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KASB-HUNAR TA'LIMI BOSHQARMASI

BUXORO DAVLAT TIBBIYOT INSTITUTI QOSHIDAGI
1-SON AKADEMIK LITSEYI

VEKTORLAR

Abiturientlar va matematika o'qituvchilari uchun uslubiy qo'llanma

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KIRISH

Q.B. Qilichovning «Vektorlar» nomli o'quv qo'llanmasi akademik litsey va kasb-hunar kollej talabalari uchun mo'ljallangan bo'lib, amaldagi dasturga mos keladi. Mazkur o'quv qo'llanma tushunarli, ravon tilda yozilgan bo'lib, uslubiy va ilmiy jihatdan qo'yilgan barcha talablarga javob beradi.

Uslubiy qo'llanmada vektorlarga ta'rif berilgan va ular ustida bajariladigan amallar o'rganilgan. Uslubiy qo'llanma 3 ta paragrafdan iborat bo'lib, har bir paragrafdan so'ng mavzularga doir bir qancha mashqlar ishlab ko'rsatilgan va ularni yechish usullariga keng o'rin berilgan bo'lib, bu usullar akademik litsey va kasb-hunar kollejlari talabalarining amaliy masalalarning yechish va tushunishlari uchun muhim ahamiyat kasb etadi. Uslubiy qo'llanmada mustaqil yechish uchun testlar berilgan bo'lib, bu testlar talabalar bilimini yanada mustahkamlaydi.

N.H. Mamatova,
fizika-matematika fanlari nomzodi, dotsent.

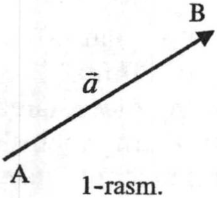
1-§. VEKTORLAR

1. Vektor ta'rifi

Vektor — matematika va fizika fanlarining asosiy tushunchalaridan biridir.

Ba'zi masalalarni yechishda faqat sonli o'lchovning berilishi yetarli bo'lmaydi. Masalan: tezligi 80 km/soat bo'lgan mashina A shahardan yo'lga chiqdi. U 2 soatdan so'ng qaysi manzilda bo'ladi?

Bu masalani yechish uchun yo'nalish ham berilishi kerak bo'ladi.



1-rasm.

Vektor — ham son (uzunlik), ham yo'nalish bilan xarakterlanadi.

***Ta'rif.* Yo'naltirilgan kesmaga vektor deyiladi** (1-rasm).

Vektor o'zining boshi (A) va uchi (B) bo'lgan nuqtalar orqali $\overrightarrow{AB}$ kabi yoki bitta kichkina harf bilan $\vec{a}$ kabi belgilanadi.

Vektor uzunligi uni tashkil etuvchi kesma uzunligiga tengdir. Vektor uzunligi $|\overrightarrow{AB}|$ yoki $|\vec{a}|$ kabi belgilanadi. Bundan vektor moduli absolut qiymat ekanligi kelib chiqadi, ya'ni $|\vec{a}| \geq 0$.

Mustaqil yechish uchun mashq:

1) Boshi C nuqtada uchi esa D nuqtada bo'lgan $\vec{b}$ vektorni yasang va uning uzunligini chizg'ich yordamida o'lchang.

2) Uzunligi 5 sm bo'lgan ikkita turli vektor chizing.

2. Nol vektor

Boshi va uchi ustma ust tushadigan yo'naltirilgan "kesma" orqali ifodalanadigan vektorga ***nol vektor*** deyiladi va $\vec{0}$ kabi belgilanadi. Demak:

$$\overrightarrow{AA}=\vec{0}, \quad |\vec{0}|=0.$$

Nol vektor yo'nalishga ega emas.

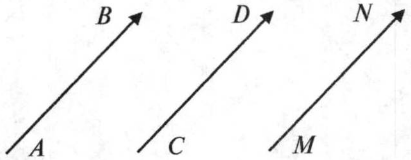
3. Teng vektorlar

Bir xil uzunlikka va bir xil yo'nalishga ega bo'lgan parallel vektorlarga ***teng*** (ayni bir) ***vektorlar*** deyiladi (2-rasm).

$$(|\overrightarrow{AB}|=|\overrightarrow{CD}|=|\overrightarrow{MN}|;$$

$$\overrightarrow{AB}\uparrow\uparrow\overrightarrow{CD}\uparrow\uparrow\overrightarrow{MN})\Rightarrow$$

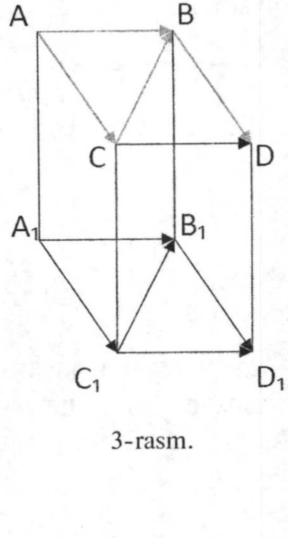
$$\Rightarrow\overrightarrow{AB}=\overrightarrow{CD}=\overrightarrow{MN}.$$



2-rasm.

Xulosa. Vektor — hammasi o'zaro parallel, bir xil uzunlik va bir xil yo'nalishga ega bo'lgan yo'naltirilgan kesmalarning butun bir sinfidir.

3-rasmdagi parallelepipedda $\overrightarrow{AB}$; $\overrightarrow{CD}$; $\overrightarrow{A_1B_1}$; $\overrightarrow{C_1D_1}$ vektorlar teng vektorlar bo'lib, ayni bir vektorning CD turli holatlaridir.



3-rasm.

Mustaqil yechish uchun mashqlar:

3-rasmdagi a) $\overrightarrow{AC}$ vektorga teng vektorlarni aniqlang.

b) $\overrightarrow{CB}$ vektorga teng bo‘lgan vektorni aniqlang.

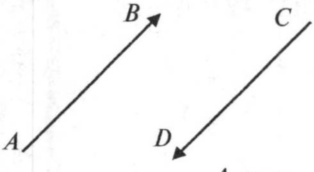
Vektorlar tenglik alomati quyidagi xossalarga ega:

1) $\vec{a}=\vec{a}$; 2) $\vec{a}=\vec{b}$ bo‘lsa, $\vec{b}=\vec{a}$ bo‘ladi; 3) $\vec{a}=\vec{b}$ va $\vec{b}=\vec{c}$ bo‘lsa, $\vec{a}=\vec{c}$ bo‘ladi:

$$(|\overrightarrow{AB}|=|\overrightarrow{CD}|, \overrightarrow{AB}\downarrow\uparrow\overrightarrow{CD}) \Rightarrow \overrightarrow{AB}=-\overrightarrow{CD}$$

4. Qarama-qarshi vektorlar

Bir xil uzunlikka ega bo‘lgan ammo qarama-qarshi yo‘nalgan ikkita parallel vektorlarga ***qarama-qarshi vektorlar*** deyiladi (4-rasm).



4-rasm.

$\vec{a}$ vektorga qarama-qarshi vektor — $\vec{a}$ kabi belgilanadi.

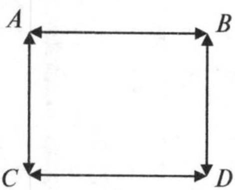
Qarama-qarshi vektorlar xossalari:

1) $\overrightarrow{AB}=-\overrightarrow{BA}$;

2) qarama-qarshi vektorlar yig‘indisi $\vec{Q}$ ga teng, ya’ni

1) $\overrightarrow{AB}+\overrightarrow{BA}=\overrightarrow{AA}=\vec{0}$; 2) $\vec{a}+(-\vec{a})=\vec{0}$;

3) $\vec{a}$ va $\vec{b}$ qarama-qarshi vektorlar bo‘lsa, $\vec{a}+\vec{b}=\vec{0}$ bo‘ladi va aksincha.



5-rasm.

Mashq. *ABCD* kvadratda qarama-qarshi vektorlarni aniqlang.

Javob: 1) $\overrightarrow{AB}$ va $\overrightarrow{DC}$; 2) $\overrightarrow{CA}$ va $\overrightarrow{BD}$ hokazo.

Bundan: 1) $\overrightarrow{AB}+\overrightarrow{DC}=\vec{0}$. 2) $\overrightarrow{CA}+\overrightarrow{BD}=\vec{0}$.

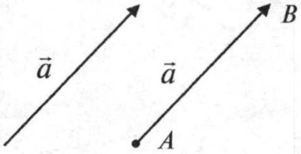
Mustaqil yechish uchun mashqlar:

1) Biror $\overrightarrow{MN}$ vektor yasang. Ixtiyoriy *K* va *P* nuqtalardan $\overrightarrow{MN}$ vektorga qarama-qarshi $\overrightarrow{KC}$ va $\overrightarrow{PD}$ vektorlarni yasang, xulosalar chiqaring.

2) 3- rasmdan 5 juft qarama-qarshi vektorlarni aniqlang.

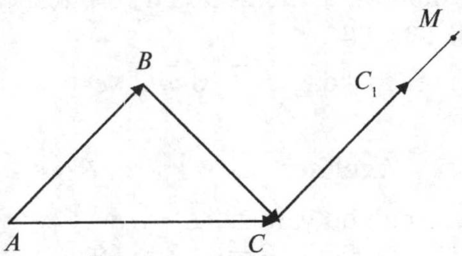
5. Vektorni nuqtadan qo'yish

$\vec{a}$ — biror vektor A esa biror nuqta bo'lsin. $\vec{a}$ vektorga teng vektorlar orasida boshi A nuqtada bo'lgan vektor mavjud va u yagonadir. Bu vektorning uchi bo'lgan B nuqta A nuqtadan boshlab, $\vec{a}$ vektorni qo'yishdan hosil bo'lgan nuqta deyiladi: $\overrightarrow{AB} = \vec{a}$ (6-rasm).



6-rasm.

Mashq. ABC uchburchakda $\overrightarrow{AB}$ vektorni C nuqtadan qo'ying.



7-rasm.

Yasash: dastlab uchburchakning C uchidan $CM \uparrow\uparrow AB$ nur o'tkazamiz, so'ng CM nurda $CC_1 = AB$ shartni qanoatlantiruvchi C_1 nuqtani topamiz (7-rasm).

Javob: $\overrightarrow{CC_1}$.

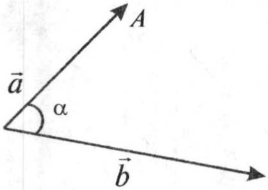
Mustaqil yechish uchun mashq. Koordinata tekisligida $A(2;1)$ va $B(5;3)$ bo'lsa, $\overrightarrow{AB}$ vektorni koordinatalar boshidan qo'ying.

6. Ikki vektor orasidagi burchak

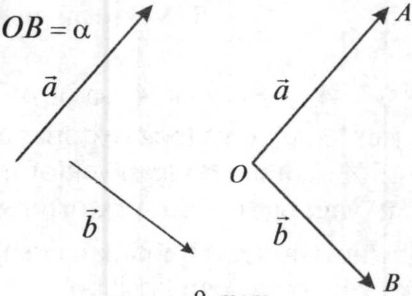
Bir nuqtadan qo'yilgan, nol bo'lmagan, **ikki vektor orasidagi burchak** deb ularni tashkil etgan kesmalar orasidagi burchakka aytiladi (8-rasm).

Agar ikki vektor boshlari turli nuqtalarda bo'lsa, bu vektorlar orasidagi burchakni o'lchash uchun ular bir nuqtadan qo'yiladi va hosil bo'lgan burchak o'lchanadi (9-rasm). Bunda: $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$.

$$\angle(\vec{a};\vec{b}) = \angle(\overrightarrow{OA};\ \overrightarrow{OB}) = \angle AOB = \alpha$$



8-rasm.

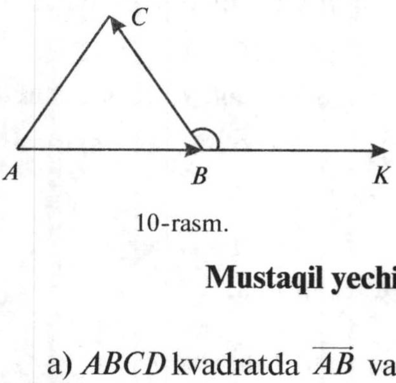


9-rasm.

Xulosalar. A) Ikkita nol bo'lmagan parallel va **yo'nalishdosh vektorlar orasidagi burchak** $0°$ ga teng.

B) Ikkita nol bo'lmagan parallel va **qarama-qarshi yo'nalgan vektorlar orasidagi burchak** $180°$ ga teng.

Mashq. Muntazam ABC uchburchakdagi $\overrightarrow{AB}$ va $\overrightarrow{BC}$ vektorlar orasidagi burchakni toping.



10-rasm.

Yechish. $\overrightarrow{AB}$ vektorni B nuqtadan qo'yamiz: $\overrightarrow{BK} = \overrightarrow{AB}$.

$\angle A = \angle B = \angle C = 60°$ (10-rasm).

$\angle(\overrightarrow{AB};\ \overrightarrow{BC}) = \angle(\overrightarrow{BK};\ \overrightarrow{BC}) = 120°$.

Javob: $120°$.

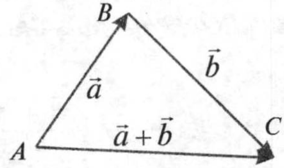
Mustaqil yechish uchun mashqlar

a) $ABCD$ kvadratda $\overrightarrow{AB}$ va $\overrightarrow{DB}$ vektorlar orasidagi burchakni toping.

b) ABC muntazam uchburchakdagi $\overrightarrow{BA}$ va $\overrightarrow{CA}$ vektorlar orasidagi burchakni toping.

7. Vektorlarni qo'shish

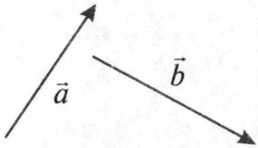
Vektor so'zining lug'aviy ma'nosi «tashuvchi», «eltuvchi» kabi ma'nolarni anglatadi. Bu ma'nodan kelib chiqib, ikkita $\vec{a}$ va $\vec{b}$ vektorlar yig'indisini topaylik; $\vec{a}$ vektor A nuqtani B nuqtaga «tashiydi», $\vec{b}$ vektor esa B nuqtani C nuqtaga «tashiydi». Ularning birgalikda bajargan «ishi»: A nuqta C nuqtaga «tashildi», ya'ni



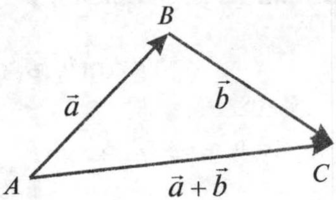
11-rasm.

$$\vec{a}+\vec{b}=\overrightarrow{AC} \text{ (11-rasm).}$$

Demak: $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$.



Ixtiyoriy va vektorlar yig'indisini topish talab qilingan bo'lsa, biror A nuqta tanlab olamiz va bu nuqtadan $\vec{a}$ vektorni qo'yamiz, ya'ni $\overrightarrow{AB}=\vec{a}$ bo'lgan B nuqtani topamiz (12-rasm). So'ng B nuqtadan boshlab $\vec{b}$ vektorni qo'yamiz, ya'ni $\overrightarrow{BC}=\vec{b}$ bo'lgan C nuqtani topamiz. $\overrightarrow{AC}$ ni yasaymiz. Bu $\overrightarrow{AC}$ vektorga $\vec{a}$ va $\vec{b}$ vektorlar yig'indisi deyiladi: $\vec{a}+\vec{b}=\overrightarrow{AC}$. Bu yig'indi vektor A nuqtani qayerda tanlab olinishiga bog'liq emas (ikki vektorni qo'shishning bu usuliga ***«uchburchak» usuli*** deyiladi).



12-rasm.

Xulosa: ***ikki vektor yig'indisi vektordir.***

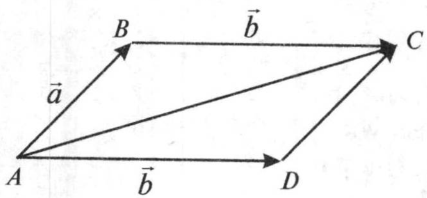
Mustaqil yechish uchun mashq. O'zingiz parallel bo'lmagan ikkita vektor tanlang va ularni qo'shing.

Vektorlarni qo'shishning bu uchburchak usulidan mashhur «UCH NUQTA QOIDASI» kelib chiqqan:

$$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC} \qquad (1)$$

Mashq: 1) $\overrightarrow{DO}+\overrightarrow{OF}=\overrightarrow{DF}$ 2) $\overrightarrow{PK}+\overrightarrow{KM}=\overrightarrow{PM}$

$\vec{O}$ va parallel bo‘lmagan ikkita $\vec{a}$ va $\vec{b}$ vektorlarni qo‘shishning ***parallelogramm usuli*** ham mavjud:



13-rasm.

Bir nuqtadan qo‘yilgan ikkita parallel va $\vec{O}$ bo‘lmagan $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorlar yig‘indisi tomonlari AB va AD bo‘lgan $ABCD$ parallelogrammning AC diagonali hosil etgan $\overrightarrow{AC}$ vektorga teng bo‘ladi: $\overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AC}$ (13-rasm).

Isbot: $\overrightarrow{AD}=\overrightarrow{BC}$ bo‘lgani uchun:

$$\overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}.$$

Bundan: nol bo‘lmagan va o‘zaro teng bo‘lmagan ikki vektor yig‘indisini parallelogramm diagonali sifatida tasavvur qilish mumkinligi kelib chiqadi.

Vektorlarni qo‘shishning xossalari

1) $\vec{a}+\vec{b}=\vec{b}+\vec{a}$; 2) $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$;

3) $\vec{a}+\vec{0}=\vec{b}$; 4) $\vec{a}+(-\vec{a})=\vec{0}$.

Tekislikda bir nechta vektor yig‘indisi quyidagicha topiladi: Avval dastlabki ikkitasi qo‘shiladi, so‘ng yig‘indi vektorga uchinchisi qo‘shiladi, so‘ng bu yig‘indi vektorga to‘rtinchisi qo‘shiladi va hokazo. (Bunga ko‘pburchak usuli deyiladi.)

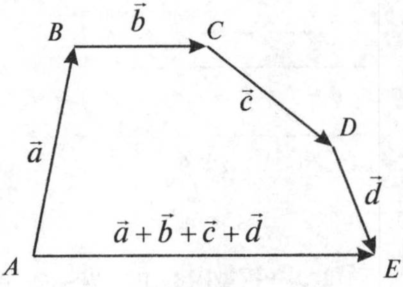
Ya’ni: $\vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}=(((\vec{a}+\vec{b})+\vec{c})+\vec{d})+\vec{e}$,

yoki:

$$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}=\overrightarrow{AE}. \quad (2)$$

Guruhlash usulini qo'llasa ham bo'ladi: $\vec{a}+\vec{b}+\vec{c}+\vec{d}=$ $=(\vec{a}+\vec{b})+(\vec{c}+\vec{d})$.

Ixtiyoriy n ta vektor uchun $\overrightarrow{A_1A_2}+\overrightarrow{A_2A_3}+\ldots+\overrightarrow{A_{n-1}A_n}=\overrightarrow{A_1A_n}$ tenglik o'rinli bo'ladi.



14-rasm.

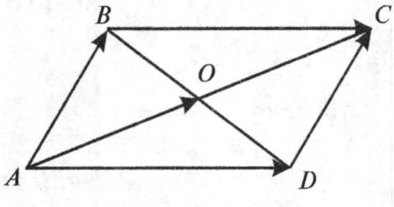
Ikki vektor ayirmasi va yig'indisi uchun

$$\| \vec{a} |-| \vec{b} \| \leq | \vec{a}-\vec{b} | \leq \| \vec{a} |+| \vec{b} \|$$

tengsizlik o'rinli bo'ladi.

Ixtiyoriy r ta yo'nalishdosh vektor uchun $|\vec{a}_1+\vec{a}_2+$ $+\vec{a}_3+\ldots+\vec{a}_n| \leq |\vec{a}_1|+|\vec{a}_2|+|\vec{a}_3|+\ldots+|\vec{a}_n|$ tengsizlik o'rinli bo'ladi.

Mashq. $ABCD$ parallelogrammda AC diagonal uzunligi 20 sm, O nuqta esa parallelogramm simmetriya markazi bo'lsa, $\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CO}$ vektor modulini hisoblang.



15-rasm.

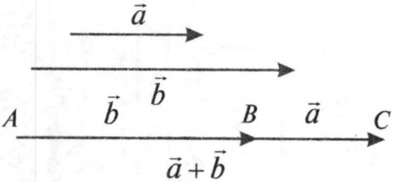
Yechish. $\overrightarrow{AD}=\overrightarrow{BC}$ bo'lgani uchun: $|\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CO}|=$ $=|\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CO}|=|\overrightarrow{AO}|=10$.

Javob: 10 sm. (15-rasm)

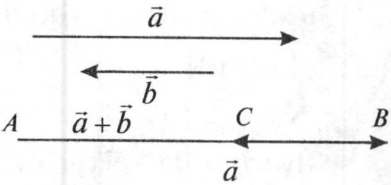
Mustaqil yechish uchun mashqlar.

a) $ABCD$ to'g'ri to'rtburchakda $AB=10$ sm va $AD=24$ sm bo'lsa, $|\overrightarrow{AD}+\overrightarrow{DC}|$ ni hisoblang.

b) $ABCDA_1B_1C_1D_1$ to'g'ri burchakli parallelepipedda $AB=$ $=3$ sm, $BC=4$ sm, $AA_1=24$ sm va M nuqta CC_1 qirra o'rtasi bo'lsa, $|\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CM}|$ ni hisoblang.

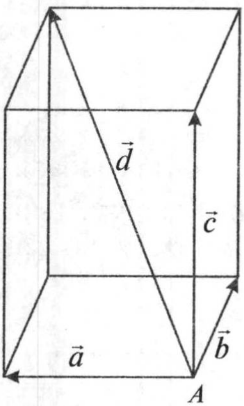


16-rasm.



17-rasm.

Parallel (kollinear) vektorlarni qo'shish ham yuqoridagi qoida asosida bajariladi (16- va 17-rasmlar):



18-rasm.

Har ikki rasmda ham $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$.

Fazoda hech bir ikkitasi o'zaro parallel bo'lmagan va bir nuqta (A) dan qo'yilgan uchta vektor yig'indisi qirralari shu vektorlardan iborat bo'lgan parallelepipedning (A uchidan chiqqan) diagonali bo'ladi (18-rasm):

$$a+\overrightarrow{BC}=\overrightarrow{AC}.$$

Bu qoidaga parallelepiped qoidasi deyiladi.

8. Vektorlarni ayirish

Ta'rif. $\vec{a}$ va $\vec{b}$ vektorlar ayirmasi deb shunday $\vec{c}$ ga aytiladiki, $\vec{c}+\vec{b}=\vec{a}$ bo'lsin.

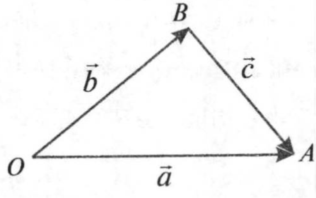
Bundan: $\overrightarrow{OA}-\overrightarrow{OB}=\overrightarrow{BA}$ (buni esda saqlash muhim).

Mashq:

1) $\overrightarrow{DM}-\overrightarrow{DK}=\overrightarrow{KM}$

2) $\overrightarrow{AP}-\overrightarrow{AQ}=\overrightarrow{QP}$

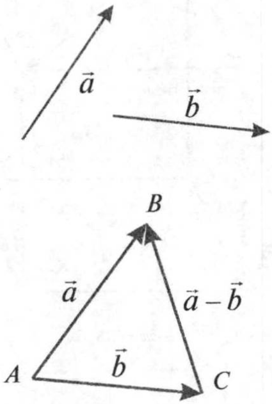
Demak, ikki vektor ayirmasini yasash uchun dastlab ular bir nuqtadan qo'yiladi, ikkinchi vektor uchini birinchi vektor uchi bilan tutashtirishdan hosil bo'lgan vektor ular ayirmasi bo'ladi:



19-rasm.

$\overrightarrow{AB}-\overrightarrow{AC}=\overrightarrow{CB}$ (3) (20-rasm).

Vektorlarni ayirishning ham **parallelogramm usuli** mavjud. $\overrightarrow{AD}-\overrightarrow{AB}=\overrightarrow{BD}$ bo'lgani uchun ikki vektor ayirmasi bu vektorlarda yasalgan parallelogramm diagonali bo'lishi kelib chiqadi (21-rasm).



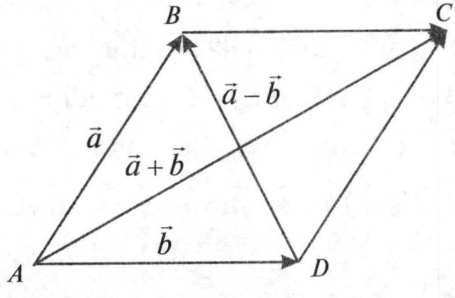
20-rasm.

Xulosa: noldan farqli $\vec{a}$ va $\vec{b}$ ($\vec{a}\nparallel\vec{b}$) vektorlar yig'indisi va ayirmasi shu vektorlarda yasalgan parallelogrammning turli diagonallari bo'ladi: $\vec{a}+\vec{b}=\overrightarrow{AC}$, $\vec{a}-\vec{b}=\overrightarrow{DB}$ (21-rasm).

Mashq: $\vec{a}$ va $\vec{b}$ nokollinear vektorlar berilgan. $|\vec{a}|=|\vec{b}|=5$ bo'lsa, $(\vec{a}+\vec{b})$ va $(\vec{a}-\vec{b})$ vektorlar orasidagi burchakni toping.

Yechish: $|\vec{a}|=|\vec{b}|=5$ shartdan bu vektorlarga yasalgan parallelogrammning romb ekanligi kelib chiqadi. $\vec{a}+\vec{b}=\vec{d}_1$ va $\vec{a}-\vec{b}=\vec{d}_2$ bo'lganligi uchun rombda ular orasidagi burchak 90°.

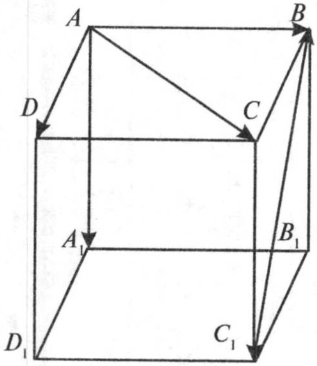
Javob: 90°.



21-rasm.

Ko‘p hollarda $\overrightarrow{AB}-\overrightarrow{CD}$ ayirmani $\overrightarrow{AB}+\overrightarrow{DC}$ yig‘indiga almashtirish ma’qul bo‘ladi (chunki: $-\overrightarrow{CD}=\overrightarrow{DC}$).

Masalan: $\overrightarrow{AB}-\overrightarrow{AM}=\overrightarrow{AB}+\overrightarrow{MA}=\overrightarrow{MA}+\overrightarrow{AB}=\overrightarrow{MB}$.



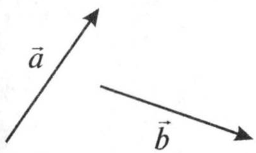
22-rasm.

Mashq: qirrasi 10 sm bo‘lgan $ABCDA_1B_1C_1D_1$ kubda $\overrightarrow{AB}+\overrightarrow{AD}-\overrightarrow{A_1A}-\overrightarrow{BC_1}$ vektor uzunligini hisoblang.

Yechish: $\overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AC}$, $-\overrightarrow{A_1A}=\overrightarrow{AA_1}=\overrightarrow{CC_1}$ va $-\overrightarrow{BC_1}=\overrightarrow{C_1B}$ ekanligidan:

$$|\overrightarrow{AB}+\overrightarrow{AD}-\overrightarrow{A_1A}-\overrightarrow{BC_1}|=$$
$$=|\overrightarrow{AB}+\overrightarrow{AD}-\overrightarrow{AA_1}-\overrightarrow{C_1B}|=$$
$$=|\overrightarrow{AC}+\overrightarrow{CC_1}-\overrightarrow{C_1B}|=|\overrightarrow{AB}|=10.$$

Javob: 10 sm (22-rasm).



23-rasm.

Mustaqil yechish uchun mashqlar.

a) 23-rasmda berilgan vektorlar $(\vec{a}-\vec{b})$ ayirmasini yasang;

b) $ABCD$ parallelogrammda $AB=3$ sm, $BC=5$ sm va $\angle A=60°$ bo‘lsa, $\overrightarrow{BA}-\overrightarrow{BC}$ vektor modulini toping.

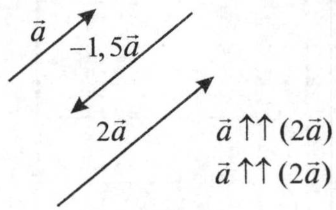
c) $\vec{a}$ va $\vec{b}$ parallel bo‘lmagan vektorlar berilgan. $|\vec{a}|=|\vec{b}|=3$ bo‘lsa, $(\vec{a}+\vec{b})$ va $(\vec{a}+\vec{b})$ vektorlar orasidagi burchakni toping.

9. Vektorni songa ko'paytirish

$\vec{a}$ — noldan farqli vektor, k — noldan farqli son bo'lsin. $k \cdot \vec{a}$ orqali quyidagi ikkita shartni qanoatlantiruvchi vektor belgilanadi:

1) $k \cdot \vec{a}$ ning uzunligi $|k| \cdot |\vec{a}|$ ga teng;

2) $k \cdot \vec{a}$ vektorga parallel va $k > 0$ bo'lganda $k \cdot \vec{a}$ va $\vec{a}$ vektorlar yo'nalishdosh, $K < 0$ bo'lganda $k \cdot \vec{a}$ va $\vec{a}$ vektorlar qarama-qarshi yo'nalgan bo'ladi (24-rasm)



24-rasm.

O'zaro parallel vektorlarga ***kollinear vektorlar*** deyiladi.

Vektorni songa ko'paytirishning asosiy xossalari: ixtiyoriy $\vec{a}$, $\vec{b}$ vektorlar va m, n sonlar uchun:

1) $m \cdot (n \cdot \vec{a}) = (m \cdot n) \cdot \vec{a}$;

2) $(m + n) \cdot \vec{a} = m\vec{a} + n\vec{a}$;

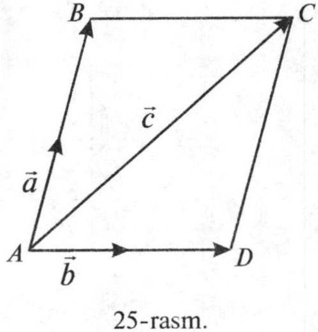
3) $m \cdot (\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$;

4) $1 \cdot \vec{a} = \vec{a}$; 5) $0 \cdot \vec{a} = \vec{0}$.

Yuqoridagi xossalar va vektorlarni qo'shish amalidan ba'zi natijalar chiqaramiz:

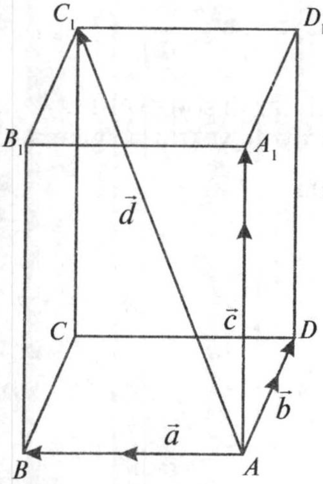
a) tekislikda o'zaro parallel bo'lmagan noldan farqli $\vec{a}$ va $\vec{b}$ vektorlar berilgan bo'lsa, bu tekislikdagi $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar orqali ifodalash mumkin: $\vec{c} = \lambda\vec{a} + \mu\vec{b}$.

Bu tenglikka $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar orqali ***yoyilmasi*** deyiladi (25-rasm). Bunda: $\overrightarrow{AB} = \lambda\vec{a}$, $\overrightarrow{AD} = \mu\vec{b}$.



25-rasm.

Bundagi $\vec{a}$ va $\vec{b}$ vektorlarga ***bazis vektorlar*** deyiladi.



26-rasm.

b) fazoda hech bir ikkitasi o'zaro parallel bo'lmagan uchta $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar mavjud. Ixtiyoriy to'rtinchi $\vec{d}$ vektorni bu uchta vektor orqali ifodalash mumkin: $\vec{d} = \lambda \cdot \vec{a} + \mu\vec{b} + \kappa\vec{c}$.

Bunda: $\lambda \cdot \vec{a} = \overrightarrow{AB}$, $\mu \cdot \vec{b} = \overrightarrow{AD}$, $\kappa \cdot \vec{c} = \overrightarrow{AA_1}$. $\vec{d} = \overrightarrow{AC_1}$. Bundagi $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlarga ***bazis vektorlar*** deyiladi (26- rasm).

c) ixtiyoriy ABC uchburchakda AM mediana bo'lsa,

$$\overrightarrow{AM} = \frac{1}{2} \cdot \left(\overrightarrow{AB} + \overrightarrow{AC}\right) \qquad (4)$$

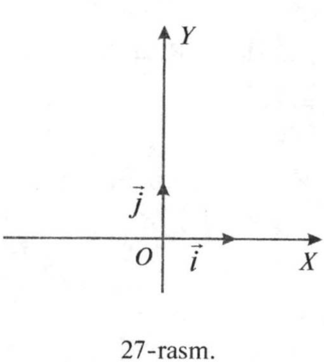
bo'ladi.

d) Ixtiyoriy ABC uchburchakda medianalar kesishgan nuqta O bo'lsa,

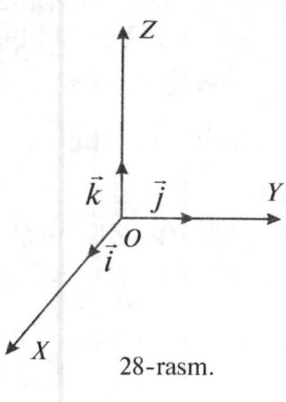
$$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \vec{O} \qquad (5)$$

bo'ladi.

e) uzunligi 1 ga teng bo'lgan vektorga ***birlik vektor*** deyiladi. ($|\vec{e}| = 1$ bo'lsa, $\vec{e}$ — birlik vektor.)



27-rasm.



28-rasm.

Ixtiyoriy $\vec{a}$ vektorni o'zining uzunligiga bo'lishdan shu vektorga yo'nalishdosh birlik vektor hosil bo'ladi: $\vec{e} = \frac{\vec{a}}{|\vec{a}|}$.

f) koordinata o'qlari bo'ylab koordinata boshidan qo'yilgan birlik vektorlarga ***koordinata vektorlari*** deyiladi va $\vec{i}$, $\vec{j}$ va $\vec{k}$ kabi belgilanadi (27- va 28- rasmlar).

Koordinata vektorlari o'zaro perpendikular bo'ladi, ya'ni $|\vec{i}\,| = |\vec{j}\,| = |\vec{k}\,| = 1$ va $\vec{i} \perp \vec{j}$, $\vec{i} \perp \vec{k}$, $\vec{j} \perp \vec{k}$.

Bundan $\vec{i} \cdot \vec{j} = \vec{i} \cdot \vec{k} = \vec{j} \cdot \vec{k} = 0$.

10. Vektorlarning o'zaro skalar ko'paytmasi

$\vec{a}$ va $\vec{b}$ vektorlarning o'zaro skalar ko'paytmasi: $\vec{a} \cdot \vec{b} = |\vec{a}\,| \cdot |\vec{b}\,| \cdot \cos\varphi$ (6) ga teng, bunda φ — $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak.

Mashq: $|\vec{a}\,| = 12$, $|\vec{b}\,| = 8$ va $\angle(\vec{a}\,;\vec{b}\,) = 120°$ bo'lsa, $\vec{a} \cdot \vec{b}$ = ?

Yechish: $\vec{a} \cdot \vec{b} = |\vec{a}\,| \cdot |\vec{b}\,| \cdot \cos\varphi = 12 \cdot 8 \cdot \cos 120° =$
$= 96 \cdot (-0{,}5) = -48$.
Javob: –48.

Vektorlarning o'zaro skalar ko'paytmasining asosiy xossalari:

1) $(\vec{a} \cdot \vec{b}) = (\vec{b} \cdot \vec{a})$; 2) $(k \cdot \vec{a}) \cdot \vec{b} = k \cdot (\vec{a} \cdot \vec{b})$;

3) $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$; 4) $\vec{a} \neq \vec{0}$ bo'lsa, $\vec{a}^2 > 0$ bo'ladi.

5) $\vec{a} = \vec{0}$ yoki $\vec{b} = \vec{0}$ yoki $\vec{a} = \vec{0}$ va $\vec{b} = \vec{0}$ bo'lsa, $\vec{a} \cdot \vec{b} = 0$.

Natijalar

a) $\vec{a}^2 = |\vec{a}\,|^2$ (7) va aksincha: $|\vec{a}\,|^2 = \vec{a}^2$ yoki $|\vec{a}\,| = \sqrt{\vec{a}^2}$ (8)

b) $\vec{a} \cdot \vec{b} = 0$ bo'lsa, $\angle(\vec{a};\vec{b}) = 90°$, ya'ni $\vec{a} \perp \vec{b}$, va aksincha: $\vec{a} \perp \vec{b}$ bo'lsa, $\vec{a} \cdot \vec{b} = 0$.

c) $\cos\varphi = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$ (9) bu ikki vektor orasidagi burchakni topish formulasidir, bunda $\varphi = \angle(\vec{a};\vec{b})$.

Mashqlar. a) $|\vec{a}|=8$, $|\vec{b}|=6$ va $\angle(\vec{a};\vec{b})=60^\circ$ bolsa, $(2\vec{a}+3\vec{b})^2$ ni hisoblang.

Yechish. $(2\vec{a}+3\vec{b})^2 = 4\vec{a}^2 + 12\vec{a}\cdot\vec{b} + 9\vec{b}^2 = 4\cdot 8^2 + 12\cdot 8\cdot 6\times\cos60^\circ + 9\cdot 6^2 = 256 + 576\cdot 0{,}5 + 324 = 868$.

Javob: 868.

b) $|\vec{a}| = 2$, $|\vec{b}| = 3$, $\angle(\vec{a};\vec{b}) = 60^\circ$ va $(3\vec{a} + \lambda\vec{b})\perp\vec{b}$ bo‘lsa, λ ni toping.

Yechish. Shartga ko‘ra $(3\vec{a}+\lambda\vec{b})\cdot\vec{b} = 0 \Rightarrow 3\vec{a}\cdot\vec{b} + \lambda\vec{b}^2 = 0 \Rightarrow$

$3\cdot|\vec{a}|\cdot|\vec{b}|\cdot\cos60^\circ + \lambda|\vec{b}|^2 = 0 \Rightarrow 3\cdot 2\cdot 3\cdot 0{,}5 + \lambda\cdot 3^2 = 0 \Rightarrow$

$9 + 9\lambda = 0 \Rightarrow \lambda = -1$.

Javob: –1.

c) Agar $\vec{m}$ va $\vec{n}$ vektorlar 120° li burchak tashkil etuvchi birlik vektorlar bo‘lsa, $2\vec{m}+4\vec{n}$ va $\vec{m}-\vec{n}$ vektorlar orasidagi burchakni toping.

Yechish. (9) ga ko‘ra $\cos\varphi = \frac{(2\vec{m}+4\vec{n})\cdot(\vec{m}-\vec{n})}{(2\vec{m}+4\vec{n})\cdot(\vec{m}-\vec{n})}$ (F) bo‘lgani uchun, dastlab, $(2\vec{m}+4\vec{n})\cdot(\vec{m}-\vec{n})$, $|(2\vec{m}+4\vec{n})|$ va $|(\vec{m}-\vec{n})|$ larni hisoblaymiz.

$(2\vec{m}+4\vec{n})\cdot(\vec{m}-\vec{n}) = 2\vec{m}^2 + 2\vec{m}\cdot\vec{n} - 4\vec{n}^2 = 2\cdot 1 + 2\cdot 1\cdot 1\cdot\cos120^\circ - 4\cdot 1 =$

$= 2 + 2\cdot(-0{,}5) - 4 = -3$.

$|2\vec{m}+4\vec{n}| = \sqrt{(2\vec{m}+4\vec{n})^2} = \sqrt{4\vec{m}^2 + 16\vec{m}\cdot\vec{n} + 16\vec{n}^2} =$

$= \sqrt{4 + 16\cdot\left(-\frac{1}{2}\right) + 16} = \sqrt{12}$.

$$|\vec{m}-\vec{n}|=\sqrt{(\vec{m}^2-2\vec{m}\cdot\vec{n}+\vec{n}^2)}=\sqrt{1-2\cdot\left(-\tfrac{1}{2}\right)+1}=\sqrt{3}.$$

Bularni (F) ga qo‘yamiz: $\cos\varphi=\frac{-3}{\sqrt{12}\cdot\sqrt{3}}=-\frac{3}{6}=-\frac{1}{2}$.

Bundan: $\varphi=\arccos\left(-\frac{1}{2}\right)=120°$.

Javob: 120°.

d) $|\vec{a}|=6, |\vec{b}|=5$ va $\angle(\vec{a};\vec{b})=45°$ bo‘lsa, $\vec{a}\cdot(3\vec{a}-\vec{b})$ ni hisoblang.

Yechish: $\vec{a}^2=|\vec{a}|^2=5^2=25$ va $\vec{a}\cdot\vec{b}=5\cdot6\cdot\cos45°=30\frac{\sqrt{2}}{2}=15\sqrt{2}$ bo‘lgani uchun:

$$\vec{a}\cdot(3\vec{a}-\vec{b})=3\vec{a}^2-\vec{a}\cdot\vec{b}=3\cdot25-15\sqrt{2}=75-15\sqrt{2}=15(5-\sqrt{2}).$$

Javob: $15(5-\sqrt{2})$.

Mustaqil yechish uchun mashqlar

a) $|\vec{a}|=2, |\vec{b}|=8$ va $\angle(\vec{a};\vec{b})=60°$ bo‘lsa, $\vec{a}\cdot\vec{b}$ ni hisoblang.

b) $|\vec{a}|=\sqrt{3}, |\vec{b}|=4$ va $\angle(\vec{a};\vec{b})=150°$ bo‘lsa, $\vec{a}\cdot\vec{b}$ ni hisoblang.

c) $|\vec{a}|=\sqrt{2}, |\vec{b}|=4$ va $\angle(\vec{a};\vec{b})=45°$ bo‘lsa, $(2\vec{a}-5\vec{b})^2$ ni hisoblang.

d) $|\vec{a}|=2, |\vec{b}|=3$ va $\angle(\vec{a};\vec{b})=120°$ bo‘lsa, $(2\vec{a}-\vec{b})\cdot(3\vec{a}+2\vec{b})$ ni hisoblang.

e) $|\vec{a}|=\sqrt{3}, |\vec{b}|=8$ va $\angle(\vec{a};\vec{b})=30°$ bo‘lsa, $(2\vec{a}+\lambda\vec{b})\perp\vec{b}$ ni toping.

2-§. VEKTOR KOORDINATALARI

1. Tekislikda vektor koordinatalari

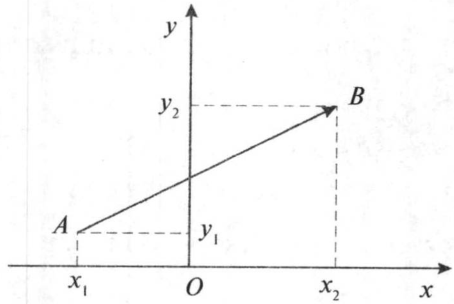
Koordinatalar tekisligida boshi $A(x_1;\ y_1)$ va uchi $B(x_2;\ y_2)$ nuqtalarda bo'lgan vektor koordinatalari deb $x_2 - x_1$ va $y_2 - y_1$ sonlarga aytiladi,

$$\overrightarrow{AB} = (x_2 - x_1;\ y_2 - y_1)$$

kabi yoki

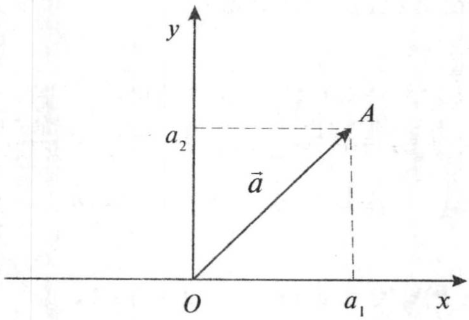
$$\overrightarrow{AB} = \{x_2 - x_1;\ y_2 - y_1\}$$

kabi belgilanadi (29-rasm).



29-rasm.

Boshi koordinata boshida uchi esa $A(a_1;\ a_2)$ nuqtada bo'lgan $\vec{a}$ vektor koordinatalari $(a_1;\ a_2)$ bo'ladi ya'ni: $\vec{a}\,(a_1;\ a_2)$ (30-rasm).



30-rasm.

Mashq:

1) Boshi $A(4;\ 7)$, uchi $B(6;\ -3)$ nuqtalarda bo'lgan $\overrightarrow{AB}$ vektor koordinatalarini toping.

Yechish: $\overrightarrow{AB}\,(x;\ y)$ bo'lsin, u holda $x = 6 - 4 = 2$ va $y = -3 - 7 = -10$ demak: $\overrightarrow{AB}\,(2;\ -10)$

Javob: $\overrightarrow{AB}\,(2;\ -10)$.

2) $A(6;\ -3)$ nuqta $\vec{a}\,(-7;\ 5)$ vektorning boshi bo'lsa, uning B uchi koordinatalarini toping.

Yechish: $B(x;\ y)$ bo'lsin, u holda $x - 6 = -7$ va $y - (-3) = 5$ bundan: $x = -1,\ y = 2$.

Javob: $B(-1;\ 2)$.

Mustaqil yechish uchun mashqlar

1) $O(0;\ 0)$ koordinatalar boshi va $A(4;\ 6)$ bo'lsa, $\overrightarrow{OA}=\vec{a}$ vektor koordinatalarini aniqlang va uni koordinatalar tekisligida tasvirlang.

2) $\vec{b}(4;-5)$ vektorni koordinatalar tekisligida chizing va $|\vec{b}|$ ni hisoblang.

3) $\vec{c}$ (-4; -3) vektorni koordinatalar tekisligida chizing va $|\vec{c}|$ ni hisoblang.

4) $B(6;\ -3)$ nuqta $\vec{a}$ (7; -2) vektor uchi bo'lsa, uning A boshi koordinatalarini toping.

5) $A(-2;\ -5)$ nuqta $\vec{a}$ (-5; 4) vektorning boshi bo'lsa, uning B uchi koordinatalarini toping.

6) $A(-6;\ 9)$ va $B(4;\ 6)$ bo'lsa, $\overrightarrow{AB}$ vektor koordinatalarini aniqlang.

Natijalar.

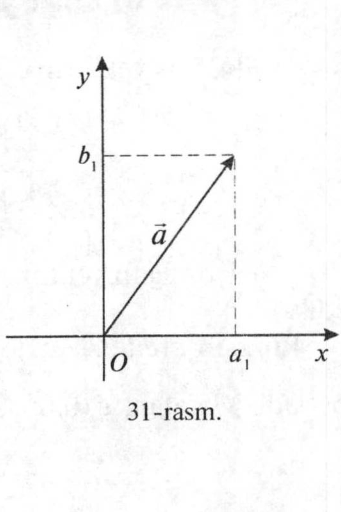
1) Koordinatalari bir xil bo'lgan vektorlar teng vektorlar bo'ladi.

2) Nol vektor koordinatalari 0 ga teng: $\vec{O}$ (0; 0).

2. Koordinatalari bilan berilgan vektor uzunligi

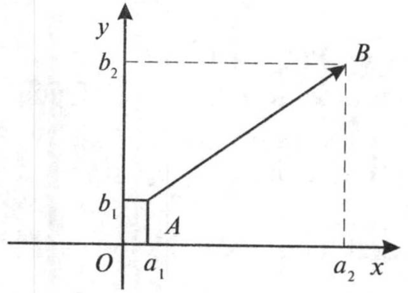
A) $\vec{a}\,(a_1;\ a_2)$ vektor uzunligi: $|\vec{a}|=\sqrt{a_1^2+a_2^2}$ (10) formula orqali topiladi (31-rasm).

B) Boshi $A(x_1;\ y_1)$, uchi $B(x_2;\ y_2)$ nuqtalarda bo'lgan vektor uzunligi esa $|\vec{a}|=\sqrt{a_1^2+a_2^2}$ (11) kabi topiladi (32-rasm).

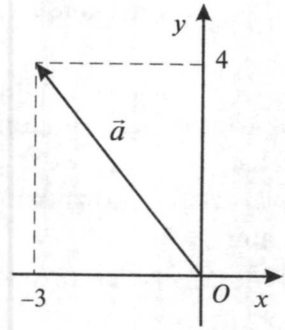


31-rasm.

Mashq: 1) $\vec{a}$ (-3; 4) vektorni chizing va uzunligini toping.



32-rasm.



33-rasm.

Yechish: (8) formulaga asosan (33-rasm).

$|\vec{a}|=\sqrt{(-3)^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$.

Javob: 5.

2) $A(4;\ 7)$ va $B(-1;\ 2)$ bo'lsa, $|\overrightarrow{AB}|$ ni aniqlang.

Yechish: (9) formulaga asosan

$|\overrightarrow{AB}|=\sqrt{(-1-4)^2+(2-7)^2}=\sqrt{25+25}=\sqrt{50}=5\sqrt{2}$.

Javob: $5\sqrt{2}$.

Mustaqil yechish uchun mashqlar

1) $\vec{a}(4;-3)$ vektorni chizing va uzunligini toping.

2) $A(-3;\ 2)$ va $B(1;\ 4)$ bo'lsa, $|\overrightarrow{AB}|$ ni aniqlang.

3. Birlik vektorlar

$|\vec{e}|=1$ bo'lgan vektorga birlik vektor deyilishini aytib o'tgan edik.

Bundan $\vec{e}(e_1;\ e_2)$ vektor uchun $e_1^2+e_2^2=1$ bo'lsa, birlik vektor bo'ladi. Masalan: $\vec{e}\ (0{,}6;\ 0{,}8)$; $\vec{n}\left(-\frac{5}{13};\ \frac{12}{13}\right)$.

$\vec{a}(a_1;\, a_2)$ vektorga yo'nalishdosh $\vec{e}(e_1;\, e_2)$ birlik vektor koordinatalari

$e_1 = \frac{a_1}{|\vec{a}|}$ va $e_2 = \frac{a_2}{|\vec{a}|}$ bo'ladi.

$\vec{a}(a_1;\, a_2)$ vektorga qarama-qarshi yo'nalgan $\vec{e}(e_1;\, e_2)$ birlik vektor koordinatalari

$e_1 = -\frac{a_1}{|\vec{a}|}$ va $e_2 = -\frac{a_2}{|\vec{a}|}$ bo'ladi.

Koordinata vektorlari: $\vec{i}\,(1;\, 0)$ va $\vec{j}\,(0;\, 1)$ (27-rasm)

Istalgan $\vec{a}(a_1;\, a_2)$ vektorni koordinata vektorlari orqali $\vec{a} = a_1 \cdot \vec{i} + a_2 \cdot \vec{i}$ kabi ifodalash mumkin va aksincha: $\vec{b} = m \cdot \vec{i} + n \cdot \vec{j}$ bo'lsa, $\vec{b}(m;\, n)$ bo'ladi.

Mashqlar: 1) $\vec{a}\,(6;\, 8)$ vektorga yo'nalishdosh birlik vektorni aniqlang.

Yechish: $|\vec{a}| = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ demak, $e_1 = \frac{6}{10} = 0{,}6$, $e_2 = \frac{8}{10} = 0{,}8$.

Javob: $\vec{e}\,(0{,}6;\, 0{,}8)$.

2) $\vec{b} = -4 \cdot \vec{i} + 8 \cdot \vec{j}$ bo'lsa, $|\vec{b}| = ?$

Yechish: $|\vec{b}| = \sqrt{(-4)^2 + 8^2} = \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5}$. Javob: $4\sqrt{5}$.

Mustaqil yechish uchun mashqlar

1) $\vec{a}(4;\, -3)$ vektorga yo'nalishdosh birlik vektorni aniqlang.

2) Birlik vektorlarni aniqlang: $\vec{m}(0{,}5;\, 0{,}5)$, $\vec{n}\left(-\frac{1}{\sqrt{2}};\, \frac{1}{\sqrt{2}}\right)$, $\vec{q}\,(0{,}3;\, 0{,}7)$, $\vec{p}\left(\frac{12}{13};\, \frac{5}{13}\right)$.

3) $\vec{a}=12\vec{i}-5\vec{j}$ vektor bilan qarama-qarshi yo'nalgan birlik vektorni aniqlang.

4. Koordinatalari bilan berilgan vektorlarni qo'shish va ayirish

A) Koordinatalari bilan berilgan $\vec{a}(a_1;\ a_2)$ va $\vec{b}(b_1;\ b_2)$ **vektorlarni qo'shish** uchun ularning mos koordinatalarini qo'shish kifoya, ya'ni:

$$\vec{a}+\vec{b}=\{a_1+b_1;\ a_2+b_2\} \qquad (12)$$

B) Koordinatalari bilan berilgan $\vec{a}(a_1;\ a_2)$ va $\vec{b}(b_1;\ b_2)$ **vektorlarni ayirish** uchun ularning mos koordinatalarini ayirish kifoya, ya'ni:

$$\vec{a}-\vec{b}=\{a_1-b_1;\ a_2-b_2\} \qquad (13)$$

Mashq. $\vec{a}(5;\ -2)$ va $\vec{b}(4;\ 7)$ bo'lsa, $\vec{a}+\vec{b}$ va $\vec{a}-\vec{b}$ vektorlar koordinatalarini toping.

Yechish. $\vec{a}+\vec{b}=\{5+4;\ -2+7\}=\{9;\ 5\}$;

$\vec{a}-\vec{b}=\{5-4;\ -2-7\}=\{1;\ -9\}$

Javob: $\vec{a}+\vec{b}=\{9;\ 5\}$, $\vec{a}-\vec{b}=\{1;\ -9\}$.

5. Koordinatalari bilan berilgan vektorni songa ko'paytirish

Koordinatalari bilan berilgan **vektorni songa ko'paytirish** uchun uning har bir koordinatasini shu songa ko'paytirish kerak:

$\vec{a}(a_1;\ a_2)$ vektor va λ ixtiyoriy son bo'lsa, $\lambda\cdot\vec{a}(\lambda a_1;\ \lambda a_2)$ bo'ladi, ya'ni $\vec{c}=\lambda\cdot\vec{a}$ bo'lsa, $\vec{c}=(\lambda a_1;\ \lambda a_2)$ bo'ladi.

$\vec{a}=(a_1;\ a_2)$ vektorga ***qarama-qarshi vektor*** $-\vec{a}(-a_1;\ -a_2)$ bo'ladi.

Mashq. 1) $\vec{a}(4; -6)$ bo'lsa $\vec{c} = 3\vec{a}$ vektor koordinatalarini toping.

Yechish. $\vec{c} = 3\vec{a}\{3 \cdot 4;\ 3 \cdot (-6)\} = \{12; -18\}$.

Javob: $\vec{c}\,(12; -18)$.

2) $\vec{a}(3; -5)$, $\vec{b}(-2; 4)$ va $\vec{c} = 2\vec{a} + 4\vec{b}$ bo'lsa, $|\vec{c}| = ?$

Yechish. Dastlab $2\vec{a}$, $4\vec{b}$ va ulardan foydalanib $2\vec{a} + 4\vec{b}$ vektor koordinatalarini topamiz: $2\vec{a}(6; -10)$, $4\vec{a}(-8; 16) \Rightarrow$ $2\vec{a} + 4\vec{b} = \{6+(-8);\ -10+16\} = \{-2;\ 6\}$. Bundan: $|2\vec{a} + 4\vec{b}| =$ $\sqrt{(-2)^2 + 6^2} = \sqrt{40} = 2\sqrt{10}$

Javob: $2\sqrt{10}$.

3) $\vec{a}(4;7)$ vektorga ***qarama-qarshi vektorni*** aniqlang.

Yechish: $-\vec{a} = -1 \cdot \vec{a}$. Bundan: $-\vec{a}(-4; -7)$.

Javob: $-\vec{a}(-4; -7)$.

Mustaqil yechish uchun mashqlar

a) $\vec{a}(2; -3)$ va $\vec{b} = -5\vec{j}$ bo'lsa, $2\vec{a} - 4\vec{b}$ vektor koordinatalarini aniqlang. b) $\vec{a} = 5\vec{i} + 2\vec{j}$ va $\vec{b} = -5\vec{j}$ bo'lsa, $6\vec{a} + 2\vec{b}$ vektor koordinatalarini aniqlang.

6. Koordinatalari bilan berilgan ikki vektorning skalar ko'paytmasi

$\vec{a}(a_1; a_2)$ va $\vec{b}(b_1; b_2)$ vektorlarning ***skalar ko'paytmasi*** deb $a_1 \cdot b_1 + a_2 \cdot b_2$ songa aytiladi. $\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2$. (14) (ikki vektorning skalar ko'paytmasi sondir)

Mashq: $\vec{a}(1; 4)$ va $\vec{b}(-6; 5)$ bo'lsa, $\vec{a} \cdot \vec{b} = ?$

Yechish: $\vec{a} \cdot \vec{b} = 1 \cdot (-6) + 4 \cdot 5 = 14$. *Javob:* 14.

Avvalgi formulalar ham o'rinli: a) $\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\cos\varphi$ (bunda $\varphi=\angle(\vec{a};\vec{b})$)

b) $\vec{a}^2=|\vec{a}|^2$ yoki $\vec{a}^2=a_1^2+a_2^2$ va aksincha: $|\vec{a}|^2=\vec{a}^2$ yoki $|\vec{a}|^2=\sqrt{\vec{a}^2}$

c) $\vec{a}\cdot\vec{b}=0$ bo'lsa, $\angle(\vec{a};\vec{b})=90^\circ$ (ya'ni $\vec{a}\perp\vec{b}$) va aksincha: $\vec{a}\perp\vec{b}$ bo'lsa, $\vec{a}\cdot\vec{b}=0$

Mashqlar: 1) $\vec{a}^2=(8;-3)$ bo'lsa, $\vec{a}^2=?$

Yechish: $\vec{a}^2=8^2+(-3)^2=64+9=73$. *Javob*: 73.

2) $\vec{a}(x-2;5),\vec{b}(4;1-x)$ va $\vec{a}\perp\vec{b}$ bo'lsa, $x=?$

Yechish: $\vec{a}\perp\vec{b}$ bo'lganidan: $\vec{a}\cdot\vec{b}=0$. Demak, $(x-2)\cdot4+5\cdot$ $\cdot(1-x)=0\Rightarrow4x-8+5-5x=0\Rightarrow-x=3\Rightarrow x=-3$. *Javob*: -3.

Mustaqil yechish uchun mashqlar

a) $\vec{a}(4;-3)$ bo'lsa, $\vec{a}^2$ ni hisoblang.

b) $\vec{a}(2;7)$ va $\vec{b}(-3;4)$ bo'lsa, $\vec{a}\cdot\vec{b}$ ni hisoblang.

c) $\vec{a}(1;x+3)$, $\vec{b}(x-2;4)$ va $\vec{a}\cdot\vec{b}=8$ bo'lsa, $x=?$

d) $\vec{a}(x-1;-2)$, $\vec{b}(x;x+2)$ va $\vec{a}\perp\vec{b}$ bo'lsa, $x=?$

7. Koordinatalari bilan berilgan ikki vektor orasidagi burchak

Ikki $\vec{a}(a_1;a_2)$ va $\vec{b}(b_1;b_2)$ vektorlar orasidagi burchak kosinusini topish formulasini quyidagicha ifodalash mumkin:

$$\cos\varphi=\frac{a_1\cdot b_1+a_2\cdot b_2}{\sqrt{a_1^2+a_2^2}\cdot\sqrt{b_1^2+b_2^2}}. \quad (15)$$

Mashq: 1) $\vec{a}(4;3)$ va $\vec{b}(5;12)$ vektorlar orasidagi burchakni toping.

Yechish: $\cos\varphi = \dfrac{a_1 \cdot b_1 + a_2 \cdot b_2}{\sqrt{a_1^2 + a_2^2} \cdot \sqrt{b_1^2 + b_2^2}} = \dfrac{4 \cdot 5 + 3 \cdot 12}{\sqrt{4^2 + 3^2} \cdot \sqrt{5^2 + 12^2}} =$

$= \dfrac{20 + 36}{5 \cdot 13} = \dfrac{56}{65}.$

Bundan: $\varphi = \arccos\frac{56}{65}$. *Javob*: $\varphi = \arccos\frac{56}{65}$.

2) $\vec{a}(-2;2)$, va $\vec{j}$ koordinata vektori orasidagi burchakni toping.

Yechish. Ma'lumki, $\vec{j}(0;1)$. (15) formulaga ko'ra,

$$\cos\varphi = \frac{a_1 \cdot b_1 + a_2 \cdot b_2}{\sqrt{a_1^2 + a_2^2} \cdot \sqrt{b_1^2 + b_2^2}} = \frac{-2 \cdot 0 + 2 \cdot 1}{\sqrt{(-2)^2 + 2^2} \cdot \sqrt{0^2 + 1^2}} =$$

$= \dfrac{2}{\sqrt{8} \cdot \sqrt{1}} = \dfrac{2}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}}$, bundan $\varphi = 45^\circ$.

Javob: 45°.

Mustaqil yechish uchun mashqlar

a) $\vec{a}(-2;3)$, va $\vec{b}(1;5)$ vektorlar orasidagi burchakni toping.

b) $\vec{a}(1;2)$ va $\vec{b}(3;-1)$ vektorlarda yasalgan parallelogramm diagonallari orasidagi burchakni toping.

8. Vektorlar kollinearligi xossalari

$\vec{a}(a_1;a_2)$ va $\vec{b}(b_1;b_2)$ vektorlar kollinear(parallel) bo'lsa, $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2}$ bo'ladi va aksincha $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2}$ bo'lsa, $\vec{a}(a_1;a_2)$ va $\vec{b}(b_1;b_2)$ vektorlar kollinear(parallel) bo'ladi.

Mashq: 1) $\vec{a}(2;\,5)$ va $\vec{b}(3;\,y-2)$ parallel bo‘lsa, $y=?$

Yechish: shartga ko‘ra $\frac{2}{5}=\frac{3}{y-2}\Rightarrow 2\cdot(y-2)=5\cdot3\Rightarrow y=9{,}5.$

Javob: 9,5.

Mustaqil yechish uchun mashqlar

a) $\vec{a}(9;\,-2)$ va $\vec{b}(x-3;\,-4)$ va $\vec{a}\parallel\vec{b}$ bo‘lsa, $x=?$

b) $\vec{a}(x+5;\,-2)$ va $\vec{b}(x+3;\,5)$ va $\vec{a}\parallel\vec{b}$ bo‘lsa, $x=?$

9. Boshqa xossalar

a) $M(x_0;\,y_0)$ nuqtadan o‘tib, $\vec{n}(A;B)$ vektorga perpendikular to‘g‘ri chiziq tenglamasi: $A(x_0-x_0)+B(y_0-y_0)=0$.

b) $M(x_0;\,y_0)$ nuqtadan o‘tib, $\vec{n}(A;B)$ vektorga parallel to‘g‘ri chiziq tenglamasi:

$$\frac{x-x_0}{A}=\frac{y-y_0}{B}.$$

c) Orasidagi burchagi α bo‘lgan $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan uchburchak yuzi $S=\frac{1}{2}\vec{a}\vec{b}tg\alpha$ (16) ga teng bo‘ladi.

d) Orasidagi burchagi α bo‘lgan $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogramm yuzi $S=\frac{1}{2}\vec{a}\vec{b}tg\alpha$ (17) ga teng bo‘ladi.

Mashq:

1) M(4;10) nuqtadan o‘tib, $\vec{n}(3;\,5)$ vektorga perpendikulyar to‘g‘ri chiziq tenglamasini tuzing.

Yechish: $A(x_0-x_0)+B(y_0-y_0)=0$ ga asosan:

$3(x-4)+5(y-10)\Rightarrow 3x+5y-62=0.$

Javob: $3x+5y-62=0.$

2) Agar $\vec{m}$ va $\vec{n}$ vektorlar 135° li burchak tashkil etsa va $\vec{m}\cdot\vec{n}=6\sqrt{2}$ bo'lsa, bu vektorlarga qurilgan uchburchak yuzini toping.

Yechish: (16) ga ko'ra $S=\frac{1}{2}\vec{m}\vec{n}\,tg135^\circ=\frac{1}{2}\cdot 6\sqrt{2}\cdot\frac{1}{\sqrt{2}}=3$.

Javob: 3.

Mustaqil yechish uchun mashqlar

a) D(5; 2) nuqtadan o'tib, $\vec{n}(3;5)$ vektorga perpendikular to'g'ri chiziq tenglamasini tuzing.

b) B(6; 4) nuqtadan o'tib, $\vec{a}(-3;6)$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

c) Orasidagi burchgi 30° bo'lgan $\vec{a}(4;-3)$ va $\vec{b}(6;8)$ vektorlarga qurilgan uchburchak yuzini hisoblang.

d) $\vec{a}\cdot\vec{b}=8\sqrt{2}$ va $\angle(\vec{a};\vec{b})=135^\circ$ bo'lsa, bu vektorlarga qurilgan parallelogramm yuzini hisoblang.

3-§. FAZODA VEKTOR KOORDINATALARI

1. Fazoda vektor koordinatalari xossalari

1) Uch o'lchovli fazoda vektor koordinatalari $\vec{a}(a_1;\ a_2;\ a_3)$ kabi bo'ladi.

2) $A(x_1;\ y_1;z_1)$ va $B(x_2;\ y_2;z_2)$ bo'lsa, $\overrightarrow{AB}$ vektor koordinatalari $\overrightarrow{AB}=\{x_2-x_1;\ y_2-y_1;z_2-z_1\}$ (18) bo'ladi.

3) $\vec{a}(a_1;\ a_2;\ a_3)$ vektor uzunligi: $|\vec{a}|=\sqrt{a_1^2+a_2^2+a_3^2}$ (19) kabi topiladi.

4) $A(x_1;\ y_1;z_1)$ va $B(x_2;\ y_2;z_2)$ bo'lsa,

$|\overrightarrow{AB}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$ (20) bo'ladi.

5) Ma'lumki, uzunligi 1 bo'lgan vektorga birlik vektor deyiladi, fazoda ham shunday: $(\vec{e}(e_1;\ e_2;\ e_3),\ |\vec{e}|=1)\Rightarrow e_1^2+e_2^2+e_3^2=1$.

6) Koordinata vektorlari: $\vec{i}$ (1; 0; 0), $\vec{j}$ (0; 1; 0) va $\vec{k}$ (0; 0; 1) (28-rasm).

7) $\vec{a}(a_1;\ a_2;\ a_3)$ vektorni koordinata vektorlari bo'yicha yoyilmasi: $\vec{a}=a_1\vec{i}+a_2\vec{j}+a_3\vec{k}$ kabi bo'ladi.

8) $\vec{a}(a_1;\ a_2;\ a_3)$ vektorga yo'nalishdosh $\vec{e}(e_1;\ e_2;\ e_3$ birlik vektor koordinatalari $e_1\frac{a_1}{|\vec{a}|}$, $e_2\frac{a_2}{|\vec{a}|}$ va $e_3\frac{a_3}{|\vec{a}|}$, (21) kabi bo'ladi.

9) $\vec{a}(a_1;\ a_2;\ a_3)$ vektorga qarama-qarshi yo'nalgan $\vec{e}(e_1;\ e_2;\ e_3)$ birlik vektor koordinatalari: $e_1=-\frac{a_1}{|\vec{a}|}$, $e_2=-\frac{a_2}{|\vec{a}|}$ va $e_3=-\frac{a_3}{|\vec{a}|}$, (22) kabi bo'ladi.

10) $\vec{a}(a_1;\ a_2;\ a_3)$ va $\vec{b}(b_1;\ b_2;\ b_3)$ bo'lsa,

$\vec{a}+\vec{b}=\{a_1+b_1;\ a_2+b_2;\ a_3+b_3\}$ (23) va

$\vec{a}-\vec{b}=\{a_1-b_1;\ a_2-b_2;\ a_3-b_3\}$ (24) bo'ladi.

11) $\vec{a}(a_1;\ a_2;\ a_3)$ vektor va λ ixtiyoriy son bo'lsa,

$\lambda\cdot\vec{a}=\{\lambda a_1;\lambda a_2;\lambda a_3\}$ (25) bo'ladi.

12) $\vec{a}(a_1;\ a_2;\ a_3)$ va $\vec{b}(b_1;\ b_2;\ b_3)$ vektorlarning skalar ko'paytmasi deb $a_1\cdot b_1+a_2\cdot b_2+a_3\cdot b_3$ songa aytiladi, ya'ni $\vec{a}\cdot\vec{b}=a_1\cdot b_1+a_2\cdot b_2+a_3\cdot b_3$. (26)

13) $\vec{a}(a_1;\ a_2;\ a_3)$ va $\vec{b}(b_1;\ b_2;\ b_3)$ vektorlar orasidagi burchak kosinusini topish formulasi quyidagicha ko'rinishga ega bo'ladi:

$$\cos\varphi = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{a_1\cdot b_1 + a_2\cdot b_2 + a_3\cdot b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}}. \quad (27)$$

14) Fazoda $\vec{a}(a_1;\ a_2;\ a_3)$ va $\vec{b}(b_1;\ b_2;\ b_3)$ vektorlar kollinearligi xossasi: $\vec{a}\parallel\vec{b}$ bo‘lsa, $\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3}$ (28) bo‘ladi.

15) $\vec{a}(a_1;\ a_2;\ a_3)$ vektor va koordinata vektorlari orasidagi burchaklar uchun quyidagi tengliklar o‘rinli bo‘ladi:

$$\cos(\angle(\vec{a};\ \vec{i})) = \frac{a_1}{|\vec{a}|} = \frac{a_1}{\sqrt{a_1^2+a_2^2+a_3^2}},$$

$$\cos(\angle(\vec{a};\ \vec{j})) = \frac{a_2}{|\vec{a}|} = \frac{a_2}{\sqrt{a_1^2+a_2^2+a_3^2}},$$

$$\cos(\angle(\vec{a};\ \vec{k})) = \frac{a_3}{|\vec{a}|} = \frac{a_3}{\sqrt{a_1^2+a_2^2+a_3^2}}.$$

Mashqlar

1) $A(5;\ 6;\ -2)$ va $B(-1;\ 5;\ 3)$ bo‘lsa, $\overrightarrow{AB}$ vektor koordinatalarini toping.

Yechish. $\overrightarrow{AB}(x; y; z)$ bo‘lsin. U holda, $x=-1-5=-6$. $y=5-6=-1$. $z=3-(-2)=5$.

Demak: $\overrightarrow{AB}(-6;\ -1;\ 5)$. *Javob*: $\overrightarrow{AB}(-6;\ -1;\ 5)$.

2) $\vec{a}=2\vec{i}-\vec{j}+2\vec{k}$ vektor uzunligini toping.

Yechish. Shartga ko‘ra $\vec{a}=(2;\ -1;\ 2)$. (19) formuladan $|\vec{a}|=\sqrt{2^2+(-1)^2+2^2}=\sqrt{4+1+4}=\sqrt{9}=3$. *Javob*: 3.

3) $\vec{a}=(3;\ 2;\ -6)$ vektorga qarama-qarshi yo'nalgan $\vec{e}(e_1;\ e_2;\ e_3)$ birlik vektorni toping.

Yechish: (19) formulaga ko'ra $|\vec{a}|=\sqrt{9+4+36}=\sqrt{49}=7$.

(22) ga asosan $e_1=-\frac{3}{7};\ e_2=-\frac{2}{7};\ e_3=\frac{6}{7}$; *Javob*: $\vec{e}=(-\frac{3}{7};\ -\frac{2}{7};\ =\frac{6}{7})$.

4) $\vec{a}(5;1;6)$ va $\vec{b}(-4;3;-1)$ bo'lsa, $\vec{c}=4\vec{a}+3\vec{b}$ vektor koordinatalarini toping.

Yechish: (25) ga ko'ra: $4\vec{a}(20;4;24)$, $3\vec{b}(-12;\ 9;\ -3)$. Bundan

(23) ga asosan: $\vec{c}=\{20+(-12);4+9;\ 24+(-3)\}=\{8;13;21\}$.

Javob: $\vec{c}(8;13;21)$.

5) $\vec{a}(1;\ 2;\ 3)$, va $\vec{b}(2;\ -3;\ 0)$ bo'lsa, $\vec{c}=2\vec{a}-5\vec{b}$ vektor modulini toping.

Yechish: (25) ga ko'ra: $2\vec{a}(2;4;6)$, $5\vec{b}(10;\ -15;\ 0)$.

Bundan (24) ga asosan:

$\vec{c}=\{2-10;\ 4-(-15);6-0\}=\{-8;19;6\}$

(19) formulaga ko'ra:

$|\vec{c}|=\sqrt{(-8)^2+19^2+6^2}=\sqrt{64+361+36}=\sqrt{461}$.

Javob: $\sqrt{461}$.

6) $\vec{a}(6;\ 2;-5)$ va $\vec{b}(1;\ -1;\ -5)$ bo'lsa, $\vec{a}(1;\ 2;\ 3)$, va $\vec{a}\cdot\vec{b}=?$

Yechish: (26) ga asosan:

$\vec{a}\cdot\vec{b}=6\cdot1+2\cdot(-1)+(-5)\cdot(-5)=6-2+25=29$. *Javob*: 29.

7) $\vec{a}(x+2;\ 13;\ 4)$, $\vec{b}(x+1;\ -2;\ 1-x)$ va $\vec{a}\perp\vec{b}$ bo'lsa, x=?

Yechish:

$\vec{a}\perp\vec{b}\Rightarrow\vec{a}\cdot\vec{b}=0\Rightarrow(x+2)\cdot(x+1)+13\cdot(-2)+4\cdot(1-x)=0\Rightarrow$

$\Rightarrow x^2+2x+x+2-26+4-4x=0 \Rightarrow x^2-x-20=0 \Rightarrow x_1=-4, x_2=5.$

Javob: -4 va 5.

8) $A(1;-2;\ 2)$, $B(1;\ 4;\ 0)$, $C(-4;\ 1;\ 1)$ va $D(-5;-5;\ 3)$ nuqtalar berilgan. $\overrightarrow{AC}$ va $\overrightarrow{BD}$ vektorlar orasidagi burchakni toping.

Yechish: (18) ga ko'ra: $\overrightarrow{AC}=\{-4-1;\ 1-(-2);\ 1-2\}=\{-5;\ 3;\ -1\}$

va $\overrightarrow{BD}=\{-5-1;\ -5-4;\ 3-0\}=\{-6;-9;3\}$.

(27) ga ko'ra,

$$\cos\varphi=\frac{\overrightarrow{AC}\cdot\overrightarrow{BD}}{\left|\overrightarrow{AC}\right|\cdot\left|\overrightarrow{BD}\right|}=\frac{-5\cdot(-6)+3\cdot(-9)+(-1)\cdot 3}{\sqrt{(-5)^2+3^2+(-1)^2}\cdot\sqrt{(-6)^2+(-9)^2+3^2}}=$$

$$=\frac{0}{\sqrt{35}\cdot\sqrt{126}}=0.$$

Demak, $\cos\varphi=0 \Rightarrow \varphi=90^\circ$. *Javob*: 90°.

9) $|\vec{a}|=6, \left|\vec{b}\right|=\sqrt{2}$ va $\angle(\vec{a};\vec{b})=135^\circ$ bo'lsa $\left|3\vec{a}+2\vec{b}\right|=?$.

Yechish: $|\vec{a}|=\sqrt{\vec{a}^2}$ va $\vec{a}\cdot\vec{b}=|\vec{a}|\cdot\left|\vec{b}\right|\cdot\cos\varphi$ formulalardan foydalanamiz.

$$\left|3\vec{a}+2\vec{b}\right|=\sqrt{(3\vec{a}+2\vec{b})^2}=\sqrt{9\vec{a}^2+12\vec{a}\vec{b}+4\overline{b^2}}=$$

$$=\sqrt{9\cdot 6^2+12\cdot 6\cdot\sqrt{2}\cdot\cos 135^\circ+4(\sqrt{2})^2}=$$

$$=\sqrt{324+72\sqrt{2}\cdot\left(-\frac{1}{\sqrt{2}}\right)+4\cdot 2}=\sqrt{260}.$$ *Javob*: $\sqrt{260}$.

Mustaqil yechish uchun mashqlar

1) $A(3;-6;\ 6)$ va $B(11;-2;-4)$ bo'lsa, $\overrightarrow{AB}$ vektor koordinatalarini toping.

2) $\vec{a}=4\vec{i}-4\vec{j}+2\vec{k}$ vektor uzunligini toping.

3) $A(4;\ 5;-2)$, $B(x-4;\ x+1;\ 3)$ va $\overrightarrow{AB}=22$ bo'lsa, $x=?$

4) $\vec{a}(3;\ 2;-6)$ vektorga yo'nalishdosh $\vec{e}(e_1;e_2;e_3)$ birlik vektorni toping.

5) $\vec{a}(2;-1;3)$ va $\vec{b}(4;-5;2)$ bo'lsa, $\vec{c}=5\vec{a}+2\vec{b}$ vektor koordinatalarini toping.

6) $\vec{a}(5;2;-3)$ va $\vec{b}(4;-2;10)$ bo'lsa, $\vec{c}=4\vec{a}-2\vec{b}$ vektor modulini toping.

7) $\vec{a}(5;3;-6)$ va $\vec{b}(10;-2;-5)$ bo'lsa, $\vec{a}\cdot\vec{b}=?$

8) $\vec{a}(y+1;3{,}5;2)$, $\vec{b}(y-1;-2;y)$ va $\vec{a}\perp\vec{b}$ bo'lsa, y=?

9) $A(2;-3;5)$, $B(1;-4;3)$, $C(-2;-3;6)$, 6) va $D(-3;-3;2)$ 2) nuqtalar berilgan.

$\overrightarrow{AC}$ va $\overrightarrow{BD}$ vektorlar orasidagi burchakni toping.

10) $\vec{a}(m+1;n-2;4)$, $\vec{b}(3;5;-2)$ va $\vec{a}\parallel\vec{b}$ bo'lsa, $m+n$=?

2. Vektorlarga doir testlardan namunalar

1) Agar $\vec{m}$ va $\vec{n}$ vektorlar 30° li burchak tashkil etsa va $\vec{m}\cdot\vec{n}=6\sqrt{3}$ bo'lsa, bu vektorlarga qurilgan parallelogramm yuzini toping.

A) 6; B) $6\sqrt{3}$; C) 18; D) $3\sqrt{3}$.

Yechish: (17) ga ko'ra $S=\vec{m}\vec{n}\,tg30°=6\sqrt{3}\cdot\frac{1}{\sqrt{3}}=6$. *Javob*: A.

2) n ning qanday qiymatlarida $\vec{a}(n;-3;1)$ va $\vec{b}(n;3n;-10)$ vektorlar o'zaro perpendikular bo'ladi.

A) O; B) −10; C) 10; D) −1, 10.

Yechish: $\vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0 \Rightarrow n \cdot n + (-3) \cdot 3n + 1 \cdot (-10) = 0 \Rightarrow$

$\Rightarrow n^2 - 9n - 10 = 0 \Rightarrow n_1 = -1,\ n_2 = 10.$ *Javob*: D.

3) Agar $\vec{a}(-1;\ 6;\ 3)$ va $\vec{b}(4;\ -5;\ 2)$ bo‘lsa, $\vec{c} = 4\vec{a} + 2\vec{b}$ vektor modulini toping.

A) $4\sqrt{117}$; B) $2\sqrt{117}$; C) $6\sqrt{117}$; D) $8\sqrt{117}$.

Yechish: (25) ga ko‘ra $4\vec{a}(-4;\ 24;\ 12)$ va $2\vec{b}(8, -10, 4)$,

(27) ga ko‘ra esa $\vec{c} = \{-4+8;\ 24+(-10);12+4\} = \{4;\ 14;16\}$ bo‘ladi.

(19) ga asosan: $|\vec{c}| = \sqrt{4^2 + 14^2 + 16^2} = \sqrt{468} = 2\sqrt{117}$. *Javob*: B.

4) $\overrightarrow{AB}(1;\ 2;\ 3)$ va $\overrightarrow{AC}(3;-10;\ 5)$ vektorlar ABC uchburchakning tomonlaridir. Shu uchburchakning AN medianasi uzunligini toping.

A) 4; B) 5; C) 6; D) 7.

Yechish: (4) ga ko‘ra $\overrightarrow{AN} = \left\{\frac{1+3}{2};\ \frac{2+(-10)}{2};\ \frac{3+5}{2}\right\} = \{2;-4;\ 4\}$.

(19) ga ko‘ra esa $|\overrightarrow{AN}| = \sqrt{2^2 + (-4)^2 + 4^2} = \sqrt{36} = 6$. *Javob*: C.

5) $\vec{a}(2;\ 6;-6)$ vektor va $M(\frac{1}{2};-\frac{5}{6};\frac{3}{2})$ nuqta berilgan. Agar $\vec{a} + 3\overrightarrow{NM} = 0$ bo‘lsa, N nuqta koordinatalarini toping.

A) $\frac{7}{6};-\frac{7}{6};\frac{7}{6}$; B) $\frac{1}{3};-\frac{7}{6};\frac{11}{6}$; C) $-\frac{7}{6};\frac{7}{6};-\frac{7}{6}$; D) $\frac{7}{6};\frac{7}{6};\frac{7}{6}$.

Yechish: $N(x;\ y;\ z)$ bo‘lsin, u holda $\overrightarrow{MN}(x-\frac{1}{2};y-(-\frac{5}{6});\ z-\frac{3}{2} \Rightarrow$

$\Rightarrow 3\overrightarrow{MN}\left(3x-\frac{3}{2};\ 3y+\frac{5}{2};\ 3z-\frac{9}{2}\right)$. $\vec{a} + 3\overrightarrow{MN} = 0 \Rightarrow \vec{a} = 3\overrightarrow{MN}$.

Bundan: $2 = 3x - \frac{3}{2} \Rightarrow x = \frac{7}{6}$

$6=3y+\frac{5}{2}\Rightarrow y=\frac{7}{6}$

$-1=3z-\frac{9}{2}\Rightarrow z=\frac{7}{6}$ demak: $N(\frac{7}{6};\frac{7}{6};\frac{7}{6})$. *Javob*: D.

6) $\vec{a}=3\vec{i}-\vec{j}$ va $b=2\vec{j}+\vec{k}$ vektorlarda yasalgan parallelogrammning diagonallari orasidagi burchakni toping.

A) 45°; B) $-\arccos\frac{5}{\sqrt{209}}$; C) $\pi-\arccos\frac{5}{\sqrt{209}}$; D) $\arccos\frac{5}{\sqrt{209}}$.

Yechish: Shartdan: $\vec{a}(3;-1;0)$ va $\vec{b}(0;2;1)$.). Parallelogramm usuliga ko'ra $\overrightarrow{d_1}=\vec{a}+\vec{b}=\{3;1;1\}$ ga asosan: $\overrightarrow{d_2}=\vec{a}-\vec{b}=\{3;-3;-1\}$. (27) ga asosan:

$$\cos\varphi=\frac{3\cdot3+1\cdot(-3)+1\cdot(-1)}{\sqrt{3^2+1^2+1^2}\cdot\sqrt{3^2+(-3)^2+(-1)^2}}=\frac{5}{\sqrt{11}\cdot\sqrt{19}}=\frac{5}{\sqrt{209}}$$

bundan: $\varphi=\arccos\frac{5}{\sqrt{209}}$. *Javob*: D.

7) $\vec{a}=3\vec{i}-2\vec{j}+6\vec{k}$ va $\vec{b}=2\vec{i}-2\vec{j}+\vec{k}$ vektorlar orasidagi burchak α bo'lsa, $\sin^2\frac{\alpha}{2}$ ning qiymatini hisoblang.

A) $\frac{5}{42}$ B) $\frac{1}{42}$ C) $\frac{11}{42}$ D) $\frac{13}{42}$

Yechish: Shartdan: $\vec{a}(3;-2;6)$ va $\vec{b}(2,-2;1)$. (27) ga asosan:

$$\cos\varphi=\frac{3\cdot2+(-2)\cdot(-2)+6\cdot1}{\sqrt{3^2+(-2)^2+6^2}\cdot\sqrt{2^2+(-2)^2+1^2}}=\frac{16}{\sqrt{49}\cdot\sqrt{9}}=\frac{16}{21}.$$

Bundan: $\sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2}=\frac{1-\frac{16}{21}}{2}=\frac{5}{42}$. *Javob*: A.

8) Agar $|\vec{a}|=2$, $|\vec{b}|=5$ va $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\frac{\pi}{3}$ ga teng bo'lsa, $2\vec{a}-3\vec{b}$ va $5\vec{a}+3\vec{b}$ vektorlarning skalar ko'paytmasini toping.

A) -145; B) -155; C) -165; D) -175.

Yechish: $\overrightarrow{a^2}=|\vec{a}|^2$ va $\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\cos\varphi$ formulalardan foydalanamiz.

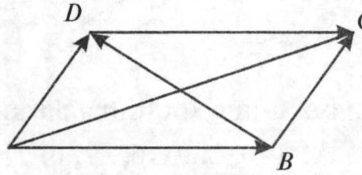
$\vec{a}^2|\vec{a}|^2=2^2=4$. $\vec{b}^2|\vec{b}|^2=5^2=25$. $\vec{a}\cdot\vec{b}=2\cdot5\cdot\cos\frac{\pi}{3}=10\cdot\frac{1}{2}=5$.

Bundan: $(2\vec{a}-3\vec{b})\cdot(5\vec{a}+2\vec{b})=10\vec{a}^2-11\vec{a}\cdot\vec{b}-6\vec{b}^2=$

$=10\cdot4-11\cdot5-6\cdot25=-165$. *Javob*: C.

9) Agar $A(-5;2;3)$, $\overrightarrow{AB}(2;-5;4)$ va $\overrightarrow{BD}(-1;-3;5)$ bo'lsa, $ABCD$ parallelogramm C uchining koordinatalari yig'indisini toping.

A) 3; B) 4;
C) 5; D) 6.



34-rasm.

Yechish: $\overrightarrow{AB}=\overrightarrow{DC}$ ekanligidan (2) ga ko'ra(34-rasm) $\overrightarrow{AC}$ vektor koordinatalarini topamiz:

$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BD}+\overrightarrow{DC}=\{2+(-1)+2;\ -5+(-3)+(-5);\ 4+5+4\}=$

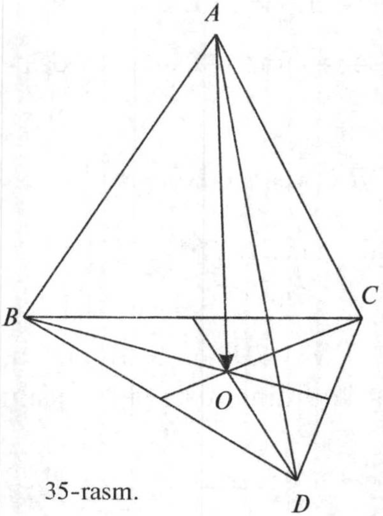
$=\{3;-13;13\}$. $C(x;y;z)$ bo'lsin, u holda $x-(-5)=3$, $y-2=-13$, $z-3=13$ bo'ladi.

Bu tengliklardan: $x=-2$, $y=-11$, $z=16$ va $x+y+z=3$.
Javob: *A*.

10) Uchburchakli piramida uchlari A(5; 2; 0), B(1;−2; 4), C(2; 4; −1), D(3;−2; 6) nuqtalarda joylashgan. O nuqta BCD uchburchak medianalari kesishgan nuqtasi. $\overrightarrow{AO}$ vektorning uzunligini toping.

A) $\sqrt{29}$; B) $\sqrt{21}$; C) $\sqrt{23}$; D) $\sqrt{22}$.

Yechish: Dastlab O nuqta koordinatalarini topamiz



35-rasm.

$x=\frac{1+2+3}{3}=2,\ y=\frac{-2+4+(-2)}{3}=0,$

$z=\frac{4+(-1)+6}{3}=3.$

Demak: $O(2;\ 0;\ 3)$ (20) ga ko‘ra (35-rasm)

$|\overrightarrow{AO}|=\sqrt{(2-5)^2+(0-2)^2+(3-0)^2}=$

$=\sqrt{9+4+9}=\sqrt{22}.$

Javob: D.

11) *x0y* tekislikda yotgan $\vec{b}$ vektor $\vec{a}(1;-3;5)$ vektorga perpendikular va $|\vec{b}|=2\sqrt{10}$ bo‘lsa, uning koordinatalarini toping.

A) $(-6;2;0)\ (6;-2;0)$; B) $(-6;-2;0)\ (6;2;0)$;

C) $(-2;6;0)\ (2;-6;0)$; D) $(2;6;0)\ (1;1;1)$.

Yechish: $\vec{b}(x;\ y;\ 0)$ bo‘lsin.(19) va (26) larga ko‘ra

$$\begin{cases}\sqrt{x^2+y^2}=2\sqrt{10}\\ 1x-3y+5\cdot 0=0\end{cases}\Rightarrow\begin{cases}x^2+y^2=40\\ x=3y\end{cases}\Rightarrow\begin{cases}(3y)^2+y^2=40\\ x=3y\end{cases}\Rightarrow\begin{cases}y^2=4\\ x=3y\end{cases}$$

bundan: $x_1=-6,\ y_1=-6,\ x_2=6,\ y_2=2$ demak: $\vec{b}(-6;-2;0)$ yoki $\vec{b}(6;\ 2;\ 0)$. *Javob*: B.

12) $A(3;-1;3)$, $B(2;1;4)$ va $C(3;2;0)$ nuqtalar berilgan. *0y* o‘qda shunday D nuqta topingki, $\overrightarrow{AC}\perp\overrightarrow{BD}$ bo‘lsin.

A) $(0;2;0)$; B) $(0;3;0)$; C) $(0;-3;0)$; D) $(0;-4;0)$.

Yechish: $D(0;\ y;\ 0)$ bo‘lsin. (18) ga ko‘ra

$\overrightarrow{AC}=\{3-3;\ 2-(-1);\ 0-3\}=\{0;\ 3;-3\}$ va

$\overrightarrow{BD}=\{0-2;\ y-1;\ 0-4\}=\{-2;\ y-1;-4\}$

shartga ko'ra: $\overrightarrow{AC}\cdot\overrightarrow{BD}=0 \Rightarrow 0\cdot(-2)+3\cdot(y-1)+(-3)\cdot(-4)=0 \Rightarrow$

$\Rightarrow\ 0+3y-3+12=0 \Rightarrow 3y=-9 \Rightarrow y=-3$. Demak, $D(0;\ -3;\ 0)$.
Javob: *C*.

13) $\vec{a}(2;\ 0)$ va $\vec{b}(2;\ 2)$ vektorlar berilgan. λ ning qanday qiymatlarida $3\vec{a}+\lambda\vec{b}$ va $\vec{b}$ vektorlar o'zaro perpendikular bo'ladi?

A) $-1{,}5$; B) $1{,}5$; C) $-2{,}5$; D) $2{,}5$.

Yechish: (25) ga ko'ra: $3\vec{a}(6;\ 0)$ va $\vec{b}(2\lambda;\ 2\lambda)$.

(23) ga ko'ra esa

$3\vec{a}+\lambda\vec{b}=\{3\cdot2+2\lambda;\ 3\cdot0+2\lambda\}=\{6+2\lambda;\ 2\lambda\}$ bo'ladi.

Shartga asosan: $(3\vec{a}+\lambda\vec{b})\cdot\vec{b}=0 \Rightarrow 2\cdot(6+2\lambda)+2\cdot2\lambda=0 \Rightarrow$

$\Rightarrow 8\lambda=-12 \Rightarrow \lambda=-1{,}5$. *Javob*: A.

14) $\overrightarrow{AB}=(0;\ 1;\ 3{,}5)$ va $\overrightarrow{BC}=(2;\ 7;\ 12{,}5)$ vektorlar parallelogrammning qo'shni tomonlari. Uning AC va BD diagonallari orasidagi burchakni toping.

A) $\arccos\left(-\frac{98}{98}\right)$; B) $\arccos\frac{98}{99}$; C) $\arccos\frac{94}{99}$; D) $\arccos\frac{46}{99}$.

Yechish: $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=\{2;\ 8;\ 16\}$, $\overrightarrow{BA}=-\overrightarrow{AB}\,(0;-1;-3{,}5)$,

$\overrightarrow{BD}=\overrightarrow{BA}+\overrightarrow{BC}=\{2;\ 6;\ 9\}$

(27) ga asosan

$$\cos\varphi=\frac{2\cdot2+8\cdot6+16\cdot9}{\sqrt{2^2+8^2+16^2}\sqrt{2^2+6^2+9^2}}=\frac{4+48+144}{\sqrt{324}\cdot\sqrt{121}}=\frac{196}{18\cdot11}=\frac{98}{99}.$$

Bundan $\varphi=\arccos\frac{98}{99}$. *Javob*: *B*.

15) $|\vec{a}|=3$, $|\vec{b}|=5$ va $\angle(\vec{a},\vec{b})=60°$. λ ning qanday qiymatlarida $2\vec{a}+\lambda\vec{b}$ va $\vec{a}$ vektorlar o‘zaro perpendikular bo‘ladi?

A) $-3,2$; B) $4,2$; C) $-4,2$; D) $-2,4$.

Yechish: Shartga ko‘ra $(2\vec{a}+\lambda\vec{b})\cdot\vec{a}=0 \Rightarrow 2\vec{a}^2+\lambda\vec{a}\cdot\vec{b}=0 \Rightarrow$

$\Rightarrow 2\cdot 3^2\lambda\cdot 3\cdot 5\cdot\cos 60°=0 \Rightarrow 18+7,5\lambda=0 \Rightarrow \lambda=-2,4$. *Javob*: D.

3. Mustaqil yechish uchun mashqlar

1. $\vec{a}(1;-2;3)$ vektorning oxiri $B(2;0;4)$ nuqta bo‘lsa, bu vektor boshining koordinatalarini toping.

A) (1; 2; 1); B) (−1; 2; 1); C) (1;−2; 1); D) (1; 2;−1).

2. $A(3;-2;5)$ va $B(-4;5;-2)$ nuqtalar berilgan. $\overrightarrow{BA}$ vektorning koordinatalarini toping.

A) $(7;-7;-7)$; B) $(-1;3;3)$; C) $(-7;7;-7)$; D) $(7;-7;7)$.

3. Agar $A(-5;2;8)$, $\overrightarrow{AB}(-3;4;1)$ va $\overrightarrow{BD}(-2;4;1)$ bo‘lsa, $ABCD$ parallelogramm C uchining koordinatalari yig‘indisini toping.

A) 8; B) 10; C) 12; D) 11.

4. Uchlari $A(0;0)$, $B(4;3)$ va $C(6;8)$ nuqtalarda bo‘lgan uchburchakning A burchagini toping.

A) $\arccos 0,9$; B) $\frac{\pi}{18}$; C) $\frac{\pi}{36}$; D) $\arccos 0,96$.

5. $\vec{a}+\vec{b}$ vektor $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchakni teng ikkiga bo‘ladi. $\vec{a}+\vec{b}$ va $\vec{a}-\vec{b}$ vektorlar orasidagi burchakni toping.

A) $\frac{\pi}{2}$; B) $\frac{\pi}{4}$; C) $\frac{\pi}{3}$; D) $\frac{\pi}{6}$.

6. $\vec{a}(4;1)$ va $\vec{b}(-2;2)$ vektorlar berilgan. $\vec{a}=\vec{c}+3\vec{b}$ bo‘lsa, $\vec{c}$ vektorning koordinatalarini toping.

A)$(-2;5)$; B)$(10;-5)$; C)$(-10;4)$; D)$(2;-5)$.

7. $\vec{a}(-2;1;4)$ vektor va $M(1;0;-1)$ nuqta berilgan. Agar $2\vec{a}+3\overrightarrow{MN}=0$ bo‘lsa, N nuqtaning koordinatalarini toping.

A) $-\frac{1}{3};\frac{2}{3};\frac{3}{5}$; B) $\frac{2}{3};-\frac{7}{3};-\frac{11}{3}$; C) $\frac{2}{3};-\frac{3}{7};-\frac{11}{3}$; D) $\frac{7}{3};-\frac{2}{3};-\frac{11}{3}$.

8. $\vec{a}(8;6)$ vektor $\vec{b}$ va $\vec{c}$ vektorlarga yoyilgan. Agar $\vec{a}=\mu\vec{b}+\lambda\vec{c}$, $\vec{c}(10;-3)$ va $\vec{b}(-2;1)$ bo‘lsa, $\mu\cdot\lambda$ ning qiymatini aniqlang.

A) 120; B) 105; C) 110; D) 115.

9. $\vec{a}(-3;2;-4)$ va $\vec{b}(4;3;-2)$ lar teng yonli uchburchakning $C(-6;4;3)$ uchidan tushirilgan vektorlar. Shu uchburchakning C uchidan CD balandlik tushirilgan. D nuqtaning koordinatalari yig‘indisini toping.

A) -1; B) 1; C) $-2{,}5$; D) 2,5.

10. $\vec{a}(1;\frac{4}{3})$ vektor berilgan. $3\cdot\vec{a}$ vektorning modulini toping.

A) 4,5; B) 3,5; C) 5; D) 5,5.

11. Agar $\vec{a}(1;2;3)$ va $\vec{b}(4;-2;9)$ bo‘lsa, $\vec{c}=\vec{a}+\vec{b}$ vektorning uzunligini toping.

A) $5\sqrt{3}$; B) $4\sqrt{3}$; C) 13; D) 11.

12. $\vec{a}(x;1;2)$ vektorning uzunligi 3 ga teng. x ning qiymatini toping.

A) 2; B) ±2; C) 0; D) 1.

13. $A(1;0;1)$, $B(-1;1;2)$ va $C(0;2;-1)$ nuqtalar berilgan. Koordinatalar boshi O nuqtada joylashgan. Agar $\overrightarrow{AB}+\overrightarrow{CD}=0$ bo‘lsa, $\overrightarrow{OD}$ vektorning uzunligini toping.

A) 4; B) 2; C) 9; D) 3.

14. $A(2;4)$, $B(3;6)$ va $C(6;14)$ nuqtalar berilgan. $|\overrightarrow{AB}+\overrightarrow{AC}|$ ni hisoblang.

A) 13; B) 12; C) 10; D) 14.

15. Muntazam uchburchak ichida olingan nuqtadan uchburchak tomonlarigacha bo'lgan masofalar mos ravishda $\vec{a}(1;\ 2;\ 3)$, $\vec{b}(1;\ 2;\ 1)$ va $\vec{c}(2;\ 3;\ 1)$ vektorlarning absolut qiymatlariga teng bo'lsa, uchburchakning balandligini toping.

A) $2\sqrt{14}+\sqrt{6}$; B) 18; C) $\sqrt{14}+\sqrt{6}$; D) 16.

16. $\vec{x}$ va $\vec{y}$ vektorlarning uzunliklari 11 va 23 ga, bu vektorlar ayirmasining uzunligi 30 ga teng. Shu vektorlar yig'indisining uzunligini toping.

A) 34; B) 64; C) 42; D) 20.

17. $\overrightarrow{AB}(-3;0;2)$ va $\overrightarrow{AC}(7;-2;2)$ vektorlar ABC uchburchakning tomonlaridir. Shu uchburchakning AN medianasi uzunligini toping.

A) 2,5; B) 3; C) $3\sqrt{6}$; D) 1,5.

18. Agar $|\vec{a}|=\sqrt{137}$, $|\vec{a}+\vec{b}|=20$ va $|\vec{a}+\vec{b}|=18$ bo'lsa, $|\vec{b}|$ ni toping.

A) $7\sqrt{2}$; B) $7\sqrt{3}$; C) $8\sqrt{2}$; D) 15.

19. $\vec{a}(2;-3;\ 4)$ va $\vec{b}(-2;-3;\ 1)$ vektorlarning skalar ko'paytmasini toping.

A) 9; B) 17; C) 13; D) 4.

20. $\vec{a}$ va $\vec{b}$ vektorlar 45° li burchak tashkil qiladi va $\vec{a}\cdot\vec{b}=4$. Shu vektorlarga qurilgan uchburchakning yuzini hisoblang.

A) 4; B) $2\sqrt{2}$; C) $4\sqrt{2}$; D) 2.

21. $\vec{a}$ va $\vec{b}$ vektorlar 30° li burchak tashkil qiladi va $\vec{a}\cdot\vec{b}=\sqrt{3}$. Shu vektorlarga qurilgan parallelogrammning yuzini hisoblang.

A) 2; B) $\frac{\sqrt{3}}{2}$; C) 1; D) $2\sqrt{3}$.

22. $\vec{a}$ va $\vec{b}$ birlik vektorlar orasidagi burchak 30°. $\left|\vec{a}+\vec{b}\right|$ni toping.

A) $\sqrt{2+\sqrt{3}}$; B) $\sqrt{3}$; C) $\sqrt{2-\sqrt{3}}$; D) 1.

23. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak 120°. Agar $|\vec{a}|=3$ va $|\vec{b}|=5$ bo‘lsa, $\left|\vec{a}-\vec{b}\right|$ning qiymati qanchaga teng bo‘ladi?

A) 2; B) 8; C) 7; D) 6.

24. $\vec{m}$, $\vec{n}$ va $\vec{p}$ birlik vektorlar berilgan. Agar $\vec{m}$ vektor $\vec{n}$ ga, $\vec{n}$ vektor $\vec{p}$ ga perpendikular bo‘lib, $\vec{p}$ va $\vec{m}$ vektorlar orasidagi burchak 60° ga teng bo‘lsa, $(2\vec{m}+\vec{p})(\vec{m}+2\vec{n})$ skalar ko‘paytmaning qiymatini toping.

A) 2; B) 2,2; C) 2,4; D) 2,5.

25. $\vec{a}(x;1;-1)$ va $\vec{b}(1;0;1)$ vektorlar uchun $(\vec{a}+3\vec{b})^2=(\vec{a}-2\vec{b})^2$ tenglik o‘rinli bo‘lsa, x ni toping.

A) 0; B) 1; C) −1; D) 0,5.

26. $\vec{p}$ va $\vec{q}$ vektorlar o‘zaro 60° burchak tashkil etadi. Agar $|\vec{p}|=1$, $|\vec{q}|=3$ bo‘lsa, $|2\vec{p}-\vec{q}|\cdot\sqrt{7}$ ni hisoblang.

A) 7; B) $2\sqrt{7}$; C) $3\sqrt{7}$; D) 14.

27. Ikki vektor yig‘indisining uzunligi 20 ga, shu vektorlar ayirmasining uzunligi 12 ga teng. Shu vektorlarning skalar ko‘paytmasini toping.

A) 16; B) 48; C) 24; D) 64.

28. $\vec{a}(2;\sqrt{2})$ va $\vec{b}(4;2\sqrt{2})$ vektorlar orasidagi burchakni toping.

A) $\frac{\pi}{4}$; B) $\frac{\pi}{3}$; C) 0 ; D) $\frac{\pi}{2}$.

29. $\vec{a}=\{1;2;1\}$ va $\vec{b}=\{2;-1;0\}$; α esa $\vec{a}+\vec{b}$ va $\vec{a}-\vec{b}$ vektorlar orasidagi burchak $\operatorname{ctg}^2\alpha$ ni hisoblang.

A) $\frac{1}{5}$; B) $\frac{1}{25}$; C) $\frac{1}{60}$; D) $\frac{1}{120}$.

30. $\vec{a}=(-4;2;4)$ va $\vec{b}=(\sqrt{2};-\sqrt{2};0)$ vektorlar berilgan bo‘lsa, $2\vec{a}$ va $\frac{\vec{b}}{2}$ vektorlar orasidagi burchakni toping.

A) $\arccos\frac{2}{3}$; B) $\frac{3\pi}{4}$; C) $\arccos\frac{5}{6}$; D) $\frac{\pi}{3}$.

31. $\overrightarrow{AB}=(3;4)$ va $\overrightarrow{AD}=(4;3)$ vektorlar parallelogrammning tomonlari bo‘lsa, uning diagonallari orasidagi burchakni toping.

A) 45°; B) 60°; C) 90°; D) 135°.

32. $A(1;-2;2)$, $B(1;4;0)$, $C(-4;1;1)$ va $D(-5;-5;3)$ nuqtalar berilgan. $\overrightarrow{AC}$ va $\overrightarrow{BD}$ vektorlar orasidagi burchakni toping.

A) 90°; B) 60°; C) 30°; D) 45°.

33. Uchlari $M(-3;3;1)$; $N(3;-5;1)$; va $E(-4;-1;-2)$ nuqtalarda bo‘lgan uchburchakning MN tomoni va EF medianasi orasidagi burchakni toping.

A) 45°; B) $\arccos 0{,}64$; C) 60°; D) $\arccos 0{,}48$.

34. $|\vec{a}|=3$, $|\vec{b}|=4$, $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak 60°. λ ning qanday qiymatida $(\vec{a}-\lambda\vec{b})$ vektor $\vec{a}$ vektorga perpendikular bo‘ladi?

A) 1; B) 2 ; C) 3; D) 1,5.

35. m ning qanday qiymatida $\vec{a}(1; m; -2)$ va $\vec{b}(m; 3; -4)$ vektorlar perpendikular bo‘ladi?

A) 2; B) −2; C) 4; D) −4.

36. Agar $|\vec{a}|=3$, $|\vec{b}|=5$ bo‘lsa, α ning qanday qiymatlarida $\vec{a}+\alpha\vec{b}$ vektor va $\vec{a}-\alpha\vec{b}$ vektorlar o‘zaro perpendikular bo‘ladi?

A) $-\frac{3}{5}<\alpha<\frac{3}{5}$; B) $-\frac{3}{5}$; C) $\frac{3}{5}$; D) $\pm\frac{3}{5}$.

37. Agar $\vec{m}$ va $\vec{n}$ o‘zaro perpendikular birlik vektorlar bo‘lsa, $\vec{a}=2\vec{m}+\vec{n}$ vektorning uzunligini toping.

A) 2; B) 3; C) $\sqrt{5}$; D) $\sqrt{3}$.

38. $A(2; 1)$ va $B(1; 2)$ nuqtalar berilgan. AB to‘g‘ri chiziqqa perpendikular va B nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

A) $x+y+2=0$; B) $x+y-3=0$; C) $x-y-4=0$; D) $x+y-4=0$.

39. $\vec{e}_1$ va $\vec{e}_2$ birlik vektorlar va $\vec{e}_1\perp\vec{e}_2$ bo‘lsa, $\left|\vec{e}_1-\frac{2(\vec{e}_1+2\vec{e}_2)}{5}\right|$ ni hisoblang.

A) 1; B) 2; C) 3; D) $\frac{1}{2}$.

40. x ning qanday qiymatlarida $\vec{a}(3; 1; 6)$ va $\vec{b}(6; 3; x)$ vektorlar parallel bo‘ladi?

A) barcha qiymatlarida; B) hech qanday qiymatida;
C) 18; D) 12.

41. $\vec{b}(3;-6; 6)$ vektorga kollinear va $\vec{a}\cdot\vec{b}=27$ tenglikni qanoatlantiruvchi $\vec{a}$ vektorni toping.

A) $\vec{a}(1;-2;-2)$; B) $\vec{a}(1; 2; 3)$; C) $\vec{a}(\frac{1}{2};-1; 1)$; D) $\vec{a}(1;-2; 2)$.

42. Qaysi m va n larda $\vec{a}(-2; m;-2)$ va $\vec{b}(-1; 3; n)$ vektorlar kollinear bo'ladi?

A) 3; −1; B) 3; 1; C) 6; −1; D) 6; 1.

43. $\vec{a}(2;-4)$, $\vec{b}(1; 2)$, $\vec{c}(1;-2)$ va $\vec{d}(-2;-4)$ vektorlardan qaysilari kollinear vektorlar?

A) $\vec{a},\vec{c}$; $\vec{b},\vec{d}$; B) $\vec{b},\vec{c}$; C) $\vec{a},\vec{d}$; D) $\vec{a},\vec{b}$.

44. $\vec{a}(1; 2;-1)$ va $\vec{b}(2; 2; 0)$ vektorlar berilgan. $\vec{c}(x; y;-6)$ vektor $2\vec{b}-3\vec{a}$ vektorga kollinear. $|\vec{c}|$ ning qiymatini toping.

A) $2\sqrt{14}$; B) 8; C) $2\sqrt{13}$; D) 13.

45. Uch o'lchovli fazoda $\vec{a}=\vec{i}+\vec{j}$ va $\vec{b}=\vec{j}+\vec{k}$ vektorlarga perpendikulyar birlik vektorning koordinatalarini ko'rsating.

A) $(1;-1; 1)$; B) $(-\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}})$;

C) $(-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}}),(\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}})$; D) $(\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}})$.

46. $\vec{i},\vec{j},\vec{k}$ koordinata o'qlari bo'ylab yo'nalgan birlik vektorlar va $\vec{a}=5\vec{i}+\sqrt{2}\vec{j}-3\vec{k}$ bo'lsa, $\vec{a}$ va $\vec{i}$ vektorlar orasidagi burchakning kosinusini toping.

A) $\frac{5}{6}$; B) $\frac{2}{3}$; C) $\frac{3}{4}$; D) $\frac{1}{2}$.

47. Ushbu $\overrightarrow{a_1}(1; 2)$, $\overrightarrow{a_2}(2; 3)$, $\overrightarrow{a_3}(3; 4)$,...,. vektorlar ketma-ketligining dastlabki nechta hadi koordinatalarining yig'indisi 120 ga teng bo'ladi?

A) 11; B) 10; C) 9; D) 8.

48. α ning qanday qiymatlarida $\vec{a}(\cos\alpha; \sin\alpha)$ va $\vec{b}(0; \cos\alpha)$ vektorlar o'zaro perpendikular bo'ladi?

A) $\pi n,\ n \in Z$; B) $\frac{\pi n}{2},\ n \in Z$; C) $\frac{\pi}{2}$; D) π.

49. $\overrightarrow{AB}(-0,5;-0,5;-3)$ va $\overrightarrow{BC}(2,5;\ 3,5;\ 9)$ vektorlar parallelogrammning qo'shni tomonlari. Uning AC va BD diagonallari orasidagi burchakni toping.

A) $\arccos\frac{54}{91}$; B) $\arccos\left(-\frac{90}{91}\right)$; C) $\arccos\frac{90}{91}$; D) $\arccos\frac{6}{7}$.

50. $\overrightarrow{AB}(-0,5;-4,5;-9)$ va $\overrightarrow{BC}(2,5;\ 7,5;\ 15)$ vektorlar parallelogrammning qo'shni tomonlari. Uning AC va BD diagonallari orasidagi burchakni toping.

A) $\arccos\frac{58}{63}$; B) $\arccos\left(-\frac{62}{63}\right)$; C) $\arccos\frac{34}{63}$; D) $\arccos\frac{62}{63}$.

Mustaqil yechish uchun mashqlarning javoblari

	0	1	2	3	4	5	6	7	8	9
		A	D	C	D	A	B	D	B	B
1	C	C	B	D	A	A	D	B	D	A
2	D	C	A	C	D	A	A	D	C	D
3	B	C	A	D	D	B	D	C	B	A
4	B	D	C	A	A	C	A	B	B	C
5	D									

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МУНДАРИЖА

VEKTORLAR

Abiturientlar hamda matematika o'qituvchilari uchun uslubiy qo'llanma

«ISTIQLOL» – Toshkent – 2015

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