ALGEBRA VA MATEMATIK ANALIZ

ASOSLARIDAN MISOL VA MASALALAR

TO'PLAMI

22.14
A-45

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ALGEBRA VA MATEMATIK ANALIZ ASOSLARIDAN MISOL VA MASALALAR TO'PLAMI

Akademik litseylar va kasb-hunar kollejlari uchun o'quv qo'llanma

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Ushbu o‘quv qo‘llanma o‘rta maxsus kasb-hunar kollejlari va akademik litsey talabalari uchun algebra va matematik analiz asoslaridan misol va masalalar to‘plamidan iborat. Hozirgi kunda o‘rganilishi juda zarur, ravon va sodda yo‘nalishda tuzilgan masala va misollarni test savollari yordamida o‘rganilishi ko‘zda tutilgan.

Qo‘llanmada eng muhim misol va masalalarning yechilish tartiblari hamda javoblari keltirilganligi o‘quvchilarga qulaylik yaratadi.

Hozirgi kun fan dasturlariga asoslanib, turli dolzarb yangiliklarni o‘z ichiga kiritgan mazkur o‘quv qo‘llanmadan kasb-hunar kollejlari va akademik litsey professor-o‘qituvchilari ham foydalanishlari mumkin.

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I BOB. TO'PLAMLAR NAZARIYASI VA MATEMATIK MANTIQ ELEMENTLARI

1-§. To'plam va uning elementlari. Bo'sh va qism to'plam

To'plam tushunchasi matematikaning boshlang'ich (ta'riflanmaydigan) tushunchalaridan biridir. U chekli yoki cheksiz ko'p obyektlar (narsalar, buyumlar, shaxslar va h.k.)ni birgalikda bir butun deb qarash natijasida vujudga keladi. Masalan, O'zbekistondagi viloyatlar to'plami; viloyatdagi tumanlar to'plami; butun sonlar to'plami; to'g'ri chiziq kesmasidagi nuqtalar to'plami; sinfdagi o'quvchilar to'plami va hokazo.

To'plamni tashkil etgan obyektlar uning *elementlari* deyiladi. To'plamlar odatda lotin alifbosining bosh harflari bilan, uning elementlari esa shu alifboning kichik harflari bilan belgilanadi. Masalan, $A = \{a, b, c, d\}$ yozuvi A to'plam a, b, c, d elementlardan tashkil topganligini bildiradi.

Elementlari soniga bog'liq holda to'plamlar chekli va cheksiz to'plamlarga ajratiladi. Elementlari soni chekli bo'lgan to'plam *chekli to'plam*, elementlari soni cheksiz bo'lgan to'plam *cheksiz to'plam* deyiladi.

1 - m i s o l. $A = \{2,4,6,8,10,12....\}$ to'plam 2 ga bo'linadigan sonlar to'plami, bu to'plam cheksiz to'plam, chunki elementlar sonini sanab bo'lmaydi.

2 - m i s o l. $B = \{x | x \in \mathrm{N}, x < 5\}$ to'plam chekli to'plam, chunki B to'plam 5 dan kichik bo'lgan barcha natural sonlardan tuzilgan, ya'ni $B = \{1,2,3,4\}$.

T a ' r i f. Birorta ham elementga ega bo'lmagan to'plam *bo'sh to'plam* deyiladi. Bo'sh to'plam $\varnothing$ orqali belgilanadi.

3-m i s o l. $x^2 + 3x + 5 = 0$ tenglama haqiqiy ildizlarga ega emas, ya'ni uning haqiqiy yechimlar to'plami $\varnothing$ dir.

T a ' r i f. Ayni bir xil elementlardan tuzilgan to'plamlar *teng to'plamlar* deyiladi.

4-m i s o l. $X = \{x|x \in N, x < 4\}$ va $Y = \{x|(x-1)(x-2)(x-3) = 0\}$ to'plamlarning har biri faqat 1, 2, 3 sonlaridan tuzilgan. Shuning uchun bu to'plamlar tengdir: $X = Y$.

T a ' r i f. Agar B to'plamning har bir elementi A to'plamning ham elementi bo'lsa, B to'plam A to'plamning *qism to'plami* deyiladi va $B \subset A$ ko'rinishida belgilanadi.

5-m i s o l. A – ikki xonali sonlar to'plami, B – ikki xonali juft sonlar to'plami bo'lsin. Har bir ikki xonali juft son A to'plamda ham mavjud. Demak, $B \subset A$.

1.1. O'zbekiston Respublikasidagi barcha viloyatlar nomlari to'plamini tuzing..

1.2. «Tenglama» so'zidagi harflar to'plamini tuzing.

1.3. O'zbekiston Respublikasining konstitutsiyasi qabul qilingan yilda qatnashgan raqamlar to'plamini tuzing.

1.4. Quyidagi to'plam elementlarini va elementlar sonini ko'rsating:

1) $\{l, f, g\}$; 2) $\{a\}$; 3) $\{a, b, c, d\}$; 4) $\varnothing$; 5) $\{0\}$; 6) $\{(a, b), (c, d)\}$; 7) $\{\{a, b, c\}, a\}$.

1.5. 5 ta elementi bor bo'lgan to'plam tuzing.

1.6. 5 ta natural son qatnashgan sonli to'plam tuzing.

1.7. Quyidagi to'plamning qaysilari chekli, qaysilari cheksiz:

1) $A = \{2; 4; 6; 8; 10; 12; 14....\}$; 2) $B = \{x|x \in N, x > 9\}$;

3) $D = \{x|x \in N, x < 66\}$; 4) $E = \{y|y \in Q, y < 2\}$.

1.8. Quyidagi to'plam elementlarini toping:

1) 5 ga karrali barcha ikki xonali natural sonlar to'plami;

2) 80 dan kichik va oxirgi raqami 2 bo'lgan barcha natural sonlar to'plami;

3) Raqamlarining yig'indisi 9 ga teng bo'lgan barcha ikki xonali sonlar to'plami;

4) Oxirgi raqami 0 bo'lgan to'rt xonali natural sonlar to'plami.

1.9. Quyidagi to‘plamlardan qaysilari bo‘sh to‘plam:

1) $\{x|x^4+1=0\}$; 2) $\{x|x\in N, x^3+1=0\}$;

3) $\{x|x\in N, x<-5\}$; 4) $\{x|x\in R, x<-2\}$;

5) $\{x|x\in R, x^3=1\}$; 6) $\{x|x\in N, -5<x<-6\}$.

1.10. Quyidagi to‘plamlar tengmi:

1) $A=\{2; 4; 6\}$ va $B=\{6; 4; 2\}$;

2) $A=\{1; 2; 3\}$ va $B=\{1; 11; 111\}$;

3) $A=\{1; 2\}, \{2; 3\}\}$ va $B=\{2; 3; 1\}$;

4) $A=\{256; 81; 16\}$ va $B=\{22; 32; 42\}$.

1.11. Quyidagi to‘plamlardan qaysilari teng:

1) $A=\{x|x\in N, x<3\}$, $B=\{x|x\in N, (x-1)(x-2)=0\}$;

2) $A=\{x|x\in N, -1<x<6\}$, $B=\{1; 2; 3; 4; 5\}$;

3) $A=\{5; 10; 15; 20; 25; 30\}$, $B=\{x|x\in N, x<25\}$;

4) $A=\{2; 3; 4; 5; 6\}$, $B=\{x|x\in N, (x-1)(x-2)(x-3)(x-4)(x-5)(x-6)=0\}$;

5) $A=\{2; 3\}$, $B=\{x|x\in N, (x-3)(x-2)=0\}$;

6) $A=\{-2; 2\}$, $B=\{x|x\in R, x^2-4=0\}$.

1.12. $D=\{125, 256, 248, 369, 265, 450, 525\}$ to‘plam berilgan. D to‘plamning

1) 2 ga bo‘linadigan; 2) 3 ga bo‘linadigan;

3) 4ga bo‘linmaydigan; 4) 5 ga bo‘linadigan;

5) 2 ga bo‘linmaydigan; 6) 3 ga bo‘linmaydigan sonlaridan tuzilgan qism to‘plamini toping.

1.13. Quyidagi to‘plamlar uchun $A\subset B$ yoki $B\subset A$ munosabatlardan qaysi biri o‘rinli:

1) $A=\{a,b,c,d\}$, $B=\{b,c,d\}$; 2) $A=\{1,2,3,4,5,6\}$, $B=\{x|x\in N, x<10\}$;

3) $A=\{a,b,c\}$, $B=\{a,c,d\}$; 4) $A=\{x|x\in N, x<5\}$, $B=\{x|x\in N, x<2\}$;

5) $A=\{x|x\in R, x>5\}$, $B=\{x|x\in N, x>4\}$

1.14. $A=\{3, 6, 9, 12\}$ to‘plamning barcha qism to‘plamlarini tuzing.

1.15. To‘plamlar jufti berilgan:

1) $A=$ {Litsey o‘quvchilari to‘plami} va $B=$ {1-kurs o‘quvchilari to‘plami};

2) C – qavariq to‘rtburchaklar to‘plami va D – to‘rtburchaklar to‘plami;

3) E – Samarqand olimlari to‘plami, F – O‘zbekiston olimlari to‘plami;

4) K – barcha tub sonlar to‘plami, M – manfiy sonlar to‘plami.

Juftlikdagi to‘plamlardan qaysi biri ikkinchisining qism to‘plami bo‘lishini aniqlang.

1.16. Munosabatning to‘g‘ri yoki noto‘g‘ri ekanligini aniqlang:

1) $\{1; 2\} \subset \{\{1; 2; 3\}; \{1; 3\}; 1; 2\}$;

2) $\{1; 2\} \supset \{\{1; 2; 3\}; \{1; 3\}; 1; 2\}$;

3) $\{1; 3\} \subset \{\{1; 2; 3\}; \{1; 3\}; 1; 2\}$;

4) $\{1; 3\} \supset \{\{1; 2; 3\}; \{1; 3\}; 1; 2\}$.

1.17. Quyidagi to‘plamlar tengmi:

1) $A = \{2; 4; 6\}$ va $B = \{6; 4; 2\}$;

2) $A = \{1; 2; 3\}$ va $B = \{1; 11; 111\}$;

3) $A = \{1; 2\}, \{2; 3\}\}$ va $B = \{2; 3; 1\}$;

4) $A = \{256; 81; 16\}$ va $B = \{22; 32; 42\}$.

2-§. To‘plamlar ustida amallar

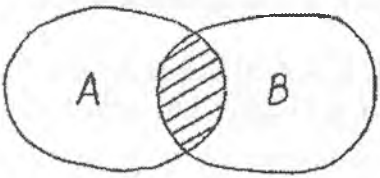
A va B to‘plamlarning ikkalasida ham mavjud bo‘lgan x elementga shu to‘plamlarning *umumiy* elementi deyiladi. A va B to‘plamlarning *kesishmasi* (yoki *ko‘paytmasi*) deb, ularning barcha umumiy elementlaridan tuzilgan to‘plamga aytiladi. A va B to‘plamlarning kesishmasi $A \cap B$ ko‘rinishda belgilanadi:

$A \cap B = \{x | x \in A$ va $x \in$ B$\}$ Eyler – Venn diagrammasi nomi bilan ataladigan chizmada (1- rasm) A va B shakllarning kesishmasi $A \cap B$ tasvirlangan (chizmada shtrixlab ko‘rsatilgan).

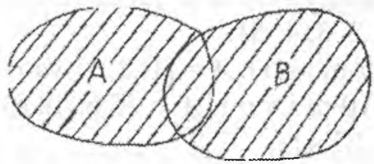
A va B to‘plamlarning *birlashmasi* (yoki *yig‘indisi*) deb, ularning kamida bittasida mavjud bo‘lgan barcha elementlardan tuzilgan to‘plamga aytiladi. A va B to‘plamlarning birlashmasi $A \cup B$ ko‘rinishida belgilanadi:

$A \cup B = \{x|\ x \in A$ yoki $x \in B\}$ (2- rasm).

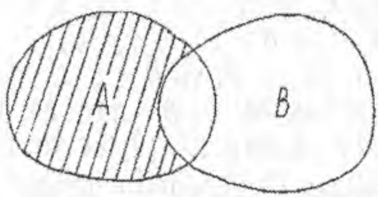
A va B to‘plamlarning *ayirmasi* deb, A ning B da mavjud bo‘lmagan barcha elementlaridan tuzilgan to‘plamga aytiladi. A va B to‘plamlarning ayirmasi $A \backslash B$ ko‘rinishda belgilanadi: $A \backslash B = \{x|x \in A$ va $x \in B\}$ (3- rasm).



1-*rasm.*



2-*rasm.*



3-*rasm.*

1-m i s o l. $A = \{a, b, c, d, e\}$ va $B = \{b, c, e, l, m\}$ to‘plamlar berilgan. Ularning kesishmasini toping.

Y e c h i s h. *b, c, e* elementlargina A va B to‘plamlarning umumiy elementlaridir. Shuning uchun: $A \cap B = \{b, c, e\}$.

2-m i s o l. $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{5, 6, 7, 8\}$ va $C = \{5, 6, 7, 8, 9, 10, 11\}$ to‘plamlarning kesishmasi ushbuga teng: $A \cap B \cap C = \{5, 6\}$.

3-m i s o l. $A = \{2, 3, 4\}$ va $B = \{7, 8, 9\}$ to‘plamlarning kesishmasi ushbuga teng: $A \cap B \neq \varnothing$.

4-m i s o l. $A = \{a, b, c, d\}$ va $B = \{c, d, e, f\}$ to‘plamlar birlashmasini toping.

$A \cup B = \{a, b, c, d, e, f\}$, chunki har ikkala to‘plamning barcha elementlari olinadi.

5-m i s o l. $A = \{a, b, c, d\}$, $B = \{a, b, c, d, e, f\}$ to'plamlarning birlashmasi: $A \cup B = \{a,b,c,d,e,f]$ ga teng.

6-m i s o l. $A = \{1,2,3,4,5,6\}$ va $B = \{4,5,6,7,8,9,10\}$ to'plamlarning birlashmasi:

$A \cup B = \{1,2,3,4,5,6,7,8,9,10\}$.

7-m i s o l. $A = \{1, 2, 3, 4\}$ va $B = \{3, 4, 5, 6, 7, 8\}$ uchun $R = A \backslash B = \{1, 2\}$.

8-m i s o l. $A = \{1, 2, 3, 4, 5\}$ va $B = \{6, 7, 8\}$ uchun $R = A \backslash B = \{1, 2, 3, 4, 5\}$.

9-m i s o l. $A = [1, 2, 3\}$ va $B = \{1, 2, 3, 4, 5\}$ uchun $R = A \backslash B = \varnothing$.

10-m i s o l. $A = \{5,6,7,8,9\}$ va $B = \{1,2,3,4,5,6\}$ uchun $B \backslash A = \{1,2,3,4\}$.

2.1. Ushbu to'plamlar kesishmasini toping:

1) $A = \{0,1,2,3,4,5,6,7,8,9\}$ va $B = \{6,7,8,9,10\}$;

2) $A = \{a, b, c, d, e\}$ va $B = \{\mathrm{d}, e, f, g, h\}$;

3) $A = \{-5, -4, -3, -2, -1, 0\}$ va $B = \{-2, -1, 0, 1, 2, 3, 4, 5\}$;

4) $A = \{5, 9, 16, 25, 36, 48\}$ va $B = \{16, 25, 38, 42, 48\}$.

2.2. $X = \{35, 30, 17, 56, 69\}$; $Y = \{7, 24, 30, 25, 56, 68\}$; $Z = \{25, 7, 24\}$ to'plamlar berilgan. Quyidagilarni toping:

1) $X \cap Y$, 2) $X \cap Z$, 3) $Y \cap Z$, 4) $X \cap Y \cap Z$;

2.3. $A = \{a, s, d, f, g\}$, $B = \{q, s, e, f, t, g\}$, $C = \{z, a, d, x, f, v, g\}$ to'plamlar berilgan. Quyidagilarni toping:

1) $A \cap B$, 2) $A \cap C$, 3) $B \cap C$, 4) $A \cap B \cap C$.

2.4. $A = \{a, w, x, r, f, v\}$, $B = \{s, q, x, f, r, v\}$, $C = \{a, s, w, c, r, v\}$ to'plamlar berilgan. Quyidagilarni toping:

1) $A \cap B$, 2) $A \cap C$, 3) $B \cap C$, 4) $A \cap B \cap C$.

2.5. Ushbu to'plamlar birlashmasini toping:

1) $A = \{1, 2, 3, 4\}$ va $B = \{5, 6, 8, 9, 7\}$; 2) $A = \{a, b, c, d, e, f\}$ va $B = \{q, e, r, f, v, c\}$;

3) $A = \{0, 1, 2, 3\}$ va $B = \{9,8,7,4,5\}$; 3) $A = \{2, 4, 6, 8, 10\}$ va $B = \{5,10,15\}$.

2.6. 1) A – 24 ning hamma natural bo'luvchilari to'plami, B – 36 ning hamma natural bo'luvchilari to'plami. $A \cap B$ to'plam elementlarini ko'rsating.

2) P ikki xonali natural sonlar to'plami, S barcha toq natural sonlar to'plami bo'lsa, $K = P \cap S$ to'plamga qaysi sonlar kiradi?

2.7. «Matematika» va «grammatika» so'zlaridagi harflar to'plamini tuzing. Bu to'plamlar kesishmasini toping.

2.8. [1; 5] va [3; 7] kesmalarning kesishmasini toping.

2.9. $P = \{a, b, c, d, e, f\}$ va $E = \{a, g, z, e, k\}$ to'plamlar birlashmasini toping.

2.10. $A = \{n | n \in N, n < 5\}$ va $B = \{n | n \in N, n > 7\}$ to'plamlar birlashmasini toping:

1) $A \cap B$; 2) $A \cup B$; 3) $A \backslash B$.

2.11. $A = \{2; 4; 6; 8;...; 40\}$, $B = \{1; 3; 5; 7;...; 37\}$, $C = \{\{a; b\}, \{c; d\}, \{e; f\}, g, h\}$ to'plamlarning har biridagi elementlar sonini aniqlang. $A \cap B$ da nechta element mavjud?

2.12. $A = \{2; 3; 4; 5; 7; 10\}$, $B = \{3; 5; 7; 9\}$, $C = \{4; 9; 11\}$ bo'lsin. Quyidagi to'plamlarda nechtadan element mavjud:

1) $A \cup (B \cap C)$; 2) $(C \cup B) \cap A$; d) $A \cup (B \cup C)$;

4) $A \cap (B \cap C)$; 5) $A \cap (B \cup C)$; 6) $B \cup (A \cap C)$.

2.13. $A = \{x | -5 < x < 10\}$, $B = \{x | x \in N, 3 < x < 15\}$ bo'lsin. $A \backslash B$ va $B \backslash A$ to'plam elementlarini toping.

2.14. P – ikki xonali natural sonlar to'plami, Q – juft natural sonlar to'plami bo'lsin. $P \backslash Q$ va $Q \backslash P$ to'plamlarni tuzing.

2.15. C – 5 ga karrali ikki xonali natural sonlar to'plami va D – 10 ga karrali ikki xonali natural sonlar to'plami bo'lsin. Quyidagilarni toping:

1) C \D, 2) D \C, 3) (C\D) ∩ (D \C).

2.16. $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $B = \{5, 6, 7, 8, 9, 10, 11\}$, C = {0, 1, 2, 3, 4, 5, 10} to'plamlar berilgan. Quyidagilarni toping:

1) (A ∪ B)\(A ∪ C); 2) (A ∪ C)\(B ∪ C); 3) (A ∩ B)\(A ∩ C);

4) (A ∩ C)\(B ∩ C); 5) (A ∪ B) ∩ (A ∪ C); 6) (A ∪ C) ∩ (B ∪ C);

7) (A\B) ∪ (B\C); 8) (C ∩ A) ∪ (B ∩ C)\(A ∪ B).

2.17. $\varnothing$, $\cup$, $\cap$, $\subset$ belgilardan foydalanib, to'plamlar orasidagi munosabatni yozing:

1) $A = \{1; 3; 5; 7\}$, $B = \{1; 5; 7\}$;

2) $A = \{\{0\}; 1; 3\}$, $B = \{1; 3\}$;

3) $A = \varnothing, B = \{k, l, m\}$;

4) $A = \{x, y, z\}, B = \{y, z, x\}$;

5) $A = \{0\}, B = \varnothing$;

6) $A = \{\{x\}, x, \varnothing\}, B = \{x\}$;

7) $A = \{\{1; 3\}; \{2; 4\}; 2; 4\}, B = \{\{1; 3\}, 2\}$;

8) $A = \{\{3\}, 3, \varnothing\}, B = \varnothing$.

2.18. To‘plamlar berilgan:

$A = \{-2; -1; 0; 1; 2; 3; 4; 5\}, B = \{3; 4; 5; 6\}, C = \{-3; -2; -1; 0; 2; 3\}$, $D = \{2; 3; 4; 5; 6; 7\}$. Quyidagilarni toping:

1) $(A \cup B) \cap (C \cup D)$; 2) $(A \cap B \cap C) \cup D$;

3) $(A \cap B) \cup (C \cap D)$; 4) $(A \cup C) \cap (A \cup B)$;

5) $(B \setminus A) \cup (A \setminus B)$; 6) $D \cup (C \setminus D)$.

I bob bo‘yicha test topshiriqlari

1. Teng to‘plamlarni ko‘rsating.

1) burchaklari to‘g‘ri burchakdan iborat bo‘lgan romblar to‘plami.

2) kvadratlar to‘plami

3) to‘g‘ri to‘rtburchaklar to‘plami

A) 1, 2 B) 1, 3 C) 3, 4 D) 1, 4

2. 14 sonini natural bo‘luvchilari to‘plamini toping.

A) {2, 7} B) {1, 2, 7} C) {2, 7, 14} D) {1, 2, 7, 14}

3. $X + Y = \{(2,3), (2,5), (2,6), (3,3), (3,5), (3,6)\}$ Dekart ko‘paytma berilgan. X va Y to‘plamlarni toping.

A) $X = \{2, 3\}, Y = \{3, 5\}$ B) $X = \{2, 3\}, Y = \{3, 6\}$

C) $X = \{2, 3\}, Y = \{3, 5, 6\}$ D) $X = \{2, 3\}, Y = \{2, 3\}$

4. $A = \{a, b, d, c\}$ $B = \{b, c, e, k\}$ to‘plamlar kesishmasini ko‘rsating.

A) $A \cap B = 0$ B) $A \cap B = \{a, b, c, d, k\}$

C) $A \cap B = \{b, c\}$ D) $A \cap B = \{a, c, k\}$

5. $A = (26, 39, 5)$ $B = (26, 39, 5, 40)$ to‘plamlar birlashmasini ko‘rsating.

A) $A \cup B = 0$ B) $A \cup B = \{26, 39, 5, 40\}$

C) $A \cup B = \{26, 39, 5\}$ D) $A \cup B = \{40\}$

6. «Matematika» so'zidagi harflar to'plamining quvvati nechaga teng?

A) $n = 10$ B) $n = 6$ C) $n = 3$ D) $n = 24$

7. Ikki A va B to'plamning birlashmasi – bu...

A) to'plamning barcha elementlari

B) B to'plamning barcha elementlari

C) A va B to'plamning barcha elementlari

D) A to'plamning B ga tegishli bo'lmagan elementlari

8. Ikki A va B to'plamning kesishmasi – bu...

A) to'plamning barcha elementlari

B) B to'plamning barcha elementlari

C) A to'plamning B ga tegishli bo'lmagan elementlari

D) Ham A, ham B to'plamlarga tegishli bo'lgan elementlar

9. Ikki A va B to'plamning ayirmasi – bu...

A) A to'plamning B to'plamga tegishli bo'lmagan elementlaridan iborat

B) Faqat A to'plamning elementlari

C) Faqat B to'plamning elementlari

D) Ham A, ham$B$ to'plamlarga tegishli bo'lgan elementlar

10. Quyidagi qaysi amal to'plamlar ustidagi amallarga kirmaydi?

A) To'plamlar birlashmasi

B) To'plamalar kesishmasi

C) Konyunksiya

D) Simmetrik ayirma

11. Qaysi to'plam qolgan to'plamlarning qism to'plami bo'ladi?

A) N B) Z C) Q D) R

12. Qaysi to'plam qolgan to'plamlarni o'z ichiga oladi?

A) N B) Q C) R D) U

13. $Z \setminus N$ ayirma nimaga teng?

A) $\{-1; -2; ...\}$ B) $\{0; -1; -2; ...\}$ C) $(-\infty; 0)$ D) To'g'ri javob yo'q

14. $(A \cup B) \setminus B$ nimaga teng ?

A) $A \setminus B$ B) $B \setminus A$ C) A D) B

15. Agar $A = (-2; 3)$ va $B = [-4; 1]$ bo‘lsa, $A \cap B$ ni toping.

A) $(-2; 1)$ B) $[-4; 3)$ C) $(-2; 1]$ D) $[-4; 3]$

16. A va B to‘plamlarning dekart ko‘paytmasi deb...

A) Shunday juftlikka aytiladiki, uning birinchi elementi A to‘plamdan, ikkinchi elementi B to‘plamdan olinadi

B) Shunday juftlikka aytiladiki, uning elementlari A va B to‘plamlardan olinadi

C) Shunday juftlikka aytiladiki, uning elementlari faqat A to‘plamdan olinadi

D) Shunday juftlikka aytiladiki, uning elementlari B to‘plamdan olinadi

17. $A = \{a,b\}$ va $B = \{c,d\}$to‘plamlar berilgan. Ularning dekart ko‘paytmasini toping.

A) $C = \{a,b,c,d\}$

B) $C = \{(a; c), (a; d), (b; c), (b; d)\}$

C) $C = \{(a; d),(b; d), (c; d),(a; c)\}$

D) $C = \{(a; d,), (b; d), (a; c)\}$

18. Ikki to‘plamning dekart ko‘paytmasi qayerda tasvirlanadi?

A) To‘g‘ri chiziqda B) Koordinata tekisligida

C) Fazoda D) Egri chiziqda

19. $A = [-3,5; -2,5]$ va $B = (-3; 0)$ to‘plamlar berilgan. $A \cup B$ ni toping.

A) $(-3; 0)$ B) $[-3,5; 0)$

C) $(-2,5; 3,5]$ D) $[-3,5; -2,5]$

20. $A = [-2; -1]$ va $B = (0; 2)$ bo‘lsa, $A \cap B$ ni toping.

A) $(0; 1]$ B) $(0; 2)$ C) $\varnothing$ D) $[-2; -1]$

21. A = (4; 5] va $B = [2; 3)$ bo‘lsa, A\B ni toping.

A) $(4; 5]$ B) $(4; 5)$ C) $[2; 3)$ D) $[2; 5]$

22. $A = [-5; 0)$ va $B = [-3; -1)$ to‘plamlar berilgan. $B \backslash A$ ni toping.

A) $[-5; 0)$ B) $[-5; -1)$ C) $\varnothing$ D) $[-3; -1)$

23. To‘g‘ri javobni ko‘rsating.

A) $99 \in N$ B) $0 \in N$ C) $-3 \in N$ D) $4,5 \in N$

24. $A = \{x/x € Z; -2 < x < 3\}$to‘plamning elementlarini ko‘rsating.

A)$\{-1; 0; 1; 2\}$ B) $\{-1; 0; 1; 2; 3\}$

C) {–2; –1; 0; 1; 2} D) {–1; 0; 1}

25. [1; 5] va [3; 7] kesmalarning kesishmasini toping.

A) [3; 5] B) [3; 6] C) [1; 7] D) [3; 7]

26. A = {1,2,3}, B = {1,3,5}, C = {1,5,9} to'plamlar berilgan. Shu to'plamlarga universal to'plamni aniqlang.

A) X = {1,2,3,4,5,9} B) X = {1, 3,4,5,9}

C) X = {1,2,3,4} D) X = {1,2,3,5,9}

27. A = {1, 2, 3, 5} va B = {1; 5} to'plamlar berilgan bo'lsa, A/B ni toping.

A) A/B = {2, 3} B) A/B = {1, 5}

C) A/B = {1, 2} D) A/B = {3, 5}

28. A = {2; 5; 7; 9} va B = {2; 4; 7} to'plamlar berilgan bo'lsa, u holda A∩B ni toping.

A) A∩B = {2; 7} B) A∩B = {Æ}

C) A∩B = {5; 9} D) A∩B = {5; 7; 9}

29. A = {2; 5; 7; 9} va B = {2; 4; 7} to'plamlar berilgan bo'lsin, u holda A/B ni toping.

A) A/B = {5; 9} B) A/B = {1; 2}

C) A/B = {Ø} D) A/B = {2; 7}

30. Agar A = {1; 2; 3; 4} va B = {1; 2} to'plamlar berilgan bo'lsa, u holda A/B ni toping.

A) A/B = {3; 4} B) A/B = {1; 2}

C) A/B = {Ø} D) A/B = {1; 2; 3)

31. A = (4; 5] va B = [2; 3) bo'lsa, A\B ni toping.

A) (4; 5] B) (4; 5) C) [2; 3) D) [2; 5]

II B O B. HAQIQIY SONLAR

1-§. Natural sonlar

1. Tub va murakkab sonlar

Narsalarni sanashda ishlatiladigan sonlar *natural sonlar* deyiladi. Barcha natural sonlar hosil qilgan cheksiz to'plam N harfi bilan belgilanadi: $N = \{1, 2, ..., n, ...\}$.

Natural sonlar to'plamida eng katta son (element) mavjud emas, lekin eng kichik son (element) mavjud, u 1 soni. 1 soni faqat 1 ta bo'luvchiga ega (1 ning o'zi). 1 dan boshqa barcha natural sonlar kamida ikkita bo'luvchiga ega (sonning o'zi va 1).

1 dan va o'zidan boshqa natural bo'luvchiga ega bo'lmagan 1 dan katta natural son *tub son* deyiladi. Masalan, 2, 3, 5, 7, 11, 13, 17, 19 sonlar 20 dan kichik bo'lgan barcha tub sonlardir. 1 dan va o'zidan boshqa natural bo'luvchiga ega bo'lgan 1 dan katta natural son *murakkab son* deyiladi. Masalan, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 sonlar 20 dan kichik bo'lgan barcha murakkab sonlardir.

Tub va murakkab sonlarga berilgan ta'riflardan 1 soni na tub, na murakkab son ekanligi ma'lum bo'ladi. Bunday xossaga ega natural son faqat 1 ning o'zidir.

1-m i s o l. Hisoblang:

$27 \times 23 - 24 \times 23 + 21 \times 19 - 18 \times 19 + 17 \times 11 - 14 \times 11$

Y e c h i s h. $27 \times 23 - 24 \times 23 + 21 \times 19 - 18 \times 19 + 17 \times 11 - 4 \times 11 = 23(27 - 24) + 19(21 - 18) + 11(17 - 14) = 23 \times 3 + 19 \times 3 + 11 \times 3 = 3(23 + 19 + 11) = 3 \times 53 = 159$.

2-m i s o l. n ning qanday eng kichik natural qiymatida $2^n + 1$ soni 33 ga qoldiqsiz bo'linadi ?

Y e c h i s h. $n = 5$ da: $2^5 + 1 = 32 + 1 = 33$.

3-m i s o l. 358 ni qanday songa bo'lganda bo'linma 17 va qoldiq 1 bo'ladi ?

Y e c h i s h. $358 = x \times 17 + 1$, $17x = 358 - 1$, $17x = 357$, $x = 357:17$, $x = 21$.

4-m i s o l. 250 va 128 sonlarini tub ko‘paytuvchilarga ajrating va kanonik shaklda yozing

Y e c h i s h.

$$\begin{array}{r|l} 250 & 2 \\ 125 & 5 \\ 5 & 5 \\ 1 & \end{array} \qquad \begin{array}{r|l} 128 & 2 \\ 64 & 2 \\ 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \end{array}$$

Demak, $250 = 2 \times 5 \times 5 = 2 \times 5^2$; $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$.

1.1. Sonlarni tub ko‘paytuvchilarga ajrating:
10; 100; 1 000; 10 000; 100 000; 1 000 000.

1.2. Sonlarni tub ko‘paytuvchilarga ajrating:
250; 300; 340; 3 700; 48 950; 4 725 000.

1.3. Sonlarni kanonik shaklda yozing:

1) 56;	2) 135;	3) 848;	4) 18 880;
5) 72;	6) 36;	7) 1 002;	8) 2 661;
9) 81;	10) 512;	11) 3 125;	12) 6 788;
13) 96;	14) 680;	15) 4 500;	16) 36 363

1.4. Sonlarning umumiy bo‘luvchilari nechta ?
1) 630 va 198; 2) 555 va 444; 3) 156 v*a*144; 4) 850 va 125;
5) 125 va 250; 6) 444 va 222; 7) 111 va 555; 8) 213 va 423.

2. Eng katta umumiy bo‘luvchi. Eng kichik umumiy karrali

$a, b \in N$ sonlarning har biri bo‘linadigan son shu sonlarning *umumiy bo‘luvchisi* deyiladi. Masalan, $a = 12$; $b = 14$ bo‘lsin. Bu sonlarning umumiy bo‘luvchilari 1; 2 bo‘ladi.

$a, b \in N$ sonlar umumiy bo‘luvchilarining eng kattasi shu sonlarning *eng katta umumiy bo‘luvchisi* (EKUB) deyiladi va $B(a; b)$ orqali belgilanadi.

Masalan, $B(12; 14) = 2$.

Agar $B(a; b) = 1$ bo‘lsa, a va b sonlar *o‘zaro tub sonlar* deyiladi.

Masalan, $B(16; 21) = 1$ bo‘lgani uchun 16 va 21 o‘zaro tub sonlardir.

$a, b \in N$ sonlarning *umumiy karralisi* (EKUK) deb, a ga ham, b ga ham bo‘linuvchi natural songa aytiladi.

a va b sonlarning umumiy karralisi ichida eng kichigi mavjud bo‘lib, u a va b sonlarining *eng kichik umumiy karralisi* deyiladi va $K(a; b)$ orqali belgilanadi.

Masalan, $K(6; 8) = 24$.

Natural sonlarning kanonik yoyilmalari bir nechta sonning eng katta umumiy bo‘luvchi va eng kichik umumiy karralilarini topishda ham qo‘llaniladi.

M i s o l. $120 = 2^3 \times 3 \times 5$, $540 = 2^2 \times 3^3 \times 5$ va $600 = 2^3 \times 3 \times 5^2$ bo‘lgani uchun

$B(120; 540; 600) = 2^2 \times 3 = 12$,

$K(120; 540; 600) = 2^3 \times 3^3 \times 5^2 = 6048$ larga ega bo‘lamiz.

1.5. Sonning bo‘luvchilarini toping.

1) 200; 2) 142; 3) 2 432; 4) 2 41432.

1.6. Sonlarning umumiy bo‘luvchilarini toping.

1) 209 va 143; 2) 143 va 2 717;

3) 209 va 2 431; 4) 2 431 va 2 717.

1.7. Sonlarning eng katta umumiy bo‘luvchisini toping.

1) 60 va 45; 2) 88, 64 va 42;

3) 120 va 180; 4) 92, 46 va 36;

5) 121 va 444; 6) 33, 120, 123 va 1 002;

7) 31 va 93; 8) 74, 60, 84 va 480;

9) 50, 75 va 100; 10) 750, 800, 865 va 1 431;

11) 74, 45 va 60; 12) 143, 209, 1 431 va 2 717.

1.8. Quyidagi sonlar o‘zaro tubmi?

1) 15 va 75; 2) 14, 16 va 19;

3) 184 va 167; 4) 63, 170 va 808;

5) 143 va 144; 6) 169 va 8443;

7) 250 va 171; 8) 181 va 121.

1.9. Sonlarning eng kichik umumiy karralisini toping.

1) 84, 42 va 21; 2) 11, 12 va 13;
3) 70, 80 va 90; 4) 50, 125 va 175;
5) 17, 51 va 289; 6) 48, 92 va 75;
7) 10, 21 va 3 600; 8) 100, 150 va 250;
9) 18, 19 va 24; 10) 80, 240 va 360;
11) 33, 36 va 48; 12) 34, 51 va 65.

1.10. Sonlarning eng katta umumiy bo‘luvchisini va eng kichik umumiy karralisini toping (natijani kanonik ko‘rinishda yozing).

1) 24, 32 va 16; 2) $7^2 \times 3$; 46 va 15;
3) 2^3, 3^4 va 7; 4) $3^2 \times 4$; 3×6 va 7×9;
5) 8, 132 va 52; 6) 34, 112 va 133;
7) 122, 15 va 1; 8) 114, 135 va 1004.

1.11. Sonlarning umumiy bo‘luvchisi nechta?

1) 18 va 54; 5) 63 va 72;
2) 42 va 56; 6) 120 va 96;
3) 96 va 92; 7) 102 va 170;
4) 84 va 120; 8) 26, 65 va 45.

1.12. Sonlarning eng katta umumiy bo‘luvchisini toping.

1) 8 804 va 8 604; 2) 6 400 va 8 400;
3) 5 444 va 11 444; 4) 87 999 va 81 000;
5) 880 va 800; 6) 795 va 2 585;
7 5348 va 4 876; 8) 42 628 va 33 124;
9) 187 va 180; 10) 71 004 va 154 452;
11) 2 165 va 3 788; 12) 1 000 va 999.

1.13. Quyidagi sonlar o‘zaro tubmi?

1) 60 va 72; 2) 55 va 71;
3) 732 va 648; 4) 111 va 11 ?

1.14. $B(a; b) \times K(a; b) = a \times b$ ($a \in N$, $b \in N$) tenglikdan foydalanib, quyidagi sonlarning eng kichik umumiy karralisini toping:

1) 822 va 963; 2) 28 va 140; 3) 75 va 1 853;
4) 644 va 904; 5) 56 va 580; 6) 23 va 1 785;
7) 100 va 1 000; 8) 419 va 854; 9) 113 va 9 881;
10) 828 va 963; 11) 897 va 9996; 12) 875 va 1 346.

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1.15. Sonlarning o‘zaro tub ekanligini isbotlang.

1) 123 va 124 007; 2) 547 va 23 147.

1.16. Sonlarning EKUB va EKUKni toping.

1) 420 va 126
2) 549493 va 122433
3) 67283 va 122433
4) 122433 va 221703
5) 476 va 1258
6) 1258 va 21114
7) 1515 va 600
8) 8104 va 5602
9) 5555 va 11110
10) 980 va 100
11) 5345 va 4856
12) 2165 va 3556
13) 5400 va 8400
14) 78999 va 80000
15) 71004 va 154452
16) 42628 va 33124
17) 795 va 2585;
18) 6663 va 887;
19) 875 va 1346;
20) 23 va 1785;
21) 75 va 1853;
22) 28 va 947;
23) 743 va 907;
24) 109 va 1005;
25) 827 va 953;
26) 56 va 953;
27) 419 va 854;
28) 113 va 9881;
29) 821 va 934;
30) 1000 va 999.

2-§. Butun va ratsional sonlar

T a ‘ r i f. Barcha natural, butun manfiy va nol sonlari birgalikda *butun sonlar* to‘plami deyiladi (4-rasm).

4-*rasm.*

Bu yerda natural sonlarga nisbatan qarama-qarshi sonlar barcha butun manfiy sonlardir, masalan, 1 va – 1, 2 va – 2, 3 va – 3,.... qarama-qarshi sonlar barcha butun manfiy sonlardir.

Butun sonlar to‘plamida faqatgina 0 soniga nisbatan qarama-qarshi bo‘lgan son yo‘q:

$$0 = 0 + 0.$$

Butun sonlar to'plamida har doim qo'shish, ayirish, ko'paytirish amallarini bajarish o'rinlidir, lekin bo'lish amali har doim bajarilavermaydi. Chunki bir butun sonni ikkinchi butun songa bo'lganda har doim ham bo'linmada butun son hosil bo'lavermaydi.

Masalan, $7:2 = 3,5$; $9:4 = 2\frac{1}{4}$,... Bu yerda hosil qilingan bo'linmadagi 3, 5; $2\frac{1}{4}$,... sonlari butun sonlar to'plamida mavjud emas. Umuman olganda $m \cdot x = n$, $m \neq 0$ ko'rinishdagi tenglamaning yechimi butun sonlar to'plamida har doim ham mavjud emas, bu tenglama har doim $x = \frac{n}{m}$ ko'rinishdagi yechimga ega bo'lishi uchun kasr tushunchasini kiritish orqali butun sonlar to'plamini kengaytirib, unga barcha manfiy va musbat kasr sonlarni qo'shish kerak. Bu $\left\{-\frac{p}{q}, 0, \frac{p}{q}\right\}$ ko'rinishdagi ratsional sonlar to'plamini hosil qilish kerak deganidir. Shundagina $mx = n$ ko'rinishdagi tenglamalar har doim yechimga ega bo'ladi. Bu yerda r va q lar natural sonlardir. Yuqoridagi mulohazalarga ko'ra ratsional songa quyidagicha ta'rif berish mumkin: $\frac{p}{q}$ *ko'rinishdagi qisqarmas kasrga ratsional son deyiladi.*

Endi kasr tushunchasini kiritish uchun foydalaniladigan misollarni ko'rib o'taylik.

Agar bir metr uzunlikdagi yog'ochni o'zaro teng ikki bo'lakka bo'linsa, u holda bo'laklarning har birining uzunligi ana shu yog'och uzunligining yarmiga teng bo'ladi va uni $\frac{1}{2}$ kabi yoziladi. Agar ana shu bir metr uzunlikdagi yog'ochni o'zaro teng uch bo'lakka bo'linsa, u holda bo'laklardan har birining uzunligi shu yog'och uzunligining uchdan biriga teng bo‘ladi va uni $\frac{1}{3}$ kabi yoziladi. Xuddi shuningdek, $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}...$

Agar bir metr uzunlikdagi yog‘ochni teng uch bo‘lakka bo‘lib, undan ikki qismini oladigan bo‘lsak, olingan uzunlikni $\frac{2}{3}$ kabi yoziladi.

Agar ana shu yog‘ochni to‘rt bo‘lakka bo‘lib, undan uch qismini olsak, olingan qism uzunlikni $\frac{3}{4}$ kabi ifodalanadi. Yuqorida qilingan mulohazalarga asoslanib kasr tushunchasining ta’rifini quyidagicha berish mumkin.

T a ‘ r i f. *Butun sonning o‘zaro teng bo‘lgan ma’lum bir ulushi shu sonning kasri deyiladi.*

Yuqorida $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}$ kasr sonlarni hosil qildik. Berilgan narsalarni yoki butun sonni qancha teng qismga bo‘linganligini ko‘rsatuvchi sonni kasrning *maxraji*, shunday qismdan nechtasi olinganligini ko‘rsatuvchi sonni kasrning *surati* deyiladi. Maxraj kasr chizig‘ining ostida, surat esa kasr chizig‘ining ustiga yoziladi.

Umumiy holda kasrni $\frac{p}{q}$ ko‘rinishda ifodalanadi. Bunda *r* – kasrning surati, *q* – kasrning maxraji deb yuritiladi. $\frac{p}{q}$ ko‘rinishdagi kasrlarga qarama-qarshi kasrlarni $-\frac{p}{q}$ ko‘rinishda ifodalanadi.

Koordinatalar o‘qida $-\frac{p}{q}$ ko‘rinishdagi kasrlar nol sonidan chapda joylashgan bo‘ladi. Biz butun sonlar to‘plamini kengaytirish orqali $-\frac{p}{q}$ va $\frac{p}{q}$ ko'rinishdagi kasrlarni hosil qildik. Natijada koordinatalar o'qida $\{-\frac{p}{q}, 0, \frac{p}{q}\}$ ko'rinishdagi sonlar to'plami hosil bo'ldi.

Bunday to'plam *ratsional sonlar to'plami* deb ataladi. Agar ratsional sonlar to'plamidagi $-\frac{p}{q}$ va $\frac{p}{q}$ kasrlarning maxrajlari $q = 1$ desak, bizga ma'lum bo'lgan butun sonlar to'plami hosil bo'ladi. Bundan ko'rinadiki, butun sonlar ratsional sonlar to'plamining xususiy bir holi ekan. Ratsional sonlar to'plami bilan koordinata to'g'ri chizig'i nuqtalari orasida o'zaro bir qiymatli moslik o'rnatish mumkinmi, degan savol tug'ilishi tabiiydir. Bu savolga quyidagicha javob berishimiz mumkin, aksincha, har bir nuqtaga bittadan ratsional soni mos keltirish mumkin emas.

Kasrlar uch xil bo'ladi:

1. To'g'ri kasrlar, 2. Noto'g'ri kasrlar, 3. O'nli kasrlar.

1. Agar kasrning surati uning maxrajidan kichik bo'lsa, bunday kasrlarni *to'g'ri kasrlar* deyiladi.

Masalan: $\frac{1}{2}, \frac{3}{4}, \frac{1}{6} ....$

2. Agar kasrning surati uning maxrajidan katta bo'lsa, bunday kasrlarni *noto'g'ri kasrlar* deyiladi.

Masalan, $\frac{5}{2}, \frac{7}{4}, \frac{17}{5} ....$

3. Agar kasrning maxraji bir va nol sonlaridan iborat bo'lsa, bunday kasrlarni *o'nli kasrlar* deyiladi.

Masalan, $\frac{1}{10} = 0{,}1; \frac{1}{100} = 0{,}01; ....$

Kasr tushunchasi kiritilganidan keyin kasrlarning tengligi tushunchasi kiritiladi. Bu tushunchani o'quvchilarga quyidagicha tushuntirish mumkin.

Faraz qilaylik, bizga bir metr uzunlikdagi kesma berilgan bo‘lsin. Agar shu kesmani teng ikkiga bo‘lsak, hosil bo‘lgan har bir kesmaning uzunligi $\frac{1}{2}$ kabi kasr bilan ifodalanadi. Endi bo‘lingan hap bir kesmani yana ikkiga bo‘lsak har bir kesmaning uzunligi $\frac{1}{4}$ kasr bilan ifodalanadi. Ana shu teng to‘rtga bo‘lingan kesmalardan ikkitasining uzunligi $\frac{2}{4}$ kasr bilan ifodalanadi. Bu esa butun kesma uzunligining teng ikkiga bo‘lgandagi $\frac{1}{2}$ kasr bilan ifodalangan qiymatiga tengdir. Shuning uchun $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}=\ldots$. Bundan ko‘rinadiki, $\frac{1}{2}$ va $\frac{2}{4}$ kasrlarning qiymatlari teng bo‘lib, ularni ifoda qilish har xildir.

O‘quvchilarga kasrlarning tengligi tushunchasini tushuntirilganidan so‘ng kasrning quyidagi xossalarini ifoda qilish mumkin.

1-x o s s a. Agar kasrning surat va maxrajini bir xil songa ko‘paytirilsa, kasrning qiymati o‘zgarmaydi: $\frac{p}{q}=\frac{p}{q}\cdot\frac{n}{n}$.

M i s o l.

1) $\frac{2}{5}=\frac{2}{5}\cdot\frac{2}{2}=\frac{4}{10}$;

2) $\frac{3}{7}=\frac{3}{7}\cdot\frac{4}{4}=\frac{12}{28}$;

3) $1=\frac{1}{1}=\frac{1}{1}\cdot\frac{4}{4}=\frac{4}{4}=\frac{4\cdot 25}{4\cdot 25}=\frac{100}{100}$.

2-x o s s a. Agar kasrning surat va maxrajini bir xil songa bo‘linsa, kasrning qiymati o‘zgarmaydi. $\frac{p:n}{q:n}=\frac{p}{q}$.

Bu yerda $n>1$ bo‘lishi kerak.

M i s o l . 1) $\frac{4}{8}=\frac{4}{4\cdot 2}=\frac{1}{2}$; 2) $\frac{15}{3}=\frac{3\cdot 5}{3}=\frac{5}{1}=5$.

3-x o s s a. Agar kasrning surat va maxrajidagi sonlar umumiy bo'luvchilarga ega bo'lmasa, u holda bunday kasr qisqarmas kasr bo'ladi. Masalan, $\frac{5}{7}, \frac{4}{5}, \frac{17}{19}, \ldots$ qisqarmas kasrlardir, chunki 5 va 7, 4 va 5, 17 va 19 sonlari o'zaro umumiy bo'luvchilarga ega emas.

1. Kasrlarni taqqoslash

1. Kasrlarni o'zaro taqqoslash uchun berilgan kasrlarni o'zaro bir xil maxrajli kasrlar holiga keltirish kerak, so'ngra ulardan qaysi birining surati katta bo'lsa, o'sha kasrning qiymati katta bo'ladi.

Masalan: $\frac{3}{4}$ va $\frac{2}{5}$; $\frac{3\cdot 5}{4\cdot 5}=\frac{15}{20}$ va $\frac{2\cdot 4}{5\cdot 4}=\frac{8}{20}, \frac{15}{20}>\frac{8}{20}$, shuning uchun $\frac{3}{4}>\frac{2}{5}$. Bu yerda kasrning surati va uning maxrajini bir xil songa ko'paytirilsa, kasrning qiymati o'zgarmaydi degan xossadan foydalandik.

2. Suratlari bir xil va maxrajlari har xil bo'lgan kasrlardan qaysi birining maxraji katta bo'lsa, o'sha kasr kichik bo'ladi. Qaysi birining maxraji kichik bo'lsa, o'sha kasr katta bo'ladi.

Masalan: $\frac{4}{15}$ va $\frac{4}{12}$ lar uchun $\frac{4}{15}>\frac{4}{12}$.

2. Kasrlarni qo'shish

Faraz qilaylik, bizga AB kesma berilgan bo'lsin, biz uni teng yettiga bo'laylik, ulardan $AC=\frac{1}{7}$, $CD=\frac{3}{7}$, $AD=\frac{4}{7}$ bo'lsin, u holda AD kesmaning qiymati AC va CD kesmalar uzunliklarining yig'indisiga teng bo'ladi, ya'ni $AD=AC+CD$. Shu bilan birga $\frac{1}{7}+\frac{3}{7}=\frac{4}{7}$.

Yuqoridagi mulohazaga ko'ra quyidagi qoidani yozishimiz mumkin.

I. Maxrajlari bir xil bo'lgan kasrlarni qo'shish uchun ularning suratlarini o'zaro qo'shib, maxrajlaridan bittasini yozish kifoya:

$$\frac{p}{q}+\frac{r}{q}=\frac{p+r}{q};\ \frac{3}{5}+\frac{1}{5}=\frac{3+1}{5}=\frac{4}{5}.$$

II. Maxrajlari har xil bo'lgan kasrlarni qo'shish uchun ularni eng kichik umumiy maxrajga keltirib, bir xil maxrajli kasrlarni qo'shish qoidasidan foydalanish kifoya:

1) $\frac{2}{5}+\frac{3}{7}=\frac{2\cdot 7}{5\cdot 7}+\frac{3\cdot 5}{7\cdot 5}=\frac{14}{35}+\frac{15}{35}=\frac{14+15}{35}=\frac{29}{35};$

2) $\frac{3}{4}+\frac{1}{6}=\frac{3\cdot 6}{4\cdot 6}+\frac{1\cdot 4}{6\cdot 4}=\frac{18}{24}+\frac{4}{24}=\frac{18+4}{24}=\frac{22}{24}=\frac{11}{12}.$

Umumiy holda: $\frac{p}{q}+\frac{r}{s}=\frac{p\cdot s}{q\cdot s}+\frac{r\cdot q}{s\cdot q}=\frac{ps+rq}{sq}.$

III. Yig'indida butun son chiqadigan kasrlarni qo'shish quyidagicha amalga oshiriladi:

1) $\frac{3}{4}+\frac{1}{4}=\frac{3+1}{4}=\frac{4}{4}=1;$

2) $\frac{1}{8}+\frac{7}{8}=\frac{1+7}{8}=\frac{8}{8}=1;$

3) $\frac{2}{4}+\frac{1}{2}=\frac{2}{4}+\frac{2\cdot 1}{2\cdot 2}=\frac{2+2}{4}=\frac{4}{4}=1.$

IV. Butun sonni kasrga qo'shish:

1) $3+\frac{1}{2}=3\frac{1}{2};$

2) $3+\frac{1}{2}=\frac{3}{1}+\frac{1}{2}=\frac{3\cdot 2}{1\cdot 2}=\frac{6+1}{2}=\frac{7}{2}=3\frac{1}{2};$

V. Aralash sonni kasrga qo'shish:

$$3\frac{3}{4}+\frac{1}{2}=3+\left(\frac{3}{4}+\frac{1}{2}\right)=3+\left(\frac{3}{4}+\frac{1\cdot 2}{2\cdot 2}\right)=3+\left(\frac{3}{4}+\frac{2}{4}\right)=$$
$$=3+\left(\frac{3+2}{4}\right)=3+\frac{5}{4}=4+\frac{1}{4}=4\frac{1}{4}.$$

VI. Aralash sonni aralash songa qo'shish:

$$2\frac{1}{3}+3\frac{1}{2}=(2+3)+\left[\frac{1}{3}+\frac{1}{2}\right]=5+\left[\frac{1\cdot 2}{3\cdot 2}+\frac{1\cdot 3}{2\cdot 3}\right]=$$

$$=5+\frac{2+3}{6}=5+\frac{5}{6}=5\frac{5}{6}.$$

Qo'shish qonunlari

1. Kasr qo'shiluvchilarning o'rni almashgani bilan yig'indi kasr sonning qiymati o'zgarmaydi :

$$\frac{a}{q}+\frac{b}{q}=\frac{a+b}{q}=\frac{b+a}{q}=\frac{b}{q}+\frac{a}{q}.$$

M i s o l.

$$\frac{3}{5}+\frac{1}{5}=\frac{3+1}{5}=\frac{1+3}{5}=\frac{1}{5}+\frac{3}{5}.$$

2. Kasr sonlarda qo'shish amaliga nisbatan gruppalash qonuni o'rinlidir:

$$\left(\frac{\grave{a}}{q}+\frac{b}{q}\right)+\frac{c}{q}=\frac{a}{q}+\left(\frac{b}{q}+\frac{c}{q}\right).$$

I s b o t i.

$$\left(\frac{a}{q}+\frac{b}{q}\right)+\frac{c}{q}=\frac{a+b}{q}+\frac{c}{q}=\frac{(a+b)+c}{q}=\frac{c+(b+a)}{q}=$$

$$=\frac{a}{q}+\frac{b+c}{q}=\frac{a}{q}+\left(\frac{b}{q}+\frac{c}{q}\right).$$

M i s o l.

$$\frac{1}{7}+\frac{3}{5}+\frac{2}{7}=\left(\frac{1}{7}+\frac{2}{7}\right)+\frac{3}{5}=\frac{1+2}{7}+\frac{3}{5}=\frac{3}{7}+\frac{3}{5}=$$

$$=\frac{3\cdot 5}{7\cdot 5}+\frac{3\cdot 7}{5\cdot 7}=\frac{15+21}{35}=\frac{36}{35}.$$

3. Kasrlarni ayirish

1. Faraz qilaylik, bizga AB kesma berilgan bo‘lib, u teng 7 bo‘lakka bo‘lingan bo‘lsin.

Ulardan $AC = \frac{1}{7}$, $CD = \frac{3}{7}$, $AD = \frac{4}{7}$ bo‘lsin. CD kesmaning qiymati $CD = AD - AC$ bo‘ladi, u holda $\frac{4}{7} - \frac{1}{7} = \frac{3}{7}$ tenglik o‘rinli.

2. Bobur ikki mashinadagi yukni $\frac{5}{7}$ soatda tushirdi. U birinchi mashinadagi yukni $\frac{3}{7}$ soatda tushirib bo‘ldi. Bobur ikkinchi mashinadagi yukni necha soatda tushirgan? $\frac{5}{7} - \frac{3}{7} = \frac{2}{7}$. Topilgan natijani to‘g‘riligini tekshirish qo‘shish amali orqali amalga oshiriladi:

$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}.$$

Endi kasrlarni ayirish uchun chiqarilgan quyidagi qoidalarni ko‘rib chiqamiz:

I. Maxrajlari bir xil bo‘lgan kasrlarni ayirish uchun ularning suratlarini o‘zaro ayirib, maxrajlardan bittasini maxraj qilib yozish kifoya.

1) $\frac{3}{5} - \frac{1}{5} = \frac{3-1}{5} = \frac{2}{5}$; 2) $\frac{4}{7} - \frac{3}{7} = \frac{4-3}{7} = \frac{1}{7}$.

II. Maxrajlari har xil bo‘lgan kasrlarni ayirish uchun ularni eng kichik umumiy maxrajga keltirib, bir xil maxrajli kasrlarni ayirish qoidasidan foydalaniladi:

$$\frac{3}{4} - \frac{2}{7} = \frac{3 \cdot 7}{7 \cdot 4} - \frac{2 \cdot 4}{7 \cdot 4} = \frac{21-8}{28} = \frac{13}{28}.$$

Umumiy holda:

$$\frac{p}{q}-\frac{r}{s}=\frac{p\cdot s}{q\cdot s}-\frac{r\cdot q}{s\cdot q}=\frac{ps-rq}{sq}.$$

III. Butun sondan kasrni ayirish:

1- u s u l. $4-\frac{2}{3}=\frac{4}{1}-\frac{2}{3}=\frac{4\cdot 3}{1\cdot 3}-\frac{2}{3}=\frac{12}{3}-\frac{2}{3}=\frac{12-2}{3}=\frac{10}{3}.$

2- u s u l. $4-\frac{2}{3}=3+\left(\frac{3}{3}-\frac{2}{3}\right)=3+\left(\frac{3-2}{3}\right)=3+\frac{1}{3}=3\frac{1}{3}.$

IV. Kasrdan butun sonni ayirish:

$$\frac{3}{7}-2=-\left(2-\frac{3}{7}\right)=-\left(\frac{2\cdot 7}{1\cdot 7}-\frac{3}{7}\right)=-\left(\frac{14}{7}-\frac{3}{7}\right)=$$

$$=-\frac{14-3}{7}=-\frac{11}{7}=-1\frac{4}{7}.$$

V. Butun sondan aralash sonni ayirish:

$$5-2\frac{1}{4}=4\frac{5}{5}-2\frac{1}{4}=(4-2)+\left(\frac{5}{5}-\frac{1}{4}\right)=2+\left(\frac{5\cdot 4}{5\cdot 4}-\frac{1\cdot 5}{4\cdot 5}\right)=$$

$$=2+\frac{20-5}{20}=2+\frac{15}{20}=2+\frac{3}{4}=2\frac{3}{4}.$$

VI. Aralash sondan butun sonni ayirish:

1) $3\frac{3}{4}-2=(3-2)+\left(\frac{3}{4}-0\right)=1+\frac{3}{4}=1\frac{3}{4};$

2) $3\frac{3}{4}-2=\frac{15}{4}-\frac{2}{1}=\frac{15}{4}-\frac{2\cdot 4}{1-4}=\frac{15-8}{4}=\frac{7}{4}=1\frac{3}{4}.$

VII. 1 sonidan kasr sonni ayirish:

$$1-\frac{3}{4}=\frac{1}{1}-\frac{3}{4}=\frac{1\cdot 4}{1\cdot 4}-\frac{3}{4}=\frac{4-3}{4}=\frac{1}{4}.$$

VIII. 1 sonidan aralash sonni ayirish:

1 -u s u l.

$$1-3\frac{1}{2}=-\left(3\frac{1}{2}-1\right)=\left[(3-1)+\left(\frac{1}{2}-0\right)\right]=-\left(2+\frac{1}{2}\right)=-2\frac{1}{2}.$$

2-u s u l. $1-3\frac{1}{2}=\frac{1}{1}-\frac{7}{2}=\frac{1\cdot 2}{1\cdot 2}-\frac{7}{2}=\frac{2-7}{2}=\frac{-5}{2}=-2\frac{1}{2}.$

4. Kasrlarni ko'paytirish

I. Kasrni butun songa ko'paytirish uchun shu butun sonni kasrning suratiga ko'paytirish kifoya:

1) $\frac{5}{17}\cdot 3=\frac{5\cdot 3}{17}=\frac{15}{17}$; 2) $\frac{2}{9}\cdot(-4)=\frac{2\cdot(-4)}{9}=-\frac{8}{9}.$

Ko'paytirish qoidasiga ko'ra $\frac{5}{17}\cdot 3,\ \frac{2}{9}\cdot(-4)$ ifodalarni quyidagicha yozish mumkin:

1) $\frac{5}{17}\cdot 3=\frac{5}{17}+\frac{5}{17}+\frac{5}{17}=\frac{5+5+5}{17}=\frac{15}{17};$

2) $\frac{2}{9}\cdot(-4)=-\frac{2}{9}\cdot 4=-\frac{2}{9}-\frac{2}{9}-\frac{2}{9}-\frac{2}{9}=-\left(\frac{2}{9}+\frac{2}{9}+\frac{2}{9}+\frac{2}{9}\right)=-\frac{8}{9}.$

II. Aralash sonni butun songa ko'paytirish uchun aralash sonni noto'g'ri kasrga aylantirib, butun sonni uning suratiga ko'paytirish kifoya:

1. *a*) $2\frac{1}{2}\cdot 3=\frac{2\cdot 2+1}{2}\cdot 3=\frac{5}{2}\cdot 3=\frac{5\cdot 3}{2}=\frac{15}{2}=7\frac{1}{2};$

b) $2\frac{1}{2}\cdot 3=2\frac{1}{2}+2\frac{1}{2}+2\frac{1}{2}=6+\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)=$

$=6+\left(\frac{1+1+1}{2}\right)=6+\frac{3}{2}=7\frac{1}{2};$

2. *a*) $3\frac{3}{4}\cdot(-2)=\frac{3\cdot 4+3}{4}\cdot(-2)=\frac{15}{4}\cdot(-2)=\frac{15\cdot(-2)}{4}=$

$=-\frac{30}{4}=-\frac{15\cdot 2}{2\cdot 2}=-\frac{15}{2}=-7\frac{1}{2}.$

b) $3\frac{3}{4}(-2)=-3\frac{3}{4}\cdot 2=-3\frac{3}{4}+\left(-3\frac{3}{4}\right)=-\left(3\frac{3}{4}+3\frac{3}{4}\right)=$

$$= -\left(6+\frac{3+3}{4}\right) = -\left(6+\frac{6}{4}\right) = -7\frac{1}{2}.$$

III. Kasrni kasrga ko‘paytirish uchun ularning suratlarini suratlariga va maxrajlarini maxrajlariga ko‘paytirish kifoya:

$$\frac{p}{q}\cdot\frac{s}{r} = \frac{p\cdot s}{q\cdot r}.$$

M i s o l.

1) $\frac{2}{7}\cdot\frac{5}{9} = \frac{2\cdot 5}{7\cdot 9} = \frac{10}{63}$; 2) $\frac{7}{1}\cdot\frac{2}{5} = \frac{7\cdot 2}{1\cdot 5} = \frac{14}{5}$.

IV. Aralash sonlarni o‘zaro ko‘paytirish uchun ularning har birini noto‘g‘ri kasrga aylantirib, suratlarini suratlariga va maxrajlarini maxrajlariga o‘zaro ko‘paytirish kifoya:

1) $2\frac{1}{3}\cdot 4\frac{2}{5} = \frac{7}{3}\cdot\frac{22}{5} = \frac{7\cdot 22}{3\cdot 5} = \frac{154}{15} = 10\frac{4}{15}$;

2) $7\frac{2}{3}\cdot 2\frac{1}{2} = \frac{23}{3}\cdot\frac{5}{2} = \frac{23\cdot 5}{3\cdot 2} = \frac{115}{6} = 19\frac{1}{6}$.

Kasrlarni ko‘paytirish o‘rin almashtirish, gruppalash va taqsimot qonunlariga bo‘ysunadi.

1. Kasrlarni ko‘paytirishda ko‘paytuvchilarning o‘rni almashgani bilan ko‘paytmaning qiymati o‘zgarmaydi:

1) $\frac{2}{3}\cdot\frac{4}{7} = \frac{2\cdot 4}{3\cdot 7} = \frac{8}{21}$;

2) $\frac{4}{7}\cdot\frac{2}{3} = \frac{4\cdot 2}{7\cdot 3} = \frac{8}{21}$.

2. Kasrlarni ko‘paytirishda ularni guruhlab ko‘paytirilsa, ko‘paytmaning qiymati o‘zgarmaydi:

1) $\left(\frac{2}{3}\cdot\frac{4}{5}\right)\cdot\frac{3}{7} = \left(\frac{2\cdot 4}{3\cdot 5}\right)\cdot\frac{3}{7} = \frac{8}{15}\cdot\frac{3}{7} = \frac{24}{105}$;

2) $\left(\frac{3}{7}\cdot\frac{2}{3}\right)\cdot\frac{4}{5} = \left(\frac{3\cdot 2}{7\cdot 3}\right)\cdot\frac{4}{5} = \frac{6}{21}\cdot\frac{4}{5} = \frac{6\cdot 4}{21\cdot 5} = \frac{24}{105}$.

3. Kasrlarni ko'paytirishda ularga taqsimot qonunini tatbiq qilinsa, ko'paytmaning qiymati o'zgarmaydi:

$$\left(\frac{a+b}{c}\right)\cdot\frac{p}{q}=\frac{(a+b)p}{cq}=\frac{ap+bp}{cq}.$$

1) $\left(\frac{4+3}{9}\right)\cdot\frac{4}{5}=\frac{(4+3)\cdot 4}{9\cdot 5}=\frac{4\cdot 4+3\cdot 4}{9\cdot 5}=\frac{16+12}{45}=\frac{28}{45};$

2) $\left(\frac{7+2}{1}\right)\cdot\frac{2}{3}=\frac{(7+2)\cdot 2}{1\cdot 3}=\frac{7\cdot 2+2\cdot 2}{1\cdot 3}=\frac{14+4}{3}=\frac{18}{3}.$

5. Kasrlarni bo'lish

Bizga butun sonlar mavzusidan ma'lumki, ikkita butun sonni o'zaro bo'lish uchun birinchisini ikkinchi sonning teskarisiga ko'paytirish kerak edi. Xuddi shuningdek, ikki kasr sonni ham o'zaro bo'lish uchun birinchi kasrni ikkinchi kasrning teskarisiga ko'paytirish kerak, masalan, $\frac{15}{27}:\frac{2}{3}=\frac{15}{27}\cdot\frac{3}{2}=\frac{45}{54}$. Bu qoidani quyidagi masala orqali o'quvchilarga tushuntirish maqsadga muvofiqdir.

M a s a l a. $\frac{6}{7}$ bo'lagi (qismi) 30 ga teng bo'lgan sonni toping.

Y e c h i s h. Noma'lum sonni x bilan belgilasak, u holda masala shartini quyidagicha yozish mumkin: $\frac{6}{7}\cdot x=30$, chunki sonning bo'lagi ko'paytirish amali yordamida topiladi. Bu tenglik bunday yechiladi: $\frac{1}{7}x=30:6=5$. Bundan $x = 5 \cdot 7 = 35$ bo'ladi, Demak, izlanayotgan son 35 ekan.

M a s a l a. Futbol maydoni yuzining $\frac{3}{4}$ qismi o'yin o'ynash uchun tayyor holga keltirildi. Bu 960 m^2 ni tashkil qiladi. Futbol maydonning yuzi qancha?

Y e c h i s h. Futbol maydonning yuzini x bilan belgilasak, shartga ko'ra bu maydonning $\frac{3}{4}$ qismi 960 m^2 edi, shuning uchun $\frac{3}{4}x = 960$ tenglik o'rinli bo'ladi. x ni topish uchun tenglamaning ikkala qismini $\frac{3}{4}$ ga bo'lish kerak. Demak, $x = 960 : \frac{3}{4} = 960 \cdot \frac{4}{3} = 320 \cdot 4 = 1280$ m^2. Futbol maydonining yuzi 1280 m^2 ekan.

Berilgan kasrning qiymati bo'yicha sonning o'zini topishda ham sonning kasrni topishdek, turli hollarni ko'rib o'tish maqsadga muvofiqdir. Sonni kasrga bo'lish ta'rifi butun sonlarni bo'lish ta'rifidek ifodalanadi. Bu qoidani o'quvchilarga alohida ta'kidlab tushuntirish maqsadga muvofiqdir. Shundan keyin kasrlarni bo'lishga doir quyidagi hollarni ko'rib chiqish foydalidir.

1. Kasrni butun songa bo'lish uchun kasrni o'z holicha, butun sonni esa teskari yozib, ularni o'zaro ko'paytirish kifoya:

$$\frac{5}{7} : 4 = \frac{5}{7} \cdot \frac{1}{4} = \frac{5 \cdot 1}{7 \cdot 4} = \frac{5}{28}.$$

2. Aralash sonni butun songa bo'lish uchun aralash sonni noto'g'ri kasrga aylantirib, so'ngra bo'lish kasrni butun songa bo'lishdek bajariladi:

$$2\frac{5}{7} : 4 = \frac{19}{7} : 4 = \frac{19}{7} \cdot \frac{1}{4} = \frac{19 \cdot 1}{7 \cdot 4} = \frac{19}{28}.$$

3. Butun sonni aralash songa bo'lish uchun butun sonni o'z holicha yozib aralash sonni noto'g'ri kasrga aylantirib, ularni o'zaro ko'paytirish kerak:

$$4 : 1\frac{3}{5} = 4 : \frac{8}{5} = 4 \cdot \frac{5}{8} = \frac{20}{8} = 2\frac{4}{8} = 2\frac{1}{2}.$$

4. Aralash sonni aralash songa bo'lish uchun ularning hap birini noto'g'ri kasrlarga aylantirib, so'ngra bo'lishni ikki kasrni o'zaro bo'lish qoidasiga ko'ra bajariladi:

$$2\frac{3}{5}:3\frac{2}{7}=\frac{13}{5}:\frac{23}{7}=\frac{13}{5}\cdot\frac{7}{23}=\frac{13\cdot 7}{5\cdot 23}=\frac{91}{115}.$$

6. O'nli kasrlar va ular bilan to'rt amalni bajarish

O'nli kasr tushunchasi XV asrda Samarqandlik olim Ali Qushchi tomonidan kiritilgan. U o'zining 1427-yilda yozgan «Hisobot san'atiga kalit», «Arifmetika kaliti» nomli kitoblarida o'nli kasr tushunchasidan foydalangan.

T a ' r i f. *Maxraji o'n yoki uning darajalari*dan *iborat bo'lgan kasr o'nli kasr deyiladi.*

O'nli kasrlarni bunday belgilash qabul qilingan:

$$\frac{1}{10}=0,1;\frac{1}{100}=0,01;\frac{1}{1000}=0,001;\frac{3}{10}=0,3;\frac{3}{1000}=0,003;2,15=2\frac{15}{100}.$$

O'nli kasrlarni maxrajsiz yozilganda verguldan o'ngdagi birinchi xonadagi raqam o'ndan birlarni, ikkinchi xonadagilari esa yuzdan birlarni va hokazolarni bildiradi. Masalan, 6,732 o'nli kasrda verguldan keyingi sonlarni turgan o'rniga qarab kasr ko'rinishda quyidagicha ifodalash mumkin: $\frac{7}{10};\frac{3}{100};\frac{2}{1000}$.

O'nli kasrlar uchun quyidagi qoidalar o'rinlidir:

1. Har bir o'nli kasr o'zidan oldingi o'nli kasrga nisbatan o'n marta kattadir. Masalan, $0,001=\frac{1}{1000}; 0,01=\frac{1}{100}; 0,1=\frac{1}{10}$.

2. O'nli kasrlarning maxrajlari 10 ning butun ko'rsatkichli darajalaridan, suratlari esa bir xonali sonlardan iborat kasrlarning yig'indisi shaklida ifodalash mumkin.

7. O'nli kasrlarni qo'shish va ayirish

Bu mavzu materialini bayon qilishdan oldin o'qituvchi o'quvchilarga maxrajlari har xil bo'lgan oddiy kasrlarni umumiy maxrajlarga keltirib qo'shish va ayirish haqidagi tushunchani misollar yordamida ko'rsatishi, so'ngra o'nli kasrlarni qo'shish va ayirish haqidagi nazariy va amaliy bilimlarni berishi maqsadga muvofiqdir.

1-q o i d a. *O'nli kasrlarni qo'shish uchun bir xil xonalari o'zaro butun sonlar kabi qo'shilib, yig'indida kasrlardagi vergulning tagiga to'g'ri keltirib butun qismi ajratiladi.*

M i s o l.

$$\begin{array}{r} 25{,}382 \\ +\ \ 7{,}200 \\ \hline 32{,}582 \end{array}$$

2-q o i d a. *O'nli kasrlarni ayirish uchun kamayuvchining tagiga ayirluvchining verguliga to'g'rilab, o'rin qiymati bir xil bo'lgan raqamlar bir-birini ostiga yozib ayriladi, so'ngra ayirmani butun qismi vergul bilan ajratiladi.*

M i s o l.

1) $\begin{array}{r} 14{,}273 \\ -\ \ 5{,}040 \\ \hline 9{,}233 \end{array}$ 2) $\begin{array}{r} 27{,}100 \\ -\ \ 3{,}236 \\ \hline 23{,}864 \end{array}$ 3) $27{,}1 - 3{,}275 = ?$

O'nli kasrlarni ayirish jarayonida quyidagi hollar bo'lishi mumkin: ayiriluvchidagi kasr xonalarining soni kamayuvchidagi kasr xonalaridan ko'p, kamayuvchi va ayiriluvchi o'nli kasrlardagi kasr xonalari soni bir xil, butun sondan o'nli kasrni ayirish, o'nli kasrdan butun sonni ayirish.

8. O'nli kasrlarni ko'paytirish

O'nli kasrlarni o'zaro ko'paytirishni oddiy kasrlarni ko'paytirish qoidasiga asoslangan holda tushuntirishi lozim.

M i s o l.

$$3{,}2\cdot 0{,}12 = 3\frac{2}{10}\cdot\frac{12}{100} = \frac{32}{10}\cdot\frac{12}{100} = \frac{32\cdot 12}{10\cdot 100} = \frac{384}{1000} = 0{,}384.$$

Ko'paytma kasrning maxrajida nechta nol bo'lsa, uni maxrajsiz yoziladigan o'nli kasrga aylantirganda shuncha kasr xonasi bo'ladi.

1) $3,2 \cdot 0,12 = \frac{384}{1000} = 0,384;$

2) $2,7 \cdot 1,3 = 2\frac{7}{10} \cdot 1\frac{3}{10} = \frac{27}{10} \cdot \frac{13}{10} = \frac{351}{10 \cdot 10} = \frac{351}{100} = 3.51.$

Ko‘rib o‘tilgan misollar asosida quyidagi qoidalar tushuntiriladi.

1-q o i d a. *O‘nli kasrlarni o‘zaro ko‘paytirish uchun ularning suratlarini suratlariga va maxrajlarini maxrajlariga ko‘paytirib, ko‘paytuvchi bilan ko‘payuvchida jami nechta kasr xonasi bo‘lsa, ko‘paytmada shuncha xona ajratiladi.* (Bu gap oddiy kasr shakdda yozilgan o‘nli kasr haqida aytilgan.)

Masalan,

$$3,4 \cdot 0,25 = 3\frac{4}{10} \cdot \frac{25}{100} = \frac{34}{10} \cdot \frac{25}{100} = 0,85.$$

2-q o i d a. *O‘nli kasrlarni o‘zaro ko‘paytirish uchun ularning verguliga e’tibor bermay, butun sonlar kabi ko‘paytirib, ko‘payuvchi va ko‘paytuvchida hammasi nechta kasr xonasi bo‘lsa, ko‘paytmaning o‘ng* tomonidan *boshlab sanab shuncha raqamni vergul bilan ajratib qo‘yiladi.*

1)
```
   3,021
×  2,51
--------
    3021
+  15105
   6042
--------
 7,58271
```

2)
```
     7,124
×    3,213
----------
     21372
      7124
+   14248
   21372
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 22,889412
```

O‘nli kasrlarni o‘zaro ko‘paytirishda ko‘paytirishning ko‘payuvchidagi yig‘indisiga nisbatan tarqatish qonunini qo‘llanishga asoslangan mulohazalarni ham olib borish foydali, buni quyidagicha sxema orqali ham ko‘rsatish mumkin. Masalan, 2,37 ni 2 ta birlik, 3 ta o‘ndan bir, 7 ta yuzdan birning yig‘indisi shaklida yozish mumkin. Yig‘indini biror songa ko‘paytirish uchun har bir qo‘shiluvchini shu songa ko'paytirish va hosil bo'lgan ko'paytmalarni qo'shish, ya'ni 2 birlikni 9 ga, 3 ta o'ndan birni 9 ga, 7 ta yuzdan birni 9 ga ko'paytirib, ko'paytmalarni o'zaro qo'shish kifoya.

$$2,37\cdot 9=9\cdot\left(1+1+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{100}+\frac{1}{100}+\frac{1}{100}+\frac{1}{100}+\frac{1}{100}+\frac{1}{100}+\frac{1}{100}\right)=$$

$$=9+9+\frac{9}{10}+\frac{9}{10}+\frac{9}{10}+\frac{9}{100}+\frac{9}{100}+\frac{9}{100}+\frac{9}{100}+\frac{9}{100}+\frac{9}{100}+\frac{9}{100}=\frac{2133}{100}=21,33.$$

O'nli kasrlarni 10 ning butun ko'rsatkichli darajalariga ko'paytirishni alohida ko'rib o'tish lozim, ya'ni o'nli kasrni 10 ga, 100 ga, 1000 ga va hokazolarga ko'paytirish uchun bu kasrda vergulni 1, 2, 3,... raqam o'ngga surish kerak. O'nli kasrlarni 0,1, 0,01, 0,001 ga va hokazolarga ko'paytirish uchun bu kasrlarda vergulni 1, 2, 3,... raqam chapga surish kifoya.

Masalan: 1) 3,7 · 100 = 3,70 · 100 = 370. Bu misolni quyidagicha tushuntirish mumkin: 3,7 ni 100 ga ko'paytirish uchun, qoidaga ko'ra, 3,7 sonidagi vergulni o'ngga qarab ikki xona surish kerak edi, ammo bizda verguldan keyin bitta son bor, xolos. Shuning uchun 7 sonidan keyin bitta nol qo'yamiz. (Bu yerda o'qituvchi o'quvchilarga 3,7 soni 3,70 soniga teng ekanligini tushuntirish va kasr holga keltirib ko'rsatish maqsadga muvofiq.)

2) 45,76 · 0,1 = 4,576. Bu misolni quyidagicha tushuntirish mumkin. Buning uchun 4576 sonini 1 soniga ko'paytirib hosil bo'lgan ko'paytmada o'ngdan chapga qarab uchta raqamni – ikkala ko'paytuvchida ular nechta bo'lsa, shuncha raqamni vergul bilan ajratamiz. Shunday mulohaza yuritib, 45,76 ni 0,01 ga ko'paytirishda 45,76 sonidan vergulni ikki raqam chapga surish kerakligini ko'rsatamiz.

Masalan: 45,76 · 0,01 = 0,4576.

9. O'nli kasrlarni bo'lish

O'nli kasrlarni bo'lish mavzusida quyidagi uch hol ko'rib o'tiladi:

1) *o'nli kasrni butun songa bo'lish.* O'nli kasrni butun songa bo'lish butun sonlarni bo'lishga o'xshash bajariladi, bunda qoldiqlar

borgan sari kichikroq ulushlarga maydalanib boradi. Masalan:

$$0{,}6 : 4 = 0{,}60 : 4 = 0{,}15.$$

1-q o i d a. *O'nli kasrlarni butun songa bo'lish uchun, butun qism bo'luvchiga yetadigan bo'lsa, butunini kasr xona almashguncha bo'lib, so'ngra bo'linmada vergul qo'yib bo'lishni davom ettirish kifoya.*

M i s o l.

1)
$$\begin{array}{r|l} -\,25{,}232 & 4 \\ \underline{24} & \overline{6{,}308} \\ -\,12 & \\ \underline{12} & \\ -\,032 & \\ \underline{32} & \\ 0 & \end{array} \qquad \begin{array}{r|l} -\,25232 & 4000 \\ \underline{2400} & \overline{6308} \\ -\,12320 & \\ \underline{12000} & \\ -\,0320 & \\ \underline{320} & \\ 0 & \end{array}$$

Yuqoridagi misol va qoidalarni tushuntirish jarayonida o'qituvchi o'quvchilarga bo'lish amalining ta'rifini va uni bajarish qoidalarini takrorlashi lozim.

2) *butun sonni o'nli kasrga bo'lish.* Bu holni ham o'qituvchi o'quvchilarga misol yordamida tushuntirishi kerak. Masalan:

$$51 : 0{,}17 = ?$$

Bu misolni yechishni oddiy kasrlarni bo'lish qoidasi asosida bajarib ko'rsatadi.

$$51 : 0{,}17 = 51 : \frac{17}{100} = (51 \cdot 100) : 17 = 5100 : 17 = 300.$$

Bu mulohazalarga ko'ra quyidagi qoidani ifodalash mumkin.

Q o i d a. *Butun sonni o'nli kasrga bo'lish uchun bo'luvchidagi o'nli kasrni butun songa aylantirish kerak. Buning uchun bo'luvchining vergul oxiriga suriladi va necha xona surilgan bo'lsa, bo'luvchining o'ng tomoniga shuncha nol qo'yiladi hamda butun sonni butun songa bo'lish kabi bajariladi.*

M i s o l.

$$351 : 2{,}7 = 3510 : 27 = 130,$$
$$25 : 6{,}25 = 2500 : 625 = 4.$$

3) *O'nli kasrni o'nli kasrga bo'lish.* Bu holda ham o'qituvchi o'quvchilarga kasrni kasrga bo'lishning umumiy qoidasini takrorlab, so'ngra o'nli kasrlarni oddiy kasrlar holiga keltirib, kasrlarni o'zaro bo'lish usulidek ko'rsatishi maqsadga muvofiqdir.

M i s o l.

$$8{,}51 : 3{,}7 = \frac{851}{100} : \frac{37}{10} = \frac{851 \cdot 10}{100 \cdot 37} = \frac{851}{10} : 37 = 85{,}1 : 37 = 2{,}3.$$

Bu misolni yana bunday yechib ko'rsatish ham mumkin:

$$8{,}51 : 3{,}7 = 8{,}51 : \frac{37}{10} = (8{,}51 \cdot 10) : 37 = 85{,}1 : 37 = 2{,}3.$$

Yuqoridagi misollardan ko'rinadiki, o'nli kasrni o'nli kasrga bo'lish uchun bo'luvchida qancha kasr xonasi bo'lsa, bo'linuvchi va bo'luvchidagi vergullarni shuncha xona o'ng tomonga so'ramiz, natijada bo'luvchi butun songa aylanadi. Buning natijasida bo'linuvchi va bo'luvchi bir xil marta ortgani uchun bo'linma o'zgarmaydi.

10. Oddiy kasrni cheksiz davriy kasrga aylantirish

2,73 o'nli kasr berilgan bo'lsin. Agar kasrning o'ng tomonidagi qismiga istalgancha nollar yozib qo'yilsa, uning qiymati o'zgarmaydi. 2,73 = 2,730 = 2,7300 =... = 2,7300...0. Shuningdek, 2,73 kasrni cheksiz ko'p nollari bo'lgan o'nli kasr ko'rinishida yozish mumkin. Masalan, 2,73 = 2,73000.... Bu yerda verguldan keyin cheksiz ko'p o'nli xonalar mavjud. Bunday o'nli kasr *cheksiz o'nli kasr* deyiladi.

Istalgan oddiy kasrni cheksiz o'nli kasr ko'rinishida yozish mumkin. Masalan, 3/14 sonini olib, uning suratini maxrajiga bo'lib ketma-ket o'nli xonalarni hosil qilamiz. Bunda istalgan natural sonni barcha o'nli xonalari nolga teng bo'lgan cheksiz o'nli kasr ko'rinishida yozish mumkinligini qayd qilib o'tamiz. Masalan, 3 = 3,00000....

borgan sari kichikroq ulushlarga maydalanib boradi. Masalan:

$$0{,}6 : 4 = 0{,}60 : 4 = 0{,}15.$$

1-q o i d a. *O'nli kasrlarni butun songa bo'lish uchun, butun qism bo'luvchiga yetadigan bo'lsa, butunini kasr xona almashguncha bo'lib, so'ngra bo'linmada vergul qo'yib bo'lishni davom ettirish kifoya.*

M i s o l .

1) $$\begin{array}{r|l} 25{,}232 & 4 \\ -\underline{24} & 6{,}308 \\ 12 & \\ -\underline{12} & \\ 032 & \\ -\underline{32} & \\ 0 & \end{array} \qquad \begin{array}{r|l} 25232 & 4000 \\ -\underline{2400} & 6308 \\ 12320 & \\ -\underline{12000} & \\ 0320 & \\ -\underline{320} & \\ 0 & \end{array}$$

Yuqoridagi misol va qoidalarni tushuntirish jarayonida o'qituvchi o'quvchilarga bo'lish amalining ta'rifini va uni bajarish qoidalarini takrorlashi lozim.

2) *butun sonni o'nli kasrga bo'lish.* Bu holni ham o'qituvchi o'quvchilarga misol yordamida tushuntirishi kerak. Masalan:

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Bu misolni yechishni oddiy kasrlarni bo'lish qoidasi asosida bajarib ko'rsatadi.

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Bu mulohazalarga ko'ra quyidagi qoidani ifodalash mumkin.

Q o i d a. *Butun sonni o'nli kasrga bo'lish uchun bo'luvchidagi o'nli kasrni butun songa aylantirish kerak. Buning uchun bo'luvchining vergul oxiriga suriladi va necha xona surilgan bo'lsa, bo'luvchining o'ng tomoniga shuncha nol qo'yiladi hamda butun sonni butun songa bo'lish kabi bajariladi.*

M i s o l.

$$351 : 2{,}7 = 3510 : 27 = 130,$$
$$25 : 6{,}25 = 2500 : 625 = 4.$$

3) *O'nli kasrni o'nli kasrga bo'lish.* Bu holda ham o'qituvchi o'quvchilarga kasrni kasrga bo'lishning umumiy qoidasini takrorlab, so'ngra o'nli kasrlarni oddiy kasrlar holiga keltirib, kasrlarni o'zaro bo'lish usulidek ko'rsatishi maqsadga muvofiqdir.

M i s o l.

$$8{,}51:3{,}7=\frac{851}{100}:\frac{37}{10}=\frac{851\cdot10}{100\cdot37}=\frac{851}{10}:37=85{,}1:37=2{,}3.$$

Bu misolni yana bunday yechib ko'rsatish ham mumkin:

$$8{,}51:3{,}7=8{,}51:\frac{37}{10}=(8{,}51\cdot10):37=85{,}1:37=2{,}3.$$

Yuqoridagi misollardan ko'rinadiki, o'nli kasrni o'nli kasrga bo'lish uchun bo'luvchida qancha kasr xonasi bo'lsa, bo'linuvchi va bo'luvchidagi vergullarni shuncha xona o'ng tomonga so'ramiz, natijada bo'luvchi butun songa aylanadi. Buning natijasida bo'linuvchi va bo'luvchi bir xil marta ortgani uchun bo'linma o'zgarmaydi.

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Istalgan oddiy kasrni cheksiz o'nli kasr ko'rinishida yozish mumkin. Masalan, 3/14 sonini olib, uning suratini maxrajiga bo'lib ketma-ket o'nli xonalarni hosil qilamiz. Bunda istalgan natural sonni barcha o'nli xonalari nolga teng bo'lgan cheksiz o'nli kasr ko'rinishida yozish mumkinligini qayd qilib o'tamiz. Masalan, 3 = 3,00000....

```
_3,00000000...| 14
 28           |0,214285714...
 ___
 _20
  14
  ___
  _60
   56
   ___
   _40
    28...
```

Shunday qilib, 3/14 = 0,214285714...

Bo‘lish davomida chiqqan barcha qoldiqlarni ketma-ket yozib chiqamiz: 2, 6, 4, 12, 8, 10, 2, 6... Bu qoldiqlarning barchasi bo‘luvchidan, ya’ni 14 sonidan kichik. Bu bo‘lishning qaysidir qismida ilgari uchragan qoldiq yana albatta uchrashi kerakligini bildiradi. Bizda yettinchi qadamda 2 qoldiq hosil bo‘lib, u birinchi qadamda paydo bo‘lgan edi. Bundan tashqari ilgari uchragan qoldiq paydo bo‘lgan zaxotiyoq undan keyingi qoldiqlar ular avval qanday tartibda bo‘lsa, shunday tartibda takrorlanadilar. Bizning misolimizda 2 qoldiqdan so‘ng 6 qoldiq, undan keyin 4, undan keyin 12 keladi va hokazo, ya’ni biz qoldiqlarning quyidagi ketma-ketligini hosil qilamiz: 2, 6, 4, 12, 8, 10, 2, 6, 4, 12, 8, 10,.... Davriy takrorlanuvchi qoldiqlar gruppasi mos ravishda sonning o‘nli yozuvidagi davriy takrorlanuvchi raqamlar gruppasiga olib keladi, ya’ni 3/14= = 0,2142857142857142857.... Sonning o‘nli yozuvida verguldan keyingi ketma-ket takrorlanib keluvchi bunday raqamlar gruppasi *davr* deb ataladi, o‘z yozuvida ana shunday davrga ega bo‘lgan chekli o‘nli kasr *davriy kasr* deyiladi. Qisqalik uchun davrni bir marta qavs ichiga olib yozish qabul qilingan: 0,2142857142857142857l4... = =0,2(142857). Agar davr verguldan keyin boshlansa, bunday kasr *sof davriy kasr* deyiladi, agar vergul va davr orasida boshqa o‘nli xonalar bo‘lsa, kasr *aralash davriy kasr* deyiladi. Masalan, 2,(23) = 2,2323232323... – sof davriy kasr, 0,2(142857) – aralash davriy kasr, 2,73 = 2,73000000... = 2,73(0) aralash davriy kasrdir.

11. Cheksiz davriy o‘nli kasrni oddiy kasrga aylantirish

Cheksiz o‘nli kasrni 10, 100, 1000 va hokazo ko‘paytirish uchun chekli o‘nli kasr holatidagi kabi vergulni bir, ikki, uch va hokazo xona o‘ngga surish kifoya. Masalan, 0,1(23) × 100 = 0,123232323... × 100 = 12, 32323232... = 12,(32). Davriy o‘nli kasrni oddiy kasrga aylantirishni quyidagi misollar orqali ko‘rib chiqaylik.

1. Sonni oddiy kasrga aylantiring:

1) 0,(13); 2) 2,(273); 3) 0,2(54); 4) 3,254(9).

Y e c h i s h. 1) $x = 0,13 = 0,131313...$ bo‘lsin. Sof davriy kasr x ni shunday songa ko‘paytiramizki, natijada vergul kasr davri qadar o‘ngga suriladi. Davrda ikkita raqam bo‘lgani uchun vergulni o‘ng tomonga ikki xona surish kerak, buning uchun esa x sonni 100 ga ko‘paytirish yetarli, u holda $100 \cdot x = 0,131313... \times 100 = 13,13131313... = 13,(13)$ $100x - x = 13,(13) - 0,(13)$. Demak, $99x = 13$, bu yerdan $x = \frac{13}{99}$.

2) $x = 2,(273)$ bo‘lsin. Bu sof davriy kasrning davrida uchta raqam bor. x ni 1000 ga ko‘paytirib, $1000x = 2273,(273)$ ni hosil qilamiz. Xuddi yuqoridagiga o‘xshash topamiz:

$1000x - x = 2273,(273) - 2,(273)$, $999x = 2271$, bundan

$$x = \frac{2271}{999} = \frac{757}{333} = 2\frac{91}{333}.$$

3) $x = 0,2(54)$ bo‘lsin. Bu aralash davriy kasrda vergulni o‘ng tomonga shunday suramizki, natijada sof davriy kasr hosil bo‘lsin. Buning uchun x ni 10 ga ko‘paytirib qo‘yish kifoya. $10x = 2,(54)$ ni hosil qilamiz.

$y = 2,(54)$ bo‘lsin va yuqoridagilarga o‘xshash bu sof davriy kasrni oddiy kasrga aylantiramiz. $y = 2,(54)$, bundan $100y = 254(54)$, $100y - y = 254(54) - 2,54$, $99y = 252$, $y = \frac{252}{99} = \frac{28}{11}$, demak, $10x = \frac{28}{11}$, bundan $x = \frac{28}{11 \cdot 10} = \frac{11}{55}$.

4) $x = 3{,}254(9)$ deb $1000x = 3254(9)$ ni hosil qilamiz. $y = 1000x$ belgilashni kiritamiz, u holda $y = 3254{,}(9)$, bu yerdan $10y - y =$ $32549(9) - 3254(9)$; $y = 3255$, $1000x = 3255$, $x = \frac{3255}{1000} = 3\frac{51}{200}$.

Endi quyidagiga e'tibor beramiz: $\frac{3255}{1000} = 3{,}255 = 3{,}255(0)$ chekli o'nli kasr yoki davrida nol bo'lgan cheksiz kasrni hosil qilamiz.

Demak, $3{,}254(9) = 3{,}255(0)$. Bu hol davrida to'qqiz bo'lgan istalgan kasr ko'rinishida yozish mumkin. Buning uchun davr oldidagi o'nli raqamni bir birlikka orttirish kifoya. Masalan, $0{,}45(9) = 0{,}46(0)$; $14{,}(9) = 15{,}(0)$.

Arifmetik to'rt amalga doir misollar yechish

1-m i s o l. $\frac{172\frac{5}{6} - 170\frac{1}{3} + 3\frac{5}{12}}{0{,}8 \cdot 0{,}25} = 29\frac{7}{12}$.

Y e c h i s h.

1) $172\frac{5}{6} - 170\frac{1}{3} + 3\frac{5}{12} = (172 - 170 + 3) + \left(\frac{5}{6} - \frac{1}{3} + \frac{5}{12}\right) =$

$= 5 + \frac{10 - 4 + 5}{12} = 5 + \frac{11}{12} = 5\frac{11}{12}$;

2) $0{,}8 \cdot 0{,}25 = \frac{8}{10} \cdot \frac{25}{100} = \frac{4}{5} \cdot \frac{1}{4} = \frac{1}{5}$.

3) $5\frac{11}{12} : \frac{1}{5} = \frac{71}{12} \cdot 5 = \frac{355}{12} = 29\frac{7}{12}$;

2-m i s o l. $\frac{\left[\left(40\frac{7}{30} - 38\frac{5}{12}\right) : 10{,}9 + \left(\frac{7}{8} - \frac{7}{30}\right) \cdot 1\frac{9}{11}\right] \cdot 4{,}2}{0{,}008} = 700.$

Y e c h i s h.

1) $40\frac{7}{30}-38\frac{5}{12}=(40-38)+\left(\frac{7}{30}-\frac{5}{12}\right)=2+\frac{14-25}{60}=$

$=1+\frac{74-25}{60}=1\frac{49}{60};$

2) $1\frac{49}{60}:10,9=\frac{109}{60}:\frac{109}{10}=\frac{109}{60}\cdot\frac{10}{109}=\frac{1}{6};$

3) $\frac{7}{8}-\frac{7}{30}=\frac{105-28}{120}=\frac{77}{120};$

4) $\frac{77}{120}\cdot1\frac{9}{11}=\frac{77}{120}\cdot\frac{20}{11}=\frac{7}{6};$

5) $\frac{1}{6}+\frac{7}{6}=\frac{8}{6}=\frac{4}{3};$

6) $\frac{4}{3}\cdot4,2=\frac{4}{3}\cdot\frac{21}{5}=\frac{28}{5};$

7) $\frac{28}{5}:0,008=\frac{28}{5}\cdot125=28\cdot25=700.$

3-m i s o l. $1\frac{7}{20}:2,7+2,7:1,35+\left(0,4:2\frac{1}{2}\right)\cdot\left(4,2-1\frac{3}{40}\right)=3$.

Y e c h i s h.

1) $1\frac{7}{20}:2,7=\frac{27}{20}:2\frac{7}{10}=\frac{27}{20}:\frac{27}{10}=\frac{27}{20}\cdot\frac{10}{27}=\frac{1}{2};$

2) $2,7:1,35=2\frac{7}{10}:1\frac{35}{100}=\frac{27}{10}:1\frac{7}{10}=\frac{27}{10}:\frac{27}{20}=\frac{27}{10}\cdot\frac{20}{27}=2;$

3) $\frac{1}{2}+2=2\frac{1}{2};$

4) $0,4:2\frac{1}{2}=\frac{2}{5}:\frac{5}{2}=\frac{2}{5}\cdot\frac{2}{5}=\frac{4}{25};$

5) $4,2-1\frac{3}{40}=4\frac{1}{5}-1\frac{3}{40}=(4-1)+\left(\frac{1}{5}-\frac{3}{40}\right)=3+\frac{8-3}{40}=$

$=3+\frac{5}{40}=3+\frac{1}{8}=3\frac{1}{8};$

6) $\frac{4}{25}\cdot3\frac{1}{8}=\frac{4}{25}\cdot\frac{25}{8}=\frac{1}{2};$

7) $2\frac{1}{2}+\frac{1}{2}=3.$

4-m i s o l. $1,7:\frac{\left(4,5\cdot1\frac{2}{3}+3,75\right)\cdot\frac{7}{135}}{5:9}-\left(0,5+\frac{1}{3}-\frac{5}{12}\right)=1\frac{17}{84}.$

Y e c h i s h.

1) $4,5\cdot1\frac{2}{3}=4\frac{1}{2}\cdot1\frac{2}{3}=\frac{9}{2}\cdot\frac{5}{3}=\frac{15}{2}=7\frac{1}{2};$

2) $7\frac{1}{2}+3\frac{75}{100}=7\frac{1}{2}+3\frac{3}{4}=(7+3)+\left(\frac{1}{2}+\frac{3}{4}\right)=$

$=10+\frac{2+3}{4}=10+\frac{5}{4}=11\frac{1}{4};$

3) $11\frac{1}{4}\cdot\frac{7}{135}=\frac{45}{4}\cdot\frac{7}{135}=\frac{7}{12};$

4) $\frac{7}{12}:\frac{5}{9}=\frac{7}{12}\cdot\frac{9}{5}=\frac{7\cdot3}{4\cdot5}=\frac{21}{20};$

5) $1,7:\frac{21}{20}=1\frac{7}{10}:\frac{21}{20}=\frac{17}{10}\cdot\frac{20}{21}=\frac{34}{21};$

6) $0,5+\frac{1}{3}-\frac{5}{12}=\frac{1}{2}+\frac{1}{3}-\frac{5}{12}=\frac{6+4-5}{12}=\frac{5}{12};$

7) $\frac{34}{21}-\frac{5}{12}=\frac{136-35}{84}=\frac{101}{84}=1\frac{17}{84}.$

5-m i s o l. $$\frac{\left(1,75:\frac{2}{3}-1,75\cdot 1\frac{1}{8}\right):\frac{7}{2}}{\left(\frac{17}{80}-0,0325\right):4}\cdot(6,79:0,7+0,3)=250.$$

Y e c h i s h.

1) $1,75:\frac{2}{3}=1\frac{75}{100}:\frac{2}{3}=1\frac{3}{4}:\frac{2}{3}=\frac{7}{4}\cdot\frac{3}{2}=\frac{21}{8}$;

2) $1,75\cdot 1\frac{1}{8}=1\frac{75}{100}\cdot 1\frac{1}{8}=1\frac{3}{4}\cdot 1\frac{1}{8}=\frac{7}{4}\cdot\frac{9}{8}=\frac{63}{32}$;

3) $\frac{21}{8}-\frac{63}{32}=\frac{84-63}{32}=\frac{21}{32}$;

4) $\frac{21}{32}:\frac{7}{12}=\frac{21}{32}\cdot\frac{12}{7}=\frac{3\cdot 3}{8}=\frac{9}{8}$;

5) $\frac{17}{80}-0,0325=\frac{17}{80}-\frac{325}{10000}=\frac{17}{80}-\frac{13}{400}=\frac{5\cdot 17-13}{400}=$

$=\frac{85-13}{400}=\frac{72}{400}=\frac{9}{50}$;

6) $\frac{9}{50}:4=\frac{9}{50}\cdot\frac{1}{4}=\frac{9}{200}$;

7) $\frac{9}{8}:\frac{9}{200}=\frac{9}{8}\cdot\frac{200}{9}=25$;

8) $6,79:0,7=6\frac{79}{100}:\frac{7}{10}=\frac{679}{100}\cdot\frac{10}{7}=\frac{97}{10}$;

9) $\frac{97}{10}+0,3=\frac{97}{10}+\frac{3}{10}=\frac{100}{10}=10$;

10) $25\cdot 10=250$.

Mustaqil yechish uchun misollar

Hisoblang (2.1–2.33):

2.1. $$\frac{1,4:\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right)}{\left(3\frac{1}{6}\cdot 6-5\frac{1}{2}\cdot 2\frac{5}{11}\right):3\frac{2}{3}}.\quad J.\,1,6.$$

2.2. $$\frac{\left(2-1\frac{3}{4}\cdot\frac{2}{7}\right)\cdot\left(1\frac{1}{3}-2\frac{1}{2}:3\frac{3}{4}\right)}{2\frac{1}{3}:\left(\frac{1}{4}-\frac{5}{8}+\frac{7}{8}\right)}.\quad J.\,\frac{3}{14}.$$

2.3. $$\frac{11\frac{2}{3}\cdot 2\frac{4}{7}}{12\frac{4}{5}\cdot 3\frac{3}{4}-4\frac{4}{11}\cdot 4\frac{1}{8}}.\quad J.\,1.$$

2.4. $$\frac{28\frac{4}{5}:14\frac{2}{5}+6\frac{3}{2}\cdot\frac{2}{3}}{1\frac{11}{16}:2\frac{1}{4}}.\quad J.\,9\frac{1}{3}.$$

2.5. $$\frac{1\frac{9}{16}\cdot 3\frac{1}{5}-9:2\frac{2}{5}}{1\frac{1}{3}\cdot\left(17\frac{7}{12}-6\frac{1}{3}\right):\frac{3}{4}}.\quad J.\,\frac{1}{16}.$$

2.6. $$\left(\frac{0,012}{5}+\frac{0,04104}{5,4}\right)\cdot 4560-42\frac{1}{3}.\quad J.\,3\frac{4}{15}.$$

2.7. $$\frac{\left(85\frac{7}{30}-83\frac{5}{18}\right):2\frac{2}{3}}{0,04}.\quad J.\,18\frac{1}{3}.$$

2.8. $\dfrac{\left(140\dfrac{7}{30}-138\dfrac{5}{12}\right):18\dfrac{1}{6}}{0,002}$. *J.* 50.

2.9. $\dfrac{\left(95\dfrac{7}{30}-93\dfrac{5}{18}\right)\cdot 2\dfrac{1}{4}+0,373}{0,2}$. *J.* 23,865.

2.10. $\dfrac{\left(12\dfrac{1}{6}-6\dfrac{1}{27}-5\dfrac{1}{4}\right)\cdot 13,5+0,111}{0,02}$. *J.* 599,3.

2.11. $\dfrac{\left(68\dfrac{7}{30}-66\dfrac{5}{18}\right):6\dfrac{1}{9}+\left(\dfrac{7}{40}+\dfrac{3}{32}\right)\cdot 4,5}{0,04}$. *J.* $38\dfrac{15}{64}$.

2.12. $\dfrac{(2,1-1,965):(1,2\cdot 0,045)}{0,00325:0,013}-\dfrac{1:0,25}{1,6\cdot 0,625}$. *J.* 6.

2.13. $\left(17\dfrac{1}{2}-8,25\cdot\dfrac{10}{11}\right)\cdot\left(11\dfrac{2}{3}:\dfrac{2}{9}+3,5\right)$. *J.* 560.

2.14. $(10,5\cdot 2,04-0,1)\cdot(6,25\cdot 0,2+0,8:0,64)$. *J.* 53,3.

2.15. $\dfrac{2\dfrac{5}{7}-\dfrac{2}{3}\cdot 2\dfrac{5}{14}}{\left(3\dfrac{1}{12}+4,375\right):9\dfrac{8}{9}}$. *J.* $\dfrac{19}{28}$.

2.16. $\dfrac{0,134+0,05}{18\dfrac{1}{6}-1\dfrac{11}{14}-\dfrac{2}{15}\cdot 2\dfrac{6}{7}}$. *J.* 0,0115.

2.17. $\dfrac{\left(58\dfrac{4}{15}-56\dfrac{7}{24}\right):0,8+2\dfrac{1}{9}\cdot 0,225}{8\dfrac{3}{4}\cdot\dfrac{3}{5}}$. *J.* $\dfrac{157}{280}$.

2.18. $\left[\dfrac{\left(6-4\frac{1}{2}\right):0,03}{\left(3\frac{1}{20}-2,65\right)\cdot 4+\frac{2}{5}}-\dfrac{\left(0,3-\frac{3}{20}\right)\cdot 1\frac{1}{2}}{\left(1,88+2\frac{1}{25}\right)\cdot\frac{1}{80}}\right]:2\frac{1}{20}$. *J.* 10.

2.19. $\left(\dfrac{0,216}{0,15}+\dfrac{2}{3}:\dfrac{4}{15}\right)+\left(\dfrac{196}{225}-\dfrac{7,7}{2\frac{3}{4}}\right)+0,695:1,39$. *J.* $2\dfrac{23}{45}$.

2.20. $\dfrac{1}{3}:\dfrac{2}{3}+0,228:\left[\left(1,5291-\dfrac{1,453662}{3-0,095}\cdot 0,305\right):0,12\right]$.

J. $\dfrac{152}{76471}$.

2.21. $\dfrac{\left[\left(6,2:0,31-\frac{5}{6}\cdot 0,9\right)\cdot 0,2+0,15\right]:0,02}{\left(2+1\frac{4}{11}\cdot 0,22:0,1\right)\cdot\frac{1}{33}}$. *J.* 1320.

2.22. $6:\dfrac{1}{3}-0,8:\dfrac{1,5}{\frac{3}{2}\cdot 0,4\cdot\frac{50}{1:\frac{1}{2}}}+\dfrac{1}{4}+\dfrac{1+\frac{1}{2}\cdot\frac{1}{0,25}}{6-\frac{46}{1+2,2\cdot 10}}$. *J.* 11.

2.23. $\dfrac{(7-6,35):6,5+9,9}{\left(1,2:3,6+1,2:0,25-1\frac{5}{16}\right):\frac{169}{24}}$. *J.* 20.

2.24. $\dfrac{\left(\frac{1}{6}+0,1+\frac{1}{15}\right):\left(\frac{1}{6}+0,1-1\frac{1}{15}\right)\cdot 2,52}{\left(0,5-\frac{1}{3}+0,25-\frac{1}{5}\right):\left(0,25-\frac{1}{6}\right)\cdot\frac{7}{13}}$. *J.* $-\dfrac{63}{84}$.

2.25. $\dfrac{2\frac{3}{4}:1,1+3\frac{1}{3}}{2,5-0,4\cdot 3\frac{1}{3}}:\dfrac{5}{7}-\dfrac{\left(2\frac{1}{6}+4,5\right)\cdot 0,375}{2,75-1\frac{1}{2}}$. $J.5.$

2.26. $\dfrac{\left(13,75+9\frac{1}{6}\right)\cdot 1,2}{\left(10,3-8\frac{1}{2}\right)\cdot\frac{5}{9}}+\dfrac{\left(6,8-3\frac{3}{5}\right)\cdot 4\frac{1}{6}}{\left(3\frac{3}{2}-3\frac{1}{6}\right)\cdot 56}-27\frac{1}{6}$. $J.\frac{2}{3}$.

2.27. $\dfrac{0,4+\left(5-0,8\cdot\frac{5}{8}\right)-5:2\frac{1}{2}}{\left[1\frac{7}{8}\cdot 8-\left(8,9-2,6\cdot\frac{2}{3}\right)\right]:23\frac{1}{2}}$. $J.8\frac{7}{10}$.

2.28. $\dfrac{0,125:0,25+1\frac{9}{16}:2,5}{(10-2:2,3)\cdot 0,46+1,6}+\left(\frac{17}{20}+1,9\right):0,5$. $J.5\frac{11}{16}$.

2.29. $\left[\dfrac{8,8077}{20-\left[28,2:(13,333\cdot+0,0001)\right]\cdot 2,004}+4,9\right]\cdot\dfrac{5}{32}$. $J.1.$

2.30. 1) 4,735 : 0,5 + 14,95 : 1,3 – 2,121 : 0,7. *J.* 17,94;
2) 589,72 : 16 – 18,305 : 7 + 0,0567 : 4. *J.* 34,256675;
3) 3,006 – 0,3417 : 34 – 0,875 : 125. *J.* 2,98895;
4) 22,5 : 3,75 + 208,45 – 2,5 : 0,004. *J.* – 410,55.

2.31. 1) (0,1955 + 0,187) : 0,085;
2) 15,76267 : (100,6 + 42697). *J.* 0,000368;
3) (86,9 + 667,6) : (37,1 + 13,2);
4) (9,09 – 900252) × (25,007 – 12,507). *J.* 11,25;

2.32. 1) (0,008 + 0,992) × (5 × 0,6 – 1,4);
2) (0,93 + 0,07) × (0,93 – 0,805). *J.* 0,125;
3) (50 000 – 1 397,3) : (20,4 + 33,603);
4) (2779,6 + 8024) : (1,98 + 2,02). *J.* 2700,9;

2.33. Davriy o'nli kasrni oddiy kasrga aylantiring:

1) 0,(3); 2) 13,0(48); 3) 2,(123);
4) 0,3(2); 5) 1,(4); 6) 2,333(45);
7) 0,71(23); 8) 2,(45); 9) 41,8519(504);
10) 11,(75); 11) 3,1(44); 12) 35,73(4845).

2.34. Ifodaning qiymatini toping:

1) $\dfrac{\left(10\frac{3}{4}-148\frac{3}{4}\right)\cdot 3}{0,2}$;

2) $\dfrac{172\frac{5}{6}-170\frac{1}{3}+3\frac{5}{12}}{0,8\cdot 0,25}$. $J.\ \dfrac{355}{12}$;

3) $\dfrac{215\frac{9}{16}-208\frac{3}{4}+3\frac{1}{2}}{0,0001:0,005}$;

4) $\left(\dfrac{0,012}{5}+\dfrac{0,04104}{5,4}\right)\cdot 4560-42\dfrac{1}{3}$. $J.\ \dfrac{49}{15}$;

5) $\dfrac{\left(85\frac{7}{30}-83\frac{5}{18}\right):2\frac{2}{3}}{0,04}$;

6) $\dfrac{\left(140\frac{7}{30}-138\frac{5}{12}\right):18\frac{1}{6}}{0,002}$. $J.\ 50$;

7) $\dfrac{\left(95\frac{7}{30}-93\frac{5}{18}\right)\cdot 2\frac{1}{4}+0,373}{0,2}$;

8) $\dfrac{\left(49\frac{5}{24}-46\frac{7}{29}\right)\cdot 2\frac{1}{3}+0,6}{0,2}$. $J.\ \dfrac{3491}{96}$;

9) $\dfrac{\left(12\frac{1}{6}-6\frac{1}{27}-5\frac{1}{4}\right)\cdot 13,5+0,111}{0,02}$;

10) $\dfrac{0,134+0,05}{18\frac{1}{6}-1\frac{11}{4}-\frac{2}{15}\cdot 2\frac{6}{7}}$. *J.* $\dfrac{112}{7}$;

11) $\dfrac{\left(6\frac{3}{5}-3\frac{3}{14}\right)\cdot 5\frac{5}{6}}{(21-1,25):2,5}$;

12) $\dfrac{2\frac{5}{8}-\frac{2}{3}\cdot 2\frac{5}{14}}{\left(3\frac{1}{12}+4,375\right):19\frac{8}{9}}$. *J.* $\dfrac{7160}{2721}$;

13) $\dfrac{\left(1\frac{1}{12}+2\frac{5}{35}+\frac{1}{24}\right)\cdot 9\frac{3}{5}+2,13}{0,4}$;

14) $\dfrac{400-1,5\cdot 18,5}{1,5\cdot 1\frac{1}{10}+1,4\cdot 1\frac{1}{2}}$. *J.* $\dfrac{1489}{15}$;

15) $\dfrac{1}{1-\dfrac{1}{1-2^{-1}}}+\dfrac{1}{1+\dfrac{1}{1+2^{-1}}}$;

16) $\dfrac{1}{1+\dfrac{1}{3-2^{-1}}}-\dfrac{1}{1-\dfrac{2}{1+2^{-1}}}$. *J.* $\dfrac{26}{7}$;

17) $\dfrac{\left(58\dfrac{4}{15}-56\dfrac{1}{24}\right):0,8+2\dfrac{1}{9}\cdot 0,225}{8\dfrac{3}{4}\cdot\dfrac{3}{5}}$;

18) $\dfrac{\left(68\dfrac{7}{30}-66\dfrac{5}{18}\right):6\dfrac{1}{9}\left(\dfrac{7}{40}+\dfrac{3}{32}\right)\cdot 4,5}{0,04}$. *J.* $\dfrac{587}{32}$;

19) $\dfrac{(2,1-1,965):(1,2\cdot 0,045)}{0,00325:0,013}-\dfrac{1:0,25}{1,6\cdot 625}$;

20) $\dfrac{\left[\left(40\dfrac{7}{30}-38\dfrac{5}{12}\right):10,9+\left(\dfrac{7}{8}-\dfrac{7}{30}\cdot 1\dfrac{9}{11}\right)\right]\cdot 4,2}{0,008}$. *J.* $\dfrac{28525}{88}$;

21) $\left[\dfrac{\left(2,4+1\dfrac{5}{7}\cdot 4,375\right)}{\dfrac{2}{3}-\dfrac{1}{6}}-\dfrac{\left(2,75-1\dfrac{5}{6}\right)\cdot 21}{8\dfrac{3}{20}-0,45}\right]:\dfrac{67}{200}$;

22) $\left[\dfrac{\left(6-4\dfrac{1}{2}\right):0,03}{\left(3\dfrac{1}{20}-2,65\right)\cdot 4+\dfrac{2}{5}}-\dfrac{\left(0,3-\dfrac{3}{20}\right)\cdot 1\dfrac{1}{2}}{\left(1,88+2\dfrac{3}{25}\right)\cdot\dfrac{1}{80}}\right]:2\dfrac{1}{20}$. *J.* $\dfrac{320}{41}$;

23) $\left[\dfrac{3:(0,2-0,1)}{2,5\cdot(0,8+1,2)}+\dfrac{(34,06-33,81)\cdot 4}{6,84:(28,57-25,15)}\right]+\dfrac{2}{3}:\dfrac{4}{21}$;

24) $\dfrac{3:\dfrac{2}{5}-0,09:\left(0,15:2\dfrac{1}{2}\right)}{0,32\cdot 6+0,03-(5,3-3,88)+0,67}$. *J.* $\dfrac{65}{12}$;

25) $1\frac{7}{20}:2,7+2,7:1,35+\left(4,2-1\frac{3}{40}\right)$;

26) $\left(10:2\frac{2}{3}+7,5:10\right)\cdot\left(\frac{3}{40}-\frac{7}{30}\cdot 0,25+\frac{157}{360}\right)$. $J.\ \frac{1793}{960}$;

27) $\left(\frac{0,216}{0,15}+\frac{2}{3}:\frac{4}{15}\right)+\left(\frac{196}{225}-\frac{7,7}{24\frac{3}{4}}\right)+0,695:1,39$. $J.\ \frac{211}{24}$;

28) $1,7:\frac{\left(4,5-1\frac{2}{3}+3,75\right)\cdot\frac{7}{35}}{\frac{5}{9}}-\left(0,5+\frac{1}{3}-\frac{5}{12}\right)$. $J.\ -\frac{169}{3060}$;

29) $\frac{1}{3}:\frac{2}{3}+0,228\cdot\left(1,5291-\frac{14,53662}{3-0,095}\cdot 0,305:0,12\right)$;

30) $\left(\frac{8,8077}{20-\left[28,2:(13,333\cdot 0,3+0,0001)\right]\cdot 2,004}+4,9\right)\cdot\frac{5}{32}$;

31) $\frac{\left[\left(6,2:0,31-\frac{5}{6}\cdot 0,9\right)\cdot 0,2+0,15\right]:0,02}{\left(2+1\frac{4}{11}\cdot 0,22:0,1\right)\cdot\frac{1}{33}}$;

32) $6:\frac{1}{3}-0,8:\frac{1,5}{\frac{3}{2}\cdot 0,4\cdot\frac{50}{1:\frac{1}{2}}}+\frac{1}{4}+\frac{1+\frac{1}{2}\cdot\frac{1}{0,25}}{6-\frac{46}{1+2,2\cdot 10}}$. $J.11$;

33) $\dfrac{\left(1,75:\dfrac{2}{3}-1,75\right)\cdot 1\dfrac{1}{8}}{\left(\dfrac{17}{80}-0,0325\right):400}:(6,79:0,7+0,3);$

34) $\dfrac{4,5:\left[47,375-\left(26\dfrac{1}{3}-18\cdot 0,75\right)\cdot 2,4:0,88\right]}{17,81:1,37-23\dfrac{2}{3}:1\dfrac{5}{6}}$. *J.* $\dfrac{121}{4}$;

35) $\left[\dfrac{\left(7-3\dfrac{1}{2}\right):0,13}{\left(4\dfrac{1}{10}-1,85\right)\cdot 4+\dfrac{1}{5}}-\dfrac{\left(0,2-\dfrac{4}{10}\right)\cdot 1\dfrac{3}{4}}{\left(1,44+2\dfrac{3}{42}\right)\cdot\dfrac{1}{90}}\right]:4\dfrac{45}{800}$.

3-§. Irratsional sonlar

1. *n*-darajali arifmetik ildiz. Ratsional ko'rsatkichli daraja

$a \geq 0$ sonning *n*-darajali *arifmetik ildizi* deb ($n \in N$), *n*-darajasi *a* ga teng bo'lgan $b \geq 0$ songa aytiladi va $b = \sqrt[n]{a}$ orqali belgilanadi. Ta'rif bo'yicha: $(\sqrt[n]{a})^n = a$.

$a > 0$, $m \in Z$ va $n \in N$ bo'lsa, $\sqrt[n]{a^m}$ soni *a* ning $r = \dfrac{m}{n}$ *ratsional ko'rsatkichli darajasi* deb ataladi, ya'ni $a^r = a^{\frac{m}{n}} = \sqrt[n]{a^m}$.

Xususan, $\sqrt[n]{a} = a^{\frac{1}{n}}$. Ratsional ko'rsatkichli darajaning *xossalari* butun ko'rsatkichli daraja xossalariga o'xshash. *a, b* – ixtiyoriy musbat sonlar, *r* va *q* – ixtiyoriy ratsional sonlar bo'lsin. U holda:

1) $(ab)^r = a^r b^r$. Haqiqatan, $r = \dfrac{m}{n}$, $n \in N$, $m \in Z$ bo'lsin. U holda:

$$\left((ab)^r\right)^n=\left((ab)^{\frac{m}{n}}\right)^n=\left(\sqrt[n]{(ab)^m}\right)^n=(ab)^m=a^m b^m=$$

$$=\left(\sqrt[n]{a^m}\right)^n\cdot\left(\sqrt[n]{b^m}\right)^n=\left(a^{\frac{m}{n}}\cdot b^{\frac{m}{n}}\right)^n=\left(a^r\cdot b^r\right)^n.$$

Xususan, $\left(\frac{a}{b}\right)^r=\frac{a^r}{b^r}$

2) $a^r\cdot a^q=a^{r+q}$ bunda $r=\frac{k}{n}$, $q=\frac{m}{n}$. Haqiqatan,

$$\left(a^{\frac{k}{n}}\cdot a^{\frac{m}{n}}\right)^n=\left(a^{\frac{k}{n}}\right)^n\cdot\left(a^{\frac{m}{n}}\right)^n=\left(\sqrt[n]{a^k}\right)^n\cdot\left(\sqrt[n]{a^m}\right)^n=$$

$$=a^k\cdot a^m=a^{k+m}=\left(a^{\frac{k+m}{n}}\right)^n=\left(a^{\frac{k}{n}+\frac{m}{n}}\right)^n.$$

3) $\frac{a^r}{a^q}=a^{r-q}$ (2) kabi isbotlanadi).

4) $(a^r)^q=a^{rq}$, bunda $r=\frac{p}{k},q=\frac{m}{n}$. Haqiqatan,

$$\left(\left(a^{\frac{p}{k}}\right)^{\frac{m}{n}}\right)^{n-k}=\left(\left(\left(a^{\frac{p}{k}}\right)^{\frac{m}{n}}\right)^n\right)^k=\left(a^p\right)^m=\left(a^{\frac{pm}{kn}}\right)^{kn}.$$

Yuqorida arifmetik ildizga ta'rif berilgan edi. $a\geq 0$ da $x=\sqrt[n]{a}$ son $x^n=a$ tenglamaning yagona nomanfiy yechimi ekanligi, shuningdek, $a\in R$ va n-toq natural son bo'lsa, $x^n=a$ tenglamaning yagona yechimga ega ekanligi quyida isbotlanadi.

$x^n=a$ tenglamaning (bu yerda $a\in R$, $n\in N$) har qanday ildizi a sonining *n-darajali ildizi* deyiladi.

1- t e o r e m a. *Har qanday $a\geq 0$ haqiqiy son uchun har doim $x^n=a$ tenglikni qanoatlantiruvchi yagona $x\geq 0$ haqiqiy son mavjud.*

2- t e o r e m a. *Agar A natural son hech bir natural sonning n-darajasi bo'lmasa, $\sqrt[n]{A}$ soni irratsionalsondir.*

I s b o t. Shart bo‘yicha A soni nomanfiy sonlarning 0^n, 1^n, 2^n,..., k^n,... n- darajalar ketma-ketligida uchramaydi, demak, $\sqrt[n]{A}$ butun son emas. U kasr ham emas. Haqiqatan, $\sqrt[n]{A} = \frac{p}{q}$ bo‘lsin, deb faraz qilaylik, bunda p va q lar o‘zaro tub va $p \neq 1$, $q \neq 0$.U holda $A = \frac{p^n}{q^n}$ p^n va q^n – o‘zaro tub, $q \neq 1$ bo‘lganidan A soni qisqarmas kasr bo‘ladi. Bu esa shartga zid.Demak, $\sqrt[n]{A}$ soni faqat irratsionaldir. Teorema isbot qilindi.

3-t e o r e m a. *Agar* $\frac{p}{q}$, $q \neq 1$, *qisqarmas kasrning surati va maxraji aniq n-daraja bo‘lmasa,* $\sqrt[n]{\frac{p}{q}}$ *ildiz irratsional sondir.*

I s b o t. Teskaricha, ildiz ratsional son, deb faraz qilaylik, ya’ni $\sqrt[n]{\frac{p}{q}} = \frac{a}{b}$, $B(a, b) = 1$. U holda $\frac{p}{q} = \frac{a^n}{b^n}$, $B(a^n, b^n) = 1$ va bundan $p = a^n$, $q = b^n$ bo‘lishi kelib chiqadi.Lekin shart bo‘yicha p va q n-darajali emas. Demak, $\sqrt[n]{\frac{p}{q}}$ – irratsional son. Teorema isbot qilindi.

4- t e o r e m a. *Haqiqiy sonlar sohasida toq darajali ildiz faqat bir qiymatli va uning uchun ushbu tenglik o‘rinli:*

$$\sqrt[2n+1]{-a} = -\sqrt[2n+1]{a}\,.$$

I s b o t. $x^{2n+1} = a$, $a \geq 0$, (1) tenglama $\forall a \in R$ uchun yagona yechimga ega ekanligini ko‘rsatamiz:

a) $a \geq 0$ bo‘lsin.U holda $\forall x < 0$ son uchun $x^{2n+1} < 0 \leq a$. Demak, (1) ning mavjudligi 1- teoremadan ko‘rinadigan, $x = \sqrt[2n+1]{-a} \geq 0$ ildizi uning yagona haqiqiy ildizidir;

b) $a < 0$ bo‘lsa, (1) ni $(-x)^{2n+1} = -a$ ko‘rinishda yozib olish mumkin. $-a > 0$ bo‘lgani uchun, a) holga ko‘ra, oxirgi tenglama va, demak, (1) tenglama ham yagona $x = \sqrt[2n+1]{-a}$ yechimga egadir.

$\forall a \in R$ uchun $x_1 = -\sqrt[2n+1]{-a}$ va $x_2 = \sqrt[2n+1]{-a}$ sonlari (1) ning ildizlari bo'ladi.Yuqorida isbotlanganlarga ko'ra, $x_1 = x_2$. Teorema isbot qilindi.

Teoremadan ko'rinadiki, $\sqrt[n]{a^n} = a$ ayniyat n ning 1 dan katta toq natural qiymatlarida, ixtiyoriy $a \in R$ uchun o'rinli. Agar $n = 2m$ (bu yerda $m \in N$) bo'lsa, $\sqrt[2m]{a^{2m}} = \sqrt[2m]{|a|^{2m}} = |a|$ bo'ladi.Demak, $a \geq 0$ bo'lsa, $\sqrt[2m]{a^{2m}} = a$ tenglik, $a < 0$ bo'lganda esa $\sqrt[2m]{a^{2m}} = -a$ tenglik o'rinli.

1-m i s o l. $\sqrt{(-7)^2} = \sqrt{|-7|^2} = |-7| = 7, ... \sqrt{(-7)^2} = \sqrt{49} = 7.$

Agar $a \leq 0$, $b \leq 0$ bo'lsa, $ab \geq 0$ va $\sqrt{ab} = \sqrt{|a||b|} = \sqrt{|a|}\sqrt{|b|}$ bo'ladi.

2-m i s o l. $\sqrt{(-3)(-12)} = \sqrt{|-3||-12|} = \sqrt{36} = 6.$

2. Arifmetik ildizlarni shakl almashtirish

Ko'paytmaning n-darajali ildizi ko'paytuvchilar n-darajali ildizlarining ko'paytmasiga teng: $\sqrt[n]{a \cdot b \cdot ... \cdot c} = \sqrt[n]{a} \cdot \sqrt[n]{b} \cdot ... \cdot \sqrt[n]{c}$, (1) bu yerda $a \geq 0$, $b \geq 0$,..., $c \geq 0$.

Haqiqatan,

$$\sqrt[n]{a \cdot b \cdot ... \cdot c} = (a \cdot b \cdot ... \cdot c)^{\frac{1}{n}} = a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} \cdot ... \cdot c^{\frac{1}{n}} = \sqrt[n]{a} \cdot \sqrt[n]{b} \cdot ... \cdot \sqrt[n]{c}. \quad (2)$$

Xususan, $\sqrt[n]{a^n b} = \begin{cases} |a|^n \sqrt{b}, \text{ agar } n - \text{juft bo'lsa}, \\ a\sqrt[n]{b}, \text{ agar } n - \text{toq bo'lsa}. \end{cases}$

Ko'paytuvchini ildiz ishorasi ostiga kiritish: $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ $(a > 0,$ $b > 0)$. (3)

Kasrdan ildiz chiqarish: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, $(a \geq 0, b \geq 0)$. (4)

Ildizni darajaga ko'tarish uchun ildiz ostidagi ifodani shu darajaga ko'tarish kifoya: $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$, $(a \geq 0)$. (5)

Haqiqatan, $\left(\sqrt[n]{a}\right)^m = \left(a^{\frac{1}{n}}\right)^m = a^{m\cdot\frac{1}{n}} = \left(a^m\right)^{\frac{1}{n}} = \sqrt[n]{a^m}$.

a sonning m- darajasining n-darajali ildizini topish uchun a ning n- darajali ildizini m- darajaga ko‘tarish kifoya, ya’ni $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ $(a > 0)$. (6)

Ildizdan ildiz chiqarish uchun ildiz ostidagi ifoda o‘zgartirilmay qoldiriladi, ildizlar ko‘rsatkichlari esa ko‘paytiriladi: $\sqrt[n]{\sqrt[m]{a}} = \sqrt[n\cdot m]{a}$ $(a \geq 0)$. (7)

Haqiqatan, $\sqrt[n]{\sqrt[m]{a}} = \left((a)^{\frac{1}{m}}\right)^{\frac{1}{n}} = a^{\frac{1}{m}\cdot\frac{1}{n}} = a^{\frac{1}{n\cdot m}} = \sqrt[nm]{a}$..

Har xil ko‘rsatkichli $\sqrt[n]{a}, \sqrt[m]{b}, ..., \sqrt[k]{c}$ ildizlarni bir xil ko‘rsatkichli ildizlarga aylantirish uchun $n, m, ..., k$ sonlarining umumiy karralisi (bo‘linuvchisi) bo‘lgan a soni topiladi. $a = nu = mv = ... = kw$ bo‘lsin, bunda $u, v, ..., w$ -qo‘shimcha ko‘paytuvchilar. Natijada ildizlar quyidagi ko‘rinishga keladi: $\sqrt[\alpha]{a^u}, \sqrt[\alpha]{b^v}, ..., \sqrt[\alpha]{c^w}$..

3-m i s o l. $\sqrt[8]{10} > \sqrt[4]{3}$, chunki $\sqrt[8]{10} > \sqrt[4]{3}, 10 > 9$.

3. Irratsional ifodalarni soddalashtirish

Sonlar, harflar va algebraik amallar (qo‘shish, ayirish, ko‘paytirish,bo‘lish, darajaga ko‘tarish va ildiz chiqarish) bilan tuzilgan ifoda *algebraik ifoda* deyiladi.

Ildiz chiqarish amali qatnashgan ifoda shu argumentga nisbatan *irratsional ifoda* deyiladi.

Masalan, $3-\sqrt{5}, \sqrt{5+\sqrt{a}}, \sqrt{a^2-\sqrt{ab}}$ ifodalar irratsional ifodalardir.

Irratsional ifodalar ustida amallar arifmetik amallar qonunlariga va ildizlar ustida amal qoidalariga muvofiq bajariladi.

4-m i s o l. Darajani ildiz ostidan chiqarishda daraja ko‘rsatkichi ildiz ko‘rsatkichiga bo‘linadi. Chiqqan bo‘linma va qoldiq mos tartibda ildiz ostidan chiqqan va ildiz ostida qolgan sonlaming daraja ko‘rsatkichlarini beradi: $\sqrt[5]{a^7b^9c^{-10}} = abc^{-25}\sqrt{a^2b^4}$.

5-m i s o l. a^u b^v... c^w ifodali maxrajni m-darajali ildiz ostidan chiqarish (kasrni irratsionallikdan qutqazish) uchun ildiz ostidagi kasrning surat va maxraji a^{m-u}, b^{m-v},..., c^{m-w} ga ko‘paytirilishi kifoya:

$$x = \sqrt[3]{\frac{a^5}{c^u d^v}} = \sqrt[3]{\frac{a \cdot c^{3-u} \cdot d^{3-v}}{c^u \cdot d^v \cdot c^{3-u} \cdot d^{3-v}}} = \sqrt{\frac{a^5 \cdot c^{3-u} \cdot d^{3-v}}{c^3 \cdot d^3}} = \frac{1}{c \cdot d}\sqrt[3]{a^5 \cdot c^{3-u} \cdot d^{3-v}}.$$

6-misol. $\sqrt[n]{a}$ $(a \geq 0)$ ildizni m-darajaga ko‘taramiz: $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$.

Agar $m = kn + 1$ bo‘lsa, $\sqrt[n]{a^{kn+l}} = a^k \cdot \sqrt[n]{a^l}$ bo‘ladi.

7- m i s o l. Ildizlarni ko‘paytirish va bo‘lish:

$$\sqrt[m]{A} \cdot \sqrt[n]{B} = \sqrt[nm]{A^n} \cdot \sqrt[nm]{B^m} = \sqrt[nm]{A^n B^m}; \frac{\sqrt[m]{A}}{\sqrt[n]{B}} = \sqrt[mn]{\frac{A^n}{B^m}}.$$

8-m i s o l. Murakkab kvadrat ildizni almashtirish:

$$\sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A + \sqrt{A^2 - B}}{2}} \pm \sqrt{\frac{A - \sqrt{A^2 - B}}{2}},$$

bunda $A > 0$, $B > 0$, $A^2 > B$.

3.1. Ifoda qiymatining butun qismini toping.

1) $\sqrt{50}$; 2) $\sqrt{70}$; 3) $\sqrt{125} + \sqrt{256}$; 4) $\sqrt{1524} + \sqrt{2555}$;

5) $2\sqrt{19} + 3\sqrt{22}$; 6) $9\sqrt{221} \cdot \frac{\sqrt{56}}{7}$; 7) $5\sqrt{88} - 2\sqrt{29}$;

8) $4\sqrt[3]{125} + \sqrt[5]{222}$; 9) $21\sqrt[3]{8} + 2\sqrt[4]{84}$; 10) $\sqrt{4045} + \sqrt[10]{1024}$.

3.2. Ifodani soddalashtiring:

1) $\frac{1}{3 - \sqrt{8}} - 2\sqrt{2} + 6$; 2) $\frac{2}{5 - \sqrt{24}} + 5\sqrt{6} - 4$;

3) $\frac{\sqrt{125}}{5} + \frac{12}{16}\sqrt{441} + 9$; 4) $\frac{15\sqrt{75}}{5} + 2\sqrt{3} - \frac{\sqrt{3}}{12}$.

3.3. Sonlarni taqqoslang:

1) $\sqrt{7}+\sqrt{3}$ *va* $\sqrt{10}$; 2) $\sqrt{24}$ *va* $\sqrt{16}+\sqrt{8}$;

3) $2\sqrt{28}+5$ *va* $\sqrt{255}$; 4) $\sqrt[3]{32}+2$ *va* $\sqrt{5}+\sqrt{2}$;

5) $\frac{12}{2\sqrt{14}}+5$ *va* $\sqrt{49}+\sqrt[3]{8}$; 6) $\sqrt[4]{625}-2\sqrt[4]{81}$ *va* $\sqrt[3]{-1}$.

3.4. Kasr maxrajidagi irratsionallikni yo'qoting:

1) $\frac{2}{1+\sqrt{2}}$; 2) $\frac{3}{2-\sqrt{3}}$; 3) $\frac{\sqrt{5}}{5+\sqrt{24}}$ 4) $\frac{2-\sqrt{2}}{\sqrt{2}+2}$;

5) $\frac{\sqrt{5}}{5+\sqrt{24}}$; 6) $\frac{\sqrt{3}}{\sqrt{5}+\sqrt{2}}$; 7) $\frac{1}{\sqrt{6}-\sqrt{5}}$; 8) $\frac{9}{\sqrt{20}-\sqrt{11}}$;

9) $\frac{21}{\sqrt{14}-\sqrt{7}}$; 10) $\frac{\sqrt{13}}{\sqrt{5}-\sqrt{12}}$; 11) $\frac{2\sqrt{2}}{\sqrt[3]{5}-\sqrt[3]{2}}$; 12) $\frac{1}{\sqrt[3]{2}+\sqrt{3}}$.

3.5. Ifodani soddalashtiring:

1) $\left(\sqrt{7}-5\right)\cdot\left(5+\sqrt{7}\right)$; 2) $\left(\sqrt{51}+2\right)\cdot\left(2-\sqrt{51}\right)$;

3) $\sqrt{2+\sqrt{9+4\sqrt{2}}}$; 4) $\sqrt{7+2\sqrt{2+\sqrt{8-2\sqrt{15}}}}$;

5) $\sqrt{2-\sqrt{8+\sqrt{7+2\sqrt{12}}}}$; 6) $\sqrt{5-\sqrt{3-\sqrt{11+\sqrt{120}}}}$;

7) $\frac{\left(\sqrt{14}-5\right)\cdot\left(5+\sqrt{14}\right)}{\left(\sqrt{21}-\sqrt{10}\right)\cdot\left(\sqrt{21}+\sqrt{10}\right)}$; 8) $\frac{\left(\sqrt{23}-4\right)\cdot\left(\sqrt{23}+4\right)}{\left(\sqrt{11}-2\right)\cdot\left(\sqrt{11}+2\right)}$.

4-§. Haqiqiy sonlar

1. Sonning moduli

a haqiqiy sonning moduli deb,

$$|a|=\begin{cases} a, & \text{agar } a\geq 0 \text{ bo'lsa,} \\ a, & \text{agar } a\leq 0 \text{ bo'lsa} \end{cases}$$

munosbat bilan aniqlanadigan $|a|$ soniga aytiladi. Uning asosiy xossalarini keltiramiz:

1) $a \le |a|$; 2) $|ab| = |a| \cdot |b|$; 3) $|a + b| \le |a| + |b|$;

4) $\left|\frac{1}{a}\right| = \frac{1}{|a|}$; 5) $|a - b| \ge |a| - |b|$; 6) $|0| = 0$.

4.1. Taqqoslang:

1) $|7{,}7|$ va 7; 2) $-|-3{,}2|$ va $-3{,}2$; 3) $|0|$ va 0;

4) $|a|$ va 0; 5) $|-22{,}2|$ va 22,2; 6) $-3|a|$ va 0;

7) $3|3|$ va 9; 8) $-|9|$ va $|-9|$; 9) a va $|a|$;

10) $|2{,}12|$ va $-(-2.12)$.

4.2. Hisoblang:

1) $|5| \times |12| + |22| : |-2|$; 2) $|-18| : |-6| + |12 - 18|$;

3) $|36 - 54| : |-2| - |9| \times |-3|$; 4) $-|24| : |-6| - |-3| \times |-8|$;

5) $|2{,}2| - |3{,}6| \times |-2|$; 6) $-|3{,}3| \times 5 - |9{,}2| : |-2|$.

4.3. Harflarning ko‘rsatilgan qiymatlarida ifodaning qiymatini hisoblang:

1) $2|a| + 3|b|$ $a = -2$, $b = 4$;

2) $|-3a| + 5|-2b|$ $a = -1$, $b = -3$;

3) $-|a| + 3|-b|$ $a = 1$, $b = 0$;

4) $|-5a| - 4|a \cdot b|$ $a = 3$, $b = -2$;

5) $\dfrac{|-5a| + |b - a|}{-2|b + 2b|}$ $a = -1$, $b = 1$;

6) $\dfrac{-8|4a \cdot b| + 3|a - b|}{|2a + 3b| \cdot 4a|b|}$ $a = -4$, $b = 1$.

4.4. Ifodani modul belgisisiz yozong:

1) $|x - 1|$; 2) $|2x - 5|$; 3) $|5 - 4x|$;

4) $|x + 5|$; 5) $|-3x + 7|$; 6) $5x + |a - 1|$;

7) $|-x - 8|$; 8) $|-9x - 7|$; 9) $|-5x - 6|$;

10) $3|x - 9|$; 11) $-6|5x - 4| + 4$; 12) $|x + 1| + |x - 1|$;

13) $|x - 4| - 3|2 + x|$; 14) $|3x - 4| - |x - 5|$;

15) $|5x - 7| + |6x - 9|$; 16) $|-x + 5| + |4x - 2|$.

2. Haqiqiy sonning butun va kasr qismi

a sonining *butun qismi* deb, *a* dan katta bo'lmagan butun sonlarning eng kattasiga aytiladi va [*a*] yoki *E* (*a*) orqali belgilanadi. O'qilishi: «*a* ning butun qismi» yoki «antye *a*» (fransuzcha *entiere* – butun).

1- m i s o l. [4,2] = [4,9] = 4; [0,2] = [0,99] = [0] = 0; [–1,1] = =[–1,6] = –2;

shu kabi $10\frac{5}{6}+5\frac{4}{6}=16\frac{3}{6}$ bo'lgani uchun $\left[10\frac{5}{6}+5\frac{4}{6}\right]=\left[16\frac{3}{6}\right]=16;$

$38 \times [0.6] = 38 \times 0 = 0;\ 10:\left[5\frac{4}{9}\right]=2$; [*E*] = 2; [–*E*] = –3

Sonning butun qismi quyidagi xossalarga ega:

1- x o s s a. $a, b\in Z$ bo'lganda, $[a + b] = [a] + [b]$ bo'ladi.

2- x o s s a. $a, b\in R$ bo'lganda, $[a + b] \geq [a] + [b]$ bo'ladi.

[8 + 20] = [8] + [20] = 28; [8,8] + [8,9] = 8 + 8 = 16;

[8,8 + 8,9] = [17,7] = 17. 16 < 17.

a – [*a*] ayirma *a* sonining *kasr qismi* deyiladi va {*a*} orqali belgilanadi:

$$\{a\} = a - [a] > 0,\ 0\leq\{a\} < 1, \text{ bunda } a = [a] + \{a\}.$$

2 - m i s o l. $\left\{15\frac{2}{7}\right\}=\frac{2}{7}$; {–19} = {–2 + 0,1} = 0,1.

4.5. Hisoblang:

1) [8,8]; 2) [5]; 3) [0]; 4) [0,6];
5) [–1,4]; 6) [–0,58]; 7) [–4,4]; 8) [0,7];
9)[–0,1]; 10) [17]; 11) [–34]; 12) [15];
13) $\left[\frac{187}{24}\right]$; 14) $\left[\frac{18}{5}\right]$; 15) $\left[\frac{200}{14}\right]$; 16) $\left[3\frac{8}{3}\right]$.

4.6. Hisoblang:

1) $25\cdot\left[\frac{1}{5}\right]$; 2) $12\cdot\left[\frac{2}{8}\right]$; 3) $4\cdot\left[1\frac{8}{10}\right]$;

4) $\left[9\frac{3}{7}+6\frac{7}{9}\right]$; 5) $\left[22\frac{4}{5}-11\frac{4}{7}\right]$; 6) $\left[\frac{33}{12}+1\frac{7}{9}\right]$;

7) $\left[15\frac{8}{11}\right]+\left[7\frac{1}{9}\right]$; 8) $\left[26\frac{7}{18}\right]-\left[5\frac{9}{12}\right]$; 9) $\left[\frac{189}{52}\right]+\left[\frac{789}{125}\right]$.

4.7. Hisoblang:

1) $\{2,2-5\}$; 2) $\{-3,5+5\}$; 3) $\{-5,5\}+\{6,2\}$; 4) $\{4,8\}-\{-3,8\}$;

5) $\left\{\frac{18}{7}+4\frac{8}{9}\right\}$; 6) $\left\{1\frac{7}{8}-\frac{9}{4}\right\}$; 7) $3\cdot\left\{5\frac{1}{2}:0,5\right\}$; 8) $\{0,7\cdot1,9\}$.

4.8. Tenglamani yeching:

1) $[2x+9]=7$; 2) $[5x-7]=6$;
3) $[-3x+8]=-4$; 4) $[9-x]=-4$;
5) $[1,5x+2,5]=4$; 6) $[0,4x-8,2]=7$;
7) $[-3,3+x]=11$; 8) $[3-x]=14$;
9) $\left[\frac{2x-5}{4}\right]=7$; 10) $\left[\frac{9-x}{3}\right]=10$.

3. Proporsiya

$a\in R$, $b\in R$ *va* $b\neq 0$ bo'lsa, $\frac{a}{b}$ ifodaga *nisbat* deyiladi.

Ikki nisbatning tengligi *proporsiya* deyiladi. Proporsiya umumiy holda

$$\frac{a}{b}=\frac{c}{d} \text{ yoki } (a:b=c:d)$$

ko'rinishda yoziladi, bunda $b\neq 0$, $d\neq 0$. a, d lar proporsiyaning *chetki* hadlari, b, c lar esa *o'rta* hadlari deyiladi.

Proporsiyaning chetki hadlari ko'paytmasi o'rta hadlari ko'paytmasiga teng, ya'ni $a\cdot d=b\cdot c$.

1-m i s o l. $3:x=5:10$ proporsiyaning noma'lum hadini toping.

Y e c h i s h. Yuqoridagi qoidaga asosan $3\cdot10=5\cdot x$, yoki $5x=30$, $x=6$;

2-m i s o l. $8:10=x:5$ proporsiyaning noma'lum hadini toping.

Y e c h i s h. $10 \cdot x = 8 \cdot 5$, yoki $10x = 40$, $x = 4$.

4.9. Quyidagi nisbatlardan proporsiya tuzish mumkinmi:

1) 21 : 28 va 36 : 48; 2) 3,5 : 21 va 1 : 6;
3) 18 : 10 va 9 : 5; 4) 0,1 : 0,02 va 4 : 0,8.

4.10. Proporsiyaning noma'lum hadini toping:

1) $14 : 5 = x : 10$; 2) $54 : 3 = x : 9$;
3) $12 : x = 6 : 9$; 4) $3 : x = 9 : 4$;
5) $x : 24 = 12 : 6$; 6) $x : 6 = 7 : 21$.
7) $38 : 15 = 19 : x$; 8) $11 : 4 = 8 : x$;
9) $x : \frac{7}{5} = \frac{8}{2} : 1\frac{4}{5}$; 10) $x : \frac{11}{6} = 1\frac{4}{5} : 2\frac{7}{9}$;
11) $5\frac{8}{12} : x = 1\frac{2}{3} : 2\frac{7}{6}$; 12) $3\frac{4}{9} : x = 0,7 : \frac{19}{5}$;
13) $4\frac{7}{12} : \frac{15}{4} = x : 0,5$; 14) $5\frac{3}{2} : 4 = x : \frac{4}{9} : 8$;
15) $2\frac{14}{20} : 0,7 = \frac{7}{9} : x$; 16) $3\frac{10}{41} : 4\frac{8}{12} = 1\frac{1}{2} : x$;
17) $14,5 : x = 4,2 : 3,3$; 18) $26,2 : x = 13,1 : 0,2$.

4.11. Proporsiyadan x ni toping:

1) $6x : 15 = 9 : 10$; 2) $14x : 5 = 21 : 8$;
3) $51 : 3x = 17 : 5$; 4) $49 : 5x = 7 : 8$;
5) $79 : 30 = 12x : 6$; 6) $33 : 7 = 12x : 5$;
7) $14 : 50 = 12 : 3x$; 8) $90 : 25 = 12 : 9x$;
9) $1\frac{7}{9}x : 5\frac{7}{2} = 2\frac{4}{8} : 6\frac{4}{5}$; 10) $6\frac{8}{12}x : 4\frac{7}{21} = 3\frac{7}{12} : 2\frac{3}{8}$;
11) $2\frac{7}{9} : 3\frac{8}{13}x = 4\frac{7}{21} : 2\frac{11}{5}$; 12) $1\frac{14}{20} : 2\frac{24}{10}x = 5\frac{10}{12} : 0,44$.

4. Protsent (foiz)lar

Berilgan sonning bir *protsenti* (foizi) deb, uning yuzdan bir qismiga aytiladi va % bilan belgilanadi.

Masalan, p sonning 1% i $\frac{p}{100}$ kasrni bildiradi.

Demak, $1\% = \frac{1}{100}, 15\% = \frac{15}{100}, 75\% = \frac{75}{100} = \frac{3}{4}$.

Protsentlarga doir 4 xil masalaga duch kelamiz:

1) sonning protsentini topish;

2) protsentiga ko'ra sonni topish;

3) ikki sonning protsent nisbatini topish;

4) murakkab protsentga doir masalalar.

1- m a s a l a. a sonining b % i bo'lgan x sonini toping.

$$b\% = \frac{b}{100}, x = \frac{ab}{100}.$$

Masalan, 520 ning 15% i quyidagicha topiladi:

$$= \frac{520 \cdot 15}{100} = \frac{7800}{100} = 78.$$

2- m a s a l a. Sonning b % i A ga teng. Shu sonni toping.

$\frac{b}{100}$ bo'lagi A ga teng bo'lgan x son $x = \frac{A \cdot 100}{b}$ bo'ladi.

Masalan, sonning 80 % i 36 bo'lsa, sonning o'zi: $x = \frac{36 \cdot 100}{80} = 45$.

3-ma s a l a. a soni b sonining necha protsentini tashkil etadi. Bu yerda a sonining b soniga nisbatini protsentlarda ifoda qilish kerak:

$$x = \frac{a}{b} \cdot 100.$$

Masalan, kollejda 1200 nafar o'quvchi bo'lib, 300 nafari qizlar. Qizlar kollej o'quvchilarining necha protsentini tashkil qiladi?

$$x = \frac{300 \cdot 100}{1200} = 25\%.$$

4- m a s a l a. Bank mijozlariga p % foyda beradi. Mijoz bankka a so'm pul topshirsa, n yildan so'ng mijozning puli necha so'm bo'ladi?

Y e c h i s h. Bankka a so'm qo'ygan mijoz 1 yildan so'ng

$$N_1 = a + \frac{a}{100} \cdot p = a\left(1 + \frac{p}{100}\right)$$

so‘m, 2 yildan so‘ng

$$N_2 = N_1 + \frac{N_1}{100} \cdot p = a\left(1 + \frac{p}{100}\right)^2$$

so‘m, 3 yildan so‘ng

$$N_3 = N_2 + \frac{N_2}{100} \cdot p = a\left(1 + \frac{p}{100}\right)^3$$

so‘mga ega bo‘ladi.

Shu jarayonni davom ettirib, mijoz n yildan so‘ng

$$N_n = a\left(1 + \frac{p}{100}\right)^n$$

so‘mga ega bo‘lishini ko‘ramiz. Oxirgi tenglik odatda *murakkab protsentlar formulasi* deb ataladi.

4.12. Kasr ko‘rinishida ifodalang:

1) 8%; 2) 19%; 3) 26%;
4) 0,25%; 5) 0,98%; 6) 5,2%;
7) 542%; 8) 38%; 9) $214\frac{2}{4}\%$;
10) $3\frac{7}{9}\%$; 11) $25\frac{12}{36}\%$; 12) 0,189%.

4.13. Foizlarda ifodalang:

1) 0,6; 2) 3,12; 3) 7,9;
4) 16; 5) $4\frac{1}{6}$; 6) $7\frac{4}{9}$;
7) 58; 8) 43; 9) 36;
10) $6\frac{5}{9}$; 11) 5,22; 12) 0,69.

4.14. 1) 1 ning 5 ga; 2) 8 ning 3 ga;
3) 6 ning 5 ga; 4) 14,5 ning 25 ga;
5) 2,2 ning 1,16 ga; 6) 0,52 ning 12 ga;

7) $41\frac{1}{5}$ ning $\frac{2}{15}$ ga; 8) $4\frac{1}{6}$ ning $2\frac{1}{6}$ ga protsent nisbatini toping.

4.15. a ning p % va q % ini toping:

1) $a = 25; p = 5, q = 4;$ 2) $a = 124; p = 24, q = 30;$

3) $a = 175; p = 4\frac{1}{6}, q = 9;$ 4) $a = 1{,}775; p = 222, q = 138.$

4.16. p % i a ga teng bo'lgan sonni toping:

1) $p = 1{,}2; a = 44;$ 2) $p = 0{,}7; a = 2{,}36;$

3) $p = 85; a = 10;$ 4) $p = 25; a = 1{,}55.$

4.17. Ishchi 600 000 so'm maosh oladi. Ishchining maoshi dastlab 20% ga oshirildi va yana 20% ga oshirildi. Ishchining maoshi necha % ga oshgan?

4.18. To'g'ri to'rtburchakning eni 100% uzaytirildi, bo'yi esa 10% qisqartirildi. Uning yuzi o'zgaradimi? Agar o'zgarsa, qanchaga o'zgaradi?

4.19. Meva quritilganda o'z og'irligining 79% ini yo'qotadi. 36 kg quritilgan meva olish uchun necha kg ho'l meva olish kerak?

4.20. 20% ga arzonlashtirilgan tovar 22000 so'mga sotildi. Tovarning dastlabki narxini toping.

4.21. Kollej binosi sirtining 84% ini bo'yash uchun 24,5 kg bo'yoq ketdi. Kollej binosi sirtining qolgan qismini bo'yash uchun qancha bo'yoq kerak bo'ladi?

4.22. Xalq banki yiliga 18% foyda to'laydi. Omonatchi kassaga 2 000 000 so'm qo'ydi. Besh yildan keyin uning kassadagi puli necha so'm bo'ladi?

II bob bo'yicha test topshiriqlari

1. Ifodaning qiymatini toping:

$18 \times 36 - 16 \times 36 + 24 \times 27 - 25 \times 24 - 21 \times 5$

A) 45 B) 15 C) 0 D) 115

2. 17827516 son quyidagi sonlardan qaysi biriga qoldiqsiz bo'linadi?

A) 3 B) 10 C) 4 D) 5

3. Quyudagi sonli ketma-ketlikdan qaysilari tub sonlardan iborat?

1) 0,3,5,7,11 2)1,3,5,7,13 3) 3,5,7,9,11

4) 2,3,5,7,17 5) 3,5,17,19,381

A) 1; 2 B) 2; 4 C) 5 D) 4

4. 840 va 264 ning umumiy bo'luvchilari nechta?

A) 9 B) 4 C) 6 D) 7

5. Qaysi juft sonlar o'zoro tub sonlardan iborat?

A) 8; 14 B) 11; 12 C)12; 35 D)12; 34

6. 21 va 35 sonlarining EKUKi bilan EKUB ining yig'indisini toping.

A) 108 B) 110 C) 112 D) 109

7. 72 va 96 sonlarinig EKUKining EKUBga nisbatini toping.

A) 10 B) 0,1 C) 9 D) 12

8. 18 va 12 sonlari EKUKining EKUBga ko'paytmasini toping.

A) 220 B) 218 C) 214 D) 216

9. 108 va 135 sonlarining EKUKining 12 va 54 sonlari EKUKga nisbatini toping.

A) 8 B) 5 C) 12 D) 6

10. Dastlabki 30 ta natural sonlar ichida 6 soni bilan o'zaro tub bo'lgan sonlar nechta?

A) 7 B) 8 C) 9 D) 10

11. [4:8] kesmada nechta o'zaro tub sonlar jufti bor?

A) 5 B) 6 C) 7 D) 4

12. 0,6 ga teskari sonni toping.

A) –0,6 B) 1,(6) C) 0,4 D) –6

13. 0,8 ga teskari bo'lgan songa qarama-qarshi sonni toping.

A) –0,8 B) 1,25 C) –1,25 D) –1,2

14. Hisoblang: 0,(8) + 0,(7).

A) 3 B) 1,(6) C) 1,25 D) 1,(5)

15. m = 0,22(23), n = 0,2(223), l = 0,222(3) sonlarini o'sish tartibida yozing.

A) $n < m < l$ B) $l < m < n$ C) $m < n < l$ D) $m < l < n$

16. Quyidagi oddiy kasr ko‘rinishida berilgan sonlardan qaysilarini chekli o‘nli kasr ko‘rinishiga keltirib bo‘lmaydi: 1) 7/32; 2) 11/160; 3) 5/48 4) 5/14.

A) 2; 3 B) 3; 4 C) 4; 1 D) 1; 2

17. Agar x natural son bo‘lsa, quyidagi sonlardan qaysi biri albatta juft son bo‘ladi.

A) $\frac{x(x+1)(x+2)}{2}$ B) $\frac{x(x+1)(x+2)}{3}$ C) $\frac{x}{2}$ D) $\frac{x(x+1)(x+2)}{4}$

18. $\left(4\frac{1}{10}-3\frac{4}{15}\right)\cdot\frac{5}{6}+4\frac{1}{10}:1\frac{1}{5}$ ni hisoblang.

A) $3\frac{5}{9}$ B) $4\frac{1}{9}$ C) $5\frac{2}{3}$ D) $7\frac{1}{5}$

19. $\frac{3}{4}\cdot1\frac{1}{7}:\frac{2}{15}\cdot12\frac{1}{4}:7\frac{1}{2}$ ni hisoblang.

A) $10\frac{1}{2}$ B) 11 C) $9\frac{1}{4}$ D) $7\frac{1}{2}$

20. $\frac{2}{3}$ va $\frac{5}{6}$ sonlar orasida maxraji 30 ga teng nechta kasr bor.

A) 1 B) 2 C) 4 D) 5

21. $\frac{3,12\cdot5,95-4,44}{2,21\cdot5,95+1,51}$ ni hisoblang.

A) 1 B) 2 C) $\frac{1}{2}$ D) $1\frac{1}{2}$

22. $\frac{244\cdot395-151}{244+395\cdot243}$ ni hisoblang.

A) 1 B) 2 C) 3 D) $1\frac{1}{2}$

23. [(1,2:36) + 0,3] × 9 ni hisoblang.

A) 148,5 B) 1,5 C) 150 D) 15

24. 3,4(3) davriy kasr qaysi oddiy kasrga teng.

A) $1\frac{13}{33}$ B) $3\frac{3}{11}$ C) $3\frac{2}{45}$ D) $3\frac{13}{30}$

III BOB. BIR O‘ZGARUVCHILI KO‘PHADLAR

1-§. Kompleks sonlar

1. Algebraik shakldagi kompleks sonlar va ular ustida amallar

Kompleks son haqida gapirishdan avval ushbu tenglamaning yechimlarini ko‘rib chiqamiz: $x^2 + 4 = 0$; tenglamani yechish jarayonida $x_1 = \sqrt{-1}$ va $x_2 = -2\sqrt{-1}$ «sonlar» hosil bo‘ladi. Haqiqiy sonlar orasida esa bunday «sonlar» mavjud emas. Bunday holatdan qutulish uchun -1 ga son deb qarash zarurati paydo bo‘ladi.

Bu yangi son hech qanday real kattalikning o‘lchamini yoki uning o‘zgarishini ifodalamaydi. Shu sababli uni *mavhum* (xayoliy, haqiqatda mavjud bo‘lmagan) *birlik* deb atash va maxsus belgilash qabul qilingan: $\sqrt{-1} = i$. Mavhum birlik uchun $i^2 = -1$ tenglik o‘rinlidir.

$a + bi$ ko‘rinishdagi ifodani qaraymiz. Bu yerda a va b lar istalgan haqiqiy sonlar, i esa mavhum birlik. $a + bi$ ifoda *haqiqiy* son a va mavhum son bi lar «kompleksi»dan iborat bo‘lgani uchun uni kompleks son deb atash qabul qilingan.

$a + bi$ ifoda *algebraik shakldagi kompleks* son deb ataladi, bu yerda $a \in R$, $b \in R$, $i^2 = -1$. Bu paragrafda $a + bi$ ni qisqalik uchun «kompleks son» deb ataymiz.

Kompleks sonlarni bitta harf bilan belgilash qulay. Masalan, $a + bi$ ni $z = a + bi$ ko‘rinishda belgilash mumkin. $z = a + bi$ kompleks sonning *haqiqiy* qismi a ni $Re(z)$ bilan, mavhum qismi b ni esa $\mathrm{Im}(z)$ bilan belgilash qabul qilingan: $a = Re(z)$, $b = \mathrm{Im}(z)$.

Agar $z = a + bi$ kompleks son uchun $b = 0$ bo‘lsa, haqiqiy son $z = a$ hosil bo‘ladi. Demak, haqiqiy sonlar to‘plami R, barcha kompleks sonlar to‘plami C ning qism to‘plami bo‘ladi: $R \in C$.

1- mi s o l. $z_1 = 1 + 4i$, $z_2 = 6 - i$, $z_3 = 4{,}2$, $z_4 = 2i$, $z_5 = 0$ kompleks sonlarning haqiqiy va mavhum qismlarini topamiz.

Y e c h i s h. Kompleks son haqiqiy va mavhum qismlarining aniqlanishiga ko‘ra, quyidagilarga egamiz:

$Re(z_1) = 1$; $Re(z_2) = 6$; $Re(z_3) = 4{,}2$; $Re(z_4) = 0$; $Re(z_5) = 0$;

$Im(z_1) = 4$; $Im(z_2) = -1$; $Im(z_3) = 0$; $Im(z_4) = 2$; $Im(z_5) = 0$.

Kompleks sonlar uchun « < », « > » munosabatlari aniqlanmaydi, lekin teng kompleks sonlar tushunchasi kiritiladi. Haqiqiy va mavhum qismlari mos ravishda teng bo‘lgan kompleks sonlar *teng kompleks sonlar* deb ataladi.

Masalan, $z_1 = 2{,}5 + \frac{4}{5}i$ va $z_2 = \frac{5}{2} + 0{,}8i$ sonlari uchun $Re(z_1) = Re(z_2) = 2{,}5$, $Im(z_1) = Im(z_2) = 0{,}8$. Demak, $z_1 = z_2$.

Bir-biridan faqat mavhum qismlarining ishorasi bilan farq qiladigan ikki kompleks son *o‘zaro qo‘shma kompleks sonlar* deyiladi. $z = a + bi$ kompleks songa qo‘shma kompleks son $\overline{z} = a - bi$ ko‘rinishda yoziladi. Masalan, $9 + 5i$ va $9 - 5i$ lar qo‘shma kompleks sonlardir: $\overline{9+5i} = 9 - 5i$. Shu kabi $\overline{z}$ soniga qo‘shma son $\overline{\overline{z}} = z$ bo‘ladi. Masalan, $\overline{\overline{9+5i}} = \overline{9-5i} = 9+5i$. a haqiqiy songa qo‘shma son a ning o‘ziga teng: $\overline{a} = \overline{a+0\cdot i} = a - 0\cdot i = a$. Lekin bi mavhum songa qo‘shma son $\overline{bi} = -bi$. Chunki $\overline{bi} = \overline{0+bi} = 0 - bi = -bi$, $a, b \in R$.

Kompleks sonlar ustida arifmetik amallar quyidagicha aniqlanadi:

1) $(a + bi) + (c + di) = (a + c) + (b + d)i$;

2) $(a + bi) - (c + di) = (a - c) + (b-d)i$;

3) $(a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i$;

4) $\frac{a+bi}{c+di} = \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}$.

2-m i s o l. $z_1 = 1 + 2i$, $z_2 = 2 - i$ kompleks sonlarni 1) qo‘shing; 2) ayiring; 3) ko‘paytiring va bo‘ling.

1) $z_1 + z_2 = (1 + 2i) + (2-i) = (1 + 2) + (2 - 1)i = 3 + i$;

2) $z_1 - z_2 = (1 + 2i)-(2-i) = (1 - 2)-(2 + 1)i = -1 - 3i$;

3) $z_1 \cdot z_2 = (1 + 2i) \cdot (2-i) = (1 \cdot 2 + 2 \cdot 1) + (1 \cdot (-1) + 2 \cdot 2)i$;

4) $\frac{z_1}{z_2}=\frac{1+2i}{2-i}=\frac{1\cdot 2+2\cdot(-1)}{2^2+(-1)^2}+\frac{2\cdot 2-1\cdot(-1)}{2^2+(-1)^2}i.$

$z = a + bi$ kompleks songa *qarama-qarshi* bo‘lgan yagona kompleks son mavjud va bu son $-z = -a - bi$ kompleks sonidan iborat.

1.1. Kompleks son z ning haqiqiy qismi Re(z) ni va mavhum qismi Im(z) ni toping:

1) $z = 2 + 3i$; 2) $z = 4 + 5i$; 3) $z = 0{,}2 - 4i$;
4) $z = -3i + 5$; 5) $z = -5 + 9i$; 6) 15;
7) $z = 22i$; 8) $z = 6 - 0{,}2i$; 9) $z = -3 - 3i$;
10) $z=\frac{4}{3}+\frac{5}{7}i$; 11) $z=\frac{9}{5}-\frac{17}{29}$; 12) $z=-\frac{5}{57}i$.

1.2. Agar:
1) Re(z) = 9, Im(z) = 4; 2) Re(z) = 5, Im(z) = 0;
3) Re(z) = – 7, Im(z) = 0,7; 4) Re(z) = – 5, Im(z) = – 6;
5) Re(z) = 22, Im(z) = – 8; 6) Re(z) = 0, Im(z) = 51 bo‘lsa, z kompleks sonini algebraik shaklda yozing.

1.3. Teng kompleks sonlarni toping:

$4+0{,}1i;\ 0{,}6-0{,}2i;\ \frac{28}{7}+\frac{1}{10}i;\ 1-0{,}7i;\ \frac{3}{5}-\frac{1}{5}i;\ 0{,}5+2i;$

$4{,}2-1{,}5i;\ \frac{1}{2}+2i;\ 5{,}5+i;\ 1-\frac{7}{10}i;\ -1{,}1-0{,}5i;\ \frac{21}{5}-\frac{3}{2}i.$

1.4. Agar:

1) $z = -5 + 5i$; 2) $z = -2i$; 3) $z = 8 + i$;

4) $z = 2 - 5i$; 5) $z = 2{,}3$; 6) $z = 0$;

7) $z = -3 - 2i$; 8) $z = 3i$; 9) $z = 12 + 4i$;

10) $z = 3 + 5i$; 11) $z = 1{,}(6)$; 12) $z = -0{,}(8) - 4{,}(6)i$ bo‘lsa, $\bar{z}$ ni toping.

1.5. Yig‘indini toping:

1) $(4 + 4i) + (6 - i)$; 2) $4 + (-1 + 2i)$;

3) $(5 + 8i) + (3 - 9i)$; 4) $(3{,}5 - 6i) + (1{,}5 - 4i)$;

5) $(9 + 2i) + (-5 - i)$; 6) $(3 + 2i) + (3 - 2i)$;
7) $(-5 + i) + (6 - 2i)$; 8) $9i + (-8 + 6i)$;
9) $-2,5 + (6,5 - 7i)$; 10) $(30 + 9i) + 6i$;
11) $-8i + (2 - 9i)$; 12) $16 + (28 - 11i)$.

1.6. Ayirmani toping:

1) $(2 + 3i) - (5 - 7i)$; 2) $(-6 + 3)i - (6,2 + i)$;
3) $(4 + 6i) - (3i + 7)$; 4) $(0,(5) + 4i) - (12 - 12i)$;
5) $(0,2 + 3i) - (4,8 + 6i)$; 6) $i - (5i + 9)$;

7) $z = 1 - i$; 8) $z = \frac{1}{2} - i\frac{\sqrt{3}}{2}$;

9) $z = \frac{\sqrt{33}}{2} - i\frac{\sqrt{11}}{2}$; 10) $z = \frac{\sqrt{3}}{2} - i\frac{1}{2}$;

11) $z = 2i$; 12) $z = \frac{1}{\sqrt{2}} - i\frac{1}{2}$.

1.7. Ko‘paytmani hisoblang:

1) $(4 + 7i)(1 + i)$; 2) $(3 + 4i)(2 - 3i)$;
3) $(4 + 7i)(1 - i)$; 4) $4 \cdot (6,2 - 2i)$;
5) $(9 - 3i)(2 - 3i)$; 6) $(5 - 2i)(2i + 5)$;
7) $(-2 + i)(2 + i)$; 8) $(-3 + i)(3 - i)$;
9) $(4,(1) + 3i)(2 + i)$; 10) $0 \cdot (4,5-i)$;
11) $(1 - i)(1 - i)$; 12) $(3 + i)(-1 + 3i)$;
13) $(1+i)\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$; 14) $\left(\frac{\sqrt{33}}{2} + i\frac{\sqrt{11}}{2}\right)\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$.

1.8. Ikki kompleks sonning bo‘linmasini toping:

1) $z_1 = 1\text{-i}$ va $z_2 = 1 - i$; 2) $z_1 = -2$ va $z_2 = \text{i}$;
3) $z_1 = 1$ va $z_2 = -i$; 4) $z_1 = -3 - 4i$ va $z_2 = 2 + 3i$;

5) $z_1 = 1 + i$ va $z_2 = \frac{1}{2} + i\frac{\sqrt{3}}{2}$;

6) $z_1 = \frac{\sqrt{33}}{2} + i\frac{\sqrt{11}}{2}$ va $z_2 = \frac{\sqrt{3}}{2} + i\frac{1}{2}$;

7) $z_1 = 2i$ va $z_2 = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$;

8) $z_1 = -i$ va $z_2 = -\sqrt{6} - \sqrt{6}i$;

9) $z_1 = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$ va $z_2 = \frac{\sqrt{2}}{2} + i\frac{\sqrt{6}}{2}$;

10) $z_1 = -\sqrt{6} - \sqrt{6}i$ va $z_2 = \frac{\sqrt{3}}{2} - i\frac{1}{2}$.

1.9. z_1, z_2 kompleks sonlar berilgan. Ular ustida quyidagi amallarni bajaring:

$$z_1 + z_2,\ z_1 - z_2,\ z_1 \cdot z_2,\ \frac{z_1}{z_2},\ \overline{z_1},\ \overline{z_2}.$$

1) $z_1 = -3 + 2i$, $z_2 = 4 - i$;

2) $z_1 = 4 + 5i$, $z_2 = 4 - 5i$;

3) $z_1 = 5 + 2i$, $z_2 = -5 - 2i$;

4) $z_1 = -3 + i$, $z_2 = -2 - 3i$;

5) $z_1 = 1{,}4 - 3i$, $z_2 = 2{,}6 - 4i$;

6) $z_1 = 3 + 8i$, $z_2 = 4 - 5i$;

7) $z_1 = 5 - 2i$, $z_2 = 3 + 4i$;

8) $z_1 = -2 + 3i$, $z_2 = 5 - 2i$;

9) $z_1 = -3 + 4i$, $z_2 = 7 - 4i$;

10) $z_1 = -2 - 4i$, $z_2 = 1 + 3i$;

11) $z_1 = 5 - 3i$, $z_2 = 8 - 4i$;

12) $z_1 = -5 + 2i$, $z_2 = 8 - 9i$;

13) $z_1 = 4 - 5i$, $z_2 = 42 - 3i$;

14) $z_1 = 14 + 3i$, $z_2 = 21 + 3i$;

15) $z_1 = 2 + 4i$, $z_2 = 7 + 4i$;

16) $z_1 = -6 + 2i$, $z_2 = 4 - i$;

17) $z_1 = -3 + 2i$, $z_2 = 5 - i$;

18) $z_1 = 4 + 2i$, $z_2 = 4 - 3i$;

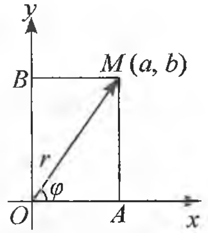
19) $z_1 = 7 + 2i$, $z_2 = 5 + i$;

20) $z_1 = -3 + 2i$, $z_2 = 1 - i$.

2. Trigonometrik shakldagi kompleks sonlar va ular ustida amallar

T a ' r i f. $\sqrt{a^2+b^2}$ manfiy bo'lmagan, haqiqiy sonni $z = a + bi$ *kompleks sonning moduli* deyiladi va $|z| = |a+b|$ ko'rinishda belgilanadi, ya'ni $|z| = \sqrt{a^2+b^2}$.

xOy to'g'ri burchakli Dekart koordinatalari sistemasini olib, Ox ni haqiqiy o'q, Oy ni esa mavhum o'q deymiz. Koordinatalari a va b haqiqiy sonlardan iborat bo'lgan M nuqtani olingan koordinatalar sistemasidagi tasvirini yasaymiz. M nuqtani $z = a + bi$ kompleks sonning geometrik tasviri deyiladi.



5-rasm.

$\overrightarrow{OM}$ vektorning Ox haqiqiy o'qning musbat yo'nalishi bilan hosil qilgan burchakni $z = a + bi$ kompleks sonning argumenti deyiladi va $\varphi = \arg z$ kabi belgilanadi.

$\overrightarrow{OM}$ vektorning uzunligini r orqali belgilasak, $r = |z| = \sqrt{a^2+b^2}$. Buni e'tiborga olsak, shakldan $OA = a = r\cos\varphi$ $OB = b = r\sin\varphi$, u holda $z = a + bi = r(\cos\varphi + i\sin\varphi)$, $\varphi = \operatorname{arctg}\frac{b}{a}$, $r(\cos\varphi + i\sin\varphi)$ ifodani $z = a + bi$ kompleks sonning *trigonometrik shakli* deyiladi.

$z = a + bi$ algebraik shakldagi kompleks sonni trigonometrik shaklga keltirishda a va b larning ishoralariga va $a = r\cos\varphi$, $b = r\sin\varphi$ tengliklarni e'tiborga olish kerak.

Trigonometrik shakldagi kompleks sonlar uchun quyidagi formulalar o'rinli:

1) $\forall(n\in N)$ $(\cos\alpha + i\sin\alpha)^n = \cos n\alpha + i\sin n\alpha$ (Muavr formulasi);

2) $\dfrac{r_1(\cos\varphi_1+\sin\varphi_1)}{r_2(\cos\varphi_2+\sin\varphi_2)} = \dfrac{r_1}{r_2}\left(\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2)\right);$

3) $\sqrt[n]{r(\cos\varphi+i\sin\varphi)} = \sqrt[n]{r}\left(\cos\dfrac{\varphi+2k\pi}{n}+i\sin\cos\dfrac{\varphi+2k\pi}{n}\right)$

(Ildiz chiqarish), $k = 0,1,2,\ldots,(n-1)$.

M i s o l l a r. 1) $z = 1 + i$ sonni trigonometrik shaklda ifodalang.

$$\Delta |z| = |1+i| = \sqrt{x^2+y^2} = \sqrt{1^2+1^2} = \sqrt{2}, \quad \varphi = \frac{\pi}{4};$$

$$\cos\varphi = \frac{x}{|z|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \quad \sin\varphi = \frac{y}{|z|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2};$$

$$1+i = |z|\left(\cos\varphi + i\sin\varphi\right) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right); \nabla$$

2) $5i$ sonni trigonometrik shaklda ifodalang.

$$|5i| = 5, \varphi = \frac{\pi}{2}, 5i = 5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right);$$

3) $x, y \in R$ deb faraz qilib, $x + 8i + (y - 3)i = 1$ tenglamadan x va y larni toping.

$$\Delta\, x + 8i + yi - 3i = 1 \Rightarrow x + yi = 1 - 5i \Leftrightarrow x = 1, y = -5. \nabla$$

4) $\dfrac{(3-4i)(2-i)}{2+i} - \dfrac{(3+4i)(2+i)}{2-i}$ ni hisoblang.

$$\Delta \frac{(3-4i)(2-i)}{2+i} - \frac{(3+4i)(2+i)}{2-i} = \frac{(3-4i)(2-i)^2}{(2+i)(2-i)} - \frac{(3+4i)(2+i)^2}{(2-i)(2+i)} =$$

$$= \frac{(3-4i)(3-4i)}{2+1^2} - \frac{(3+4i)(3+4i)}{2^2+1^2} = \frac{1}{5}\left[(-7-24i) - (-7+24i)\right] = \frac{48}{5} i. \nabla$$

5) $z_1 = a_1 + b_1 i$, $z_2 = a_2 + b_1 i$, bo'lsa, $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$ bo'lishini isbotlang.

$$\Delta\, z_1 z_2 = (a_1 a_2 - b_1 b_2) + (a_1 a_2 + b_1 b_2)i;$$

$$\overline{z_1 \cdot z_2} = \left(a_1 a_2 - b_1 b_2\right) + \left(a_1 b_2 + a_2 b_1\right)i;$$

$$\overline{z_1} \cdot \overline{z_2} = \left(\left(a_1 - b_1 i\right) + \left(a_2 + b_2 i\right)\right) = \left(a_1 a_2 - b_1 b_2\right) + \left(a_1 b_2 + a_2 b_1\right)i;$$

bulardan $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$ kelib chiqadi. ∇

6) Amallarni bajaring: $\dfrac{(1+i)^n}{(1-i)^{n-1}}$.

$$\Delta \frac{(1+i)^n}{(1-i)^{n-1}} = \left[\frac{1+i}{1-i}\right]^{n-1} \cdot (1+i) = \left[\frac{(1+i)^2}{(1+i)(1-i)}\right]^{n-1} (1+i) =$$

$$= \left(\frac{2i}{2}\right)^{n-1} (1+i) = i^{n-1}(1+i). \nabla$$

7) $z^2 + \bar{z} = 0$ tenglamani yeching.

Δ $z = a+bi$ bo'lsin, u holda $\bar{z} = a - bi$ bo'ladi.

$$(a + bi)^2 + (a - bi) = 0 \Rightarrow a^2 + 2abi - b^2 + a - bi = 0 \Rightarrow$$

$$(a^2 + a - b^2) + (2ab-b)i = 0 \Leftrightarrow$$

$$\begin{cases} a^2 + a - b^2 = 0 \\ 2ab - b = 0 \end{cases} \Rightarrow b(2a-1) = 0 \Rightarrow \left(b = 0, a = \frac{1}{2}\right);$$

a) $b = 0$ bo'lganda, $a^2 + a = 0 \Rightarrow (a_1 = 0 \vee a_2 = -1)$

b) $a = \frac{1}{2}$ bo'lganda, $b^2 = \frac{3}{4} \Rightarrow \left(b_1 = \frac{\sqrt{3}}{2}, b_2 = -\frac{\sqrt{3}}{2}\right)$.

Demak, $z_1 = 0, z_2 = -1, z_3 = \frac{1}{2} + i\frac{\sqrt{3}}{2}, z_4 = \frac{1}{2} - i\frac{\sqrt{3}}{2}$ lar berilgan tenglamaning ildizlari bo'ladi. ∇

8) $z = 1 - i$ sonni trigonometrik shaklda yozing.

Δ $a = 1$, $b = -1$ bo'lgani uchun:

$$r = \sqrt{a^2 + b^2} = \sqrt{1+1} = \sqrt{2}, \cos\varphi = \frac{a}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \sin\varphi = -\frac{1}{\sqrt{2}} =$$

$$= -\frac{\sqrt{2}}{2} = \frac{b}{2} \Rightarrow \varphi \arg z = \frac{7\pi}{4}, z = 1 - i = \sqrt{2}\left(\cos\frac{7\pi}{4} - i\sin\frac{7\pi}{4}\right). \nabla$$

9) Agar $z + \frac{1}{z} = 1$ bo'lsa, $z^{1990} + \frac{1}{z^{1990}}$ ni hisoblang.

$$\Delta\ z + 1 = 1 \Rightarrow z^2 - z + 1 = 0 \Rightarrow z_1 = \frac{1}{2} - i\frac{\sqrt{3}}{2}, z_2 = \frac{1}{2} + i\frac{\sqrt{3}}{2}.$$

z_1^{1990} ni hisoblash uchun z_1 ni trigonometrik shaklga keltiramiz.

$|z_1| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$ (Shu kabi $|z_2| = 1$).

$$\varphi_1 = \arg z_1 = \text{arctg}\left(-\sqrt{3}\right) = \frac{5\pi}{3}, \text{ chunki } \sin\varphi_1 = \frac{1}{2}, \cos\varphi_1 = -\frac{\sqrt{3}}{2},$$

shu kabi $\varphi_2 = \arg z_2 = \text{arctg}\sqrt{3} = \frac{\pi}{3}$.

Demak,

$$z_1 = \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} \Rightarrow z_1^{1990} = \left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{2}\right)^{1990} =$$

$$= \cos\frac{5\cdot 1990\pi}{3} + i\sin\frac{5\cdot 1990\pi}{3} = \cos\frac{2\pi}{3}\cdot i\sin\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2},$$

$$\frac{1}{z_1^{1990}} = z_1^{-1990} = -\frac{1}{2} - i\frac{\sqrt{3}}{3}, \; z_1^{1990} + \frac{1}{z_1^{1990}} = -1.$$

Shu kabi $z_1^{1990} + \frac{1}{z_1^{1990}}$ ni hisoblashni o'quvchiga qoldiramiz. ∇

10) $\sqrt[3]{-2+2i}$ ni hisoblang.

Δ $a = -2, b = 2$ bo'lgani sababli $r = \sqrt{8}, \cos\varphi = -\frac{\sqrt{2}}{2}, \sin\varphi = \frac{\sqrt{2}}{2}$.

Bundan

$$\varphi = \frac{3\pi}{4}, -2+2i = \sqrt{8}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right),$$

$$\sqrt[3]{-2+2i} = \sqrt[3]{\sqrt{8}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)} =$$

$$= \sqrt[3]{\sqrt{8}}\left(\cos\frac{\frac{3\pi}{4} + 2k\pi}{3} + i\sin\frac{\frac{3\pi}{4} + 2k\pi}{3}\right), \quad k = 0, 1, 2.$$

$$\alpha_0 = \sqrt{2}\left(\cos\frac{\frac{3\pi}{4}}{3} + i\sin\frac{\frac{3\pi}{4}}{3}\right) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{3\pi}{4}\right) =$$

$$=\sqrt{2}\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=1+i.$$

$$\alpha_1=\sqrt{2}\left(\cos\frac{\frac{3\pi}{4}+2\pi}{3}+i\sin\frac{\frac{3\pi}{4}+2\pi}{3}\right)=\sqrt{2}\left(\cos\frac{11\pi}{4}-i\sin\frac{11\pi}{4}\right)=$$

$$=\sqrt{2}\left(\cos\left(\pi-\frac{\pi}{12}\right)+i\sin\left(\pi-\frac{\pi}{12}\right)\right)=\sqrt{2}\left(\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}\right);$$

$$a_2=\sqrt{2}\left(\cos\frac{\frac{3\pi}{4}+4\pi}{3}+i\sin\frac{\frac{3\pi}{4}+4\pi}{3}\right)=\sqrt{2}\left(\cos\frac{19\pi}{12}+i\sin\frac{19\pi}{12}\right)=$$

$$=\sqrt{2}\left(\cos\left(2\pi-\frac{5\pi}{12}\right)+i\sin\left(2\pi-\frac{5\pi}{12}\right)\right)=\sqrt{2}\left(\cos\frac{5\pi}{12}-i\sin\frac{5\pi}{12}\right).\nabla$$

1.10. Quyidagi sonlarni trigonometrik shaklda yozing:

1) 2; 2) –2; 3)$2i$; 4) $-2i$;

5)$1+i$; 6) $-1+i$; 7) $2-2i$; 8) $1+\sqrt{3}i$;

9) $-1+\sqrt{3}i$; 10) $\sqrt{3}-i$; 11) $-\sqrt{3}-i$; 12) $\frac{\sqrt{3}}{2}+\frac{1}{2}i$;

13) $\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}$; 14) $-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}$ 15) $2+\sqrt{3}+i$.

1.11. Kompleks sonni trigonometrik shaklda yozing:

1) $z=1-i$; 2) $z=1-i$; 3) $z=3+i$;

4) $z=-1+3i$ 5) $z=-2$; 6) $z=i$;

7) $z=1$; 8) $z=-i$; 9) $z=1+i$;

10) $z=\frac{1}{2}+i\frac{\sqrt{3}}{2}$; 11) $z=\frac{\sqrt{33}}{2}+i\frac{\sqrt{11}}{2}$; 12) $z=\frac{\sqrt{3}}{2}+i\frac{1}{2}$;

13) $z=2i$; 14) $z=\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$; 15) $z=-i$;

16) $z=-\sqrt{6}-\sqrt{6}i$; 17) $z=-3-4i$; 18) $z=2+3i$;

19) $z=\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$; 20) $z=\frac{\sqrt{2}}{2}+i\frac{\sqrt{6}}{2}$; 21) $z=3i$;

22) $z=3$; 23) $z=\frac{\sqrt{33}}{2}+i\frac{\sqrt{11}}{2}$; 24) $z=-2-3i$;

25) $z=-\sqrt{6}-\sqrt{6}i$; 26) $z=\frac{\sqrt{3}}{2}-i\frac{1}{2}$; 27) $z=2\sqrt{2}+i$;

28) $z=1+2+3i$; 29) $z=2+i$; 30) $z=\sqrt{2}-i\sqrt{2}$.

1.12. Quyidagi kompleks sonlarni trigonometrik shaklda yozing: 1) $3+i$; 2) $4-i$; 3) $-2+i$; 4) $-1-2i$; 5) $2+i$.

1.13. 1) Arar $z+\frac{1}{2}=\cos\alpha$ bo‘lsa, $z^n+\frac{1}{2^n}=2\cos n\alpha$ ekanligini isbotlang.

1.14. Quyidagi ayniyatlarni isbotlang:

1) $(1+i)^n=2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}+i\sin\frac{n\pi}{4}\right)$;

2) $(1-i)^n=2^{\frac{n}{2}}\left(\cos\frac{7n\pi}{4}+i\sin\frac{7n\pi}{4}\right)$;

3) $\left(\sqrt{3}-i\right)^n=2^n\left(\cos\frac{n\pi}{6}-i\sin\frac{n\pi}{6}\right)$;

4) $\left(1+i\sqrt{3}\right)^n=2^n\left(\cos\frac{n\pi}{3}+i\sin\frac{n\pi}{6}\right)$;

5) $\left(\frac{1+i\mathrm{tg}\alpha}{1-i\mathrm{tg}\alpha}\right)^n=\frac{1+i\mathrm{tg}n\alpha}{1-i\mathrm{tg}n\alpha}$.

1.15. Quyidagilarni hisoblang:

1) $(1+i)^{1990}$; 2) $\left(1+i\sqrt{3}\right)^{150}$; 3) $\left(\sqrt{3}+i\right)^3$; 4) $\left(\frac{1-i\sqrt{3}}{1+i}\right)^{12}$;

5) $\left(\dfrac{\sqrt{3}+i}{1+(-i)}\right)^{30}$; 6) $\left(\dfrac{1+i\sqrt{3}}{1-i}\right)^{20}$; 7) $\dfrac{\left(1-i\sqrt{3}\right)^{15}}{(1-i)^{20}}+\dfrac{\left(-1-i\sqrt{3}\right)^{15}}{(1+i)^{20}}$.

1.16. Komleks sonlarning ildizlarini toping:

1) $\sqrt[3]{1}$; 2) $\sqrt[3]{i}$; 3) $\sqrt[3]{1+i}$; 4) $\sqrt[4]{1+i}$;

5) $\sqrt[4]{1-i}$; 6) $\sqrt{-\sqrt{3}+i}$; 7) $\sqrt{\dfrac{1-i}{\sqrt{3}+i}}$;

8) $\sqrt[3]{\dfrac{-1+i}{1-i\sqrt{3}}}$; 9) $\sqrt[6]{\dfrac{1-i}{1-i\sqrt{3}}}$; 10) $\sqrt[6]{\dfrac{1-i}{1+i\sqrt{3}}}$.

2-§. Algebraik ifodalar

1. Natural ko'rsatkichli daraja va uning xossalari

Agar sonli ifodadagi ayrim yoki barcha sonlar harflar bilan almashtirilsa, *harfiy ifoda* hosil bo'ladi.

To'rt matematik amal, butun darajaga ko'tarish va butun ko'rsatkichli ildiz chiqarish ishoralari orqali birlashtirilgan harflar va sonlardan iborat ifodalar *algebraik ifoda* deyiladi. Agar algebraik ifodada sonlar va harflarning ildiz ishoralari qatnashmasa, u *ratsional algebraik ifoda*, ildiz ishoralari qatnashsa, *irratsional algebraik ifoda* deyiladi. Agar ratsional ifodada harfli ifodaga bo'lish amali qatnashmasa, u *butun algebraik ifoda* deyiladi.

M i s o l l a r. 1) $6b-3a+dc$ – *butun algebraik ifoda*;

2) $\dfrac{a-b}{2b}$ – *kasr algebraik ifoda*;

3) $2a+\sqrt{b}$ – *irratsional algebraik ifoda;*

4) $(a-b)^2=(b-a)^2$ – *ayniyat.*

Har biri a ga teng bo'lgan $n(n\geq 2)$ ta ko'paytuvchining ko'paytmasi a sonining *n- darajasi* deyiladi va a^n deb belgilanadi, ya'ni:

$$a^n = \underbrace{a \cdot a \cdot ... \cdot a}_{n\ ta}.$$

Natural ko'rsatkichli daraja quyidagi xossalarga ega:

1^0. $a^0 = 1$;

2^0. $a^1 = a$;

3^0. $a^m \cdot a^n = a^{m+n}$; $m, n \in N$.

4^0. $a^m : a^n = a^{m-n}$; $m, n \in N$.

5^0. $(a^m)^n = a^{m \cdot n}$; $m, n \in N$.

6^0. $(a \cdot b)^m = a^m \cdot b^m$; $m \in N$.

7^0. $(\frac{a}{b})^m = \frac{a^m}{b^m}$; $a, b \in R$, $b \neq 0, m \in N$.

1-m i s o l. $8 \cdot 32 \cdot 5^5 \cdot 125$ ni hisoblang.

Y e c h i s h.

$$8 \cdot 32 \cdot 5^5 \cdot 125 = 2^3 \cdot 2^5 \cdot 5^5 \cdot 5^3 = 2^8 \cdot 5^8 = (2 \cdot 5)^8 = 10^8 = 100\ 000\ 000.$$

2-m i s o l. $a^2 \cdot b^5 \cdot a^4 \cdot b^3$ ifodani soddalashtiring.

Y e c h i s h.

$$a^2 \cdot b^5 \cdot a^4 \cdot b^3 = a^{2+4} \cdot b^{5+3} = a^6 \cdot b^8;$$

3-m i s o l. $3a^5 \cdot b^2 \cdot a^4 \cdot 5b^3$ ifodani soddalashtiring.

Ye c h i s h.

$$3a^5 \cdot b^2 \cdot a^4 \cdot 5b^3 = 3 \cdot 5 \cdot a^{5+4} \cdot b^{2+3} = 15a^9 \cdot b^5.$$

2.1. Ifodani *a* asosi daraja ko'rinishida yozing:

1) $a^3 \cdot a^7$;	2) $a^2 \cdot a^5 \cdot a^3$;	3) $a^4 \cdot a^2 \cdot a^7$;
4) $a^8 : a^7$;	5) $a^4 : a^2 : a^3$;	6) $a^{12} : a^7 : a^4$;
7) $-a^2 \cdot a^3$;	8) $-a^5 \cdot a^5 : a^3$;	9) $a^7 \cdot (-a^5) : a^3$;
10) $(a^2)^3$;	11) $(a^5 \cdot a^3)^4$;	12) $a^2 \cdot (a^5 : a^3)^3$;
13) $((a^2 \cdot a^5)^2)^3$;	14) $(a^2)^3 \cdot (a^5)^2$;	15) $(a^3)^2 : (a^4)^3$;

2.2. Hisoblang:

1) $2^4 \cdot 5^2$; 2) $2^4 \cdot 128$; 3) $3^4 \cdot 4^2$;

4) $\frac{27 \cdot 55}{3^3 \cdot 11}$; 5) $\frac{32 \cdot 3^2}{27 \cdot 2^3}$; 6) $\frac{3^4 \cdot 34}{27 \cdot 17}$;

7) $\frac{2^5 \cdot 55 \cdot 81}{128 \cdot 121 \cdot 3^5}$; 8) $\frac{26^2 \cdot 55}{13^3 \cdot 7^4} \cdot \frac{14^4 \cdot 4}{11 \cdot 2^6}$;

9) $\frac{2^2 \cdot 55^2}{3^4 \cdot 2^8} \cdot \frac{12^4 \cdot 2}{5^2 \cdot 4}$; 10) $\frac{10^2 \cdot 25}{3^4 \cdot 5^4} : \frac{10^4 \cdot 625}{27 \cdot 5^6}$;

11) $\frac{6^2 \cdot 5^5}{3^3 \cdot 7^4} \cdot \frac{21^3 \cdot 4}{3^2 \cdot 10^6}$; 12) $\frac{8^3 \cdot 7}{16^3 \cdot 7^2} : \frac{2^4 \cdot 128}{49 \cdot 2^6}$.

2.3. Algebraik ifodani soddalashtiring:

1) $(2a + 3b) + (7a + 4b)$; 2) $(9a + 7b) + (5a - 6b)$;
3) $(12a - 6b) + (7b - 13a)$; 4) $(4a - 3b) + (9a - 7b)$;
5) $(-7a - 3b) - (-3a - 5b)$; 6) $(-7a + 5b) + (-4a - 3b)$.

2.4. Algebraik Ifodani soddalashtiring:

1) $5(3a + 5b) + 4(5a + b)$; 2) $4(2a + 7b) + 4(a + 6b)$;
3) $-2(4a + b) - 3(a - 2b)$; 4) $8(-3a - 2b) + 4(6a + 4b)$;
5) $1,5(4a + 6b) - 2,5(4a + 2b)$; 6) $5,5(a - 4b) - 4,5(6a - 6b)$;

2. Butun ko‘rsatkichli darajaning xossalari

Har qanday a haqiqiy sonning α *butun ko‘rsatkichli darajasi* yoki α- *darajasi* deb, a^α songa aytilishini bilamiz, bunda a – daraja asosi, α – daraja ko‘rsatkichi,

$$a^\alpha = \begin{cases} a, \text{ agar } \alpha = 1 \text{ bo‘lsa,} \\ \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ marta}}, \text{ agar } \alpha = n, \ n \in N, \ n \geq 2 \text{ bo‘lsa.} \end{cases}$$

Har qanday $a \neq 0$ haqiqiy sonning nolinchi darajasi 1 ga teng, $a^0 \neq 1$. Nolning nolinchi darajasi, ya’ni 0° ma’noga ega emas.

Ixtiyoriy $a \neq 0$ haqiqiy sonning butun manfiy n ko‘rsatkichli darajasi $\frac{1}{a^n}$ sonidan iborat, $a^{-n} = \frac{1}{a^n}$; 0^{-n} ifoda ma’noga ega emas.

B u t u n k o ʻ r s a t k i c h l i d a r a j a n i n g x o s s a l a r i (a, b – noldan farqli haqiqiy sonlar, α, β – butun sonlar).

1) $(ab)^\alpha = a^\alpha b^\alpha$;

Haqiqatan, $\alpha = n \in N$ boʻlsa, haqiqiy sonlarni koʻpaytirishning asosiy qonunlariga muvofiq:

$$(ab)^\alpha = (ab)^n = \underbrace{(ab)(ab)...(ab)}_{n} =$$

$$= \underbrace{a \cdot a \cdot a \cdot ... \cdot a}_{n} \cdot \underbrace{b \cdot b \cdot b \cdot ... \cdot b}_{n} = a^n b^n = a^\alpha \cdot b^\alpha,$$

agar $\alpha = 0$ boʻlsa, $(ab)^\alpha = (ab)^0 = 1 = 1 \cdot 1 = a^0 b^0 = a^\alpha b^\alpha$;

agar $\alpha = -n$, $n \in N$ boʻlsa, $(ab)^\alpha = (ab)^{-n} = \dfrac{1}{(ab)^n} = \dfrac{1}{a^n b^n}$.

2) $\left(\dfrac{a}{b}\right)^\alpha = \dfrac{a^\alpha}{b^\alpha}$;

3) $a^a a^b = a^{a+b}$;

Haqiqatan, agar $\alpha = n$, $\beta = m$, $n \in N$, $m \in N$ boʻlsa, u holda:

$$a^\alpha a^\beta = a^n a^m = \underbrace{a \cdot a \cdot ... \cdot a}_{n} \cdot \underbrace{a \cdot a \cdot ... \cdot a}_{m} =$$

$$= \underbrace{a \cdot a \cdot a \cdot ... \cdot a}_{m+n} = a^{m+n} = a^{\alpha+\beta}.$$

$\alpha = n, \beta = -m$ va $\alpha = -n, \beta = m$ boʻlgan hollar ham shu kabi isbotlanadi. $\alpha = -n, \beta = -m$ holning isbotini quyidagicha bajarish mumkin:

$$a^\alpha a\beta = a^{-n} a^{-m} = \frac{1}{a^n} \cdot \frac{1}{a^m} = \frac{1}{a^n a^m} = \frac{1}{a^{n+m}} = a^{-(n+m)} =$$

$$= a^{-n-m} = a^{(-n)+(-m)} = a^{\alpha+\beta}.$$

4) $\dfrac{a^\alpha}{a^\beta} = a^{\alpha-\beta}$;

5) $(a^a)^b = a^{ab}$.

Xususan, $a = n$, $b = m$, n, $m \in N$ boʻlganda:

$$(a^\alpha)^\beta = (a^n)^m = \underbrace{a^n \cdot a^n \cdot ... \cdot a^n}_{m} = \underbrace{a \cdot a \cdot ... \cdot a}_{nm} = a^{\alpha\beta}.$$

2.5. Hisoblang:

1) $3^{-5} \cdot 4^{-2} \cdot 9^{-1} \cdot 16^{-1}$; 2) $5^{-5} \cdot 4^{-2} \cdot 25^{-1} \cdot 2^{-1}$;

3) $27^{-1} \cdot 4^{-2} \cdot 9^{-1} \cdot 8^{-1}$; 4) $8^{-4} \cdot 3^{-2} \cdot 6^{-1} \cdot 21^{-1}$;

5) $3^{-2} \cdot 15^{2} \cdot 9^{-1} \cdot 5^{-2}$; 6) $7^{-3} \cdot 4^{-4} \cdot 14^{2} \cdot 2^{-3}$;

7) $3^{8} \cdot 7^{-2} \cdot 21^{-2} \cdot 9^{-2}$; 8) $20^{-5} \cdot 2^{-2} \cdot 25^{-1} \cdot 16^{2}$;

9) $(10^{-5}) \cdot 2^{-2} \cdot 5^{4}$; 10) $5^{-3} \cdot 6^{-2} \cdot (5^{-1} \cdot 6^{2})^{2}$;

11) $(20^{-5} \cdot 2^{-2}) : (25^{-1} \cdot 16^{2})$; 12) $(11^{-2} \cdot 2^{-2}) : (22^{-2} \cdot 4^{3})$;

2.6. Hisoblang:

1) $8^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} \cdot 2 \cdot 3^{\frac{3}{2}}$; 2) $9^{\frac{1}{2}} \cdot 49^{\frac{1}{2}} \cdot 12 \cdot 3^{\frac{3}{2}}$;

3) $4^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} \cdot 8 \cdot 3^{2}$; 4) $27^{\frac{1}{2}} \cdot 32^{\frac{1}{2}} \cdot 2^{5} \cdot 3^{\frac{3}{2}}$;

5) $12^{\frac{1}{2}} : 9^{\frac{1}{2}} : 2 : 3^{\frac{3}{2}}$; 6) $25^{\frac{1}{2}} : 9^{\frac{1}{2}} : 5^{\frac{5}{2}} : 3^{\frac{3}{2}}$;

7) $6^{\frac{5}{2}} : 3^{\frac{5}{2}} : 2^{\frac{9}{2}} : 3^{\frac{3}{2}}$; 8) $10^{\frac{1}{2}} : 9^{\frac{1}{2}} : 10 : 3^{\frac{3}{2}}$;

9) $\left(2^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \cdot 2 \cdot 3^{\frac{3}{2}}\right)^{2}$; 10) $\left(10^{\frac{1}{2}} \cdot 4^{\frac{1}{2}} \cdot 25 \cdot 5^{\frac{3}{2}}\right)^{4}$;

11) $\left(6^{\frac{1}{3}} \cdot 5^{\frac{1}{2}} \cdot 6^{2} \cdot 5^{\frac{3}{2}}\right)^{6}$; 12) $\left(4^{\frac{3}{1}} \cdot 9^{\frac{1}{2}} \cdot 2^{2} \cdot 18^{\frac{3}{2}}\right)^{3}$.

2.7. Ifodani *a* asosi daraja ko‘rinishida yozing:

1) $a^{\frac{1}{2}} \cdot a^{\frac{5}{2}}$; 2) $a^{\frac{1}{2}} \cdot a^{5} \cdot a^{-3}$; 3) $a^{-4} \cdot a^{\frac{3}{2}} \cdot a^{\frac{1}{3}}$;

4) $a^{\frac{4}{5}} : a^{-\frac{1}{10}}$; 5) $a^{\frac{7}{2}} : a^{\frac{5}{2}} : a^{-3}$; 6) $a^{-4} : a^{-2} : a^{\frac{4}{3}}$;

7) $-a^{\frac{1}{7}} \cdot a^{-3}$; 8) $-a^{\frac{1}{4}} \cdot a^{-2} : a^{1\frac{4}{5}}$; 9) $a^{-\frac{5}{7}} \cdot \left(-a^{\frac{7}{8}}\right) : a^{3}$;

10) $\left(a^{\frac{5}{6}}\right)^{3}$; 11) $\left(a^{\frac{4}{5}} \cdot a^{\frac{5}{4}}\right)^{20}$; 12) $a^{3} \cdot \left(a^{\frac{5}{7}} : a^{\frac{4}{6}}\right)^{\frac{5}{2}}$;

13) $\left(\left(a^{2\frac{4}{5}}\cdot a^{\frac{4}{6}}\right)^{\frac{7}{3}}\right)^{\frac{7}{28}}$; 14) $\left(a^{\frac{4}{10}}\right)^{-2\frac{2}{4}}\cdot\left(a^{4\frac{7}{5}}\right)^{\frac{15}{27}}$;

15) $\left(a^{-\frac{4}{5}}\right)^{20}:\left(a^{-4}\right)^{\frac{7}{20}}$.

3. Birhadlar

Butun musbat darajali harf, son yoki ulardan tuzilgan ko'paytuvchilar ko'paytmasidan iborat butun algebraik ifoda *birhad* deyiladi.

Koeffitsiyentlari bilangina farq qiladigan birhadlar *o'xshash birhadlar* deyiladi. Masalan, 5,5ab va 7ab lar o'xshash birhadlardir.

Har qanday birhad turli ko'rinishda yozilishi mumkin. Masalan,

$$5\cdot a^8\cdot b^3=2{,}5\cdot 2a^8\cdot b^3=5a^3\cdot b^2\cdot a\cdot a^4\cdot b=\ldots$$

Birhaddagi barcha harflar darajalarining yig'indisi shu birhadning *darajasi* deyiladi.

Son yoki bitta harf ham birhaddir. Masalan, x; y; z; 0; $\frac{7}{5}$; 2,(41).

Berilgan birhadda ko'paytirishni daraja bilan almashtirib, dastlab o'zgarmas sonni, so'ngra unda qatnashgan harflarni tegishli tartibda yozilsa, hosil bo'lgan ifodaga birhadning *standart* ko'rinishi deyiladi. Harflar oldidagi sonli ko'paytuvchiga birhadning *koeffitsiyenti* deyiladi.

Masalan, $3abc\cdot 5ac\cdot\frac{2}{13}bc$ birhadning *standart* shakli $\frac{30}{13}a^2b^2c^3$ bo'ladi.

2.8. Birhadning koeffitsiyentini toping:

1) $5a^5\cdot 3a^3$; 2) $4a^3\cdot\left(3a^6\right)^2$; 3) $(2a)^5\cdot 7a^3$;

4) $2{,}5x^5\cdot 3x^3$; 5) $0{,}5x^2\cdot\frac{5}{2}y^3$; 6) $\frac{16}{5}x^3\cdot 2{,}2y^3$;

7) $\left(4x^4\right)^3\cdot(0{,}4y)^3$; 8) $\left(5x^2\right)^3\cdot\left(1{,}5y^3\right)^4$;

9) $(2x^3)^2\cdot\left(\frac{9}{2}y^2\right)^3$;

10) $\left(1\frac{5}{6}x^7\right)^3\cdot\left(\frac{7}{2}y^8\right)^3$;

11) $6x^4\cdot5{,}4y^3\cdot5z^7$;

12) $8x^7\cdot5y^3\cdot4{,}4z^4$;

13) $\frac{2}{5}x^4\cdot\frac{44}{5}y^5\cdot25z^3$;

14) $10\frac{15}{61}x^2\cdot312\frac{1}{2}y^8$.

2.9. Birhadlarni ko'paytiring:

1) $(3a^5)\cdot(9a^3)$;

2) $(-2a^3)\cdot(7a^6)$;

3) $(-12a^{-2})\cdot(10a^3)$

4) $(4{,}5x^5)\cdot(2x^3)$;

5) $(5{,}5x^2y)\cdot\left(\frac{5}{2}y^3x\right)$;

6) $\left(\frac{26}{10}x^3y\right)\cdot(-3{,}3y^3x)$;

7) $(-5x^4y)^3\cdot(4yx^2)^3$;

8) $(-5x^2y)^3\cdot(1{,}5y^3x^5)^4$;

9) $(12x^3a^2)^2\cdot\left(\frac{5}{2}y^2a\right)^3$;

10) $\left(4\frac{2}{3}x^8y\right)^2\cdot\left(\frac{5}{3}y^8x\right)^3$;

11) $4x^4\cdot5{,}4y^3\cdot5(xy)^7$;

12) $(2x)^7\cdot(5y)^3\cdot(4xy)^4$;

13) $\left(\frac{3}{5}xy\right)^4\cdot\left(\frac{5}{9}xy\right)^3\cdot25(xy)^3$;

14) $20\frac{25}{30}x^2\cdot363\frac{1}{2}y^8$.

2.10. Birhadlarni bo'ling:

1) $(3a^5):(3a^3)$;

2) $(-36a^9):(9a^5)$;

3) $(-12a^{-2}):(4a^3)$;

4) $(25x^5):(5x^3)$;

5) $(161yx^2):\left(\frac{5}{7}y^3x\right)$;

6) $\left(\frac{30}{25}x^3y\right):(-60y^3x)$;

7) $(-5x^4y)^3:(5yx^2)^3$;

8) $(-5x^2y)^3:(-5y^3x^5)^4$;

9) $(12x^3a^2)^2:\left(\frac{5}{12}x^2a\right)^3$;

10) $\left(1\frac{2}{3}x^8y\right)^2:\left(\frac{5}{3}y^8x\right)^3$;

11) $\left(4x^4\cdot 5,4y^3\right):(xy)^7$;

12) $(20xy)^5:\left(5y^3\cdot 4x\right)^4$;

13) $\left(\frac{3}{5}y^4\cdot\frac{5}{9}x\right)^3:(3xy)^3$:

14) $20\frac{25}{30}x^2:311\frac{3}{20}y^8$;

15) $\frac{5}{3}x^3y:3\frac{3}{2}x^2y^2$;

16) $4\frac{5}{3}x^2a^3:3\frac{2}{5}x^8a^2$.

2.11. Ko'phadlarni qo'shing:

1) $\left(7a^5\right)+\left(3a^5\right)$;

2) $\left(-3a^3\right)+\left(4a^3\right)$;

3) $\left(-14a^{-2}\right)+\left(12a^{-2}\right)$;

4) $\left(-2,5x^5\right)+\left(-2x^5\right)$;

5) $\left(3,3x^2y\right)+\left(\frac{5}{2}yx^2\right)$;

6) $\left(\frac{26}{10}x^3y\right)+\left(-3,3yx^3\right)$;

7) $\left(-2x^4y\right)^3+\left(4yx^4\right)^3$;

8) $\left(-4x^2y\right)^2+\left(0,5yx^2\right)^3$;

9) $\left(12x^3a^2\right)^2+\left(\frac{5}{2}x^2a^{\frac{4}{3}}\right)^3$;

10) $\left(-\frac{2}{3}x^8y\right)^2+\left(\frac{5}{3}x^8y\right)^2$;

11) $4x^4+5,4x^4+5x^4$;

12) $(2x)^3+\left(5x^3\right)+(4x)^3$;

13) $\left(\frac{3}{5}xy\right)^4+\left(\frac{2}{5}xy\right)^4+25(xy)^4$;

14) $1\frac{5}{7}x^2+3\frac{2}{7}x^2+\frac{4}{14}x^2$.

2.12. Ko'phadlarni ayiring:

1) $\left(12a^5\right)-\left(9a^5\right)$;

2) $\left(-6a^2\right)-\left(14a^3\right)$;

3) $\left(-21a^{-2}\right)-\left(3a^{-2}\right)$;

4) $\left(-3,5x^5\right)-\left(-2x^5\right)$;

5) $\left(4,3x^2y\right)-\left(\frac{5}{3}yx^2\right)$;

6) $\left(-\frac{4}{5}x^3y\right)-\left(-3,3x^3y\right)$;

7) $\left(-4x^4y^3\right)-\left(-4x^4y^3\right)$;

8) $\left(-8x^2y^3\right)-\left(1,5x^2y^3\right)$;

9) $\left(2x^3a^2\right)^2-\left(\frac{3}{2}x^3a^{\frac{4}{3}}\right)^3$;

10) $\left(-\frac{5}{3}x^8y\right)^2-\left(-\frac{5}{3}x^8y\right)^2$;

11) $4x^4-5,4x^4-\left(-5x^4\right)$; 12) $\left(-2x\right)^3-\left(-5x\right)^3-\left(-4x\right)^3$;

13) $\left(\frac{3}{5}xy\right)^4-\left(\frac{1}{5}xy\right)^4-25\left(-xy\right)^4$; 14) $1\frac{3}{4}x^2-2\frac{1}{6}x^2-\frac{4}{5}x^2$.

4. Ko'phadlar

Ikki yoki undan ortiq birhadlarning yig'indisiga *ko'phad* deyiladi. Demak, ko'phad bu birhadlarning algebraik yig'indisidan iborat bo'lar ekan. Masalan:

$9+x+x^2$, $\left(x-y\right)^2$, $x^3+5ax^2-a^2$, $x^3+5ax^2-a^3$,

$\frac{x}{5}+\frac{y}{2}-\frac{z}{6}$, $\left(ax+by\right)^2\left[\left(x-y^4+3\left(a^x+b^3y\right)\right)\right]$.

$\frac{1}{x-y}$, $\frac{x}{y}$, $\frac{x-5y-3}{x}+x^2-5y^2$ ifodalarning maxrajlarida argument qatnashgani sababli ko'phad bo'la olmaydi.

x^2+6x-3, $-x^3-2x^2+x-4$ lar bir argumentli ko'phadlarga misol bo'ladi.

Ko'phad birhadlarning yig'indisidan iborat. Ko'phad tarkibidagi eng katta darajali birhadning daraja ko'rsatkichi shu *ko'phadning darajasi* deyiladi. Ko'phadni darajasi pasayib borish tartibida yozish, ko'phadni standart shaklda yozish deyiladi:

$$a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n.$$

Agar ko'phadning hamma o'xshash hadlari keltirilgan bo'lib, standart shaklda yozilgan bo'lsa, bu shakl *ko'phadning kanonik shakli* deyiladi.

M i s o l. $(x-2)^2+x^3-2x^2+1$ ko'phadni kanonik shaklga keltiring.

Y e c h i s h. $x^2-4x+4+x^3-2x^2+1=x^3-x^2-4x+5$ ko'phad kanonik ko'rinishga keltirildi.

Ikkita ko'phad $P(x)=a_0x^n+a_1x^{n-1}+\ldots+a^n$ va $Q(x)=b_0x^n+b_1x^{n-1}+\ldots+b_n$ o'zaro teng deyiladi, agar bir xil darajali noma'lumlar oldidagi koeffitsiyentlar teng, ya'ni $a_0=b_0$, $a_1=b_1\ldots a_n=b_n$ bo'lsa, bu holda $P(x)=Q(x)$ deb yoziladi.

Ko‘phadlarni qo‘shish, ayirish, ko‘paytirish mumkin. Natijada yana ko‘phad hosil bo‘ladi:

$(x^3 - 2x^2 + 3) + (x^4 - 2x^2 + 1) = x^4 + x^3 - 4x^2 + 4,$

$(3x^3 - 2x^2 + 1) - (x^3 - 2x^2 + 4) = 2x^3 - 3,$

$(x^2 - x)(x^3 + 1) = x^5 - x^4 + x^2 - x.$

2.13. Ko‘phadlarni kanonik shaklga keltiring:

1) $3x^3 - 2x^2 + x - 1 + (x + 2)^2$; 3) $x^3 + 2x^2 + x + x(x - 1)^2$;

2) $x^4 - 3x + (x - 3)^3$; 4) $(x - 1)^3 - x^3 + 3x^2 - 3x + 1$.

2.14. $P(x) = 2x^2 - 3x + 5$ bo‘lsa, $P(-2)$, $P(1/2)$, $P(3)$ ni toping.

2.15. $P(x) = x^3 - 4x^2 + x$ bo‘lsa, $P(-1)$, $P(1/2)$, $P(2)$ ni toping.

2.16. $P(x)$ va $Q(x)$ ko‘phadlar teng bo‘lsa, noma’lum koeffitsiyentlarni toping.

1) $P(x) = ax^5 + 2x^6 + 3x^2 - 1$; $Q(x) = 3x^6 + bx^2 - 1$.

2) $P(x) = ax^3 + 2x + 3$; $Q(x) = 4x^3 + bx + 3$.

2.17. $P(x)$ va $Q(x)$ berilgan. $P(x) \pm Q(x)$, $P(x) \cdot Q(x)$ ko‘phadlarni toping.

1) $P(x) = x^2 - 1$; $Q(x) = x^3 + x$;

2) $P(x) = x - 2$; $Q(x) = 2x^2 + 3x$

3) $P(x) = x + 3$; $Q(x) = 3x^2 + 5x - 6$;

4) $P(x) = x^2 + 2x$; $Q(x) = x^2 + x + 1$;

5) $P(x) = (x - 2)^2$; $Q(x) = -x^2 + 2x + 1$;

6) $P(x) = (2x - 1)^2$; $Q(x) = 3x^2 + 3x + 1$.

J a v o b l a r:

2.13. 2) $x^4 + x^3 - 9x^2 + 24x - 27$. **2.14.** 2) $P(\frac{1}{2}) = 4$.

2.15. 2) $P(\frac{1}{2}) = -\frac{3}{8}$. **2.16.** 2) $a = 4$, $b = 2$. **2.17.** 2) $P + Q = 2x^2 + 4x - 2$. $P - Q = -2x^2 - 2x - 2$. $P \cdot Q = 2x^3 - x^2 - 6x$.

Ko‘phadlarni bo‘lish

Berilgan $P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ ko‘phadni $Q(x) = b_0x^m + b_1x^{m-1} + \ldots + b_m$ ko‘phadga bo‘lish talab qilinsin. Agar shunday $S(x)$ va $R(x)$ ko‘phadlar mavjud bo‘lib,

$$P(x) = Q(x) \cdot S(x) + R(x) \qquad (1)$$

tenglik o'rinli bo'lsa, $P(x)$ – bo'linuvchi, $Q(x)$ – bo'luvchi $S(x)$ – bo'linma va $R(x)$ – qoldiq ko'phadlar deyiladi. Bu yerda $R(x)$ ning daraja ko'rsatkichi, $Q(x)$ daraja ko'rsatkichidan kichik bo'ladi. $R(x) = 0$ bo'lsa, $P(x)$ ko'phad $Q(x)$ ga qoldiqsiz bo'linadi deyiladi, aks holda bo'lish qoldiqli deyiladi (yoki bo'linmaydi deyiladi).

Bo'linma $S(x)$ va qoldiq $R(x)$ ni topishda «aniqmas koeffitsiyentlar usuli» yoki «burchakli bo'lish» usulidan foydalanish mumkin.

Bo'luvchi $Q(x)$ va bo'linma $S(x)$ daraja ko'rsatkichlarining yig'indisi $P(x)$ daraja ko'rsatkichiga tengligini hisobga olgan holda, $P(x) = Q(x) \cdot S(x) + R(x)$ tenglikni $S(x)$ va $R(x)$ koeffitsiyentlari noma'lum bo'lgan shaklda yozamiz. Ikki ko'phad tengligidan (qavslarni ochib, ma'lum amallarni bajargandan keyin) foydalanib, noma'lum koeffitsiyentlarni topish uchun chiziqli tenglamalar sistemasini hosil qilamiz. Bunday sistema yagona yechimga ega bo'ladi. Buni misolda ko'ramiz.

1-misol. $P(x) = x^3 + 2x^2 - 1$ ko'phadni $Q(x) = x^2 + x + 2$ ko'phadga bo'lamiz.

Bo'linmani $S(x) = c_0x + c_1$ ko'rinishda qidiramiz. $Q(x)$ ni darajasi 2 ga $P(x)$ ning darajasi 3 ga teng, demak, $S(x)$ning darajasi birga teng bo'lishi kerak, qoldiqni $R(x) = d_0x + d_1$ ko'rinishda qidiramiz. (1) tenglikni yozamiz: $x^3 + 2x^2 - 1 = (x^2 + x + 2)(c_0x + c_1) + d_0x + d_1$.

Bundan ko'rinadiki, $c_0 = 1$ bo'lishi kerak. Qavslarni ochib, o'xshashlarini keltirib, $x^3 + 2x^2 - 1 = x^3 + (1 + c_1)x^2 + (2 + c_1 + d_0)x + (2c_1 + d_1)$ tenglikni hosil qilamiz. Mos koeffitsiyentlarni tenglashtirib,

$$\begin{cases} 1 = 1, \\ 1 + c_1 = 2, \\ 2 + c_1 + d_0 = 0, \\ 2c_1 + d_1 = -1 \end{cases}$$

sistemaga ega bo'lamiz, uni yechib, $c_1 = 1$, $d_0 = -3$, $d_1 = -3$ ni topamiz. Bo'linma $S(x) = x + 1$ va qoldiq $R(x) = -3x - 3$ ekan.

«Burchakli bo‘lish» usulini misolda ko‘ramiz.

2-m i s o l. Ushbu ifodaning butun qismini ajratamiz. Quyidagi bo‘lishni bajaramiz:

$$
\begin{array}{l|l}
4x^2 - 15ax^3 + 20a^2x^2 + 5a^4 & x^2 - 2ax + 4a^2 \\
\cline{2-2}
\underline{4x^2 - 8ax^3 + 16a^2x^2} & 4x^2 - 7ax - 10a^2 \\
-7ax^3 + 4a^2x^2 + 5a^4 & \\
\underline{-7ax^3 + 14a^2x^2 - 28a^3x} & \\
-10a^2x^2 + 28a^3x + 5a^4 & \\
\underline{-10a^2x^2 + 20a^3x - 40a^4} & \\
\qquad 8a^3x + 45a^4 &
\end{array}
$$

Butun qism $4x^2 - 7ax - 10a^2$ bo‘lib, qoldiq $8a^3 x + 45a^4$ ga teng ekan.

Berilgan $P(x)$ va $Q(x)$ ko‘phadning eng katta umumiy bo‘luvchisini topish uchun Yevklid algoritmidan foydalanish mumkin. $P(x)$ ni $Q(x)$ ga bo‘lib, qoldiq $R_1(x)$ ni hosil qilamiz, $Q(x)$ ni $R_1(x)$ ga bo‘lib $R_2(x)$ qoldiqni va hokazo hosil qilamiz. Qoldiqlarning darajalari pasayib boradi va oxiri 0 ga teng qoldiqqa ega bo‘lamiz. Undan oldingi 0 dan farqli, qoldiq berilgan ko‘phadlarning eng katta umumiy bo‘luvchisi bo‘ladi.

3-m i s o l. $P(x) = x^3 - 4x^2 + 4x - 1$ va $Q(x) = x^2 - x$ ko‘phadlarning eng katta umumiy bo‘luvchisi topilsin.

Y e c h i s h:

1) $$
\begin{array}{l|l}
x^3 - 4x^2 + 4x - 1 & x^2 - x \\
\cline{2-2}
\underline{x^3 - x^2} & x - 3 \\
-3x^2 + 4x - 1 & \\
\underline{-3x^2 + 3x} & \\
x - 1 = R_1(x) &
\end{array}
$$

2) $$
\begin{array}{l|l}
x^2 - x & x - 1 \\
\cline{2-2}
\underline{x^2 - x} & x \\
\quad 0 &
\end{array}
$$

Eng katta umumiy bo‘luvchi $x - 1$ ga teng.

2.18. «Noma'lum koeffitsiyentlar usuli» dan foydalanib, bo'linma va qoldiqni toping:

1) $P(x) = x^3 - 3x^2 + 5$, $Q(x) = x + 2$;

2) $P(x) = 2x^3 + 5x - 3$, $Q(x) = x^2 + 3x$;

3) $P(x) = 3x^4 - 5x^2 + 1$, $Q(x) = x^2 + 3$;

4) $P(x) = 4x^4 + 3x^3 - x$, $Q(x) = 2x^2 + 3x - 1$.

2.19. «Burchakli bo'lish» usulidan foydalanib, $P(x)$ ni $Q(x)$ ga bo'ling:

1) $P(x) = 3x^4 - 3x^2 + 5$, $Q(x) = 2x^2 - x$;

2) $P(x) = x^5 + 10x^4 - 14x^3 + 16x^2 - x + 3$, $Q(x) = 2x^2 - 3x$;

3) $P(x) = 5x^5 - 7x^3 + 4x - 5$, $Q(x) = x^2 - 3$;

4) $P(x) = 6x^6 - 5x^4 + 7x^2 - 3x$, $Q(x) = 2x^3 + 3x$.

2.20. Yevklid algoritmidan foydalanib, ko'phadlarning eng katta umumiy bo'luvchisini toping:

1) $P(x) = x^4 - 4x^3 + 1$, $Q(x) = 2x^3 + 2x^2 + 1$;

2) $P(x) = x^5 + x^4 - x^2 - 2x - 1$, $Q(x) = 3x^4 + 2x^3 + x^2 + 2x - 2$;

3) $P(x) = x^5 + x^4 - x^3 - 3x^2 - 3x - 1$, $Q(x) = x^4 - 2x^3 - x^2 - 2x + 1$;

4) $P(x) = x^6 + 2x^4 - 4x^3 - 3x^2 + 8x - 5$, $Q(x) = x^5 + x^2 - x + 1$.

Javoblar: **2.18.** 2) bo'linma $2x - 6$, qoldiq $23x - 3$. 4) bo'linma $2x^2 - \frac{3}{2}x - \frac{5}{4}$, qoldiq $\frac{5}{4}x - \frac{5}{4}$. **2.19.** 2) bo'linma $\frac{x^3}{2} + \frac{23}{4}x^2 + \frac{13}{8}x + \frac{167}{16}$, qoldiq $\frac{485}{16}x + 3$. 4) bo'linma x, qoldiq $2x^4 - 5x^3 - x^2 + 7x - 5$.

5. Bezu teoremasi va Gorner-Ruffini sxemasi

$P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n+2} + ... + a_{n-1}x + a_n$ ko'phadni $Q(x) = x\text{-}a$ ikkihadga bo'lsak: $P(x) = (x - a) \cdot S(x) + R$ ni hosil qilamiz.

Qoldiq R ning darajasi 0 ga teng bo'lgan ko'phad, chunki uning darajasi bo'luvchi $Q(x)$ ning darajasidan kichik.

1-teorema (Bezu). (Etyen Bezu (1730–1783) – fransuz matematigi) Ko'phad $P(x)$ ni x-a ga bo'lganda qoldiq R ko'phadning $x = a$ dagi qiymatiga teng, ya'ni $R = P(a)$.

I s b o t. $P(x) = (x - a) \cdot S(a) + R$ tenglikda x ning o'rniga a ni qo'yib topamiz:

$P(a) = R$, teorema isbotlandi.

N a t i j a. Agar $R = 0$ bo'lsa, $P(x)$ ko'phad $x - a$ ga qoldiqsiz bo'linadi, $R \neq 0$ bo'lsa,

$P(x)$ ko'phad $x - a$ ga qoldiqli bo'linadi (bo'linmaydi).

1-m i s o l. $P(x) = 3x^3 - 4x^2 + 5$ ni $x - 3$ ga bo'lganda qoldiqni toping.

Y e c h i s h. $R = P(3) = 3 \cdot 3^3 - 4 \cdot 3^2 + 5 = 81 - 36 + 5 = 50$.

2-m i s o l. $P(x) = x^n$-a^n ni x-a ga qoldiqsiz bo'linishini ko'rsating.

Y e c h i s h. $R = P(a) = a^n$-$a^n = 0$.

3-m i s o l. $P(x) = x^{2n} + a^{2n}\, x + a$ ga bo'linmasligini ko'rsating.

Y e c h i s h. $R = P(-a) = a^{2n} + a^{2n} = 2a^{2n} \neq 0$.

Bezu teoremasidan $P(x)$ ko'phadni $ax + b$ ikkihadga bo'lishdan hosil bo'ladigan qoldiq R ning $P(-\frac{b}{a})$ ga tengligi kelib chiqadi.

4-m i s o l. $P(x) = 2x^3 - 3x^2 + 5x + 3$ ni $2x + 1$ ga bo'lganda qoldiq nimagateng?

Y e c h i s h. $R = P\left(-\frac{1}{2}\right) = 2 \cdot \left(-\frac{1}{2}\right)^3 - 3 \cdot \left(-\frac{1}{2}\right)^2 + 5 \cdot \left(-\frac{1}{2}\right) + 3 =$

$$= -\frac{1}{4} - \frac{3}{4} - \frac{5}{2} + 3 = \frac{-1-3-10+12}{4} = -\frac{1}{2}.$$

Endi $P(x)$ ko'phadni $Q(x) = x$-a ikkihadga bo'lganda bo'linma $S(x)$ ning koeffitsiyentlarini aniqlashga o'tamiz.

$$a_0 x^n + a_1 x^{n-1} + \ldots + a_n = (x - a) \cdot S(x) + R \qquad (*)$$

tenglikda $S(x)$ bo'linmani $S(x) = b_0 x^{n-1} + b_1 x^{n-2} + \ldots + b_{n-1}$ ko'rinishda qidiramiz. (*) tenglikda x ning bir xil darajalari oldidagi koeffitsiyentlarini tenglashtirib, quyidagiga ega bo'lamiz:

$a_0 = b_0$, $a_1 = b_1 - ab_0$, $a_2 = b_2 - ab_1 \ldots$ $a_{n-1} = b_{n-1} - ab_{n-2}$, $a_n = R - ab_{n-1}$.

Bu tengliklardan ketma-ket noma'lum koeffitsiyentlarni topamiz: $b_0 = a_0$, $b_1 = ab_0 + a_1$, $b_2 = ab_1 + a_2$,...,$b_{n-1} = ab_{n-2} + a_{n-1}$, $R=ab_{n-1} + a_n$. topilganlarni quyidagi jadvalga joylashtirish maqsadga muvofiq bo'ladi.

a_0	a_1	a_2	...	a_{n-1}	a_n
$b_0 = a_0$	$b_1 =$ $= ab_0 + a_1$	$b_2 =$ $= ab_1 + a_2$	...	$b_{n-1} =$ $= ab_{n-2} + a_{n-1}$	$R =$ $= a_0b_{n-1} + a_n$

Keltirilgan usul Gorner (Xorner Uilyam (1786–1837) – ingliz matematigi) sxemasi deb ataladi.

5-m i s o l. Gorner sxemasi yordamida $P(x) = x^5 - 3x^3 + 5x - 4$ ko'phadni $x + 2$ ga bo'lganda bo'linma va qoldiqni toping.

Y e c h i s h. Jadvalning birinchi qatorida $P(x)$ ning koeffitsiyentlari 1, 0, –3, 0, 5, –4 ni joylashtiramiz. Ikkinchi qatoriga $a = -2$ ni qo'yib topamiz:

	$a_0 = 1$	$a_1 = 0$	$a_2 = -3$	$a_3 = 0$	$a_4 = 5$	$a_5 = -4$
$a = -2$	$b_0 = 1$	$b_1 = -2$	$b_2 = 1$	$b_3 = -2$	$b_4 = 9$	$R = -22$

Topilgan koeffitsiyentlarga ko'ra bo'linma $S(x) = x^4 - 2x^3 + x^2 - 2x + 9$, qoldiq $R = -22$ ga teng.

2.21. $P(x)$ ko'phad $Q(x)$ ikkihadga bo'linadimi?

1) $P(x) = x^{80} - 4x + 3$, $Q(x) = x - 1$;

2) $P(x) = x^{80} - 4x + 3$, $Q(x) = x + 1$;

3) $P(x) = x^{80} - 4x + 3$, $Q(x) = x^2 - 1$;

4) $P(x) = x^{80} - 4x + 3$, $Q(x) = x^2 + 1$.

2.22. 1) $x^4 - 3x + 1$ ni $x - 2$ ga,

2) $3x^5 - 4x^3 + 2x - 1$ ni $x + 2$ ga,

3) $6x^6 - 6x^3 + 2x - 2$ ni $x - 1$ ga,

4) $x^4 + 2x^3 - 3x + 2$ ni $2x + 1$ ga va $2x - 3$ ga bo'lganda qoldiqni toping.

5) m ning qanday qiymatlarida $3x^3 - 4x^2 - m$ ko'phad $x - 1$ ga bo'linadi?

2.23. Gorner-Ruffini sxemasi yordamida $P(x)$ ni $Q(x)$ ga bo'lganda bo'linma va qoldiqni toping.

1) $P(x) = x^3 - 3x^2 + 5x - 4$, $Q(x) = x + 2$;

2) $P(x) = 2x^4 - 3x^2 + 5$, $Q(x) = x - 2$;

3) $P(x) = 5x^5 - 4x^3 + 8x$, $Q(x) = x - 1$;

4) $P(x) = x^6 + 3x^5 + 4x^2 + 3$, $Q(x) = x + 1$.

Javoblar: **2.21.** 2) ha. **2.22.** 2) –69. 5) $m = -2$. **2.23.** 2) bo'linma $2x^3 + 4x^2 + 3x + 6$, qoldiq 17. 4) bo'linma $x^5 + 2x^4 - 4x^3 + 4x^2$, qoldiq 3.

Ko'phadning ildizi

T a ' r i f. Agar $P(x) = a_0x^n + a_1x^{n-1} + ... + a_n$ bo'lib, $P(a) = 0$ bo'lsa, a son $P(x)$ ko'phadning ildizi deyiladi.

M i s o l. $P(x) = x^3 - 3x^2 + 5x - 3$ ko'phad uchun 1 son ildiz bo'ladi.

Haqiqatan, $P(1) = 1 - 3 + 5 - 3 = 0$; a son $P(x) = a_0x^n + ... + a_n$ ko'phadning ildizi bo'lishi uchun, $P(x)$ ning $x - a$ ga bo'linishi zarur va yetarlidir.

I s b o t. 1. Z a r u r i y l i g i. a son $P(x)$ ning ildizi bo'lsin, u holda ta'rifga ko'ra $P(a) = 0$ bo'ladi. Bezu teoremasiga asosan esa $R = P(a) = 0$. Bu esa $P(x)$ ning $x - a$ ga bo'linishini bildiradi.

2. Y e t a r l i l i g i. $P(x)$ ko'phad $x - a$ ga bo'linsin, u holda $R = 0$ bo'ladi. Yana Bezu teoremasiga asosan $P(a) = R = 0$ bo'lib, a son $P(x)$ uchun ildiz ekanligini bildiradi. $x^n + a_1x^{n-1} + ... + a_n = 0$ tenglamaning butun ildizi ozod had a_n ning butun bo'luvchisidir. Agar $\frac{p}{q}$ ko'rinishidagi ratsional son ildiz bo'lsa, u holda p ozod had a_n ning, q esa bosh koeffitsiyent 1 ning bo'luvchisi bo'lishi zarur.

2.24. Tenglamaning butun ildizlarini toping:

1) $x^3 + 3x^2 + 3x - 2 = 0$; 2) $x^3 - 2x^2 - x - 6 = 0$;

3) $x^4 - x^2 + 3x + 3 = 0$; 4) $x^3 - 3x^2 + 3x - 1 = 0$.

2.25. Tenglamaning barcha ildizlarini toping:

1) $x^3 + x^2 - 4x - 4 - 0$; 2) $x^3 - 3x^2 - 4x + 12 = 0$;

3) $x^3 + 5x^2 + 5x + 1 = 0$; 4) $x^3 + x^2 + 6x + 6 = 0$.

Javoblar: **2.24.** 2) 3; 4) 1. **2.25.** 2) –2; 2: 3; 4) –1.

6. Ratsional ifodalarni ayniy shakl almashtirish

Biror $X(x_1,..., x_n)$ algebraik ifodani *aynan almashtirish* deb, uni, umuman olganda, X ga o'xshamaydigan shunday $Y(x_1,..., x_n)$ algebraik ifodaga almashtirish tushuniladiki, barcha $x_1,..., x_n$ qiymatlarda X va Yqiymatlari teng bo'lsin.

Masalan, $A(x)=\frac{(x^2+1)(x-1)(x+3)}{(x^2-1)(x+3)}$, $B(x)=\frac{x^2+1}{x+1}$, $C(x)=\frac{(x^2+1)(x-1)(x+3)}{(x^2-1)(x+3)}$ larda $A(x)$ ifoda barcha $x\neq -1$, $x\neq 1$ qiymatlarda, $B(x)$ ifoda $x \neq -1$ qiymatlarda, $C(x)$ esa $x \neq 1$, $x \neq -1$, $x \neq -3$ qiymatlarda aniqlangan. Ularning umumiy mavjudlik sohasi $x^I \pm 1$, $x\neq -3$ qiymatlardan iborat, unda ular bir xil qiymatlar qabul qilishadi, ya'ni *aynan tengdir*.

Umumiy mavjudlik sohasida bir ratsional ifodani unga aynan teng ifoda bilan almashtirish shu ifodani *ayniy almashtirish* deyiladi.

Ayniy almashtirishlardan tenglamalarni yechish, teoremalar va ayniyatlarni isbotlash kabi masalalarni yechishda foydalaniladi. Ayniy almashtirishlar kasrlarni qisqartirish, qavslarni ochish, umumiy ko'paytuvchini qavsdan tashqariga chiqarish, o'xshash hadlarni ixchamlash va shu kabilardan iborat bo'ladi. Ayniy almashtirishlarda arifmetik amallarning xossalaridan foydalaniladi.

Quyidagi ayniyatlar o'rinli:

1) $(AB)^n = A^n B^n$; 2) $A^m A^n = A^{m+n}$; 3) $(A^m)^n = A^{mn}$;

4) $\frac{A}{B}+\frac{C}{D}=\frac{AD+BC}{BD}, B\neq 0, D\neq 0$; 5) $\frac{A}{B}\cdot\frac{C}{D}=\frac{AC}{BD}, B\neq 0, D\neq 0$;

6) $\frac{A}{B}:\frac{C}{D}=\frac{AD}{BC}, B\neq 0, C\neq 0, D\neq 0$; 7) $\frac{AC}{BC}=\frac{A}{B}, B\neq 0, C\neq 0$;

8) $\frac{A^m}{A^n}=\begin{cases} A^{m-n}, m>n \\ 1, m=n, A\neq 0 \end{cases}$ da; 9) $|AB|=|A|\cdot|B|$; 10) $|A^n|=|A|^n$.

Agar x_1 va x_2 $ax^2 + bx + c = 0$ tenglamaning ildizlari bo‘lsa, u holda $ax^2 + bx + c = a(x - x_1)(x - x_2)$ tenglik o‘rinli bo‘ladi.

Qisqa ko‘paytirish formulalari va ba’zi umumlashtirilganlari:

$(a \pm b)^2 = a^2 \pm 2ab + b^2;$

$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3;$

$(a + b)(a - b) = a^2 - b^2;$

$(a + b)(a^2 - ab + b^2) = a^3 + b^3;$

$(a - b)(a^2 + ab + b^2) = a^3 - b^3;$

$(a \pm b)^4 = a^4 \pm 4a^3b + 6a^2b^2 \pm 4ab^3 + b^4;$

$(a \pm b)^5 = a^5 \pm 5a^4b + 10a^3b^3 \pm 10a^2b^3 + 5ab^4 \pm b^5;$

$a^4 - b^4 = (a - b)(a^3 + a^2b + ab^2 + b^3) = (a - b)(a + b)(a^2 + b^2);$

$a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4).$

Daraja bilan amallar: $a \cdot \underbrace{a \cdot \ldots \cdot a}_{n\ ta} = a^n$.

$$a^n \cdot a^k = a^{n+k}; \qquad \frac{a^n}{a^k} = a^{n-k}; \qquad a^0 = 1;$$

$$a^{-n} = \frac{1}{a^n}; \qquad \frac{1}{a^{-n}} = a^n; \qquad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n;$$

$$\left(a^n b^k\right)^m = a^{nm} b^{km}.$$

M i s o l l a r. 1) Kasrni qisqartiring: $\frac{3x+3y}{12x} \cdot \frac{4x^2}{x^2 - y^2}$. Ketma-ket amallarni bajarib, topamiz: $\frac{3(x+y)}{12x} \cdot \frac{4x^2}{(x+y)(x-y)} = \frac{x}{x-y}$.

2) Ifodani soddalashtiring: $\frac{x^3 - 2x^2 + 5x + 26}{x^3 - 5x^2 + 17x - 13}$.

Suratdagi ko‘phadning butun ildizlari ozod had 26 ning bo‘luvchilari -26, -3, -2, -1, 1, 2, 13, 26 orasida bo‘lishi mumkin. Bu sonlarni ketma-ket $x^3 - 2x^2 + 5x + 26$ ko‘phaddagi x o‘rnida qo‘yib,

$= -2$ ildiz ekanligini aniqlaymiz. Shuning uchun (bu ko‘phadni $x + 2$ ga bo‘lib, bo‘linma $x^2 - 4x + 1$ ni topamiz):

$$x^3 - 2x^2 + 5x + 26 = (x + 2)(x^2 - 4x + 1).$$

Shunga o‘xshash $x^3 - 5x^2 + 17x - 13 = (x - 1)(x^2 - 4x + 1)$.
Bularni berilgan kasrning surat va maxrajga qo‘yib, topamiz:

$$\frac{x^3-2x^2+5x+26}{x^3-5x^2+17x-13}=\frac{(x+2)(x^2-4x+1)}{(x-1)(x^2-4x+1)}=\frac{x+2}{x-1}.$$

2.26. Hisoblang:

1) $\frac{116^8 \cdot 87^4}{58^9 \cdot 174^3}$; 2) $\frac{51^2 \cdot 16 \cdot 125}{102 \cdot 5^2 \cdot 153}$;

3) $\frac{16^6 \cdot 12^3}{54^5 \cdot 80^3}$; 4) $\frac{50^2 \cdot 6^2 \cdot 35}{100^3 \cdot 5^2 \cdot 21^2}$;

5) $\frac{11^6 \cdot 81^4}{27^5 \cdot 121^3}$; 6) $\frac{32^2 \cdot 27 \cdot 125}{18 \cdot 4^2 \cdot 15^2}$.

2.27. Quyidagi kasrlarni qisqartiring:

1) $\frac{48a^7b^3c}{64a^8bc^4}$; 2) $\frac{a^{n+1}b^{m+2}}{a^nb^m}$; 3) $\frac{x^{m-1}y^{n-3}}{x^my^n}$;

4) $\frac{12a^mb^{m-2}}{27a^{m+n}b^m}$; 5) $\frac{7x^my^{m+n}}{21x^{m-2}y^m}$; 6) $\frac{6a^mb^{n+3}}{9a^{m-2}b^{n+1}}$;

7) $\frac{a^{-3}b^4}{9}\cdot\left(\frac{3}{a^{-2}b^3}\right)^{-3}$; 8) $\left(\frac{c}{10a^5b^2}\right)^{-1}\cdot\left(5a^3bc^2\right)^{-2}$;

9) $\left(\frac{x^2y^{-3}}{6z}\right)^{-3}\cdot\left(\frac{x^2y^{-2}}{9z}\right)^2$.

2.28. Kasrlarni qisqartiring:

1) $\frac{1-a^3}{1+a+a^2}$; 2) $\frac{8-a^3}{2-a}$; 3) $\frac{a^2+8}{a^2-2a+4}$;

4) $\frac{a^3+27}{a+3}$; 5) $\frac{a^3-1}{a^2+a+1}$; 6) $\frac{9a^2-4}{3a+2}$;

7) $\frac{a^2+6a+9}{a+3}$; 8) $\frac{b^2-6b+9}{b-3}$.

2.29. Ifodani soddalashtiring:

1) $\frac{x^2+x-6}{x^2+4x+3}$; 2) $\frac{x^4-x^3+x-1}{x^3+1}$;

3) $\frac{x^3-x^2-x+1}{x^3+x^2-x-1}$; 4) $\frac{x^3+8}{3x+6}$;

5) $\frac{x^2-5x+6}{x^2-7x+12}$; 6) $\frac{x^3-1}{x^2+x+1}$;

7) $\frac{x^3+1}{x^2-x+1}$; 8) $\frac{a^2+7a+12}{a^2+6a+8}$;

9) $\frac{a^4+7a^2+10}{a^4+6a^2+5}$; 10) $\frac{x^3-x^2-7x+7}{x^2-7}$;

11) $\left(x-\frac{1+x^2}{x-1}\right):\frac{x^2+2x+1}{x-1}$; 12) $\frac{1-b^1+b^{-2}}{1-b+b^2}$.

2.30. Ayniyatni isbotlang:

1) $\frac{(n+1)4-16n^2}{n^3+5n^2-5n-1}=n-1$; 2) $\frac{a^3-9a}{2a+10}:\frac{3-a}{a^2+5a}=-\frac{a^2(a+3)}{2}$.

2.31. Ifodani soddalashtiring:

1) $\frac{m^3-n^3}{m^2-n^2}-\frac{m^2+n^2}{m+n}-\frac{m^2n+mn^2}{m^2+n^2+2nm}$;

2) $\frac{a^2-1}{an+n^2}\cdot\frac{1}{n-1}\cdot\frac{a-an^3-n^4+n}{1-a^2}$;

3) $\frac{a}{a^2+b^2}-\frac{b(a-b)^2}{a^4-b^4}$;

4) $\frac{2a}{a^2-4b^2}+\frac{b+3}{2b^2+6b-ab-3a}$;

5) $\frac{b}{ab-2a^2}-\frac{2+2b}{b^2+b-2ab-2a}$;

6) $\left[\frac{x^3+y^3}{xy^3}:\left(\frac{x-y}{y^2}+\frac{1}{x}\right)\right]:\frac{x(x-y)\cdot 2+4x^2y}{x+y}$;

7) $\left[\frac{(y-x)^2}{x^2}-\frac{(x+y)^2-4xy}{x^2-xy}\right]^2\cdot\frac{x^4}{x^2y^2-y^4}$;

8) $\left(\frac{2x+5y}{4x^2-y^2}-\frac{1}{2x-5y}\right):\frac{y^2}{4x^2-y^2}$;

9) $\frac{3a^2+2ax-x^2}{(3x+a)(a+x)}-2+10\cdot\frac{ax-3x^2}{a^2-9x^2}$.

Javoblar: **2.26.** 2) 40. **2.27.** 2) ab^2; 4) $\frac{4}{9a^nb^2}$; 6) $\frac{2}{3}a^2b^2$; 8) $\frac{2}{5ac^5}$;

2.28. 2) $4+2a+a^2$; 4) a^2-3a+9; 6) $a-1$; 8) $b-a$,

2.29. 2) $x-1$; 4) $\frac{1}{3}(x^2-2x+4)$; 6) $x-1$; 8) $\frac{a+3}{a+2}$; 10) $x-1$.

2.31. 2) $\frac{n^2+n+1}{n}$; 4) $\frac{1}{a+26}$; 6) $\frac{1}{xy}$; 8) $-\frac{24}{2x-5y}$.

7. Ratsional ifodalarning kanonik shakli

T a ' r i f: Argument (o'zgaruvchi)larga nisbatan *ratsional ifoda* deb o'zgarmas miqdorlardan va o'zgaruvchilarni qo'shish, ayirish, ko'paytirish va bo'lish amallari yordamida tuzilgan ifodaga aytiladi.

Masalan:

$\frac{ax+by}{cx+b}$; $\frac{x^2}{3}+x-\frac{1}{2}$; $\frac{\frac{x}{a}+\frac{y}{b}+\frac{ab}{x+y}}{a^2+b^2+\frac{1}{x^2}+\frac{1}{2}}$ va hokazolar ratsional ifodalarga misol bo‘ladi. Ushbu $\frac{3^x-3^{-x}}{3^x+3^{-x}}$; $\operatorname{tg}\left(\frac{x-1}{x+1}\right)\cdot\frac{\cos x-1}{\sin x-1}$ ifodalar ratsional bo‘la olmaydi, chunki argumentlar ustida transsendent amallar bajarilishi lozim.

T a ’ r i f: Ikkita ko‘phadning nisbati $\frac{P(x,y,...,z)}{Q(x,y,...,z)}$ *algebraik* yoki *ratsional kasr* deyiladi.

Har qanday ko‘phadni maxraji birga teng bo‘lgan ratsional kasr deb tushunish mumkin. Shunday qilib, ko‘phadlar to‘plami ratsional kasr to‘plamining qism to‘plamidir.

Kasr maxraji noldan farqli bo‘ladigan argumentlarning barcha qiymatlari to‘plami algebraik kasrning *aniqlanish sohasi* deyiladi.

Masalan, $\frac{3x}{(x^2-1)(x^2+1)}$ kasrning aniqlanish sohasi $x\neq\pm1$ bo‘ladigan barcha haqiqiy sonlar to‘plamidan iborat.

Agar algebraik kasrning surat va maxrajida mumkin bo‘lgan hamma amallar bajarilib, standart shaklda (darajalari o‘suvchi yoki kamayuvchi tartibda) yozilgan bo‘lsa, ratsional kasr *kanonik shaklga keltirilgan* deyiladi.

Ratsional *kasrlarni soddalashtirish* deb surat va maxrajida qavslarni ochish, umumiy ko‘paytuvchilarni qavsdan tashqariga chiqarish, o‘xshash hadlarni ixchamlash (keltirish), surat va maxrajda umumiy bo‘lgan ko‘paytuvchilarga qisqartirish va shu kabilardan iborat bo‘ladi.

M i s o l. $\left(\frac{a-b}{2b-a}-\frac{a^2+b+b-2}{a^2-ab-2b^2}\right):\frac{4a^4+4a^2b+b^2-4}{a^2+b+ab+a}$ ifoda soddalashtirilsin.

Y e c h i s h. Qavs ichidagi ikkinchi kasr maxrajini $a^2-ab-2b^2=-(a+b)(2b-a)$ ko‘rinishida va bo‘luvchini $\frac{(2a^2+b)^2-4}{a(a+b)+(a+b)}=\frac{(2a^2+b-2)(2a^2+b+2)}{(a+b)(a+1)}$ ko‘rinishida yozib, ketma-ket amallarni bajaramiz:

$$\left(\frac{a-b}{2b-a}+\frac{a^2+b^2+b-2}{(a+b)(2b-a)}\right):\frac{(2a^2+b-2)(2a^2+b+2)}{(a+b)(a+1)}=$$

$$=\frac{a^2-b^2+a^2+b^2+b-2}{(a+b)(2b-a)}\cdot\frac{(a+b)(a+1)}{(2a^2+b-2)(2a^2+b+2)}=$$

$$\frac{a+1}{(2b-a)(2a^2+b+2)}.$$

2.32. Quyidagi ifodalarni soddalashtiring:

1) $\left(a^2-(b-c)^2\right):\frac{a+b-c}{a+b+c}$;

2) $\left(\frac{2a+b}{a(a+2b)}-\frac{6b}{4b^2-a^2}\right)\cdot\left(2b+3a-\frac{6a^2}{2a-b}\right)$;

3) $\frac{a^2+a-2}{a-3}\cdot\left(\frac{(a+2)^2-a^2}{4a^2-4}-\frac{3}{2a-b}\right)$;

4) $\frac{2a^2b^2-\frac{1}{2}}{a^3+2a^2}:\frac{2ab-1}{2(a^2+2a)}$.

Javoblar: **2.32.** 2) $\frac{a}{6}$; 4) $\frac{2ab+1}{a}$.

2.33. Quyidagi ifodalarni soddalashtiring:

1) $\left(2x+1-\frac{1}{1-2x}\right):\left(2x-\frac{4x^2}{2x-1}\right)$;

2) $\left(\frac{2}{2x+y}-\frac{1}{2x-y}-\frac{3}{y^2-4x^2}\right)\cdot\left(\frac{y^2}{8x^2}-\frac{1}{2}\right)$;

3) $\frac{a-2}{a^2+2a}:\left(\frac{a}{a^2-2a}-\frac{a^2+4}{a^3-4a}-\frac{1}{a^2+2a}\right)$;

4) $\left(\frac{25}{a^2+5a+25}-\frac{2a}{5-a}-\frac{a^3+25a^2}{a^3+125}\right)\cdot\left(a-5+\frac{15a}{a-5}\right)$;

5) $\dfrac{\frac{a^2}{b^2}-\frac{a}{b}+\frac{b^2}{a^2}-\frac{b}{a}}{\left(\frac{1}{a}-\frac{1}{b}\right)^2\cdot\left(\frac{a}{b}+\frac{b}{a}+1\right)}$;

6) $\left(\frac{3a}{a+6}-\frac{2a}{a^2+12a+36}\right):\frac{3a+16}{a^2-36}+\frac{6(a-6)}{a+6}$;

7) $\left(x-\frac{1+x^2}{x-1}\right):\frac{x^2+2x+1}{x-1}$;

8) $\left(\frac{5m}{m+3}-\frac{14m}{m^2+6m+9}\right):\frac{5m+1}{m^2-9}+\frac{3(m-3)}{m+3}$;

9) $\left(m^2-\frac{1+m^4}{m^2-1}\right):\frac{m^2+1}{m+1}$;

10) $\left(b^2-\frac{1+b^4}{b^2+1}\right):\frac{1-b}{1+b^2}$;

11) $\left(\frac{3a}{a-4}+\frac{10a}{a^2-8a+16}\right):\frac{3a-2}{a^2-16}-\frac{4(a+4)}{a-4}$;

12) $\left(\frac{2x}{x-5}+\frac{x}{x^2-10x+25}\right)\cdot\frac{2x-9}{x^2-25}-\frac{5(x+5)}{x-5}$;

13) $\left(\frac{4a}{4-a^2}-\frac{a-2}{4+2a}\right)\cdot\frac{4}{a+2}-\frac{a}{2-a}$;

14) $\frac{x^3-8}{x^2+2x+4}-\frac{x^2-4}{x-2}$;

15) $\frac{a^2+ab+b^2}{a^3-b}-\frac{a^2-ab+b^2}{a^3+b^3}$;

16) $\frac{5x+6}{x^2-4}-\frac{x}{x^2-4}:\frac{x}{x^2-4}-\frac{x+2}{x-2}$;

17) $\left(\frac{1}{a(a+1)}+\frac{1}{(a+1)(a+2)}\right)\cdot\frac{a^2+2a}{8}$;

18) $\left(\frac{a^2-4}{a^2+4}\right)^2+\left(\frac{4a}{a^2+4}\right)^2$;

19) $\frac{x^3-2x^2}{3x+3}:\frac{x^2-4}{3x^2+6x+3}$;

20) $\left(\frac{1}{m^2-m}-\frac{1}{m-1}\right)\cdot\frac{m}{m+2}+\frac{m}{m^2-4}$;

21) $\left(a^3-3a^2b+3ab^2-b^3\right)\cdot(a+b):\left(\frac{a^3+b^3}{a+b}-ab\right)$;

22) $\frac{x^3+y^3}{x+y}:\left(x^2+y^2\right)+\frac{2y}{x+y}-\frac{xy}{x^2-y^2}$;

23) Agar $a=7+\sqrt{3}$ va $b=7-\sqrt{3}$ bo'lsa,

$\frac{a^3-b^3}{a^2-b^2}:\frac{a^2+ab+b^2}{a^3+3a^2b+3ab^2+b^2}$ ning qiymatini toping;

24) $\left(2a+\frac{2ab}{a-b}\right)\cdot\left(\frac{ab}{a+b}-a\right):\frac{4,5a^2}{a^2-b^2}$;

25) $\frac{a^2}{a^2-1}+\frac{1}{a+1}:\left(\frac{1}{2-a}+\frac{2}{a^2-2a}\right)$;

26) $\frac{a^3+b^3}{a^2-ab+b^2}\cdot(a-b)\cdot\frac{a^3-b^3}{a^2+ab+b^2}\cdot(a+b)$;

27) $a^2b^2\left(\frac{1}{(a+b)^2}\cdot\left(\frac{1}{a^2}+\frac{1}{b^2}\right)+\frac{2}{(a+b)^3}\cdot\left(\frac{1}{a}+\frac{1}{b}\right)\right)$;

28) $\left(\dfrac{2}{1-x^2}-\dfrac{2}{(x-1)^2}\right)\cdot(1-x)^2-\dfrac{4}{1+x}$;

29) $\dfrac{abc}{bc+ac+ab}-\left(\dfrac{a-1}{a}+\dfrac{b-1}{b}-\dfrac{c-1}{c}\right):\left(\dfrac{1}{a}+\dfrac{1}{b}-\dfrac{1}{c}\right)$.

8. Chiziqli, kvadrat tenglamaga keltiriladigan tenglamalar

T a ' r i f. Agar tenglamaning chap va o'ng qismlari noma'lum o'zgaruvchiga nisbatan chiziqli funktsiyalardan iborat bo'lsa, bunday tenglama *chiziqli tenglama* deyiladi.

Umumiy ko'rinishi: $ax + b = cx + d$, bu yerda a, b, c, d – ma'lum sonlar, x – noma'lum son. Tenglamani yechish uchun noma'lum qatnashgan ifodani chapga, aniq ifodani o'ngga yig'amiz:

$$ax - cx = d - b,$$

$$x(a-c) = d-b, \quad \text{bundan} \quad x=\frac{d-b}{a-c}.$$

1) agar $a - c \neq 0$ bo'lsa, tenglama 1 ta yechimga ega;

2) $a - c = 0$, $b - d \neq 0$ bo'lsa, tenglama yechimga ega emas;

3) $a - c = 0$, $b - d = 0$ bo'lsa, tenglama cheksiz ko'p yechimga ega bo'ladi.

Chiziqli tenglamalar $|5x - 7| = 13$ ko'rinishda ham berilishi mumkin. Bunday tenglama absalyut miqdor belgisi ostidagi tenglama deyiladi va u quyidagicha yechiladi:

1) $5x - 7 = 13$
$5x = 13 + 7$
$5x = 20$
$x = 4.$

2) $5x - 7 = -13$
$5x = -13 + 7$
$5x = -6$
$x = -\dfrac{6}{5}.$

Topilgan ikkala yechim ham tenglamani qanoatlantiradi.

Kvadrat uchhadni olamiz:

$$y = ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\left(x^2 + 2\cdot\frac{b}{2a}x + \frac{c}{a}\right) =$$

$$= a\left(x^2 + 2\cdot\frac{b}{2a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}\right) = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right] =$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}.$$

T a ' r i f. $ax^2 + bx + c = 0$ (1) ko'rinishidagi tenglama *kvadrat tenglama* deyiladi. Bunda a, b, c berilgan sonlar bo'lib, $a \neq 0$, x – noma'lum sondir. Bu tenglamani yechish uchun tenglikning chap tarafidagi uchhaddan to'la kvadrat ajratamiz:

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a} = 0, \text{ bundan } \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Tenglikning ikkala tarafini kvadrat ildizdan chiqarsak, $\frac{b^2 - 4ac}{4a^2} \geq 0$ bo'lishi kerak. $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$, bundan

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$D = b^2 - 4ac$ diskriminant deb belgilasak, (1) kvadrat tenglamaning yechimi shu diskriminantga bog'liq.

1) agar $D > 0$ bo'lsa, (1) tenglama 2 ta haqiqiy ildizga ega bo'ladi:

$$x_1 = \frac{-b - \sqrt{D}}{2a}; \quad x_2 = \frac{-b + \sqrt{D}}{2a};$$

2) agar $D < 0$ bo'lsa, (1) tenglama haqiqiy sonlar to'plamida yechimga ega emas;

3) agar $D = 0$ bo'lsa, (1) tenglama bitta haqiqiy yechimga ega bo'ladi:

$$x = \frac{-b}{2a}.$$

To'la kvadrat tenglamaning koeffitsiyentlariga ma'lum shartlar qo'yish natijasida chala kvadrat tenglamalarni hosil qilamiz. $ax^2 + bx + c = 0$ to'la kvadrat tenglama ko'rinishi,

1) $a \neq 0,\ b = 0,\ c = 0$ desak, (1) tenglama $ax^2 = 0$ bo'ladi. Bu tenglamaning yechimi $x_1 = x_2 = 0$ dan iborat.

2) $a \neq 0,\ c \neq 0,\ b = 0$ bo'lsa, (1) tenglama $ax^2 + c = 0$ bo'ladi. Bu tenglamaning yechimi: $x^2 = -\frac{c}{a}$. Agar $\frac{c}{a} < 0$ bo'lsa, $-\frac{c}{a} > 0$ bo'lib, tenglama 2 ta $x_{1,\ 2} = \pm\sqrt{\frac{c}{a}}$ haqiqiy yechimga ega. Agar $\frac{c}{a} > 0$ bo'lsa, $-\frac{c}{a} < 0$ bo'lib, tenglamaning haqiqiy yechimi yo'q.

3) $a \neq 0,\ b \neq 0,\ c = 0$ bo'lsa, (1) tenglama $ax^2 + bx = 0$ bo'ladi. Bu tenglamaning yechimi:

$$x(ax+b) = 0 \Rightarrow \begin{cases} x_1 = 0, \\ x_2 = -\frac{b}{a}. \end{cases}$$

Agar (1) tenglamaning 2-hadi koeffitsiyenti juft son bo'lsa, hisoblashda yengil bo'ladigan quyidagi formulalarni keltirib chiqaramiz:

$b=2k;\ ax^2+2kx+c=0;\ D_1 = k^2 + ac;\quad x_1 = \frac{-k-\sqrt{D_1}}{a};\quad x_2 = \frac{-k+\sqrt{D_1}}{a}.$

Kvadrat tenglama ildizlarini uning diskriminantiga ko'ra tekshirishni quyidagi jadval orqali tushuntirilsa, o'quvchilarning mantiqiy fikrlash qobiliyatlari ortadi:

$D = b - 4ac > 0$	Agar $c > 0$ bo'lsa,	$b < 0$ bo'lsa, ikkala ildiz musbat, $b > 0$ bo'lsa, ikkala ildiz manfiy.
	$c < 0$ bo'lsa, ikkala ildiz har xil bo'ladi	$b < 0$ bo'lsa, ikkala ildiz musbat, $b > 0$ bo'lsa, ikkala ildiz manfiy.
	$c = \begin{cases} x_1 = 0 \\ x_2 = -\frac{b}{a} \end{cases}$	$b > 0$ bo'lsa, ildizlardan biri nolga teng, ikkinchisi esa manfiy bo'ladi, $b < 0$ bo'lsa, ildizlardan biri nolga teng, ikkinchisi esa musbat bo'ladi.
$D = b - 4ac = 0$		$b > 0$ bo'lsa, ikkala ildiz manfiy bo'ladi; $b < 0$ bo'lsa, ikkala ildiz musbat bo'ladi.

T a ' r i f. Agar (1) tenglamaning birinchi hadi koeffitsiyenti 1 ga teng bo'lsa, $x^2 + bx + c = 0$ tenglama *keltirilgan kvadrat tenglama* deyiladi. Keltirilgan kvadrat tenglamalarni Viyet teoremasi yordamida ildizlarini topish mumkin: $x^2 + px + q = 0$

T e o r e m a. *Agar keltirilgan kvadrat tenglama haqiqiy ildizga ega bo'lsa, bu ildizlarning yig'indisi qarama-qarshi ishora bilan olingan x oldidagi koeffitsiyentga, ularning ko'paytmasi esa shu tenglamaning ozod hadiga teng, ya'ni*:

$$\begin{cases} x_1 + x_2 = -p, \\ x_1 \cdot x_2 = q. \end{cases}$$

Kvadrat tenglamaga keltiriladigan tenglamalar

T a ' r i f. 1. $ax^4 + bx^2 + c = 0$ tenglama *bikvadrat tenglama* deyiladi. Bunda $x^2 = z$ almashtirish kiritsak, u $az^2 + bz + c = 0$ ko'rinishga keladi. Bu tenglamani yechib, topilgan yechimni almashtirishga qo'yamiz.

2) $\left(\frac{x}{x+a}\right)^2 - \left(\frac{x+a}{x}\right) = b$, o'zaro teskari noma'lumni o'z ichiga olgan tenglama. Bunda $\left(\frac{x}{x+a}\right)^2 = z$ desak, $z - \frac{1}{z} = b$; bundan: $z^2 - bz - 1 = 0$, bu kvadrat tenglamani yechib almashtirishga qo'ysak, berilgan tenglama yechimi chiqadi.

3. $ax^4 - bx^3 + cx^2 + dx + c = 0$ ko'rinishidagi 4-darajali tenglama ham to'la kvadratga ajratish yo'li bilan kvadrat tenglama ko'rinishiga keltirib yechiladi.

T a ' r i f. $a^n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0 = 0$ tenglama koeffitsiyentlarining $a_n = a_0$, $a_{n-1} = a_1$, $a_{n-2} = a_2$... tengliklari o'rinli bo'lsa, bunday tenglamani *qaytma tenglama* deyiladi.

Bu tenglama ham ayniy almashtirishlar bajarish natijasida kvadrat tenglama ko'rinishiga keladi.

Yuqoridagi tenglamalarning har birini amalda ko'ramiz.

1) $3x^2-5x+2=0;$ $D=25-4\cdot3\cdot2=1>0;$

$$x_1=\frac{5-1}{2\cdot3}=\frac{2}{3};\quad x_2=\frac{5+1}{2\cdot3}=1;$$

2) $3x^2+2x-1=0;$ $D_1=1+3\cdot1=4;$

$$x_1=\frac{-1-2}{3}=-1;\quad x_2=\frac{-1+2}{3}=\frac{1}{3};$$

3) $x^2+4x-3=0;$ $x_1+x_2=-4;$ $x_1\cdot x_2=-3$

$D=4+3=7;$ $x_1=-2-\sqrt{7};$ $x_2=-2+\sqrt{7}.$

Viyet teoremasidan foydalanib tanlash yo‘li bilan ham ildizlarni topish mumkin.

4) $x^2-8x-9=0;$ $x_1+x_2=8,$ $x_1x_2=-9.$

Agar $x_1=-1,$ $x_2=9$ desak, Viyet teoremasi sharti bajariladi.

5) $x^4-4x^2-5=0;$ $x^2=z$

$z^2-4z-5=0;$

$z_1+z_2=4;$

$z_1z_2=-5,$ demak, $z_1=-1,$ $z_2=5;$ z_1-chet ildiz $x^2=5;$ $x_{1,2}=\pm\sqrt{5}$ tenglamaning yechimi.

6) $\left(\frac{x}{x+1}\right)^2-\left(\frac{x+1}{x}\right)^2=\frac{3}{2};$ $x\neq-1,$ $x\neq0;$

$$\left(\frac{x}{x+1}\right)^2=z;\quad z-\frac{1}{2}=\frac{3}{2};\quad 2z^2-3z-2=0;$$

$$D=9+4\cdot2\cdot2=25;$$

$$z_1=\frac{3-5}{4}=-\frac{1}{2};\quad z_2=\frac{3+5}{4}=2;$$

$$\left(\frac{x}{x+1}\right)^2=2;\quad \frac{x}{x+1}=\sqrt{2};$$

z_1 - chet ildiz.

1) $x = \sqrt{2}x + \sqrt{2}; \quad x\left(1-\sqrt{2}\right) = \sqrt{2};$

$$x_1 = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}\left(1+\sqrt{2}\right)}{1-2} = -2-\sqrt{2}; \quad x_1 = -2-\sqrt{2};$$

2) $\frac{x}{x+1} = -\sqrt{2}; \quad x = -\sqrt{2}x - \sqrt{2}; \quad x\left(1+\sqrt{2}\right) = -\sqrt{2};$

$$x_2 = -\frac{\sqrt{2}}{1+\sqrt{2}} = -\frac{\sqrt{2}\left(1-\sqrt{2}\right)}{1-2} = 2-\sqrt{2};$$

$$x_2 = 2-\sqrt{2}.$$

7) $x^4 + 6x^3 + 5x^2 - 12x + 3 = 0$

$x^4 + 6x^3 + 9x^2 - 4x^2 - 12x + 3 = 0$

$(x^2 + 3x)^2 - 4(x^2 + 3x) + 3 = 0$

$x^2 + 3x = z; \quad z^2 - 4z + 3 = 0.$

Tanlash yo'li bilan: $z_1 = 1$, $z_2 = 3$.

1) $x^2 + 3x = 1$; $x^2 + 3x - 1 = 0$; $D = 9 + 4 = 13$.

$$x_1 = \frac{-3-\sqrt{3}}{2}; \quad x_2 = \frac{-3+\sqrt{3}}{2};$$

2) $x^2 + 3x = 3$; $x^2 + 3x - 3 = 0$; $D = 9 + 12 = 21$.

$$x_3 = \frac{-3-\sqrt{21}}{2}; \quad x_4 = \frac{-3+\sqrt{21}}{2}.$$

8) $2x^4 + 3x^3 - 16x^2 + 3x + 2 = 0$.

Tenglamaning har ikkala tomonini $x^2 \neq 0$ ga bo'lamiz:

$2x^2 + 3x - 16 + \frac{3}{x} + \frac{2}{x^2} = 0$ bundan

$$2\left(x^2 + \frac{1}{x^2}\right) + 3\left(x + \frac{1}{x}\right) - 16 = 0,$$

$x + \frac{1}{x} = z$ desak, $x^2 + \frac{1}{x^2} = z^2 - 2$,

$2(z^2-2)+3z-16=0$; $2z^2+3z-20=0$.

$$D=9+4\cdot2\cdot20=169;\quad z_1=\frac{-3-13}{4}=-4;\quad z_2=\frac{10}{4}=\frac{5}{2}.$$

1) $x+\dfrac{1}{x}=-4$; $x^2+4x+1=0$; $D=4-1=3$.

$x_1=-2-\sqrt{3}$; $x_2=-2+\sqrt{3}$;

2) $x+\dfrac{1}{x}=\dfrac{5}{2}$; $2x^2-5x+2=0$; $D=25-4\cdot2\cdot2=9$.

$x_3=\dfrac{5-3}{4}=\dfrac{1}{4}$; $x_4=\dfrac{5+3}{4}=\dfrac{8}{4}=2$.

2.34. Tenglamalarni yeching:

1) $4x+7=5x-3$;
2) $12x+10=-2x+38$;
3) $-9x+7=-5+3x$;
4) $-22+6x=7x-12$;
5) $\dfrac{1}{3}(3x-2)=\dfrac{1}{4}x+\dfrac{2}{3}$;
6) $\left(\dfrac{x-4}{7}+\dfrac{3}{4}\right)\cdot2\dfrac{2}{3}=x-2$;
7) $8\left(\dfrac{x}{4}-1\right)=x+6x+12$;
8) $2x-2,5=1.5(x-1)$;
9) $7,5x-15=10,5(x-2)$;
10) $\dfrac{1}{2}(x-1)=4,2(x-1)$;
11) $9(x-1)=(x-1)(x+2)$;
12) $x(x+2)=(x+2)(x-6)$;
13) $(x-2)(x-9)=0$;
14) $(x-1)(x+4)=0$;
15) $\dfrac{1}{3}x-x^2=1-x^2$;
16) $\dfrac{1}{3}x(x+1)=\dfrac{1}{3}(x+1)^2$;
17) $(x+1)(x-2)=(x+6)(x-5)$;
18) $x^2=(x+2)(x-1)$;
19) $(x+3)^2=(x-2)(x-4)$;
20) $(x+2)(x+1)^2=(x+2)(x-1)^2$.

2.35. Tenglamalarni yeching:

1) $x^2+5x+4=0$;
2) $x^2+7x+6=0$;
3) $3x^2-5x+2=0$;
4) $4x^2+5x+1=0$;

5) $2x^2+3x=0$; 6) $5x^2-2x=0$;

7) $4x^2+1=0$; 8) $2x^2+3=0$;

9) $2x^2-3x+5=0$; 10) $3x^2+4x+2=0$;

11) $x^2(x+1)(x-2)(x-5)=0$; 12) $x^2(x+2)(x-1)(x-3)=0$;

13) $(x+3)^2(x-2)(x-4)=0$;

14) $(x+2)(x+1)^2\left(x+\frac{1}{2}\right)(x-2)^2=0$;

15) $(2x-1)(x-4)(x-3)=0$; 16) $(3x+2)(x^2-1)(x-2)=0$.

2.36. Tenglamalarni yeching:

1) $(x-2)(x^2-9)=0$; 2) $(x^2-1)(x+4)=0$;

3) $\frac{1}{3}x-\frac{4}{9}x^2=1-x$; 4) $\frac{1}{3}x(x+1)=(x+1)^2$;

5) $x\left(\frac{x}{4}-1\right)=x^2+x+1$; 6) $2x-2,5=x(x-1)$;

7) $3x-7,5=x(x-2)$; 8) $\frac{1}{2}x(x-1)=(x-1)^2$;

9) $x(x-1)=(x-1)(x+2)$; 10) $x(x+2)=(x+3)(x-6)$.

9. Bir o'zgaruvchili ratsional va irratsional tenglamalarni yechish usullari

$\frac{P(x)}{Q(x)}=0$ ko'rinishidagi tenglama *kasr-ratsional tenglama* deyiladi, bu yerda $P(x)$ va $Q(x)$ lar ko'phadlar bo'lib, $Q(x)$ ning darajasi kamida 1 ga teng.

$\frac{P(x)}{Q(x)}=0$ tenglamani yechish uchun $P(x)=0$ tenglamaning

$Q(x) \neq 0$ shartni qanoatlantiradigan yechimlarini topish kifoya, ya'ni $\frac{P(x)}{Q(x)}=0$ tenglama va $\begin{cases} P(x)=0 \\ Q(x)\neq 0 \end{cases}$ sistema teng kuchlidir.

Masalan, 1) $\frac{x+3}{x-3}+\frac{x}{x+3}=2$ kasrning surat va maxrajini bir xil ifodaga ko'paytiramiz. Bu kasrning asosiy xossasi. $\frac{(x+3)^2}{x^2-9}+\frac{x(x-3)}{x^2-9}=\frac{2(x^2-9)}{x^2-9}$ tenglama xossasidan tenglikning ikkala tarafini $x^2-9\neq 0$ ifodaga ko'paytiramiz. $x^2+6x+9+x^2-3x=$ $=2x^2-18$ hosil bo'ladi. Soddalashtirsak, $3x=-27$, $x=-9$. Bunda ham tenglikning bir tomonidan ikkinchi tomoniga ifodalarni qarama-qarshi ishora bilan o'tkazish xossasidan foydalandik. Topilgan yechim $x\neq\pm 3$ shartni qanoatlantiradi. Demak, javob: $x=-9$.

2) $\frac{x+2}{x-2}-\frac{x(x-4)}{x^2-4}=\frac{x-2}{x+2}-\frac{4(3+x)}{4-x^2}$;

$$\frac{(x+2)^2}{x^2-4}-\frac{x(x-4)}{x^2-4}=\frac{(x-2)^2}{x^2-4}+\frac{4(3+x)}{x^2-4};$$

$$x^2-4\neq 0$$

$$x^2+4x+4-x^2+4x=x^2-4x+4+12+4x.$$

$$x^2-8x+12=0.$$

Viyet teoremasidan foydalansak, $x_1=2$; $x_2=6$; $x\neq\pm 2$ bo'lgani uchun javob: $x=6$.

2.37. Tenglamani yeching:

1) $\left(\frac{4x-1}{5}+\frac{3}{4}\right):2\frac{1}{2}=7\frac{3}{4}$; 2) $\left(2\frac{3}{4}x+\frac{1}{3}\right)\cdot 3\frac{1}{2}=\frac{x+2}{3}$;

3) $\left(\frac{3x-2}{4}+\frac{4}{5}\right):3\frac{1}{2}=3\frac{3}{4}$; 4) $\left(6\frac{2}{3}-2x\right)\cdot\frac{7}{20}=\frac{13-2x}{5}$;

5) $\left(x+\frac{3}{4}\right)\cdot 2\frac{2}{3}=\frac{10-x}{3}+\frac{4}{5}$; 6) $\left(\frac{5x-2}{4}+2\frac{1}{4}\right)\cdot 1\frac{1}{3}=\frac{8-x}{8}$;

7) $\left(\frac{4-x}{4}+6\right)\cdot 1\frac{2}{3}=\frac{5x+1}{4}$;

8) $\left(\frac{1}{3}+\frac{x-2}{4}\right)\cdot 2\frac{1}{2}=\frac{x-3}{7}+\frac{2}{3}$;

9) $\left(2x-\frac{1}{4}\right)\cdot 3\frac{3}{4}=\frac{x-2}{4}+\frac{1}{3}$;

10) $\left(\frac{x+8}{3}-\frac{1}{8}\right)\cdot 2\frac{1}{4}=\frac{2-3x}{5}+\frac{1}{6}$;

11) $\left(\frac{x+2}{3}+2\frac{1}{3}\right):2\frac{1}{3}=\frac{x+2}{4}$;

12) $17-\frac{x-3}{4}=\left(\frac{x+2}{3}+\frac{1}{4}\right)\cdot 1\frac{2}{3}$;

13) $\left(9-\frac{x-4}{2}\right):3\frac{1}{2}=\frac{8-x}{2}+\frac{1}{3}$;

14) $10-\frac{x-1}{2}=\left(\frac{7-2x}{3}+\frac{4}{5}\right)\cdot 2\frac{1}{3}$;

15) $15-\frac{x-2}{3}=\left(\frac{5-x}{2}+\frac{2}{3}\right)\cdot 1\frac{1}{2}$;

16) $8+\frac{x-2}{3}=\left(\frac{8-2x}{2}+\frac{7}{6}\right):1\frac{1}{2}$;

17) $\left(\frac{x+9}{4}-\frac{2}{3}\right)\cdot\frac{1}{6}=\frac{x+2}{4}+\frac{1}{3}$;

18) $\left(8-\frac{x-6}{2}\right)\cdot 1\frac{2}{3}=\frac{2x-3}{2}+2\frac{1}{6}$;

19) $\left(8-\frac{x+3}{3}\right)\cdot 1\frac{2}{3}=\frac{x-2}{3}+\frac{4}{5}$;

20) $\left(2x-\frac{2}{5}\right)\cdot 2\frac{2}{3}=\frac{3x-1}{2}+\frac{1}{3}$;

21) $\left(x-\frac{2}{3}\right):3\frac{1}{2}=\frac{x-2}{3}+4\frac{1}{3}$;

22) $\left(9-\frac{x-2}{6}\right)\cdot 2\frac{1}{4}=\frac{2x-1}{2}+1\frac{1}{2}$;

23) $\left(7+\frac{x-2}{3}\right)\cdot 4\frac{3}{4}=\frac{3x-2}{3}+\frac{3}{4}$;

24) $22-\frac{x-4}{2}=\left(\frac{7-2x}{3}+\frac{5}{6}\right)\cdot 3\frac{1}{2}$;

25) $\left(\frac{2x+2}{3}-2\frac{1}{4}\right):1\frac{1}{3}=\frac{2x-4}{3}+\frac{3}{4}$;

26) $\left(10-\frac{x-3}{4}\right)\cdot 2\frac{1}{2}=\left(\frac{x+6}{3}+1\frac{3}{4}\right):1\frac{1}{3}$.

2.38. Tenglamani yeching:

1) $\frac{x^2+5x+4}{2x^2-3x+2}=0$;

2) $\frac{3x^2-5x+2}{x^2+x-6}=0$;

3) $\frac{1}{x+2}-\frac{2}{x-2}=0;$ 4) $\frac{2}{x-1}+\frac{1}{x+2}=1;$

5) $\frac{4x^2-4x-3}{x+3}=0;$ 6) $\frac{x-7}{(4-x)(2x+1)}=0;$

7) $\frac{4x^2+x-3}{5x^2+9x-9}=0;$ 8) $\frac{2x^2-3x-2}{x-1}=0;$

9) $\frac{(x+3)(x-5)}{x+1}=0;$ 10) $\frac{2x-4x^2+56}{x-3}=0;$

11) $\frac{4x-x^2-3}{(x+1)(x-2)}=0;$ 12) $\frac{3x^2-5x-8}{2x^2-5x-3}=0;$

13) $\frac{2+9x-5x^2}{3x^2-2x-1}=0;$ 14) $\frac{2+7x-4x^2}{3x^2+2x-1}=0;$

15) $\frac{(x^2-x)(x+2)}{x-3}=0;$ 16) $\frac{x-3}{x(x+2)(x+3)}=0;$

17) $\frac{3}{x^2-1}-\frac{1}{2}=\frac{3}{2x-2};$ 18) $\frac{(x^2-4)(16-x^2)}{x(x+1)}=0;$

19) $\frac{1}{2}x(x+1)=(x+1)^2;$ 20) $\frac{9}{2x+2}+\frac{x}{x-1}=\frac{1-3x}{2-2x};$

21) $\frac{x+3}{2}+\frac{2x-3}{7}=x-7;$ 22) $\frac{2}{2x-4}+\frac{x}{x-2}=\frac{2+6x}{4-2x};$

23) $\frac{2+2x}{x-2}=\frac{x-2}{x+2}+4\frac{1}{3}+x;$ 24) $\left(\frac{2x}{3}-\frac{x-2}{6}\right)=\frac{2x-1}{2}+2\frac{1}{2};$

25) $\left(7+\frac{x-2}{3-x}\right)=\frac{3x-9}{3+x}+\frac{3}{4};$ 26) $\frac{x-4}{2-3x}=\frac{7-2x}{3x+2}+\frac{5}{6}-\frac{4}{9x^2-4}.$

10. Parametrik usulda berilgan kasr-ratsional tenglamalarni yechish

Parametrik usuldagi tenglamalarni yechish degan so‘z tenglamada qatnashayotgan parametrlarning yo‘l qo‘yiladigan barcha qiymatlariga mos keluvchi ildizlarni topish demakdir.

1-m i s o l. $\frac{5}{ax-4}=\frac{1}{9x-a}$ tenglamani yeching.

Bu tenglama ma’noga ega bo‘lishi uchun $ax-4\neq 0$ va $9x-a\neq 0$ bo‘lishi kerak. Tenglamaning har ikkala tomonini $(ax-4)\cdot(9x-a)$ ga ko‘paytirsak, $45x-ax=5a-4$ tenglama hosil bo‘ladi, bundan:

$$45x-ax=5a-4,\ x(45-a)=5a-4.\ (*)$$

Endi biz a ning qanday qiymatlarida $9x-a=0$ va $ax-4=0$ tengliklar o‘rinli bo‘lishini topamiz. $x=\frac{a}{9}$ va $x=\frac{4}{a}$, $a\neq 0$. Bu qiymatlarni (*) tenglamaga qo‘ysak, a ga nisbatan kvadrat tenglama hosil bo‘ladi:

1) $\frac{a}{9}(45-a)=5a-4,$

$45a-a^2=45a-36,$

$a^2=36,\ a=\pm 6.$

2) $\frac{4}{a}(45-a)=5a-4,$

$180-4a=5a^2-4a,$

$a^2=36,\ a=\pm 6.$

Agar parametr $a=\pm 6$ qiymatni qabul qilsa, berilgan tenglama maxraji nolga teng bo‘lib, u ma’noga ega bo‘lmaydi, shu sababli $(45-a)x=5a-4$. (*) tenglama berilgan tenglamaga teng kuchli bo‘lganligi uchun, $a\neq\pm 6$ shartga ko‘ra, bu tenglamani quyidagicha yechamiz:

1. a) agar $45-a\neq 0$ bo‘lsa, $a\neq 45$ bo‘ladi. Bu holda (*) tenglama bitta yechimga ega bo‘ladi;

b) agar $45-a=0$ bo‘lsa, (*) tenglama $0\cdot x=221$ bo‘ladi, bu holda tenglama yechimga ega emas. Javob: $x=\frac{5a-4}{45-a}$, $a\neq 45$ va $a=\pm 6$.

2. Agar $a=45$ bo‘lsa, tenglama yechimga ega emas.

3. Agar $a=\pm 6$ bo‘lsa, tenglama ma’noga ega bo‘lmaydi.

2-m i s o l. $\frac{1}{2n+nx}+\frac{1}{x^2-2x}=\frac{2(n+3)}{x^3-4x}$ tenglamani yeching.

J a v o b : 1) agar $n=-4$ bo'lsa, $x=8$; 2) agar $n=-2$ bo'lsa, $x=4$; 3) agar $n=-1$ bo'lsa, $x=1$; 4) agar $n=1$ bo'lsa, $x=3$.

3-m i s o l. $\frac{1}{x-1}+\frac{1}{x-b}=1+\frac{1}{b}$ tenglamani yeching.

J a v o b: $x_1=b+1, x_2=\frac{2b}{b+1}, b\neq 0, b\neq 1$.

Agar $b=-1$ bo'lsa, $x=0$. Agar $b=1$ bo'lsa, $x=2$.

2.39. Tenglamani yeching:

1) $\left(\frac{3x-1}{2}+\frac{2}{5}\right):1\frac{1}{2}=a$; 2) $\left(1\frac{1}{3}-2x\right)\cdot\frac{7}{10}=\frac{14}{5-ax}$;

3) $\left(x+\frac{1}{4}\right)\cdot 4\frac{2}{4}=\frac{5-x}{3}+\frac{4a}{5}$; 4) $\left(\frac{2x-2}{4}+3\frac{1}{4}\right)\cdot 4\frac{1}{3}=\frac{a-x}{8}$;

5) $\left(\frac{2-x}{4}+5\right)\cdot 1\frac{1}{2}=\frac{2ax+1}{4}$; 6) $\left(\frac{1}{3}+\frac{ax-2}{6}\right)\cdot 2\frac{1}{2}=\frac{x+3}{4}+\frac{2}{5}$;

7) $\left(2ax-\frac{1}{2}\right)\cdot 1\frac{3}{4}=\frac{x-2}{2}+\frac{1}{3}$; 8) $\left(\frac{ax+8}{3}-\frac{1}{2}\right)\cdot 2\frac{1}{3}=\frac{2-3ax}{2}$;

9) $\left(\frac{ax+2}{3}+1\frac{1}{3}\right):3\frac{1}{3}=\frac{ax+2}{4}$; 10) $17-\frac{x-a}{4}=\left(\frac{x+2}{3}+\frac{1}{2}\right)\cdot 2\frac{2}{3}$.

2.40. Tenglamani yeching:

1) $\frac{4x^2-4x-3}{x+a}=0$; 2) $\frac{x-7}{(a-x)(2ax+1)}=0$;

3) $\frac{4x^2+x-3}{5x^2+9x-a}=0$; 4) $\frac{2x^2-3x-2}{ax-1}=0$;

5) $\frac{(x+3)(ax-5)}{ax+1}=0$; 6) $\frac{2x-4x^2+56}{ax-3}=0$;

7) $\frac{4x-x^2-3}{(x+a)(ax-2)}=0$; 8) $\frac{3x^2-5x-8}{2x^2-ax-3}=0$;

9) $\frac{2+9x-5x^2}{ax^2-2x-1}=0$; 10) $\frac{2+7x-4x^2}{3ax^2+2ax-1}=0$.

11. Irratsional tenglamalarni yechish

Irratsional son tushunchasi maktab matematika kursining VIII sinfida o'tiladi. O'quv qo'llanmada irratsional tenglamaga ta'rif berib, uni yechish usullari ko'rsatilgan. O'quv qo'llanmadagi ta'rif quyidagicha ifodalanadi.

T a ' r i f. *Noma'lumlari ildiz ishorasi ostida bo'lgan tenglamalar irratsional tenglamalar deyiladi.*

Bu ta'rifni kengroq ma'noda quyidagicha ham berish mumkin. *Noma'lumlari ildiz ishorasi ostida yoki kasr ko'rsatkichli daraja belgisi ostida bo'lgan tenglama irratsional tenglama deyiladi.*

Masalan: $\sqrt{4-5x}=6$, $x^{\frac{1}{2}}-7=0$, $\sqrt[3]{x-a}+\sqrt[3]{x-b}$,

$\sqrt{2x+8}+\sqrt{x+5}=7$, $\sqrt{1-\sqrt{x^4-x^2}}=x-1$ va hokazo.

Irratsional tenglamani yechishdan avval uning aniqlanish sohasini topish lozim.

1-m i s o l. $\sqrt{3x-6}+\sqrt{1+x}=2$ tenglamaning aniqlanish sohasi topilsin.

Y e c h i s h. $3x-6\geq 0$ va $1+x\geq 0$, bu tengsizliklardan: $x\geq 2$ va $x\geq -1$. Demak, bu tenglamaning aniqlanish sohasi $x\geq 2$ bo'ladi. Haqiqatdan ham, bu tenglama yechilsa, uning ildizi 2 ga teng yoki undan katta son chiqishi uning aniqlanish sohasidan ko'rinadi.

2-m i s o l. $\sqrt{x-1}-\sqrt{3-x}=\sqrt{x+2}$ tenglamaning aniqlanish sohasi topilsin. $x-1\geq 0$, $3-x\geq 0$, $x+2\geq 0$, bu tengsizliklardan $x\geq 1$, $x\leq 3$, $x\geq -2$. Bularga ko'ra tenglamaning aniqlanish sohasi $1\leq x\leq 3$ bo'ladi, bu tenglama ildizi 1 soni bilan 3 soni orasida bo'ladi, deganidir.

Irratsional tenglamalar ayniy shakl almashtirishlar orqali ratsional tenglama ko'rinishiga keltiriladi. Irratsional tenglamalarni yechish uchun eng ko'p ishlatiladigan shakl almashtirish – berilgan tenglikning har ikkala tomonini bir xil darajaga ko'tarish va $\sqrt{f(x)}\cdot\sqrt{g(x)}=\sqrt{f(x)g(x)}$, $\frac{\sqrt{f(x)}}{\sqrt{g(x)}}=\sqrt{\frac{f(x)}{g(x)}}$ kabi usullardir. Bunday shakl almashtirishlarni bajarish jarayonida yechilayotgan tenglama uchun chet ildiz hosil bo'lishi mumkin, chunki bu ayniy tengliklarning o'ng tomonlarining aniqlanish sohasi chapga qaraganda kengroqdir.

T e o r e m a. *Agar $f(x) = g(x)$ tenglamaning har ikkala qismini kvadratga ko'tarilsa, berilgan tenglama uchun chet ildiz hosil bo'ladi, bu chet ildiz $f(x) = -g(x)$ tenglamaning ildizidir.*

I s b o t i. Agar $f(x) = g(x)$ tenglamaning har ikkala tomonini kvadratga ko'tarsak, $[f(x)]^2 = [g(x)]^2$ yoki $[f(x)]^2 - [g(x)]^2 = 0$. Bu degan so'z $[f(x) - [g(x)][f(x) + g(x)] = 0$ deganidir. Bunda quyidagi ikki hol bo'lishi mumkin:

1) agar $f(x) - g(x) = 0$ bo'lsa, $f(x) + g(x)\neq 0$ u holda $f(x) = g(x)$ bo'ladi; 2) agar $f(x) + g(x) = 0$ bo'lsa, $f(x) - g(x)^{1}0$ u holda $f(x) = -g(x)$ bo'ladi. Demak, hosil bo'ladigan chet ildiz yoki $[f(x)]^2 - [g(x)]^2 = 0$. tenglamaning ildizi bo'ladi.

3-m i s o l. $4x = 7$ tenglama berilgan bo'lsin. Bu tenglamaning har ikkala tomonini kvadratga ko'tarsak, $16x^2 = 49$ bo'ladi. Bundan $(16x^2 - 49 = 0) => (4x - 7)(4x + 7) = 0$.

1) agar $4x - 7 = 0$ bo'lsa, $4x + 7 \neq 0$, bundan $x = \frac{7}{4} = 1\frac{3}{4}$;

2) agar $4x + 7 = 0$ bo'lsa, $4x - 7 \neq 0$, bundan $x = -\frac{7}{4} = -1\frac{3}{4}$;

Bunda $x = -1\frac{3}{4}$ chet ildizdir, haqiqatan ham, $4\cdot\left(-\frac{7}{4}\right) = 7$ bundan

$-7 = 7$, bu $x = -1\frac{3}{4}$ yechim tenglamani qanoatlantirmaydi deganidir. Bu chet ildiz $4x = 7$ tenglamaning har ikkala tomonini kvadratga ko'tarish natijasida hosil bo'ladi. Matematika kursida irratsional tenglamalarni yechish quyidagi ikkita usul orqali amalga oshiriladi:

1) irratsional tenglamaning har ikkala tomonini bir xil darajaga ko'tarish;

2) yangi o'zgaruvchilar kiritish.

Irratsional tenglamalarning ikkala tomonini bir xil darajaga ko'tarish usuli quyidagi ketma-ketlik asosida amalga oshiriladi:

a) berilgan irratsional tenglama $\sqrt[n]{f(x)} = \sqrt[n]{g(x)}$ ko'rinishga keltiriladi;

b) bu tenglamaning ikkala tomoni n darajaga ko'tariladi;

d) natijada $f(x) = g(x)$ ratsional tenglama hosil bo'ladi;

e) $f(x) = g(x)$ ratsional tenglama yechiladi va tekshirish orqali chet ildiz aniqlanadi.

4-m i s o l. $\sqrt{3x+4} = x$ tenglama yechilsin.

Ye c h i s h. Tenglamaning aniqlanish sohasini topamiz: $x \geq 0$ va $3x + 4 \geq 0$, bundan $x \geq -\frac{4}{3}$.

1 - u s u l. Har ikkala tomonini kvadratga ko'tarsak:

$$\left[\left(\sqrt{3x+4}\right)^2 = x^2\right] \Rightarrow \left|3x+4 = x^2\right|..$$

Bundan $x^2 - 3x - 4 = 0$ tenglamani hosil qilamiz. Uning yechimlari $x_1 = 4$ va $x_2 = -1$, $x_2 = -1$ yechim $\sqrt{3x+4} = x$ tenglama uchun chet ildizdir, chunki u tenglamani qanoatlantirmaydi.

2 - u s u l.

$$\left[\left(\sqrt{3x+4}\right)^2 = x^2\right] \Rightarrow \left(3x+4 = x^2\right) \Rightarrow \left\{\left[x^2 - (3x+4)\right] = 0\right\} \Rightarrow$$

$$\Rightarrow \left(x - \sqrt{3x+4}\right)\left(x + \sqrt{3x+4}\right) = 0.$$

1) agar $x-\sqrt{3x+4}=0$ bo‘lsa, $x+\sqrt{3x+4}\neq 0$ bo‘ladi, bundan $x=\sqrt{3x+4}$ berilgan tenglama hosil bo‘ladi, buning yechimi $x = 4$ bo‘ladi;

2) agar $x+\sqrt{3x+4}=0$ bo‘lsa, $x-\sqrt{3x+4}\neq 0$ bo‘ladi, bundan $x=-\sqrt{3x+4}$ bo‘ladi, buning yechimi $x = -1$ dir.

Demak, $\sqrt{3x+4}=x$ tenglamaning har ikkala tomonini kvadratga ko‘tarish natijasida $x = -1$ chet ildiz hosil bo‘ladi, $x = 4$ esa uning haqiqiy yechimi bo‘ladi.

Irratsional tenglamalarni yangi o‘zgaruvchilar kiritish usuli bilan ham yechiladi.

5-m i s o l. $\sqrt[5]{(x-5)^2}-\sqrt[5]{x-5}=6$ tenglamani yeching.

Y e c h i s h. Bu tenglamaning aniqlanish sohasi $(-\infty, \infty)$. Agar $\sqrt[5]{x-5}=y$ deb belgilasak, tenglama $y^2 - y - 6 = 0$ ko‘rinishga keladi. Bu tenglama $y_1 = 3$ va $y_2 = -2$ yechimlarga ega. Bunga ko‘ra $(\sqrt[5]{x-5}=3)\Rightarrow(\sqrt[5]{x-5})^5=3^5; x - 5 = 243; x = 248.$ $\sqrt[5]{x-5}=-2$ tenglama aniqlanish sohasiga tegishli bo‘lgan ildizga ega. Demak, $x_1 = 248$, $x_2 = -27$ tenglamaning yechimi bo‘lar ekan.

6-m i s o l. $\sqrt{x+4}+\sqrt{x+20}=8$ tenglama yechilsin.

Y e c h i s h. 1) aniqlanish sohasini topamiz: $x+4\geq 0$ va $x+20\geq 0$, bulardan $x\geq -4$ va $x\geq -20$ bo‘ladi. Bundan $x\geq -4$ qiymat olinadi.

2) berilgan tenglamaning har ikkala tomonini kvadratga ko‘taramiz:

$$\left(\sqrt{x+4}+\sqrt{x+20}\right)^2=8^2,$$

$$x+4+2\sqrt{x+4}\cdot\sqrt{x+20}+x+20=64,$$

$$2x+2\sqrt{x+4}\cdot\sqrt{x+20}=40,$$

$$\left(\sqrt{(x+4)(x+20)}\right)^2=(20-x)^2,$$

$$(x+4)(x+20)=400-40x+x^2,$$

$$x^2+24x+80=400-40x+x^2,$$

$64x = 320,$

$x = 5.$

Bu tenglamani yana quyidagi usul bilan ham yechish mumkin:

$$\left(\sqrt{x+4}+\sqrt{x+20}\right)^2 = 8^2, \left[\left(\sqrt{x+4}+\sqrt{x+20}\right)^2 - 8^2 = 0\right] \Rightarrow$$

$$\Rightarrow \left[\left(\sqrt{x+4}+\sqrt{x+20}\right)-8\right]\left[\left(\sqrt{x+4}\right)\left(\sqrt{x+20}\right)+8\right] = 0.$$

1) $\sqrt{x+4}+\sqrt{x+20} = 8,$ buning yechimi $x = 5$;

2) $\sqrt{x+4}+\sqrt{x+20} = -8,$ bu tenglama yechimga ega emas.

7-m i s o l. $\sqrt{2x+3} = a,$ parametrik ko'rinishdagi irratsional tenglama yechilsin.

Bu yerda tenglamaning aniqlanish sohasiga nisbatan $a > 0$ shartni qo'yish etarli bo'ladi. $(\sqrt{2x+3})^2 = a^2$, $2x + 3 = a^2$, bundan $2x = a^2 - 3$ yoki $x = \dfrac{a^2-3}{2}$ yechim hosil bo'ladi.

T e k s h i r i s h. $\sqrt{2\dfrac{a^2-3}{2}+3} = a, \quad \sqrt{a^2} = a, \quad a = a.$

8-m i s o l. $\sqrt{x^2+4x+4}+\sqrt{x^2-10x+25} = 10$ irratsional tenglamani yeching.

Y e c h i s h. Bu tenglamani $\sqrt{(x+2)^2}+\sqrt{(x-5)^2} = 10$ yoki $|x + 2| + |x - 5| = 10$ ko'rinishga keltirib, so'ngra yechamiz.

a) agar $x < -2$ bo'lsa, $-x - 2 - x + 5 = 10$, bundan $-2x = 7$ yoki $x = -3{,}5$;

b) agar $-2 \le x \le 5$ bo'lsa, $x + 2 - x + 5 = 10$, yoki $7 = 10$, bu holda tenglama yechimga ega emas;

d) agar $x > 5$ bo'lsa, $x + 2 + x - 5 = 10$, bundan $2x = 13$ yoki $x = 6{,}5$.

J a v o b. $x = -3{,}5$ va $x = 6{,}5$

9-m i s o l. $\sqrt{1-4x}+2 = \sqrt{x^2-6x+9}$ tenglamani yeching.

Y e c h i s h. Bu tenglamani $\sqrt{1-4x}+2=\sqrt{(x-3)^2}$ ko'rinishda yozib olamiz, u holda: $\sqrt{1-4x}+2=|x-3|$. Bu tenglamaning aniqlanish sohasi $1-4x\geq 0$ yoki $x\leq\frac{1}{4}$ bo'ladi. Aniqlanish sohasi $x\leq\frac{1}{4}$ bo'lgani uchun $\sqrt{1-4x}+2=3-x$. bo'ladi.

$$\left(\sqrt{1-4}+2=1-x\right)\Rightarrow\left(1-4x=1-2x+x^2\right)\Rightarrow$$

$$\Rightarrow\left(x^2+2x=0\right)\Rightarrow\left(x_1=-2\ \ va\ \ x_2=0\right).$$

Tekshirish. $\sqrt{1-4\cdot(-2)}+2=\sqrt{(-2)^2-6(-2)+9}; 5=5.$

10-m i s o l. $1-\frac{1}{x}=\sqrt{1-\frac{1}{x}\sqrt{4-\frac{7}{x^2}}}$ tenglama yechilsin.

Yechish.

1) bu tenglamaning aniqlanish sohasini topamiz, $-\frac{1}{x}<1$ bo'ladi;

2) berilgan tenglamaning har ikkala tomonini kvadratga ko'tarib, $\frac{2}{x}-\frac{1}{x^2}=\frac{1}{x}\sqrt{4-\frac{7}{x^2}}$ tenglamani hosil qilamiz;

3) bu tenglamaning har ikkala tomonini yana kvadratga ko'tarsak, $\frac{4}{x^2}-\frac{4}{x^3}+\frac{1}{x^4}=\frac{4}{x^2}-\frac{7}{x^2}$ yoki $\frac{8}{x^4}-\frac{4}{x^3}=0$;

4) oxirgi tenglamani yechamiz: $8x^3-4x^2=0$; $4x^3(2-x)=0$;

a) agar $4x^3=0$ bo'lsa, $2-x\neq 0$, bundan $x_{1,2,3}=0$;

b) agar $2-x=0$ bo'lsa, $4x^3\neq 0$, bundan $x_4=2$.

T e k s h i r i s h. $1-\frac{1}{2}=\sqrt{1-\frac{1}{2}\sqrt{4-\frac{7}{4}}},\ \ \frac{1}{2}=\frac{1}{2}.$

J a v o b. $x=2$.

11-m i s o l. $\sqrt{x^2-8x+7}+2\sqrt{x^2-18x+17}=2\sqrt{x^2-32x+31}$ tenglamani yeching.

Y e c h i s h. 1) bu ko'rinishdagi tenglamalarni yechish uchun ildiz ostidagi ifodalarni ko'paytuvchilarga ajratamiz:

$$\sqrt{(x-1)(x-7)}+2\sqrt{(x-1)(x-17)}=2\sqrt{(x-1)(x-31)}.$$

2) aniqlanish sohasini topamiz:

$$\begin{cases}(x-1)(x-7)\geq 0,\\ (x-1)(x-17)\geq 0,\\ (x-1)(x-31)\geq 0,\end{cases} \text{ bulardan } \begin{cases}x\leq 1\\ x\geq 31.\end{cases}$$

3) tenglama ko'rinishini quyidagicha yozib olamiz:

$$\sqrt{|x-1|}\cdot\sqrt{|x-7|}+2\sqrt{|x-1|}\cdot\sqrt{|x-7|}=2\sqrt{|x-1|}\cdot\sqrt{|x-31|}$$

yoki $\sqrt{|x-1|}\left(\sqrt{|x-7|}+2\sqrt{|x-17|}-2\sqrt{|x-31|}\right)=0$ bundan:

$a)\left(\sqrt{|x-1|}=0\right)\Rightarrow(x=1);$

$b)\sqrt{|x-7|}+2\sqrt{|x-17|}-2\sqrt{|x-31|}=0.$

Agar $x\leq 1$ bo'lsa, tenglamani $\sqrt{7-x}+2\sqrt{17-x}-2\sqrt{31-x}=0$ ko'rinishda yozib olish mumkin. Bundan:

$$\sqrt{7-x}+2\sqrt{17-x}=2\sqrt{31-x}.$$

4) hosil qilingan tenglamaning har ikkala tomonini kvadratga ko'tarib ixchamlasak, $15x^2-482x-497=0$ tenglama hosil bo'ladi, uni yechsak, $x_1=-1$ va $x_2=\frac{497}{15}$ yechimlar topiladi, ammo $x_2=\frac{497}{15}$ yechim berilgan tenglamaning $x\leq 1$ aniqlanish sohasida yotmaydi, shuning uchun bu hol uchun yechim $x=-1$ bo'ladi. Agar $x\geq 31$ bo'lsa, tenglama $\sqrt{x-7}+2\sqrt{x-17}-2\sqrt{x-31}=0$ ko'rinishni oladi, bunday tenglamani yechishni bilamiz.

J a v o b: $x_1=-1$ va $x_2=1$.

12-m i s o l. $\sqrt[3]{x+45}-\sqrt[3]{x-16}=1$ irratsional tenglama yechilsin.

Y e c h i s h. *1-usul.* $\sqrt[3]{x+45}=u$, $\sqrt[3]{x-16}=v$ deb belgilasak, bulardan $x + 45 = u^3$ va $x - 16 = v^3$ hamda $u - v = 1$ bo'ladi. Bulardan ushbu sistemani hosil qilamiz:

$$\begin{cases} u^3 - v^3 = 61, \\ u - v = 1. \end{cases}$$

$$(u - v)(u^2 + uv + v^2) = 61;\ u^2 + uv + v^2 = 61;\ u = v + 1;$$

$$(v + 1)^2 + (v + 1)v + v^2 = 61;\ 3v^2 + 3v - 60 = 0;\ v^2 + v - 20 = 0;$$

$$v_{1,2} = \frac{-1 \pm \sqrt{1+80}}{2} = \frac{-1 \pm 9}{2};$$

$$v_1 = -5;\ v_2 = 4;\ x_1 = v_1^3 - 45 = -109;$$

$$x_2 = v_2^3 + 16 = 4^3 + 16 = 64 + 16 = 80.$$

2-usul. Tenglamaning ikkala tomonini kubga ko'taramiz:

$$\left(\sqrt[3]{x+45} - \sqrt[3]{x-16}\right)^3 = 1^3$$

yoki

$$x + 45 - x + 16 - 3 \cdot \sqrt[3]{(x+45)(x-16)}\left(\sqrt[3]{x+45} - \sqrt[3]{x-16}\right) = 1$$

$61 - 3\sqrt[3]{(x+45)(x-16)} = 1,$ bundan $\sqrt[3]{(x+45)(x-16)} = 20,$ $(x + 45)(x - 16) = 8000,$ $x^2 + 29x - 8720 = 0.$ Bu tenglamani yechsak, $x_1 = -109$ va $x_2 = 80$ yechimlar kelib chiqadi.

13-m i s o l. $\sqrt[3]{2-x} = 1 - \sqrt{x-1}$ tenglama yechilsin.

Y e c h i s h. $\sqrt[3]{2-x} = u$ va $\sqrt{x-1} = v$ desak, u holda $u^3 = 2 - x,$ $v^2 = x - 1,$ $v + u = 1,$ $v \geq 0.$ Bu tengliklardan $\begin{cases} u^3 + v^3 = 1 \\ v + u = 1 \end{cases}$ sistemani hosil qilib uni yechamiz:

$v = 1 - u;$ $u^3 + u^2 - 2u = 0;$ $u(u^2 + u - 2) = 0,$ bundan $u_1 = 0;$ $u_2 = -2;$ $u_3 = 1;$ $v_1 = 1;$ $v_2 = 3;$ $v_3 = 0$ kelib chiqadi.

Bularga asosan:

1) $\sqrt[3]{2-x} = 0,$	2) $\sqrt[3]{2-x} = -2,$	3) $\sqrt[3]{2-x} = 1,$
$2 - x = 0,$	$2 - x = -8,$	$2 - x = 1,$
$x_1 = 2.$	$x_2 = 10.$	$x_3 = 1.$

Javob. $x_1 = 2; x_2 = 10; x_3 = 1$.

2.41. Tenglamani yeching:

1) $1+\sqrt{2x-2} = x$; 2) $\sqrt{4-x}+\sqrt{5+x} = 3$;

3) $\sqrt{2x+1} = 2\sqrt{x}-\sqrt{x-3}$; 4) $\sqrt{1+x\sqrt{x^2+24}} = x+1$;

5) $\sqrt{a+x}+\sqrt{a-x} = \sqrt{2x}$; 6) $\sqrt{x-3}+\sqrt{x-18} = x$;

7) $\sqrt{x+4}+\sqrt{20+x} = 8$; 8) $\sqrt{3x+4}+\sqrt{x-4} = 2\sqrt{x}$;

9) $\sqrt{x+6}-\sqrt{x+1} = 1$; 10) $\sqrt{15-x}+\sqrt{3-x} = 6$;

11) $1+\sqrt{1+x\sqrt{x^2-24}} = x$; 12) $\sqrt{3x+7}-\sqrt{x+1} = 2$;

13) $\sqrt[3]{1+\sqrt{x}}+\sqrt[3]{1-\sqrt{x}} = 2$; 14) $\sqrt{x+\sqrt{x+11}}+\sqrt{x-\sqrt{x+11}} = 14$.

Javoblar: **2.41.** 1) 3; 2) 4 va –5; 3) 4; 4) 0 va 5; 5) a va $-a$; 6) 7; 7) 5; 8) 4; 9) 3; 10) – 1; 11) 7; 12) –1 va 3; 13) 0; 14) 5.

12. Parametrli irratsional tenglamalarni yechish

1-m i s o l. $\sqrt{x-1} = x-a$ tenglama yechilsin.

Y e c h i s h. Berilgan tenglamani quyidagicha yozib olamiz:

$$x-1-\sqrt{x-1}+1-a=0. \qquad (1)$$

Agar bu tenglamada $\sqrt{x-1} = y$ desak, $x-1 = y^2$ bo'ladi, u holda tenglamani quyidagicha yozish mumkin:

$$y^2-y+1-a=0, \qquad y_{1,2} = \frac{1}{2} \pm \sqrt{\frac{1}{4}-(1-a)} =$$

$$= \frac{1}{2} \pm \sqrt{\frac{1-4+4a}{4}} = \frac{1}{2} \pm \frac{\sqrt{4a-3}}{2};$$

$$y_1 = \frac{1+\sqrt{4a-3}}{2}; \qquad y_2 = \frac{1-\sqrt{4a-3}}{2}.$$

(1) tenglama faqat $a \geq \frac{3}{4}$ bo‘lgandagina yechimga ega bo‘ladi, ya’ni:

$$\sqrt{x-1} = \frac{1-\sqrt{4a-3}}{2}, \qquad (2)$$

$$\sqrt{x-1} = \frac{1+\sqrt{4a-3}}{2}. \qquad (3)$$

(2) tenglama $1-\sqrt{4a-3} \geq 0$ bo‘lganida yechimga ega bo‘ladi. (2) va (3) ni yechsak, $\frac{3}{4} \leq a \leq 1$ tengsizlik hosil bo‘ladi, u holda tenglama quyidagi ko‘rinishdagi ikkita haqiqiy har xil yechimga ega bo‘ladi:

$$x_1 = \frac{2a+1+\sqrt{4a-3}}{2}; \qquad x_2 = \frac{2a+1-\sqrt{4a-3}}{2}.$$

Agar $a > 1$ bo‘lsa, tenglama $x_{1,2} = \frac{2a+1-\sqrt{4a-3}}{2}$ yechimga ega bo‘ladi, agar $a < \frac{3}{4}$ bo‘lsa, tenglama yechimga ega bo‘lmaydi.

2-m i s o l. $\sqrt{3x-2} + \sqrt{x+2} = a$ tenglama yechilsin.

Y e c h i s h. Bu tenglamaning aniqlanish sohasi

$$\begin{cases} 3x-2 \geq 0, \\ x+2 \geq 0 \end{cases} \Rightarrow \begin{cases} x \geq \frac{2}{3}, \\ x \geq -2 \end{cases} \Rightarrow x \geq \frac{2}{3}$$

bo‘ladi, u holda berilgan tenglama $a_1 \geq \frac{2\sqrt{6}}{3}$ bo‘lgandagina yechimga ega bo‘ladi, agar $a < \frac{2\sqrt{6}}{3}$ bo‘lsa, yechimga ega emas:

$$\sqrt{3x-2} = a - \sqrt{x+2} \qquad (1)$$

$$3x-2 = a^2 - 2a\sqrt{x+2} + x + 2$$

$$2x-4 = a^2 - 2a\sqrt{x+2}$$

$$2x+4+2a\sqrt{x+2}-a^2-8=0$$

$$2(x+2)+2a\sqrt{x+2}-a^2-8=0.$$

Agar $\sqrt{x+2}=y$ desak, $2y^2+2ay-a^2-8=0$ yoki $2y^2+2ay-(a^2+8)=0$, bundan

$$y_{1,2}=\frac{-a\pm\sqrt{a^2+2a^2+16}}{2}=\frac{-a\pm\sqrt{3a^2+16}}{2};$$

$$\sqrt{x+2}=\frac{-a-\sqrt{3a^2+16}}{2}; \qquad (2)$$

$$\sqrt{x+2}=\frac{-a+\sqrt{3a^2+16}}{2}; \qquad (3)$$

$a\ge\frac{2\sqrt{6}}{3}$ da (2) tenglama yechimga ega emas, (3) tenglama esa $x=\frac{2a^2+4-a\sqrt{3a^2+16}}{2}$ yechimga ega bo‘ladi.

3-m i s o l. $\sqrt{x^2-ax+2}=x-1$ tenglama yechilsin.

Y e c h i s h. $\begin{cases}x-1\ge 0,\\ x^2-ax+2=(x-1)^2\end{cases}\Leftrightarrow\begin{cases}x\ge 1,\\ (a-2)x=1.\end{cases}$

Agar $a=2$ bo‘lsa, sistema yechimga ega emas, agar $a\ne 2$ bo‘lsa, $\{\ \ge$ ____ hosil bo‘ladi, u holda $\frac{1}{a-2}\ge 1$ yoki $2<a\le 3$ bo‘ladi.

Javob. Agar $2<a\le 3$ bo‘lsa, $x=\frac{1}{a-2}$; agar $a\le 2$, $a>3$ bo‘lsa, tenglama yechimga ega emas.

4-m i s o l. $\sqrt{a-\sqrt{a+x}}=x$ tenglama yechilsin.

Y e c h i s h.

$$\begin{cases} x \geq 0 \\ a + x \geq 0 \\ a - \sqrt{a+x} \geq 0 \end{cases} \Rightarrow \begin{cases} x \geq 0 \\ x \geq -a \\ a \geq \sqrt{a+x} \end{cases} \Rightarrow \begin{cases} x \geq 0 \\ x \geq -a \\ a^2 \geq a + x \end{cases} \Rightarrow \begin{cases} x \geq 0 \\ x \geq -a \\ x \leq a^2 - a. \end{cases}$$

Agar $a \geq 1$ bo'lsa, tenglamaning aniqlanish sohasi $0 \leq x \leq a^2 - a$ bo'ladi. $a - \sqrt{a+x} = x^2$ yoki $\sqrt{a+x} = a - x^2$, bu tenglama $a - x^2 \geq 0$ bo'lgandagina yechimga ega, shuning uchun $a + x = a^2 - 2ax^2 + x^4$ yoki

$$a^2 - (2x^2 + 1)a + (x^4 - x) = 0$$

$$a_{1,2} = \frac{2x^2 + 1 \pm \sqrt{(2x^2+1)^2 - 4x^2 + 4x}}{2};$$

$$a_1 = x^2 - x; \quad a_2 = x^2 + x + 1; \quad a_1 = x^2 - x$$

tenglama $a^2 - x^2 \geq 0$ shart uchun $x = \dfrac{-1 \pm \sqrt{4a-3}}{2}$.

5-m i s o l. $\sqrt[3]{a+x} + \sqrt[3]{a-x} = \sqrt[3]{2a}$ tenglama yechilsin.

Y e c h i s h. Bu tenglamani yechish uchun uning har ikki tomonini uchinchi darajaga ko'taramiz:

$$a + x + a - x + 3\sqrt[3]{a^2 - x^2}\left(\sqrt[3]{a+x} + \sqrt[3]{a-x}\right) = 2a;$$

$$2a + 3\sqrt[3]{a^2 - x^2} \cdot \sqrt[3]{2a} = 2a;$$

$$3\sqrt[3]{a^2 - x^2} \cdot \sqrt[3]{2a} = 0;$$

$$\sqrt[3]{a - x^2} = 0; \quad a^2 - x^2 = 0; \quad x_{1,2} = \pm a.$$

6-m i s o l. $\sqrt[3]{(a+x)^2} + 4\sqrt[3]{(a-x)^2} = 5\sqrt[3]{a^2 - x^2}$ tenglama yechilsin.

Y e c h i s h. Tenglamaning har ikkala tomonini $\sqrt[3]{(a-x)^2}$ ga bo'lamiz, bu yerda $x \neq a$:

$$\sqrt[3]{\left(\frac{a+x}{a-x}\right)^2}+4=5\sqrt[3]{\frac{a+x}{a-x}};$$

$$\sqrt[3]{\frac{a+x}{a-x}}=t; \quad \frac{a+x}{a-x}=t^3;$$

$$t^2-5t+4=0; \quad t_1=4; \quad t_2=1.$$

$$1)\ \frac{a+x}{a-x}=64; \quad 65x=63a; \quad x_1=\frac{63a}{65};$$

$$2)\ \frac{a+x}{a-x}=1; \quad x_2=0.$$

7-m i s o l. $\sqrt[3]{a+\sqrt{x}}+\sqrt[3]{a-\sqrt{x}}=\sqrt[3]{b}$ tenglama yechilsin.

Y e c h i s h. Bu tenglamaning har ikki tomonini $(a+b)^3=a^3+3a^2b+3ab^2=a^3+b^3+3ab(a+b)$ formula bo‘yicha kubga ko‘taramiz:

$$a+\sqrt{x}+a-\sqrt{x}+3\sqrt[3]{a^2-x}\cdot\left(\sqrt[3]{a+\sqrt{x}}+\sqrt[3]{a-\sqrt{x}}\right)=b.$$

$\sqrt[3]{a+\sqrt{x}}+\sqrt[3]{a-\sqrt{x}}=\sqrt[3]{b}$ bo‘lgani uchun $2a+3\sqrt[3]{a^2-x}\cdot\sqrt[3]{b}$

$\sqrt[3]{a^2-x}=\frac{b-2a}{3\sqrt[3]{b}};\ a^2-x=\frac{(b-2a)^3}{27b},\ b\neq 0$ bo‘lsa,

$$x=a^2-\frac{(b-2a)^3}{27b}.$$

8-m i s o l. $\sqrt[4]{a-x}+\sqrt[4]{b-x}=\sqrt[4]{a+b-2x}$ tenglama yechilsin.

Y e c h i s h. Tenglamaning aniqlanish sohasi:

$$\begin{cases} a-x\geq 0 \\ b-x\geq 0 \\ a+b-2x\geq 0. \end{cases}$$

Endi $\sqrt[4]{a-x}=t,\ \sqrt[4]{b-x}=z$ belgilash kiritamiz, u holda $t^4+z^4=a+b-2x=(a-x)+(b-x)$. Bularni tenglamaga qo‘yamiz:

$$t+z=\sqrt[4]{t^4+z^4};$$

$$(t+z)^4=t^4+z^4;$$

$$t^4+4t^3z+6t^2z^2+4tz^3+z^4=t^4+z^4;$$

$$tz\left(2t^2+3tz+2z^2\right)=0;$$

$$1)\ t_1=0,\quad z_1=0,\quad (a-x=0)\Rightarrow x_1=a;$$

$$(b-x=0)\Rightarrow x_1=b;$$

2) $2t^2+3tz+2z^2=0$ tenglama yechilsa, qolgan yechimlar hosil bo‘ladi.

9-m i s o l. $\sqrt[n]{\dfrac{a-x}{b+x}}+\sqrt[4]{\dfrac{b+x}{a-x}}=2$ tenglama yechilsin.

Y e c h i s h. Bu tenglamaning yo‘l quyiladigan qiymatlar sohasi $x\neq a$ va $x\neq b$. Bundan tashqari, $\dfrac{a-x}{b+x}>0$ va $\dfrac{b+x}{a-x}>0$. Agar bo‘lsa, $a-x>0$ va $b+x>0$ yoki $a-x<0$ va $b+x<0$ bo‘ladi, bulardan: $x<a$ va $x>-b$.

$\dfrac{a-x}{b+x}=z^n$ desak, $\left(z+\dfrac{1}{z}=2\right)\Rightarrow\left(z^2-2z+1=0\right)$,

$z_{1,2}=1;\ \dfrac{a-x}{b+x}=1;\ a-x=b+x$ yoki $x=\dfrac{a-b}{2}$.

T e k s h i r i s h.

$$\sqrt[n]{\frac{a-\frac{a-b}{2}}{b+\frac{a-b}{2}}}+\sqrt[n]{\frac{b+\frac{a-b}{2}}{a-\frac{a-b}{2}}}=\sqrt[n]{\frac{a+b}{a+b}}+\sqrt[n]{\frac{a+b}{a+b}}=1+1=2.$$

10-m i s o l. $(x-a)\sqrt{x}-(x+a)\sqrt{b}=b\left(\sqrt{x}-\sqrt{b}\right)$ tenglama yechilsin.

Y e c h i s h. Tenglamaning chap tomonidagi ifodaning qavslarini ochib va o‘ng tomonidagi ifodani qarama-qarshi ishora bilan chap tomonga o‘tkazib, berilgan tenglamani quyidagi ko‘rinishda yozamiz:

$$x\sqrt{x}-a\sqrt{x}-x\sqrt{b}-a\sqrt{b}-b\left(\sqrt{x}-\sqrt{b}\right)=0;$$

$$x\left(\sqrt{x}-\sqrt{b}\right)-a\left(\sqrt{x}+\sqrt{b}\right)-b\left(\sqrt{x}-\sqrt{b}\right)=0;$$

$$\left(\sqrt{x}-\sqrt{b}\right)(x-b)-a\left(\sqrt{x}+\sqrt{b}\right)=0;$$

$$\left(\sqrt{x}-\sqrt{b}\right)^2\left(\sqrt{x}+\sqrt{b}\right)-a\left(\sqrt{x}+\sqrt{b}\right)=0;$$

$$\left(\sqrt{x}+\sqrt{b}\right)\left[\left(\sqrt{x}-\sqrt{b}\right)^2-a\right]=0.$$

$\sqrt{x}+\sqrt{b}\neq 0$ bo‘lsa, $\left(\sqrt{x}+\sqrt{b}\right)^2-a=0$ bo‘ladi, bundan $\left(\sqrt{x}-\sqrt{b}\right)^2=a$ hosil bo‘ladi.

1) agar $a<0$ bo‘lsa, bu tenglama yechimga ega emas;

2) agar $a>0,\ b>0$ bo‘lsa, $\sqrt{x}-\sqrt{b}=\pm\sqrt{a},\ \sqrt{x}=\sqrt{b}\pm\sqrt{a}$ bo‘ladi, $x_{1,2}=\left(\sqrt{b}\pm\sqrt{a}\right)^2$.

3) agar $b=0$ va $a>0$ bo‘lsa, $x_1=0,\ x_2=a$ yechimlar bo‘ladi.

11-m i s o l. $\dfrac{\sqrt[n]{a+x}}{a}+\dfrac{\sqrt[n]{a+x}}{x}=\dfrac{\sqrt[n]{x}}{b}$ tenglama yechilsin.

Y e c h i s h. Bunda $n\in N$ bo‘lib, $x>0,\ a+x>0,\ a\neq 0,\ b\neq 0,\ x\neq 0$. Tenglamaning chap tomonidagi $\sqrt[n]{a+x}$ ni qavsdan chiqaramiz:

$$\sqrt[n]{a+x}\left(\frac{1}{a}+\frac{1}{x}\right)=\frac{\sqrt[n]{x}}{b},\quad \sqrt[n]{a+x}\cdot\frac{a+x}{ax}=\frac{\sqrt[n]{x}}{b}\cdot\sqrt[n]{\left(\frac{a+x}{x}\right)^{n+1}}=\frac{a}{b}.$$

Bu tenglikning chap tomoni noldan katta, shuning uchun o‘ng tomoni ham noldan katta bo‘ladi, ya’ni $\dfrac{a}{b}>0$.

$$\left(\frac{a+x}{x}\right)^{\frac{n+1}{n}}=\frac{a}{b};\quad \frac{a+x}{x}=\left(\frac{a}{b}\right)^{\frac{n+1}{n}};\quad \frac{a}{x}+1=\left(\frac{a}{b}\right)^{\frac{n}{n+1}};$$

$$\frac{a}{x}=\left(\frac{a}{b}\right)^{\frac{n}{n+1}}-1;\quad a=\left[\left(\frac{a}{b}\right)^{\frac{n}{n+1}}-1\right]x.$$

Agar $\left(\frac{a}{b}\right)^{\frac{n}{n+1}} - 1 \neq 0,\ a \neq 0$ bo‘lsa, $x = \frac{a}{\left(\frac{a}{b}\right)^{\frac{n}{n+1}} - 1}$. (1)

1) agar $a > 0,\ \frac{a}{b} > 1$ bo‘lsa, $a > b > 0$ bo‘ladi;

2) agar $a < 0,\ \frac{a}{b} < 1$ bo‘lsa, $0 > a > b$ bo‘ladi.

Bu holda (1) yechim n ning juft hollari uchun o‘rinli bo‘ladi.

2.42. Quyidagi tenglamalarni yeching:

1) $\sqrt{x-3} = x - a$.

J.: agar $2,75 \leq a \leq 3$ bo‘lsa, $x = \frac{1}{2}\left(2a + 1 \pm \sqrt{4a-11}\right)$. agar $a \geq 3$ bo‘lsa, $x = \frac{1}{2}\left(2a + 1 + \sqrt{4a-11}\right)$;

2) $\sqrt{x-a} = b - x$.

J.: agar $b \geq a$ bo‘lsa, $x = \frac{1}{2}\left(2b + 1 - \sqrt{4b - 4a + 1}\right)$; agar $b < a$ bo‘lsa, yechimga ega emas.

3) $\sqrt{x^2 - ax + 3a} = 2 - x$.

J.: agar $-4 \leq a \leq 4$ bo‘lsa, $x = \frac{3a-4}{a-4}$; agar $a < -4,\ a \geq 4$ bo‘lsa, yechimga ega emas.

4) $\sqrt{2x-4} + \sqrt{x+7} = a$.

J.: agar $a \geq 3$ bo‘lsa, $x = 3a^2 + 11 - 2a\sqrt{2a^2 + 18}$; agar $a < 3$ bo‘lsa, yechimga ega emas.

5) $\sqrt{2x-1} + \sqrt{x-2} = a$.

J.: agar $0,5\sqrt{6} \leq a \leq \sqrt{3}$ bo‘lsa, $x = 3a^2 - 1 \pm 2a\sqrt{2a^2 - 3}$; agar $a > \sqrt{3}$ bo‘lsa, $x = 3a^2 - 1 + 2a\sqrt{2a^2 - 3}$; agar $a < 0,5\sqrt{6}$ bo‘lsa, yechimga ega emas.

6) $\sqrt{2x^2-2ax+1}=x-2.$

J.: agar $a \geq \frac{9}{4}$ bo‘lsa, $x=a-2+\sqrt{a^2-4a+7}$; agar $a<\frac{9}{4}$ bo‘lsa, yechimga ega emas.

7) $\sqrt{3x-a}=a-2x.$

J.: agar $a \geq 0$ bo‘lsa, $x=\frac{1}{8}\left(4a+3-\sqrt{8a+9}\right)$; agar $a<0$ bo‘lsa, yechimga ega emas.

8) $\sqrt{x-1}+\sqrt{3-x}=a.$

J.: agar $\sqrt{2} \leq a \leq 2$ bo‘lsa, $x=\left(2 \pm \sqrt{4a^2-a^4}\right):2$; agar $a \leq \sqrt{2}$, $a>2$ bo‘lsa, yechimga ega emas.

9) $x+\sqrt{1-x}=a.$

J.: agar $x_{1,2}=\frac{1-2a \pm \sqrt{5-4a}}{2}$, $a>\sqrt{2}$ bo‘lsa, yechimga ega emas.

13. Noma’lum absolut miqdor belgisi ostida qatnashgan tenglamalarni yechish

Absolut miqdor ta’rifiga ko‘ra x sonining absolut miqdori quyidagicha aniqlanadi:

$$|x|=\begin{cases} x, & agar \quad x>0 \quad bo‘lsa, \\ -x, & agar \quad x<0 \quad bo‘lsa, \\ 0, & agar \quad x=0 \quad bo‘lsa. \end{cases}$$

Masalan. $|5|=5$, $|-2|=2$;...

T a ’ r i f. *Agar tenglamadagi noma’lum soni absolut qiymati belgisi bilan kelsa, bunday tenglama absolut miqdor belgisi ostidagi tenglama deyiladi.*

Masalan, $|3x-1|=4$. $|2x-1|=|5x-7|$, $|5x-7|=13$.

Bu ko'rinishdagi tenglamalarni quyidagi usullar bilan yechiladi.

1-m i s o l. $|5x-7|=13$.

Y e c h i s h.

1-u s u l.

1) $5x-7=13$

$5x=13+7$

$5x=20$

$x=4.$

2) $5x-7=-13$

$5x=-13+7$

$5x=-6$

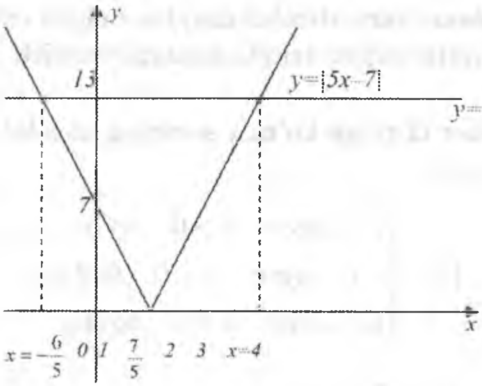
$x=-\frac{6}{5}=-1\frac{1}{5}.$

T e k s h i r i s h. $20-7=13$, $13=13$. Demak, $x=4$, $x=-6/5$ sonlari berilgan tenglamaning ildizlari bo'ladi.

2-u s u l. (Grafik usul). $y=5x-7$ funksiya grafigini chizamiz, ularning kesishish nuqtasining abssissasi berilgan tenglamaning yechimi bo'ladi.

Quyidagi jadvalni tuzamiz:

X	0	1	2	3	4	5	6	–1	–2	–3	–4
Y	7	2	3	8	13	18	23	12	17	22	27



6-*rasm.*

$y=|5x-7|$ funksiyaning grafigini yasaymiz. Bu grafikning x o'qidan yuqorida yotgan qismini o'zgarishsiz qoldiramiz. Uning uchun $5x-7>0$, shu sababli $|5x-7|=5x-7$ bo'ladi. Bu grafikning abbsissalar o‘qidan pastda yotgan qismiga shu o‘qqa nisbatan simmetrik akslantiramiz. Bu holda $5x-7<0$ bo‘ladi, ya’ni $|5x-7| = -(5x-7)$. Natijada $y=5x-7$ funksiya grafigi $y=13$ chiziq bilan ikki nuqtada kesishadi, kesishish nuqtalarning abssissalari $x=4$ va $x=-1\frac{1}{5}$ nuqtalardan iborat bo‘ladi, ana shu nuqtalar $|5x-7|=13$ tenglamaning yechimi bo‘ladi.

3-u s u l. (Oraliqlar metodi). Absolut miqdor belgisi ostidagi $|5x-7|$ ifoda $x=\frac{7}{5}$ da nolga aylanadi. Sonlar to‘g‘ri chizig‘ida $x=\frac{7}{5}$ nuqtani belgilab, bu nuqtadan chapda $\left(-\infty;\frac{7}{5}\right)$ va o‘ngda $\left(\frac{7}{5};\infty\right)$ olingan qiymatlarga ko‘ra $|5x-7|$ ifodani absolut miqdor belgisiz quyidagicha yozish mumkin:

$$|5x-7|=\begin{cases}-5x+7, & \text{birinchi } \left(-\infty;\frac{7}{5}\right) \text{ oraliqda,}\\ 5x-7, & \text{ikkinchi } \left(\frac{7}{5};\infty\right) \text{ oraliqda.}\end{cases}$$

Bularga ko‘ra tenglamani quyidagi ikki ko‘rinishda yozish mumkin:

1) $-5x+7=13$	2) $5x-7=13$
$-5x=13-7$	$5x=13+7$
$-5x=6$	$5x=20$
$x=-\frac{6}{5}=-1\frac{1}{5}.$	$x=4.$

2-m i s o l. $|7x-1|=21-9x$.

Y e c h i s h.

1) $7x-1=21-9x$	2) $7x-1=-(21-9x)$
$7x+9x=21+1$	$7x-1=9x-21$

$16x = 22$ $\qquad\qquad$ $9x - 7x = 21 - 1$

$x = \frac{22}{16} = \frac{11}{8}, \ x = 1\frac{3}{8}.$ $\qquad\qquad$ $2x = 20, \ x = 10.$

T e k s h i r i s h.

$$7 \cdot \frac{11}{8} - 1 = 21 - 9\frac{11}{8}, \quad \frac{77-8}{8} = \frac{168-99}{8}.$$

Demak, $x = \frac{11}{8}$ soni berilgan tenglama yechimi ekan.

3-m i s o l. $|x - 1| + |x + 1| = 2$ tenglamani yeching.

Y e c h i s h. Bu tenglamada $x - 1 = 0$ va $x + 1 = 0$, demak, ular $x = 1$ va $x = -1$ yechimlarga ega bo‘ladi. Sonlar to‘g‘ri chizig‘ida $x = 1$ va $x = -1$ nuqtalarni belgilaymiz, bu holda sonlar to‘g‘ri chizig‘i uchta oraliqqa ajraladi. Birinchi oraliq $(-\infty, \ -1)$, ikkinchi oraliq $[-1,1]$, uchinchi oraliq $(1,\infty)$ dan iboratdir. $|x - 1|$ va $|x + 1|$ ifodalarning har birini hosil qilingan oraliqlarda absolut miqdor belgisiz quyidagicha yozish mumkin:

1) agar $x \leq -1$ bo‘lsa, $|x - 1| + |x + 1| = 2$ tenglama $-x+1-x-1=2$ bo‘ladi, bundan $-2x = 2$ yoki $x = -1$ yechimga ega bo‘lamiz;

2) agar $-1 \leq x \leq 1$ bo‘lsa, $|x-1|+|x+1| = 2$ tenglama $-x+1+x+1=2$ bo‘ladi, bundan $2x = 2$ yoki $x = 1$ bo‘ladi. Demak, $x = -1$ va $x = 1$ yechimlarga ega bo‘ladi.

4-m i s o l. $2x^2 - 5x - 3\,|x - 2| = 0$ tenglamani yeching.

Y e c h i s h.

1) agar $x<2$ bo‘lsa, $2x^2-5x-3|x-2|$ tenglama $2x^2-5x+3x-6=0$ yoki $x^2-x-3 = 0$ ko‘rinishni oladi, uni yechsak, $x_{1,2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 3} = \frac{1}{2} \pm \frac{\sqrt{13}}{2}$,

ya’ni $x_1 = \frac{1+\sqrt{13}}{2}$ va $x_2 = \frac{1-\sqrt{13}}{2}$ yechimlar hosil qilinadi.

Bunda: $x_1 = \frac{1+\sqrt{13}}{2}$ yechim qaralayotgan sohada yotmaydi, shuning uchun $x_2 = \frac{1+\sqrt{13}}{2}$ $(-\infty; 2)$ oraliq uchun yechim bo‘ladi;

2) agar $x \geq 2$ bo'lsa, berilgan tenglamadan $2x^2 - 5x - 3x + 6=0$ hosil bo'ladi yoki ushbu $x^2 - 4x + 3 = 0$ ko'rinishni oladi, uni yechib, $x_1 = 1$ va $x_2 = 3$ yechimlarga ega bo'lamiz. Bundagi $x_1 = 1$ yechim qaralayotgan oraliqda yotmaydi, shuning uchun $(2,\infty)$ oraliq uchun yechim $x_2 = 3$ bo'ladi. Demak. $2x^2 - 5x - 3|x - 2| = 0$ tenglamaning yechimi $x_1 = 1 - \frac{\sqrt{13}}{2}$, $x_2 = 3$ bo'ladi.

2.43. Quyidagi tenglamalarni yeching:

1) $|2x+5| = 5x-3;$ 2) $|5x+10| = -x+8;$

3) $-|2x+3| = -6+2x;$ 4) $-|15+9x| = 5x-7;$

5) $|(2x-5)| = \frac{1}{5}x + \frac{2}{3};$ 6) $\left(\frac{x-4}{2} + \frac{3}{4}\right) = |x-2|;$

7) $\left|\frac{x}{4} - 1\right| = x + 5x + 22;$ 8) $|2x-2,3| = 2,5(x-1);$

9) $|1,5x-5| = 1,5(x-2);$ 10) $\frac{5}{2}|x-1| = 2,5(x-1);$

11) $|x-1| = (x-1)(x+2);$ 12) $x|x+2| = (x+2)(x-6);$

13) $|x-2|(x-9) = 0;$ 14) $|x-1|(x+4) = 0;$

15) $\frac{1}{3}x - x^2 = |1-x^2|;$ 16) $\frac{1}{3}|x(x+1)| = \frac{1}{3}(x+1)^2;$

17) $(x+1)(x-2) = |(x+6)(x-5)|;$ 18) $x^2 = (|x|+2)(|x|-1);$

19) $(x+3)^2 = |(x-2)(x-4)|;$ 20) $|x+2|(x+1)^2 = |x+2|(x-2)^2.$

2.44. Quyidagi tenglamalarni yeching:

1) $x^2 + 5|x| + 4 = 0;$ 2) $x^2 + 7|x| + 6 = 0;$

3) $3x^2 - 5|x| + 2 = 0;$ 4) $4x^2 + 5|x| + 1 = 0;$

5) $2x^2 + 3|x| = 0;$ 6) $5x^2 - 2|x| = 0;$

7) $4x^2 + 1 = |x|;$ 8) $2x^2 + 3 = |x|;$

9) $2x^2 - 3|x| + 5 = 0;$ 10) $3x^2 + 4|x| + 2 = 0.$

2.45. Quyidagi tenglamalarni yeching:

1) $\left(\frac{3x-2}{4}+\frac{4}{5}\right):3\frac{1}{2}=\left|3\frac{3}{4}\right|$;

2) $\left(6\frac{2}{3}-2x\right)\cdot\frac{7}{20}=\left|\frac{13-2x}{5}\right|$;

3) $\left(x+\frac{3}{4}\right)\cdot 2\frac{2}{3}=\left|\frac{10-x}{3}+\frac{4}{5}\right|$;

4) $\left(\frac{x-2}{4}+2\frac{1}{4}\right)\cdot 1\frac{1}{3}=\left|\frac{5-x}{8}\right|$;

5) $\left(\frac{5-x}{2}+2\right)\cdot 1\frac{4}{6}=\left|\frac{2x+1}{3}\right|$;

6) $\left(\frac{4}{3}+\frac{x-2}{2}\right)\cdot 1\frac{1}{2}=\frac{x+3}{8}+\left|\frac{2}{10}\right|$.

14. Chiziqli va kvadrat tengsizliklar

Agar x ga bog'liq bo'lgan $A(x)$ va $B(x)$ ifodalar quyidagi munosabatlardan $A(x) > B(x)$, $A(x) \geq B(x)$, $A(x) < B(x)$, $A(x) \leq B(x)$ birini qanoatlantirsa, bir noma'lumli tengsizlik berilgan deyiladi. Bu ifodalarning ikkala tomoni ma'noga ega bo'ladigan x ning qiymatlari to'plami tengsizliklarning *mavjudlik sohasi* deyiladi. O'zgaruvchi x ning tengsizlikni qanoatlantiradigan qiymatlar to'plami *tengsizlikning yechimi* deyiladi.

$2x - 6 \leq 0$ bo'lsin, bundan $2x \leq 6 => x \leq 3$ bo'lib, tengsizlikning yechimi $x \in (-\infty; 3)$ bo'ladi.

Tengsizliklarning yechimini topishda quyidagi qoidalarga rioya qilish lozim:

1) tengsizlikning ikkala tomoniga bir xil ifodani qo'shish yoki ayirishdan tengsizlik ishorasi o'zgarmaydi;

2) tengsizlikning ikkala tomonini bir xil musbat ifodaga ko'paytirish yoki bo'lishdan tengsizlik ishorasi o'zgarmaydi;

3) tengsizlikning ikkala tomonini bir xil manfiy ifodaga ko'paytirsak yoki bo'lsak, tengsizlik ishorasi teskarisiga o'zgaradi, ya'ni $A(x) > B(x)$ bo'lsa:

1) $A(x) + C(x) > B(x) + C(x)$;

2) $C(x) > 0$ bo'lsa, $A(x) \cdot C(x) > B(x) \cdot C(x)$ va $\frac{A(x)}{C(x)} > \frac{B(x)}{C(x)}$;

3) $C(x) < 0$ bo'lsa, $A(x) \cdot C(x) < B(x) \cdot C(x)$ va $\frac{A(x)}{C(x)} < \frac{B(x)}{C(x)}$

bo'ladi.

Chiziqli tengsizliklar

Soddalashtirishdan keyin $ax > b$, $ax \geq b$, $ax < b$, $ax \leq b$ ko'rinishlardan biriga keltirilishi mumkin bo'lgan tengsizlik *chiziqli* (birinchi darajali) *tengsizlik* deyiladi.

M i s o l. $\frac{2x-1}{2} - 3 > x - \frac{x+3}{3}$ tengsizlikni yeching.

Y e c h i s h. Ikkala tomonini 6 ga ko'paytirib $6x-3-18>6x-2x-6$ ni, bundan esa $2x>15$ ni hosil qilamiz. Ikkala tomonini 2 ga bo'lib, $x>7{,}5$ ni topamiz. Yechim: $x \in (7{,}5;\ \infty)$.

2.46. Tengsizliklarni yeching:

1) $4(x-2) \leq 2x-5$; 2) $5-6(x+1) \geq 2x+3$;

3) $3x-7 < 4(x+2)$; 4) $7-6x \geq \frac{1}{3}(9x-1)$;

5) $1{,}5(x-4)+2{,}5x < x+6$; 6) $1{,}4(x+5)+1{,}6x > 9+x$;

7) $\frac{x-1}{3} - \frac{x-4}{2} \leq 1$; 8) $\frac{x+4}{5} - \frac{x-1}{4} \geq 1$;

9) $\frac{2x-5}{4}-\frac{3-2x}{5}<1;$ 10) $\frac{x+2}{4}+x<3;$

11) $3(x+2)+\frac{2}{3}x<4x+5;$ 12) $\frac{5x+6}{3}+2\le 3x-\frac{x}{2};$

13) $\frac{2}{2+x}>0;$ 14) $\frac{5}{x-3}<0;$

15) $\frac{x}{x+3}>1;$ 16) $\frac{2x}{x-5}<2.$

Javoblar:

2) $x \le -0{,}5$; 4) $x\le\frac{22}{27}$; 6) $x>1$; 8) $x\le 1$;

10) $x<2$; 12) $x>4{,}8$; 14) $x<3$; 16) $x<5$.

Kvadrat tengsizliklar

$ax^2+bx+c>0,\quad (ax^2+bx+c\ge 0),$

$ax^2+bx+c<0,\quad (ax^2+bx+c\le 0)$ ko‘rinishidagi yoki shu

ko‘rinishga keltirilishi mumkin bo‘lgan tengsizlik *kvadrat tengsizlik* deyiladi (bunda x – o‘zgaruvchi, a, b, c – o‘zgarmas sonlar).

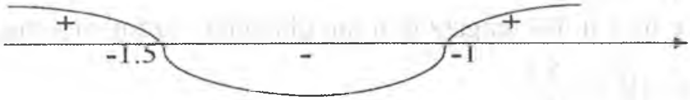
Kvadrat tengsizlikni yechishda quyidagilarga amal qilish kerak. $ax^2 + bx + c < 0$ kvadrat uchhadni $a(x - x_1)(x - x_2) < 0$ ko‘rinishida tasvirlaymiz (x_1 va x_2 ($x_1 < x_2$) kvadrat uchhadlarning nollari).

$a(x - x_1)(x - x_2) < 0$ ning yechimi $a > 0$ bo‘lganda $x \in (x_1, x_2)$, $a < 0$ bo‘lganida $x \in (-\infty; x_1)\cup(x_2; \infty)$ bo‘ladi, chunki $ax^2 + bx + c$ ning ishorasi a ning qiymatiga qarab u yoki bu oraliqning ishorasi bilan bir xil bo‘ladi; $a(x - x_1)(x - x_2) > 0$ bo‘lganda, aksincha.

Agar $ax^2 + bx + c$ uchhadning diskriminanti $D < 0$ bo‘lsa, $ax^2 + bx + c > 0$ tengsizlik $a > 0$ bo‘lganda x ning barcha qiymatlarida o‘rinli, $a < 0$ bo‘lsa, yechimga ega emas. Amalda bu qoidaning qo‘llanishini misollarda ko‘rib chiqamiz.

M i s o l l a r. 1) $2x^2 + 5x + 3 > 0$ tengsizlikni yeching.

Y e c h i s h. Kvadrat uchhadning ildizlarini topib, tengsizlikni $2\left(x+\frac{3}{2}\right)(x+1)>0$ ko'rinishida yozamiz. Kvadrat uchhadning aniqlanish sohasi $(-\infty;\ \infty)$ ekanligini bilgan holda, uni $x_1=-\frac{3}{2}$, $x_2=-1$ nuqtalar yordamida oraliqlarga ajratamiz: $\left(-\infty;-\frac{3}{2}\right)$, $\left(-\frac{3}{2};-1\right)$ va $(-1;\ \infty)$. Bu oraliqlarni sonlar o'qida tasvirlaymiz (7-rasm):

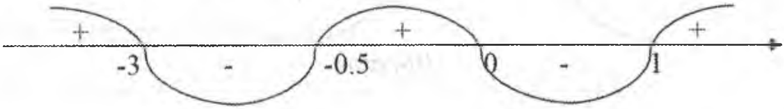


7-rasm.

$2\left(x+\frac{3}{2}\right)(x+1)>0$ – tengsizlikda ikkala qavsning ishorasi chapdagi oraliqda hamma vaqt musbat bo'ladi, undan bitta oldingi oraliqda esa qavslarning ishorasi qarama-qarshi bo'lib, umumiy ishora minus bo'ladi, keyingisida musbat bo'ladi va hokazo. Tengsizlik yechimi $x\in\left(-\infty;\frac{3}{2}\right)\cup(-1;\infty)$ bo'ladi. Bu usuldan ko'paytuvchilar (qavslar) soni ko'p bo'lganda ham foydalanish mumkin.

2) $x(x+3)\left(x+\frac{1}{2}\right)(x-1)<0$ tengsizlikni yeching.

Y e c h i s h. Bu tengsizlikda chap tomondagi ifodaning nollari $-3, -\frac{1}{2}, 0, 1$ bo'ladi, shuning uchun yechim tasviri quyidagicha bo'ladi (8-rasm):



8-rasm.

Ifoda manfiy qiymatlarni $\left(-3;-\frac{1}{2}\right)$ va (0; 1) oraliqlarda qabul qiladi.

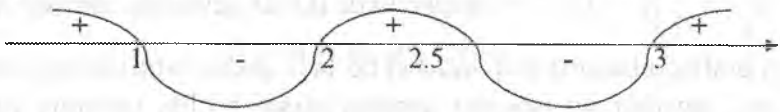
Yechim: $x\in\left(-3;-\frac{1}{2}\right)\cup(0;1)$.

Keltirilgan usuldan (u *intervallar usuli* deyiladi) kasr ifoda bo'lganda ham foydalanish mumkin.

3) $\frac{2x^2-7x+5}{x^2-5x+6}\geq 0$ tengsizlikni yeching.

Y e c h i s h. Bu tengsizlikni quyidagicha yozamiz (9-rasm):

$\frac{2(x-1)\left(x-\frac{5}{2}\right)}{(x-2)(x-3)}\geq 0$ va sonlar o'qida belgilab topamiz:



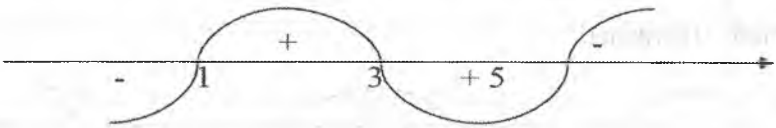
9-rasm.

Yechim. $x\in\left(\left(-\infty;1\right]\cup\left(2;\frac{5}{2}\right]\cup(3;\infty)\right)$.

4) $\frac{x+7}{x-5}+\frac{3x+1}{2}\leq 0$ tengsizlikni yeching.

Y e c h i s h.

$$\frac{2x+14+3x^2+x-15x-5}{2(x-5)}\leq 0,\ \frac{3x^2-12x+9}{2(x-5)}\leq 0,\ \frac{3(x-1)(x-3)}{2(x-5)}\leq 0.$$



10-rasm.

Yechim: $x\in(-\infty;1]\cup[3;5)$.

2.47. Tengsizliklarni yeching:

1)$x^2+5x+4\geq 0$; 2)$x^2+7x+6\leq 0$;

3)$3x^2-5x+2\leq 0$; 4)$4x^2+5x+1\leq 0$;

5)$2x^2+3x\geq 0$; 6)$5x^2-2x\leq 0$;

7)$4x^2+1>0$; 8)$2x^2+3<0$;

9)$2x^2-3x+5<0$; 10)$3x^2+4x+2>0$;

11)$x^2(x+1)(x-2)(x-5)>0$; 12)$x^2(x+2)(x-1)(x-3)<0$;

13)$(x+3)^2(x-2)(x-4)\leq 0$; 14)$(x+2)(x+1)^2\left(x+\frac{1}{2}\right)(x-2)^2\geq 0$;

15)$(2x-1)(x-4)(x-3)\geq 0$; 16)$(3x+2)(x^2-1)(x-2)\leq 0$;

17)$\frac{x^2+5x+4}{2x^2-3x+2}\leq 0$; 18)$\frac{3x^2-5x+2}{x^2+x-6}\geq 0$;

19)$\frac{1}{x+2}-\frac{2}{x-2}<0$; 20)$\frac{2}{x-1}+\frac{1}{x+2}<1$.

Javoblar:

2) $x\in[-1;-6]$; 4) $x\in[-0,25;-1]$; 6) $x\in[0;0,4]$;

8) $x\in$; 10) $x\in(-\infty;\infty)$; 12)$x\in(-\infty;0)\cup(0;-2)\cup(1;3)$;

14) $x\in(-\infty;-2)\cup(-0,5;2)\cup(2;\infty)$; 16) $x\in[-1,5;-1]\cup[1;2]$;

18) $x\in(-\infty;-3)\cup\left[\frac{2}{3};1\right]\cup(2;\infty)$;

20) $x\in(-\infty;-2)\cup\left(\frac{1-\sqrt{6}}{2};1\right)\cup\left(\frac{1+\sqrt{6}}{2};\infty\right)$.

2.48. Tengsizliklarni yeching:

1) $(x-2)(x^2-9)>0$; 2) $(x^2-1)(x+4)<0$;

3) $\frac{(x+3)(x-5)}{x+1} \le 0;$ 4) $\frac{x-7}{(4-x)(2x+1)} > 0;$

5) $\frac{4x^2-4x-3}{x+3} \ge 0;$ 6) $\frac{2x^2-3x-2}{x-1} < 0;$

7) $\frac{(x^2-x)(x+2)}{x-3} \ge 0;$ 8) $\frac{1}{3}x - \frac{4}{9}x^2 \ge 1-x;$

9) $\frac{1}{3}x(x+1) \le (x+1)^2;$ 10) $x(1-x) > 1-x;$

11) $\frac{1}{3}x(x+1) \le x(x-1);$ 12) $x\left(\frac{x}{4}-1\right) \le x^2+x+1;$

13) $2x-2,5 > x(x-1);$ 14) $3x+7,5 > x(x-2);$

15) $\frac{1}{2}x(x-1) \le (x-1)^2;$ 16) $\frac{9}{2x+2} + \frac{x}{x-1} \ge \frac{1-3x}{2-2x};$

17) $\frac{3}{x^2-1} - \frac{1}{2} < \frac{3}{2x-2};$ 18) $\frac{3x^2-5x-8}{2x^2-5x-3} > 0;$

19) $\frac{4x^2+x-3}{5x^2+9x-9} < 0;$ 20) $\frac{2+7x-4x^2}{3x^2+2x-1} \le 0;$

21) $\frac{2+9x-5x^2}{3x^2-2x-1} \ge 0;$ 22) $\frac{2x-4x^2+56}{x-3} \le 0;$

23) $\frac{4x-x^2-3}{(x+1)(x-2)} \le 0;$ 24) $\frac{(x^2-4)(16-x^2)}{x(x+1)} \ge 0;$

25) $\frac{1}{2}x(x+1) \le (x+1)^2;$ 26) $\frac{x-3}{x(x+2)(x+3)} > 0;$

27) $\frac{x+3}{2} + \frac{2x-3}{7} < x-7;$ 28) $\frac{1}{3}(3x-2) < \frac{1}{4}x + \frac{2}{3};$

29) $x(x-1) < (x-1)(x+2);$ 30) $\left(\frac{x-4}{7} + \frac{3}{4}\right) \cdot 2\frac{2}{3} \le x-2.$

III bob bo‘yicha test topshiriqlari

1. Kompleks sonlarni ayiring: $(3 + 4i) - (3 - i)$.

A) $2i$ B) $3i$ C) 6 D) 0

2. Kompleks sonlarni qo‘shing: $(5 - 6i) + (3 + 6i)$.

A) 2 B) 0 C) 8 D) $12i$

3. Kompleks sonlarni ko‘paytiring: $(2 - 3i)(3i + 2)$.

A) 12 B) 13 C) 14 D) 15

4. Soddalashtiring: $-6 - 2(2 - y) - 2y + 2$.

A) 8 B) $-8 - 4y$ C) $8 - 4y$ D) -8

5. Soddalashtiring: $(1 - 2a)^2 + (1 + 2a)(2a - 1)$.

A) $8a^2 - 4a$ B) $-2a$ C) $-2a + 2$ D) $4a^2 - 2a$

6. Ko‘paytuvchilarga ajrating: $(x^2 + 1)^2 - 4x^2$.

A) $(x^2 + 1) - (x - 1)^2$ B) $x^2(x^2 - 2)$

C) $(x - 1)^2(x + 1)^2$ D) $(x^2 - 2)(x^2 + 1)$

7. Kasrni qisqartiring: $\frac{x^2+3xy}{9y^2-x^2}$.

A) $\frac{x}{x+3y}$ B) $-\frac{x}{x-3y}$ C) $\frac{x}{3y+x}$ D) $\frac{x}{3y-x}$

8. Soddalashtiring: $\left(m^2-\frac{1+m^4}{m^2-1}\right):\frac{m^2+1}{m+1}$.

A) $m - 1$ B) $\frac{1}{m-1}$ C) $\frac{1}{m+1}$ D) $\frac{1}{1-m}$

9. Ushbu $\frac{4^{a+1}-2^{2a-1}}{2^{2a}}$ ning qiymati 9 dan qancha kam?

A) 4 B) 3,5 C) 3 D) 5,5

10. Tenglamaning nechta ildizi bor: $\frac{2}{x}=x+2$?

A) 3 B) 2 C) 1 D) Ildizi yo‘q.

11. Agar $(x-5)\left(\frac{1}{5}x+4\right)=0$ bo'lsa, $\frac{1}{5}x+4$ qanday qiymatlar qabul qiladi?

A) faqat 0 B) faqat –20 C) 0 yoki 5 D) 0 yoki 8

12. Agar $x^2 + x - 1 = 0$ tenglamaning ildizlari x_1 va x_2 bo'lsa, $x_1^3 + x_2^3$ ning qiymati qanchaga teng bo'ladi?

A) 1 B) 3 C) 2 D) –4

13. a ning qanday qiymatlarida $|a + 2| = -a - 2$ tenglik o'rinli bo'ladi?

A) $a < -2$ B) $a = -3$ C) $a = -2$ D) $a \in \phi$

14. $\dfrac{a^2+\frac{1}{a}}{a+\frac{1}{a}-1}$ ni soddalashtiring.

A) $a-1$ B) a^2-a+1 C) a^2+a+1 D) a^2+a-1

15. Quyidagi ifodalardan qaysi biri –1 ga teng?

A) $((-1)^2)^3$ B) $(-(-1)^2)^3$ C) $((-1)^3)^2$ D) $(-(-1)^3)^3$

16. $\dfrac{5\cdot 4^{16}-4\cdot 2^{30}}{(4)^{16}}$ ni hisoblang.

A) 16 B) 5 C) 4 D) 3

17. Soddalashtiring: $\dfrac{1,6^2-1,6\cdot 0,8+0,4^2}{1,4^2-0,2^2}$.

A) 1 B) 0 C) 0,75 D) 0,7

18. Soddalashtiring: $\left(\dfrac{-16x^{31}}{9y^3}\right)^3 : \left(\dfrac{8x^{23}}{3y^2}\right)^4$.

A) $-\dfrac{y}{x}$ B) $-\dfrac{x}{y}$ C) $\dfrac{x}{9y}$ D) $-\dfrac{y}{9x}$

19. $\dfrac{0,04^{-2}\cdot 125^4\cdot 0,2^{-1}}{4\cdot 25^8}$ ni hisoblang.

A) $\frac{1}{4}$ B) $1\frac{1}{2}$ C) 0,5 D) 0,2

20. $(x + 3)^2 - 2|x + 3| - 3 = 0$ tenglama ildizlarining yig‘indisi nechaga teng?

A) –6 B) –5 C) –4 D) 4

21. Tengsizlikni yeching: $(x + 2)(x + 3) > 0$.

A) $(-\infty; 2) \cup (3; \infty)$ B) $(-\infty; -3) \cup (2; \infty)$
C) $(-\infty; -2) \cup (3; \infty)$ D) $(-\infty; \infty)$

22. Tengsizlikni yeching: $(x + 2)(x - 3) > 0$.

A) $(-\infty; 2) \cup (3; \infty)$ B) $(-\infty; -3) \cup (2; \infty)$
C) $(0; \infty)$ D) $(-\infty; -2) \cup (3; \infty)$

23. Qo‘sh tengsizlikni yeching: $0 < \frac{3x-1}{2x+5} < 1$.

A) $\left(-\frac{5}{2}; 6\right)$ B) $\left(\frac{1}{3}; \infty\right)$
C) $\left(-\infty; -\frac{5}{2}\right) \cup \left(\frac{1}{3}; 6\right)$ D) $\left(-\frac{5}{2}; \infty\right)$

24. $\frac{1}{x} < 1$ tengsizlikning $(-3; 3)$ oraliqdagi butun yechimlari sonini toping.

A) 7 B) 5 C) 3 D) 2

25. Nechta tub son $3 < \frac{5x-1}{2x-3} < 5$ tengsizlikning yechimi bo‘ladi?

A) 0 B) 1 C) 2 D) 3

26. Agar $a > b > c$ bo‘lsa, $|a - b| + |c - a| - |b - c|$ ni soddalashtiring.

A) $a - 2b$ B) $2c$ C) $2a$ D) $2a - 2b$

27. Agar $x > y > z$ bo‘lsa, $|x - y| - |z - y| - |z - x|$ ni soddalashtiring.

A) $2x$ B) $2y - 2x$ C) $2z - 2y$ D) $2y - 2z$

28. Ushbu $x^2 + 3|x| - 40 = 0$ tenglamaning ildizlari ko‘paytmasini toping.

A) –40 B) 40 C) –32 D) –64

29. $x^2 + |x| - 2 = 0$ tenglamaning nechta ildizi bor?

A) 4 B) 1 C) 2 D) 3

30. Tengsizlikni yeching: $|x - 1| \geq 2$.

A) $(-\infty; -1)$ B) $[-1; 3]$

C) $(-\infty; -1] \cup [3; \infty)$ D) $[1; 3]$

31. Tengsizlikni yeching: $|x - 1| \leq 2$.

A) yechimga ega emas B) $(-\infty; -1) \cup (3; \infty)$

C) $[-1; 3]$ D) $[1; 3]$

32. Soddalashtiring: $\dfrac{x^2 + 2x^2 + x}{(x+1)^2}$.

A) $\dfrac{x+2}{x-1}$ B) $\dfrac{x+2}{x+1}$ C) $\dfrac{x-2}{x-1}$ D) x

33. a ning qanday qiymatlarida $3(x + 1) = 4 + ax$ tenglamaning ildizi –1 dan katta bo‘ladi?

A) $(0; \infty)$ B) $(4; \infty)$ C) $(-\infty; 0)$ D) $(-\infty; 3)$

34. $x^2 + |x| = \dfrac{7}{4}$ tenglamaning eng katta va eng kichik ildizlari ayirmasini toping.

A) $\sqrt{2}$ B) $2\sqrt{2} - 1$ C) $2\sqrt{2}$ D) 2

MUHAMMAD AL-XORAZMIYNING ASARLARI HAQIDA QISQACHA MA'LUMOT

Matematika darslarida tarixiy ma'lumotlar, qomusiy allomalarimizning hayoti va ijodini o'rganish matematika darslarini insonparvarlashtirishga yordam beradi, deb o'ylaymiz.

Abu Abdulloh Muhammad ibn Muso al-Xorazmiy Xorazmda taxminan 783-yilda tug'ilgan. Al-Xorazmiy «Al-jabr va al-muqobala haqida qisqacha kitob» asari bilan algebra faniga asos soldi. Shu asar tufayli olim nomining lotincha shaklida «algoritm» termini paydo bo'lgan. Al-Xorazmiy Bag'doddagi «Bayt ul-bikma» (Donishmandlar uyi)da rasadxona, kutubxona va barcha ilmiy tekshirish ishlariga rahbarlik qildi. Al-Xorazmiyning 10 ta asari bizgacha yetib kelgan:

1. «Hind hisobi haqida» (*«Fi hisab al-hind»*).

Bu asarni XII asrda Ispaniya olimi Batlik Adelard arab tilidan lotin tiliga tarjima qildi. Keyinchalik Bonkompani, K.Fogel, LSevilskiylar tadqiq qildilar.

Risola 8 ta bobdan iborat bo'lgan: 1) natural sonlarni «hind raqamlari» hisoblangan 0,1,2,3,4,5,6,7,8,9 lar yordamida yozish; 2) sonlarni qo'shish va ayirish; 3) ikkiga bo'lish va ikkiga ko'paytirish qoidalari; 4) ko'paytirish amali va uni 9 raqami yordamida tekshirish; 5) bo'lish; 6) kasrlar hisobi; 7) kasrlami ko'paytirish; 8) musbat sonlardan kvadrat ildiz chiqarish.

Bu risola hisob bo'yicha qo'llanma sifatida Yaqin va o'rta Sharq hamda G'arbiy Yevropaga katta ta'sir ko'rsatdi. Lotin tiliga tarjimasida «Al-Xorazmiy» so'zi Algorithmus (*Algoritmus*) deb yozildi va jahon faniga yangi «algoritm» atama sifatida kiritildi.

2. «Al-jabr va al-muqobila haqida qisqacha kitob» (*«Al-kitab al-muxtasar fi hisab al-jabr va-1-muqabala»*).

Bu asarni XII asrda Ispaniya olimlari Kremonalik Gerardo va Batlik Adelardlar arab tilidan lotin tiliga, keyinchalik Gans va Grantlar ingliz tiliga tarjima qilish asosida o'rgandilar. Bu risola 27 ta bobdan iborat bo'lib: 1–6-boblari musbat koeffitsiyentli chiziqli va kvadrat tenglamalarni hal qilishga bag'ishlangan.

7–9-boblarda 4–6-boblardagi qoidalar handasa(geometriya) usullari bilan isbotlanadi. Bunda X va b kesmalar bilan, $X \cdot b$ ko'paytma esa shu kesmalar yordamida chizilgan to'g'ri to'ttburchak, c-to'g'ri to'rtburchaklardan tuzilgan tekis shakl kabi ifodalanadi. 10-bobda ko'phadlarni ko'paytirish qoidalari berilgan. 11-bobda kvadratik irratsional miqdorlarni sonli misollar yordamida qo'shish, ayirish va ko'paytirish amallari keltirilgan.12-bobda olti xil ko'rinishdagi kvadratik tenglamalarga keltirilib hal qilinadigan masalalar ko'riladi.

13-bobda bisob usullarida yechiladigan turli xil masalalar beriladi.

14-bobda bitimlar haqidagi masalalar hal qilinadi.

15-bob geometriyaga bag'ishlangan bo'lib, kvadrat, uchburchak, romb, doira, aylana uzunligi, parallelepiped, uchburchakli prizma, aylanma silindr, uchburchakli va to'rtburchakli piramidalar, doiraviy konus, kesik piramida va konuslar hajmlarini hisoblash, uchburchak va to'rtburchaklarni tasniflash muammolari ko'riladi.

16–23-boblarda vasiyatlar va merosni taqsimlash masalalari islom dini huquqshunosligi asosida hal qilinadi. Ular chiziqli tenglamalarga keltirilib ishlanadi.

24–27-boblarda merosni taqsimlashdagi murakkab masalalar (merosxo'r merosni qoldiruvchidan avval vafot etgan) ko'riladi.

3. «Al-Xorazmiy ziji» (ya'ni jadvallari) yoki (*«Al-Ma'mun ziji»*).

Bu risola 37 ta bobdan iborat bo'lib, uni XII asrda Ispaniya olimi Batlik Adelard arab tilidan lotin tiliga, keyinchalik X. Zuter nemis tiliga, B. Kopelevich rus tiliga tarjima qildilar.

4. «Asturlobiyalarni qo'llash haqida kitob» (*«Kitab al-amal bi-l-astrulabat»*). Bu risolada astronomiyaga oid 43 ta masala hal qilingan.

5. «Asturlob yordamida azimutni aniqlash» (*«Ma'rif as-samt bi-l-astrulab»*). X asrda yashagan Ibn Nadimnitig «Fixrist» asarida tilga olinadi.

6. «Quyosh soati tekisligida soatni ko‘rish haqida» (*«Amal as-saat fabasit ar-ruxama»*).

7. «Astrulobni yasash haqida kitob» (*«Kitab amal as-astrulab»*). U *X* asrda yashagan Ibn Nadimning «Fixrist» asarida tilga olinadi.

8. «Quyosh soati haqida kitob» (*«Kitob ar-ruxama»*).Bu risola haqidagi ma’lumot Ibn Nadimning «Fixrist»ida bor.

9. «Jo‘g‘rofiya kitobi» (*«Kitob surati al-ard»*). Mjik tomonidan arab tilidagi matn chop qilingan.

10. «Yahudiylar yeralari va bayramlari haqida risola» (*«Risola fi istixroj ta’rix yahud va a’yodihim»*). Bu risola AQSH olimi E.Kennedi tomonidan o‘rganildi.

Al-Xorazmiyning bizgacha yetib kelgan 10 ta risolasidan quydagi 3 ta katta kashfiyot haqida aytish mumkin:

1. «Hind hisobi haqida»gi risolasida o‘nlik pozitsion sanoq tizimining oltmishlikdan ustun ekanligini ko‘rsatgan va bu asarni lotin tiliga tarjimasi orqali o‘nli pozitsion sanoq tizimi tarqalgan.

2. «Al-jabr va al-muqobala haqida qisqacha kitob»ida aljabrni astronomiyaning yordamchi qismidan mustaqil fan darajasiga ko‘tardi, 6 ta chiziqli va kvadrat tenglamalami tasniflagan.

3. Al-Xorazmiy o‘z shogirdlari bilan orasidagi masofasi 35 km bo‘lgan Tadmor va ar-Rakka shaharlaridan o‘tuvchi Yer sharining 1° li meridiani uzunligini hisobladi va u 6,72 km ga teng ekanligini topgan.

Al-Xorazmiy 850-yilda Bag‘dodda vafot etgan.

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