z\}$ dan iborat.

Agar $T=\sqrt{13}$ deb olsak, istalgan $n$ butun son uchun $\sqrt{13}n+T=\sqrt{13}(n+1)\in D$ va $\sqrt{13}n-T=\sqrt{13}(n-1)\in D$.

Shunday qilib, aniqlanish sohasiga tegishli istalgan $x$ soni va istalgan $n$ butun soni uchun quyidagi tenglik o'rinli.

$\cos\frac{2\pi(\sqrt{13}n+\sqrt{13})}{\sqrt{13}} = \cos 2\pi(n+1) = 1 = \cos 2\pi n = \cos\frac{2\pi(\sqrt{13}n)}{\sqrt{13}}$, $\sqrt[4]{\log_5 1}=0$, ya'ni $f(x+\sqrt{13})=f(x)$, shuning uchun $f(x)$ funksiya $\sqrt{13}$ davrli davriy funksiyadir.

5(00-5-43). $y=\sin|x|$ funksiyaning eng kichik davrini ko'rsating.

A) $2\pi$ B) $\pi$ C) davriy emas D) $\frac{\pi}{2}$ E) $3\pi$

**Yechish.** Faraz qilaylik T soni berilgan funksitaning davrlaridan biri bo'lsin. T>0 deb olish mumkin(aks holda –T>0 davrni qarash kerak). $y(\frac{\pi}{2})=1$ bo'lganligi sababli $y(\frac{\pi}{2}+T)=1$ bo'ladi. $\frac{\pi}{2}$ va $\frac{\pi}{2}+T$ sonlar musbat bo'lganligi sababli $\frac{\pi}{2}+T=\frac{\pi}{2}+2\pi n$ bo'ladi( $y=\sin x$, $x>0$ bo'lganda bo'ladi), ya'ni T soni $2\pi n$ ko'rinishga egadir. Bunda $n$ qandaydir natural son. U holda $y(-\frac{\pi}{2}+T)=y(-\frac{\pi}{2})=$

$=\sin(\frac{\pi}{2})=1$ bajarilishi lozim. Bu esa no'tog'ridir, chunki istalgan natural $n$ soni uchun $y(-\frac{\pi}{2}+2\pi n)=\sin\left|2\pi n-\frac{\pi}{2}\right|=\sin(2\pi n-\frac{\pi}{2})=-1$. Demak, berilgan funksiya davriy emas ekan.

**Javob.** davriy emas (C)

## Mustaqil yechish uchun masalalar

6(96-13-14). Ushbu $y = ctg\frac{x}{3} + tg\frac{x}{2}$ funksiyaning eng kichik musbat davrini toping. A) $6\pi$ B) $2\pi$ C) $3\pi$ D) $12\pi$ E) $5\pi$

7(97-12-41). Quyidagi funksiyalardan qaysi birining eng kichik davri $2\pi$ gat eng?

A) $f(x) = \cos^2 x - \sin^2 x$ B) $f(x) = ctg\frac{x}{2}\sin\frac{x}{2}$ C) $f(x) = 2\sin\frac{x}{2}\cos\frac{x}{2}$

D) $f(x) = \cos^2 x + 3\sin^2 x$ E) $f(x) = tg2x - \cos 2x$

8(02-3-41). Quyidagi funksiyalardan qaysi biri davriy emas?

A) $y = \sin\sqrt{x}$ B) $y = \sqrt{\sin x}$ C) $y = |\sin|x||$ D) $y = \sin^2 x$ E) $y = \sqrt[3]{\sin^2 x}$

**9-misol.** $f(x) = \cos^2 x + \cos^2(1+x) - 2\cos 1\cos x\cos(1+x) - \frac{1}{2}$ funksiya davriy funksiyami?

10(96-12-105). Funksiyaning eng kichik musbat davrini toping. $y = tg\frac{x}{3} - 2\sin x + 3\cos 2x$ A) $6\pi$ B) $3\pi$ C) $4\pi$ D) $9\pi$ E) $2\pi$

11(97-4-38). $y = \cos(8x+1)$, $y = \sin(4x+3)$, $y = tg8x$ va $y = tg(2x+4)$ funksiyalar uchun eng kichik umumiy davrni toping. A) $2\pi$ B) $\pi$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{4}$ E) $\frac{\pi}{6}$

12(98-5-54). Ushbu $y = 13\sin^2 3x$ funksiyaning eng kichik musbat davrini toping.

A) $\frac{2\pi}{3}$ B) $\frac{\pi}{3}$ C) $\frac{13\pi}{2}$ D) $\frac{\pi}{4}$ E) $\frac{13\pi}{6}$

13(01-6-29). Ushbu $f(x) = \left(2 + \sin\frac{x}{2}\right)\cdot\left(1 - \cos\frac{x}{4}\right)\cdot tg\frac{x}{3}$ funksiyaning eng kichik musbat davrini toping. A) $\frac{\pi}{2}$ B) $2\pi$ C) $3\pi$ D) $4\pi$ E) $1{,}5\pi$

14(03-2-34). $y = 1 - 8\sin^2 x\cos^2 x$ funksiyaning eng kichik musbat davrini toping.

A) $2\pi$ B) $\pi$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{4}$ E) davriy funksiya emas

15(03-6-55). $f(x) = \cos\frac{3x}{2} - \sin\frac{x}{3}$ funksiyaning eng kichik musbat davrini toping.

A) $6\pi$ B) $\frac{4\pi}{3}$ C) $8\pi$ D) $10\pi$ E) $12\pi$

## 14-§. Oddiy funksional tenglamalarni yechish

Funksional tenglamalar bilan darslikdagi funksiyaning juftligi, toqligi,davriyligi bilan tanish bo'lganligingiz sababli tanishsiz.

$$f(-x)=f(x)$$
$$f(-x)=-f(x)$$
$$f(x+T)=f(x)$$

Faqat u yerda funksiyaning bu xossalari bu nom bilan yuritilmagan edi. Umuman, funksional tenglamalar-bu noma'lum funksiya topilishi lozim bo'lgan,qandaydir bog'lanishdir.

Masalan, $f(x+1)+f(x)=x$

$2f(1+x)+1=xf(x)$

$xf(x)+f(\frac{2}{4-x})=\frac{1}{x}$ lar funksiyonal tenglamalardir.

**1-misol.** $f(x)+2\cdot f(\frac{1}{x})=x$ (1) shartni qanoatlantiruvchi $f(x)$ funksiyani toping.

**Yechish.** $\frac{1}{x}=t$ almashtirishni olaylik $(x\neq 0)$. U holda $x=\frac{1}{t}$ bo'ladi.Buni tenglamaga hisobga olsak, $f(\frac{1}{t})+2\cdot f(t)=\frac{1}{t}$, yoki uni $x$ bilan almashtirsak, $f(\frac{1}{x})+2\cdot f(x)=\frac{1}{x}$ (2)

(1)va (2) tenglamani sistema sifatida qarasak

$$\begin{cases} f(x)+2f(\frac{1}{x})=x \\ 2f(x)+f(\frac{1}{x})=\frac{1}{x} \end{cases}$$

(2) tenglamani -2 ga ko'paytirib, (1) ga qo'shsak, $f(x)-4f(x)+2f(\frac{1}{x})-2f(\frac{1}{x})=x-\frac{2}{x}$, $-3f(x)=\frac{x^2-2}{x}$, $f(x)=\frac{x^2-2}{x}:(-3)=\frac{2-x^2}{3x}$

**Javob.** $f(x)=\frac{2-x^2}{3x}$

**2-misol.** $2\cdot f(1-x)+1=x\cdot f(x)$ tenglamani qanoatlantiruvchi $f(x)$ funksiyani toping.

**Yechish.** $1-x=t$ almashtirib olaylik. U holda $x=1-t$. Buni berilgan tenglamaga qo'ysak, $2f(t)+1=(1-t)\cdot f(1-t)$

$t$ ni $x$ ga almashtirsak $2f(x)+1=(1-x)f(1-x)$

Berilgan tenglamadan $f(1-x)=\frac{1}{2}(xf(x)-1)$ ni topib tenglamalarni bir-biriga qo'ysak

$2f(x)+1=(1-x)\cdot\frac{1}{2}\cdot(xf(x)-1)$ Bundan $f(t)=\frac{x-3}{x^2-x+4}$

**Javob.** $f(x)=\frac{x-3}{x^2-x+4}$

**3-misol.** $f(x)+x\cdot f(\frac{x}{2x-1})=2$ tenglamani qanoatlantiruvchi *f(x)* funksiyani toping.

**Yechish.** $\frac{x}{2x-1}=t$ deb olaylik. U holda $x=t\cdot(2x-1),\Rightarrow x=\frac{t}{2t-1}$ bo'ladi.

Bularni berilgan tenglamaga qo'yib $t$ ni $x$ bilan almashtirib,berilgan tenglama bilan birga tenglamalar sistemasi sifatida qarasak,

$$\begin{cases} f(x)+x\cdot f(\frac{x}{2x-1})=2 \\ f(\frac{x}{2x-1})+\frac{x}{2x-1}\cdot f(x)=2 \end{cases}$$

Ikkinchi tenglamadan $f(\frac{x}{2x-1})=2-\frac{x}{2x-1}f(x)$ bo'ladi. Buni birinchi tenglamaga qo'ysak $f(x)+x\cdot(2-\frac{x}{2x-1}f(x))=2$ dan $f(x)=\frac{4x-2}{x-1}$

**Javob.** $f(x)=\frac{4x-2}{x-1}$

**4-misol.** Agar $f(x)=\frac{a^x}{a^x+\sqrt{a}}$ bo'lsa, $f(\frac{1}{1988})+f(\frac{2}{1988})+...+f(\frac{1987}{1988})$ yig'indini hisoblang.

**Yechish.** $f(x)+f(1-x)=1$ tenglik to'g'ri.Haqiqatdan, $f(x)=\frac{a^x}{a^x+\sqrt{a}}$,

$f(1-x)=\frac{a^{1-x}}{a^{1-x}+\sqrt{a}}$,

$$f(x)+f(1-x)=\frac{a^x}{a^x+\sqrt{a}}+\frac{a^{1-x}}{a^{1-x}+\sqrt{a}}=\frac{a^x+a^x\cdot\sqrt{a}+a+a^{1-x}\cdot\sqrt{a}}{(a^x+\sqrt{a})(a^{1-x}+\sqrt{a})}=\frac{2a+\sqrt{a}\cdot(a^x+a^{1-x})}{a+a^x\cdot\sqrt{a}+a^{1-x}\cdot\sqrt{a}+a}=$$

$$\frac{2a+\sqrt{a}(a^x+a^{1-x})}{2a+\sqrt{a}(a^x+a^{1-x})}=1$$

Bundan ketma-ket hosil qilamiz.

$f(\frac{1}{1988})+f(1-\frac{1}{1988})=1,$

$f(\frac{2}{1988})+f(1-\frac{2}{1988})=1$

..............................

1987 dona qo'shiluvchini yuqoridagi kabi gruppalaganda 993 tasi 1 ga teng.Lekin

$$f(\frac{994}{1988})=f(\frac{1}{2})=\frac{a^{\frac{1}{2}}}{a^{\frac{1}{2}}+\sqrt{a}}=\frac{\sqrt{a}}{2\sqrt{a}}=\frac{1}{2}$$

Demak, $f(\frac{1}{1988})+f(\frac{2}{1988})+...+f(\frac{1987}{1988})=993+\frac{1}{2}=993,5$

**Javob.** 993,5

5(98-6-16). Agar $f(x)$ funksiya uchun $x\in(-\infty;\infty)$ da $f(x+3)=-\frac{1}{f(x+1)}$ tenglik bajarilsa, $\frac{f(4)}{f(0)}$ ni toping. A)1 B)2 C)3 D) 4 E) 5

**Yechish.** 1) $x=1$ bo'lsa, $f(1+3)=-\frac{1}{f(1+1)}$ yoki $f(4)=-\frac{1}{f(2)}$.

2) $x=-1$ bo'lsa, $f(-1+3)=-\frac{1}{f(-1+1)}$; $f(2)=-\frac{1}{f(0)}$ $f(0)=-\frac{1}{f(2)}$

3) $\frac{f(4)}{f(0)}=-\frac{1}{f(2)}:(-\frac{1}{f(2)})=1$

**Javob.** 1 (A)

6(98-11-65). Agar $(x-2)f(x-2)+f(2x)+f(x+2)=x+6$ bo'lsa, $f(4)=?$
A)13 B) 2 C) 3 D) 4 E)41

**Yechish.** $x=2$ bo'lsa, $(2-2)f(2-2)+f(4)+f(4)=2+6$ $2\cdot f(4)=8$, $f(4)=4$

**Javob.** 4 (D)

7(99-1-14) Agar $f(\frac{ax-b}{bx-a})=x^{50}+x^{49}+x^{48}+...+x^2+x+1$ $(|a|\neq|b|)$ bo'lsa, $f(1)=?$
A)1 B) 2 C) 51 D) 4 E)5

**Yechish.** $\frac{ax-b}{bx-a}=1$ dan $x=-1$ bo'ladi. U holda
$f(1)=(-1)^{50}+(-1)^{49}+(-1)^{48}+...+(-1)^2+(-1)+1=1-1+1-1+...+1-1+1=0+1=1$

**Javob.** 1 (A)

**8-misol.** Agar $f(x)=x^2$ va $y(x)=2x-1$ bo'lsa, $x$ ning nechta qiymatlarida $f(y(x))=y(f(x))$ bo'ladi?
A)∅ B) 1 C)2 D) 3 E)4

**Yechish.** 1) $f(y(x))=y^2(x)=(2x-1)^2=4x^2-4x+1$

2) $y(f(x))=2\cdot f(x)-1=2\cdot x^2-1$. U holda $4x^2-4x+1=2x^2-1$ tenglik hosil bo'ladi. Bundan $2x^2-4x+2=0$, $(x-1)^2=0$, $x=1$

**Javob.** 1ta (B)

**9-misol.** $y=f(x)$ funksiya uchun barcha $x\in R$ larda $2f(x)+f(1-x)=3x^2$ tenglik bajariladi. $f(5)$ ni toping.

**Yechish. 1-usul.** Berilgan tenglikda $x=5$, $x=-4$ qo'ysak, quyidagi tenglamalar sistemasi hosil bo'ladi:

$\begin{cases}2f(5)+f(-4)=75,\\2f(-4)+f(5)=48\end{cases}$ buni yechib, $f(5)=34$ ekanligi topiladi

**2-usul.** $2f(x)+f(1-x)=3x^2$ da $1-x=t$ deb olsak, $x=t-1$ bo'ladi. $2f(1-t)+f(t)=3(1-t)^2$ ni $x$ bilan almashtirib,qo'yib sistemadan $f(x)$ ni topamiz.

$$\begin{cases} 2f(x)+f(1-x)=3x^2 \\ 2f(1-x)+f(x)=3(1-x)^2 \end{cases}$$

$$f(x)=\frac{-6x^2+3(1-x)^2}{-3}=\frac{-6x^2+3-6x+3x^2}{-3}=\frac{-3x^2-6x+3}{-3}=x^2+2x-1$$

$f(x)=x^2+2x-1$ bunda $x=5$ bo'lsa, $f(5)=5^2+2\cdot 5-1=34$ $f(5)=34$

**Javob.** 34

## Mustaqil yechish uchun masalalar

**10(03-2-6).** $y=f(x)$ funksiyaning aniqlanish sohasi $[-1;2]$ dan iborat. $y=f(1+x)$ funksiyaning aniqlanish sohasini toping.

A) $[-2;-1]$ B) $[-2;1]$ C) $[-4;2]$ D) $[-1;0]$ E) $[0;3]$

**11(03-3-48).** Agar $f(x)=\frac{2x+1}{3x-1}$ bo'lsa, $f(\frac{1}{x})+f(\frac{x}{9})$ funksiyani aniqlang.

A) $\frac{1}{3}$ B) $\frac{x}{3}$ C) $-\frac{x}{3}$ D) $-\frac{1}{3}$ E) $\frac{1}{3x-1}$

**12(03-9-42).** Agar $f(1+x)=3-2x$ va $f(\varphi(x))=6x-3$ bo'lsa, $\varphi(x)$ funksiyani aniqlang.

A) $4-3x$ B) $3x-4$ C) $4x+3$ D) $4x-3$ E) $6x-8$

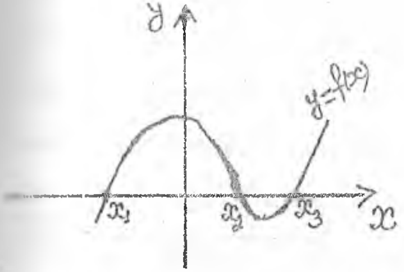
**13-misol.** Agar $f(\frac{3x-1}{x+2})=\frac{x+1}{x-1}$ va $x\neq -2;1$ bo'lsa, $f(x)$ ni toping.

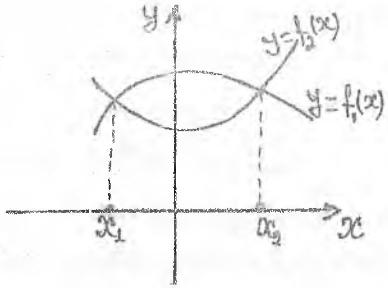
**14-misol.** $f(x)=ax^2+bx+c$ bo'lsin. U holda $f(x+3)-3f(x+2)+3f(x+1)-f(x)=0$ ekanligini isbotlang.

**15-misol.** Tenglamalar sistemasini qanoatlantiruvchi $f(x)$ va $g(x)$ funksiyalarni toping. $\begin{cases} f(2x+1)+g(x-1)=x, \\ f(2x+1)-2g(x-1)=2x^2 \end{cases}$

## 15-§. Funksiyaning grafigi yordamida yechiladigan tenglamalarni yechish

$f(x)=0$ tenglamaning haqiqiy ildizlari sonini va ularning qiymatlarini aniqlashda grafiklar keng qo'llaniladi.
Haqiqatdan $y=f(x)$ funksiyaning grafigini chizish bilan bu grafikning OX o'qi bilan kesishish nuqtasi $f(x)=0$ tenglamaning haqiqiy ildizlari bo'ladi.

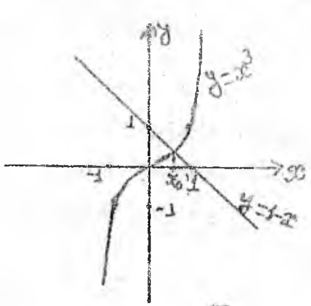



Agar $y=f(x)$ funksiyaning grafigini chizish qiyinroq bo'lsa, tenglamani $f_1(x)=f_2(x)$ ko'rinishida shunday yozish kerakki $y=f_1(x)$ va $y=f_2(x)$ funksiyalarning grafiklarini chizish oson bo'lsin. $y=f_1(x)$ va $y=f_2(x)$ funksiyalarning grafiklari kesishish nuqtalarini absissalari tenglamaning ildizlari bo'ladi.

Agar bu grafiklar kesishmasa tenglama haqiqiy ildizlariga ega bo'lmaydi.
Grafiklarni qanchalik aniq chizsak tenglamaning ildizlari shunchalik aniq topiladi.

**1-misol.** $x^3+x-1=0$ tenglamaning haqiqiy ildizlari sonini aniqlang.
**Yechish.** Tenglamani $x^3=1-x$ ko'rinishida yozib olaylik. $y=x^3$ va $y=1-x$ funkiyalarning grafiklarini bitta koordinatalar tekisligida yasaymiz.



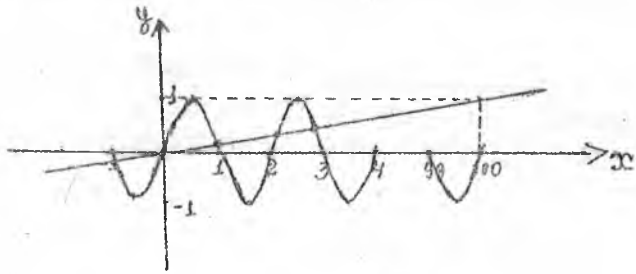
| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y=x^3$ | -8 | -1 | 0 | 1 | 8 |
| $y=1-x$ | 3 | 2 | 1 | 0 | -1 |

Grafikning kesishish nuqtasi bitta bo'lganligi sababli bitta ildizga ega.

**Javob.** 1 ta

**2-misol.** $100\sin\pi x = x$ tenglamaning ildizlari sonini aniqlang.

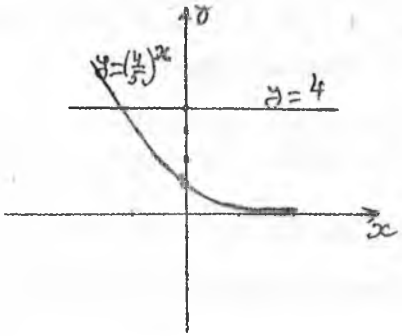
**Yechish.** Tenglamani $\sin\pi x = \frac{x}{100}$ ko'rinishida yozib olaylik va $y=\sin\pi x$, $y=\frac{x}{100}$ funksiyalarni grafigini sxematik chizaylik. $y=\sin\pi x$ va $y=\frac{x}{100}$ funksiyalar toq bo'lganligi sababli ikki grafik koordinatalar boshiga nisbatan simmetrik bo'ladi. Shuning uchun bu tenglamani $x \geq 0$ bo'lgan hol uchun yechish yetarli. $y=\frac{x}{100}$ funksiyaning qiymati 1 dan katta ($x>100$) bo'lsa, funksiyaning grafiklari kesishmaydi($\sin\pi x \leq 1$). $y=\sin\pi x$ funksiyaning davri T=2 bo'lib, uzunligi davrga teng bo'lgan oraliqda to'gri chiziq sinusoidani ikkita nuqtada kesib o'tadi. Shuning uchun $[0;100]$ kesmada bunday kesishish nuqtalari $2\cdot\frac{100}{2}=100$ ta bo'ladi. Xuddi shunday $[-100;0]$ kesmada ham bunday nuqtalar 100 ta bo'ladi. Koordinata boshi ikki marta hisoblagani sababli berilgan tenglama 199 ta ildizga ega bo'ladi. Bulardan biri 0 aniq qiymat. Qolganlari taqribiy qiymat hisoblanadi.




3(96-7-35). Tenglamaning nechta ildizi bor? $e^{-x}=x-2$

A)1 B)2 C)3 D) ildizi yo'q E)aniqlab bo'lmaydi

**Yechish.** $y_1=e^{-x}=(\frac{1}{e})^x$ va $y_2=x-2$ funksiyalarning grafiklarini bitta koordinata tekisligida yasaymiz.



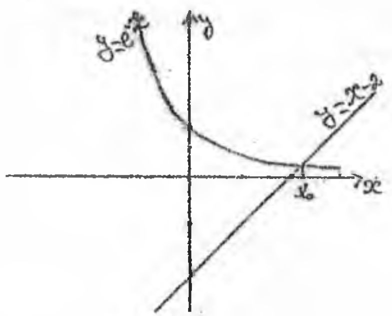

$y_1 = (\frac{1}{e})^x$ funksiya $R$ da kamayuvchi, $y_2 = x-2$ funksiya $R$ da o'suvchi.Bitta ildizga ega.

**Javob.** 1 (A)

**4(98-5-30).** Ushbu $(\frac{4}{5})^x = 4$ tenglamaning yechimi qaysi oraliqqa tegishli?

A) $(-\infty;-1)$ B) $(0;1)$ C) $[2;\infty)$ D) $(-1;0)$ E) $(1;2)$

**Yechish.** $y_1 = (\frac{4}{5})^x$ va $y_2 = 4$ funksiyalarning grafiklarini bitta koordinata tekisligida yasaymiz.




Berilgan tenglama bitta ildizga ega bo'lib, $x_0 < 0$ ekanligi grafikdan ko'rinadi.

Javoblardan A) yoki D) to'g'ri bo'lishi mumkin.Buning uchun tenglamaning ildizi $x_0 = \log_{\frac{4}{5}} 4$ bilan -*1* ni taqqoslaymiz. $4 > \frac{5}{4}$ tengsizlik to'g'ri.

$y = \log_{\frac{4}{5}} x$ funksiyaning kamayuvchiligidan $\log_{\frac{4}{5}} 4 < \log_{\frac{4}{5}} \frac{5}{4}$ yoki $x_0 < -1$

**Javob.** $(-\infty;-1)$ (A)

## Mustaqil yechish uchun misollar

5(99-7-31). Tenglamani yechimi qaysi oraliqqa tegishli? $(\frac{2}{3})^x = 2$

A) $(-\infty;-2)$ B)(-2;0) C) $(1;\infty)$ D) (-2;-1) E) (0;1)

6(00-9-30). Tenglama nechta ildizga ega? $3^{-x} = 4+x-x^2$

A)$\varnothing$ B)1 C)2 D)3 E)4

7(00-10-20). Tenglamaning nechta ildizi bor? $|\log_5 x| = -x+5$

A)1 B)$\varnothing$ C)5 D)2 E)3

8(01-4-1). Ushbu $\sin x = x^2 - x + 0,75$ tenglamaning ildizlari qaysi kesmaga tegishli?

A)$[0;\pi]$ B)$[-\pi;0]$ C)$[\pi;2\pi]$ D)$\left[\frac{3\pi}{2};2\pi\right]$ E)ildizlari yo'q

9(01-9-43). Ushbu $x^2+8=\log_2(x+1)+6x$ tenglamaning nechta ildizi bor?

A)2 B)3 C)1 D)$\varnothing$ E)ildizini topib bo'lmaydi.

**10**(01-12-10). Ushbu $2^x+0,5=|\sin x|$ tenglamaning manfiy yechimlari nechta?

A)2 B)14 C)$\varnothing$ D)15 E)cheksiz ko'p

**11**(01-12-44). Ushbu $|x|\cdot(x^2-4)=-1$ tenglama nechta ildizga ega?

A)1 B)2 C)3 D)4 E) $\varnothing$

**12**(02-1-57). $1+x-x^2=|x^3|$ tenglama nechta haqiqiy yechimga ega?

A)1 B)2 C)3 D)4 E)yechimga ega emas

13(03-1-38). $\frac{3}{x}=x^2-6x+7$ tenglamaning nechta ildizi bor?

A)0 B)1 C)2 D)3 E)4

14(03-2-59). $\sin x=\log_2 x$ tenglamani nechta ildizi bor?

A)ildizi yo'q B)1 C)2 D)4 E)cheksiz ko'p

15(03-5-24). $2^x=x^3$ tenglama nechta haqiqiy ildizga ega?

A)2 B)1 C)3 D) $\varnothing$ E)aniqlab bo'lmaydi.

16(03-12-56) $x^2\cdot 7^x+1>7^x+x$ tengsizlikni yeching.

A)$(1;\infty)$ B)$(-1;0)$ C)$(-1;1)$ D)$(-\infty;0)\cup(1;\infty)$ E) $(-1;1)\cup(1;\infty)$

17(00-3-37). tenglama nechta ildizga ega? $\log_2(2+x)=\frac{x^2}{2}$

A)2 B)1 C)3 D)0 E)4

## 16-§. Hosilaga ega bo'lmagan funksiyalarga doir misollar yechish namunalari

**1-ta'rif.** $y=f(x)$ funksiya biror oraliqda aniqlangan bo'lib $x$-shu oraliqning nuqtasi va $h\neq 0$ shunday son bo'lsinki, $x+h$ ham berilgan oraliqqa tegishli bo'lsin.U holda

$\frac{f(x+h)-f(x)}{h}$ ayirmali nisbatning $h\to 0$ dagi limiti (agar bu limit mavjud bo'lsa) $f(x)$ funksiyaning $x$ nuqtadagi hosilasi deb aytiladi va $f'(x)$ kabi belgilanadi, ya'ni

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

**2-ta'rif.** $\lim_{h\to 0-0}\frac{f(x+h)-f(x)}{h}$ ni $y=f(x)$ funksiyani $x$ nuqtadagi chap hosilasi deb aytiladi va u quyidagicha belgilanadi:

$$f'_-(x)=\lim_{h\to 0-0}\frac{f(x+h)-f(x)}{h}$$

$\lim_{h\to 0+0}\frac{f(x+h)-f(x)}{h}$ ni $y=f(x)$ funksiyani $x$ nuqtadagi o'ng hosilasi deb aytiladi va quyidagicha belgilanadi:

$$f'_+(x)=\lim_{h\to 0+0}\frac{f(x+h)-f(x)}{h}$$

**Teorema.** $y=f(x)$ funksiyaning $x$ nuqtadagi hosilasi $f'(x)$ mavjud bo'lishi uchun,bu nuqtada chap va o'ng hosilalar $f'_-(x)$, $f'_+(x)$ mavjud bo'lib ya'ni $f'_+(x)=f'_-(x)$ bo'lishi zarur va yetarli.

**1-misol.** $f(x)=|x|$ funksiyaning hosilasini toping.

A)1 B)-1 C) $\begin{cases}1, agar & x\geq 0 \quad bo'lsa,\\ -1, agar & x<0 \quad bo'lsa\end{cases}$

D) $\begin{cases}1, agar & x>0 \quad bo'lsa,\\ x=0 & da \quad hosila \quad mavjud \quad emas\\ -1, agar & x<0 \quad bo'lsa\end{cases}$ E)hisoblab bo'lmaydi.

**Yechish.** Bizga ma'lumki, $|x|=\begin{cases}x, agar & x\geq 0,\\ -x, agar & x<0\end{cases}$ $f(x)=|x|$ funksiyani $x=0$ nuqtadagi hosilasini hisoblab topaylik.

$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{|0+h|-|0|}{h}=\lim_{h\to 0}\frac{|h|}{h}$ $x=0$ nuqtadagi chap va o'ng hosilalarini hisoblaylik.

$f'_-(0)=\lim\limits_{h\to 0-0}\frac{|h|}{h}=\lim\limits_{h\to 0-0}\frac{-h}{h}=-1,\quad f'_+(0)=\lim\limits_{h\to 0+0}\frac{|h|}{h}=\lim\limits_{h\to 0+0}\frac{h}{h}=1,\quad -1\neq 1$ bo'lganligi sababli $f(x)=|x|$ funksiyani $x=0$ nuqtada hosilasi mavjud emas.

Agar $x>0$ bo'lsa $f'(x)=(x)'=1$ agar $x<0$ bo'lsa $f'(x)=(-x)'=-1$

Shunday qilib $f'(x)=(|x|)'=\begin{cases}1, agar & x>0,\\ x=0 \quad da & mavjud \quad emas\\ -1, agar & x<0\end{cases}$

**Javob.** $\begin{cases}1, agar \quad x>0 \quad bo'lsa,\\ x=0 \quad da \quad hosila \quad mavjud \quad emas\\ -1, agar \quad x<0 \quad bo'lsa\end{cases}$ (D)

2(00-5-45). $y=x+|x|$ funksiyaning hosilasini toping.

A)0 B)2 C) $\begin{cases}0, agar & x<0,\\ 2, agar & x\geq 0\end{cases}$ D) $\begin{cases}0, agar & x<0\\ mavjud & emas, agar x=0,\\ 2, agar & x>0\end{cases}$ E) hosila mavjud emas.

**Yechish.** $y=x+|x|=\begin{cases}0, agar & x<0,\\ 2x, agar & x\geq 0\end{cases}$ $x<0$ bo'lsa $y'=0$. $x>0$ bo'lsa $y'=2$.

$x=0$ nuqtada hosilani hisoblab ko'raylik.

$y'(0)=\lim\limits_{h\to 0}\frac{y(0+h)-y(0)}{h}=\lim\limits_{h\to 0}\frac{h+|h|}{h}$. Demak $y'(0)$ ni qiymati $|h|$ ni qiymatiga bog'liq. Shuning uchun chap va o'ng hosilalarni hisoblaylik.

$$y'_-(0)=\lim_{h\to 0-0}\frac{h+|h|}{h}=\lim_{h\to 0-0}\frac{h-h}{h}=0$$

$$y'_+(0)=\lim_{h\to 0+0}\frac{h+|h|}{h}=\lim_{h\to 0+0}\frac{h+h}{h}=2 \qquad 0\neq 2$$

$x=0$ nuqtada chap va o'ng hosilalar teng bo'lmaganliklari sababli bu nuqtada hosila mavjud emas.

**Javob.** $y'=\begin{cases}0, agar & x<0\\ mavjud & emas, agar \quad x=0,\\ 2, agar & x>0\end{cases}$ **(D)**

**3(03-7-28).** $f(x)=|x^2-14x+45|,\quad f'(9)=?$

A)0 B)4 C)2 D)7 E) mavjud emas

**Yechish.** $f(x)=|x^2-14x+45|=|(x-5)(x-9)|$ $x=9$ nuqtadagi hosilani ta'rifdan foydalanib topamiz.

$f'(9)=\lim\limits_{h\to 0}\frac{|(9+h-5)(9+h-9)|-|(9-5)(9-9)|}{h}=\lim\limits_{h\to 0}\frac{|4+h|\cdot|h|}{h}$ $x=9$ nuqtadagi chap va o'ng hosilalarni hisoblaylik.

$$f'_{-}(9)=\lim_{h\to 0-0}\frac{|4+h|\cdot|h|}{h}=\lim_{h\to 0-0}\frac{|4+h|\cdot(-h)}{h}=-4 \qquad f'_{+}(9)=\lim_{h\to 0+0}\frac{|4+h|\cdot|h|}{h}=\lim_{h\to 0+0}\frac{|4+h|\cdot h}{h}=4$$

$-4\neq 4$

Demak, $x=9$ nuqtada hosila mavjud emas

**Javob.** Mavjud emas (E)

**4-misol.** $f(x)=x|x|, \qquad f'(x)=?$

**Yechish.** $f(x)=x|x|=\begin{cases} x^2, agar & x\geq 0, \\ -x^2, agar & x<0 \end{cases}$

$x>0$ bo'lsa, $f'(x)=(x^2)'=2x.$ $\quad x<0$ bo'lsa, $f'(x)=(-x^2)'=-2x$

$x=0$ nuqtada hosilani ta'rif bo'yicha topaylik.

$$f'(0)=\lim_{h\to 0}\frac{(0+h)|0+h|-0|0|}{h}=\lim_{h\to 0}\frac{h|h|}{h}=\lim_{h\to 0}|h|=|0|=0$$

$x=0$ nuqtada chap va o'ng hosilalar ham $0$ga teng. Demak,

$$f'(x)=\begin{cases} 2x, agar & x\geq 0 \\ -x^2, agar & x<0 \end{cases}$$

5(01-2-33). Funksiyani hosilasini aniqlang. $y=|x+1|$

A) $\begin{cases} 1, agar & x\geq -1 \\ -1, agar & x<-1 \end{cases}$ B) $\begin{cases} 1, agar & x>-1 \\ x=-1da & hosila \quad mavjud \quad emas \\ -1, agar & x<-1 \end{cases}$ C) 2 D) 1 E) -1

**Yechish.** Modul ostidagi ifodani nolini topaylik. $x+1=0,\ x=-1$. Agar $x\geq -1$ bo'lsa u holda $y=|x+1|=x+1$. Agar $x+1<0$ bo'lsa, $y=|x+1|=-(x+1)=-x-1$ bo'ladi. Berilgan funksiyani quyidagi ko'rinishda yozib olaylik.

$$y=|x+1|=\begin{cases} x+1, agar & x\geq -1 \\ -x-1, agar & x<-1 \end{cases}$$

$x>-1$ va $x<-1$ bo'lsa berilgan funksiyani hosilasi osongina topiladi.

$x=-1$ nuqtada, ya'ni funksiya yo'nalishini o'zgartiradigan nuqtada hosilani ta'rifdan foydalanib hisoblaylik. $y'(-1)=\lim\limits_{h\to 0}\frac{|-1+h+1|-|-1+1|}{h}=\lim\limits_{h\to 0}\frac{|h|}{h}$

$x=-1$ nuqtada chap va o'ng hosilalarni hisoblaylik.

$$y'_{-}(-1)=\lim_{h\to 0-0}\frac{|h|}{h}=\lim_{h\to 0-0}\frac{-h}{h}=-1, \quad y'_{+}(-1)=\lim_{h\to 0+0}\frac{|h|}{h}=\lim_{h\to 0+0}\frac{h}{h}=1$$

Demak, $x=-1$ nuqtada berilgan funksiyaning chap va o'ng hosilasi teng bo'lmaganligi sababli bu nuqtada hosila mavjud emas.

**Javob.** $\begin{cases} 1, agar & x>-1 \\ x=-1da & hosila \quad mavjud \quad emas \\ -1, agar & x<-1 \end{cases}$ (B)

## Mustaqil yechish uchun masalalar

**6-misol.** $y=|x^2-3x+2|$ funksiyaning $x=1$ va $x=2$ nuqtadagi hosilasini toping.

**7-misol.** $y=3|x+3|$ funksiyaning $x=-3$ nuqtadagi hosilasini toping.

**8-misol.** $f(x)=\begin{cases} x, agar & x\geq 1 \\ x^2, agar & x<1 \end{cases}$ funksiyaning $x=1$ nuqtada hosilasi mavjud emasligini isbotlang.

**9-misol.** $f(x)=|\log_2 x|$ funksiyani hosilasini toping.

**10-misol.** Ushbu $f(x)=\begin{cases} x^2, agar & x\leq 1 \\ ax+b, agar & x>1 \end{cases}$ funksiya berilgan. $a$ va $b$ ning qanday qiymatlarida $f(x)$ funksiya $x=1$ nuqtada hosilaga ega bo'ladi.

**11**(03-6-21) $f(x)=|x^2-14x+45|$, $f'(6)=?$

A)0 B)5 C)2 D)7 E)mavjud emas.

**12-misol.** $f(x)=\sqrt[3]{x^2}$ funksiya $x=0$ nuqtada hosilaga ega emasligini isbotlang.

$$f'_-(9)=\lim_{h\to 0-0}\frac{|4+h|\cdot|h|}{h}=\lim_{h\to 0-0}\frac{|4+h|\cdot(-h)}{h}=-4 \qquad f'_+(9)=\lim_{h\to 0+0}\frac{|4+h|\cdot|h|}{h}=\lim_{h\to 0+0}\frac{|4+h|\cdot h}{h}=4$$

$-4\neq 4$

Demak, $x=9$ nuqtada hosila mavjud emas

**Javob.** Mavjud emas (E)

**4-misol.** $f(x)=x|x|$, $f'(x)=?$

**Yechish.** $f(x)=x|x|=\begin{cases}x^2, agar & x\geq 0,\\ -x^2, agar & x<0\end{cases}$

$x>0$ bo'lsa, $f'(x)=(x^2)'=2x$. $x<0$ *bo'lsa*, $f'(x)=(-x^2)'=-2x$

$x=0$ nuqtada hosilani ta'rif bo'yicha topaylik.

$$f'(0)=\lim_{h\to 0}\frac{(0+h)|0+h|-0|0|}{h}=\lim_{h\to 0}\frac{h|h|}{h}=\lim_{h\to 0}|h|=|0|=0$$

$x=0$ nuqtada chap va o'ng hosilalar ham $0$ga teng.Demak,

$$f'(x)=\begin{cases}2x, agar & x\geq 0\\ -x^2, agar & x<0\end{cases}$$

5(01-2-33). Funksiyani hosilasini aniqlang. $y=|x+1|$

A) $\begin{cases}1, agar & x\geq -1\\ -1, agar & x<-1\end{cases}$ B) $\begin{cases}1, agar & x>-1\\ x=-1da & hosila \quad mavjud \quad emas\\ -1, agar & x<-1\end{cases}$ C)2 D)1 E)- 1

**Yechish.** Modul ostidagi ifodani nolini topaylik. $x+1=0$, $x=-1$. Agar $x\geq -1$ bo'lsa u holda $y=|x+1|=x+1$. Agar $x+1<0$ bo'lsa, $y=|x+1|=-(x+1)=-x-1$ bo'ladi. Berilgan funksiyani quyidagi ko'rinishda yozib olaylik.

$$y=|x+1|=\begin{cases}x+1, agar & x\geq -1\\ -x-1, agar & x<-1\end{cases}$$

$x>-1$ va $x<-1$ bo'lsa berilgan funksiyani hosilasi osongina topiladi.

$x=-1$ nuqtada,ya'ni funksiya yo'nalishini o'zgartiradigan nuqtada hosilani ta'rifdan foydalanib hisoblaylik. $y'(-1)=\lim\limits_{h\to 0}\frac{|-1+h+1|-|-1+1|}{h}=\lim\limits_{h\to 0}\frac{|h|}{h}$

$x=-1$ nuqtada chap va o'ng hosilalarni hisoblaylik.

$$y'_-(-1)=\lim_{h\to 0-0}\frac{|h|}{h}=\lim_{h\to 0-0}\frac{-h}{h}=-1,\quad y'_+(-1)=\lim_{h\to 0+0}\frac{|h|}{h}=\lim_{h\to 0+0}\frac{h}{h}=1$$

Demak, $x=-1$ nuqtada berilgan funksiyaning chap va o'ng hosilasi teng bo'lmaganligi sababli bu nuqtada hosila mavjud emas.

**Javob.** $\begin{cases}1, agar & x>-1\\ x=-1da & hosila \quad mavjud \quad emas\\ -1, agar & x<-1\end{cases}$ (B)

## Mustaqil yechish uchun masalalar

**6-misol.** $y=|x^2-3x+2|$ funksiyaning $x=1$ va $x=2$ nuqtadagi hosilasini toping.

**7-misol.** $y=3|x+3|$ funksiyaning $x=-3$ nuqtadagi hosilasini toping.

**8-misol.** $f(x)=\begin{cases} x, agar & x\geq 1 \\ x^2, agar & x<1 \end{cases}$ funksiyaning $x=1$ nuqtada hosilasi mavjud emasligini isbotlang.

**9-misol.** $f(x)=|\log_2 x|$ funksiyani hosilasini toping.

**10-misol.** Ushbu $f(x)=\begin{cases} x^2, agar & x\leq 1 \\ ax+b, agar & x>1 \end{cases}$ funksiya berilgan. $a$ va $b$ ning qanday qiymatlarida $f(x)$ funksiya $x=1$ nuqtada hosilaga ega bo'ladi.

**11**(03-6-21) $f(x)=|x^2-14x+45|$, $f'(6)=?$

A)0 B)5 C)2 D)7 E)mavjud emas.

**12-misol.** $f(x)=\sqrt[3]{x^2}$ funksiya $x=0$ nuqtada hosilaga ega emasligini isbotlang.

## 17-§. $u(x)^{v(x)}$ funksiya [illegible] nashgan misollarni yechish

### 1. $u(x)^{v(x)}=u(x)^{z(x)}$ ko'rinishidagi tenglamalarni yechish.

$u(x)^{v(x)}=u(x)^{z(x)}$ (1) tenglama daraja- ko'rsatkichli tenglama deyiladi. Agar $u(x)>0$ va $u(x)\neq 1$ bo'lsa, u holda (1) tenglama ko'rsatkichli tenglamadek, ko'rsatkichlarini tenglashtirish bilan yechiladi: $v(x)=z(x)$

Agar $u(x)\leq 0$ yoki $u(x)=1$ bo'lsa, bir qancha hollarni qarashga to'g'ri keladi. Buni aniq misollarda qaraylik.

**1-misol.** $(2x-5)^{2x^2-x}=(2x-5)^{14x-28}$ tenglamani yeching.

**Yechish. 1)** $2x-5=1$ bo'lsin. $2x=6$, $x=3$. Bu holda tenglama $1^{2x^2-x}=1^{14x-28}$ ko'rinishni oladi. $2x=6$ tenglamaning ildizi berilgan tenglamani ham ildizi bo'ladi.

2) $2x-5=-1$ bo'lsin. U holda $2x=4$, $x=2$. Bu holda $x=2$ bo'lsa, $(-1)^{2\cdot4-2}=(-1)^{28-28}$, $1=1$. $x=2$ berilgan tenglamaning ildizi bo'ladi.

3) $2x-5=0$ bo'lsin. $x=2.5$. U holda $0^{2\cdot2.5-2.5}=0^{14\cdot2.5-28}$, $0=0$. $x=2.5$ berilgan tenglamaning ildizi bo'ladi.

4) $2x-5>0$, $2x-5\neq 1$ bo'lsin. U holda $2x^2-x=14x-28$, $2x^2-15x+28=0$ kvadrat tenglama hosil bo'ladi. Bu tenglamani yechib, $x_1=3.5$, $x_2=4$ ildizlarini topamiz. Oxirgi topilgan ikkala ildiz berilgan tenglamani qanoatlantiradi.

**Javob.** $x_1=2$, $x_2=2.5$, $x_3=3$, $x_4=3.5$, $x_5=4$

**2-misol.** $|x-2|^{x^2-2x}=|x-2|^{5x-10}$ tenglamani yeching.

**Yechish.** Aniqlanish sohasi $x\neq 2$. Berilgan tenglamadan $|x-2|^{x^2-7x+10}=1$ kelib chiqadi. Bundan $|x-2|^{x^2-7x+10}=1$ ikkala qismini *10* asosga ko'ra logarifmlasak

$$(x^2-7x+10)\lg|x-2|=\lg 1.$$

$$\begin{cases} x^2-7x+10=0 \\ \lg|x-2|=0 \end{cases} \quad \begin{cases} x_1=2, x_2=5 \\ |x-2|=1 \end{cases} \quad \begin{cases} x_1=2, x_2=5 \\ x-2=1 \\ x-2=-1 \end{cases} \quad \begin{cases} x_1=2, x_2=5 \\ x_3=3 \\ x_4=1 \end{cases}$$

Topilgan ildizlardan $x=5, x=3, x=1$ ildizlar tenglamaning ildizi bo'ladi.

**Javob.** $x=5, x=3, x=1$.

(1) tenglamani yechish uchun o'quv qo'llanmalarda turli xil qarash mavjud. Ba'zi adabiyotlarda faqat $u(x)>0$ hol bilan chegaralanadilar. Bu [illegible] ma'noda to'g'ridir. DTM „Axborotnoma"larida (1) tenglamani $u(x)>0$ bo'lgan holdagi yechimi bilan chegaralangan.

[illegible] Tenglama yechimlari [illegible]

A) 1 B) 2 C) 3 D) 4 E) 5

**Yechish.** Aniqlanish sohasi $x^2-6x+8\neq 0,\quad x\neq 2,\ x\neq 4$.
Berilgan tenglama $|x^2-6x+8|^{6-x-1}=0$ tenglamaga teng kuchli,yoki $|x^2-6x+8|^{5-x}=0$.
Bundan $(5-x)\lg|x^2-6x+8|=0$
1) *5-x=0, x=5*
2) $\lg|x^2-6x+8|=0,\quad |x^2-6x+8|=1,\quad \left[\begin{array}{l}x^2-6x+8=1\\ x^2-6x+8=-1\end{array}\right.\quad \left[\begin{array}{l}x^2-6x+7=0\\ (x-3)^2=0\end{array}\right.\quad \left[\begin{array}{l}x=3\pm\sqrt{2}\\ x=3\end{array}\right.$
Demak, berilgan tenglama 4 ta $x_{1,2}=3\pm\sqrt{2}\quad x_3=3,\quad x_4=5$ ildizlarga ega.
**Javob.** 4 (D)
4(98-4-46). Tenglamani ildizlari yig'indisini toping. $x^{\sqrt{x}}=\sqrt{x}^{x}$
A)5 B)10 C)11 D)4 E)8
**Yechish.** Aniqlanish sohasi *x>0*. Berilgan tenglamani ikkala qismini *10* asosga ko'ra logarifmlab ketma -ket hosil qilamiz. $\sqrt{x}\lg x-\frac{x}{2}\lg x=0.\quad (\sqrt{x}-\frac{x}{2})\lg x=0$

$$\left[\begin{array}{l}\sqrt{x}-\frac{x}{2}=0\\ \lg x=0\end{array}\right.\quad \left[\begin{array}{l}\sqrt{x}=\frac{x}{2}\\ x=1\end{array}\right.\quad \left[\begin{array}{l}x=\frac{x^2}{4}\\ x=1\end{array}\right.\quad \left[\begin{array}{l}x=0,x=4\\ x=1\end{array}\right..$$

Topilgan ildizlardan tenglamani faqat $x_1=1,x_2=4$ ildizlar qanoatlantiradi.
$x_1+x_2=1+4=5$
**Javob.** 5 (A)

## 2. $u(x)^{v(x)}>u(x)^{v(x)}$ ko'rinishidagi tengsizliklarni yechish

$a^b-a^c$ ifoda *a>1* bo'lsa,*b-c* ifoda bilan bir xil ishoraga ega.Agar *0<a<1* bo'lsa, $a^b-a^c$ va *b-c* ifodalar qarama qarshi ishoralarga ega.
Har ikkala holni bitta holga birlashtirish mumkin. Demak, $a^b-a^c$ ifoda *(a-1)(b-c)* ifodalar bilan bir xil ishoraga ega.

**5-misol.** $(x^2+x+1)^x<1$ tengsizlikni yeching.
**Yechish.** 1-usul. Aniqlanish sohasini topaylik. $x^2+x+1$ kvadrat uchhad diskriminanti *D=-3<0* va bosh koeffitsienti *a=1>0* bo'lganligi sababli barcha *x* lar uchun $x^2+x+1>0$ tengsizlik bajariladi.Demak, aniqlanish sohasi R dan iborat. $(x^2+x+1)^x<(x^2+x+1)^0$
Ikki holni qaraymiz.
**1-hol.** $\begin{cases}x^2+x+1>1\\ x<0\end{cases}\quad \begin{cases}x(x+1)>0\\ x<0\end{cases}\quad \begin{cases}x+1<0\\ x<0\end{cases}\quad \begin{cases}x<-1\\ x<0\end{cases}\quad x<-1$

**2-hol.** $\begin{cases} x^2+x+1<1 \\ x>0 \end{cases}$ $\begin{cases} x(x+1)<0 \\ x>0 \end{cases}$ $\begin{cases} x+1<0 \\ x>0 \end{cases}$ $\begin{cases} x<-1 \\ x>0 \end{cases}$ $\varnothing$

**Javob.** $x\in(-\infty;-1)$

**2-usul.** Aniqlanish sohasi $R$ dan iborat. $(x^2+x+1)^x-(x^2+x+1)^0$ ifodaning ishorasi yuqoridagi ko'rsatmaga asosan $(x^2+x+1-1)(x-0)$ ifodani ishorasi bilan bir xil bo'ladi.

$(x^2+x)x<0$ tengsizlikni yechamiz.

$x^2(x+1)<0$ $\begin{cases} x\neq 0 \\ x+1<0 \end{cases}$ $\begin{cases} x\neq 0 \\ x<-1 \end{cases}$ $x<-1$

**Javob.** $x\in(-\infty;-1)$

6(96-3-88). Tengsizlik $x$ ning qanday qiymatlarida o'rinli?

$(x+2)^{\log_2(x^2+1)}<(x+2)^{\log_2(2x+9)}$

A) $(-4,5;\infty)$ B) $(-2;4)$ C) $(4;\infty)$ D) $(-1;4)$ E) $(-2;-1)$

**Yechish. 1-usul.** Aniqlanish sohasi: $\begin{cases} 2x+9>0 \\ x+2>0 \end{cases}$ $\begin{cases} x>-4,5 \\ x>-2 \end{cases}$ $x>-2$

**1-hol.**

$\begin{cases} x+2>1 \\ \log_2(x^2+1)<\log_2(2x+9) \end{cases}$ $\begin{cases} x>-1 \\ x^2+1<2x+9 \end{cases}$ $\begin{cases} x>-1 \\ x^2-2x-8<0 \end{cases}$ $\begin{cases} x>-1 \\ (-2;4) \end{cases}$ $(-1;4)$

**2-hol.**

$\begin{cases} 0<x+2<1 \\ \log_2(x^2+1)>\log_2(2x+9) \end{cases}$ $\begin{cases} -2<x<-1 \\ x^2+1>2x+9 \end{cases}$ $\begin{cases} -2<x<-1 \\ (x+2)(x-4)>0 \end{cases}$ $\begin{cases} (-2;-1) \\ (-\infty;-2)\cup(4;\infty) \end{cases}$ $\varnothing$

**Javob.** *(-1;4)* (D)

**2-usul.** Yuqoridagi ikki holni bitta holga almashtirish mumkin. Yuqoridagi ko'rsatmaga asosan

$(x+2)^{\log_2(x^2+1)}-(x+2)^{\log_2(2x+9)}<0$ $(x+2-1)(\log_2(x^2+1)-\log_2(2x+9))<0$

$(x+1)\log_2\frac{x^2+1}{2x+9}<0$ $(x+1)(2-1)(\frac{x^2+1}{2x+9}-1)<0$ $\frac{(x+1)(x+2)(x-4)}{2(x+4,5)}<0$

Oxirgi tengsizlikni intervallar usuli bilan tengsizlikni aniqlanish sohasini hisobga olib yechsak *(-1;4)*

**Javob.** *(-1;4)* (D)

7(96-13-29). Tengsizlik $x$ ning qanday qiymatlarida o'rinli?

$(x-2)^{\log_3(x^2-5x+5)}<(x-2)^{\log_3 [illegible]}$

A) $(3;\infty)$ B) $(2;4)$ C) $(\frac{5+\sqrt{5}}{2};4)$ D) $(-\infty;2)\cup(4;\infty)$ E) $(-\infty;\frac{5-\sqrt{5}}{2})\cup(\frac{5+\sqrt{5}}{2};\infty)$

**Yechish.** Aniqlanish sohasini topaylik. $\begin{cases} x-2>0 \\ x^2-5x+5>0 \\ x-3>0 \end{cases}$ $\begin{cases} x>2 \\ (-\infty;\frac{5-\sqrt{5}}{2})\cup(\frac{5+\sqrt{5}}{2};\infty) \\ x>3 \end{cases}$ $(\frac{5+\sqrt{5}}{2};\infty)$

**1-hol.**

$$\begin{cases} x-2>1 \\ \log_2(x^2-5x+5)<\log_2(x-3) \end{cases} \quad \begin{cases} x>-1 \\ x^2-5x+5<x-3 \\ x-3>0 \end{cases} \quad \begin{cases} x>-1 \\ x^2-6x+8<0 \\ x>3 \end{cases} \quad \begin{cases} x>-1 \\ (2;4) \\ x>3 \end{cases} \quad (3;4)$$

**2-hol.** $\begin{cases} 0<x-2<1 \\ \log_2(x^2-5x+5)>\log_2(x-3) \end{cases}$ $\begin{cases} 2<x<3 \\ x^2-5x+5>x-3 \\ x-3>0 \end{cases}$ $\begin{cases} 2<x<3 \\ x^2-6x+8>0 \\ x>3 \end{cases}$ $\varnothing$

Aniqlanish sohasini hisobga olsak tengsizlikning yechimi $(\frac{5+\sqrt{5}}{2};4)$

**Javob.** $(\frac{5+\sqrt{5}}{2};4)$ (C)

## 3. $\log_{u(x)} v(x) \le 0$ ko'rinishidagi tengsizliklarni yechish

$\log_a b$ ifoda $a>1$ bo'lsa, $b-1$ ifoda bilan bir xil ishoraga ega bo'ladi. Agar $0<a<1$ bo'lsa, $b-1$ bilan qarama-qarshi ishoraga ega bo'ladi. Har ikkala holni bitta holga birlashtirish mumkin. $\log_a b$ ifoda $(a-1)(b-1)$ ifoda bilan bir xil ishoraga ega bo'ladi. Faqat aniqlanish sohasini hisobga olish lozim.

**8-misol.** $\log_{x^2-6x+8}(x-4)>0$ tengsizlikni yeching.

**Yechish.** 1) Aniqlanish sohasini topaylik. $\begin{cases} x^2-6x+8>0 \\ x^2-6x+8\ne 1 \\ x-4>0 \end{cases}$

$$\Leftrightarrow \begin{cases} (x-2)(x-4)>0 \\ x^2-6x+7\ne 0 \\ x>4 \end{cases} \Leftrightarrow \begin{cases} x>2 \\ x\ne 3\pm\sqrt{2} \\ x>4 \end{cases} \Leftrightarrow (4;3+\sqrt{2})\cup(3+\sqrt{2};\infty)$$

2) yuqoridagi ko'rsatmaga asosan $\log_{x^2-6x+8}(x-4)$ va $(x^2-6x+8-1)(x-4-1)$ ifodalar bir xil ishoraga ega. $(x^2-6x+7)(x-5)>0$ tengsizlikni yechimi berilgan tengsizlikni aniqlanish sohasini hisobga olganda $(4;3+\sqrt{2})\cup(5;\infty)$

**Javob.** $(4;3+\sqrt{2})\cup(5;\infty)$

**9-misol.** $\log_{2-x}(x+2)\log_{x+3}(3-x)\le 0$ tengsizlikni yeching.

**Yechish.** 1) Aniqlanish sohasini topaylik. $\begin{cases}2-x\ne 1\\2-x>0\\x+2>0\\x+3>0\\x+3\ne 1\\3-x>0\end{cases}\Leftrightarrow\begin{cases}x\ne 1\\x<2\\x>-2\\x>-3\\x\ne -2\\x<3\end{cases}\Leftrightarrow(-2;1)\cup(1;2)$

2) yuqoridagi ko'rsatmaga asosan $(2-x-1)(x+2-1)(x+3-1)(3-x-1)\le 0$, $(x-1)(x+1)(x+2)(x-2)\le 0$. Bu tengsizlikni berilgan tengsizlikning aniqlanish sohasini hosobga olgan holdagi yechimi $(-2;-1]\cup(1;2)$

**Javob.** $(-2;-1]\cup(1;2)$

## 4. $y=u(x)^{v(x)}$ funksiyaning hosilasi

10(02-10-28) $y=x^x$ funksiyaning hosilasini toping.

A) $x^x(1+\ln x)$ B) $x^{x-1}\cdot\dfrac{\ln x+1}{\ln x}$ C) $x^x$ D) $x^x\ln x$ E) $x^{x-1}$

**Yechish.** $y=x^x=e^{x\ln x}$ ekanligidan

$y'=e^{x\ln x}(x\ln x)'=x^x(x'\ln x+x(\ln x)')=x^x(\ln x+1)$

**Javob.** $x^x(\ln x+1)$ (A)

**Umuman,** $y=u(x)^{v(x)}$ **funksiyaning hosilasi uchun**

$$y'=u(x)^{v(x)}\left(v'(x)\ln u(x)+v(x)\frac{1}{u(x)}u'(x)\right)$$

**formula o'rinli.**

**11-misol.** $y=(\sin x)^{\cos x}$ funksiyaning hosilasini toping.

**Yechish.** Yuqoridagi formulaga asosan:

$$y'=(\sin x)^{\cos x}\left(-\sin x\ln\sin x+\cos x\frac{1}{\sin x}\cos x\right)=(\sin x)^{\cos x}(-\sin x\ln\sin x+ctg\,x\cos x).$$

12(00-10-55). Sistemani yechimlarini ifodalovchi nuqtalar orasidagi masofani toping.$(x>0)$ $\begin{cases} x^{\sqrt{y}} = y \\ y^{\sqrt{y}} = x^4 \end{cases}$

A) $\sqrt{7}$ B) 4 C) $\sqrt{10}$ D) $2\sqrt{2}$ E) 9

13(98-10-78). Tenglamaning nechta ildizi bor? $|x^2-2x-1|^{x-7} = |x^2-2x-1|$

A) 1 B) 2 C) 3 D) 4 E) 5

14(00-8-6). Tenglamani yeching. $x^{\log_3 x^2+\log_3^2 x-10} = \frac{1}{x^2}$

A) $1;9;\frac{1}{81}$ B) $1;9$ C) $1;\frac{1}{81}$ D) $9;\frac{1}{81}$ E) $4;1;\frac{1}{81}$

15(96-9-30). Tengsizlik $x$ ning qanday qiymatlarida o'rinli?

$x^{\log_{0,3}(x^2-5x+4)} < x^{\log_{0,3}(x-1)}$

A) $\varnothing$ B) $(4;\infty)$ C) $(5;\infty)$ D) $(-\infty;1)$ E) $(3;\infty)$

16(96-12-88). $x$ ning qanday qiymatlarida tengsizlik o'rinli?

$(x-2)^{\log_{\frac{1}{2}}(x^2-5x+5)} < (x-2)^{\log_{\frac{1}{2}}(x-3)}$

A) $(-\infty;2)\cup(4;\infty)$ B) $(2;4)$ C) $(\frac{5+\sqrt{5}}{2};4)$ D) $(4;\infty)$ E) $(-\infty;\frac{5-\sqrt{5}}{2})\cup(\frac{5+\sqrt{5}}{2};\infty)$

**17. Tenglamalarni yeching.**

1) $(x-3)^{x^2-x} = (x-3)^2$ 2) $(x^2-5x+7)^{x-2} = 1$ 3) $|x-3|^{x^2-x} = (x-3)^2$ 4) $|x|^{x^2-2x} = 1$

**18. Funksiyalarni hosilasini toping.**

1) $f(x) = x^{\operatorname{tg} x}$ 2) $g(x) = (x^2+2x+3)^{\ln x}$

## 18-§. Modulli tenglama va tengsizliklarni yechish

Ta'rif. $a$ sonining moduli deb, son o'qida sanoq boshidan shu songa mos keluvchi nuqtagacha bo'lgan masofaga aytiladi va $|a|$ deb belgilanadi.

$a$ sonning modulini quyidagicha yozish mumkin.

$$|a| = \begin{cases} a, & agar \quad a \geq 0 \quad bo'lsa \\ -a, & agar \quad a < 0 \quad bo'lsa \end{cases}$$

### 1. Son modulining xossalari

1. $|a| = |-a|$ 2. $|a| \geq a$, $|a| \geq -a$ 3. $|ab| = |a| \cdot |b|$ 4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, $bunda \quad b \neq 0$

5. $|a+b| \leq |a| + |b|$

6. $|a+b| \geq |a-b|$ 7. $|a-b| \geq ||a|-|b||$ 8. $\sqrt[2n]{a^{2n}} = |a|$

9. a) $|a+b| = |a| + |b|$ faqat $ab \geq 0$ bo'lganda va faqat shundagina bo'ladi;

b) $|a+b| < |a| + |b|$ faqat $ab < 0$ bo'lganda va faqat shundagina bo'ladi;

v) $|a| + |b| = a + b$ tenglik $a \geq 0$ va $b \geq 0$ bo'lganda bo'ladi.

10. a) $|a-b| = |a| - |b|$ faqat $(a-b)b > 0$ bo'lganda va faqat shundagina bo'ladi;

b) $|a-b| > |a| - |b|$ faqat $(a-b)b < 0$ bo'lganda va faqat shundagina bo'ladi;

11. Agar $n$ toq natural son bo'lsa, u holda $|a^n| = |a|^n$ bo'ladi.

12. Agar $n$ juft natural son bo'lsa, u holda $|a^n| = a^n$ bo'ladi.

13. $|a| < c$, $(c > 0) \Leftrightarrow -c < a < c$ .

14. $|a| > c$, $(c > 0) \Leftrightarrow \left[\begin{array}{l} a > c \\ a < -c \end{array}\right.$

15. $|a| = |b| \Rightarrow a = \pm b$

### 2. Modulli tenglamalar

1. $|f(x)| = f(x) \Leftrightarrow f(x) \geq 0$

2. $|f(x)| = -f(x) \Leftrightarrow f(x) \leq 0$

3. $|f(x)| = |g(x)| \Leftrightarrow \left[\begin{array}{l} f(x) = g(x) \\ f(x) = -g(x) \end{array}\right.$

4. $|f(x)| = a, (a \geq 0) \Leftrightarrow \left[\begin{array}{l} f(x) = a \\ f(x) = -a \end{array}\right.$

### 3. Modulli tengsizliklar

1. $|f(x)| < a, (a > 0) \Leftrightarrow -a < f(x) < a$

2. $|f(x)| > a, (a > 0) \Leftrightarrow \left[\begin{array}{l} f(x) > a \\ f(x) < -a \end{array}\right.$

3. $|f(x)| < |g(x)| \Leftrightarrow f^2(x) < g^2(x) \Leftrightarrow (f(x) - g(x))(f(x) + g(x)) < 0$

1(03-1-23). $|x^2 + 3x + 2| = |x^2 + 2x + 5| + |x - 3|$ tenglamani yeching.

A) 3;5 B) 4;6 C) $[3;\infty)$ D) $[0;3)$ E) $[3;10]$

**Yechish.** $a=x^2+2x+5$, $b=x-3$ deb olsak, $a+b=x^2+3x+2$ bo'ladi. $|a+b|=|a|+|b|$ tenglik bajarilmoqda. Demak, berilgan tenglama $ab\ge 0$ ga teng kuchli. Endi $(x^2+2x+5)(x-3)\ge 0$ tengsizlikni yechamiz. $x^2+2x+5=(x+1)^2+4>0$ bo'lganligi sababli oxirgi tengsizlik $x-3\ge 0$ gat eng kuchli. Bundan $x\ge 3$ javobni olamiz.

**Javob.** $[3;\infty)$ (C)

**2-misol.** Tenglamani yeching. $\left|\frac{x}{x-1}\right|+|x|=\frac{x^2}{|x-1|}$

**Yechish.** $\frac{x^2}{x-1}=\frac{x+x^2-x}{x-1}=\frac{x+x(x-1)}{x-1}=\frac{x}{x-1}+x$, $\frac{x^2}{|x-1|}=\frac{|x^2|}{|x-1|}=\left|\frac{x^2}{x-1}\right|$ bo'lganligi sababli berilgan tenglamani quyidagi ko'rinishda yozish mumkin.

$\left|\frac{x}{x-1}\right|+|x|=\left|\frac{x}{x+1}+x\right|$. U holda 9-a xossaga asosan berilgan tenglama $\frac{x}{x-1}\cdot x\ge 0$ tengsizlikka teng kuchli. Bundan $\frac{x^2}{x-1}\ge 0$, $x\in(1;\infty)$

**Javob.** $x\in(1;\infty)$

**3-misol.** $|x^3-1|+|2-x^3|=1$ tenglamani yeching

**Yechish.** $1=2-x^3+x^3-1$ bo'lganligi sababli berilgan tenglamani $|x^3-1|+|2-x^3|=|x^3-1+2-x^3|$ ko'rinishida yozish mumkin. 9-a xossaga asosan berilgan tenglama $(x^3-1)\cdot(2-x^3)\ge 0$ tenglamaga teng kuchli. $x^3=t$ deb belgilasak, $(t-1)\cdot(2-t)\ge 0$ ni yechimi $1\le t\le 2$ dan iborat. Belgilashni hisobga olsak, $1\le x^3\le 2$ $1\le x\le\sqrt[3]{2}$

**Javob.** $1\le x\le\sqrt[3]{2}$

**4-misol.** $|x^2-x|<|2-x|+|x^2-2|$ tengsizlikni yeching.

**Yechish.** $(x^2-x)-(2-x)=x^2-2$ bo'lganligi sababli, berilgan tengsizlikni $|x^2-2|>|x^2-x|-|2-x|$ ko'rinishda yozish mumkin. 10-b xossaga asosan berilgan tengsizlik $(2-x)(x^2-2)<0$ tengsizlikka teng kuchli. Yoki $(x-2)(x-\sqrt{2})(x+\sqrt{2})>0$ bu tengsizlikni yechimi $-\sqrt{2}<x<\sqrt{2}$, $x>2$

**Javob.** $-\sqrt{2}<x<\sqrt{2}$, $x>2$

$|x+a|+|x+b|=c$ tenglamada $c>0$; $ab<0$ bo'lsa, ya'ni $a$ va $b$ lar har xil ishorali bo'lib, $|a|+|b|=c$ bo'lsa, yechim $[-a;-b]$ yoki $[-b;-a]$ oraliqlardan iborat bo'ladi.

$|x+a|+|x+b|=c$ tenglamada $c>0$; $ab>0$ bo'lsa, ya'ni $a$ va $b$ lar bir xil ishorali bo'lib, $a$ va $b$ larda moduli kattasidan moduli kichigini ayirganda $c$ ga teng bo'lsa, u holda yechimlar $[-a;-b]$ yoki $[-b;-a]$ oraliqlardan iborat bo'ladi.

Agar $|x+a|+|x+b|=c$ tenglamada $|a-b|<c$ bo'lsa, tenglama $[a;b]$ kesmada yotmaydigan ikkita ildizga ega bo'ladi.

**Agar** $|x+a|+|x+b|=c$ **tenglamada** $|a-b|>c$ **bo'lsa, tenglama yechimga ega bo'lmaydi.**

**5-misol.** $|x+2|+|x+5|=3$ tenglamani yeching.

**Yechish.** $a=2$, $b=5$ desak, $|b-a|=|5-2|=3$ bo'ladi. U holda tenglamaning yechimi $[-5;-2]$ kesmadan iborat bo'ladi.

**6-misol.** $|x-1|+|x-4|=3$ tenglamani yeching.

**Yechish.** $a=-1$, $b=-4$ $|-4-(-1)|=3$ bo'lganligi sababli tenglama $[1;4]$ kesmadan iborat yechimga ega bo'ladi.

**7-misol.** $|x+2|+|x-3|=5$ tenglamani yeching.

**Yechish.** $a=-2$, $b=3$ $|a|+|b|=c$ bo'lganligi sababli berilgan tenglamaning yechimi $[-2;3]$ kesmadan iborat bo'ladi.

8(96-3-79). Tenglamaning ildizlari yig'indisini toping. $|x+3|+|x-1|+|x-4|=6$

A)ildizi yo'q B)0 C)-4 D)1 E)-2

**Yechish.** $a=3$, $b=-4$ har xil ishorali va $|b-a|=|-4-3|=7$ bo'lganligi sababli

berilgan tenglamada $\begin{cases}|x+3|+|x-4|\geq 7\\ |x-1|\geq 0\end{cases}$ ko'rinib turibdiki, $|x+3|+|x-1|+|x-4|\leq 6$

bo'lishi mumkin emas. Demak tenglama ildizga ega emas.

**Javob.** ildizi yo'q (A)

9(96-12-77). Tenglamaning ildizlari yig'indisini toping. $|x+4|+|x-2|+|x-3|=7$

A)2 B)ildizi yo'q C)0 D)-2 E)-1

**Yechish.** $|4|+|-3|=7$ bo'lgani uchun $|x+4|+|x-3|=7$ tenglama $[-4;3]$ dagi istalgan $x$ uchun bajariladi. Demak $\begin{cases}|x+4|+|x-3|=7\\ |x-2|=0\end{cases}$ shartlar bajarilishi kerak. U holda,

$$\begin{cases}|x+4|+|x-3|=7\\ |x-2|=0\end{cases}\Leftrightarrow\begin{cases}x\in[-4;3]\\ x=2\end{cases}\Leftrightarrow x=2$$

**Javob.** 2 (A)

10(96-9-20). Tenglama nechta ildizga ega? $|x+2|+|x|+|x-2|=4$

A)ildizi yo'q B)cheksiz ko'p C)1 ta D)2 ta E)4 ta

**Yechish.** $|2|+|-2|=4$ bo'lganligi sababli $|x+2|+|x|+|x-2|=4$ tenglik bajarilishi uchun $\begin{cases}|x+2|+|x-2|=4\\ |x|=0\end{cases}$ shart bajarilishi kerak.

$$\begin{cases}|x+2|+|x-2|=4\\ |x|=0\end{cases}\Leftrightarrow\begin{cases}x\in[-2;2]\\ x=0\end{cases}\Leftrightarrow x=0$$

**Javob.** 1 ta (C)

11(96-13-20). Tenglamani ildizlari nechta? $|x-4|+|x-1|+|x+2|=6$

A)ildizi yo'q B)2 C)3 ta D)1 ta E)cheksiz ko'p

**Yechish.** $a=4,\quad b=-2\quad |a-b|=6$ bo'lganligi sababli

$$\begin{cases}|x-4|+|x-2|=6\\ |x-1|=0\end{cases} \Leftrightarrow \begin{cases}x\in[2;4]\\ x=1\end{cases} \Leftrightarrow \varnothing$$

**Javob.** ildizi yo'q (A)

**12-misol.** $|x+5|+|x^2-9|+|x-3|=8$ tenglamani yeching.

**Yechish.** $|5|+|-3|=8$ bo'lganligi sababli $|x+5|+|x-3|=8$ tenglamaning yechimlari $[-5;3]$ kesmadan iborat bo'ladi. Yuqoridagi tenglama yechimga ega bo'lishi uchun quyidagi shart bajarilishi kerak: $\begin{cases}|x+5|+|x-3|=8\\ |x^2-9|=0\end{cases} \Leftrightarrow \begin{cases}x\in[-5;3]\\ x^2-9=0\end{cases} \Leftrightarrow \begin{cases}x\in[-5;3]\\ x=\pm3\end{cases} \Leftrightarrow x=\pm3$

**Javob.** $x=\pm3$

$|x-a|-|x-b|=c$ **tenglamada**

1) **agar** $|a-b|>|c|$ **bo'lsa, tenglama** $(a;b)$ **intervalga tegishli bo'lgan bitta yechimga ega bo'ladi;**
2) **agar** $|a-b|=|c|$ **bo'lsa, tenglama cheksiz ko'p yechimga ega bo'ladi;**
3) **agar** $|a-b|<|c|$ **bo'lsa, tenglama yechimga ega bo'lmaydi.**

## Mustaqil yechish uchun misollar

**13.** $|x+1|+|x-3|=8$ tenglama haqidagi quyidagi mulohazalardan qaysi biri to'g'ri?
A) Faqat bitta ildizi bor B) Ildizlari turli ishorali C) Ildizi yo'q D) Har ikkala ildizi manfiy E) Har ikkala ildizi musbat
**Javob.** $x_1=-3;\quad x_2=5$ (B)

14(99-10-23). Agar $x>y>0$ bo'lsa, $\left|\sqrt{xy}-\frac{x+y}{2}\right|+\left|\frac{x+y}{2}+\sqrt{xy}\right|$ ni soddalashtiring.

A) $x-y$ B) $2\sqrt{xy}$ C) $-2\sqrt{xy}$ D) $x+y$ E) $y-x$

15(98-4-24). Ushbu $|x^2-8x+7|=-7+8x-x^2$ tenglamaning barcha natural yechimlari yig'indisini toping.
A)8 B)40 C)25 D)28 E)aniqlab bo'lmaydi

16(99-8-4). Tengsizlikni qanoatlantiruvchi $x$ ning eng kichik natural qiymatini toping. $|x+1|+|x-4|>7$
A)1 B)3 C)6 D)5 E)2

**17-misol.** $|x+4|+|x^2-9|+|x-3|=7$ tenglamani yeching.

**Javob.** $x=\pm3$

**18-misol.** $|7-2x|=|5-3x|+|x+2|$ tenglamani yeching

**Javob.** $-2\le x\le\frac{5}{3}$

## 19-§. Aralash misollar

1(96-3-80). Ushbu *31323334...7980* sonning raqamlari yig'indisini toping.

A)473 B)480 C)460 D)490 E)453

**Yechish.1-usul.** *31323334...7980* sonini quyidagi tartibda yozaylik.

....313233 ...39
40414243 ...49
50515253 ...59
........ .......... ..
70717273 ...7980

Birinchi qatorga 30 ni qo'shaylik.

30313233...39
40414243...49
50515253...59
.....................
70717273...7980

Har bir qatordagi raqamlar yig'indisini topaylik.

$$\left.\begin{array}{l} 3\cdot 10+(1+2+3+...+9) \\ 4\cdot 10+(1+2+3+...+9) \\ 5\cdot 10+(1+2+3+...+9) \\ ..................................... \\ 7\cdot 10+(1+2+3+...+9)+8 \end{array}\right| +$$

Barcha qatordagi sonlarni qo'shsak,

$$(3+4+5+...+7)\cdot 10+(1+2+3+...9)\cdot 5+8=\frac{3+7}{2}\cdot 5\cdot 10+\frac{1+9}{2}\cdot 9\cdot 5+8=10\cdot 25+5\cdot 9\cdot 5+8=483$$

*483-3=480*

**2-usul.** $\overline{1234...ab}$ sonning raqamlar yig'indisi quyidagi formula bilan hisoblanadi.

$$S(\overline{ab})=5a^2+ab+41a+\frac{b(b+1)}{2}$$

Berilgan misol uchun

$$S(80)-S(30)=5\cdot 8^2+8\cdot 0+41\cdot 8+\frac{0(0+1)}{2}-(5\cdot 3^2+3\cdot 0+41\cdot 3)+\frac{0(0+1)}{2}=5\cdot 64+328-45-123=480$$

**Javob.** 480 (B)

2(00-9-15). $\overline{ABC}+\overline{MN}=\overline{FEDP}$ ($\overline{MN}$-ikki xonali son, $\overline{ABC}$-uch xonali son) $F^{M+N}+A^{P}$ ni hisoblang.

A)Aniqlab bo'lmaydi B)1 C)2 D)9 E)10

**Yechish.** $\overline{ABC}$-uch xonali sonni $\overline{MN}$-ikki xonali songa qo'shganda to'rt xonali $\overline{FEDP}$ son hosil bo'lsa $F=1$ bo'ladi. Masalan: $985+90=1075$. $M$ va $N$ ni topish shart emas,chunki $1^{M+N}=1$ bo'ladi. Endi $A$ ni topamiz.Uch xonali sonni ikki xonali songa qo'shganda to'rt xonali son hosil bo'lishi uchun $\overline{ABC}\geq 901$ bo'lishi lozim.Bundan $A=9$ ekanligi kelib chiqadi.U holda $F^{M+N}+A^F=1^{M+N}+9^1=1+9=10$

**Javob.** 10 (E)

**3**(99-5-6). $\overline{abc}+\overline{dec}=\overline{fkmc}$ ($\overline{abc}$ va $\overline{dec}$ - uch xonali sonlar, $\overline{fkmc}$ to'rt xonali son). $f^{a+d}+(b+d)^c$ ni hisoblang.

A)aniqlab bo'lmaydi B)1 C)2 D)3 E)4

**Yechish.** $\overline{abc}+\overline{dec}=\overline{fkmc}$ tenglikdan $c=0$ ekanligi kelib chiqadi.Endi $f$ ni aniqlaymiz.Raqamlari takrorlanmaydigan $\overline{abc}$ va $\overline{dec}$ sonlarni qo'shganda raqamlari takrorlanmaydigan to'rt xonali son hosil bo'lganligi sababli $f=1$ bo'ladi. Natijada $f^{a+d}+(b+d)^c=1^{a+d}+(a+d)^0=1+1=2$

**Javob.** 2 (C)

**4**(98-12-70). Agar $a$ toq son bo'lsa,quyidagi sonlardan qaysi biri albatta toq son bo'ladi?

A) $a^2+27$ B) $5(a+13)$ C) $a^8$ D) $\frac{a(a+3)}{2}$ E) $\frac{(a+1)(a+7)}{2}$

**Yechish.**Toq sonning har qanday natural ko'rsatkichli darajasi toqdir.Shuning uchun $a^8$ soni $a$ toq son bo'lsa toq son bo'ladi.

**Javob.** $a^8$ (C)

**5**(03-2-69). Agar $m\in N$ bo'lsa,quyidagi keltirilgan sonlarning qaysi biri doimo juft bo'ladi?

A) $m(m+6)$ B) $m^2+18m$ C) $\frac{m^2-16}{m^2+4}$ D) $m^2+13m$ E) $m^4+8$

**Yechish.** $m\in N$ bo'lganligi uchun $m=2k$ $m=2k-1$ ko'rinishda yozish mumkin.

U holda 1) $m=2k$ bo'lsa $m^2+13m=4k^2+13\cdot 2k=2k(2k+13)$ juft 2) $m=2k-1$ bo'lsa

$$m^2+13m=(2k-1)^2+13\cdot(2k-1)=4k^2-4k+1-26k-13=4k^2-30k-12=2\cdot(2k^2-15k-6)$$

Juft son

**Javob.** $m^2+13m$ (D)

**6**(99-8-25). Ikkita natural sonni 5ga bo'lganda mos ravishda 1 va 3 qoldiq hosil bo'ladi.Bu sonlar kvadratlarining yig'indisini 5ga bo'lganda qoldiq nechaga teng bo'ladi?

A)4 B)1 C)2 D)3 E)0

**Yechish.** $m=5k+1$  $n=5q+3$  $m^2+n^2=?$

$m^2+n^2=(5k+1)^2+(5q+3)^2=25k^2+10k+1+25q^2+30q+9=25(k^2+q^2)+10(k+3q)+10=$

$5(5(k^2+q^2)+2(k+3q)+2)+0=5e+0$

**Javob.** 0 (E)

7(01-7-8). *2146,1991* va *1805* sonlarining har birini qanday natural songa bo'lganda qoldiqlari bir xil chiqadi?

A)7 B)13 C)21 D)31 E)37

**Yechish.**1) *2146-1991=155* 2)*1991-1805=186* , $155=5\cdot31$, $186=2\cdot93=2\cdot3\cdot31$

**Javob.** 31 (D)

8(99-3-2). Hisoblang: *1-3+5-7+9-11+...+97-99*

A)-46 B)-48 C)-50 D)-52 E)-54

**Yechish.** *(1-3)+(5-7)+(9-11)+...+(97-99)=-2+(-2)+(-2)+...+(-2)=-* $2\cdot25$ *=-50*

**Javob.** *-50* (C)

9(01-1-2). Yig'indini hisonlang: *4-7+8-11+12-15+...+96-99*

A)-75 B)-80 C)-72 D)-63 E)-60

**Yechish.** *S=(4-7)+(8-11)+(12-16)+...+(96-99);* Bunday juftliklar soni *4* dan *96* gacha *4* ga karrali sonlar soniga teng.Bunday sonlar $4\cdot n=96$ dan *n=24* ta bo'ladi.U holda

*S=-3+(-3)+(-3)+...+(-3)=-3*$\cdot24$*=-72*

**Javob.** *-72* (C)

10(98-12-14). $m=\frac{1107}{1109}, n=\frac{2216}{2220}$ sonlar uchun quyidagi munosabatlardan qaysi biri to'g'ri?

A)$m<n$ B)$m>n$ C)$m=n$ D)$n=m+1$ E)$n=\frac{2m+2}{2220}$

**Yechish.** Berilgan sonlarning teskarisini taqqoslaymiz.

$\frac{1}{m}=\frac{1109}{1107}=1+\frac{2}{1107}$; $\frac{1}{n}=\frac{2220}{2216}=\frac{1110}{1108}=1+\frac{2}{1108}$ $\frac{2}{1107}>\frac{2}{1108}$ bo'lganligi uchun $\frac{1}{m}>\frac{1}{n}$ *m<n*

**Javob.** *m<n* (A)

11(98-12-62). Hisoblang: $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+...+\frac{1}{999\cdot1000}$

A)0,750 B)1,125 C)0,998 D)1,450 E)0,999

**Yechish.** Yig'indidagi qo'shiluvchilarning maxrajlari bir-biridan 1 ga farq qiladi va ko'paytuvchilarning ikkinchisi bilan navbatdagi kasrning maxraji boshlangan.

$(1\cdot2; 2\cdot3; 3\cdot4; ...; 999\cdot1000)$ har bir qo'shiluvchini quyidagicha yozamiz.

$\frac{1}{1\cdot2}=\frac{1}{1}-\frac{1}{2}$; $\frac{1}{2\cdot3}=\frac{1}{2}-\frac{1}{3}$; $\frac{1}{3\cdot4}=\frac{1}{3}-\frac{1}{4}$;... $\frac{1}{999\cdot1000}=\frac{1}{999}-\frac{1}{1000}$ Bularni hisobga olsak, $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+...+\frac{1}{999\cdot1000}=$

$$(1-\frac{1}{2})+(\frac{1}{2}-\frac{1}{3})+(\frac{1}{3}-\frac{1}{4})+...+(\frac{1}{999}-\frac{1}{1000})=1-\frac{1}{1000}=\frac{999}{1000}=0,999$$

**Javob.** 0,999 (E)

12(00-2-4). Hisoblang: $\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\frac{1}{99}+\frac{1}{143}+\frac{1}{195}$

A) $\frac{4}{15}$ B) $\frac{7}{15}$ C) $\frac{17}{45}$ D) $\frac{11}{15}$ E) $\frac{2}{15}$

**Yechish.** Berilgan yig'indidagi kasrning maxrajini ko'paytuvchilarga quyidagicha ajrataylik: $\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\frac{1}{7\cdot9}+\frac{1}{9\cdot11}+\frac{1}{11\cdot13}+\frac{1}{13\cdot15}$

Kasrlarning maxrajlari bir -biridan 2ga farq qiladi. Ular uchun quyidagi tenglik o'rinli:

$\frac{1}{3\cdot5}=\frac{1}{2}\cdot(\frac{1}{3}-\frac{1}{5})$; $\frac{1}{5\cdot7}=\frac{1}{2}\cdot(\frac{1}{5}-\frac{1}{7})$; $\frac{1}{7\cdot9}=\frac{1}{2}\cdot(\frac{1}{7}-\frac{1}{9})$; $\frac{1}{9\cdot11}=\frac{1}{2}\cdot(\frac{1}{9}-\frac{1}{11})$;

$\frac{1}{11\cdot13}=\frac{1}{2}\cdot(\frac{1}{11}-\frac{1}{13})$; $\frac{1}{13\cdot15}=\frac{1}{2}\cdot(\frac{1}{13}-\frac{1}{15})$ Bularni hisobga olsak

$$\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\frac{1}{7\cdot9}+\frac{1}{9\cdot11}+\frac{1}{11\cdot13}+\frac{1}{13\cdot15}=$$

$$\frac{1}{2}(\frac{1}{3}-\frac{1}{5})+\frac{1}{2}(\frac{1}{5}-\frac{1}{7})+\frac{1}{2}(\frac{1}{7}-\frac{1}{9})+\frac{1}{2}(\frac{1}{9}-\frac{1}{11})+\frac{1}{2}(\frac{1}{11}-\frac{1}{13})+\frac{1}{2}(\frac{1}{13}-\frac{1}{15})=\frac{1}{2}(\frac{1}{3}-\frac{1}{15})=\frac{1}{2}\cdot\frac{4}{15}=\frac{2}{15}$$

**Javob.** $\frac{2}{15}$ (E)

**Yuqoridagi 2 ta misolni yechishda diqqat bilan e'tibor bersak 1-misolda $(1-\frac{1}{2})$ qo'shiluvchilardagi birinchi qo'shiluvchi 1, oxirgi qo'shiluvchidagi $(\frac{1}{999}-\frac{1}{1000})$ ikkinchi qo'shiluvchi $\frac{1}{1000}$ sonini bilish yig'indini hisoblash uchun yetarli ekanligi ko'rinib turibdi, chunki qolgan qo'shiluvchilarning yig'indisi 0 ga teng bo'ladi.** $1-\frac{1}{1000}=\frac{999}{1000}=0,999$.

**Umuman olganda** $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+...+\frac{1}{n\cdot(n+1)}=1-\frac{1}{n+1}=\frac{n}{n+1}$ **tenglik to'g'ri. Bunday misollarni uch-to'rttasini mustaqil yechish bilan ularni tez yechish ko'nikmasini hosil qilasiz.**

13(01-7-43). $\frac{2}{5\cdot7}+\frac{2}{7\cdot9}+\frac{2}{9\cdot11}+...+\frac{2}{73\cdot75}$ ni hisoblang.

A)$\frac{16}{75}$ B)$\frac{28}{75}$ C)$\frac{1}{5}$ D)$\frac{14}{75}$ E)$\frac{2}{5}$

**Yechish.** Ko'paytuvchilarni qavsdan tashqariga chiqarsak oldingi misollar kabi yechiladi. Bu misollarni birdaniga quyidagicha ham yechish mumkin.

$\frac{2}{5\cdot7}+\frac{2}{7\cdot9}+\frac{2}{9\cdot11}+...+\frac{2}{73\cdot75}=$

$\frac{7-5}{5\cdot7}+\frac{9-7}{7\cdot9}+\frac{11-9}{9\cdot11}+...+\frac{75-73}{73\cdot75}=(\frac{1}{5}-\frac{1}{7})+(\frac{1}{7}-\frac{1}{9})+(\frac{1}{9}-\frac{1}{11})+...+(\frac{1}{73}-\frac{1}{75})=\frac{1}{5}-\frac{1}{75}=\frac{15-1}{75}=\frac{14}{75}$

**Javob.** $\frac{14}{75}$ (D)

14(02-3-16). $\frac{x}{3}+\frac{x}{15}+\frac{x}{35}+\frac{x}{63}+\frac{x}{99}+\frac{x}{143}=12$ tenglamani yeching.

A)26 B)13 C)18 D)16 E)24

**Yechish.** $x\cdot(\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\frac{1}{7\cdot9}+\frac{1}{9\cdot11}+\frac{1}{11\cdot13})=12$

$x\cdot(\frac{1}{2}(1-\frac{1}{3})+\frac{1}{2}(\frac{1}{3}-\frac{1}{5})+\frac{1}{2}(\frac{1}{5}-\frac{1}{7})+\frac{1}{2}(\frac{1}{7}-\frac{1}{9})+\frac{1}{2}(\frac{1}{9}-\frac{1}{11})+\frac{1}{2}(\frac{1}{11}-\frac{1}{13}))=12$

$x\cdot(\frac{1}{2}\cdot1-\frac{1}{2}\cdot\frac{1}{13})=12$; $\frac{x}{2}(1-\frac{1}{13})=12$; $\frac{x}{2}\cdot\frac{12}{13}=12$; $\frac{6x}{13}=12$; $x=12\cdot\frac{13}{6}=26$; $x=26$

**Javob.** 26 (A)

15(98-4-13). Agar $5^x+5^{-x}=7$ bo'lsa, $25^x+25^{-x}$ ning qiymati qancha bo'ladi?

A)47 B)49 C)51 D)29 E)38

**Yechish. 1-usul.** $5^x+5^{-x}=7$ bo'lsa, $(5^x+5^{-x})^2=7^2$ bo'ladi. Bundan

$5^{2x}+2\cdot5^x\cdot5^{-x}+5^{-2x}=49$ $25^x+25^{-x}=49-2=47$

**2-usul.** $25^x+25^{-x}=(5^{2x}+2\cdot5^x\cdot5^{-x}+5^{-2x})-2\cdot5^x\cdot5^{-x}=(5^x+5^{-x})^2-2=7^2-2=47$

**Javob.** 47 (A)

16(98-12-72). Agar $49^x+49^{-x}=7$ bo'lsa, $7^x+7^{-x}$ ni toping.

A)4 B)$\sqrt{7}$ C)$\sqrt{5}$ D)14 E)3

**Yechish.** $7^x+7^{-x}=A$ bo'lsin. Bu tenglikni ikkala qismini kvadratga ko'tarsak

$(A>0)$ $(7^x+7^{-x})^2=A^2$ Bundan $49^x+2\cdot7^x\cdot7^{-x}+49^{-x}=A^2$; $(49^x+49^{-x})+2=A^2$

$7+2=A^2$ $A=\sqrt{9}=3$ $A=3$

**Javob.** 3 (E)

17(98-7-26)* $-2a^2-2b^2$ ni $a+b$ va $ab$ orqali ifodalang

A)$4ab-2(a+b)^2$ B)$2(a+b)^2-4ab$ C)$4ab+2(a+b)^2$ D)$-4ab-2(a+b)^2$

E)$2(a+b)^2-2ab$

**Yechish. 1-usul.** Javoblarni tekshirish orqali yechish. (Mustaqil tekshiring).

**2-usul.** $-2a^2-2b^2=-2(a^2+b^2)=-2(a^2+2ab+b^2)+4ab=-2(a+b)^2+4ab$

**Javob.** $4ab-2(a+b)^2$ (A)

18(99-2-9)*. $\alpha$ va $\beta$ irratsional sonlar ($\alpha\neq\beta$), $\alpha+\beta$ esa ratsional son. Quyidagilardan qaysi biri ratsional son bo'ladi?

A) $\alpha-2\beta$ B) $\alpha^2+2\alpha\beta+\beta^2$ C) $\frac{\alpha+2\beta}{2}$ D) $2\alpha+\beta$ E) $\alpha-3\beta$

**Yechish.** Agar $\alpha$ va $\beta$ irratsional sonlar, $\alpha+\beta$ esa ratsional son bo'lsa ratsional sonning kvadrati ratsional son bo'ladi. Shuning uchun $\alpha^2+2\alpha\beta+\beta^2=(\alpha+\beta)^2$-ratsional son bo'ladi.

**Javob.** $\alpha^2+2\alpha\beta+\beta^2$ **(B)**

19(99-6-35)*. $a=2^5+2^{-5}$ va $b=2^5-2^{-5}$ bo'lsa, $a^2-b^2$ nimaga teng?

A)0 B)2 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E)4

**Yechish. 1-usul.**

$a^2-b^2=(2^5+2^{-5})^2-(2^5-2^{-5})^2=2^{10}+2\cdot2^5\cdot2^{-5}+2^{-10}-2^{10}+2\cdot2^5\cdot2^{-5}-2^{-10}=4$

**2-usul.** $a^2-b^2=(a-b)(a+b)=(2^5+2^{-5}-2^5+2^{-5})(2^5+2^{-5}+2^5-2^{-5})=2\cdot2^{-5}\cdot2\cdot2^5=4$

**Javob.** 4 **(E)**

20(99-6-40). $a^2+\frac{9}{a^2}=22$ bo'lsa, $a-\frac{3}{a}$ nimaga teng?

A)3 B)-3 C)2 D)±4 E)1

**Yechish.** $a^2+\frac{9}{a^2}=22$; $(a-\frac{3}{a})^2+2a\cdot\frac{3}{a}=22$ $(a-\frac{3}{a})^2=22-6$ $(a-\frac{3}{a})^2=16$

$a-\frac{3}{a}=\pm4$

**Javob.** (D)

21(01-8-7). Agar $a+\frac{1}{a}=3$ bo'lsa, $\frac{a^6+1}{a^3}$ nimaga teng?

A)27 B)24 C)18 D) $21\frac{1}{3}$ E)aniqlab bo'lmaydi

**Yechish.** 1) $\frac{a^6+1}{a^3}=a^3+\frac{1}{a^3}$

2) $(a+\frac{1}{a})^3=3$ $a^3+\frac{1}{a^3}+3\cdot a\cdot\frac{1}{a}(a+\frac{1}{a})=27$ $a^3+\frac{1}{a^3}+3\cdot3=27$ $a^3+\frac{1}{a^3}=18$

**Javob.** 18 (C)

22(02-5-7). Agar $a-\frac{1}{a}=\sqrt{7}$ bo'lsa, $a^4+\frac{1}{a^4}$ nimaga teng?

A)81 B)79 C)49 D)63 E)77

**Yechish. 1-usul.** 1) $a-\frac{1}{a}=\sqrt{7}$ ni ikkala qismini kvadratga ko'taramiz. $(a-\frac{1}{a})^2=(\sqrt{7})^2$ , $a^2-2a\cdot\frac{1}{a}+\frac{1}{a^2}=7$ $\quad a^2+\frac{1}{a^2}=9$

2) $a^2+\frac{1}{a^2}=9$ ikkala qismini kvadratga ko'taramiz. $(a^2+\frac{1}{a^2})^2=9^2$

$a^4+2a^2\cdot\frac{1}{a^2}+\frac{1}{a^4}=81$ $\quad a^4+\frac{1}{a^4}=79$

**Javob.** 79 (B)

**2-usul.** $a^4+\frac{1}{a^4}=(a^2+\frac{1}{a^2})^2-2=((a-\frac{1}{a})^2+2)^2-2=((\sqrt{7})^2+2)^2-2=9^2-2=81-2=79$

**Javob.** 79 (B)

23(98-4-11). Agar $x$ natural son bo'lsa, quydagi sonlardan qaysi biri albatta juft son bo'ladi?

$A)\frac{x(x+1)(x+2)}{2}$ $\quad B)\frac{x(x+1)(x+2)}{3}$ $\quad C)\frac{x}{2}$ $\quad D)\frac{x(x+1)(x+2)}{4}$ $\quad E)\frac{x(x+1)(x+2)}{6}$

**Yechish.** $x(x+1)(x+2)$ – bu ketma-ket keluvchi 3 ta natural sonning ko'paytmasidir. Ketma-ket keluvchi 3 ta natural sonning ko'paytmasidir 6 ga bo'linadi, ya'ni $x(x+1)(x+2)=6k$, $\frac{x(x+1)(x+2)}{3}=\frac{6k}{3}=2k$ – bu juft sondir.

**Javob.** $\frac{x(x+1)(x+2)}{3}$ (B)

24(98-7-28). Qisqartiring. $\frac{x^4+1}{x^2+x\sqrt{2}+1}$

$A)x^3+1$ $\quad B)x^2-x\sqrt{2}-1$ $\quad C)x^2-2\sqrt{2}x+1$ $\quad D)x^2-1$ $\quad E)x^2-x\sqrt{2}+1$

**Yechish.** $\frac{x^4+1}{x^2+x\sqrt{2}+1}=\frac{(x^2+1)^2-2x^2}{x^2+x\sqrt{2}+1}=\frac{(x^2+1-\sqrt{2}x)(x^2+1+\sqrt{2}x)}{x^2+1+\sqrt{2}x}=x^2-\sqrt{2}x+1$

**Javob.** $x^2-2\sqrt{2}x+1$ (C)

25(98-4-3). Ifodaning qiymatini hisoblang.

$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+...+\frac{1}{\sqrt{1599}+\sqrt{1600}}$

A)32 B)41 C)39 D)34 E)28

**Yechish.** Ifodadagi har bir kasrning maxrajini irratsionallikdan qutqaraylik.

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+...+\frac{1}{\sqrt{1599}+\sqrt{1600}}=\frac{\sqrt{1}-\sqrt{2}}{(\sqrt{1}+\sqrt{2})(\sqrt{1}-\sqrt{2})}+\frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}+...$$

$$+\frac{\sqrt{1599}-\sqrt{1600}}{(\sqrt{1599}+\sqrt{1600})(\sqrt{1599}-\sqrt{1600})}=\frac{\sqrt{1}-\sqrt{2}}{-1}+\frac{\sqrt{2}-\sqrt{3}}{-1}+...+\frac{\sqrt{1599}-\sqrt{1600}}{-1}=-1+40=39$$

**Javob.** 39 (C)

26(00-10-54). Ifodaning qiymatini toping. $\sqrt{2^3\sqrt{5^3\sqrt{2^3\sqrt{5^3}}}}$

*A)17 B)12 C)14 D)41 E)20*

**Yechish.** Berilgan ifodaning qiymatini $x$ bilan belgilaylik. $\sqrt{2^3\sqrt{5^3\underbrace{\sqrt{2^3\sqrt{5^3\ldots}}}_{x}}}=x$, $\sqrt{2^3\sqrt{5^3x}}=x$ tenglamani yechaylik.

$8\cdot\sqrt{125x}=x^2$, $125x=\frac{x^4}{64}$, $x^3=125\cdot64$; $x=20$

**Javob.** 20 (E)

**27**(00-8-4). $5n^3-5n$ ifoda istalgan natural $n$ da quydagi sonlardan qaysi biriga qoldiqsiz bo'linadi? *A)30 B)22 C)25 D)45 E)60*

**Yechish.** Berilgan ifodani shakl almashtiraylik. $5n^3-5n=5n(n^2-1)=5(n-1)n(n+1)=5\cdot6k=30k$

**Javob.** 30 (A)

**28**(97-9-85). Agar $\begin{cases}x^3-3x^2y=y^3+20\\3xy=7\end{cases}$ bo' lsa, $\frac{x-y}{3}$ ni hisoblang.

A)3 B)2 C)1 D)0 E)6

**Yechish.** Tenglamalar sistemaidagi tenglamalarni hadma-had qo'shaylik.

$x^3-3x^2y+3xy^2-y^3=27$, $(x-y)^3=27$, $x-y=3$, $\frac{x-y}{3}=1$

**Javob.** 1 (C)

**29**(97-5-30). Hisoblang. *arcsin(sin10)*

A)$\pi-10$ B)$2\pi-10$ C)$3\pi-10$ D)$\frac{3\pi}{2}-10$ E)$\frac{2\pi}{3}-10$

**Yechish. 1-usul.** Bizga ma'lumki, agar $\alpha\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ bo'lsa, $\arcsin(\sin\alpha)=\alpha$ formula o'rinli. Keltirish formulasiga asosan $\sin10=\sin(3\pi-10)$ tenglik o'rinli hamda $3\pi-10\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ shart bajariladi. U holda *arcsin(sin10)* $=\arcsin(\sin(3\pi-10))=3\pi-10$

**2-usul.** *arcsin(sin10)=*$\alpha$ bo'lsin. $\alpha\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$.

$\sin\alpha-\sin10=0$, $2\sin\frac{\alpha-10}{2}\cos\frac{\alpha+10}{2}=0$. **Bundan,**

1) $\sin\frac{\alpha-10}{2}=0$, $\frac{\alpha-10}{2}=\pi n$, $\alpha=10+2\pi n$, ildizlar orasida $\alpha\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ shartni qanoatlantiruvchi ildiz mavjud emas.

2) $\cos\frac{\alpha-10}{2}=0$, $\frac{\alpha-10}{2}=\frac{\pi}{2}+\pi n$, $\alpha=\pi-10+2\pi n$ ildizlardan $n=1$ bo'lsa, $\alpha=3\pi-10$ ildiz $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ kesmaga tegishli bo'ladi.

**Javob.** $3\pi-10$ (C)

...-36). Agar $\frac{ab}{a+b}=1$, $\frac{ac}{a+c}=2$ va $\frac{bc}{b+c}=3$ bo'lsa, $\frac{ab}{c}$ ning qiymatini toping. A) $\frac{6}{25}$ B) $-\frac{15}{58}$ C) $\frac{21}{40}$ D) $-\frac{12}{25}$ E) $\frac{18}{65}$

...chish. Berilgan tengliklarning har birini teskarisini qarab sistema ...aylik.

$$\begin{cases} \frac{a+b}{ab}=1 \\ \frac{a+c}{ac}=\frac{1}{2} \\ \frac{b+c}{bc}=\frac{1}{3} \end{cases} \Leftrightarrow \begin{cases} \frac{1}{a}+\frac{1}{b}=1 \\ \frac{1}{a}+\frac{1}{c}=\frac{1}{2} \\ \frac{1}{b}+\frac{1}{c}=\frac{1}{3} \end{cases}$$

tenglamalarni hadma-had qo'shib, shakl ...shtirsak $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{11}{12}$ bo'ladi. Oxirgi tenglikdan sistemadagi ...nglamalarni hisobga olsak, $c=-12$, $b=\frac{12}{5}$, $a=\frac{12}{7}$ hosil bo'ladi. U holda ... $=-\frac{12}{35}$ bo'ladi.

**Javob.** $-\frac{12}{25}$ (D)

...(00-10-75). Agar $\{a_n\}$ ketma-ketlik uchun $a_1=0$, $a_2=1$, ..., $a_{n+2}=a_{n+1}-a_n$ ...ani ma'lum bo'lsa, $a_{885}$ ni toping .A) 1 B)0 C)-1 D)2 E)3

**Yechish.** Ketma- ketlikni bir nechta hadlarini hisoblaylik.

$a_3=a_2-a_1=1-0=1$,

$a_4=a_3-a_2=1-1=0$, $a_5=a_4-a_3=0-1=-1$, $a_6=a_5-a_4=-1-0=-1$ xuddi shu ...artibda $a_7=0$, $a_8=1$, $a_9=1$ larni hosil qilamiz. $\underbrace{0;1;1;0;-1;-1}_{6};0;1;1...$

$885=6\cdot147+3$, $a_{885}=a_{6\cdot147+3}=a_3=1$

**Javob.** 1 (A)

...(98-5-13). $k$ ning qanday qiymatida $y=\sqrt{kx^2+2x-1}$ funksiya $(-1;\frac{1}{3})$ oraliqda aniqlanmagan? A) 4 B) 5 C) 3 D) -3 E) -4

**Yechish.** Agar $kx^2+2x-1<0$ bo'lsa, berilgan kvadrat funksiya aniqlanmagan bo'ladi. $x\in(-1;\frac{1}{3})$ bo'lganligi uchun $x=-1$ bo'lsa,

$k(-1)^2+2\cdot(-1)-1=k-3=0$, $k=3$

**Javob.** 3 (C)

...(98-12-92). Ushbu $y=\sqrt{\sin x}+\sqrt{16-x^2}$ funksiyaning aniqlanish ...ohasiga tegishli $x$ ning butun qiymatlari nechta? A) 3 B) 4 C) 5 D) 2 E) 1

**Yechish.** Berilgan funksiya ma'noga ega bo'lishi uchun $\sin x\geq0$ va $16-x^2\geq0$ shartlar bajarilishi lozim.

$\begin{cases}\sin x \ge 0\\ 16-x^2 \ge 0\end{cases} \Leftrightarrow \begin{cases}2\pi n \le x \le \pi + 2\pi n, & n \in Z\\ -4 \le x \le 4\end{cases} \Leftrightarrow [-4;-\pi]\cup[0;\pi]$. Oxirgi oraliqqa tegishli butun sonlar $-4;0;3$ - 3 ta

**Javob.** 3 ta (A)

34(00-6-52). Hisoblang. $\cos\frac{\pi}{5}-\cos\frac{2\pi}{5}$

$A)\frac{\sqrt{2}-1}{2}$ $B)\frac{1}{3}$ $C)\frac{1}{\sqrt{3}}$ $D)\frac{\sqrt{3}-1}{2}$ $E)\frac{1}{2}$

**Yechish.** Berilgab ifodani bir necha marotiba shakl almashtiramiz.

$$\cos\frac{\pi}{5}-\cos\frac{2\pi}{5} = -2\sin\frac{\frac{\pi}{5}+\frac{2\pi}{5}}{2}\sin\frac{\frac{\pi}{5}-\frac{2\pi}{5}}{2} = 2\sin\frac{3\pi}{10}\sin\frac{\pi}{10} = 2\sin\frac{\pi}{10}\cos(\frac{\pi}{2}-\frac{3\pi}{10}) =$$

$$= 2\sin\frac{\pi}{10}\cos\frac{2\pi}{10} = 2\sin\frac{\pi}{10}\cos\frac{\pi}{10}\cos\frac{2\pi}{10}\cdot\frac{1}{\cos\frac{\pi}{10}} = 2\sin\frac{2\pi}{10}\cos\frac{2\pi}{10}\cdot\frac{1}{2\cos\frac{\pi}{10}} = \sin\frac{4\pi}{10}\cdot\frac{1}{2\cos\frac{\pi}{10}} =$$

$$\cos\left(\frac{\pi}{2}-\frac{2\pi}{5}\right)\cdot\frac{1}{2\cos\frac{\pi}{10}} = \cos\frac{\pi}{10}\cdot\frac{1}{2\cos\frac{\pi}{10}} = \frac{1}{2}$$

**Javob.** $\frac{1}{2}$ (E)

35(98-6-17). Ushbu $y=2^{x+\frac{1}{x}}$ funksiyaning qiymatlar sohasini toping. $A)(-\infty;\infty)$ $B)(0;\infty)$ $C)[2;\infty)$ $D)[4;\infty)$ $E)\left(0;\frac{1}{4}\right]\cup[4;\infty)$

**Yechish. 1-usul.** $y=2^t>0$ bo'lganligi sababli $y>0$ bo'ladi. Agar $x>0$ bo'lsa, $x+\frac{1}{x}\ge 2$ bo'ladi. U holda $y=2^{x+\frac{1}{x}}\ge 2^2=4$, $y\ge 4$. Agar $x<0$ bo'lsa, $x+\frac{1}{x}\le -2$ bo'ladi. U holda $y=2^{x+\frac{1}{x}}\le 2^{-2}=\frac{1}{4}$, $y\le\frac{1}{4}$. Demak $y\in\left(0;\frac{1}{4}\right]\cup[4;\infty)$

**2-usul.** Berilgan funksiyaga $y$ ni parametr sifatida qarab, $x$ ga nisbatan yechaylik. $x+\frac{1}{x}=\log_2 y$, $x^2-\log_2 y\cdot x+1=0$ kvadrat tenglama ildizga ega bo'lishi uchun uning diskriminanti nomanfiy bo'lishi lozim. $D=\log_2^2 y-4\ge 0$ tengsizlikni yechaylik. $\left[\begin{array}{l}\log_2 y\ge 2\\ \log_2 y\le -2,\end{array}\right. \Leftrightarrow \left[\begin{array}{l}y\ge 4\\ 0<y\le\frac{1}{4}\end{array}\right.$

**Javob.** $\left(0;\frac{1}{4}\right]\cup[4;\infty)$ (E)

36(98-12-85). Tengsizlikni qanoatlantiruvchi manfiy sonlar nechta?

$(x-2-x^2)\cdot(2x+\frac{1}{e})^3\cdot\log_{\pi\ldots}\left(1-\frac{x^2}{\pi}\right)\ge 0$

A)cheksiz ko'p B)1 C)∅ D)2 E) aniqlab bo'lmaydi.

**Yechish.** 1) $-x^2+x-2$ uchhadda $a=-1<0$, $D=-7<0$ bo'lganligi sababli barcha $x$ uchun $-x^2+x-2<0$ bo'ladi. U holda berilgan tengsizlik quyidagi tengsizlikka teng kuchli. $(2x+\frac{1}{e})^4\log_{(x+1)^2+1}\left(1-\frac{x^2}{\pi}\right)\le 0$. Agar $x\ne -\frac{1}{2e}$ bo'lsa, $(2x+\frac{1}{e})^4>0$ bo'ladi. U holda oxirgi tengsizlik $\log_{(x+1)^2+1}\left(1-\frac{x^2}{\pi}\right)\le 0$ ko'rinishni oladi. Bundan

$1-\frac{x^2}{\pi}\le 1,\quad x^2\ge 0,\quad x\in R$

2) aniqlanish sohasini hisobga olsak $1-\frac{x^2}{\pi}>0$, $|x|<\sqrt{\pi}$. $\begin{cases}x\in R\\ |x|<\sqrt{\pi}\end{cases}\Leftrightarrow |x|<\sqrt{\pi}$

**Javob.** cheksiz ko'p (A)

**37(02-11-12).** $arcsin\frac{x}{2}+2\arccos x=\pi$ tenglama nechta ildizga ega?

A)1 B)2 C) yechimi yo'q D)3 E)cheksiz ko'p yechimga ega

**Yechish.** Tenglamani $2\arccos x=\pi-\arcsin\frac{x}{2}$ ko'rinishda yozib, ikkala qismini kosinusini hisoblaylik. $\cos(2\arccos x)=\cos(\pi-\arcsin\frac{x}{2})$. Chap qismi $\cos(2\arccos x)=2\cos^2(\arccos x)-1=2x^2-1$. O'ng qismi $\cos(\pi-\arcsin\frac{x}{2})=-\cos(\arcsin\frac{x}{2})=-\sqrt{1-\frac{x^2}{4}}$. U holda $2x^2-1=-\sqrt{1-\frac{x^2}{4}}$ bo'ladi. Bu irratsional tenglamani shakl almashtirsak, $16x^4-15x^2=0$ bo'ladi. Bu tenglamaning $x=0$ ildizi berilgan tenglamani qanoatlantiradi. Qolgan ikkita ildizi $x=\pm\frac{\sqrt{15}}{4}$ chet ildiz.

**Javob.** 1 ta (A)

**38(02-3-2).** $125^6\cdot15^4\cdot2048^2$ ko'paytmaning qiymati necha xonali son bo'ladi? A)24 B) 26 C)22 D) 23 E) 25

**Yechish.** $125=5^3$, $15=3\cdot5$, $2048=2^{11}$ larni hisobga olsak $125^6\cdot15^4\cdot2048^2=5^{18}\cdot3^4\cdot5^4\cdot2^{22}=5^{22}\cdot2^{22}\cdot81=10^{22}\cdot81$. Oxirgi natijadan bu son 24 xonali ekanligi ko'rinib turibdi.

**Javob.** 24 (A)

**39(01-5-3).** Hisoblang. $\sqrt[3]{5\sqrt{2}+7}-\sqrt[3]{5\sqrt{2}-7}$

A)2 B)1 C)3 D)4 E)5

**Yechish.** $\sqrt[3]{5\sqrt{2}+7}-\sqrt[3]{5\sqrt{2}-7}=x$ belgilash kiritaylik va ikkala tomonini $(a-b)^3=a^3-b^3-3ab(a-b)$ formuladan foydalanib kubga ko'taramiz. $5\sqrt{2}+7-(5\sqrt{2}-7)-3\cdot\sqrt[3]{5\sqrt{2}+7}\cdot\sqrt[3]{5\sqrt{2}-7}(\sqrt[3]{5\sqrt{2}+7}-\sqrt[3]{5\sqrt{2}-7})=x^3$, $14-3x=x^3$ yoki

$x^3+3x-14=0$. Bu tenglamani ildizi $x=2$ ekanligini osongina topish mumkin.

**Javob.** 2 (A)

**40**(03-1-36). $\dfrac{5^{x^2}-5}{3\sin x+4\cos x-2\pi}\ge 0$ tengsizlikni yeching.

A) $[-1;1]$ B) $\left[1;\dfrac{\pi}{2}\right]$ C) $[-1;\pi]$ D) $[0;\pi]$ E) $[1;\pi]$

**Yechish.** $y=3\sin x+4\cos x-2\pi=\sqrt{3^2+4^2}\sin(x+\varphi)-2\pi=5\sin(x+\varphi)-2\pi<0$ bo'lganligi sababli barcha $x$ uchun tengsizlikning maxraji manfiydir. U holda berilgan tengsizlikdan $5^{x^2}-5\le 0$ kelib chiqadi. Bundan $5^{x^2}\le 5^1, x^2\le 1, |x|\le 1$.

**Javob.** $[-1;1]$ (A)

**41**(03-2-2). Agar $m^2+n^2=p^2+q^2=1$ va $mp+nq=0$ bo'lsa $mn+pq$ ning qiymatini toping. A)1 B)0 C)2 D)4 E)0,5

**Yechish. 1-usul.** $m=\cos\alpha,\ n=\sin\alpha,\ p=\sin\alpha,\ q=-\cos\alpha$ almashtirishlar olsak $m^2+n^2=p^2+q^2=1$ va $mp+nq=0$ tengliklar bajariladi. U holda $mn+pq=\cos\alpha\sin\alpha-\sin\alpha\cos\alpha=0$

**Javob.** 0 (B)

**2-usul.** Agar $m^2+n^2=p^2+q^2=1$ va $mp+nq=0$ bo'lsa, $(m^2+n^2-1)^2+(p^2+q^2-1)^2+2(mp+nq)^2=0$ bo'ladi. Bu tenglikni shakl almashtiraylik.

$$m^4+n^4+1+2m^2n^2-2m^2-2n^2+p^4+q^4+1+2p^2q^2-2p^2-2q^2+2m^2p^2+4mpnq+2n^2q^2=0$$

yoki $(m^2+p^2-1)^2+(n^2+q^2-1)^2+2(mn+pq)^2=0$ bo'ladi. Bundan $m^2+p^2-1=0,\quad n^2+q^2-1=0,\quad mn+pq=0$ bo'ladi.

**Javob.** 0 (B)

**42**(99-4-52). Hisoblang. $\cos\dfrac{\pi}{7}\cdot\cos\dfrac{3\pi}{7}\cdot\cos\dfrac{5\pi}{7}$

A) $\dfrac{1}{8}$ B) $-\dfrac{1}{16}$ C) $-\dfrac{\sqrt{3}}{8}$ D) $\dfrac{1}{16}$ E) $-\dfrac{1}{8}$

**Yechish.** Keltirish formulasi, ikkilangan burchak sinusi formulalarini bir necha marotiba qo'llaymiz.

$$\cos\frac{\pi}{7}\cdot\cos\frac{3\pi}{7}\cdot\cos\frac{5\pi}{7}=\frac{1}{2\sin\frac{\pi}{7}}\left(2\sin\frac{\pi}{7}\cos\frac{\pi}{7}\right)\cdot\cos\frac{3\pi}{7}\cdot\cos\left(\pi-\frac{2\pi}{7}\right)=-\frac{1}{2\sin\frac{\pi}{7}}\cdot\sin\frac{2\pi}{7}\cdot\cos\frac{2\pi}{7}\times$$

$$\times\cos\left(\pi-\frac{4\pi}{7}\right)=\frac{1}{4\sin\frac{\pi}{7}}\cdot\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}=\frac{1}{8\sin\frac{\pi}{7}}\cdot\sin\frac{8\pi}{7}=\frac{1}{8\sin\frac{\pi}{7}}\cdot\sin\left(\pi+\frac{\pi}{7}\right)=-\frac{1}{8}$$

**Javob.** $-\dfrac{1}{8}$ (E)

43(00-4-44). Agar $0 \le \beta \le \frac{\pi}{4}$ bo'lsa, $tg\beta = \left|\frac{a^2-5a+4}{a^2-4}\right|$ o'rinli bo'ladigan $a$ ning barcha qiymatlarini toping.

A) $[2,5;\infty)$ B) $[0;\infty)$ C) $[0;1,6]\cup[2,5;\infty)$ D) $[0;1,5]\cup[3,6;\infty)$ E) $[-3;1,6)\cup[2,5;\infty)$

**Yechish.** $y=tgx$ funksiya I-chorakda o'suvchi funksiya bo'lganligi sababli $0 \le \beta \le \frac{\pi}{4}$ tengsizlikdan $tg0 \le tg\beta \le tg\frac{\pi}{4}$ yoki $0 \le tg\beta \le 1$ tengsizlik kelib chiqadi. Oxirgi tengsizlikka $tg\beta$ ning qiymatini qo'ysak,

$\left|\frac{a^2-5a+4}{a^2-4}\right| \le 1$ tengsizlik kelib chiqadi. Buni yechamiz.

$\frac{|a^2-5a+4|}{|a^2-4|} \le 1$, $\begin{cases} |a^2-5a+4| \le |a^2-4| \\ |a^2-4| \ne 0 \end{cases} \Leftrightarrow \begin{cases} (a^2-5a+4)^2-(a^2-4)^2 \le 0 \\ a \ne \pm 2 \end{cases}$ Birinchi

tengsizlikni yechaylik. $((a^2-5a+4)-(a^2-4))((a^2-5a+4)+(a^2-4)) \le 0$ ,

$a(a-1,6)(a-2,5) \ge 0$, $[0;1,6]\cup[2,5;\infty)$

**Javob.** $[0;1,6]\cup[2,5;\infty)$ (C)

## Mustaqil yechish uchun masalalar

44(00-3-15). Hisoblang:

$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots+\frac{1}{13\cdot 15}$

A) $\frac{11}{15}$ B) $\frac{7}{30}$ C) $\frac{8}{15}$ D) $\frac{7}{15}$ E) $\frac{2}{5}$

45(00-8-57). Yig'indini hisoblang: $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{99\cdot 100}$

A) $\frac{1}{9}$ B) $\frac{1}{10}$ C) $\frac{1}{100}$ D) $\frac{1}{99}$ E) $\frac{99}{100}$

46(00-9-10). Hisoblang: $\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\frac{1}{99}+\ldots+\frac{1}{255}$

A) $\frac{7}{51}$ B) $\frac{2}{15}$ C) $\frac{2}{25}$ D) $\frac{3}{35}$ E) $\frac{7}{40}$

47(99-5-1). Hisoblang: $\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\ldots+\frac{1}{182}$

A) $\frac{11}{42}$ B) $\frac{10}{33}$ C) $\frac{1}{4}$ D) $\frac{12}{35}$ E) $\frac{15}{56}$

48(03-1-9). $\frac{1}{8}+\frac{1}{24}+\frac{1}{48}+\frac{1}{80}$ yig'indini hisoblang.

A)0.1 B)0,2 C)0,4 D)0.6 E)0.8

49(03-8-29). $1+\frac{1}{10\cdot 11}+\frac{1}{11\cdot 12}+\frac{1}{12\cdot 13}+\frac{1}{13\cdot 14}+\frac{1}{14\cdot 15}+\frac{1}{15\cdot 16}$ ni hisoblang.

A) $1\frac{3}{80}$ B) $1,16$ C) $1\frac{3}{40}$ D) $1\frac{7}{80}$ E) $1\frac{13}{80}$

**50**(02-9-6)*. Agar $a+\frac{1}{a}=3$ bo'lsa, $\frac{a^4+1}{2a^2}$ ning qiymati nimaga teng?
A)3,5 B)4 C)5,5 D)7 E)10

**51**(00-6-7)*. Agar $a-\frac{1}{a}=\frac{2}{3}$ bo'lsa, $\frac{a^4+1}{a^2}$ ning qiymatini toping.
A) $2\frac{4}{9}$ B) $1\frac{1}{2}$ C) $1\frac{5}{9}$ D) $2\frac{5}{9}$ E) $4\frac{2}{3}$

**52**(03-8-44)*. Agar $a+a^{-1}=3$ bo'lsa, $a^2+a^{-2}$ ni hisoblang.
A)7 B)4 C)9 D)13 E)12

**53**(03-8-45)*. Agar $a+a^{-1}=5$ bo'lsa, $a^3+a^{-3}$ ni hisoblang.
A)110 B)70 C)80 D)90 E)100

**54**(98-12-25). $a^2+b^2$ ni *ab* va *a+b* orqali ifodalang.
A) $(a+b)^2-2ab$ B) $(a+b)^2-ab$ C) $(a+b)^2-4ab$ D) $(a+b)ab$ E) $(a+b)^2+2ab$

55(00-9-19). *Agar x, y, z va t ketma-ket keladigan natural son bo'lsa, quydagilarning qaysi biri albatta juft son bo'ladi?* A) $\frac{x+y+z}{3}$
B) $\frac{xyzt}{24}$ C) $\frac{xyz}{6}$ D) $\frac{x(x^2-1)}{3}$ E) $\frac{y(y^2-1)}{2}$

56(98-12-26). Qisqartiring. $\frac{x^2-x+1}{x^4+x^2+1}$
A) $\frac{1}{x^2+x+1}$ B) $\frac{1}{x^2-2x-1}$ C) $\frac{1}{x^2-x+1}$ D) $\frac{1}{x^2-x-1}$ E) $\frac{1}{x^2-2x+1}$

57(99-9-22). Hisoblang. $\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+...+\frac{1}{\sqrt{9}+\sqrt{8}}$
*A)2 B)3 C)4 D)1 E)5*

58(00-2-23). Yig'indini hisoblang.
$\frac{1}{\sqrt{1}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+...+\frac{1}{\sqrt{79}+\sqrt{81}}$
*A)6 B)5 C)3 D)2 E)4*

59(98-3-60). $\sin^2 x-\frac{1}{2}\sin 2x-2\cos^2 x\geq 0$ $(x\in[0;2\pi])$ tengsizlik $x$ ning qanday qiymatlarida o'rinli?
A) $\left[arctg2;\frac{3\pi}{4}\right]\cup\left[\pi+arctg2;\frac{7\pi}{4}\right]$ B) $\left[arctg2;\frac{3\pi}{4}\right]$ C) $\left[\pi+arctg2;\frac{7\pi}{4}\right]$
D) $\left[\frac{3\pi}{4};\pi+arctg2\right]$ E) $\left[\frac{7\pi}{4};2\pi\right]$

60(00-7-10). Agar $a^2-3ab+b^2=44$ va $a^2+ab+b^2=28$ ga teng bo'lsa, $a^2-ab+b^2$ ning qiymati nechaga teng bo'ladi?
*A) 14 B18 C)12 D19 E)15*

61(97-12-43). $k$ ning qanday qiymatlarida $f(x)=kx-\sin x$ funksiya o'zining aniqlanish sohasida kamayadi? *A)* $k\le 1$ *B)* $k>-1$ *C)* $k<0$ *D)* $k>0$ *E)* $0<k<1$

62(00-10-52). Hisoblang. $\cos 24^0-\cos 84^0-\cos 12^0+\sin 42^0$

*A)* $\frac{1}{2}$ *B)* $\frac{1}{3}$ *C)* $\frac{\sqrt{5}-1}{4}$ *D)* $\frac{\sqrt{3}}{2}$ *E)* $\frac{1}{\sqrt{3}}$

63(00-10-48). Kasrni qisqartiring.

$$\frac{x^3-1}{x^4+x^2+1}$$

*A)* $\frac{x-1}{x^2-x+1}$ *B)* $\frac{x}{x+2}$ *C)* $\frac{x+1}{x^2-x+1}$ *D)* $\frac{x-2}{x^2-x-1}$ *E)* $\frac{x+2}{x^2-x-1}$

64(02-10-5). $\sqrt[3]{9+2\sqrt{20}}+\sqrt[3]{9-2\sqrt{20}}$ ning qiymatini toping.

*A)*3 *B)*1 *C)*4 *D)*2 *E)*$2\sqrt[3]{2}$

65(00-4-14). Agar $a$, $b$, $c$ va $d$ turli raqamlar bo'lib, $a+b+c=7$, $(a+b)^2=d$ va $abc\ne 0$ bo'lsa, $\frac{c^2-c}{a+b}$ ning qiymatini toping.

A) aniqlab bo'lmaydi B)1 C)2 D)3 E)4

66(03-12-56). $x^2 7^x+1>7^x+x$ tengsizlikni yeching.

*A)* $(1;\infty)$ *B)* $(-1;0)$ *C)* $(-1;1)$ *D)* $(-\infty;0)\cup(1;\infty)$ *E)* $(-1;1)\cup(1;\infty)$

67(99-5-57). $[-10;10]$ oraliqdagi nechta butun son $y=2^{\cos x}\cdot\sqrt{x^3\cdot\sin^2\left(\frac{\pi x}{3}\right)\cdot e^{-x}}$ funksiyaning aniqlanish sohasiga tegishli?

*A)* 10 *B)* 11 *C)* 12 *D)* 13 *E)* 14

68(00-5-30). Hisoblang. $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$

*A)* $-\frac{1}{2}$ *B)* $\frac{1}{2}$ *C)* $\frac{1}{4}$ *D)* $\frac{1}{8}$ *E)* $-\frac{1}{4}$

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# MUNDARIJA

QURBONOV NORMUROD XOLMAMATOVICH

# MAXSUS YO`L BILAN YECHILADIGAN ALGEBRAIK MASALALAR

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$$P(x)=a_0x^n+a_1x^{n-1}+\dots+a_{n-1}x+a_n$$

$$\begin{cases} x_1+x_2+\dots+x_n=-\dfrac{a_1}{a_0} \\ x_1x_2+x_1x_3+\dots+x_{n-1}x_n=\dfrac{a_2}{a_0} \\ x_1x_2x_3+\dots+x_{n-2}x_{n-1}x_n=-\dfrac{a_3}{a_0} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ x_1x_2\cdot\ldots\cdot x_n=(-1)^n\cdot\dfrac{a_n}{a_0} \end{cases}$$

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