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OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

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MATEMATIK ANALIZDAN MISOL VA MASALALAR TO'PLAMI

(O'quv qo'llanma)

II- qism

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ANNOTATSIYA

Mazkur qo'llanma universitetlarning fizika fakultetlari uchun mo'ljallangan. Undagi misol va masalalarni tanlashda Mirzo Ulug'bek nomidagi O'zbekiston Milliy universiteti nazariy fizika kafedrasida ishlab chiqilgan o'quv dasturiga amal qilindi. Qo'llanmada keltirilgan misol va masalalar matematik analizning sonli va funksional qatorlar, ko'p argumentli funksiyalar, ikki va uch karrali integrallar, egri chiziqli va sirt integrallari, Fure qatori va parametrga bog'liq bo'lgan integrallar kabi bo'limlariga taaluqlidir.

Qo'llanmada keltirilgan misol va masalalar ko'rsatma va javoblar bilan ta'minlangan.

Taqrizchilar: dots. Jumaniyozov Q.
dots. Ataxanov Q.

Mas'ul muxarrir: prof. Abdumalikov A.A.

I Bob
Qatorlar.
1§ Sonli qatorlar.

Quyidagi qatorlarning umumiy hadini yozing.

1. $1-\frac{1}{4}+\frac{1}{9}-\frac{1}{10}+....$

2. $\frac{1}{2}-\frac{1}{6}+\frac{1}{12}-\frac{1}{20}+\frac{1}{30}+....$

3. $1-\frac{1\cdot 3}{1\cdot 4}+\frac{1\cdot 3\cdot 5}{1\cdot 4\cdot 7}-\frac{1\cdot 3\cdot 5\cdot 7}{1\cdot 4\cdot 7\cdot 10}+....$

4. $2+\frac{2^2}{1\cdot 2}+\frac{2^3}{1\cdot 2\cdot 3}+\frac{2^4}{1\cdot 2\cdot 3\cdot 4}+....$

5. $1-\frac{2\cdot 4}{1\cdot 3}+\frac{2\cdot 4\cdot 6}{1\cdot 3\cdot 5}+....$

Quyidagi qatorlarning dastlabki n ta hadlar yiğindisi S_n, qator yiğindisi S ni toping.

6. $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+...+\frac{1}{(3n-2)(3n+1)}+...$

7. $\frac{3}{1^2\cdot 2^2}+\frac{5}{2^2\cdot 3^2}+...+\frac{2n+1}{n^2(n+1)^2}+...$

8. $\frac{5}{6}+\frac{13}{36}+...+\frac{3^n+2^n}{6^n}+...$

9. $\frac{3}{4}+\frac{5}{36}+...+\frac{2n+1}{n^2(n+1)^2}+...$

10. $\frac{2}{1\cdot 2\cdot 3}+\frac{2}{2\cdot 3\cdot 4}+...+\frac{2}{n(n+1)(n+2)}+...$

Quyidagi qatorlarning qaysilari zaruriy shartga kõra uzoqlashuvchi bõladi.

11. $1-1+1-1+...+(-1)^{n-1}$

12. $0.001+\sqrt{0.001}+\sqrt[3]{0.001}+...+\sqrt[n]{0.001}+...$

13. $\frac{3}{4}+\frac{5}{6}+...+\frac{2n+1}{2n+2}+...$

14. $\sqrt{\frac{3}{2}}+\sqrt{\frac{4}{3}}+...+\sqrt{\frac{n+2}{n+1}}+...$

15. $\frac{1}{2\sqrt{2}}+\frac{1}{4\sqrt{4}}+\frac{1}{6\sqrt{6}}+...+\frac{1}{2n\sqrt{2n}}+...$

16. $\frac{2}{3}+\frac{4}{9}+\frac{6}{27}+...+\frac{2n}{3^n}+...$

Taqqoslash belgisi yordamida qator yaqinlashuvchiligini aniqlang.

17. $\frac{1}{2}+\frac{2}{5}+\frac{3}{10}+...+\frac{n}{n^2+1}+...$

18. $\frac{3}{1\cdot 4}+\frac{5}{4\cdot 9}+\frac{7}{9\cdot 16}+...+\frac{2n+1}{n^2(n+1)^2}+..$

19. $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+...+\frac{1}{\sqrt{n+1}}+...$

20. $Sin\frac{\pi}{2}+Sin\frac{\pi}{4}+...+Sin\frac{\pi}{2^n}+...$

21. $1+\frac{1}{2\cdot 5}+\frac{1}{3\cdot 5^2}+...+\frac{1}{n\cdot 5^{n-1}}+...$

22. $\frac{1}{\ln 2}+\frac{1}{\ln 3}+...+\frac{1}{\ln(n+1)}+...$

Dalamber belgisi yordamida qatorlarning yaqinlashuvchanligini tekshiring.

23. $\sum_{n=1}^{\infty}\frac{n^3}{3^n}$

24. $\sum_{n=1}^{\infty}\frac{2\cdot 5\cdot 8\cdot ...\cdot(3n-1)}{1\cdot 6\cdot 11\cdot ...\cdot(5n-4)}$

25. $\sum_{n=1}^{\infty}\frac{1\cdot 3\cdot 5\cdot ...\cdot(2n-1)}{3^n\cdot n!}$

26. $\sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2}$

27. $\sum_{n=1}^{\infty}\frac{(2n+1)!}{(3n+4)\cdot 3^n}$

28. $\sum_{n=1}^{\infty}\frac{(2n+1)!!}{1\cdot 4\cdot ...\cdot(3n+1)}$

29. $\sum_{n=1}^{\infty}\frac{2\cdot 5\cdot ...\cdot(3n+2)}{2^n(n+1)!}$

30. $\sum_{n=1}^{\infty}\frac{(2n+1)!!}{3^n\cdot n!}$

31. $\sum_{n=1}^{\infty}\frac{(3n)!}{(n!)^3\cdot 4^{3n}}$

32. $\sum_{n=1}^{\infty}\frac{n!(2n+1)!}{(3n)!}$

Koshi belgisi yordamida qatorlarning yaqinlashuvchanligini tekshiring.

33. $\sum_{n=1}^{\infty}\left(\frac{3}{n}\right)^n$

34. $\sum_{n=1}^{\infty}\left(\frac{an}{n+2}\right)^n$

35. $\sum_{n=1}^{\infty}2^n\left(\frac{n}{n+1}\right)^{n^2}$

36. $\sum_{n=1}^{\infty}\left(\frac{\sqrt{n}+2}{\sqrt{n}+3}\right)^{n^2}$

37. $\sum_{n=1}^{\infty} 3^{n+1}\left(\frac{n+2}{n+3}\right)^{n^2}$

38. $\sum_{n=1}^{\infty}\left(\frac{n^2+5}{n^2+6}\right)^{n^2}$

39. $\sum_{n=1}^{\infty}\left(\frac{n-1}{n+1}\right)^{n^2+4n+1}$

40. $\sum_{n=1}^{\infty}\left(\frac{n-1}{n+1}\right)^{\sqrt{n^2+3n+1}}$

41. $\sum_{n=1}^{\infty}\left(\frac{2n-1}{2n+1}\right)^{n(n-1)}$

42. $\sum_{n=1}^{\infty} 3^{-n}\left(\frac{n+1}{n}\right)^{n^2}$

43. $\sum_{n=1}^{\infty}\frac{n^n}{(\ln(n+1))^{\frac{n}{2}}}$

44. $\sum_{n=1}^{\infty}\left(\frac{6n+1}{5n-3}\right)^{\frac{n}{2}}\left(\frac{5}{6}\right)^{\frac{2n}{3}}$

Ishorasi ōzgaruvchan qatorlarni yaqinlashuvchanligini tekshiring.

45. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n\sqrt[3]{n}}$

46. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+2}{n}$

47. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n-1)^3}$

48. $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt[4]{n}}$

49. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2n+1}{n(n+1)}$

50. $\sum_{n=1}^{\infty}(-1)^n\frac{\ln n}{n}$

51. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(2n+1)!!}{2\cdot 5\cdot 8\cdot\ldots\cdot(3n-1)}$

52. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1\cdot 4\cdot 7\cdot\ldots\cdot(3n-2)}{7\cdot 9\cdot 11\cdot\ldots\cdot(2n+5)}$

53. $\sum_{n=1}^{\infty}\frac{(-1)^n Cos\frac{\pi}{n}}{n}$

54. $\sum_{n=1}^{\infty}\frac{(-1)^n Sin\frac{\pi}{n}}{n}$

2§ Funksional qatorlar

Quyidagi funksiyalarni darajali qatorga yoying.

55. $y=\frac{x^2}{\sqrt{1-x^2}}$

56. $y=Cos^2 x$

57. $y=Sin^3 x$

58. $y=Sin^6 x$

59. $y=\frac{x+\ln(1-x)}{x^2}$

60. $y=(1+x)\ln(1+x)$

61. $y=arcSinx^3$

62. $y=arctg\frac{2-2x}{1+4x}$

63. $y=arcSinx$

Funksional qatorlarning yaqinlashish sohasini toping.

64. $\sum_{n=1}^{\infty} \ln^n x$

65. $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

66. $\sum_{n=1}^{\infty} \frac{x^n}{1+x^{2n}}$

67. $\sum_{n=1}^{\infty} \frac{x^n}{n+\sqrt{n}}$

68. $\sum_{n=1}^{\infty} Sin\frac{x}{2^n}$

69. $\sum_{n=1}^{\infty} x^n tg\frac{x}{2^n}$

70. $\sum_{n=1}^{\infty} e^{-n^2 x}$

71. $\sum_{n=1}^{\infty} \frac{nx}{e^{nx}}$

72. $\sum_{n=1}^{\infty} \frac{1}{n^x}$

73. $\sum_{n=1}^{\infty} \frac{n}{n+1} \ln^n\left(x^2+2\right)$

Darajali qatorlarning yaqinlashish sohasini toping.

74. $\sum_{n=1}^{\infty} \frac{x^n}{n(n+2)}$

75. $\sum_{n=1}^{\infty} \frac{\ln(n+1)}{n} x^{n+1}$

76. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-1}}{3^{n-1} n\sqrt{n}}$

77. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{2n+1}$

78. $\sum_{n=1}^{\infty} \frac{(n-1)(x+3)^n}{3^{n+1}}$

79. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-2)^{2n}}{n \cdot 4^n}$

80. $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n^2} x^n$

81. $\sum_{n=1}^{\infty} c^{\sqrt{n}} x^n, c>0$

82. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n$

83. $\sum_{n=1}^{\infty} c^{\ln n} x^n, c>0$

84. $\sum_{n=1}^{\infty} \frac{3^{-\sqrt{n}} x^n}{\sqrt{n^2+1}}$

85. $\sum_{n=1}^{\infty} \frac{x^n}{1+c^n}, c \geq 0$

Quyidagi funksional qatorlarning absalyut va shartli yaqinlashish sohasini toping.

86. $\sum_{n=1}^{\infty} \frac{1}{x^n}$

87. $\sum_{n=1}^{\infty} \frac{1}{n} Sin\frac{\pi x}{n}$

88. $\sum_{n=1}^{\infty} e^{-nx}$

89. $\sum_{n=1}^{\infty} \frac{Cos\pi x n}{n \ln^2(n+1)}$

90. $\sum_{n=1}^{\infty} \frac{1}{n(x+2)^n}$

91. $\sum_{n=1}^{\infty} \frac{\ln^n x}{n^2}$

92. $\sum_{n=1}^{\infty} (5-x^2)^n$

93. $\sum_{n=1}^{\infty}\frac{1}{n^{x}}$

94. $\sum_{n=1}^{\infty}n^{-\ln x^{2}}$

95. $\sum_{n=1}^{\infty}n^{2}e^{-nx^{3}}$

96. $\sum_{n=1}^{\infty}\frac{n}{\ln^{n}(x+2)}$

97. $\sum_{n=1}^{\infty}\frac{tg^{n}x}{n^{2}+4}$

98. $\sum_{n=1}^{\infty}\frac{n}{n+1}\ln^{n}(x^{2}+2)$

99. $\sum_{n=1}^{\infty}\frac{n}{n^{2}+4}\left(\frac{x+2}{2x+1}\right)^{n}$

100. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\left(\frac{2x}{x^{2}+1}\right)^{n}$

101. $\sum_{n=1}^{\infty}\frac{2^{n}Sin^{n}x}{n(n+1)}$

102. $\sum_{n=1}^{\infty}\left(1+\frac{x}{n}\right)^{n}n^{-x}$

Veyershtrass belgisidan foydalanib berilgan oraliqda qatorning tekis yaqinlashuvchanligini tekshiring.

103. $\sum_{n=1}^{\infty}\frac{x}{1+n^{4}x^{2}},0\le x<\infty$

104. $\sum_{n=1}^{\infty}\frac{\sqrt{1-x^{2n}}}{2^{n}};-1\le x\le 1$

105. $\sum_{n=1}^{\infty}\frac{n^{2}}{\sqrt{n!}}\left(x^{n}-x^{-n}\right);\frac{1}{2}\le x\le 2$

106. $\sum_{n=1}^{\infty}x^{2}e^{-nx};0\le x<\infty$

107. $\sum_{n=1}^{\infty}\frac{1}{2n-1}\left(\frac{x-1}{x+1}\right)^{n};\ x\in[1;2]$

Quyidagi funksiyalarni darajali qatorga yoying.

108. $y=e^{2x}$

109. $y=e^{-x^{2}}$

110. $y=\begin{cases}\dfrac{e^{x}-1}{x},x\ne 0\\ 1,x=0\end{cases}$

111. $y=Cos^{2}x$

112. $y=(x-tgx)Cosx$

113. $y=\ln(10+x)$

114. $y=x\ln(1+x)$

115. $y=\sqrt{1+x^{2}}$

116. $y=\sqrt[3]{8-x^{3}}$

117. $y=\frac{1}{\sqrt[3]{1+x^{3}}}$

Hadma-had integrallash va differensiyallash yordamida qator yigindisini topish.

118. $\frac{x^{3}}{3}+\frac{x^{7}}{7}+\ldots+\frac{x^{4n-1}}{4n-1}+\ldots$

119. $x+\frac{x^{5}}{5}+\ldots+\frac{x^{4n-3}}{4n-3}+\ldots$

120. $\frac{x^2}{1\cdot 2}-\frac{x^3}{2\cdot 3}+\ldots+\frac{x^{n+1}}{n(n+1)}+\ldots$

121. $f(x)=e^{-x}+2e^{-2x}+\ldots+ne^{-nx}+\ldots$, bo'lsa $\int_{\ln 2}^{\ln 3} f(x)dx$ ni hisoblang.

122. $f(x)=1+2\cdot 3x+\ldots+n\cdot 3^{n-1}x^{n-1}+\ldots$, bo'lsa $\int_{0}^{0,125} f(x)dx$ ni hisoblang.

123. . $f(x)=\frac{1}{2}tg\frac{x}{2}+\frac{1}{4}tg\frac{x}{4}+\ldots+\frac{1}{2^n}tg\frac{x}{2^n}+\ldots$, bo'lsa $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} f(x)dx$ ni hisoblang.

II Bob

Kõp õzgaruvchili funksiyalar

1§ Quyidagi funksiyalarning aniqlanish sohasini toping.

124. $z=\frac{1}{x+y}$

125. $z=\frac{y}{x}$

126. $z=\sqrt{3-x^2-y^2}$

127. $z=\sqrt{1-\frac{x^2}{9}-\frac{y^2}{4}}$

128. $z=\sqrt{x}+\sqrt{y}$

129. $z=\sqrt{-x}+\sqrt{y}$

130. $z=\frac{x}{\sqrt{x-y}}+\frac{y}{\sqrt{x+y}}$

131. $z=arcCos\frac{x^2+y^2}{9}$

132. $z=\sqrt{R^2-x^2-y^2}+\frac{1}{\sqrt{x^2+y^2-r^2}}$

133. $z=\sqrt{x-\sqrt{y}}$

134. $z=\frac{\sqrt{4x-y^2}}{\ln(1-x^2-y^2)}$

135. $z=\ln\left(\frac{x^2}{9}-\frac{y^2}{4}-1\right)$

Kõp õzgaruvchili funksiyaning limitini hisoblash.

136. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{\sqrt{1+x^2(y^2-1)}}{x^2+y^2}$

137. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{Sin(x^4\cdot y^2)}{(x^2+y^2)^2}$

138. . $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{(x^2+y^2)x^2y^2}{1-Cos(x^2+y^2)}$

139. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{e^{\frac{1}{x^4+y^4}}}{x^4+y^4}$

140. $\lim\limits_{\substack{x\to 0\\ y\to 0}}(1+x^2y^2)^{\frac{1}{x^2+y^2}}$

141. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{(x+y)^2}{(x^2+y^4)^2}$

142. $\lim\limits_{\substack{x\to 0\\ y\to 0}}(x^2+y^2)Sin\frac{1}{x^2+y^2}$

143. $f(x,y)=\begin{cases}\dfrac{x^2+2xy-3y^2}{x^3-y^3}, x\neq y\\ \dfrac{4}{3}, x=y\end{cases}$

Quyidagi funksiyalarning hususiy hosilalarini toping.

144. $z=2axy-x^2-y^2$

145. $z=\dfrac{x}{y}$

146. $z=x^3t-t^3x$

147. $u=\dfrac{y}{x}+\dfrac{x}{z}-\dfrac{z}{y}$

148. $S=axe^{-t}+bt$ (a,b õzgarmas sonlar)

149. $z=e^{\frac{\sin x}{x}}$

150. $z=(3xy^3-x^2+5)^4$

151. $z=\sqrt[3]{e^y}$

152. $z=Sin\dfrac{x}{y}Cos\dfrac{y}{x}$

153. $u=x^{yz}$

154. $u=x^{y^z}$

155. $u=Sin(xy-yz)$

156. $u=\left(\dfrac{y}{x}\right)^x$

157. $f(x,y)=x+y+\sqrt{x^2+y^2}$ $f_x'(4;3)$ va $f_y'(4;3)$ larni toping

158. $f(x;y;z)=Sin^2(3x+2y-z)$ $f_x'(1;-1;1), f_y'(1;1;4), f_z'\left(-\dfrac{1}{2};0;-1\right)$ larni toping

159. $f(x;y;z)=\ln(1+x+y^2+z^2)\,f_x'(1;1;1)+f_y'(1;1;1)+f_z'(1;1;1)=?$

160. $f(x;y)=\sqrt{xy+\dfrac{x}{y}}$; $f_x'(2;1), f_y'(2;1)$ larni toping

161. $f(x;y;z)=\ln(xy+z)$; $f_x'(1;2;0), f_y'(1;2;0), f_z'(1;2;0)$ larni toping

162. $z=\ln(e^x+e^y)$ bõlsa, $\dfrac{\partial z}{\partial x}+\dfrac{\partial z}{\partial y}=1$ tenglikning õrinli ekanini kõrsating.

163. $z=x^y$ bõlsa, $\dfrac{x}{y}\dfrac{\partial z}{\partial x}+\dfrac{1}{\ln x}\dfrac{\partial z}{\partial y}=2z$ tenglikning õrinli ekanini kõrsating.

164. $z=x\ln\dfrac{y}{x}$ bõlsa, $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=z$ tenglikning õrinli ekanini kõrsating.

165. $S=e^{\frac{x}{t^2}}$ bõlsa, $2x\dfrac{\partial S}{\partial x}+t\dfrac{\partial S}{\partial t}=0$ tenglikning õrinli ekanini kõrsating.

2§ O'zgaruvchilarni almashtirish.

O'zgaruvchilarni almashtirish yordamida differensial tenglamalarni yozing

166. $x^2y''+xy'+y=0$, agar $x=e^t$

167. $(1-x^2)y''-xy'+n^2y=0$, agar $x=\cos t$

168. $(1+x^2)^2y''=0$ agar $x=tgt$,$y=u(t)/\cos t$

169. $y'''-x^3y''+xy'=0$,agar $x=1/t$, $y=u(t)/t$

y ni erkli o'zgaruvchi, x ni uning funksiyasi sifatida olib, quyidagi tenglamalarni yozing.

170. $y'y'''-3(y'')^2=0$

171. $y''+(e^y-x)y'^3=0$

172. $y''+y'-y'^3=0$

Quyidagi tenglamalarni qutb koordinatalari r va φ ga o'tib yozing.

173. $\frac{\partial y}{\partial x}=\frac{x+y}{x-y}$

174. $(xy'-y)^2=2xy(1+y'^2)$

175. $(x^2+y^2)y''=(x+yy')^3$

Quyidagi tenglamalarni yangi o'zgaruvchi u va v ni kiritib hisoblang.

176. Agar $u=x+y$,$v=x-y$ bo'lsa $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}$ tenglamani eching.

177. Agar $u=x$,$v=x^2+y^2$ bo'lsa $y\frac{\partial z}{\partial x}-x\frac{\partial z}{\partial y}=0$ tenglamani eching.

Quyidagi tenglamalarni yangi o'zgaruvchi u va v ga o'tib yozing.

178. Agar $u=\ln x$ va $v=\ln(y+\sqrt{1+y^2})$ bo'lsa, $x\frac{\partial z}{\partial x}-\sqrt{1+y^2}\frac{\partial z}{\partial y}=xy$

179. $(x+y)\frac{\partial z}{\partial x}-(x-y)\frac{\partial z}{\partial y}=0$

180. $u=\ln\sqrt{x^2+y^2}$, $v=\operatorname{arctg}\frac{y}{x}$

181. $2\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-\frac{\partial^2 z}{\partial y^2}+\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$, agar $u=x+2y+2$, $v=x-y-1$

182. $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$, agar $u=\frac{x}{x^2+y^2}$, $v=-\frac{y}{x^2+y^2}$

183. $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}+x+y=0$, $u=x+y$, $v=y-x$, $w=xy-z$

184. $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=4x$, $u=x$, $y=x-y$, $w=xy-z$

185. $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0$, $u=\frac{y}{x}$, $v=y$, $w=x-y+z$

186. $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=z^2, u=x, y=\frac{1}{y}-\frac{1}{x}, w=\frac{1}{z}-\frac{1}{x}$

187. $(xy+z)\frac{\partial z}{\partial x}+(1-y^2)\frac{\partial z}{\partial y}=x+yz$, u=yz-x, v=xz-y, w=xy-z

188. $y\frac{\partial^2 z}{\partial y^2}+2\frac{\partial z}{\partial y}=\frac{2}{x}, u=\frac{x}{y}$, v=x, w=xz-y

189. $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=0, u=x+y, v=\frac{y}{x}, w=\frac{z}{x}$

3§ Kop argumentli funksiyalar ekstremumi.

Quyidagi funksiyalarni ekstremumga tekshiring.

190. $z=2xy-3x^2-2y^2+10$

191. $z=4(x-y)-x^2-y^2$

192. $z=x^2+xy+y^2+x-y+1$

193. $z=x^3y^2(12-x-y),(x>0,y>0)$

194. $z=x^3+y^3-3xy$

195. $z=x^4+y^4-2x^2+4xy-2y^2$

196. $z=x^2+(y-1)^2$

197. $z=(x-y+1)^2$

198.

199. $z=2x^4+y^4-x^2-2y^2$

200. $z=xy+\frac{50}{x}+\frac{20}{y},(x>0,y>0)$

201. $z=xy\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}},(a>0,b>0)$

202. $z=x^4+y^4-36xy$

203. $z=3x^2y+y^3-18x-30y$

204. $z=1-\sqrt{x^2+y^2}$

205. $z=x^2+xy+y^2-4\ln x-10\ln y$

206. $z=xy\ln(x^2+y^2)$

207. $z=(x^2+y^2)e^{-(x^2+y^2)}$

208. $u=x^2+y^2+z^2-xy+x-2z$

209. $u=x+\frac{y^2}{4x}+\frac{z^2}{y}+\frac{2}{z},(x>0,y>0,z>0)$

210. $u=x^2+y^2+z^2+2x+4y-6z$

211. $u = xy^2z^3(a - x - 2y - 3z), (a > 0)$

212. $u = \frac{a^3}{x} + \frac{x^3}{y} + \frac{y^3}{z} + \frac{z^3}{b}, (x > 0, y > 0, z > 0, a > 0, b > 0)$

Oshkormas funksiyaning ekstremal qiymatlarini toping.

213. $y^3 - ay - \sin x = 0 \quad 0 \le x \le \pi$

214. $(y - x)^3 + x + 6 = 0$

215. $x^3 + xy + y^2 = 27$

216. $x^2 + y^2 + z^2 - 2x + 2y - 4z - 10 = 0$

217. $x^2 + y^2 + z^2 - xz - yz + 2x + 2y + 2z - 2 = 0$

218. $x^3 - y^3 - 3x + 4y + z^2 + z - 8 = 0$

Shartli ekstremumni aniqlang.

219. $x + y = 1$ da $z = xy$

220. $x^2 + y^2 = 2a^2$ da $z = xy$

221. $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{a^2}$ da $z = \frac{1}{x} + \frac{1}{y}$

222. $x - y = \frac{\pi}{4}$ da $z = \cos^2 x + \cos^2 y$

223. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ da $u = x + y + z$

224. $\begin{cases} x + y + z = 5 \\ xy + xz + yz = 8 \end{cases}$ da $u = xyz$

225. $x^2 + y^2 + z^2 = 1$ da $u = x - 2y + 2z$

226. $4x^2 + y^2 = 25$ da $z = x^2 + 12xy + 2y^2$

227. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ da $u = x^2 + y^2 + z^2, (a > b > c > 0)$

228. $x + 2y + 3z = a$ da $u = xy^2z^3$

229. $x^2 + y^2 = 2, y + z = 2$ da $u = xy + yz, (x > 0, y > 0, z > 0)$

230. $x^2 + y^2 + 2z^2 = 22$ da $u = 2x + y - z + 1$

231. $tgx - 3tgy = 0$ da $z = x - y, |x| < \frac{\pi}{2}, |y| < \frac{\pi}{2}$

232. Agar $n \geq 1$ va $x \geq 0, y \geq 0$ bolsa tengsizlikni isbotlang

$\frac{x^n + y^n}{2} \geq \left(\frac{x+y}{2}\right)^n$, korsatma: $x + y = S$ sharti bilan $z = \frac{1}{2}(x^n + y^n)$ funksiyani minimumini aniqlang

Funksiyaning berilgan sohadagi eng katta va eng kichik qiymatlarini toping.

233. $z = x^2 - y^2, x^2 + y^2 \leq 4$

234. $z = x^2 + 2xy - 4x + 8y, 0 \leq x \leq 1, 0 \leq y \leq 2$

235. $z = e^{-x^2-y^2}(2x^2 + 3y^2), x^2 + y^2 \leq 4$

236. $z = x - 2y - 3, 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq x + y \leq 1$

237. $z = x^2 + y^2 - 12x + 16y, x^2 + y^2 \leq 25$

238. $z = x^2 - xy + y^2, |x| + |y| \leq 1$

239. $u = x^2 + 2y^2 + 3z^2, x^2 + y^2 + z^2 \leq 100$

240. $u = x + y + z, x^2 + y^2 \leq z \leq 1$

241. $z = xy - x^2 y - \frac{xy^2}{2}, 0 \leq x \leq 1, 0 \leq y \leq 2$

242. $z = \frac{xy}{2} - \frac{x^2 y}{6} - \frac{xy^2}{8}, x \geq 0, y \geq 0, \frac{x}{3} + \frac{y}{4} \leq 1$

243. $u = xy + yz + zx, x^2 + y^2 + z^2 \leq a^2$

4§ Teylor formulasi.

244. f(x,y)=x^3+$2y^3$-xy funksiyani h va k darajalari bo`yicha yozing.

245. f(x,y) =x^3+y^2-6xy-39x+18y+4 funksiyaning x=5, y=6 dan x=5+h, y=6+k ga ko`chirganda xosil bo`ladigan orttirmani toping.

246. f(x,y) =$2x^2$-xy-y^2-6x-3y+5 funksiyani A(1,-2) nuqta atrofida Teylor formulasiga yoying.

247. f(x,y,z) =x^3+y^3+z^3-3xy funksiyani A(1,-2) nuqta atrofida Teylor formulasiga yoying.

248. f(x,y) =x^2y+xy^2-2xy funksiyaning A(1,-1) nuqtadan B(1+l,-1+k)nuqtaga o`tgandagi orttirmasini toping.

249. f(x,y) =xy funksiyani A(1,1) nuqta atrofida Teylor formulasiga yoying. (ikkinchi tartibgacha)

250. Z=sinx sin y funksiyni (x-π4) va (y-π/4) larning darajalari bo`yicha qatorga yoying (2-tartibgacha)

Quydagi fynksiyalarni Makloren qatoriga yoying.

251. $\frac{1}{1-x-y+xy}$

252. $Z=\ln(1+x)\ln(1+y)$

253. $Z=\sin(x^2+y^2)$

254. $Z= e^x \sin y$

255. $Z= e^x \cos y$

Quyidagi ifodalarni Teylor formulasidan foydalanib 2-tartibgacha aniqlikda taqriban hisoblang.

256. a) $(0,95)^{2,01}$ b) $\sqrt{1,03}\cdot\sqrt[3]{0,98}$

III Bob
Hosmas integrallar
1§ Hosmas integrallar.

Quyidagi hosmas integrallarni hisoblang.

257. $\int_1^{\infty}\frac{dx}{x^4}$

258. $\int_1^{\infty}\frac{dx}{\sqrt{x}}$

259. $\int_0^{\infty}e^{-ax}dx, (a>0)$

260. $\int_{-\infty}^{+\infty}\frac{dx}{x^2+2x+2}$

261. $\int_2^{\infty}\frac{\ln x}{x}dx$

262. $\int_1^{\infty}\frac{dx}{x^2(x+1)}$

263. $\int_0^{\infty}\frac{x}{(1+x)^3}dx$

264. $\int_{\sqrt{2}}^{\infty}\frac{dx}{x\sqrt{x^2-1}}$

265. $\int_0^{\infty}x^3e^{-x^2}dx$

266. $\int_0^{\infty}xSinxdx$

267. $\int_0^{\infty}e^{-x}Sinxdx$

268. $\int_0^{\infty}e^{-ax}Cosbxdx$

269. $\int_0^{\infty}e^{-ax}Sinbxdx$

270. $\int_1^{\infty}\frac{arctgx}{x^2}dx$

271. $\int_{-\infty}^{+\infty}\frac{dx}{(x^2+1)^2}$

272. $\int_1^{\infty}\frac{\sqrt{x}}{(1+x)^2}dx$

273. $\int_{-\infty}^{+\infty}\frac{dx}{(x^2+x+1)^2}$

274. $\int_0^{\infty}\frac{x^2+1}{x^4+1}dx$

275. $\int_0^{\infty}\frac{x\ln x}{(1+x^2)^2}dx$

276. $\int_2^{\infty}\frac{dx}{(x^2-1)^2}$

277. $\int_0^1\frac{dx}{\sqrt{1-x^2}}$

278. $\int_0^2\frac{dx}{x^2-4x+3}$

279. $\int_1^2\frac{xdx}{\sqrt{x-1}}$

280. $\int_0^1 x\ln xdx$

281. $\int_0^1 \frac{dx}{x\ln^2 x}$

282. $\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}, (a<b)$

283. $\int_a^b \frac{xdx}{\sqrt{(x-a)(b-x)}}$

284. $\int_3^5 \frac{x^2dx}{\sqrt{(x-3)(5-x)}}$

285. $\int_0^1 \frac{dx}{1-x^2+2\sqrt{1-x^2}}$

286. $\int_{-1}^1 \frac{3x^2+2}{\sqrt[3]{x^2}}dx$

287. $\int_{-1}^1 \frac{x-1}{\sqrt[3]{x^5}}dx$

288. $\int_{-1}^0 \frac{e^{\frac{1}{x}}}{x^3}dx$

289. $\int_0^1 \frac{e^{\frac{1}{x}}}{x^3}dx$

290. $\int_0^1 \frac{dx}{x^3-5x^2}$

291. $\int_0^1 \ln x dx$

Quyidagi integrallarning yaqinlashuvchanligini tekshiring.

292. $\int_0^\infty \frac{x}{x^3+1}dx$

293. $\int_1^\infty \frac{x^3+1}{x^4}dx$

294. $\int_0^{+\infty} \frac{x^2dx}{x^4-x^2+1}$

295. $\int_1^\infty \frac{dx}{x^3\sqrt{x^2+1}}$

296. $\int_0^2 \frac{dx}{\ln x}$

297. $\int_0^{+\infty} x^{p-1}e^{-x}dx$

298. $\int_2^\infty \frac{dx}{x\ln\ln x}$

299. $\int_e^\infty \frac{dx}{x(\ln x)^{\frac{2}{3}}}$

300. $\int_{-1}^\infty \frac{dx}{x^2+\sqrt[3]{x^4+1}}$

301. $\int_0^\infty \frac{x^a}{1+x^b}dx, (b\geq 0)$

302. $\int_0^{+\infty} \frac{\ln(1+x)}{x^p}dx$

303. $\int_0^{+\infty} \frac{x^n arctgx}{2+x^n}dx, (n\geq 0)$

304. $\int_0^1 \frac{\sqrt{x}}{\sqrt{1-x^4}}dx$

305. $\int_0^1 \frac{x^3dx}{\sqrt[3]{(1-x^2)^5}}$

306. $\int_0^{+\infty} \frac{dx}{x^p+x^q}$

307. $\int_0^1 \frac{\sqrt{x}dx}{e^{\sin x}-1}$

308. $\int_0^1 \frac{dx}{e^x-Cosx}$

309. $\int_0^{\frac{\pi}{2}} \frac{\ln Sinx}{\sqrt{x}}dx$

310. $\int_{\frac{\pi}{2}}^{\infty}\frac{Sinx}{x^2}dx$

Quyidagi integrallarni absolyut va shartli yaqinlashuvchiligini tekshiring.

311. $\int_{0}^{\infty}\frac{Sinx}{x}dx$

312. $\int_{0}^{\infty}\frac{\sqrt{x}Cosx}{x+100}dx$

313. $\int_{0}^{\infty}x^{p}Sin(x^{q})dx,(q\neq 0)$

314. $\int_{0}^{\infty}\frac{x^{p}Sinx}{1+x^{q}}dx,(q\geq 0)$

315. $\int_{0}^{+\infty}\frac{Sin\left(x+\frac{1}{x}\right)}{x^{n}}dx$

Quyidagi integrallarni hisoblang.

316. v.p. $\int_{0}^{\infty}\frac{dx}{x^2-3x+2}$

317. v.p. $\int_{\frac{1}{2}}^{2}\frac{dx}{x\ln x}$

318. v.p. $\int_{-\infty}^{\infty}\frac{1+x}{1-x}dx$

319. v.p. $\int_{-\infty}^{\infty}arctgxdx$

IV Bob
Ikki karrali integrallar
1§ Quyidagi ikki karrali integrallar hisoblansin:

320. $\iint_{D}xydxdy \quad D:(0\leq x\leq 1,0\leq y\leq 2)$

321. $\iint_{D}e^{x+y}dxdy \quad D:(0\leq x\leq 1,0\leq y\leq 1)$

322. $\iint_{D}\frac{x^2}{1+y^2}dxdy \quad D:(0\leq x\leq 1,0\leq y\leq 1)$

323. $\iint_{D}\frac{dxdy}{(1+x+y)^3} \quad D:(0\leq x\leq 1,0\leq y\leq 1)$

324. $\iint_{D}\frac{ydxdy}{(1+x^2+y^2)^{\frac{3}{2}}} \quad D:(0\leq x\leq 1,0\leq y\leq 1)$

325. $\iint_{D}x\sin(x+y)dxdy \quad D:\left(0\leq x\leq \pi,0\leq y\leq \frac{\pi}{2}\right)$

326. $\iint_{D}x^2ye^{xy}dxdy \quad D:(0\leq x\leq 1,0\leq y\leq 2)$

327. $\iint_{D}x^2y\cos(xy^2)dxdy \quad D:\left(0\leq x\leq \frac{\pi}{2},0\leq y\leq 2\right)$

Quyidagi takroriy integrallar hisoblansin:

328. $\int_0^2 dy \int_0^1 (x^2 + 2y)dx$

329. $\int_1^4 dx \int_1^2 \frac{dy}{(x+y)^2}$

330. $\int_1^2 dx \int_1^x \frac{x^2 dy}{y^2}$

331. $\int_{-3}^{3} dy \int_{y^2-4}^{5} (x+2y)dx$

332. $\int_0^{2\pi} d\varphi \int_{a\sin\varphi}^{a} rdr$

333. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^{3\cos\varphi} r^2 \sin^2 \varphi dr$

334. $\int_0^1 dx \int_0^{\sqrt{1-x^2}} \sqrt{1-x^2-y^2}\,dy$

335. $\int_0^1 dx \int_{x^2-1}^{1-x} xydy$

336. $\int_0^1 dx \int_{x^2}^{x} xy^2 dy$

337. $\int_0^a dx \int_0^{\sqrt{ax}} dy$

338. $\int_1^2 dy \int_0^{\ln y} e^x dx$

Berilgan egri chiziqlar bilan chegaralangan yoki ma'lum shartlar bilan berilgan (D) soha uchun $\iint_D f(x,y)dxdy$ ikki karrali integralda integrallash chegaralari ikki hil tartibda qoyilsin:

339. D – uchburchak O(0,0), A(1,0), B(1,1)

340. D – uchburchak O(0,0), A(1,1), B(1,-1)

341. D – trapetsiya O(0,0), A(1,0), B(1,2), C(0,1)

342. $D = \{(x,y): x^2 + y^2 \le 1\}$

343. $D = \{(x,y): x^2 + y^2 \le y\}$

344. $D = \{(x,y): x^2 + y^2 \ge 1, 0 \le y \le 1, 0 \le x \le 1\}$

345. $D = \{(x,y): |x| + |y| \le 1\}$

346. $D = \{(x,y): x^2 + y^2 \ge 2x + 2y - 1, 0 \le x \le 1, 0 \le y \le 1\}$

347. $D = \left\{(x,y): 0 < y \le \frac{1}{x}, y \ge 0, x \ge 0, y - 2x \le 0, y - \frac{1}{2x} \ge 0\right\}$

348. $D - 2x = \sin \pi y, y = (1+x)^2, y = 0$ chiziqlar bilan chegaralangan

349. $D = \{(x,y): y^2 \le 2x + 4, y^2 \ge 4x + 4\}$

Berilgan egri chiziqlar bilan chegaralangan yoki ma'lum shartlar bilan berilgan (D) soha uchun $\iint_D f(x,y)dxdy$ ikki karrali integralda integrallash chegaralari qoyilsin:

350. Tomonlari $x=0, y=0, x+y=2$ chiziqlar bilan chegaralangan uchburchak

351. $D=\{(x,y): x+y\le 1, x-y\le 1, x\ge 0\}$

352. $D=\{(x,y): y\ge x^2, y\le 4-x^2\}$

353. $D-y=x^2, y=\sqrt{x}$ parabolalar bilan chegaralangan soha

354. Tomonlari $D-y=x, y=2x, x+y=6$ bilan chegaralangan uchburchak

355. Tomoni $D-y=x, y=x+3, y=-2x+1, y=-2x+5$ bo'lgan parallelogram

Quyidagi ikki karrali integralda integrallash tartibi o'zgartirilsin:

356. $\int_0^1 dy \int_y^{\sqrt{y}} f(x,y)dx$

357. $\int_{-1}^{1} dx \int_0^{\sqrt{1-x^2}} f(x,y)dy$

358. $\int_{-2}^{2} dx \int_{-\frac{1}{2}\sqrt{4-x^2}}^{\frac{1}{2}\sqrt{4-x^2}} f(x,y)dy$

359. $\int_1^2 dx \int_x^{2x} f(x,y)dy$

360. $\int_0^2 dx \int_{2x}^{6-x} f(x,y)dy$

361. $\int_1^2 dx \int_{2-x}^{\sqrt{2x-x^2}} f(x,y)dy$

362. $\int_0^{2a} dx \int_0^{\sqrt{8ax}} f(x,y)dy$

363. $\int_0^2 dx \int_0^{(x-1)^2} f(x,y)dy$

364. $\int_3^7 dy \int_{\frac{9}{y}}^{3} f(x,y)dx + \int_7^9 dy \int_{\frac{9}{y}}^{10-y} f(x,y)dx$

365. $\int_0^3 dy \int_{\sqrt{9-y^2}}^{\sqrt{25-y^2}} f(x,y)dx$

Ko'rsatilgan (D) soha uchun $\iint_D f(x,y)dxdy$ integralda qutb koordinatalarga $(x=r\cos\varphi, y=r\sin\varphi)$ o'tib, integrallash chegaralari qo'yilsin:

366. $D=\{(x,y): x^2+y^2\le a^2\}$

367. $D=\{(x,y): x^2+y^2\le ax^2\}(a>0)$

368. D – ikki $x^2+y^2\le ax$ va $x^2+y^2\le by$ aylananing umumiy qismi

369. $D-y=x, y=0, x=1$ chiziqlar bilan chegaralangan soha

370. $D-x^2+y^2=4$ aylanadan $x+y=2$ to'g'ri chiziq bilan ajratilgan segment

371. $D-(x^2+y^2)^2=a^2(x^2-y^2)$ Bernulli lemniskatasining o'ng sirtmog'ining ichki qismi

372. D – O(0,0), A(1,1), B(-1,1) uchburchak

373. D – O(0,0), A(0,1), B(1,0), C(1,1) kvadrat

374. $D=\{(x,y):(x-1)^2+y^2\le 1, 0\le x\le 1\}$

375. $D=\{(x,y):x^2+y^2\ge 1, 0\le x\le 1, 0\le y\le 1\}$

376. $D=\{(x,y):|x-1|+|y|\le 1\}$

Quyidagi integrallarda qutb koordinatalarga $(x=r\cos\varphi, y=r\sin\varphi)$ ötib, integrallash chegaralari ikki hil tartibda qöyilsin:

377. $\int_0^1 dx\int_0^1 f(x,y)dy$

378. $\int_0^1 dx\int_{1-x}^{\sqrt{1-x^2}} f(x,y)dy$

379. $\int_0^2 dx\int_x^{x\sqrt{3}} f\left(\sqrt{x^2+y^2}\right)dy$

Qutb koordinatalariga ötib, quyidagi ikki karrali integrallar hisoblansin:

380. $\iint_{x^2+y^2\le a^2}\sqrt{x^2+y^2}\,dxdy$

381. $\iint_{a^2\le x^2+y^2\le 4a^2}\sin\sqrt{x^2+y^2}\,dxdy$

382. $\int_0^R dx\int_0^{\sqrt{R^2-x^2}}\ln\left(1+x^2+y^2\right)dy$

383. $\iint_D arctg\frac{y}{x}dxdy$ $D=\left\{(x,y):x^2+y^2\ge 1, x^2+y^2\le 9, y\ge\frac{x}{\sqrt{3}}, y\le x\sqrt{3}\right\}$

384. $\iint_{x^2+y^2\le ax}\frac{xdxdy}{\sqrt{a-x^2-y^2}}$

Yangi U va V özgaruvchilar kiritilib, quyidagi integrallar hisoblansin:

385. $\iint_D\left(x^2y^2+y^2\right)dxdy$ $D=\left\{(x,y):\frac{1}{x}\le y\le\frac{2}{x}, x\le y\le 3x\right\}$

386. $\iint_D\frac{(x+y)^2}{x}dxdy$ $D=\left\{(x,y):1-x\le y\le 3-x, \frac{x}{2}\le y\le 2x\right\}$

387. $\iint_{|x|+|y|\le 1}\left(|x|+|y|\right)dxdy$

388. $\iint_{\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1}\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}dxdy$

389. $\iint_D x^2dxdy$ $D=\{(x,y):x^3\le y\le 2x^3, x\le 2y\le 6x\}$

2§ Yuzalarni hisoblash.

Quyidagi chiziqlar bilan chegaralangan yuza hisoblansin:

390. $xy = a^2, x + y = \frac{5}{2}a, (a > 0)$

391. $y^2 = 2px + p^2, y^2 = -2qx + q^2, \ (p > 0, q > 0)$

392. $x = 0, y = 0, x + y = 1$

393. $y = x, y = 5x, x = 1$

394. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

395. $y = \sqrt{x}, y = 2\sqrt{x}, x = 4$

Qutb koordinatalariga ōtib, quyida berilgan egri chiziqlar bilan chegaralangan yuzalar hisoblansin:

396. $(x^2 + y^2)^2 = 2a^2(x^2 - y^2)$

397. $(x^2 + y^2)^2 = 2a^2(x^2 - y^2); x^2 + y^2 \geq a^2$

398. $(x^2 + y^2)^2 = 2ax^3$

399. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x}{h} + \frac{y}{k}$

400. $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = \frac{xy}{c^2}$

401. $\left(\frac{x}{a} + \frac{y}{b}\right)^3 = \frac{x^2}{h^2}$

402. $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = \frac{x^2}{c^2}$

403. $(x + y)^3 = axy$

404. $(x + y)^5 = ax^2y^2$

405. $x^3 + y^3 = 2xy \ x + y = a, x + y = b, y = \alpha x, y = \beta x, (0 < a < b, 0 < \alpha < \beta)$

406. $xy = a^2, xy = 2a^2, y = x, y = 2x, (x > 0, y > 0)$

407. $y^2=2px$, $y^2=2qx$, $x^2=2ry$, $x^2=2sy$ $(0<p<q;0<r<s)$

408. $xy=p$, $xy=q$, $y^2=ax$, $y^2=bx$, $(0<p<q;\ 0<a<b)$

409. $x^2=2py$, $x^2=2qy$, $y=\alpha x$, $y=\beta x$, $(0< p< q;\ 0<\alpha <\beta)$

410. $x^2+y^2=ay$, $x^2+y^2=by, x+\alpha=y$, $x=\beta y$, $(0< a<b;\ 0<\alpha<\beta)$

3§ Hajmlarni hisoblash

Quydagi sirtlar bilan chegaralangan hajmlar topilsin:

411. $x=4$, $y=4$, $z= x^2+y^2+1$

412. x=0, y=0, z=0, x=a, y=b, $z = \frac{x^2}{2p} + \frac{y^2}{2q}$.

413. y=0, z=0, 3x+y=6, 3x+2y=12, x+y+z=6.

414. $z=x^2+y^2$, $x+y=1$, x=0, y=0, z=0.

415. $y = \sqrt{x}$, $y = 2\sqrt{x}$, z=0, x+z=6.

416. $z=x^2+y^2$, z=0, y=1, y=2x, y=6-x.

417. $z=4-x^2$, x=0, y=0, z=0, 2x+y=4 ($x \geq 0$)

418. z=1+x+y, z=0, x+y=1, x=0, y=0.

419. $z = x^2+y^2$, $y=x^2$, $y=1$, z=0.

420. z=xy, x+y+z=1, z=0.

421. $z=4-y^2$, $y = \frac{x^2}{2}$, z=0.

422. $y=\ln x$, $y=\ln^2 x$, z=0, y+z=1.

423. $z=\cos x \cdot \cos y$, x=0, y=0, z=0, $x + y = \frac{\pi}{2}$.

424. $-x \leq y \leq x$, $x^2+y^2 \leq az \leq 2x^2+2y^2$, $z \leq h$.

425. $0 \leq z \leq 1-y^2$, $0 \leq x \leq 2-z$

Quyidagi sirtlar bilan chegaralangan jismlarning hajmlarini qutb koordinatalar sistemasida hisoblang.

426. $z^2=xy$, $x^2+y^2=a^2$., $x^2+y^2+z^2=a^2$, $x^2+y^2=ax$.

427. $z=x^2+y^2$, $x=x^2+y^2$, $x^2+y^2=2x$, z=0

428. $z = e^{-(x^2+y^2)}$, z=0, $x^2+y^2=R^2$

429. $x^2+y^2=R^2$, $z = e^{-(x^2+y^2)}$

Qatorlarning yaqinlashuvchanligini tekshiring

430. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$ (z>0).

431. $z^2=xy$, x+y=a, x+y=b ($0 < a < b$).

432. $z=x^2+y^2$, $xy=a^2$, $xy=2a^2$, $y = \frac{x}{2}$, y=2x, z=0.

433. z=xy, $x^2=y$, $x^2=2y$, $y^2=x$, $y^2=2x$, z=0.

4§ Sirt yuzini xisoblash.

434. $z^2=2xy$, agar $0 \leq x \leq a$, $0 \leq y \leq b$.

435. $z = \sqrt{x^2 + y^2}$, agar $x^2+y^2 \leq 2ax$.

436. $2z=x^2$, agar $x \leq 2y \leq 4x$, $x \leq 2\sqrt{2}$.

437. $x^2+y^2+z^2=a^2$, agar $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$ ($b \leq a$).

438. $x^2+z^2 \leq a^2$, $z^2+y^2 \leq a^2$.

439. $\begin{cases} x^2+y^2=R^2 \\ z=xy \end{cases}$ sirtlar bilan chegaralangan sirt yuzi hisoblansin.

440. $\begin{cases} x^2+y^2=2a^2xy \\ x^2+y^2=2az \end{cases}$ sirtlar bilan chegaralangan sirt yuzi hisoblansin.

5§ Ikki karrali integralning mehanikaga tatbig`i.

Quyidagi chiziqlar bilan chegaralangan va zichligi ρ bo`lgan plastinka massasi topilsin.

441. $y=x^2$, $x+y=2$, $y-x=2$ $(x>0)$, agar $\rho=x+2$.

442. $y=x$, $x-3y=1$, $y=3$, $y=1$ agar $\rho=y$.

443. $x^2+y^2=4x$, $x^2+y^2=4$ $(xy\ge 0)$, agar $\rho=x$.

444. $\frac{x^2}{9}+\frac{y^2}{4}=1, \frac{x^2}{18}+\frac{y^2}{8}=1$, agar $\rho=4x^2+9y^2$.

Ushbu egri chiziq bilan chegaralangan bir jinsli plastinkaning og`irlik markazi topilsin.

445. $ay=x^2$, $x+y=2a$ $(a>0)$

446. $\sqrt{x}+\sqrt{y}=\sqrt{a}$, $x=0$, $y=0$.

447. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$, $x=0$, $y=0$ $(x\ge 0, y\ge 0)$.

448. $y=\sin x$, $y=0$, $x=\frac{\pi}{4}$.

449. $y^2=x^2-x^4$ $(x\ge 0)$.

450. $y=x^2$, $y=3x^2$, $y=3x$.

451. $\frac{x}{a}+\frac{y}{b}=1, \frac{x}{a}+\frac{y}{b}=2, \frac{x}{a}=\frac{y}{b}, \frac{3x}{a}=\frac{y}{b}$.

452. $x=a(t-\sin t)$, $y=a(1-\cos t)$, $y=0$, $x=\pi a$ $(0\le x\le \pi a)$.

453. $r=a(1+\cos\varphi)$, $0\le x\le \pi$, $\varphi=0$.

454. $(x^2+y^2)^2=2a^2xy$, $x\ge 0, y\ge 0$.

455. $r^2=a^2\cos 2\varphi$, $0\le\varphi\le\frac{\pi}{4}$, $\varphi=0$.

456. $r=9\cos\varphi$, $r=4\cos\varphi$.

Ushbu egri chiziq bilan chegaralangan yuzaning OX va OY o`qlariga nisbatan I_x va I_y inersiya momenti topilsin (zichligi $\rho=1$ deb qaralsin).

457. $\frac{x}{a}+\frac{y}{h}=1$. $\frac{x}{b}+\frac{y}{h}=1$. $y=0$ $(a>0.\ b>0.\ h>0)$.

458. $(x-a)^2+(y-a)^2=a^2$, $x=0, y=0$ $(0\le x\le a)$.

459. $r=a(1+\cos\varphi)$.

460. $xy=a^2$, $xy=2a^2$, $x=2y$, $2x=y$ $(x>0, y>0)$

461. $x=0$, $y=0$, $x=q$, $y=b$

Quyidagi chiziqlar bilan chegaralangan, massasi M bo`lgan bir jinsli plastinkaning to`g`ri chiziqqa nisbatan inertsiya momentini hisoblang.

462. $x^2+y^2=R^2$ doira markazidan o`tuvchi va doira tekisligida yotuvchi to`g`ri chiziqqa nisbatan.

463. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ katta va kichik yarim o`qqa nisbatan.

464. $y=\sin x$, $0\le x\le\pi$, $y=0$, $y=1$ to`g`ri chiziqqa nisbatan.

465. $ay=x^2$, $x+y=2a$ har bir koordinat o`qiga nisbatan.

466. $r=(1+\cos\varphi)$ qutb o`qiga nisbatan.

467. $r=a^2\cos 2\varphi$ qutb o`qiga nisbatan

V Bob

Uch karrali integrallar.

1§ Uch karrali integrallar.

Quiyidagi uch karrali integrallar hisoblansin.

468. $\int_0^{*}dx\int_0^{*}dy\int_0^{y}xyzdz$;

469. $\int_0^{*}dx\int_0^{*}dy\int_0^{xy}x^3y^2zdz$;

470. $\int_1^{2}dy\int_y^{2}dx\int_0^{\frac{1}{xy}}\frac{dz}{x(1+x^2y^2z^2)}$;

471. $\int_{-1}^{0}dy\int_y^{0}dz\int_0^{xy}y^2\cos xdx$;

472. $\int_0^{e-1}dx\int_0^{e-1}dy\int_e^{x+y+e}\frac{\ln(z-x-y)}{(x-e)(x+y-e)}dz$

Quiyidagi uch karrali integrallar ko`rsatilgan sohada hisoblansin.

473. $\iiint_V xy^2z^3dxdydz$, $z=xy$, $y=x$, $x=1$, $z=0$.

474. $\iiint_V xyzdxdydz$, $x^2+y^2+z^2=1$, $x=0$, $y=0$, $z=0$.

475. $\iiint_V\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\right)dxdydz$, $\frac{xx^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

476. $\iiint_V\sqrt{x^2+y^2}dxdydz$, $x^2+y^2=z^2$, $z=1$.

477. $\iiint_V\frac{dxdydz}{(1+x+y+z)^3}$, $x+y+z=1$, $x=0$, $y=0$, $z=0$.

478. $\iiint_V y\cos(z+x)dxdydz$, $y=\sqrt{x}$, y=0, z=0, $x+z=\frac{\pi}{2}$

Quiyidagi uch karrali integrallarda integrallash chegaralari almashtirilsin.

479.

480. $\int_0^1 dx\int_0^{1-x} dy\int_0^{x+y} f(x,y,z)dz$.

481. $\int_{-1}^{1} dx\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\int_{\sqrt{x^2+y^2}}^{1} f(x,y,z)dz$.

482. $\int_0^1 dx\int_0^1 dy\int_0^{x^2+y^2} f(x,y,z)dz$.

483. $\int_{-1}^{1} dx\int_0^{4-x} dy\int_0^{4-x-y} f(x,y,z)dz$.

484. $\int_0^R dx\int_0^{\sqrt{R^2-x^2}} dy\int_0^{x^2+y^2} f(x,y,z)dz$.

Quiyidagi integrallarni sferik yoki silindrik koordinatalar sistemasiga o'tish yordamida hisoblang.

485. $\iiint_V \sqrt{x^2+y^2+z^2}\,dxdydz$, $x^2+y^2+z^2\le z$.

486. $\int_0^1 dx\int_0^{\sqrt{1-x^2}} dy\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} z^2dz$.

487. $\int_0^1 dx\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\int_0^a dz$.

488. $\int_0^2 dx\int_0^{\sqrt{2x-x^2}} dy\int_0^a z\sqrt{x^2+y^2}\,dz$

489. $\int_{-R}^{R} dx\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} dy\int_0^{\sqrt{R^2-x^2-y^2}} (x^2+y^2)dz$

490. $\int_0^1 dx\int_0^{\sqrt{1-x^2}} dy\int_0^{\sqrt{1-x^2-y^2}} \sqrt{x^2+y^2+z^2}\,dz$

491. $\iiint_V (x^2+y^2)dxdydz$, $x^2+y^2=2z$, z=2.

492. $\iiint_V (x^2+y^2)dxdydz$, $z\ge 0, r^2\le x^2+y^2+z^2\le R^2$.

493. $\iiint_V \frac{dxdydz}{\sqrt{x^2+y^2+(z-2)^2}}$, $x^2+y^2+z^2\le 1$.

494. $\iiint_V \frac{dxdydz}{\sqrt{x^2+y^2+(z-2)^2}}$, $x^2+y^2\le 1$, $-1\le z\le 1$.

Quyidagi sirtlar bilan chegaralangan jismning hajmi topilsin.

495. $z=4-y^2$, $z=y^2+2$, x=-1, x=2.

496. $z=x^2+y^2$, $z=x^2+2y^2$, $y=x$, $y=2x$, $x=1$.

497. $z=x^2+y^2$, $z=2x^2+2y^2$, $y=x^2$, $y=x$.

498. $z=\ln(x+2)$, $z=\ln(6-x)$, $x=0$, $x+y=2$, $x-y=2$.

499. $z=x^2+y^2$, $z^2=xy$.

500. $z=x+y$, $z=xy$, $x+y=1$, $x=0$, $y=0$.

Sferik yoki silindrik koordinatalar sistemasiga o`tib, quyidagi sirtlar bilan chegaralangan jism hajmi hisoblansin.

501. $x^2+y^2+z^2=2az$, $x^2+y^2\le z$.

502. $(x^2+y^2+z^2)^2=a^2(x^2+y^2-z^2)$

503. . $x^2+y^2+z^2)^3=3xyz$.

504. $x^2+y^2+z^2=a^2$, $x^2+y^2+z^2=b^2$, $x^2+y^2=z^2$, $(z\ge 0)$, $(0< a< b)$.

505. $(x^2+y^2+z^2)^2=axyz$.

506. $x^2+y^2+z^2=4$, $x^2+y^2=3z$

506. $(x^2+y^2+z^2)^2=a^3x$.

507. $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\right)^2=\frac{x}{h}$;

508. $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\right)^2=\frac{x^2}{a^2}+\frac{y^2}{b^2}$;

509. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$; $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$.

510. $\left(\frac{x}{a}+\frac{y}{b}+\frac{z}{c}\right)^2=\frac{x}{h}+\frac{y}{k}$; $(x\ge 0, y\ge 0, z\ge 0)$.

511. $\left(\frac{x}{a}+\frac{y}{b}+\frac{z}{c}\right)^4=\frac{xyz}{abc}$; $(x\ge 0, y\ge 0, z\ge 0)$.

Quyidagi sirtlar bilan chegaralangan va zichligi ρ bo`lgan jism massasi topilsin.

512. $Z=x^2+y^2$, $Z^2+x^2+Y^2=6$, $Z>0$ agar $\rho=z$.

513. $z=\frac{x^2}{4}+\frac{y^2}{9}$, $z=2y$, agar $\rho=x^2$

514. $z=x^2+y^2$, $z=2y$, agar $\rho=y$

515. $x+y+z=2$, $x=0$, $y=0$, $z=0$, agar $\rho=x+y+z$

Quyidagi sirtlar bilan chegaralangan bir jinsli jismning og'irlik markazi topilsin.

516. $x=0$, $y=0$, $z=0$, $x=2$, $y=4$, $x+y+z=8$

517. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\le 1$, $x\ge 0$, $y\ge 0$, $z\ge 0$

518. $z=\frac{y^2}{2}$, $x=0$, $y=0$, $z=0$, $2x+3y-12=0$

519. $y=\sqrt{x}$, $y=2\sqrt{x}$, $z=0$, $x+z=6$

520. $z=\frac{x^2+y^2}{2a}$, $x^2+y^2+z^2=3a^2$, $(z\geq 0)$

521. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$, $z=c$

522. $z=x^2+y^2$, $x+y=a$, $x=0$, $y=0$, $z=0$

523. $x^2=2pz$, $y^2=2px$, $x=0$, $y=0$, $z=0$

524. $x^2+y^2+z^2=a^2$, $x\geq 0$, $y\geq 0$, $z\geq 0$

525. $x^2+y^2=2z$, $z=1$

526. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, $x=0$, $y=0$, $z=0$

527. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$,

528. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$, $z=c$

Quyidagi sirtlar bilan chegaralangan bir jinsli jismning inersiya momentini toping.

529. $x=0$, $x=a$, $y=0$, $y=b$, $z=0$, $z=c$ koordinatalar o'qlariga nisbatan

530. $y=\sqrt{x}$, $x=2\sqrt{y}$, $z=0$, $z+x=4$

531. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

532. $x^2+y^2-ax=0$, $z^2=2ax$, $z=0$, $(z>0)$

533. $z=\frac{x^2+y^2}{2a}$, $z=1$ Ox o'qiga nisbatan

534. $x+y+z=2$, $z=0$, $x^2+y^2=2$, $(z>0)$

535. $x^2+y^2+z^2=1$, $x^2+y^2=z^2$, Oz o'qiga nisbatan.

VI Bob

Egri chiziqli integrallar

1§ I tur egri chiziqli integrallar.

Quyidagi I tur egri chiziqli integrallar hisoblansin.

536. $\int_L \frac{ds}{x-y}$, $A(0,-2), B(4,0)$ L-AB chiziq.

537. $\int_L xyds$ L: $A(0,0), B(4,0), C(4,2), D(0,2)$, nuqtalarda bo'lgan to'g'ri to'rtburchak

538. $\int_L yds, L: \begin{cases} x^2=2py \\ y^2=2px \end{cases}$ lar bilan chegaralangan yoy

539. $\int_L (x^2+y^2)^n ds, L=\{(x,y): x=a\cos t, y=a\sin t\}$

540. $\int_L xyds, L = \left\{(x,y): \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, x \geq 0, y \geq 0\right\}$

541. $\int_L \sqrt{2y}ds, L = \{(x,y): x = a(t - \sin t), y = a(1 - \cos t), 0 \leq t \leq 2\pi\}$

542. $\int_L (x - y)ds, L = \{(x,y): x^2 + y^2 = ax\}$

543. $\int_L x\sqrt{x^2 - y^2}ds$, $L = \{(x,y): (x^2 + y^2)^2 = a^2(x^2 - y^2), x \geq 0\}$

544. $\int_L arctg\frac{x}{y}ds, L = \{(\rho,\varphi): \rho = 2\varphi, \rho \leq R\}$

545. $\int \frac{z^2 ds}{x^2 + y^2}; L = \{x,y): x = a\cos t, y = a\sin t, z = at, 0 \leq t \leq 2\}$

546. $\int_L xyzds, L = \left\{(x,y,z): x^2 + y^2 + z^2 = R, x^2 + y^2 = \frac{R^2}{4}, x \geq 0, y \geq 0, z \geq 0\right\}$

547. $\int_L (2z - \sqrt{x^2 + y^2})ds$,

$L = \{(x,y,z): x = t\cos t, y = t\sin t, z = t, 0 \leq t \leq 24\}$

548. $\int_L (x + y)ds$, $L\{(x,y,z): x^2 + y^2 + z^2 = R, y = x, x \geq 0, y \geq 0, z \geq 0\}$

549. $\int_L (x + y)ds$, L-uchlari 0(0,0), A(1,0), B(0,1) nuqtalarda bo'lgan uchburchak

550. $\int_L y^2 ds$, $L = \{(x,y): x = a(t - \sin t), y = a(1 - \cos t): 0 \leq t \leq 2\pi\}$

551. $\int_L (x^2 + y^2)ds, L = \{(x,y): x = a(\cos t + t\sin t), y = a(\sin t - t\cos t), 0 \leq t \leq 2\pi\}$

552. $\int(x^{\frac{4}{3}} + y^{\frac{4}{3}})ds, L = \left\{(x,y): x^{2/3} + y^{2/3} = a^{2/3}\right\}$

553. $\int_L \sqrt{x^2 + y^2}ds$, $L = \{(x,y): x^2 + y^2 = ax\}$

554. $\int xyds, L = |x| + |y| = a(a > 0)$ kvadrat konturi

555. $\int (x + y)ds, L = \pi^2 = a^2\cos 2\varphi$ lemniskataning o'ng tomoni bo'lagi

556. $\int |y|ds, L = \pi = a(2 + \cos\varphi)$ egri chiziq

557. $\int (x^3 + y^3)ds, L = \{(x,y): (x^2 + y^2)^2 = a^2xy, x \geq 0, y \geq 0\}$

2§ Egri chiziqli integralning tatbig'i.

Quyidagi tenglamalar bilan berilgan egri chiziqlar yoyining uzunligi topilsin.

558. $x = 3t, y = 3t^2, z = 2t^3$, O(0,0,0)dan A(3,3,2) gacha

559. $x = e^{-t}\cos t, y = e^{-t}\sin t, z = e^{-t}, 0 < t < +\infty$

560. $y = a\arcsin\frac{x}{a};\ z = \frac{a}{4}\ln\frac{a-x}{a+x}$; O(0,0,0)dan A$(x_0, y_0, z_0)$ gacha

561. $(x-y)^2 = a(x+y), x^2 - y^2 = \frac{9}{8}z^2$, O(0,0,0) dan A$(x_0, y_0, z_0)$ gacha

562. $x^2 + y^2 = cz, \frac{y}{x} tg\frac{z}{c}$: O(0,0,0) dan A$(x_0, y_0, z_0)$ gacha

Zichligi q bo'lgan bir jinsli yoyning inersiya momenti

563. L=$\{(x,y); x + 2y = 3, 1 \le x \le 2\}$; $\upsilon = OX$

564. L=$\{(x,y); y^2 = x, 1 \le x \le 2\}, \upsilon = 0x$

565. L=$\left\{(x,y,z); x = a\cos t, y = a\sin t, z = \frac{ht}{2\pi}; 0 \le t \le 2\pi\right\}$; a)$\upsilon = OX$;

b)$\upsilon = OY$; c)$\upsilon = OZ$

566. L=$\left\{(x,y); x = a(t - \sin t), y = a(1 - \cos t), 0 \le t \le \frac{\pi}{2}\right\}$, $\upsilon = 0X$

567. L=$\{(x,y); x^{2/3} + y^{2/3} = a^{2/3}, x \ge 0, y \ge 0\}$; a)$\upsilon = 0X$; b)$\upsilon = 0Y$

568. L=$\sqrt{x} + \sqrt{Y} = \sqrt{a}, 0 \le X \le a\}$: $\upsilon = 0X$

Bir jinsli yoy L ning og'irlik markazining koordinatalari topilsin.

569. L=$\{(x,y) : x^2 + y^2 = R^2, x \ge 0, y \ge 0\}$

570. L=$\left\{(x,y) : y = \frac{a}{2}(e^{\frac{x}{a}} + e^{-\frac{x}{a}}), -a \le x \le a\right\}$

571. L=$\left\{(x,y) : x = \frac{1}{4}y^2 - \frac{1}{2}\ln y, 1 \le y \le 2\right\}$

572. L=$\{(x,y) : x = a(t - \sin t, y = a(1 - \cos t), 0 \le t \le \pi\}$

573. L=$\{(x,y,z) : x = a\cos t, y = a\sin t, z = bt, 0 \le t \le 2\pi\}$

3§ II tur egri chiziqli integrallar.

Quyidagi ikkinchi tur egri chiziqli integrallar hisoblansin.

574. $\int(x^2 - 2xy)dx + (y^2 - 2xy)dy, L = \{(x,y) : y = x^2, -1 \le x \le 1\}$

575. $\int x^2 + y^2)dx + (x^2 - y^2)dy, L = \{(x,y) : y = 1 - |1 - x|, 0 \le x \le 2\}$

576. $\oint(x+y)dx + (x-y)dy, L = \left\{(x,y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right\}$

577. $\int(2a - y)dx + xdy, L = \{(x,y) : x = a(t - \sin t), y = a(1 - \cos t), 0 \le t \le 2\pi)\}$

578. $\oint \frac{(x+y)dx-(x-y)dy}{x^2+y^2}; L=\{(x,y): x^2+y^2=a^2\}$

579. $\oint \frac{dx+dy}{|x|+|y|}$; uchlari A(1,0),B(0,1),C(-1,0),D(0,-1) nuqtalarda bo'lgan kvadrat

580. $\int \sin ydx + \sin xdy$, uchlari A$(0,\pi)$, B$(\pi,0)$ nuqtalarda bo'lgan kesma

581. $\oint arctg\frac{y}{x}dy - dx$,

582. $\int xy^2dx - x^2ydy, L=\{(x,y): 2(x+y)=(x-y)^2\}$; A(0,2)dan B(2,0) gacha

583. $\int_L yzdx + z\sqrt{R^2-y^2}dy + xydz$ dy+xydz,

$$L=\left\{(x,y,z): x=R\cos t, y=R\sin t, z=\frac{at}{2\pi}, 0\le z\le a\right\}$$

584. $\int y^2dx + z^2dy + x^2dz, L=\begin{cases} x^2+y^2+z^2=a^2, z\ge 0 \\ x^2+y^2=ax, a>0 \end{cases}$ Viviani sirti

585. $\int yzdx + aydz - azdy, L=x^2+y^2=z^2, y^2+x^2=ax, y\ge 0, z\ge 0$; egri chiziqlarning A(0,0,0), B(a,0,a) orasidagi bo'lagi

Integral ostidagi ifoda to'liq defferensial ekanligini tekshirib berilgan egri chiziqli integrallar hisoblansin

586. $\int_{(-1,2)}^{(2,3)} xdy + ydx$

587. $\int_{(0,0)}^{(2,1)} 2xydx + x^2dy$

588. $\int_{(3,4)}^{(5,12)} \frac{xdx+ydy}{x^2+y^2}$

589. $\int_{(1,1,1)}^{(2,3,-4)} xdx + y^2dy - z^3dz$

590. $\int_{(1,2,3)}^{(3,2,1)} yzdx + zxdy + xydz$

591. $\int_{(0,1)}^{(1,0)} (3x^2-2xy+y^2)dx - (x^2-2xy)dy$

592. $\int_{(1,1)}^{(2,3)} 2x(y^2-2)dx + 2y(x^2+1)dy$

593. $\int_{(1,1,1)}^{(2,3,4)} (2xy + y^2 + yz^2)dx + (x^2 + 2xy + xz^2)dy + 2xyzdz$

594. $\int_{(-1,1,-1)}^{(1,1,2)} x(y^2+z^2)dx + y(x^2+z^2)dy + z(x^2+y^2)dz$

595. $\int_{(1,1,1)}^{(2,2,2)} yzx^{yz+1}dx + zx^{yz}\ln x dy + yx^{yz}\ln x dz$

596. $\int_{(-1,5)}^{(2,2)} \frac{xdy - ydx}{x^2+y^2}$; OX o'qi bilan kesishmaydigan qismi

Quyidagi misollarda to'liq differensial bo'yicha boshlang'ich funksiya topilsin!

597. du=x^2 dx+y^2 dy

598. du=4(x^2-y^2) (xdx-ydy)

599. $du = \frac{x}{y\sqrt{x^2+y^2}}dx - \frac{x^2+\sqrt{x^2+y^2}}{y^2\sqrt{x^2+y^2}}dy$

600. du=$(\frac{x-2y}{(y-x)^2}+x)dx + (\frac{y}{(y-x)^2}-y^2)dy$

601. du=(2xcosy-y^2 sin x)dx + (2y cos x − x^2 − sin y)dy

602. du=$\frac{2x(1-e^y)}{(1+x^2)^2}dx + (\frac{e^y}{1+x^2}+1)dy$

603. du=(x^2+2xy − y^2)dx + (x^2 − 2xy − y^2)dy

604. du=$\frac{(x^2+2xy+5y^2)dx + (x^2-2xy+y^2)dy}{(x+y)^3}$

605. du=$(2xyz+\frac{1}{z})dx + (y^2z - \frac{1}{z^2})dy + (x^2y - \frac{x}{z^2} + \frac{2y}{z^3})dz$

606. du= (x^2 −2yz)dx + (y^2 − 2xz)dy+)z^2 − 2xy)dz

607. du=$(1-\frac{1}{y}+\frac{y}{z})dx + (\frac{x}{z}+\frac{x}{y^2})dy - \frac{xy}{z^2}dz$

608. du=$(2xy+z^2+yz) + (x^2+2yz+xz)dy + (y^2+2xz+xy)dz$

4§ Grin formulasi.

Quyidagi egri chiziqli integrallardan ikki karrali integralga o'ting.

609. $\oint_L (1-x^2)ydx + x(1+y^2)dy$

610. $\lim_{y\to 0}\int_0^1 \frac{x}{y^2} e^{-\frac{x^2}{y^2}}\,dx$, ifodada integral ostida limitga o`tish mumkinmi?

611. $F(y)=\int_0^1 \ln\sqrt{x^2+y^2}\,dx$, y=0 da funksiya xosilasini Leybnits shartiga asosan hisoblash mumkinmi?

612. Tenglikni $\int_0^b \frac{dx}{1+ax}=\frac{1}{a}\ln(1+ab)$ ifodani parametr bo`yicha differinsiallash yordamida $\int_0^b \frac{xdx}{(1+ax)^2}=\frac{1}{a^2}\ln(1+ab)-\frac{b}{a(1+ab)}$ formulani hosil qiling.

613. $\int_0^b \frac{dx}{(x^2+a^2)^2}$ integralni $\int_0^b \frac{dx}{x^2+a^2}=\frac{1}{a}arctg\frac{b}{a}$ tenglikdan foydalanib hisoblang.

614. $\int_0^{\frac{\pi}{2}} \frac{dx}{(a^2\cos^2 x+b^2\sin^2 x)^2}$ integralni $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}=\frac{\pi}{2|ab|}$, tenglikdan foydalanib hisoblang.

F'(x) ni hisoblang.

615. $F(x)=\int_x^{x^2} e^{-xy^2}\,dy$

616. $F(x)=\int_{\sin x}^{\cos x} e^{x\sqrt{1-y^2}}\,dy$

617. $F(x)=\int_{x^2}^{x^3+x} \frac{\sin xy}{y}\,dy$

618. $F(x)=\int_0^x \frac{\ln(1+xy)}{y}\,dy$

619. $F(x)=\int_0^x f(y+x,y-x)dy$

620. $F(x)=\int_0^{x^2} dy\int_{y-x}^{y+x}\sin(y^2+z^2-x^2)dz$

621. $F''_{xx}(x,y)$ ni hisoblang $F(x,y)=\int_{\frac{x}{y}}^{xy}(x-yz)f(z)dz$

VII Bob

Parametrga bog'liq integrallar

1§ Parametrga bog'liq integrallarni xisoblash.

Quyidagi integrallarni parametr bo`yicha differinsiallash yordamida hisoblang.

622. $\int_0^1 \frac{arctgax}{x\sqrt{1-x^2}}dx$

623. $\int_0^{\pi} \frac{\ln(1+a\cos x)}{\cos x}(a^2<1)$

624. $\int_0^{\frac{\pi}{2}} \ln\left(\frac{1+a\sin x}{1-a\sin x}\right)\frac{dx}{\sin x}(a^2<1)$

625. $\int_0^{\frac{\pi}{2}} \ln(a^2\cos^2 x + b^2\sin^2 x)dx$

626. $\int_0^{\pi} \ln(1-2a\cos x + a^2)dx$

627. $\int_0^{\frac{\pi}{2}} \frac{arctg(atgx)}{tgx}dx$

628. $\frac{arctgx}{x} = \int_0^1 \frac{dy}{1+x^2y^2}$. formuladan foydalanib $\int_0^1 \frac{arctgx}{x}\frac{dx}{\sqrt{1-x^2}}$ ni hisoblang.

629. $\int_0^1 x^n dx = \frac{1}{n+1}$, tenglikdan foydalanib $\int_0^1 \frac{x^b - x^a}{\ln x}dx(a>-1, b>-1)$ integralni hisoblang.

Integralni hisoblang.

630. a) $\int_0^1 \sin\left(\ln\frac{1}{x}\right)\frac{x^b - x^a}{\ln x}dx$; b) $\int_0^1 \cos\left(\ln\frac{1}{x}\right)\frac{x^b - x^a}{\ln x}dx$, (a>0,b>0)

Parametrga bog'liq hosmas integrallar.

Quyidagi integrallarning aniqlanish sohasini toping.

631. $\int_0^{+\infty} \frac{e^{-\alpha x}}{1+x^2}dx$ **632.** $\int_{\pi}^{+\infty} \frac{x\cos x}{x^p + x^q}dx$ **633.** $\int_0^{+\infty} \frac{\sin x^{\alpha}}{x^p}dx$

Quyidagi funksiyalarni uzluksizlikka tekshiring.

634. $F(\alpha) = \int_0^{+\infty} \frac{xdx}{2+x^{\alpha}}$, $\alpha > 2$ da.

635. $F(\alpha) = \int_0^{+\infty} \frac{\cos x}{x^{\alpha}}dx$, $\alpha > 0$ da.

636. $F(\alpha)=\int_0^{+\infty}\alpha e^{-x\alpha^2}dx$, $-\infty<\alpha<+\infty$ da.

637. $F(\alpha)=\int_0^{\pi}\frac{\sin x}{x^3(\pi-x)^{\alpha}}dx$, $0<\alpha<2$ da.

638. Agar $F(\alpha)=\int_0^{+\infty}e^{-\alpha y^2}dy$ bo'lsa f'(x)ni hisoblang

639. Funksiyaning $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ tenglamani qanoat-lantirishini isbotlang.

640. $\int_0^{\infty}e^{-pt}dt=\frac{1}{p}$ (p>0) formuladan foydalanib $\int_0^{\infty}t^2e^{-pt}dt$ integralni hisoblang.

Integralni hisoblang

641. $\int_0^{+\infty}\left(\frac{e^{-ax}-e^{-bx}}{x}\right)^2dx$ (a>0,b>0)

642. $\int_0^{\infty}\frac{e^{-ax}-e^{-bx}}{x}\sin mxdx$ (a>0,b>0)

643. $\int_0^{\infty}\frac{e^{-ax}-e^{-bx}}{x}\cos mxdx$ (a>0,b>0)

644. $\int_0^1\frac{\ln(1-a^2x^2)}{x^2\sqrt{1-x^2}}dx$ ($a^2<1$)

645. $\int_0^1\frac{\ln(1-a^2x^2)}{\sqrt{1-x^2}}dx$ ($a^2<1$)

646. $\int_0^{\infty}e^{-ax}\frac{\cos bx-\cos cx}{x}dx$ (a>0)

647. $\int_0^{\infty}\frac{arctgax}{x^2\sqrt{1-x^2}}dx$

648. $\int_0^{\infty}\frac{\ln(a^2+x^2)}{b^2+x^2}dx$

649. $\int_0^{\infty}\frac{arctgax\,arctgbx}{x^2}dx$

650. $\int_0^{\infty}\frac{\ln(1+a^2x^2)\ln(1+b^2x^2)}{x^4}dx$

Quyidagi integrallarni Eyler – Puasson formulasi yordamida hisoblang

651. $\int_{-\infty}^{+\infty}e^{-(ax^2+2bx+c)}dx$ ($a>0, ac-b^2>0$)

652. $\int_0^{+\infty}e^{-\left(x^2+\frac{a^2}{x^2}\right)}dx$ (a>0,b>0)

653. $\int_0^{\infty}\frac{e^{-ax^2}-e^{-bx^2}}{x^2}dx$ (a>0)

654. $\int_0^{+\infty}e^{-ax^2}\cos bxdx$ (a>0)

655. $\int_0^{+\infty}xe^{-ax^2}\sin bxdx$ (a>0)

656. $\int_0^{+\infty}x^{2n}e^{-ax^2}\cos 2bxdx$ ($n\in N$)

Dirixle va Furullani integrali yordamida hisoblang

657. $\int_0^{\infty}\frac{e^{-ax^2}-\cos bx}{x^2}dx$ (a>0)

658. $\int_0^{\infty}\frac{\sin ax\cos bx}{x}dx$ (a>0)

659. $\int_0^{\infty}\frac{\sin^3 ax}{x}dx$ **660.** $\int_0^{\infty}\frac{(\sin\ ax)^2}{x}dx$

$\frac{1}{a^2+x^2}=\int_0^{\infty}e^{-(a^2+x^2)t}dt$ (a>0,b>0) formuladan foydalanib Laplas intergalini hisoblang

661. a)$L=\int_0^{\infty}\frac{\cos bx}{a^2+x^2}dx$ (a>0,b>0), b)$L_1:=\int_0^{\infty}\frac{x\sin bx}{a^2+x^2}dx$ (a>0,b>0)

$\frac{1}{\sqrt{x}}=\frac{2}{\sqrt{\pi}}\int_0^{\infty}e^{-xy^2}dy$ (x>0) formuladan foydalanib Laplas intergalini hisoblang

662. a) $\int_0^{\infty}\sin x^2dx=\frac{1}{2}\int_0^{\infty}\frac{\sin x}{\sqrt{x}}dx$, b) $\int_0^{\infty}\cos x^2dx=\frac{1}{2}\int_0^{\infty}\frac{\cos x}{\sqrt{x}}dx$

VIII Bob
EYLER INTEGRALI

Quyidagi integrallarni Eyler integrali yordamida hisoblang :

663. $\int_0^1\sqrt{x-x^2}dx$

664. $\int_0^a x^2\sqrt{a-x^2}dx$ (a>0)

665. $\int_0^{\infty}\frac{4\sqrt{x}}{(x+1)^2}dx$

666. $\int_0^{\infty}\frac{dx}{1+x^3}$

667. $\int_0^{\infty}\frac{x^2}{1+x^4}dx$

668. $\int_0^{\frac{\pi}{2}}\sin^6 x\cos^4 xdx$

669. $\int_0^{\infty}\frac{1}{n\sqrt{1-x^n}}dx$ (n>1)

670. $\int_0^{+\infty}x^{2n}e^{-x^2}dx$ ($n\in N$)

Quyidagi integrallarni Eyler integrallari orqali ifodalang:

671. $\int_0^{\infty}\frac{x^{m-1}dx}{1+x^4}$ (n>0)

672. $\int_0^{\infty}\frac{x^{m-1}dx}{(1+x)^4}$ (n>0)

673. $\int_0^{\infty}\frac{x^m dx}{(a+bx^n)^p}$ (a>0,b>0,n>0)

674. $\int_0^1\frac{dx}{n\sqrt{1-x^m}}$ (m>0)

675. $\int_0^{\frac{\pi}{2}}\sin^m x\cos^n xdx$

676. $\int_0^{\frac{\pi}{2}}tg^n xdx$

677. $\int_0^{+\infty}e^{-x^n}dx$ (n>0)

678. $\int_0^{+\infty}x^m e^{-x^n}dx$ (n>0)

679. $\int_0^1 (\ln\frac{1}{x})^p dx$

680. $\int_0^\infty x^p e^{-ax} \ln x dx$ (a>0)

681. $\int_0^\infty \frac{x^{p-1}\ln x}{1+x} dx$

682. $\int_0^\infty \frac{x^{p-1}\ln^2 x}{1+x} dx$

683. $\int_0^{+\infty} \frac{x\ln x}{1+x^2} dx$

684. $\int_0^1 \ln\Gamma(x) dx$

685. $\int_a^{a+1} \ln\Gamma(x) dx$ (a>0)

IX Bob
Fure qatorlari
1§ Funksiyalarni Fure qatorlariga yoyish

686. $\sin^2 x$

687. $\cos^3 x$

688. $\sin^4 x$

689. $f(x)=\begin{cases} A, 0<x<l \\ \frac{A}{2}, x=l \\ 0, 0<x<2l \end{cases}$

690. f(x)=x X= (-π,π)

691. f(x)=|x| X=[-1,1]

692. f(x)=2x , X-=(0,1)

693. $f(x)=e^{ax}$ X=(-h,h)

694. f(x)-=cosax $X\in Z$ x= (-π,π)

695. f(x)-=sinax $X\in Z$ x= (-π,π)

696. f(x)=x, X=(-π,π);

697. f(x)=shax X=(-π,π)

698. f(x)=x+sgnx, X=(-π,π)

α parametr boyicha hosila olib I(α) integralni hisoblang, agar:

699. $I(\alpha) = \int_0^{\frac{\pi}{2}} \ln(\alpha^2-\sin^2\varphi)d\varphi$, α>1

700. $I(\alpha)=\int_0^{\pi} \ln\frac{1+\alpha\cos x}{1-\alpha\cos x}\cdot\frac{dx}{\cos x}$, $|\alpha|<1$

Furullani formulasidan foydalanib integralni hisoblang:

701. $\int_0^\infty \frac{\cos^2 ax-\cos^2 bx}{x} dx$

702. $\int_0^\infty \frac{e^{-ax^2}-e^{-bx^2}}{x} dx$

Dirixle integralidan foydalanib integralni hisoblang:

703. $\int_0^\infty \frac{1-\cos\alpha x}{x^2} dx$

704. $\int_0^\infty \frac{x-\sin x}{x^3} dx$

Eyler-Puasson integralidan foydalanib isbotlang:

705. $\int_{-\infty}^{\infty} e^{-(\alpha x^2+2\beta x)} dx = \sqrt{\frac{\pi}{\alpha}} e^{\frac{\beta^2}{\alpha}}$, α>0;

706. $\int_{-\infty}^{\infty} \frac{e^{-ax^2} - e^{-bx^2}}{x^2} dx = 2\sqrt{\pi}\,(\sqrt{b} - \sqrt{a})$, a>0,b>0;

Fure integrali orqali ko'rsating:

707. $f(x)=\begin{cases} 1, agar |x| < \tau \\ 0, agar |x| > \tau \end{cases}$

708. $f(x)=\begin{cases} 1-\frac{|x|}{a}, agar |x| \le a, \\ 0, agar |x| > a \end{cases}$

709. f(x)=sign(x-a)-sign(x-b), b>a;

710. $f(x)=\frac{1}{(x^2+a^2)}$, $a \neq 0$

711. $f(x)=\begin{cases} \sin x, agar |x| \le \frac{\pi}{2} \\ 0, agar |x| > \frac{\pi}{2} \end{cases}$

712. $f(x)= e^{-a|x|} \sin bx$, a>0

713. $f(x)= e^{-a|x|} \cos bx$, a>0

714. $f(x)=e^{-x^2}$

715. $f(x)=xe^{-x^2}$

F(x) fuksiyaning Fure tasvirini toping.

716. $f(x)=\begin{cases} 1, agar, |x| \le 1 \\ 0, agar, |x| > 1 \end{cases}$

717. $f(x) = \begin{cases} \cos x, agar |x| \le \pi, \\ 0, agar |x| > \pi \end{cases}$

718. $f(x) = \begin{cases} \sin x, agar |x| \le \pi, \\ 0, agar |x| > \pi \end{cases}$

719. $f(x) = \begin{cases} \cos x, agar x \in [0, \pi[\\ 0, agar x \in [0, \pi] \end{cases}$

720. $f(x)= xe^{-a|x|}$,a>0

721. $f(x)= e^{\frac{-x^2}{2}}$

722. $f(x)= e^{\frac{-x^2}{2}} \cos ax$

723. $f(x) = \begin{cases} x \sin x, agar |x| \le \pi, \\ 0, agar |x| > \pi \end{cases}$

724. $f(x)=\begin{cases} 1, agar, 0 < |x| \le 2 \\ 0, agar, |x| > 2 va |x| \le 1 \end{cases}$

725. $f(x)=\begin{cases} x^2, agar, |x| \le 1, \\ 0, agar, |x| > 1 \end{cases}$

726. $f(x) = \begin{cases} x^2 \cos x^2, agar, |x| \le \pi \\ 0, agar, |x| > \pi \end{cases}$, $-\infty < \alpha < +\infty$ da

1 Bob
Qatorlar
1§ Sonli qatorlar.

Quyidagi qatorlarning umumiy hadini yozing.

1. $a_n = \dfrac{(-1)^{n-1}}{n^2}$

2. $a_n = \dfrac{(-1)^{n-1}}{n(n+1)}$

3. $a_n = (-1)^{n-1}\dfrac{(2n-1)!!}{1\cdot 4\cdot \ldots \cdot (3n-2)}$

4. $a_n = \dfrac{2^n}{n!}$

5. $a_n = (-1)^{n-1}\dfrac{(2n)!!}{(2n-1)!!}$

Quyidagi qatorlarning dastlabki n ta hadlar yiğindisini S_n, qator yiğindisi S ni toping.

6. $S_n = \dfrac{1}{3}\left(1 - \dfrac{1}{3n+1}\right), S = \dfrac{1}{3}$

7. $S_n = 1 - \dfrac{1}{(n+1)^2}, S = 1$

8. $S_n = 1 + \dfrac{1}{2} - \dfrac{1}{2^n} - \dfrac{1}{2\cdot 3^n}, S = 1{,}5$

9. $S_n = 1 - \dfrac{1}{(n+1)^2}, S = 1$

10. $S_n = \dfrac{1}{2} - \dfrac{1}{(n+1)(n+2)}, S = \dfrac{1}{2}$

Quyidagi qatorlarning qaysilari zaruriy shartga kōra uzoqlashuvchi bōladi.

11. uzoqlashuvchi
12. uzoqlashuvchi
13. uzoqlashuvchi
14. uzoqlashuvchi
15. yaqinlashuvchi
16. yaqinlashuvchi

Dalamber belgisi yordamida qatorlarning yaqinlashuvchanligini tekshiring.

23. Yaqinlashuvchi
24. Yaqinlashuvchi

25. Yaqinlashuvchi
26. Uzoqlashuvchi
27. Uzoqlashuvchi
28. Yaqinlashuvchi
29. Uzoqlashuvchi
30. Yaqinlashuvchi
31. Yaqinlashuvchi
32. Yaqinlashuvchi

Koshi belgisi yordamida yaqinlashuvchanligini tekshiring

33. yaqinlashuvchi
34. yaqinlashuvchi
35. yaqinlashuvchi
36. yaqinlashuvchi
37. uzoqlashuvchi
38. yaqinlashuvchi
39. yaqinlashuvchi
40. Aniq emas
41. yaqinlashuvchi
42. yaqinlashuvchi
43. a ning har qanday qiymatida yaqinlashuvchi
44. yaqinlashuvchi

Ishorasi ōzgaruvchan qatorlarning yaqinlashuvchanligini tekshiring.

45. Absalyut yaqinlashuvchi
46. Uzoqlashuvchi
47. Absalyut yaqinlashuvchi
48. Shartli yaqinlashuvchi
49. Shartli yaqinlashuvchi
50. Shartli yaqinlashuvchi
51. Absalyut yaqinlashuvchi
52. Uzoqlashuvchi
53. Shartli yaqinlashuvchi
54. Absalyut yaqinlashuvchi

2§ Funksional qatorlar

Quyidagi funksiyalarni darajali qatorga yoying.

55. $x^2+\sum_{n=1}^{\infty}\frac{(2n-1)!!}{2^n n!}x^{2n+2},-1<x<1$

56. $1+\sum_{n=1}^{\infty}(-1)^n\frac{2^{2n-1}}{(2n)!}x^{2n},-\infty<x<\infty$

57. $\frac{3}{4}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3^{2n-1}}{(2n+1)!}x^{2n+1},$
$-\infty<x<\infty$

58. $-\sum_{n=1}^{\infty}\frac{x^{n-1}}{n+1},-1<x<1$

59. $x+\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{n+1}}{n(n+1)},-1\le x\le 1$

60. $x^3+\sum_{n=1}^{\infty}\frac{(2n-1)!!}{(2n)!!}\cdot\frac{x^{4n+3}}{2n+1},-1\le x\le 1$

61. $arctg2+\sum_{n=1}^{\infty}\frac{(-1)^n 2^{2n-1}}{2n-1}x^{2n-1}$

62. $x+\sum_{n=1}^{\infty}\frac{(2n-1)!!}{(2n)!!}\cdot\frac{x^{2n+1}}{(2n+1)},\ -1\le x\le 1$

Funksional qatorlarning yaqinlashish sohasini toping.

63. $e^{-1}<x<e$

64. $-1\le x<1$

65. $x\neq\pm 1$

66. $-1\le x<1$

67. har qanday x da

68. $-2<x<2$

69. $x>0$

70. $x\ge 0$

71. $x>1$

72. $|x|<\sqrt{e-2}$

Darajali qatorlarning yaqinlashish sohasini toping.

73. $-1\le x\le 1$

74. $-1\le x<1$

75. $-\sqrt{3}\le x\le\sqrt{3}$

76. $-1<x\le 1$

77. $-6<x<0$

78. $0\le x\le 4$

79. $-\frac{1}{e}<x<\frac{1}{e}$

80. $c<1$ da $-1\le x\le 1$, $c=1$ da $-1<x\le 1$, $c>1$ da $-1<x<1$

81. $-4<x<4$

82. $c<e^{-1}$ da $-1\le x\le 1$, $e^{-1}\le c<1$ da $-1\le x<1$, $c\ge 1$ da $-1<x<1$

83. $-1\le x\le 1$

84. $c\le 1$ da $-1<x<1$, $c>1$ da $-c<x<c$

Quyidagi funktsional qatorlarning absalyut va shartli yaqinlashish sohasini toping.

85. $|x|>1$ da absalyut yaqinlashuvchi

86. $x\in R$ da absalyut yaqinlashuvchi

87. $x>0$ da absalyut yaqinlashuvchi

88. $x\in R$ da absalyut yaqinlashuvchi

89. $x<-3$ va $x>-1$ da absalyut yaqinlashuvchi, $x=-3$ da shartli yaqinlashuvchi

90. $\frac{1}{e}\le x\le e$ da absalyut yaqinlashuvchi

91. $2<|x|<\sqrt{6}$ da absalyut yaqinlashuvchi

92. $x>1$ da absalyut yaqinlashuvchi

93. $|x| > |e|$ da absalyut yaqinlashuvchi

94. $x \neq 0$ da absalyut yaqinlashuvchi

95. $-2 < x < -2 + e^{-1}$ va $x > e - 2$ da absalyut yaqinlashuvchi

96. $|x - R\pi| \leq \frac{\pi}{4}$, $R \in Z$ kesmada absalyut yaqinlashuvchi

97. $|x| < \sqrt{e-2}$ da absalyut yaqinlashuvchi

98. $|x| > 1$ da absalyut yaqinlashuvchi

99. $x \neq 1$ da absalyut yaqinlashuvchi $x = -1$ da shartli yaqinlashuvchan

100. $|x - k\pi| \leq \frac{\pi}{6}$, $k \in Z$ kesmada absalyut yaqinlashuvchi

101. $x > 1$ da absalyut yaqinlashuvchi

Quyidagi funksiyalarni darajali qatorga yoying.

108. $1 + 2x + \frac{(2x)^2}{2!} + \ldots + \frac{(2x)^{n-1}}{(n-1)!} + \ldots$

109. $1 - x^2 + \frac{x^4}{2!} - \ldots + (-1)^{n-1} \frac{x^{2(n-1)}}{(n-1)!} + \ldots$

110. $1 + \frac{x}{2!} + \frac{x^2}{3!} + \ldots + \frac{x^{n-1}}{n!} + \ldots$

111. $1 - \left[x^2 - \frac{(2x)^4}{2 \cdot 4!} + \ldots + (-1)^{n-1} \frac{2^{2n-1} x^{2n}}{(2n)!} + \ldots \right]$

112. $-\frac{2x^3}{3!} + \frac{4x^5}{5!} - \ldots + (-1)^n \frac{2nx^{2n+1}}{(2n+1)!} + \ldots$

113. $\ln 10 + \left[\frac{x}{10} - \frac{x^2}{2 \cdot 10^2} + \ldots + (-1)^{n+1} \frac{x^n}{n \cdot 10^n} + \ldots \right]$

114. $x^2 - \frac{x^3}{2} + \ldots + (-1)^n \frac{x^{n+1}}{n} + \ldots$

115. $1 + \left[\frac{x^2}{2} - \frac{1}{2} \cdot \frac{x^4}{4} + \ldots + (-1)^{n+1} \frac{1 \cdot 3 \cdot \ldots \cdot (2n-3) x^{2n}}{2 \cdot 4 \cdot \ldots \cdot (2n-2) \cdot 2n} + \ldots \right]$

116. $2 - 2\left[\frac{1}{3}\left(\frac{x}{2}\right)^3 + \frac{2}{3^2 \cdot 2!}\left(\frac{x}{2}\right)^6 + \ldots + \frac{2 \cdot 5 \cdot \ldots \cdot (3n-4)}{3^n \cdot n!}\left(\frac{x}{2}\right)^{3n} + \ldots \right]$

117. $1-\left[\frac{1}{3}x^3-\frac{1\cdot 4}{3^2\cdot 2!}x^6+\ldots+(-1)^{n+1}\frac{1\cdot 4\cdot\ldots\cdot(3n-2)}{3^n\cdot n!}x^{3n}+\ldots\right]$

II Bob

Kop argumentli funksiyalar

1§ Quyidagi funksiyalarning aniqlanish sohasini toping.

124. y=x to'g'ri chiziq nuqtalaridan tashqari tekislikning barcha nuqtalari

125. Oy o'qi nuqtalaridan tashqari tekislikning barcha nuqtalari.

126. Radiusi $\sqrt{3}$,markazi koordinata boshida bo'lgan doira

127. $\frac{x^2}{9}+\frac{y^2}{4}=1$ning ichki va tashqi nuqtalar to'plami.

128. Birinchi koordinata choragi.

129. ikkinchi koordinata choragi.

130. Koordinat burchaklari bissektrisalarining o'ng vertical ichki burchaklari.

131. Markazi koordinata boshida radiusi 3ga teng doira.

132. tekislikning $x^2+y^2=R^2$, $x^2+y^2=r^2$ aylana orasidagi qismi

133. $x\geq 0$, $y\geq 0$, $x\geq\sqrt{y}$

134. $y^2=4x$, va $x^2+y^2=1$ chiziqlar orasidagi qismi.

135. Tekislikning $\frac{x^2}{9}-\frac{y^2}{4}=1$ giperbola chiziqlari orasidagi qismi

136. 0

137. 0

138. 0

139. 0

140. 1

141. 0

142. 1

143. 4/3

144. $z'_x=2ay-2x$, $z'_y=2ax-2y$

145. $z'_x=\frac{1}{y}$, $z'_y=-\frac{x}{y^2}$

146. $z'_x=3x^2t-t^3$, $z'_t=x^3-3xt^3$

147. $u'_x=-\frac{y}{x^2}+\frac{1}{z}$, $u'_y=\frac{1}{x}+\frac{z}{y^2}$, $u'_z=-\frac{x}{z^2}-\frac{1}{y}$

148. $S'_t=-axe^{-t}-b$, $S'_x=ae^{-t}$

149. $z'_x=e^{\sin(x/y)}\cos\frac{y}{x}(-\frac{y}{x^2})$, $z'_y=\frac{1}{x}e^{\sin\frac{y}{x}}\cos\frac{y}{x}$

150. $z'_x=4(3xy^3-x^2+5)(3y^3-2x)$, $z'_y=4(3xy^3-x^2+5)(gxy^2)$

151. $z'_x=-\frac{y}{x^2}e^{\frac{y}{x}}$, $z'_y=\frac{1}{x^2}e^{\frac{y}{x}}$

152. $z'_x=\frac{1}{y}\cos(x/y)\cos(y/x)+\frac{y}{x^2}\sin(x/y)\sin(y/x)$, $z'_y=-\frac{y}{x^2}\cos(x/y)\cos(y/x)-\frac{1}{x}\sin(y/x)\sin(x/y)$

153. $u'_x=yzx^{yz-1}$, $u'_y=zx^{yz}\ln x$, $u'_z=yx^{yz}\ln x$

154. $u'_x=y^z x^{y^z-1}$, $u'_y=x^{y^z}\ln x z^y=z^y$, $u'_z=y^z x^{yz}\ln x\ln y$

155. $u'_x=y\cos(xy+yz)$, $u'_y=(x+z)\cos(xz+yz)$, $u'_z=y\cos(xy+yz)$

156. $u'_x=-\frac{z}{x}\left(\frac{y}{x}\right)^z$, $u'_x=\frac{z}{y}\left(\frac{y}{x}\right)^z$, $u'_x=\left(\frac{y}{x}\right)^z\ln\frac{y}{x}$

157. $f'_x(4;3)=9/5$, $f'_y(4;3)=8/5$

158. $f'_x(1;-1;0)=0$, $f'_y(1;-1;4)=2\sin 2$, $f'_z(-1/2;0;-1)=-\sin(-1)$

159. 3/2

160. $f'_x(2;1)=1/2$, $f'_y(2;1;)=0$,

161. $f'_x(1;2;0)=1$, $f'_y(1;2;0)=1/2$, $f'_z(1;2;0)=1/2$

2§ O'zgaruvchilarni almashtirish.

O'zgaruvchilarni almashtirish yordamida differensial tenglamalarni yozing.

166.

167. $\frac{d^2y}{dt^2}+y=0$

168. $\frac{d^2y}{dt^2}+n^2y=0$

169. $\frac{d^2y}{dt^2}=0$

170. $t^5\frac{d^3y}{dt^3}+(3t^4+1)\frac{d^2y}{dt^2}+\frac{dy}{dt}=0$

171. $x^{\prime\prime}=0$

172. $x''+x=e^y$

173. $x''+x'^2+x^3=0$

174. $\frac{dr}{d\varphi}=r$

175. $r'^2=\frac{1-\sin 2\varphi}{\sin 2\varphi}r^2$

176. $r(2r'^2+r^2-rr'')=r'^3r^2$

177. $z=\varphi(x,y)$

178. $z=\varphi(x^2,y^2)$

Quyidagi tenglamalarni yangi o'zgaruvchi u va v ga o'tib yozing.

179. $\frac{dz}{du}+\frac{dz}{dv}=e^u\sin v$

180. $\frac{dz}{du}=\frac{dz}{dv}$

181. $3\frac{d^2z}{du^2}+\frac{dz}{du}=0$

182. $\frac{d^2z}{du^2}+\frac{d^2z}{dv^2}=0$

183. $\frac{dw}{du}=0$

183. $\frac{dw}{du}=2u$

184. $\frac{dw}{du}=\frac{w}{v}$

185. $\frac{dw}{du}=0$

186. $\frac{dw}{dv}=0$

187. $\frac{d^2w}{du^2}=0$

188. $\frac{d^2w}{dv^2} = 0$

Kõp argumentli funksiyalar ekstremumi
Quyidagi funksiyalarni ekstremumga tekshiring.

190. $(0;0)$ - maksimum nuqtasi

191. $(2;-2)$ - maksimum nuqtasi

192. $(-1;1)$ - minimum nuqtasi

193. $(6;4)$ - maksimum nuqtasi

194. $(1;1)$ - minimum nuqtasi

195. $(\sqrt{2};-\sqrt{2})$ va $(-\sqrt{2};\sqrt{2})$ nuqtalarida $z_{max} = -8$

196. $(0;1)$ - minimum nuqtasi

197. $x - y + 1 = 0$ tõgri chiziq nuqtalarida $z = 0$ qat'iy bõlmagan minimum

198. $(1;1)$ - minimum nuqtasi

199. $(0;0)$ nuqtada $z_{max} = 0$; $x = \pm\frac{1}{2}; y = \pm 1$ da $z_{min} = -1\frac{1}{8}$; $x = 0; y = \pm 1$ da egar $z = -1$ va $x = \pm\frac{1}{2}; y = 0$ da egar. Agar $x = 0; y = \pm 1$ bõlsa $z = -1$ egar; agar $x = \pm\frac{1}{2}; y = 0$ bõlsa $z = \frac{1}{8}$ egar.

200. $(5;2)$ - minimum nuqtasi

201. $\frac{x}{a} = -\frac{y}{b} = \pm\frac{1}{\sqrt{3}}$ da $z_{min} = -\frac{ab}{3\sqrt{3}}$; $\frac{x}{a} = \frac{y}{b} = \pm\frac{1}{\sqrt{3}}$ da $z_{max} = \frac{ab}{3\sqrt{3}}$

202. $(3;3)$ va $(-3;-3)$ - minimum nuqtalari; $(0;0)$ - egarli nuqta

203. $(1;3)$ - minimum nuqta, $(-1;-3)$ - maksimum nuqta, $(3;1),(-3;-1)$ - egarli nuqta

204. $(0;0)$ - maksimum nuqta

205. $(1;2)$ - minimum nuqta

206. $x = y = \pm\frac{1}{\sqrt{2e}}$ da $z_{max} = -\frac{1}{2e}$; $x = -y = \pm\frac{1}{\sqrt{2e}}$ da $z_{max} = \frac{1}{2e}$; $x = 0; y = \pm 1$ va $x = \pm 1; y = 0$ nuqtalarda ekstremum yõq.

207. $(0;0)$ nuqtada $z_{max} = 0$; $x^2 + y^2 = 1$ da qat'iy bõlmagan maksimum

208. $\left(-\frac{2}{3};-\frac{1}{3};1\right)$ - minimum nuqta

209. $\left(\frac{1}{2};1;1\right)$ - minimum nuqta

210. $(-1;-2;3)$ - minimum nuqta

211. $x=y=\frac{a}{7}$ da $u_{min}=\frac{a^7}{7^7}$; $y=0;x\neq 0;z\neq 0;x+2y+3z\neq a$ da $u=0$ qat'iy bōlmagan ekstremum

212. $x=\frac{1}{2}\sqrt[15]{16a^{14}b};y=\frac{1}{4}\sqrt[15]{16a^4b};z=\frac{1}{2}\sqrt{\frac{a^5b^7}{4}}$ da $u_{min}=\frac{15a}{4}\sqrt[15]{\frac{a}{16b}}$;

Oshkormas funksiyaning ekstremal qiymatlarini toping.

213. $\left(\frac{\pi}{2};\frac{1}{2}a+\frac{1}{2}\sqrt{a^2+4}\right)$ - maksimum nuqta, $\left(\frac{\pi}{2};\frac{1}{2}a-\frac{1}{2}\sqrt{a^2+4}\right)$ - minimum nuqta

214. $\left(-6-\frac{1}{3\sqrt{3}};-6+\frac{2}{3\sqrt{3}}\right)$ - maksimum nuqta, $\left(-6+\frac{1}{3\sqrt{3}};-6-\frac{2}{3\sqrt{3}}\right)$ - minimum nuqta

215. $(-3;6)$ - maksimum nuqta, $(3;-6)$ - minimum nuqta

216. $x=1;y=-1$ da $z_1=-2$ minimum va $z_2=6$ maksimum

217. $x=y=-\left(3+\sqrt{6}\right)$ da $z_{max}=-\left(4+2\sqrt{6}\right)$; $x=y=-\left(3-\sqrt{6}\right)$ da $z_{max}=2\sqrt{6}-4$

218. Tenglama bilan ifodalanuvchi funksiyalardan biri $x=-1;y=2$ da $(z_{max}=-2)$ maksimumga ega, ikkinchisi $x=-1;y=2$ da $(z_{min}=1)$ minimumga ega, ikkala funksiya $4x^3-4y^2-12x+16y-33=0$ egri chiziq nuqtalarida chegaraviy ekstremumga ega.

Shartli ekstremumini toping.

219. $x=\frac{1}{2}$, $y=\frac{1}{2}$ da $z_{max}=\frac{1}{4}$

220. (a,a) va $(-a,-a)$ nuqtalarda $z_{max}=a^2$

221. $(a,-a)$ va $(-a,a)$ nuqtalarda $z_{min}=-a^2$

222. $\left(-a\sqrt{2},-a\sqrt{2}\right)$nuqtada $z_{min}=a^2$; $\left(a\sqrt{2},a\sqrt{2}\right)$ nuqtada $z_{max}=\frac{\sqrt{2}}{a}$

223. $x=\frac{\pi}{8}+\frac{\pi k}{2}$, $y=-\frac{\pi}{8}+\frac{\pi k}{2}$, $k\in z$ da $z=1+\frac{(-1)^k}{\sqrt{2}}$ ekstremum (agar k juft maksimum, agar k toq minimum)

224. $(3,3,3)$ nuqtada $u_{max}=9$

226. $\left(-\frac{1}{3},\frac{2}{3},-\frac{2}{3}\right)$ nuqtada $u_{max}=-3$; $\left(\frac{1}{3},-\frac{2}{3},\frac{2}{3}\right)$ nuqtada $u_{min}=3$

227. $x=\pm1\frac{1}{8}, y=\pm4$ da $z_{max}=106\frac{1}{4}$; $x=\pm2, y=\mp3$ da $z_{min}=-50$

228. $x=0, y=0, z=\pm c$ da $u_{min}=c^2$; $x=\pm a, y=0, z=0$ da $u_{max}=a^2$

229. $x=y=z=\frac{a}{6}$ da $u_{max}=\left(\frac{a}{6}\right)^6$

$x=y=\frac{1}{\sqrt{6}}, z=-\frac{2}{\sqrt{6}}; x=z=\frac{1}{\sqrt{6}}, y=-\frac{2}{\sqrt{6}}; y=z=\frac{1}{\sqrt{6}}, x=-\frac{2}{\sqrt{6}}$; da

$u_{min}=-\frac{1}{3\sqrt{6}}$;

$x=y=-\frac{1}{\sqrt{6}}, z=\frac{2}{\sqrt{6}}; x=z=-\frac{1}{\sqrt{6}}, y=\frac{2}{\sqrt{6}}; y=z=-\frac{1}{\sqrt{6}}, x=\frac{2}{\sqrt{6}}$ da

$u_{max}=\frac{1}{3\sqrt{6}}$

230. $(4,2,-1)$ da $u_{max}=12$; $(-4;-2;-1)$ da $u_{min}=-10$

231. $\left(\frac{\pi}{3};\frac{\pi}{6}\right)$ nuqtada $z_{max}=\frac{\pi}{3}$; $\left(-\frac{\pi}{3};-\frac{\pi}{6}\right)$ nuqtada $z_{min}=-\frac{\pi}{3}$

Funksiyaning berilgan sohadagi eng kichik va eng katta qiymatini toping.

233. $(2,0)$ va $(-2,0)$ nuqtalarda, $\sup z=4$, $(0,2)$ va $(0,-2)$ nuqtalarda $\inf z=-4$

234. $(1,2)$ nuqtada $\sup z=17$, $(1,0)$ nuqtada $\inf z=-3$

235. $(0,\pm1)$ nuqtada $\sup z=\frac{3}{e}$, $(0,0)$ nuqtada $\inf z=0$

236. $\inf z=-5, \sup z=-2$

237. $\inf z=-75, \sup z=125$

238. $\inf z=0, \sup z=1$

239. $\inf u=0, \sup u=300$

240. $\inf u = -\frac{1}{2}, \sup u = 1+\sqrt{2}$

241. $(1,2)$ nuqtada $\inf z = -2$, $\left(\frac{1}{3},\frac{2}{3}\right)$ nuqtada $\sup z = \frac{2}{27}$

242. $\left(1,\frac{4}{3}\right)$ nuqtada $\sup z = \frac{2}{9}$, butun chegarada $\inf z = 0$

243. $x = y = z = \frac{a}{\sqrt{3}}$ da $\sup u = a^2$;

$x^2 + y^2 + z^2 = a^2, x + y + z = 0$ ning butun aylanasida $\inf u = -\frac{a^2}{2}$;

sup u=a^2

4§ Teylor formulasi

244. $x^3+2y^2-xy+h(3x^2-y)+k(6y^2-x)+3xh^2-hk+6yk^2+h^3+2k^3$

245. $\Delta z = 15h^2-6hk+k^2+h^3$

245. $f(x,y)=5+2(x-1)^2-(x-1)(y+2)-(y+2)^2$

246. $f(x,y,z)=3[(x-1)^2+(y-1)^2+(z-1)^2-(x-1)(y-1)-(x-1)(z-1)-(z-1)(x-1)] + (x-1)^3+(xy1)^3+(z-1)^3-3(x-1)(y-1)(z-1)$

247. $\Delta f(-1,1)=h-3k+(-h^2-2hk+k^2)+(h^2k+hk^2)$

248. $x^y=1+(x-1)-(x-1)(y-1)+R_2(1+D(x-1),1+D(y+1))(0<D<1$) bu yerda

$R2(x,y) = \frac{1}{6}x^y[(\frac{y}{x}dx + \ln x dy)^3 + 3(\frac{y}{x}dx + \ln x du)(-\frac{y}{x^2}dx^2 + \frac{2}{x}dxdy) +$

$+(\frac{2y}{x^3}dx^3 - \frac{3}{x^2}dx^2dy)$ va dx=x-1, dy=y-1

249. $z=\frac{1}{2}+\frac{1}{2}(x-\frac{\pi}{4})+\frac{1}{2}(y-\frac{\pi}{4})-\frac{1}{4}[(x-\frac{\pi}{4})^2-2((x-\frac{\pi}{4})(y-\frac{\pi}{4})+(y-\frac{\pi}{4})^2 -$

$-\frac{1}{6}[\cos\xi\cos\eta(x-\frac{\pi}{4})^3+3\sin\xi\cos\eta(x-\frac{\pi}{4})^2(y-\frac{\pi}{4}) + 3\cos\xi\sin\eta(x-\frac{\pi}{4})^2(y-\frac{\pi}{4})$

$+\sin\xi\cos\eta(y-\frac{\pi}{4})\ ]$

251. $1+(x+y)+\ldots+\frac{x^{n+1}-y^{n+1}}{x-y}+\ldots$

252. $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}(-1)^{n+m}\frac{x^n y^m}{nm}(|x|<1,|y|<1)$

253. $\sum_{n=0}^{\infty}(-1)^n\frac{(x^2+y^2)^{2n+1}}{(2n+1)!}$

254. $\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(-1)^n\frac{x^n y^{2m+1}}{n!(2m+1)!}(|x|<\infty,|y|<\infty)$

255. $\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(-1)^n\frac{x^n y^{2m}}{n!(2m)!}(|x|<\infty,|y|<\infty)$

256. a)1,0081; b)0,902

III Bob
Hosmas integrallar
1§ Hosmas integrallar

Quyidagi hosmas integrallarni hisoblang.

257. $\frac{1}{3}$

258. uzoqlashuvchi

259. $\frac{1}{a}$

260. π

261. uzoqlashuvchi

262. $1-\ln 2$

263. $\frac{1}{2}$

264. $\frac{\pi}{4}$

265. $\frac{1}{2}$

266. uzoqlashuvchi

267. $\frac{1}{2}$

268. agar $a>0$ bōlsa $\frac{a}{a^2+b^2}$; agar $a\le 0$ bōlsa uzoqlashuvchi bōladi

269. agar $a>0$ bōlsa $\frac{b}{a^2+b^2}$; agar $a\le 0$ bōlsa uzoqlashuvchi bōladi

270. $\frac{\pi}{4}+\frac{1}{2}\ln 2$

271. $\frac{\pi}{2}$

272. $\frac{1}{2}+\frac{\pi}{4}$

273. $\frac{4\pi}{3\sqrt{3}}$

274. $\frac{\pi}{\sqrt{2}}$

275. 0

276. $\frac{1}{3}+\frac{1}{4}\ln 3$

277. $\frac{\pi}{2}$

278. uzoqlashuvchi

279. $\frac{8}{3}$

280. $-\frac{1}{4}$

281. 1

282. π

283. $\frac{1}{2}\pi(a+b)$

284. $\frac{33\pi}{2}$

285. $\frac{\pi}{3\sqrt{3}}$

286. $14\frac{4}{7}$

287. uzoqlashuvchi

288. $-\frac{2}{e}$

289. uzoqlashuvchi

290. uzoqlashuvchi

291. -1

Quyidagi qatorlarning yaqinlashuvchiligini tekshiringyaqinlashuvchi

292. uzoqlashuvchi

293. yaqinlashuvchi

294. yaqinlashuvchi

295. uzoqlashuvchi

296. $p > 0$ da yaqinlashuvchi

297. uzoqlashuvchi

298. yaqinlashuvchi

299. yaqinlashuvchi

300. agar $a > -1, b - a > 1$ bõlsa yaqinlashuvchi

301. $1 < p < 2$ da yaqinlashuvchi

302. agar $m > -2, n - m > 1$ bõlsa yaqinlashuvchi

303. yaqinlashuvchi

304. uzoqlashuvchi

305. agar $\min(p,q) < 1, \max(p,q) > 1$ bõlsa yaqinlashuvchi

306. yaqinlashuvchi

307. uzoqlashuvchi

308. yaqinlashuvchi

Quyidagi integrallarning shartli va absalyut yaqinlashuvchiligini tekshiring.

309. yaqinlashuvchi

310. absalyut yaqinlashmaydi

311. absalyut yaqinlashmaydi

312. agar $-1 < \frac{p+1}{q} < 0$ bõlsa absalyut yaqinlashuvchi; agar $0 \le \frac{p+1}{q} < 1$ bõlsa shartli yaqinlashuvchi

313. agar $p > -2, q > p+1$ bõlsa absalyut yaqinlashuvchi; agar $p > -2, p < q \le p+1$ bõlsa shartli yaqinlashuvchi $0 < n < 2$ da yaqinlashuvchi

Quyidagi integrallarni hisoblang.

314. $\ln\frac{1}{2}$

315. 0

316. π

317. 0

IV Bob

Ikki karrali integrallar

1§ Quyidagi ikki karrali integrallar hisoblansin:

318. 1

319. $(e-1)^2$

320. $\frac{\pi}{12}$

321. $\ln\frac{4}{3}$

322. $\ln\frac{2+\sqrt{2}}{1+\sqrt{3}}$

323. $\pi-2$

324. 2

325. $-\frac{\pi}{16}$

Quyidagi takrorfy integrallar hisoblansin:

326. $4\frac{2}{3}$

327. $\ln\frac{25}{24}$

328. $\frac{9}{4}$

329. 50,4

330. $\frac{\pi a^2}{2}$

331. 2,4

332. $\frac{\pi}{6}$

333. $-\frac{1}{24}$

334. $\frac{1}{40}$

335. $\frac{2}{3}a^{\frac{3}{2}}$

336. $\frac{1}{2}$

Berilgan egri chiziqlar bilan chegaralangan yoki ma'lum shartlar bilan berilgan (D) soha uchun $\iint_D f(x,y)dxdy$ ikki karrali integralda integrallash chegaralari ikki hil tartibda qōyilsin:

337. $\int_0^1 dx\int_0^x f(x,y)dy = \int_0^1 dy\int_y^1 f(x,y)dx$

338. $\int_0^1 dx\int_{-x}^{x} f(x,y)dy = \int_{-1}^{0} dy\int_{-y}^{1} f(x,y)dx + \int_0^1 dy\int_y^1 f(x,y)dx$

339. $\int_0^1 dx\int_0^{x+1} f(x,y)dy = \int_0^1 dy\int_0^1 f(x,y)dx + \int_1^2 dy\int_{y-1}^1 f(x,y)dx$

340. $\int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)dy = \int_{-1}^{1} dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)dx$

341. $\int_{-\frac{1}{2}}^{\frac{1}{2}} dx \int_{\frac{1}{2}-\sqrt{\frac{1}{4}-x^2}}^{\frac{1}{2}+\sqrt{\frac{1}{4}-x^2}} f(x,y)dy = \int_{0}^{1} dy \int_{-\sqrt{y-y^2}}^{\sqrt{y-y^2}} f(x,y)dx$

342. $\int_{0}^{1} dx \int_{\sqrt{1-x^2}}^{1} f(x,y)dy = \int_{0}^{1} dy \int_{\sqrt{1-y^2}}^{1} f(x,y)dx$

343. $\int_{-1}^{0} dx \int_{-x-1}^{x+1} f(x,y)dy + \int_{0}^{1} dx \int_{x-1}^{1-x} f(x,y)dy = \int_{-1}^{0} dy \int_{-y-1}^{y+1} f(x,y)dx + \int_{0}^{1} dy \int_{y-1}^{1-y} f(x,y)dx$

344. $\int_{1}^{2} dx \int_{0}^{1-\sqrt{1-(x-1)^2}} f(x,y)dy = \int_{0}^{1} dx \int_{1+\sqrt{1-(y-1)^2}}^{2} f(x,y)dy$

345. $\int_{0}^{\frac{1}{\sqrt{2}}} dx \int_{\frac{x}{2}}^{2x} f(x,y)dy + \int_{\frac{1}{\sqrt{2}}}^{\sqrt{2}} dx \int_{\frac{x}{2}}^{\frac{1}{x}} f(x,y)dy = \int_{0}^{\frac{\sqrt{2}}{2}} dy \int_{\frac{y}{2}}^{2y} f(x,y)dx + \int_{\frac{\sqrt{2}}{2}}^{\sqrt{2}} dy \int_{\frac{y}{2}}^{\frac{1}{y}} f(x,y)dx$

346. $\int_{0}^{1} dy \int_{-1+\sqrt{y}}^{\frac{1}{2}\sin\pi y} f(x,y)dx = \int_{-1}^{0} dx \int_{0}^{(1+x)^2} f(x,y)dy + \int_{0}^{\frac{1}{2}} dx \int_{\frac{1}{\pi}\arcsin 2x}^{1-\frac{1}{\pi}\arcsin 2x} f(x,y)dy$

347. $\int_{-2}^{-1} dx \int_{-\sqrt{2x+4}}^{\sqrt{2x+4}} f(x,y)dy + \int_{-1}^{0} dx \int_{-\sqrt{4x+4}}^{-\sqrt{2x+4}} f(x,y)dy + \int_{-1}^{0} dx \int_{\sqrt{4x+4}}^{\sqrt{2x+4}} f(x,y)dy = \int_{-2}^{2} dy \int_{\frac{y^2-4}{2}}^{\frac{y^2-4}{4}} f(x,y)dx$

Berilgan egri chiziqlar bilan chegaralangan yoki ma'lum shartlar bilan berilgan (D) soha uchun $\iint_D f(x,y)dxdy$ ikki karrali integralda integrallash chegaralari qo'yilsin:

348. $\int_{0}^{2} dx \int_{0}^{2-x} f(x,y)dy$

349. $\int_{0}^{1} dx \int_{x-1}^{0} f(x,y)dy$

350. $\int_{-\sqrt{2}}^{\sqrt{2}} dx \int_{x^2}^{4-x^2} f(x,y)dy$

351. $\int_{0}^{1} dx \int_{x^3}^{\sqrt{x}} f(x,y)dy$

352. $\int_{0}^{2} dx \int_{x}^{2x} f(x,y)dy + \int_{2}^{3} dx \int_{x}^{6-x} f(x,y)dy$

353. $\int_{-\frac{2}{3}}^{\frac{1}{3}} dx \int_{1-2x}^{x+3} f(x,y)dy + \int_{\frac{1}{3}}^{\frac{2}{3}} dx \int_{x}^{x+3} f(x,y)dy + \int_{\frac{2}{3}}^{\frac{5}{3}} dx \int_{x}^{5-2x} f(x,y)dy$

Quyidagi ikki karrali integralda integrallash tartibi ōzgartirilsin:

354. $\int_{0}^{1} dx \int_{x^2}^{x} f(x,y)dy$

355. $\int_{0}^{1} dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)dx$

356. $\int_{-\sqrt{2}}^{\sqrt{2}} dy \int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} f(x,y)dx$

357. $\int_{1}^{2} dy \int_{1}^{y} f(x,y)dx + \int_{2}^{4} dy \int_{\frac{y}{2}}^{2} f(x,y)dx$

358. $\int_{0}^{4} dy \int_{0}^{\frac{y}{2}} f(x,y)dx + \int_{4}^{6} dy \int_{0}^{6-y} f(x,y)dx$

359. $\int_{0}^{1} dy \int_{2-y}^{1+\sqrt{1-y^2}} f(x,y)dx$

360. $\int_{0}^{1} dy \int_{\arcsin y}^{\pi-\arcsin y} f(x,y)dx - \int_{-1}^{0} dy \int_{\pi-\arcsin y}^{2\pi+\arcsin y} f(x,y)dx$

361. $\int_{0}^{1} dy \int_{0}^{1-\sqrt{y}} f(x,y)dx + \int_{0}^{1} dy \int_{1+\sqrt{y}}^{2} f(x,y)dx$

362. $\int_{1}^{3} dx \int_{\frac{9}{x}}^{10-x} f(x,y)dy$

363. $\int_{0}^{1} dx \int_{\sqrt{9-x^2}}^{3} f(x,y)dy + \int_{3}^{4} dx \int_{0}^{3} f(x,y)dy + \int_{4}^{5} dx \int_{0}^{\sqrt{25-x^2}} f(x,y)dy$

Kōrsatilgan (D) soha uchun $\iint_D f(x,y)dxdy$ integralda qutb koordinatalariga $(x = r\cos\varphi, y = r\sin\varphi)$ ōtib, integrallash chegaralari qōyilsin:

364. $\int_{0}^{2a} d\varphi \int_{0}^{a} rf(r\cos\varphi, r\sin\varphi)dr$

365. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_{0}^{a\cos\varphi} rf(r\cos\varphi, r\sin\varphi)dr$

366. $\int_{0}^{-\arctg\frac{a}{b}} d\varphi \int_{0}^{b\sin\varphi} rf(r\cos\varphi, r\sin\varphi)dr + \int_{-\arctg\frac{a}{b}}^{\frac{\pi}{2}} d\varphi \int_{0}^{a\cos\varphi} rf(r\cos\varphi, r\sin\varphi)dr$

367. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_{0}^{\frac{1}{\sin\varphi}} rf(r\cos\varphi, r\sin\varphi)dr$

368. $\int_{0}^{\frac{\pi}{2}} d\varphi \int_{\sqrt{2}\sin\left(\varphi-\frac{\pi}{4}\right)}^{2} rf(r\cos\varphi, r\sin\varphi)dr$

369. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\varphi \int_{0}^{a\sqrt{\cos 2\varphi}} rf(r\cos\varphi, r\sin\varphi)dr$

370. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\varphi \int_{0}^{\frac{1}{\sin\varphi}} rf(r\cos\varphi, r\sin\varphi)dr$

371. $\int_{0}^{\frac{\pi}{4}} d\varphi \int_{0}^{\frac{1}{\cos\varphi}} rf(r\cos\varphi, r\sin\varphi)dr + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_{0}^{\frac{1}{\sin\varphi}} rf(r\cos\varphi, r\sin\varphi)dr$

372. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} d\varphi \int_{0}^{2\cos\varphi} rf(r\cos\varphi, r\sin\varphi)dr + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\varphi \int_{0}^{\frac{1}{\cos\varphi}} rf(r\cos\varphi, r\sin\varphi)dr +$

$+ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_{0}^{2\cos\varphi} rf(r\cos\varphi, r\sin\varphi)dr$

373. $\int_{0}^{\frac{\pi}{4}} d\varphi \int_{1}^{\frac{1}{\cos\varphi}} rf(r\cos\varphi, r\sin\varphi)dr + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_{1}^{\frac{1}{\sin\varphi}} rf(r\cos\varphi, r\sin\varphi)dr$

374. $\int_{-\frac{\pi}{4}}^{0} d\varphi \int_{0}^{\frac{2}{\cos\varphi-\sin\varphi}} rf(r\cos\varphi, r\sin\varphi)dr + \int_{0}^{\frac{\pi}{4}} d\varphi \int_{0}^{\frac{2}{\cos\varphi+\sin\varphi}} rf(r\cos\varphi, r\sin\varphi)dr$

Quyidagi integrallarda qutb koordinatalariga $(x = r\cos\varphi, y = r\sin\varphi)$ ōtib, integrallash chegaralari ikki hil tartibda qōyilsin:

375. $\int\limits_0^{\frac{\pi}{4}} d\varphi \int\limits_0^{\frac{1}{\cos\varphi}} rf(r\cos\varphi, r\sin\varphi)dr + \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int\limits_0^{\frac{1}{\sin\varphi}} rf(r\cos\varphi, r\sin\varphi)dr =$

$$= \int\limits_0^1 rdr \int\limits_0^{\frac{\pi}{2}} f(r\cos\varphi, r\sin\varphi)d\varphi + \int\limits_1^{\sqrt{2}} rdr \int\limits_{\arccos\frac{1}{r}}^{\arcsin\frac{1}{r}} f(r\cos\varphi, r\sin\varphi)d\varphi$$

376. $\int\limits_0^{\frac{\pi}{2}} d\varphi \int\limits_{\frac{1}{\sqrt{2}\cos\left(\varphi-\frac{\pi}{4}\right)}}^{1} rf(r\cos\varphi, r\sin\varphi)dr = \int\limits_{\frac{1}{\sqrt{2}}}^{1} rdr \int\limits_{\frac{\pi}{4}-\arccos\frac{1}{2\sqrt{2}}}^{\frac{\pi}{4}+\arccos\frac{1}{2\sqrt{2}}} f(r\cos\varphi, r\sin\varphi)d\varphi$

377. $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} d\varphi \int\limits_0^{\frac{2}{\cos\varphi}} rf(r)dr = \frac{\pi}{12}\int\limits_0^{2\sqrt{2}} rf(r)dr + \int\limits_{2\sqrt{2}}^{4}\left(\frac{\pi}{3} - \arccos\frac{2}{r}\right) rf(r)dr$

Qutb koordinatalariga ōtib, quyidagi ikki karrali integrallar hisoblansin:

378. $\frac{2\pi a^3}{3}$

379. $-6\pi^2$

380. $\frac{\pi}{4}\left[(1+R^2)\ln(1+R^2) - R^2\right]$

381. $\frac{\pi^2}{6}$

382. $\frac{2}{3}a^2$

Yangi U va V ōzgaruvchilar kiritilib, quyidagi integrallar hisoblansin:

383. $\frac{3}{2} + \frac{7}{6}\ln 3$

384. $\frac{26\ln 2}{3}$

385. $\frac{4}{3}$

386. $\frac{2}{3}\pi ab$

387. $\frac{215}{2^7}$

2§ Yuzalarni hisoblash

Quyidagi chiziqlar bilan chegaralangan yuza hisoblansin:

388. $\left(\frac{15}{8} - 2\ln 2\right)a^2$

389. $\frac{2}{3}(p+q)\sqrt{pq}$

390. $\frac{1}{2}$

391. 2

392. πab

393. $\frac{16}{3}$

Qutb koordinatalariga otib, quyida berilgan egri chiziqlar bilan chegaralangan yuzalar hisoblansin:

394. $2a^2$

395. $\frac{3\sqrt{3}-\pi}{3}a^2$

396. $\frac{5}{8}\pi a^2$

397. $\frac{\pi ab}{4}\left(\frac{a^2}{h^2}+\frac{b^2}{k^2}\right)$

398. $\frac{a^2b^2}{2c^2}$

399. $\frac{1}{10}\frac{a^2b}{h^4}$

400. $\frac{\pi}{2}\frac{a^3b}{c^2}$

401. $\frac{a^2}{60}$

402. $\frac{a^2}{1260}$

403. $\frac{2}{3}$

404. $\frac{(\beta-\alpha)(b^2-a^2)}{2(\alpha+1)(\beta+1)}$

405. $\frac{a^2}{2}\ln 2$

406. 4/3(q-p)(S-r)

407. 1/3(q-p)$\ln\frac{b}{a}$

408. 1/6(q^2-p^2) (b^3-a^3)

409. 1/4(a^2-b^2)$\left[\frac{(\alpha-\beta)(1-\alpha\beta)}{(1+\alpha^2)(1+\beta^2)}+arctg\frac{\alpha-\beta}{1+\alpha\beta}\right]$

3§ Hajmlarni hisoblash.

Quyidagi sirtlar bilan chegaralangan hajmlar topilsin.

410.

411. $186\frac{2}{3}$

412. $\frac{ab}{6}(\frac{a^2}{p}+\frac{b^2}{q})$

413. 12

414. 1/6

415. $\frac{48}{5}\sqrt{6}$

416. $78\frac{15}{32}$

417. $13\frac{1}{3}$

418. 5/6

419. 88/105

420. $\frac{17}{12}-2\ln 2$

421. $12\frac{4}{21}$

422. 3e-8

423. $\pi/4$

424. $\frac{\pi ah^2}{16}$

425. $\frac{32}{15}$

Quyidagi sirtlar bilan chegaralangan jismlarini qutib koordinatalar sistemasida hisoblang

426. $\frac{4}{3\sqrt{\pi}}\Gamma^2(\frac{4}{3})a^3$

427. $\frac{4}{3\sqrt{\pi}}a^2(\frac{\pi}{2}-\frac{2}{3})$

428. $\frac{45}{32}\pi$

429. $\pi(1-e^{-R^2})$

Quyidagi sirtlar bilan chegaralangan jismlar topilsin.

430. $\frac{1}{3}\pi(2-\sqrt{2})$

431. $\frac{\pi(b^3-a^3)}{12}$

432. $4.5a^4$

433. ¾

4§ Sirt yuzini hisoblash.

Sirt yuzini hisoblang.

434. $\frac{4\sqrt{2}}{3}(a+b)\sqrt{ab}$

435. $\pi\sqrt{2}a^2$

436. 13

437. $8a^2\arcsin(b/a)$

438. $16a^2$

439. $\frac{2\pi}{3}\left[\left(1+R^2\right)^{\frac{3}{2}}-1\right]$

440. $\frac{a^2}{9}(20-3\pi)$

5§ Ikki karrali integrallarning mexanikaga tadbig'i.

441. 79/12

442. 64/3

443. $2\pi-4$

444. 1620π

Ushbu egri chiziq bilan chegaralangan bir jinsli plastinkaning og'irlik markazi topilsin.

445. $x_0=-a/2$, $y_0=-8a/5$

446. $x_0=y_0=a/5$

447. $x_0=y_0=\frac{256}{315\pi}a$

448. $x_0=(1-\frac{\pi}{4})(\sqrt{2}+1)$,

$y_0=\frac{1}{8}(\frac{\pi}{2}-1)(\sqrt{2}+2)$

449. $x_0=3\pi/16$, $y_0=0$

450. $x_0=13/8$, $y_0=39/10$

451. $x_0=7a/12$, $y_0=35b/36$

452. $x_0=\frac{\pi}{12}+\frac{8a}{9\pi}$, $y_0=\frac{5a}{6}$

453. $x_0=\frac{5a}{6}$, $y_0=\frac{16a}{9\pi}$

454. $x_0=y_0=\frac{\pi a}{8}$

455. $x_0=\frac{\pi a}{4\sqrt{2}}$, $y_0=\frac{a}{6}\left(1+\frac{5}{\sqrt{2}}\ln 3\right)$

456. $x_0=133/26$, $y_0=0$

Ushbu egri chiziq bilan chegaralangan yuzaning OX va OY o`qlariga nisbatan I_x va I_y inersiya momenti topilsin (zichligi $\rho=1$ deb qaralsin).

457. $J_x=\frac{|a-b|h^3}{12}$, $J_y=\frac{h|a^3-b^3|}{12}$

458. $J_x=J_y=\frac{3\pi a^4}{4\sqrt{2}}$

459. $J_x=21\pi a^4/32$, $J_y=49\pi a^4/32$

460. $J_x=J_y=9a^4/8$

461. $J_x=J_y=a^2b/3$

Quyidagi chiziqlar bilan chegaralangan, massasi M bo`lgan bir jinsli plastinkaning to`g`ri chiziqqa nisbatan inersiya momentini hisoblang.

462. $\frac{MR^2}{4}$

463. $\frac{Mb^2}{4};\frac{1}{4}Ma^2$

464. $\frac{M}{36}(44-9\pi)$

465. $\frac{47Ma^2}{14};\frac{7}{10}Ma^2$

466. $\frac{7}{16}Ma^2$

467. $\frac{3\pi-8}{48}Ma^2$

V Bob

Uch karrali integrallar.

Quyidagi uch karrali integrallar hisoblansin

468. $\frac{a^6}{48}$

469. $\frac{a^{11}}{110}$

470. $\frac{\pi}{8}(1-\ln 2)$

471. $\frac{1}{2}(1-\sin 1)$

472. 2e-5

Quyidagi uch karrali integrallar ko`rsatilgan sohada hisoblansin.

473. 1/364

474. 1/48

475. $\frac{4\pi abc}{5}$

476. $\pi/6$

477. $\frac{1}{2}(\ln 2-\frac{5}{8})$

478. $\frac{\pi^2}{16} - \frac{1}{2}$

Quyidagi uch karrali integrallarda integrallash chegaralari qo'yilsin

479. $\int_0^1 dx \left\{ \int_0^{1-x} f(x,y,z)dy + \int_x^1 dz \int_{z-x}^{1-x} f(x,y,z)dy \right\}$ **480.** $\int_0^1 dz \int_{-a}^{a} dy \int_{-\sqrt{z^2-y^2}}^{\sqrt{z^2-y^2}} f(x,y,z)dx$

Quiyidagi integrallar sferik yoki silindrik koordinatalar sistemasiga o'tish yordamida hisoblang

484. $\pi/10$

485. $\frac{\pi}{10}(2\sqrt{2}-1)$

486. $\frac{a\pi}{2}$

487. $\frac{8a^2}{9}$

488. $\frac{4\pi R^5}{15}$

489. $\frac{\pi}{8}$

490. $\frac{16\pi}{3}$

491. $\frac{4\pi}{15}(R^5 - r^5)$

492. $\frac{2\pi}{3}$

493. $\pi\left[3\sqrt{10} + \ln\frac{\sqrt{2}-1}{\sqrt{10}-3} - \sqrt{2} - 8\right]$

Quyidagi sirtlar bilan chegaralangan jismning hajmi topilsin.

494. 8

495. 7/12

496. 3/35

497. 4(4-3ln3)

498. $\pi/96$

499. 7/24

Sferik yoki silindrik koordinatalar sistemasiga o'tib, quyidagi sirtlar bilan chegaralangan jism hajmi hisoblansin

500. πa^3

501. $\frac{\pi^2 a^3}{4\sqrt{2}}$

502. 1/2

503. $\frac{\pi}{3}(2-\sqrt{2})(b^3 - a^3)$

504. $\frac{a^3}{360}$

505. $\frac{19}{6}\pi$ va $\frac{15}{2}\pi$

506. $\frac{a^3}{3}\pi$

507. $\frac{\pi}{3}\frac{a^2bc}{h}$

508. $\frac{\pi^2}{4}abc$

509. $\frac{5\pi}{12}abc(3-\sqrt{5})$

510. $\frac{abc}{60}(\frac{a}{h}+\frac{b}{k})(\frac{a^2}{h^2}+\frac{b^2}{k^2})$ **511.** $\frac{abc}{554400}$

Quyidagi sirtlar bilan chegaralangan va zichligi ρ bo'lgan jism massasi topilsin.

512. $\frac{11\pi}{6}$ **514.** $\frac{\pi}{2}$

513. 54π **515.** 4/3

Quyidagi sirtlar bilan chegaralangan bir jinsli jismning og'irlik markazi topilsin.

516. $(\frac{14}{15};\frac{26}{15};\frac{8}{3})$ **521.** $\left(0;0;\frac{3}{4}c\right)$

517. $\left(\frac{3}{8}a;\frac{3}{8}b;\frac{3}{8}c\right)$

522. $\left(\frac{2}{5}a,\frac{2}{5}a;\frac{7}{30}a^2\right)$

518. $\left(\frac{16}{5};\frac{12}{5};\frac{8}{5}\right)$

523. $\left(\frac{17}{18}p;0;\frac{17}{176}p\right)$

519. $\left(\frac{18}{7};\frac{15}{16}\sqrt{6};\frac{12}{7}\right)$

524. $\left(\frac{a}{2};\frac{a}{2};\frac{a}{2}\right)$

520. $\left(0;0;\frac{5a}{83}(6\sqrt{3}+5)\right)$

525. $\left(0;0;\frac{55+\sqrt{3}}{130}\right)$

526. $J_{xy}=\frac{abc^3}{60}$; $J_{yz}=\frac{a^3bc}{60}$; $J_{zx}=\frac{ab^3c}{60}$;

527. $J_{xy}=\frac{4\pi abc^3}{15}$; $J_{zy}=\frac{4\pi ba^3}{15}$; $J_{xz}=\frac{4\pi ab^3}{15}$;

528. $J_{xy}=\frac{\pi abc^3}{5}$; $J_{zy}=\frac{\pi ba^3}{5}$; $J_{xz}=\frac{\pi ab^3}{5}$;

Quyidagi sirtlar bilan chegaralangan bir jinsli jismning inersiya momentini toping

529. M=abc; $J_{0x}=\frac{M}{3}(a^2+b^2)$; $J_{0y}=\frac{M}{3}(a^2+c^2)$; $J_{0z}=\frac{M}{3}(c^2+b^2)$;

530. $J_{0x}=\frac{2^8101}{63}\rho$; $J_{0y}=\frac{2^{10}41}{315}\rho$; $J_{0z}=\frac{2^941}{315}\rho$;

531. $M=\frac{4}{3}\pi abc\rho$ $J_{0x}=\frac{1}{5}M(b^2+c^2)$; $J_{0y}=\frac{1}{5}M(a^2+c^2)$; $J_{0z}=\frac{1}{5}M(a^2+b^2)$

532. $M=\frac{8\sqrt{2}a^3}{15}\rho$; $J_{0x}=\frac{8a^2}{21}$; $J_{0x}=\frac{80a^2M}{3}$; $J_{0x}=\frac{4a^2}{9}M$;

533. $J_{0x}=\frac{5\pi\rho}{6}$

534. $J_{0z}=4\pi\rho$

535. $J_{0x}=\frac{4\pi}{15}\left(4\sqrt{2}-5\right)$

VI Bob

Egri chiziqli integrallar

1§ 1 tur egri chiziqli integrallar

536. $\sqrt{5}\ln 2$

537. 24

538. $\frac{P^2}{3}(5\sqrt{5}-1)$

539. $2\pi a^{2n+1}$

540. $\frac{ab(a^2+ab+b^2)}{3(a+b)}$

541. $4\pi a\sqrt{a}$

542. $\pi a^2/2$

543. $\frac{2a^3\sqrt{2}}{3}$

544. $\frac{1}{2}\left[(R^2+4)^{\frac{3}{2}}-8\right]$

545. $\frac{8a\pi^3\sqrt{2}}{3}$

546. $\frac{R^4\sqrt{3}}{32}$

547. $\frac{2\sqrt{2}}{3}\left[\left(1+2\pi^2\right)^{\frac{3}{2}}-1\right]$

548. $R^2\sqrt{2}$

549. $1+\sqrt{2}$

550. $\frac{216a^3}{15}$

551. $2\pi^2a^3(1+2\pi^2)$

552. $4a^{\frac{7}{3}}$

553. $2a^2$

554. 0

555. $a^2\sqrt{2}$

556. $64\ a^2$

557. $2\pi a^3$

2§ Egri chiziqli integrallar tatbig'i

Quyidagi tenglamalar bilan berilgan egri chiziqlar yoyining uzunligi topilsin.

558. 5

559. $\sqrt{3}$

560. $|x_0|+|z_0|\quad(|x_0|<a)$

561. $\frac{3}{4\sqrt{2}}\left(\sqrt[3]{\frac{3z_0^4}{a}}+2\sqrt[3]{\frac{az_0^4}{3}}\right)$

562. $\left(1+\frac{2z_0}{3c}\right)\sqrt{cz_0}$

563. $\frac{7\sqrt{5}\rho}{24}$

564. $\frac{1}{2}\rho\left[\frac{17}{16}\sqrt{\frac{9}{2}}-\frac{9}{16}\sqrt{\frac{5}{4}}-\frac{1}{128}\ln\left(\frac{17}{8}+\sqrt{\frac{9}{2}}\right)+\frac{1}{128}\ln\left(\frac{9}{8}+\sqrt{\frac{5}{4}}\right)\right]$

565. $J_{0x}=J_{0y}=\left(\frac{a^2}{2}+\frac{h^2}{3}\right)\sqrt{4\pi^2a^2+h^2}\,\rho$; $J_{oz}=a^2\sqrt{4\pi^2a^2+h^2}\,\rho$;

566. $\frac{128}{15}-\frac{74}{15}\sqrt{2}$

567. $J_{0x}=J_{0y}=\frac{3}{8}a^3\rho$

568. $\frac{a^3\rho}{128\sqrt{2}}\left(\frac{89\sqrt{2}}{3}+95\ln(\sqrt{2}-1)\right)$

Bir jinsli yoy L ning og'irlik markazi koordinatalari topilsin.

569. $x_0=y_0=2R/\pi$

570. $x_0=0$; $y_0=\frac{a\left(e^1+4e^{-1}-1\right)}{4e(e^2-1)}$

571. $x_0=\frac{27-24\ln 2}{8(3+2\ln 2)}$; $y_0=\frac{20}{3(3+2\ln 2)}$

572. $x_0=y_0=4a/3$

573. $x_0=0$; $y_0=0$; $z_0=\pi b$

3§ II tur egri chiziqli integrallar.

Quyidagi ikkichi tur egri chiqli integrallar hisoblansin.

574. –14/15

575. 4/3

576. 0

577. $-2\pi a^2$

578. -2π

579. 0

580. 0

581. $\frac{\pi}{4}-1$

582. $\frac{4}{35}$

583. 0

584. $-\frac{\pi}{4}a^3$

585. $\frac{14}{15}a^3$

Integral ostidagi ifoda to'liq differensial ekanligini tekshirib, berilgan egri chiziqli integrallar hisoblansin

586. 8

587. 4

588. $\ln\frac{13}{5}$

589. $-53\frac{7}{12}$

590. 0

591. 1

592. 37

593. 123

594. 3

595. 15

596. $\operatorname{arctg}(-\frac{2}{3})$

Quyidagi misollarda to'liq differensial bo'yicha boshlang'ich funksiya topilsin

597. $\frac{x^3-y^3}{3}+c$

598. $(x^2-y^2)^2+c$

599. $\frac{\sqrt{x^2+y^2}+1}{y}+c$

600. $\ln|x-y|+\frac{y}{x-y}+\frac{x^2}{2}-\frac{y^3}{3}+c$

601. $x^2\cos y+y^2\cos x+c$

602. $\frac{e^y-1}{1+x^2}+y+c$

603. $\frac{x^3}{3}+x^2y+xy^2-\frac{y^3}{3}$

604. $-\frac{2y^2}{(x+y)^2}+\ln|x+y|+c$

605. $\frac{x}{z}+x^2yz-\frac{y}{z^2}+c$

606. $\frac{1}{3}(x^3+y^3+z^3)+2xyz+c$

607. $x-\frac{x}{y}+\frac{xy}{z}+c$

608. $x^2y+y^2z+z^2x+xzy+c$

4§ Grin formulasi.

Quyidagi egri chiziqli integrallardan ikki karrali integralga o'ting.

609. $\iint_D (x^2+y^2)dxdy$

610. mumkin emas

613. $\frac{b}{8a^4}\left(\frac{5a^2+3b^2}{(a^2+b^2)^2}+\frac{3}{ab}arctg\frac{b}{a}\right)$

614. $\frac{\pi(a^2+b^2)^2}{4|ab|^3}$

F(x) ni hisoblang.

615. $2xe^{-x^3}-e^{-x^3}-\int_x^{x^2} y^2e^{-yx^3}dy$

616. $-e^{x|\sin x|}\sin x+e^{x|\cos x|}\cos x+\int_{\sin x}^{\cos x}\sqrt{1-y^2}e^{x\sqrt{1-y^2}}dy$

617. $\left(\frac{1}{x}+\frac{1}{a+b}\right)\sin x(b+x)-\left(\frac{1}{x}+\frac{1}{a+x}\right)\sin x(x+a)$

618. $\frac{x}{2}\ln(1+x^2)$

620. $f(x,-x)+2x\int_0^x f_2'(u,v)dy$ bu erda u=y+x, v=y-x

621. $x(2-3y^2)f(x,y)+\frac{x}{y}f(\frac{x}{y})+x^2y(1-y^2)f'(x,y)$

VII Bob

Parametrga bog'liq integrallar

1§ Parametrga bog'liq integrallarni hisoblash.

Quyidagi integrallarni parametr bo'yicha differensiallash yordamida hisoblang.

622. $\frac{\pi}{2}\ln\left(a+\sqrt{1+a^2}\right)$

623. $\pi\arcsin a$

624. $\pi\arcsin a$

625. $\pi\ln\frac{a+b}{2}$

626. 0, agar $1\geq|a|$; $\pi\ln a^2$,agar $|a|>1$

627. $\frac{\pi}{2}\,signa\ln(1+|a|)$

628. $\frac{\pi}{2}\ln\left(1+\sqrt{2}\right)$

629. $\ln\frac{1+b}{1+a}$

Integralni hisoblang.

630. a)$\operatorname{arctg}\frac{b-a}{1+(a+1)(1+b)}$; b) $\frac{1}{2}\ln\frac{b^2+2b+2}{a^2+2a+2}$

Parametrga bog'liq xosmas integrallar.
Quyidagi integrallarning aniqlanish sohasini toping.

631. $a\geq 0$

632. $\max(p,q)>1$

633. $\left|\frac{p-1}{q}\right|<1$

Quyidagi funksiyalarning uzluksizligini tekshiring.

634. Uzluksiz

635. Uzluksiz

636. Uzluksiz

637. x=0 da uzilishga ega

638. $-\int_x^{\infty} y^2 e^{-xy^2}\,dy = -e^{-x^3}$

640. $\frac{2}{\rho^3}$

Integralni hisoblang.

641. $\ln\frac{(2a)^{2a}(2b)^{2b}}{(b+a)^{2a+2b}}$

642. $\operatorname{arctg}\,arctg\frac{b}{a}-arctg\frac{a}{m}(m\neq 0)$

643. $\frac{1}{2}\ln\frac{b^2+m^2}{a^2+m^2}$

644. $\pi\left(\sqrt{1-a^2}-1\right)$

645. $\pi\arcsin a$

646. $\frac{1}{2}\ln\frac{b^2+a^2}{a^2+c^2}$

647. $\frac{\pi}{2}signa\left(1+\sqrt{1+a^2}+|a|\right)$

648. $\frac{\pi}{|b|}\ln\left(|b|+|a|\right)(b\neq 0)$

649. $\frac{\pi}{2}\ln\frac{(a+b)^{a+b}}{a^a b^b}(a>0,b>0)$

650. $\frac{2\pi}{3}\left(ab(a+b)+a^3\ln a+b^3\ln b-(a^3+b^3)\ln(a+b)\right)(a>0,b>0)$

Quyidagi integrallarni Eyler-Puasson formulasi yordamida hisoblang.

651. $\sqrt{\frac{\pi}{a}}e^{-\frac{qc-\frac{b^2}{a}}{a}}$

652. $\frac{\sqrt{\pi}e^{-2a}}{2}$

653. $\sqrt{\pi}(\sqrt{b}-\sqrt{a})$

654. $\frac{1}{2}\sqrt{\frac{\pi}{a}}e^{\frac{-b^2}{4a}}$

655. $\frac{b\sqrt{\pi}}{4a\sqrt{a}}e^{\frac{-b^2}{4a}}$

656. $(-1)^n\frac{\sqrt{\pi}}{2^{2n+1}}\frac{d^{2n}}{db^{2n}}\left(e^{-b^2}\right)$

Dirixle va Fure integrali yordamida hisoblang.

657. $\pi\frac{|b|}{2}-\sqrt{\pi a}$

658. 0, agar $|a|<|b|$; $(\pi/4)$signa, agar $|a|=|b|$;$(\pi/2)$signa agar $|a|>|b|$

659. $(\pi/4)$signa

660. $|a|\,\pi/2$

$\frac{1}{a^2+x^2}=\int_0^\infty e^{-t(a^2+x^2)}dt$ (a>0,b>0) formuladan foydalanib Laplas intergalini hisoblang

661. a) $\frac{\pi}{2a}e^{-ab}$; b) $\frac{\pi}{2a}e^{+ab}$

$\frac{1}{\sqrt{x}}=\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-xy^2}dy$ (x>0) formuladan foydalanib Laplas intergalini hisoblang

662. a) $\frac{1}{2}\sqrt{\frac{\pi}{2}}$; b) $\frac{1}{2}\sqrt{\frac{\pi}{2}}$

VIII Bob
Eyler integrali

Quyidagi integrallarni Eyler integrali yordamida hisoblang :

663. $\frac{\pi}{8}$

664. $\frac{\pi\sqrt{16}}{}$

665. $\frac{\pi}{2\sqrt{2}}$

666. $\frac{2\pi}{3\sqrt{3}}$

667. $\frac{\pi}{2\sqrt{2}}$

668. $\frac{3\pi}{512}$

669. $\frac{\pi}{n\sin\frac{\pi}{n}}$

670. $\frac{1}{2^{n+1}}\sqrt{\pi}(2n-1)!!$

Quyidagi integrallarni Eyler integrallari orqali ifodalang.

671. $\frac{\pi}{n\sin\frac{m\pi}{n}}$, (0<m<n)

672. B(n-m,m), (0<m<n)

673. $\frac{a^{-p}}{n}\left(\frac{a}{b}\right)^{\frac{m+1}{n}}B\left(\frac{m+1}{n},p-\frac{m+1}{n}\right),(0<\frac{m+1}{n}<p)$

674. $\frac{1}{m}B\left(\frac{1}{m},1-\frac{1}{n}\right),(n<0 yokin>1)$

675. $\frac{1}{2}B\left(\frac{m+1}{2},\frac{n+1}{2}\right)$, (m>-1 yoki n>-1)

676. $\frac{\pi}{2\cos\frac{n\pi}{2}}$, (n<1)

677. $\frac{1}{n}\Gamma\left(\frac{1}{n}\right)$, (n > 0)

678. $\frac{1}{|n|}\Gamma\left(\frac{m+1}{n}\right)$, $(\frac{m+1}{n}>0)$

679. $\Gamma(p+1)$, (p>-1)

680. $\frac{d}{dx}\left[\frac{\Gamma(p+1)}{a^{p+1}}\right]$, (p>-1)

681. $-\frac{\pi^2\cos p\pi}{\sin^2 p\pi}$, (0<p<1)

682. $\pi^3\frac{1+\cos^2\cos p\pi}{\sin^3 p\pi}$, (0<p<1)

683. $\frac{2}{27}\pi^2$

684. $\ln\sqrt{2\pi}$

685. $\ln\sqrt{2\pi}+a(\ln a-1)$

IX Bob
Fure qatorlari
1§ Funksiyalarni Fure qatorlariga yoying.

686. $\frac{1}{2}-\frac{1}{2}\cos 2x$

687. $\frac{3}{4}\cos x+\frac{1}{4}\cos 3x$

688. $\frac{3}{8}-\frac{1}{2}\cos 2x+\frac{1}{8}\cos 4x$

689. $\frac{A}{2}+\frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)}\sin\frac{2n-1}{l}\pi x$

690. $2\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}$

691. $\frac{1}{2}-\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\pi(2n-1)x}{(2n-1)^2}$

692. $1-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin 2n\pi x}{n}$

693. $2\text{shah}\left[\frac{1}{2ah}+\sum_{n=1}^{\infty}(-1)^n\frac{ah\cos\frac{n\pi x}{h}-n\pi a h\sin\frac{n\pi x}{h}}{(ah)^2+(n\pi)^2}\right]$

694. $\frac{2\sin a\pi}{\pi}\left[\frac{1}{2a}+\sum_{n=1}^{\infty}(-1)^{n+1}\frac{a\cos nx}{n^2-a^2}\right]$

695. $\frac{2\sin a\pi}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n\sin nx}{n^2-a^2}$

696. $a+l+\frac{2l}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{na\pi}{l}-\frac{nx\pi}{l}\right)$

697. $\frac{2\sin a\pi}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n\sin nx}{n^2+a^2}$

698. $\frac{2}{\pi}\sum_{n=1}^{\infty}(1+(-1)^{n+1}(1+\pi))\frac{\sin nx}{n}$

α parametr bo'yicha hosila olib I(α) integralni hisoblang, agar:

699. $\pi\ln\frac{\alpha+\sqrt{\alpha^2-1}}{2}$

700. $2\pi\text{arcsin}a$

Furullani formulasidan foydalainib integralni hisoblang:

701. $0.5\ln\frac{b}{a}$

702. $0.5\ln\frac{b}{a}$

Dirixle integralidan foydalanib integralni hisoblang

703. $\frac{\pi|\alpha|}{2}$

704. $\frac{5\pi}{32}$

Fure integrali yordamida f(x) funksiyani ifodalang.

707. $\frac{2}{\pi}\int_0^{+\infty}\frac{\sin\gamma y}{y}\cos xydy$

708. $\frac{2}{\pi}\int_0^{+\infty}\frac{1-\cos ay}{y^2}\cos xydy$

709. $\frac{2}{\pi}\int_0^{+\infty}\frac{\sin(\gamma(x-a))-\sin(\gamma(x-b))}{y}dy$

710. $\frac{2}{|a|}\int_0^{+\infty}e^{-|a|y}\cos xydy$

711. $\frac{2}{\pi}\int_0^{+\infty}\frac{\sin\pi y}{1-y^2}\sin xydy$

712. $\frac{2}{\pi}\int_0^{+\infty}\frac{\cos\frac{\pi y}{2}}{1-y^2}\sin xydy$

713. $\frac{2}{\pi}\int_0^{+\infty}\frac{y\sin\pi y}{((y-b)^2+a^2)((y+b)^2+a^2)}dy$

714. $\frac{1}{\sqrt{\pi}}\int_0^{+\infty}e^{-\frac{y^2}{4}}\cos xydy$

715. $\frac{1}{2\sqrt{\pi}}\int_0^{+\infty}ye^{-\frac{y^2}{4}}\sin xydy$

716. $\sqrt{\frac{2}{\pi}}\frac{\sin y}{y}$

717. $\sqrt{\frac{2}{\pi}}\frac{y\sin\pi y}{1-y^2}$

718. $-i\sqrt{\frac{2}{\pi}}\frac{y\sin\pi y}{1-y^2}$

719. $-\frac{i}{\sqrt{2\pi}}\frac{ye^{i\pi y}}{1-y^2}$

720. $-i\sqrt{\frac{8}{\pi}}\frac{ay}{(y^2+a^2)^2}$

721. $e^{-\frac{y^2}{2}}$

722. $e^{-\frac{y^2+a^2}{2}}$ chay

723. $\frac{1}{\sqrt{2\pi}}\frac{\pi(1-y^2)\cos\pi y+2y\sin\pi y}{(1-y^2)^2}$

724. $\sqrt{\frac{2}{\pi}}\frac{\sin 2y-\sin y}{y}$

725. $\sqrt{\frac{2}{\pi}}\frac{2y\cos y-(y^2-2)\sin y}{y^3}$

726. $-\sqrt{\frac{2}{\pi}}\frac{2\pi\cos\pi y(1-y^4)-\sin\pi y(6y+2y^3-\pi^2(y-2y^3-y^5))}{(1-y^2)^3}$

ADABIYOTLAR

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