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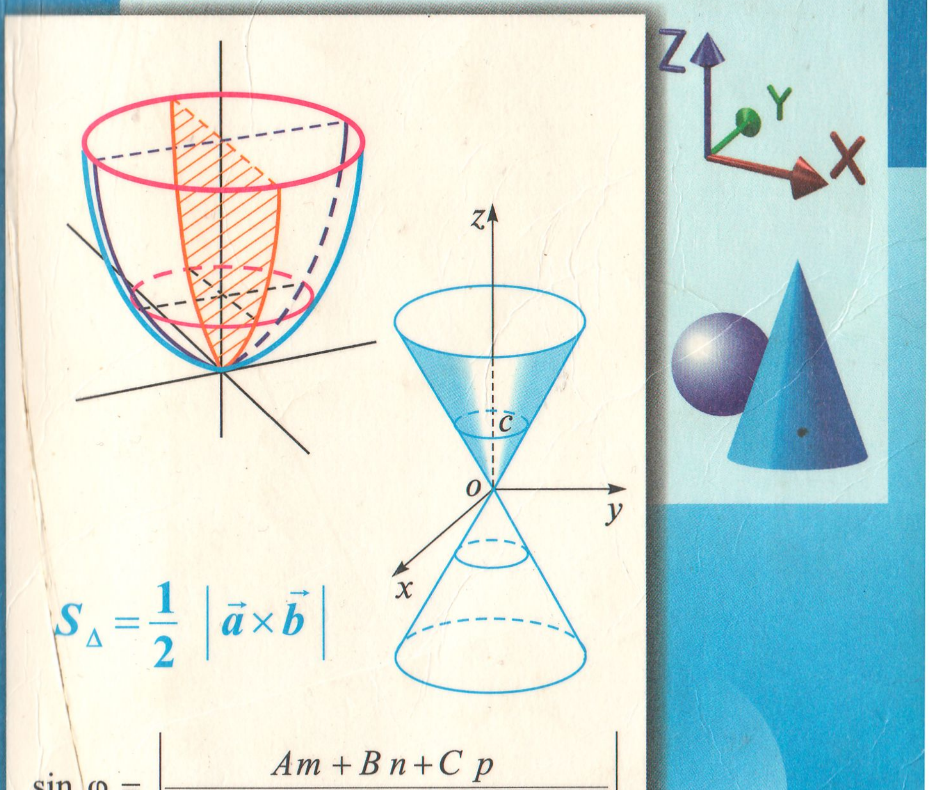

$$S_{\Delta}=\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$$

$$\sin\varphi=\left|\frac{Am+Bn+Cp}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{m^2+n^2+p^2}}\right|$$

O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

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CHIZIQLI ALGEBRA VA ANALITIK GEOMETRIYADAN MASALALAR YECHISH

O‘zbekiston Respublikasi Oliy va o‘rta maxsus ta’lim vazirligi oliy texnika o‘quv yurtlarining 5580100 — Arxitektura ixtisosliklari uchun «Oliy matematika» kursidan o‘quv qo‘llanma sifatida tavsiya etgan

«TURON-IQBOL»
TOSHKENT
2006

T a q r i z ch i l a r: **B. X. Xo'jayorov** — Samarqand iqtisodiyot va servis instituti oliy matematika kafedrasi mudiri, fizika-matematika fanlari doktori, professor.

E. Davronov — SamDAQI oliy matematika kafedrasi dotsenti, fizika-matematika fanlari nomzodi.

O'quv qo'llanma oliy texnika o'quv yurtlari talabalari uchun mo'ljallangan. Mazkur qo'llanma 5 bobdan iborat. Dastlabki uchta bobida chiziqli algebra elementlari, 4-bobda tekislikda analitik geometriya, 5- bobda fazoda analitik geometriya qaralgan bo'lib, birinchi kursda amaliy mashg'ulot darslarida o'tiladigan «Oliy matematika» kursini o'z ichiga oladi.

Har bir paragraf boshida zarur bo'lgan qisqacha nazariy tushunchalar, keltirilgan misol-masalalar yetarlicha sharhlar bilan yechib ko'rsatilgan. Paragraf oxirida talabalarning mustaqil shug'ullanishlari uchun misol-masalalar berilgan. Ularning javoblari har bir bobning oxirida keltirilgan.

O $\frac{1602050000-46}{M361(04)-2006}$ 2006

ISBN 978-9943-14-006-6

SO'ZBOSHI

Mazkur «Chiziqli algebra va analitik geometriyadan masalalar yechish» o'quv qo'llanmasi oliy texnika o'quv yurtlari talabalariga mo'ljallab yozilgan bo'lib, undan boshqa ixtisoslikdagi o'quv yurtlari talabalari ham foydalanishlari mumkin.

Qo'llanmada keltirilgan mavzular oliy texnika o'quv yurtlarining barcha mutaxassisliklari uchun oliy matematika fanining hozirgi paytdagi dasturiga mos keladi. U talabalar va o'qituvchilar uchun amaliy mashg'ulotlar darslarida hamda mustaqil o'rganishda foydali qo'llanma bo'lib xizmat qiladi deb umid qilamiz.

Unda har bir mavzuga doir nazariy ma'lumotlar berilgan va tipik misollar yechib ko'rsatilgan hamda mustaqil bajarish uchun yetarlicha mashqlar berilgan. Ularning javoblari esa har bir bob uchun alohida keltirilgan

Mualliflar Samarqand iqtisodiyot va servis instituti «Oliy matematika» kafedrasi mudiri, fizika-matematika fanlari doktori, professor B.X. Xo'jayorov va SamDAQI «Oliy matematika» kafedrasi dotsenti E.D. Davronovga xolisona taqrizlari uchun hamda magistrant B. Mardonovga matnni kompyuterda tayyorlashdagi yordami uchun o'z minnatdorchiklarini bildiradilar.

Qo'llanma bo'yicha hamkasblarimizning fikr-mulohazalarini minnatdorchilik bilan qabul qilamiz.

Mualliflar

I bob. DETERMINANTLAR, MATRITSALAR VA CHIZIQLI TENGLAMALAR SISTEMALARI

1- §. Determinantlar

1⁰. Ikkinchi va uchinchi tartibli determinantlar. To'rtta sondan tuzilgan

$$A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$$

jadval *ikkinchi tartibli kvadrat matritsa*, $a_1b_2 - a_2b_1$ son esa bu matritsaning *determinanti* yoki *ikkinchi tartibli determinant* deyiladi. U quyidagicha belgilanadi:

$$\det A = |A| = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1, \qquad (1)$$

bu yerda: a_1, a_2, b_1, b_2 — determinantning elementlari; ulardan a_1, b_1 va a_2, b_2; a_1, a_2 va b_1, b_2; a_2, b_2 va a_2, b_1 lar, mos ravishda, *birinchi va ikkinchi satrlar, birinchi va ikkinchi ustunlar; bosh va yordamchi diagonallar elementlari* deyiladi. Satr va ustunlar determinantning qatorlari ham deb aytiladi.Matritsalar haqida to'liqroq ma'lumot 3- § da beriladi.

1-misol. $\begin{vmatrix} 5 & 7 \\ 6 & 13 \end{vmatrix}$ determinantni hisoblang.

► $\begin{vmatrix} 5 & 7 \\ 6 & 13 \end{vmatrix} = 5 \cdot 13 - 6 \cdot 7 = 65 - 42 = 23.$ ◄

Determinantning xossalaridan foydalanish uni hisoblashni osonlashtiradi.

Determinantning xossalari:

1. Satrlarni mos ustunlar bilan almashtirilsa, determinantning qiymati o'zgarmaydi.

2. Ikkita parallel qator o'rinlari o'zaro almashtirilsa, determinantning faqat ishorasi o'zgaradi.

3. Biror qator elementlarining umumiy ko'paytuvchisini determinant belgisidan tashqariga chiqarish mumkin.

4. Determinantning biror qatori elementlarini noldan farqli songa ko'paytirib, unga parallel boshqa qatorning mos elementlariga qo'shilsa, determinantning qiymati o'zgarmaydi.

5. Quyidagi hollarda determinant nolga teng:
- biror qatori nollardan iborat bo'lsa;
- ikkita palallel qatori bir xil bo'lsa;
- ikkita parallel qatori elementlari proporsional bo'lsa.

Bu xossalar *istalgan tartibli determinant uchun ham o'rinlidir.*

2- misol. Determinantni hisoblang: $\begin{vmatrix} 4 & 1998 \\ -2 & 3996 \end{vmatrix}$

► $\begin{vmatrix} 4 & 1998 \\ -2 & 3996 \end{vmatrix} = 2 \cdot 1998 \cdot \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = 10 \cdot 1998 = 19980.$ ◄

Ushbu

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

ko'rinishdagi jadval *uchinchi tartibli kvadrat matritsa* $a_1b_2c_3 + {}$ $+a_2b_3c_1 + a_3a_1c_2 - a_3b_2c_1 - a_1c_2b_3 - a_2b_1c_3$ son bu matritsaning *determinanti* yoki *uchinchi tartibli determinant deyiladi.* U quyidagicha belgilanadi:

$$\det A = |A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 -$$
$$-a_3b_2c_1 - a_1c_2b_3 - a_2b_1c_3. \qquad (2)$$

Uchinchi tartibli determinant, ko'pincha, quyidagicha hisoblanadi: (2) dagi musbat va manfiy qo'shiluvchilar 1-rasmda

(+) (−)

1-rasm.

ko‘rsatilgani kabi uchtadan elementlarni ko‘paytirib hosil qilinadi:

3-misol. Determinantni hisoblang:

$$\blacktriangleright \begin{vmatrix} 2 & 4 & 1 \\ -1 & 3 & -2 \\ 3 & 2 & 3 \end{vmatrix} = 18 - 24 - 2 - 9 + 8 + 12 = 3. \blacktriangleleft$$

Uchinchi tartibli determinant berilgan elementining *minori* deb, shu element turgan satr va ustunni o‘chirishdan hosil bo‘lgan ikkinchi tartibli determinantga aytiladi. Shu elementning *algebraik to‘ldiruvchisi* deb uning $(-1)^k$ soniga ko‘paytirilgan minoriga aytiladi. Bu yerda k — berilgan element turgan satr va ustun tartib raqamlarining yig‘indisi. Uchinchi tartibli determinantni

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad (3)$$

ko‘rinishda yozsak, a_{ij} eliementning minori M_{ij}, algebraik to‘ldiruvchisi A_{ij} deb belgilanadi: $A_{ij} = (-1)^{i+j} M_{ij}$.

Agar $i + j$ juft son bo‘lsa, a_{ij} element juft o‘rinda, aks holda toq o‘rinda turibdi deyiladi. Juft o‘rindagi element uchun $A_{ij} = M_{ij}$, toq o‘rindagi element uchun $A_{ij} = -M_{ij}$. Masalan,

$$A_{12} = -M_{12} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}; \quad A_{31} = M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}.$$

Determinantning yana bir muhim xossasini keltiramiz.

6. Determinant o‘zining istalgan qatori elementlari bilan ularga mos algebraik to‘ldiruvchilar ko‘paytmalarining yig‘indisiga teng. Masalan, (3) determinantning birinchi satr elementlari bo‘yicha yoyilmasi: $\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$.

Determinant biror qatori elementlari bilan unga parallel boshqa qator elementlari algebraik to‘ldiruvchilari ko‘paytmalarining yig‘indisi esa nolga teng. Masalan, (3) determinant uchun

$$\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = a_{12}A_{13} + a_{22}A_{23} + a_{32}A_{33} = 0.$$

Uchinchi tartibli determinantning birinchi satr elementlari bo‘yicha yoyilmasi quyidagicha yoziladi:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1A_{11} + b_1A_{12} + c_1A_{13} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}.$$

4-misol. Determinantni birinchi ustun elemenlari bo‘yicha yoyish yordamida hisoblang:

$$\begin{vmatrix} 3 & 4 & 15 \\ 2 & 25 & 12 \\ 0 & 2 & 1 \end{vmatrix}.$$

$$\blacktriangleright \begin{vmatrix} 3 & 4 & 15 \\ 2 & 25 & 12 \\ 0 & 2 & 1 \end{vmatrix} = 3\cdot\begin{vmatrix} 25 & 12 \\ 2 & 1 \end{vmatrix} - 2\cdot\begin{vmatrix} 4 & 15 \\ 2 & 1 \end{vmatrix} + 0\cdot\begin{vmatrix} 4 & 15 \\ 25 & 12 \end{vmatrix} =$$

$$= 3(25 - 24) - 2(4 - 30) = 3 + 52 = 55. \blacktriangleleft$$

5-misol. Tenglanani yeching:

$$\begin{vmatrix} x & x+1 \\ -4 & x+1 \end{vmatrix} = 0. \blacktriangleright (x+1)\begin{vmatrix} x & 1 \\ -4 & 1 \end{vmatrix} = (x+1)(x+4) = 0$$

$\Leftrightarrow$

1) $x + 1 = 0, \quad x = -1;$

2) $x + 4 = 0, \quad x = -4.$ $\blacktriangleleft$ *Javobi:* $-4; -1$.

2⁰. *n*-tartibli determinantlar. Quyidagi

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

ko'rinishdagi n^2 ta sondan iborat jadval n-tartibli kvadrat matritsa deyiladi. Bu matritsaning *determinanti* yoki *n -tartibli determinant* deb,

$$\Delta=\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

kabi belgilanuvchi songa aytiladi.

Determinantning yuqorida keltirilgan barcha xossalari n-tartibli determinant uchun ham o'rinlidir. n-tartibli determinantni hisoblashda quyidagi usullar qo'llaniladi.

Tartibni pasaytirish (yoki yoyish) usuli. Bu usulda determinant biror qatorning elementlari bo'yicha yoyiladi. Odatda, yoyishdan oldin bu qatorning faqat bitta noldan farqli elementi qoldiriladi.

1-misol. Determinantni hisoblang:

$$D=\begin{vmatrix} 4 & 5 & 12 & 8 \\ -8 & 2 & -7 & -10 \\ 2 & 1 & 3 & 3 \\ 0 & 4 & -3 & 2 \end{vmatrix}.$$

► Uchinchi satrni (-2) ga ko'paytirib, 1- satrga, 4 ga ko'paytirib, 2- satrga qo'shamiz va hosil bo'lgan determinantni 1-ustun elementlari bo'yicha yoyamiz:

$$D=\begin{vmatrix} 0 & 1 & 6 & 2 \\ 0 & 6 & 5 & 2 \\ 2 & 1 & 3 & 3 \\ 0 & 4 & -3 & 2 \end{vmatrix}=2(-1)^{3+1}\begin{vmatrix} 1 & 6 & 2 \\ 6 & 5 & 2 \\ 4 & -3 & 2 \end{vmatrix}=2\cdot 2\cdot\begin{vmatrix} 1 & 6 & 1 \\ 6 & 5 & 1 \\ 4 & -3 & 1 \end{vmatrix}=$$

$$=4\cdot\begin{vmatrix} 1 & 6 & 1 \\ 5 & -1 & 0 \\ 3 & -9 & 0 \end{vmatrix}=4\begin{vmatrix} 5 & -1 \\ 3 & -9 \end{vmatrix}=4(-45+3)=-168. \blacktriangleleft$$

Uchburchakli ko'rinishga keltirish usuli. Bu usulda determinant diagonallaridan birining bir tomonidagi barcha elementlar nollar bo'lgan ko'rinishga keltiriladi.

2-misol. Determinantni hisoblang:

$$D=\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & 2 \\ 1 & 1 & -1 & 3 \\ 1 & 1 & 1 & -1 \end{vmatrix}.$$

► Birinchi satrni qolgan barcha satrlardan ayirib quyidagini hosil qilamiz:

$$D=\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & -2 \end{vmatrix}=1\cdot(-2)(-2)(-2)=-8. \blacktriangleleft$$

Rekurrent munosabatlar usuli. Bu usul berilgan determinantni xuddi shu shakldagi quyi tartibli determinantlar yordamida ifodalash mumkin bo'lganida qo'llaniladi.

3-misol. Ushbu beshinchi tartibli Vandermond determinantini hisoblang:

$$D_5 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ a_1 & a_2 & a_3 & a_4 & a_5 \\ a_1^2 & a_2^2 & a_3^2 & a_4^2 & a_5^2 \\ a_1^3 & a_2^3 & a_3^3 & a_4^3 & a_5^3 \\ a_1^4 & a_2^4 & a_3^4 & a_4^4 & a_5^4 \end{vmatrix}.$$

Ikkinchi va uchinchi tartibli Vandermond determinantlari:

$$D_2 = \begin{vmatrix} 1 & 1 \\ a_1 & a_2 \end{vmatrix} = a_2 - a_1;$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ a_1 & a_2 & a_3 \\ a_1^2 & a_2^2 & a_3^2 \end{vmatrix} = (a_2 - a_1)(a_3 - a_1)(a_3 - a_2)$$

dan ko'rinadiki, D_5 ham $a_i - a_j \ (5 \geq i \geq j \geq 1)$ ko'rinishdagi barcha ayirmalarning ko'paytmasiga teng bo'ladi:

$$D_5 = (a_2 - a_1)(a_3 - a_1)(a_3 - a_2)(a_4 - a_1)(a_4 - a_2)(a_4 - a_3)(a_5 - a_1)\times$$
$$\times(a_5 - a_2)(a_5 - a_3)(a_5 - a_4). \blacktriangleleft$$

Shu usulda n-tartibli Vandermond determinantini ham hisoblash mumkin (mustaqil bajarib ko'ring!).

Mustaqil bajarish uchun mashqlar

1.1. Determinantni hisoblang:

1) $\begin{vmatrix} -1 & 4 \\ -5 & 2 \end{vmatrix}$; 2) $\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix}$; 3) $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$;

4) $\begin{vmatrix} 5 & 3 \\ 6 & 4 \end{vmatrix}$; 5) $\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\beta & \cos\beta \end{vmatrix}$; 6) $\begin{vmatrix} x+6 & 9 \\ 4 & x+6 \end{vmatrix}$.

1.2. Tenglamani yeching:

1) $\begin{vmatrix} 2x-1 & 3 \\ 3x-4 & 2 \end{vmatrix} = 0$; 2) $\begin{vmatrix} 4 & 1-2x \\ 3 & 5+x \end{vmatrix} = 0$;

3) $\begin{vmatrix} \cos 8x & -\sin 5x \\ \sin 8x & \cos 5x \end{vmatrix} = 0$; 4) $\begin{vmatrix} \sin 4x & \cos 3x \\ -\cos 4x & \sin 3x \end{vmatrix} = 0$;

5) $\begin{vmatrix} 3x & 6x-9 \\ 1 & x-2 \end{vmatrix} = 0$; 6) $\begin{vmatrix} x-1 & 6 \\ 4 & x+1 \end{vmatrix} = 0$.

1.3. Determinantni hisoblang:

1) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$; 2) $\begin{vmatrix} 0 & x & 0 \\ x & 1 & x \\ 0 & x & 0 \end{vmatrix}$;

3) $\begin{vmatrix} a+x & x & x \\ x & b+x & x \\ x & x & c+x \end{vmatrix}$;

④ $\begin{vmatrix} a^2+1 & \alpha\beta & \alpha\gamma \\ \alpha\beta & \beta^2+1 & \beta\gamma \\ \alpha\gamma & \beta\gamma & \gamma^2+1 \end{vmatrix}$; 5) $\begin{vmatrix} 1 & 1 & x \\ 1 & 1 & x^2 \\ x^2 & x & 1 \end{vmatrix}$.

1.4. Tenglamani yeching:

1) $\begin{vmatrix} x+1 & 1 & 2 \\ 6 & x & 1 \\ x+4 & 2 & 0 \end{vmatrix} = 0$; 2) $\begin{vmatrix} x & x+1 \\ -4 & x+1 \end{vmatrix} = 0$;

3) $\begin{vmatrix} x & x+1 & x+2 \\ x+3 & x+4 & x+5 \\ x+6 & x+7 & x+8 \end{vmatrix} = 0.$

1.5. Tengsizlikni yeching:

1) $\begin{vmatrix} 3 & -2 & 1 \\ 1 & x & -2 \\ -1 & 2 & -1 \end{vmatrix} < 0;$ 2) $\begin{vmatrix} 2 & x+2 & -1 \\ 1 & 1 & -2 \\ 5 & -3 & x \end{vmatrix} > 0.$

1.6. Ayniyatni isbotlang:

1) $\begin{vmatrix} a_1+b_1x & a_1-b_1x & c_1 \\ a_2+b_2x & a_2-b_2x & c_2 \\ a_3+b_3x & a_3-b_3x & c_3 \end{vmatrix} = 2x\cdot\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix};$

2) $\begin{vmatrix} a_1+b_1x & a_1x+b_1 & c_1 \\ a_2+b_2x & a_2x+b_2 & c_2 \\ a_3+b_3x & a_3x+b_3 & c_3 \end{vmatrix} = (1-x^2)\cdot\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix};$

3) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x).$

1.7. Determinantni hisoblang:

1) $\begin{vmatrix} 2 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ 3 & -1 & 2 & 3 \\ 3 & 1 & 5 & 1 \end{vmatrix};$ 2) $\begin{vmatrix} 2 & 3 & -3 & 4 \\ 2 & 1 & -1 & 2 \\ 6 & 2 & 1 & 0 \\ 2 & 3 & 0 & -5 \end{vmatrix};$

3) $\begin{vmatrix} 3 & -1 & 4 & 2 \\ 5 & 2 & 0 & 1 \\ 0 & 2 & 1 & -3 \\ 6 & -2 & 9 & 8 \end{vmatrix};$ 4) $\begin{vmatrix} 0 & -a & -b & -d \\ a & 0 & -c & -e \\ b & c & 0 & 0 \\ d & e & 0 & 0 \end{vmatrix};$

5) $\begin{vmatrix} 0 & b & c & d \\ b & 0 & d & c \\ c & d & 0 & b \\ d & c & b & 0 \end{vmatrix}.$

1.8. n-tartibli determinantni uchburchakli ko'rinishga keltirish usuli bilan hisoblang:

1) $\begin{vmatrix} 1 & 2 & 3 & \ldots & n \\ -1 & 0 & 3 & \ldots & n \\ -1 & -2 & 0 & \ldots & n \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ -1 & -2 & -3 & \ldots & 0 \end{vmatrix};$ 2) $\begin{vmatrix} 3 & 2 & 2 & \ldots & 2 \\ 2 & 3 & 2 & \ldots & 2 \\ 2 & 2 & 3 & \ldots & 2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 2 & 2 & 2 & \ldots & 3 \end{vmatrix}.$

1.9. n-tartibli determinantni rekurrent munosabatlar usuli bilan hisoblang:

1) $\begin{vmatrix} 0 & 1 & 1 & \ldots & 1 \\ 1 & a_1 & 0 & \ldots & 0 \\ 1 & 0 & a_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & 0 & 0 & \ldots & a_n \end{vmatrix};$ 2) $\begin{vmatrix} 2 & 1 & 0 & \ldots & 0 \\ 1 & 2 & 1 & \ldots & 0 \\ 0 & 1 & 2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 2 \end{vmatrix}.$

2- §. n noma'lumli *n* ta chiziqli tenglama sistemasini yechish. Kramer qoidasi

n noma'lumli n ta chiziqli tenglama sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n. \end{cases} \qquad (1)$$

Bu sistema kamida bitta yechimga ega bo'lsa, *birgalikdagi sistema*, yechimga ega bo'lmasa, *birgalikdamas sistema* deyiladi. Birgalikdagi sistema yagona yechimga ega (*aniq sistema*) yoki cheksiz ko'p yechimga ega (*aniqmas sistema*) bo'lishi mumkin. Quyidagi determinantlarni tuzamiz:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} b_1 & a_{12} & \dots & a_{1n} \\ b_2 & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ b_n & a_{n2} & \dots & a_{nn} \end{vmatrix}, \dots,$$

$$\Delta_n = \begin{vmatrix} a_{11} & a_{12} & \dots & b_1 \\ a_{21} & a_{22} & \dots & b_2 \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & b_n \end{vmatrix}.$$

Bu yerda sistema determinanti Δ (1) dagi noma'lumlarning koeffitsiyentlaridan, $\Delta_k\left(k=\overline{1,n}\right)$ esa Δ da k- ustunni ozod hadlar ustuni bilan almashtirishdan hosil bo'ladi.

Agar $\Delta \neq 0$ bo'lsa, (1) sistema birgalikda va yagona yechimga ega, ya'ni aniq sistema bo'ladi. Bu yechim

$$x_1 = \frac{\Delta_1}{\Delta}, \quad x_2 = \frac{\Delta_2}{\Delta}, \quad \dots, \quad x_n = \frac{\Delta_n}{\Delta} \qquad (2)$$

formulalar bilan topiladi. Sistemani yechishning bu usuli *Kramer qoidasi* deyiladi.

1- misol. Tenglamalar sistemasini yeching:

$$\begin{cases} 2x - 3y = 1, \\ 3x + 4y = 10. \end{cases}$$

► Δ, Δ_1, Δ_2 determinantlarni hisoblaymiz:

$$\Delta = \begin{vmatrix} 2 & -3 \\ 3 & 4 \end{vmatrix} = 8 + 9 = 17,$$

$$\Delta_1 = \begin{vmatrix} 1 & -3 \\ 10 & 4 \end{vmatrix} = 4 + 30 = 34; \quad \Delta_2 = \begin{vmatrix} 2 & 1 \\ 3 & 10 \end{vmatrix} = 17.$$

$\Delta \neq 0$ bo'lgani uchun sistema birgalikda va yagona yechimga ega (aniq sistema). Bu yechimni topamiz:

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{34}{17} = 2, \quad x_2 = \frac{\Delta_2}{\Delta} = \frac{17}{17} = 1.$$

Javobi: (2 ; 1). ◄

2- misol. Tenglamalar sistemasini yeching:

$$\begin{cases} 3x - y + 2z = 3, \\ -2x + y + 3z = 3, \\ x - 3y + 4z = -1. \end{cases}$$

► $\Delta = \begin{vmatrix} 3 & -1 & 2 \\ -2 & 1 & 3 \\ 1 & -3 & 4 \end{vmatrix} = 12 - 3 + 12 - 2 + 27 - 8 = 38.$ $\Delta \neq 0.$

Sistema yagona yechimga ega. Yechimni Kramer formulalari yordamida topamiz:

$$\Delta_1 = \begin{vmatrix} 3 & -1 & 2 \\ 3 & 1 & 3 \\ 1 & -1 & 4 \end{vmatrix} = 12 + 3 - 18 + 2 + 27 + 12 = 38;$$

$$\Delta_2 = \begin{vmatrix} 3 & 3 & 2 \\ -2 & 3 & 3 \\ 1 & -1 & 4 \end{vmatrix} = 76; \quad \Delta_3 = \begin{vmatrix} 3 & -1 & 3 \\ -2 & 1 & 3 \\ 1 & -3 & -1 \end{vmatrix} = 38;$$

$$x = \frac{\Delta_1}{\Delta} = \frac{38}{38} = 1; \quad y = \frac{\Delta_2}{\Delta} = \frac{76}{38} = 2; \quad z = \frac{\Delta_3}{\Delta} = \frac{38}{38} = 1.$$

Javobi: (1, 2, 1). ◄

Agar sistema determinanti $\Delta = 0$ bo'lib:

$\Delta_1 = \Delta_2 = \dots = \Delta_n = 0$ bo'lsa, (1) sistema cheksiz ko'p yechimlarga ega (aniqmas sistema);

$\Delta_1, \Delta_2, \dots, \Delta_n$ lardan birortasi noldan farqli bo'lsa, sistema yechimga ega emas (birgalikdamas sistema).

Ushbu bir jinsli

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = 0, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = 0, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = 0 \end{cases}$$

sistema $\Delta \neq 0$ da yagona $x_1 = x_2 = \ldots = x_n = 0$ nol (*trivial*) yechimga ega, $\Delta = 0$ bo'lganida esa noldan farqli (*notrivial*) cheksiz ko'p yechimlarga ega. Bir jinsli sistemalarni tekshirish va yechish istalgan algebraik tenglamalar sistemalarini yechishga bag'ishlangan bobda qaraladi.

Mustaqil bajarish uchun mashqlar

2.1. Tenglamalar sisitemasini yeching:

1) $\begin{cases} 3x - 4y = 1, \\ 2x - 7y = -8; \end{cases}$ 2) $\begin{cases} 2x_1 + 3x_2 = 1, \\ 3x_1 + 5x_2 = 3; \end{cases}$

3) $\begin{cases} 2ax - 3by = 0, \\ 3ax - 6by = ab; \end{cases}$ 4) $\begin{cases} 3x_1 + x_2 = 4, \\ 2x_1 + 4x_2 = 1; \end{cases}$

5) $\begin{cases} x - y = 3, \\ -2x + 2y = 1; \end{cases}$ 6) $\begin{cases} x = 2y + 1, \\ y = \dfrac{x}{2} - 0{,}5. \end{cases}$

2.2. Tenglamalar sistemasini yeching:

1) $\begin{cases} 2x + y = 5, \\ x + 3z = 16, \\ 5y - z = 10; \end{cases}$ 2) $\begin{cases} 3x_1 + 2x_2 + x_3 = 5, \\ 2x_1 - x_2 + x_3 = 6, \\ x_1 + 5x_2 = -3; \end{cases}$

3) $\begin{cases} 2x - y + 3z = 9, \\ 3x - 5y + z = -4, \\ 4x - 7y + z = 5; \end{cases}$ 4) $\begin{cases} x - y - 2z = 6, \\ 2x + 3y - 7z = 16, \\ 5x + 2y + z = 16; \end{cases}$

5) $\begin{cases} 7x + 2y + 3z = 15, \\ 5x - 3y + 2z = 15, \\ 10x - 11y + 5z = 36; \end{cases}$ 6) $\begin{cases} 4x_1 + 4x_2 + 5x_3 + 5x_4 = 0, \\ 2x_1 + 3x_2 - x_4 = 10, \\ x_1 + x_2 - 5x_3 = -10, \\ 3x_2 + 2x_3 = 1; \end{cases}$

7) $\begin{cases} 2x_1 - x_2 + 3x_3 + 2x_4 = 4, \\ 3x_1 + 3x_2 + 3x_3 + 2x_4 = 6, \\ 3x_1 - x_2 - x_3 - 2x_4 = 6, \\ 3x_1 - x_2 + 3x_3 - x_4 = 6. \end{cases}$

3- §. Matritsalar

1°. Matritsa tushunchasi. Matritsalar ustida chiziqli amallar. m ta satr va n ta ustundan iborat

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix} = (a_{ij}), \quad (i = \overline{1,m}; \quad j = \overline{1,n})$$

ko'rinishdagi jadval ($m\times n$)-*o'lchovli to'g'ri burchakli matritsa* yoki ($m\times n$)-*matritsa* deyiladi.

Faqat nollardan iborat bo'lgan matritsa *nol-matritsa* deyiladi va u ko'pincha O harfi bilan belgilanadi..

$m = n$ bo'lsa, A matritsa *n-tartibli kvadrat matritsa* deyiladi. Kvadrat matritsaning determinanti noldan farqli, ya'ni det $A \neq 0$ bo'lsa, u *xosmas* (*maxsusmas*), det$A = 0$ da esa *xos (maxsus)* matritsa deyiladi. Kvadrat matritsa uchun *diagonal, skalar, birlik* (u ko'pincha E harfi bilan belgilanadi) *matritsa* tushunchalari mavjud, ularni 3-tartibli matritsa misolida keltiramiz:

$$\begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}; \quad \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}; \quad E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

A matritsada satrlarni mos ustunlar bilan almashtirishdan hosil bo'lgan A^T matritsa A ga *transponirlangan* matritsa deyiladi. Agar $A = A^T$ bo'lsa, A — *simmetrik matritsa* deyiladi. Matritsa bitta satrdan iborat bo'lsa *satr-matritsa*, bitta ustundan iborat bo'lsa *ustun-matritsa* yoki *vektor* ham deyiladi. Ustun-matritsaning transponirlangani satr-matritsa bo'ladi va, aksincha.

Mos elementlari teng bo'lgan bir xil o'lchamli matritsalar *teng matritsalar* deyiladi. Bir xil o'lchamli matritsalarni qo'shish (ayirish) mumkin. Buning uchun ularning mos (bir xil o'rindagi) elementlarini qo'shish (ayirish) kerak. Istalgan matritsani songa ko'paytirish mumkin. Buning uchun ularning mos (bir xil o'rindagi) elementlarini qo'shish (ayirish) kerak.

Istalgan matritsani songa ko'paytirish mumkin. Buning uchun uning barcha elementlarini shu songa ko'paytirish kerak.

1-misol. $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 1 & 2 \\ 2 & 3 & -4 \end{pmatrix}$ matritsalar berilgan. $C = 3A + 2B$ va C^T matritsalarni toping.

▶ $$C = 3\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix} + 2\begin{pmatrix} -1 & 1 & 2 \\ 2 & 3 & -4 \end{pmatrix} =$$

$$= \begin{pmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \end{pmatrix} + \begin{pmatrix} -2 & 2 & 4 \\ 4 & 6 & -8 \end{pmatrix} = \begin{pmatrix} 1 & 8 & 13 \\ 4 & 9 & -2 \end{pmatrix};$$

$$C^T = \begin{pmatrix} 1 & 4 \\ 8 & 9 \\ 13 & -2 \end{pmatrix}. ◀$$

Agar A matritsaning satrlar soni B matritsaning ustunlar soniga teng bo'lsa, A ni B ga ko'paytirish mumkin: $(m\times k)$-o'lchamli

$A = (a_{ij})$ matritsani $(k\times n)$-o'lchamli $B = (b_{ij})$ matritsaga ko'paytirishdan $(m\times n)$-o'lchamli $C = (c_{ij}) = AB$ matritsa hosil bo'ladi. Ko'paytirish «satrni ustunga» qoidasi bo'yicha quyidagicha bajariladi: $C = (c_{ij})$ matritsaning c_{ij} elementi A ning i-satr elementlarini B ning j-ustuni mos elementlariga ko'paytirib qo'shishdan hosil bo'ladi:

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{ik}b_{kj}, \quad \left(i = \overline{1,m}; \qquad j = \overline{1,n}\right).$$

Matritsalarni ko'paytirish amali uchun o'rin almashtirish (kommutativlik) qonuni o'rinli emas: $AB \neq BA$.

Matritsalarni ko'paytirish amalining xossalari:

1) $A(CB) = (AB)C$; 2) $(A + B)C = AC + BC$;
3) $(\lambda A)B = \lambda(AB)$; 4) $AE = EA = A$;
5) $AO = OA = O$; 6) $(AB)^T = B^T A^T$;
7) $\det(AB) = \det A \cdot \det B$.

2-misol.

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 3 \\ 1 & 2 \end{pmatrix}$$

matritsalar berilgan. AB va BA matritsalarni toping.

▶ «Satrni ustunga» qoidasi bo'yicha ko'paytiramiz:

$$AB = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -2 & 3 & 0 \\ 1 & 2 & -1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1\cdot(-2)+2\cdot 1 & 1\cdot 3+2\cdot 2 & 1\cdot 0+2\cdot(-1) \\ 0\cdot(-2)+1\cdot 1 & 0\cdot 3+1\cdot 2 & 0\cdot 0+1\cdot(-1) \\ 1\cdot(-2)+0\cdot 1 & 1\cdot 3+0\cdot 2 & 1\cdot 0+0\cdot(-1) \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 7 & -2 \\ 1 & 2 & -1 \\ -2 & 3 & 0 \end{pmatrix}.$$

(3×2)-matritsani (2×3)-matritsaga ko‘paytirib, 3 tartibli kvadrat matritsa hosil qildik. BA matritsani hisoblab ko‘ramiz:

$$B\cdot A=\begin{pmatrix}-2 & 3 & 0\\ 1 & 2 & -1\end{pmatrix}\cdot\begin{pmatrix}1 & 2\\ 0 & 1\\ 1 & 0\end{pmatrix}=$$

$$=\begin{pmatrix}-2+0+0 & -4+3+0\\ 1+0-1 & 2+2-0\end{pmatrix}=\begin{pmatrix}-2 & -1\\ 0 & 4\end{pmatrix}.$$

Demak, $AB\neq BA$. ◀

3-misol. $f(A)$ matritsaviy ko‘phadning A matritsaga bog‘liq qiymatini toping:

$$f(A)=A^2-5A+6E;\ A=\begin{pmatrix}2 & -1\\ 1 & 3\end{pmatrix}.$$

▶ $$f(A)=A^2-5A+6E=\begin{pmatrix}2 & -1\\ 1 & 3\end{pmatrix}\cdot\begin{pmatrix}2 & -1\\ 1 & 3\end{pmatrix}-5\cdot\begin{pmatrix}2 & -1\\ 1 & 3\end{pmatrix}+$$

$$+6\cdot\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}4-1 & -2-3\\ 2+3 & -1+9\end{pmatrix}-\begin{pmatrix}10 & -5\\ 5 & 15\end{pmatrix}+\begin{pmatrix}6 & 0\\ 0 & 6\end{pmatrix}=$$

$$=\begin{pmatrix}3 & -5\\ 5 & 8\end{pmatrix}-\begin{pmatrix}10 & -5\\ 5 & 15\end{pmatrix}+\begin{pmatrix}6 & 0\\ 0 & 6\end{pmatrix}=$$

$$=\begin{pmatrix}3-10+6 & -5-10+0\\ 5-5+0 & 8-15+6\end{pmatrix}=\begin{pmatrix}-1 & -15\\ 0 & -1\end{pmatrix}.$$

Javobi: $f(A)=\begin{pmatrix}-1 & -15\\ 0 & -1\end{pmatrix}$. ◀

2⁰. Teskari matritsa. Chiziqli tenglamalar sistemasini matritsa usuli bilan yechish. Agar A xosmas kvadrat matritsa (ya’ni $\Delta=$ $=\det A\neq 0$) bo‘lsa, u holda shunday A^{-1} matritsa mavjudki, uning uchun

$$A\cdot A^{-1}=A^{-1}\cdot A=E$$

tenglik o‘rinli bo‘ladi, bu yerda E — birlik matritsa. A^{-1} *matritsa A ga teskari matritsa* deyiladi. Teskari matritsaning xossalari:

1. $\det A^{-1}=\frac{1}{\det A}$. 2. $(AB)^{-1}=B^{-1}\cdot A^{-1}$.

3. $(A^{-1})^T=(A^T)^{-1}$. 4. $(A^V)^T\cdot A=A\cdot(A^V)^T=\det A\cdot E$,

A^V *matritsa* $\det A$ determinant elementlarining algebraik to‘ldiruvchilaridan tuzilgan matritsa bo‘lib, *A ga biriktirilgan matritsa* deyiladi. Oxirgi xossadan

$$A^{-1}=\frac{1}{\det A}(A^V)^T$$

yoki $$A=\begin{pmatrix}a_{11} & a_{12} & \dots & a_{1n}\\ a_{21} & a_{22} & \dots & a_{2n}\\ \dots & \dots & \dots & \dots\\ a_{n1} & a_{n2} & \dots & a_{nn}\end{pmatrix}$$ bo‘lsa,

$$A^{-1}=\frac{1}{\det A}\begin{pmatrix}A_{11} & A_{21} & \dots & A_{n1}\\ A_{12} & A_{22} & \dots & A_{n2}\\ \dots & \dots & \dots & \dots\\ A_{1n} & A_{2n} & \dots & A_{nn}\end{pmatrix}. \quad (1)$$

Bu — teskari matritsani topish formulasidir.

Ushbu n noma’lumli n ta chiziqli tenglama sistemasini qaraylik:

$$\begin{cases}a_{11}x_1+a_{12}x_2+\dots+a_{1n}x_n=b_1,\\ a_{21}x_1+a_{22}x_2+\dots+a_{2n}x_n=b_2,\\ \dots\dots\dots\dots\dots\dots\dots\dots\\ a_{n1}x_1+a_{n2}x_2+\dots+a_{nn}x_n=b_n.\end{cases} \quad (2)$$

Sistema noma’lumlarining koeffitsiyentlaridan tuzilgan matritsa yuqorida yozilgan A matritsadan iborat. Yana

$$B=\begin{pmatrix}b_1\\ b_2\\ \dots\\ b_n\end{pmatrix},\qquad X=\begin{pmatrix}x_1\\ x_2\\ \dots\\ x_n\end{pmatrix}$$

ustun-matritsalarni kiritsak, (2) sistemani

$$AX = B \qquad (3)$$

matritsaviy tenglama shaklida yozish mumkin. A xosmas matritsa, ya'ni $\det A \neq 0$ bo'lsa, A^{-1} mavjud va bu tenglamani chapdan A^{-1} ga ko'paytirib,

$$X = A^{-1} \cdot B$$

ni olamiz. Bu (2) sistema yechimining matritsaviy yozuvidir. Chiziqli tenglamalar sistemasini yechishning bu usuli *matritsa usuli* deyiladi.

4-misol. Tenglamalar sistemasini matritsa usuli bilan yeching:

$$\begin{cases} 3x_1 - 2x_2 + x_3 = 6, \\ x_1 + 2x_2 - x_3 = 2, \\ 3x_1 - x_2 + x_3 = 7. \end{cases}$$

▶ A, B, X matritsalarni tuzamiz va $\det A$ ni hisoblaymiz:

$$A = \begin{pmatrix} 3 & -2 & 1 \\ 1 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 6 \\ 2 \\ 7 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix};$$

$$\det A = \begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & -1 \\ 3 & -1 & 1 \end{vmatrix} = 6 + 6 - 1 - 6 - 3 + 2 = 4;$$

$\det A \neq 0$ bo'lgani uchun A — xosmas matritsa va A^{-1} mavjud. Uni (1) formula bo'yicha topamiz:

$$A_{11} = \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} = 2 - 1 = 1; \; A_{12} = -\begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = -4;$$

$$A_{13} = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = -7; \; A_{21} = -\begin{vmatrix} -2 & 1 \\ -1 & 1 \end{vmatrix} = 1;$$

$$A_{22} = \begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix} = 0; \; A_{23} = -\begin{vmatrix} 3 & -2 \\ 3 & -1 \end{vmatrix} = -3;$$

$$A_{31} = \begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix} = 0; \; A_{32} = -\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} = 4;$$

$$A_{33} = \begin{vmatrix} 3 & -2 \\ 1 & 2 \end{vmatrix} = 8; \; A^{-1} = \frac{1}{4} \cdot \begin{pmatrix} 1 & 1 & 0 \\ -4 & 0 & 4 \\ -7 & -3 & 4 \end{pmatrix};$$

$$X = \frac{1}{4} \cdot \begin{pmatrix} 1 & 1 & 0 \\ -4 & 0 & 4 \\ -7 & -3 & 8 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 2 \\ 7 \end{pmatrix} = \frac{1}{4} \cdot \begin{pmatrix} 6+2+0 \\ -24+0+28 \\ -42-6+56 \end{pmatrix} = \frac{1}{4} \cdot \begin{pmatrix} 8 \\ 4 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix};$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \Leftrightarrow x_1 = 2; \; x_2 = 1; \; x_3 = 2.$$

Javobi: (2; 1; 2). ◀

5-misol. $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}$ matritsalar berilgan. $XA = B$ matritsaviy tenglamani yeching.

▶ $X \cdot A = B \;\Rightarrow\; X \cdot A \cdot A^{-1} \;\Rightarrow\; X = B \cdot A^{-1}$.

(1) formuladan foydalansak:

$$X = B \cdot A^{-1} = B \cdot \frac{1}{\det A} \cdot \begin{pmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix} \cdot \frac{1}{2} \cdot \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} =$$

$$= \frac{1}{2}\begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix} \cdot \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \frac{1}{2} \cdot \begin{pmatrix} -3 & -1 \\ -7 & -1 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{7}{2} & \frac{1}{2} \end{pmatrix};$$

$$Javobi: X = \begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{7}{2} & \frac{1}{2} \end{pmatrix}. \quad ◀$$

Mustaqil bajarish uchun mashqlar

3.1. Matritsalar ustida amallarni bajaring:

1) $A=\begin{pmatrix}2 & 1 & -1\\ 0 & 1 & -1\end{pmatrix},\qquad B=\begin{pmatrix}-1 & 2 & 1\\ 2 & 0 & -2\end{pmatrix}$

bo‘lsa, $3A+4B$ ni toping;

2) $A=\begin{pmatrix}3 & -2\\ 5 & -4\end{pmatrix},\qquad B=\begin{pmatrix}3 & 4\\ 2 & 5\end{pmatrix}$

bo‘lsa, AB, BA, $\det(AB)$ va $\det(BA)$larni toping.

3.2. Amallarni bajaring:

1) $\begin{pmatrix}2 & -3\\ 4 & -6\end{pmatrix}\cdot\begin{pmatrix}9 & -6\\ 6 & -4\end{pmatrix};$

2) $\begin{pmatrix}4 & 3\\ 7 & 5\end{pmatrix}\cdot\begin{pmatrix}-28 & 93\\ 38 & -126\end{pmatrix}\cdot\begin{pmatrix}7 & 3\\ 2 & 1\end{pmatrix};$

3) $\begin{pmatrix}1 & 1 & 2\\ 1 & 3 & 1\\ 4 & 1 & 1\end{pmatrix}^2;$

4) $\begin{pmatrix}1 & 2 & 1 & 0\\ 0 & 2 & 1 & 1\\ 1 & -2 & 2 & 1\\ 2 & -1 & 2 & -1\end{pmatrix}\cdot\begin{pmatrix}1\\ 2\\ 1\\ -1\end{pmatrix};$

5) $\begin{pmatrix}2 & -3 & 3 & 5\end{pmatrix}\cdot\begin{pmatrix}1\\ 3\\ -2\\ 5\end{pmatrix};$

6) $\begin{pmatrix}0 & 0 & 1\\ 1 & 1 & 2\\ 2 & 2 & 3\\ 3 & 3 & 4\end{pmatrix}\cdot\begin{pmatrix}-1 & -1\\ 2 & 2\\ 1 & 1\end{pmatrix}\cdot\begin{pmatrix}4\\ 1\end{pmatrix};$

7) $\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\\ 1 & 3 & 2\end{pmatrix}\cdot\begin{pmatrix}1\\ -2\\ 3\end{pmatrix};$

8) $\begin{pmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{pmatrix}\cdot\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix};$

9) $\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}^n,\qquad (\lambda\in R).$

3.3. $f(A)$ matritsiaviy ko‘phadning A matritsaga bog‘liq qiymatini toping.

1) $f(x)=x^2+5,\quad A=\begin{pmatrix}1 & 3\\ 2 & 0\end{pmatrix};$

2) $f(x)=x^2-3x+1,\quad A=\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}.$

3.4. $A\cdot X=B$ tenglamadan x, y, z larni toping, bunda:

1) $A=\begin{pmatrix}2 & -3 & 0\\ 0 & -2 & 2\\ 5 & 0 & -2\end{pmatrix},\quad B=\begin{pmatrix}-1\\ 0\\ 3\end{pmatrix},\quad X=\begin{pmatrix}x\\ y\\ z\end{pmatrix};$

2) $A=\begin{pmatrix}0 & 2 & 1\\ -2 & 1 & 0\\ 3 & 0 & -5\end{pmatrix},\quad B=\begin{pmatrix}5\\ -4\\ 4\end{pmatrix},\quad X=\begin{pmatrix}x\\ y\\ z\end{pmatrix}.$

4- §. Matritsaning rangi. Elementar almashtirishlar

Ushbu

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}=(a_{ij}),\quad (i=\overline{1,m};\quad j=\overline{1,n})$$

to'g'ri burchakli matritsa berilgan bo'lsin. Bu martitsada qandaydir k ta satr va k ta ustunni ajratamiz ($k\le m$, $k\le n$). A matritsaning bu satrlar va ustunlarning kesishgan o'rinlarida turgan elementlaridan tuzilgan k-tartibli determinant *A matritsaning k-tartibli minori* deyiladi. A matritsaning barcha minorlari soni $C_m^k\cdot C_n^k$ ga teng, bunda:

$$C_m^k=\frac{m!}{k!\cdot(m-n)!},\ C_n^k=\frac{n!}{k!\cdot(n-k)!}.$$

A matritsaning noldan farqli barcha minorlarini qaraymiz. *A matritsaning rangi* deb uning noldan farqli minorlarining eng katta tartibiga aytiladi. Nol matritsaning rangi nolga teng deb qabul qilinadi. Matritsadagi tartibi matritsaning rangiga teng noldan farqli har qanday minor *bazis minor* deyiladi. A matritsaning rangi $r(A)$ yoki rang(A) kabi belgilanadi. Agar $r(A)=r(B)$ bo'lsa, A va B *ekvivalent matritsalar* deyiladi va $A\sim B$ kabi yoziladi.

Matritsaning rangini hisoblashda, juda ko'p determinantlarni hisoblab o'tirmaslik uchun, elementar almashtirishlardan foydalaniladi. Matritsaning *elementar almashtirishlari* deb quyidagilarga aytiladi:

1) barcha satrlarni mos ustunlar bilan yoki ustunlarni mos satrlar bilan almashtirish;
2) satrlar (ustunlar) o'rinlarini almashtirish;
3) barcha elementlari nollardan iborat satrni (ustunni) o'chirsh;
4) satrni noldan farqli songa ko'paytirish;
5) bir satrning (ustunning) elementlariga boshqa satrning (ustunning) elementlarini noldan farqli songa ko'paytirib qo'shish.

Elementar almashtirishlar natijasida matritsaning rangi o'zgarmaydi, ya'ni ekvivalent matritsalar hosil bo'ladi.

1-misol. Matritsaning rangini aniqlang:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix}.$$

► Berilgan matritsada satrlar elementlari proporsional bo'lganligi uchun barcha ikkinchi va uchinchi tartibli minorlar nolga teng. Birinchi tartibli minorlar, ya'ni elementlarning o'zi, noldan farqli bo'lganligi uchun bu matritsaning rangi 1 ga teng. ◄

2-misol. Matritsaning rangini va bazis minorlarini toping:

$$\begin{pmatrix} 3 & 5 & 7 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}.$$

► $$\begin{pmatrix} 3 & 5 & 7 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}\sim\begin{pmatrix} 4 & 8 & 12 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}\sim\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}\sim$$

$$\sim\begin{pmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}\sim\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}.$$

Bu yerda quyidagi elementar almashtirishlarni bajardik: birinchi satr elementlariga uchinchi satr elementlarini qo'shdik; birinchi satrning hosil bo'lgan elementlarini 4 ga bo'ldik; birinchi satrga ikkinchi satr elementlarini -1 ga ko'paytirib qo'shdik; hosil bo'lgan nollardan iborat birinchi satrni o'chirdik. Oxirgi matritsaning rangi, demak, berilgan matritsaning ham rangi 2 ga teng, chunki,

masalan, $\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}=3-2=1\ne 0.$

Qolgan bazis minorlar: $\begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix}$, $\begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix}$. ◄

3- misol. Matritsaning rangini toping:

$$A=\begin{pmatrix}1&2&1&3&4\\3&4&2&6&8\\1&2&1&3&4\end{pmatrix}.$$

Berilgam matritsada ketma-ket elementar almashtirishlar bajaramiz:

$$\begin{pmatrix}1&2&1&3&4\\3&4&2&6&8\\1&2&1&3&4\end{pmatrix}\sim\begin{pmatrix}1&2&1&3&4\\3&4&2&6&8\\0&0&0&0&0\end{pmatrix}\sim$$

$$\sim\begin{pmatrix}1&2&1&3&4\\1&0&0&0&0\\0&0&0&0&0\end{pmatrix}\sim\begin{pmatrix}1&2&1&3&4\\1&0&0&0&0\end{pmatrix}.$$

Ko‘rinib turibdiki, $r(A)=2$, chunki $\begin{vmatrix}1&2\\1&0\end{vmatrix}\neq 0.$ ◀

4- misol. Matritsaning rangini va bazis minorlarini toping:

$$A=\begin{pmatrix}0&2&-4\\-1&-4&5\\3&1&7\\0&5&-10\\2&3&0\end{pmatrix}.$$

▶ Ketma-ket elementar almashtirishlar bajarib, quyidagilarni olamiz:

$$\begin{pmatrix}0&2&-4\\-1&-4&5\\3&1&7\\0&5&-10\\2&3&0\end{pmatrix}\sim\begin{pmatrix}1&4&-5\\2&3&0\\3&1&7\\0&5&-10\\0&2&-4\end{pmatrix}\sim\begin{pmatrix}1&4&-5\\0&-5&10\\0&-11&22\\0&5&-10\\0&2&-4\end{pmatrix}\sim$$

$$\sim\begin{pmatrix}1&4&-5\\0&1&-2\\0&1&-2\\0&1&-2\\0&1&-2\end{pmatrix}\sim\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\\0&0&0\\0&0&0\end{pmatrix}.$$

Oxirgi matritsaning rangi, demak, berilgan matritsaning ham rangi ikkiga teng: $r(A)=2$.

Bazis minor bitta: $\begin{vmatrix}1&0\\0&1\end{vmatrix}$. ◀

Mustaqil bajarish uchun mashqlar

4.1. Matritsaning rangini aniqlang va bazis minorlarini toping:

1) $A=\begin{pmatrix}1&0&0&0&5\\0&0&0&0&0\\2&0&0&0&11\end{pmatrix}$;

2) $A=\begin{pmatrix}1&0&2&0&0\\0&1&0&2&0\\2&0&4&0&0\end{pmatrix}$;

4.2. Matritsaning rangini aniqlang:

1) $\begin{pmatrix}2&-1&3&-2&4\\4&-2&5&1&7\\2&-1&1&8&2\end{pmatrix}$; 2) $\begin{pmatrix}1&2&3&6\\2&3&1&6\\3&1&2&6\end{pmatrix}$;

3) $\begin{pmatrix}1&3&5&7&9\\1&-2&3&-4&5\\2&11&12&25&22\end{pmatrix}$; 4) $\begin{pmatrix}25&31&17&43\\75&94&53&132\\75&94&54&134\\25&32&20&48\end{pmatrix}$;

5) $\begin{pmatrix}47&-67&35&201&155\\26&98&23&-294&86\\16&-428&1&1284&52\end{pmatrix}$; 6) $\begin{pmatrix}3&1&1&4\\0&4&10&1\\1&7&17&3\\2&2&4&3\end{pmatrix}$.

Mustaqil bajarish uchun berilgan mashqlarning javoblari

1- §. 1.1. 1) 18. 2) $4ab$. 3) 1. 4) 2. 5) $\cos(\alpha+\beta)$. 6) x^2+12x. **1.2.** 1) 2. 2) $-1{,}7$. 3) $\frac{\pi}{6}+\frac{\pi n}{3}, \quad n\in Z$. 4) $\frac{\pi}{2}+\pi n, \quad n\in Z$. 5) 1; 3. 6) -5; 5. **1.3.** 1) 0. 2) 0. 3) $abc+(ab+bc+ac)x$. 4) $\alpha^2+\beta^2+\gamma^2+1$. 5) $(x^2-x)^2$. **1.4.** 1) 2; $-6{,}5$. 2) -1; -5. 3) $x\in R$. **1.5.** 1) $x>4$. 2) $-6<x<-4$. **1.7.** 1) 0. 2) 48. 3) 223. 4) $(be-cd)^2$. 5) $(b+c+d)(b+c-d)$. **1.8.** 1) $n!$. 2) $2n+1$. **1.9.** 1) $-a_1a_2\ldots a_n\left(\frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_n}\right)$. 2) $n+1$.

2- §. 2.1. 1) (3; 2). 2) (-4; 3). 3) $(-b;\ -\frac{2}{3}a)$. 4) ($-1{,}5$; $-0{,}5$). 5) $\varnothing$. 6) cheksiz ko'p yechimga ega. **2.2.** 1) (1; 3; 5). 2) (2; -1; 1). 3) $\varnothing$. 4) (3; 1; -1). 5) (2; -1; 1). 6) (1; -1; 2; -2). 7) (2; 0; 0; 0).

3- §. 3.1. 1) $\begin{pmatrix} 2 & 11 & 1 \\ 8 & 3 & -11 \end{pmatrix}$. 2) $AB=\begin{pmatrix} 5 & 2 \\ 15 & 20 \end{pmatrix}$; $BA=\begin{pmatrix} 29 & -6 \\ 31 & -4 \end{pmatrix}$; $\det(AB)=70$; $\det(BA)=70$. **3.2.** 1) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. 2) $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. 3) $\begin{pmatrix} 10 & 6 & 5 \\ 8 & 11 & 6 \\ 9 & 8 & 10 \end{pmatrix}$. 4) $\begin{pmatrix} 6 \\ 5 \\ -2 \\ 3 \end{pmatrix}$. 5) (12). 6) $\begin{pmatrix} 5 \\ 15 \\ 25 \\ 35 \end{pmatrix}$. 7) $\begin{pmatrix} 6 \\ 7 \\ 1 \end{pmatrix}$. 8) $\begin{pmatrix} a_1x_1+b_1x_2+c_1x_3 \\ a_2x_1+b_2x_2+c_2x_3 \\ a_3x_1+b_3x_2+c_3x_3 \end{pmatrix}$. 9) $\begin{pmatrix} \lambda^n & n\lambda^{n-1} \\ 0 & \lambda^n \end{pmatrix}$. **3.3.** 1) $\begin{pmatrix} 12 & 3 \\ 2 & 11 \end{pmatrix}$. 2) $\begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix}$. **3.4.** 1) $x=1$; $y=1$; $z=1$. 2) $x=3$; $y=2$; $z=1$.

4- §. 4.1. 1) $r(A)=2$; bazis minor: $\begin{vmatrix} 1 & 5 \\ 2 & 11 \end{vmatrix}$. 2) $r(A)=2$; bazis minorlar: $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$, $\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}$, $\begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix}$, $\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$, $\begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix}$, $\begin{vmatrix} 1 & 0 \\ 0 & 4 \end{vmatrix}$, $\begin{vmatrix} 0 & 2 \\ 4 & 0 \end{vmatrix}$.

4.2. 1) 2. 2) 3. 3) 2. 4) 3. 5) 2. 6) 2.

II bob. VEKTORLAR ALGEBRASI

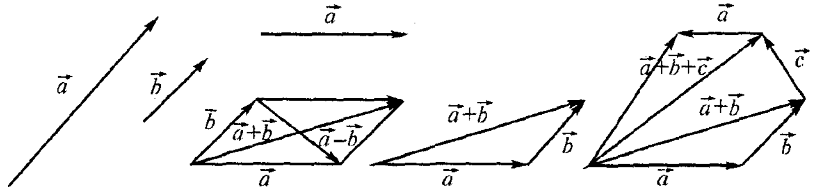
1- §. Vektorlar va ular ustida chiziqli amallar. Vektorning fazodagi to'g'ri burchakli koordinatalari

1°. Vektorlar va ular ustida chiziqli amallar. Fan va texnikada uchraydigan miqdorlarni (kattaliklarni), asosan, ikki turga ajratish mumkin: *skalar* va *vektor* miqdorlar. Skalar miqdor o'z son qiymati bilan to'la aniqlanadi. Vektor miqdor esa kattaligi(moduli)dan tashqari yo'nalishi bilan ham aniqlanadi. Masalan, uzunlik, yuz, hajm, zichlik, massa, temperatura va h.k.lar skalar miqdorlar; tezlik, tezlanish, kuch, kuch momenti, elektr (magnit) maydon kuchlanganligi kabi miqdorlar esa vektor miqdorlardir. Vektor miqdorlarni o'rganish uchun vektorlardan foydalaniladi.

Vektor (aniqrog'i *geometrik vektor*) deb yo'nalgan kesmaga aytiladi. Vektor boshi va oxirini ko'rsatgan holda yoki bitta harf bilan belgilanadi. Masalan, $\overline{AB}$ yoki $\vec{a}$ vektor (2-*rasm*). Bunda A nuqta vektorning *boshi*, B nuqta esa *oxiri* deyiladi. $\overline{AB}=\vec{a}$ vektorning uzunligi uning *moduli* (yoki *absolut qiymati*) deyilib, $|\overline{AB}|$, $AB=a=|\vec{a}|$ kabi belgilanadi. Boshi va oxiri ustma-ust tushuvchi vektor *nol vektor* deyilib, $\vec{0}$ kabi belgilanadi. Uning moduli nolga teng, yo'nalishi aniqlanmagan. $\overline{AB}$ va $\overline{BA}$ o'zaro qarama-qarshi vektorlar deyiladi:

$$\overline{BA}=-\overline{AB}, \qquad \overline{AB}+\overline{BA}=\vec{0}.$$

Bir to'g'ri chiziqda yoki o'zaro parallel to'g'ri chiziqlarda yotuvchi vektorlar *kollinear vektorlar* deyilib, $\vec{a}$ va $\vec{b}$ ning kollinearligi $\vec{a}\parallel\vec{b}$ kabi ko'rsatiladi. Nol vektor har qanday vektorga kollinear deb hisoblanadi. Kollinear vektorlar bir xil yoki qarama-qarshi yo'nalgan bo'lishi mumkin.



2- *rasm.* 3- *rasm.* 4- *rasm.*

$\vec a$ va $\vec b$ vektorlar teng modulga ega, kollinear va bir xil yo'nalgan bo'lsa, ular *o'zaro teng vektorlar* deyilib, $\vec a=\vec b$ kabi yoziladi. Bu ta'rifdan vektorni fazoda (tekislikda) o'z-o'ziga parallel ko'chirish mumkin ekanligi kelib chiqadi.

Bitta tekislikda yoki o'zaro parallel tekisliklarda yotuvchi vektorlar *komplanar vektorlar* deyiladi. O'z-o'ziga parallel ko'chirib, kollinear vektorlarni bitta to'g'ri chiziqqa, komplanar vektorlarni bitta tekislakka joylashtirish mumkin. Shuning uchun ikki vektorga parallelogramm yoki uchburchak qurish uchun ular kollinear bo'lmasligi, uch vektorga parallelepiped yoki piramida qurish uchun ular komplanar bo'lmasligi kerak.

Vektorlarni qo'shish, ayirish va songa ko'paytirish amallari vektorlar ustida *chiziqli amallar* deyiladi. Vektorlarni qo'shish uchun *parallelogramm qoidasi* (3- rasm) yoki uchburchak qoidasidan (4- rasm) foydalaniladi. Keyingi usul yordamida ikkitadan ko'p vektorlarni ham qo'shish mumkin, bu holda qo'shish usuli *ko'pburchaklar qoidasi* ham deyiladi (4- rasm). Vektorlarni qo'shish quyidagi xossalarga ega:

1. $\vec a+\vec 0=\vec a$. 2. $\vec a+\vec b=\vec b+\vec a$.

3. $\vec a+(\vec b+\vec c)=(\vec a+\vec b)+\vec c$. 4. $\vec a+(-\vec a)=\vec 0$.

Kuchlarni ifodalovchi vektorlarning yig'indisi shu kuchlarning teng ta'sir etuvchisidan iborat vektorga teng.

$\vec a$ vektordan $\vec b$ vektorning *ayirmasi* deb, $\vec b$ vektor bilan yig'indisi $\vec a$ vektorni beradigan $\vec c=\vec a-\vec b$ vektorga aytiladi (3- rasm): $\vec c+\vec b=\vec a$, $\vec c$ vektor kamayuvchi $\vec a$ vektor tomonga qarab yo'nalgan bo'lishini unutmaslik kerak.

$\vec a$ vektorning λ songa *ko'paytmasi* deb moduli $|\lambda|\ |\vec a|$ ga teng, yo'nalishi esa $\lambda>0$ bo'lsa, $\vec a$ bilan bir xil, $\lambda<0$ bo'lganida $\vec a$ ga qarama-qarshi bo'lgan vektorga aytiladi. Bektorni songa ko'paytirish amali quyidagi xossalarga ega:

1. $\vec a\cdot 0=0\cdot\vec a=\vec 0$.
2. $\lambda(\vec a+\vec b)=\lambda\vec a+\lambda\vec b$.
3. $(\lambda_1+\lambda_2)\vec a=\lambda_1\vec a+\lambda_2\vec b$.
4. $\lambda_1(\lambda_2\vec a)=\lambda_2(\lambda_1\vec a)$.

Moduli (uzunligi) 1 ga teng vektor *birlik vektor* deyiladi. $\vec a$ vektor bo'ylab yo'nalgan birlik vektor, ko'pincha, $\vec a^0$ kabi belgilanib, u $\vec a^0=\frac{\vec a}{|\vec a|}$ munosabatdan topiladi.

Agar $\vec b$ vektorning Ox o'qi bilan tashkil etgan burchagi φ bo'lsa, uning bu o'qqa proeksiyasi: $\text{pr}_{Ox}\ \vec b=|\vec b|\cdot\cos\varphi$ formula bilan topiladi (41-bet, 10- rasmga q.).

Quyidagi xossa o'rinli: $\text{pr}_{Ox}(\vec a+\vec b)=\text{pr}_{Ox}\ \vec a+\text{pr}_{Ox}\ \vec b$.

1-misol. $ABCD$ parallelogrammda $\overline{AB}=\vec a$, $\overline{AD}=\vec b$ deb belgilangan. M nuqta parallelogramm diagonallarining kesishish nuqtasi. $\overline{MA}$, $\overline{MB}$, $\overline{MC}$, $\overline{MD}$ larni $\vec a$ va $\vec b$ orqali ifodalang.

▶ Vektorlar yig'indisi va ayirmasi ta'rifiga asosan 5- rasmdan:

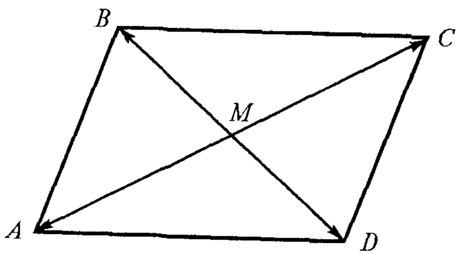
$$\overrightarrow{AB}=\overrightarrow{DC}=\vec a,\ \overrightarrow{AD}=\overrightarrow{BC}=\vec b,$$

$$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=\vec a+\vec b,\ \overrightarrow{DB}=\vec a-\vec b,$$

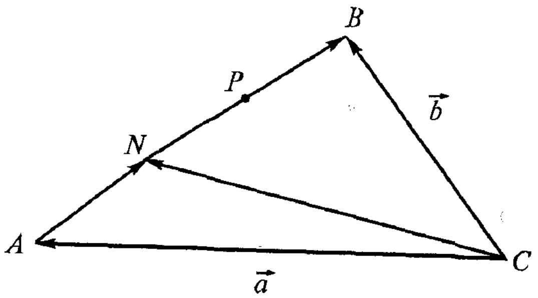
$$\overrightarrow{MA}=-\tfrac{1}{2}\overrightarrow{AC}=-\tfrac{1}{2}(\overrightarrow{AB}+\overrightarrow{BC})=-\tfrac{1}{2}(\overrightarrow{AB}+\overrightarrow{BD})=-\tfrac{1}{2}(\vec a+\vec b);$$

$$\overrightarrow{MB}=\tfrac{1}{2}\overrightarrow{DB}=\tfrac{1}{2}(\vec a-\vec b);\qquad \overrightarrow{MC}=\tfrac{1}{2}\overrightarrow{AC}=\tfrac{1}{2}(\vec a+\vec b);$$

$$\overrightarrow{MD}=\tfrac{1}{2}\overrightarrow{BD}=-\tfrac{1}{2}\overrightarrow{DB}=-\tfrac{1}{2}(\vec a-\vec b)=\tfrac{1}{2}(\vec b-\vec a).$$ ◀



5- rasm.



6- rasm.

2-misol. *ABC* ucburchakda *AB* tomon *N* va *P* nuqtalar bilan uchta teng qismga bo'lingan: *AN= NP= PB.* Agar $\overline{CA}=\vec{a}$, $\overline{CB}=\vec{b}$ vektorlar berilgan bo'lsa, $\overline{CN}$ vektorni toping.

▶ *ABC* uchburchakni va berilgan vektorlarni shaklda tasvirlaymiz (6- rasm). $\overline{AB}=\vec{b}-\vec{a}$ bo'lganidan:

$$\overline{AN}=\frac{1}{3}(\vec{b}-\vec{a}); \quad \overline{CN}=\overline{CA}+\overline{AN}=\vec{a}+\frac{1}{3}(\vec{b}-\vec{a})=\frac{2}{3}\vec{a}+\frac{1}{3}\vec{b}. \blacktriangleleft$$

2°. Bazis. Nuqtaning va vektorning koordinatalari. Fazoda istalgan tartiblangan uchta $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$ nokomplanar vektorlar *bazis* deyiladi. Har qanday $\vec{a}$ vektor ular orqali yagona ravishda ifodalanadi: $\vec{a}=x_1\vec{e}_1+x_2\vec{e}_2+x_3\vec{e}_3$.

Bunda x_1, x_2, x_3 sonlar vektorning $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ bazisdagi *koordinatalari* deyiladi. Tekislikda istalgan ikkita $(\vec{e}_1, \vec{e}_2)$ nokollinear

vektor *bazis* deyiladi va tekislikdagi istalgan $\vec{a}$ vektorni yagona ravishda $\vec{a}=x_1\vec{e}_1+x_2\vec{e}_2$ deb yozish mumkin. To'g'ri chiziqda (son o'qida) istalgan noldan farqli $\vec{e}$ vektor bazis deyiladi va har qanday $\vec{a}$ vektorni $\vec{a}=x\vec{e}$ deb yozish mumkin.

$\vec{a}$, $\vec{b}$, $\vec{c}$ ..., $\vec{d}$ vektorlar sistemasi *chiziqli bog'liq* deyiladi, agar kamida biri noldan farqli *k, m, n, ..., l* sonlar topish mumkin bo'lib, ular uchun

$$k\vec{a}+m\vec{b}+n\vec{c}+\ \dots+l\vec{d}=\vec{0}$$

tenglik bajarilsa. Bu tenglik faqat $k=m=n=\dots=0$ bo'lganda bajarilsa, *chiziqli erkli sistema* deyiladi. Vektorlar sistemasi chiziqli bog'liq bo'lsa, ulardan birini qolganlari orqali *chiziqli ifodalash* mumkin, masalan, $l\neq 0$ bo'lsa: $\vec{d}=p\vec{a}+q\vec{b}+\dots+g\vec{c}$.

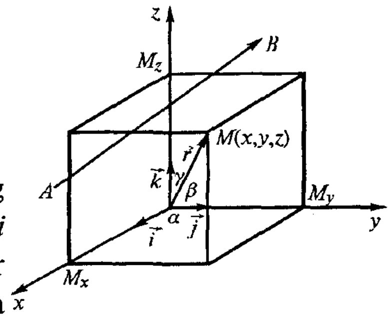
Bu holda $\vec{d}$ vektor $\vec{a}$, $\vec{b}$, ..., $\vec{c}$ vektorlarning *chiziqli kombinatsiyasi* (yoki $\vec{a}$, $\vec{b}$, ..., $\vec{c}$ lar orqali *yoyilmasi*) deyiladi.

Agar $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$ lar o'zaro perpendikular birlik vektorlar bo'lsa, $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ bazis *to'g'ri burchakli bazis* deyilib, bu holda ular uchun $\vec{e}_1=\vec{i}$, $\vec{e}_2=\vec{j}$, $\vec{e}_3=\vec{k}$ belgilashlar ishlatiladi:

$$\vec{a}=x_1\vec{i}+x_2\vec{j}+x_3\vec{k}. \qquad (1)$$

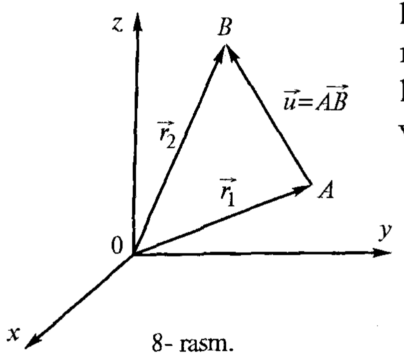
Umumiy *O* nuqtaga ega, o'zaro perpendikular *Ox, Oy, Oz* koordinata o'qlari va *M* nuqta berilgan bo'lsin (7- rasm). $\vec{r}=\overline{OM}$ vektor *M* nuqtaning *radius-vektori*, uning o'qlardagi *proyeksiyalari*

$$\begin{aligned} pr_{Ox}\vec{r}&=OM_x=x,\\ pr_{Oy}\vec{r}&=OM_y=y,\\ pr_{Oz}\vec{r}&=OM_z=z \end{aligned} \qquad (2)$$



7- rasm.

esa *M* nuqtaning yoki $\vec{r}$ vektorning *to'g'ri burchakli koordinatalari* deyiladi. Ular orqali radius-vektor $\vec{r}\{x;\ y;\ z\}$ kabi yoziladi. Koordinata o'qlarining birlik vektorlari $\vec{i}, \vec{j}, \vec{k}$



8- rasm.

lar *ortlar* deyilib, ular orqali radius-vektor $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ ko'rinishda ifodalanadi. Radius vektorning moduli (uzunligi)

$$r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2} \qquad (3)$$

formula orqali, yo'nalishi esa

$$\cos\alpha = \frac{x}{r}, \ \cos\beta = \frac{y}{z},$$

$$\cos\gamma = \frac{z}{r}$$

yo'naltiruvchi kosinuslar yordamida aniqlanadi, bunda $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

Boshi $A(x_1, y_1, z_1)$ nuqtada va oxiri $B(x_2, y_2, z_2)$ nuqtada bo'lgan $\vec{u} = \overrightarrow{AB}$ vektor uchun (8- rasm):

$$\vec{r}_1 + \overrightarrow{AB} = \vec{r}_2; \quad \vec{u} = \overrightarrow{AB} = \vec{r}_2 - \vec{r}_1 = \overrightarrow{AB}\{x_2 - x_1; \ y_2 - y_1; \ z_2 - z_1\}$$

yoki

$$\vec{u} = \overrightarrow{AB} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}; \qquad (5)$$

$$u = |\vec{u}| = AB = |\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}; \qquad (6)$$

$$\cos\alpha = \frac{x_2 - x_1}{AB}, \quad \cos\beta = \frac{y_2 - y_1}{AB}, \quad \cos\gamma = \frac{z_2 - z_1}{AB}; \qquad (7)$$

$$\text{pr}_{Ox}\overrightarrow{AB} = x_2 - x_1; \quad \text{pr}_{Oy}\overrightarrow{AB} = y_2 - y_1; \quad \text{pr}_{Oz}\overrightarrow{AB} = z_2 - z_1. \ (8)$$

3-misol. Uchta $\vec{e}_1(1; 0; 0)$, $\vec{e}_2(1; 1; 0)$, $\vec{e}_3(1; 1; 1)$ nokomplanar vektorlar berilgan. $\vec{a} = -2\vec{i} - \vec{k}$ vektorning $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ bazisdagi koordinatalarini toping va $\vec{a}$ ni shu bazis bo'yicha yoying.

▶ Istalgan vektorni bazis bo'yicha yoyish mumkin bo'lganidan:

$$\vec{a} = x_1\vec{e}_1 + x_2\vec{e}_2 + x_3\vec{e}_3;$$

$$-2\vec{i} - \vec{k} = x_1 \cdot (1; 0; 0) + x_2 \cdot (1; 1; 0) + x_3 \cdot (1; 1; 1);$$

$$(x_1; 0; 0) + (x_2; x_2; 0) + (x_3; x_3; x_3) = (-2; 0; -1);$$

bundan:

$$x_1 + x_2 + x_3 = -2, \qquad x_2 + x_3 = 0, \qquad x_3 = -1.$$

$$\begin{cases} x_1 + x_2 + x_3 = -2, \\ x_2 + x_3 = 0, \\ x_3 = -1. \end{cases} \qquad \begin{cases} x_3 = -1, \\ x_2 = 1, \\ x_1 = -2. \end{cases}$$

Demak, $\vec{a} = -2\vec{e}_1 + \vec{e}_2 - \vec{e}_3$. ◀

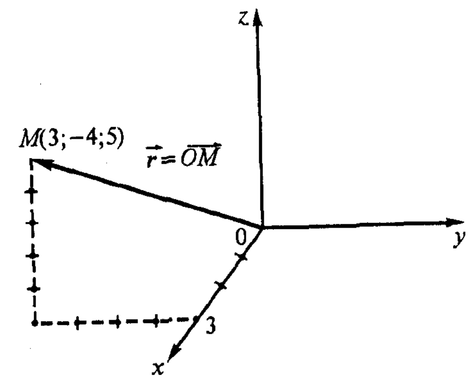
4-misol. $M(3; -4; 5)$ nuqtani yasang, uning radius-vektori modulini va yo'nalishini aniqlang.

▶ $M(3; -4; 5)$ nuqtani yasaymiz (9- rasm) va (1) — (3) formulalarga ko'ra radius-vektorini yozamiz, moduli va yo'nalishini topamiz:

$$\vec{r} = \overrightarrow{OM} = 3\vec{i} - 4\vec{j} + 5\vec{k} = \vec{r}(3; -4; 5);$$

$$r = \sqrt{3^2 + (-4)^2 + 5^2} = 5\sqrt{2};$$

$$\cos\alpha = \frac{x}{r} = \frac{3}{5\sqrt{2}}; \qquad \cos\beta = \frac{y}{r} = \frac{-4}{5\sqrt{2}};$$



9- rasm.

$$\cos\gamma = \frac{z}{r} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

Radius-vektorning Ox, Oy, Oz koordinata o‘qlari bilan tashkil etgan burchaklari:

$$\alpha = \arccos\frac{3}{5\sqrt{2}};\ \beta = \arccos\left(-\frac{4}{5\sqrt{2}}\right);\ \gamma = 45°. ◀$$

5-misol. Parallelogrammning uchta uchi: $A(1; -2; 3)$, $B(3; 2; 1)$ va $C(6; 4; 4)$ berilgan. To‘rtinchi D uchini va BD diagonalining uzunligini toping.

▶ Parallelogrammning xossasiga ko‘ra AD va BC tomonlar parallel va teng. Bunda $D(x; y; z)$ desak,

$$\overline{AD} = \overline{BC};\qquad \overline{AD}\{x-1;\ y+2;\ z-3\} = \overline{BC}\{6-3;\ 4-2;\ 4-1\};$$

$$x-1=3;\quad x=4;$$
$$y+2=2;\quad y=0;$$
$$z-3=3;\quad z=6.$$

kelib chiqadi. Demak, $D(4; 0; 6)$.

BD diagonalning uzunligi $\overline{BD}\{4-3;\ 0-2;\ 6-1\} = \overline{BD}\{1; -2; 5\}$ vektorning uzunligiga teng bo‘lganligidan:

$$BD = \left|\overline{BD}\right| = \sqrt{1^2+(-2)^2+5^2} == \sqrt{30}.\quad BD = \sqrt{30}. ◀$$

Mustaqil bajarish uchun mashqlar

1.1. Vektor tengliklarning to‘g‘riligini analitik va geometrik usullarda isbotlang:

1) $\vec a + \frac{\vec b-\vec a}{2} = \frac{\vec a+\vec b}{2}$. 2) $\vec a - \frac{\vec a-\vec b}{2} = \frac{\vec a-\vec b}{2}$.

1.2. $\overline{AD}$, $\overline{BE}$ va $\overline{CF}$ lar ABC uchburchakning medianalari. $\overline{AD}+\overline{BE}+\overline{CF}=\vec 0$ tenglikning bajarilishini isbotlang.

1.3. ABC uchburchakda AP kesma BAC burchakning bissektrisasi, P nuqta BC tomonda yotadi. Agar $\overline{AB}=\vec b$, $\overline{AC}=\vec c$ bo‘lsa, $\overline{AP}$ ni toping.

1.4. $\overline{AB}=\vec a+2\vec b$, $\overline{BC}=-4\vec a-\vec b$, $\overline{CD}=-5\vec a-3\vec b$ bo‘lsa, $ABCD$ ning trapetsiya ekanini isbotlang.

1.5. $\overline{OA}=\vec a$, $\overline{OB}=\vec b$, $\overline{OC}=\vec c$ nokomplanar vektorlarga yasalgan parallelepipedning $\vec a+\vec b-\vec c$, $\vec a-\vec b+\vec c$, $\vec a-\vec b-\vec c$ va $\vec b-\vec a-\vec c$ vektor-diagonallarini yasang.

1.6. Uchta nokomplanar $\vec m$, $\vec n$, $\vec p$ birlik vektorlar uchun $(\vec m,\hat{\ }\vec n)=30°$, $(\vec n,\hat{\ }\vec p)=60°$ bo‘lsa, $\vec u=\vec m+2\vec n-3\vec p$ vektorni yasang va uning modulini toping.

Ko‘rsatma: $\vec m$, $2\vec n$ va $-3\vec p$ larga yasalgan siniq chiziqda $\vec m$ ni $\left(-3\vec p\right)$ bilan kesishguncha davom ettiring.

1.7. $OACB$ to‘g‘ri to‘rtburchaknung OA va OB tomonlari bo‘ylab $\vec i$ va $\vec j$ birlik vektorlar qo‘yilgan. Agar $OA=3$, $OB=4$, M va N nuqtalar BC va AC kesmalarning o‘rtalari bo‘lsa, $\overline{OA}$, $\overline{AC}$, $\overline{CB}$, $\overline{OC}$, $\overline{OM}$, $\overline{ON}$, $\overline{MN}$ vektorlarni $\vec i$ va $\vec j$ orqali ifodalang.

1.8. $OACB$ teng yonli trapetsiyada $\angle BOA=60°$, $OB=BC=CA=2$, M va N nuqtalar BC va AC tomonlarning o‘rtalari. $\overline{AC}$, $\overline{OM}$, $\overline{ON}$, $\overline{MN}$ vektorlarni $\overline{OA}$ va $\overline{OB}$ bo‘ylab qo‘yilgan $\vec m$ va $\vec n$ birlik vektorlar yordamida ifodalang.

1.9. Tekislikda $A(0; -2)$, $B(4; 2)$ va $C(4; -2)$ nuqtalar berilgan. Koordinatalar boshida $\overline{OA}$, $\overline{OB}$ va $\overline{OC}$ kuchlar qo‘yilgan. Ularning $\overline{OM}$ teng ta’sir etuvchisini yasang, uning koordinata o‘qlariga proyeksiyalarini va kattaligini toping. $\overline{OA}$, $\overline{OB}$, $\overline{OC}$ va $\overline{OM}$ vektorlarni koordinata o‘qlari birlik vektorlari $\vec i$ va $\vec j$ orqali ifodalang.

1.10. $OABCDE$ muntazam oltiburchakning tomoni 3 ga teng. $\overline{OA}$, $\overline{AB}$, $\overline{BC}$ larning birlik vektorlarini $\vec m$, $\vec n$, $\vec p$ deb, ular orasidagi bog‘lanishni toping. $\overline{OB}$, $\overline{EO}$, $\overline{OD}$ va $\overline{DA}$ larni $\vec m$, $\vec n$ va $\vec p$ orqali ifodalang.

1.11. $M(2, 3, -6)$ nuqtani yasang, uning radius-vektori uzunligini va yo‘nalishini aniqlang.

1.12. $\vec{r} = \overline{OM} = 6\vec{i} - 6\vec{k}$ vektorni yasang, uning uzunligini va yo‘nalishini aniqlang.

1.13. $A(-1, 0, 1)$ va $B(1, -6, 4)$ nuqtalar berilgan. $\overline{AB} = \vec{u}$ vektorni, uning koordinata o‘qlaridagi proyeksiyalarini yasang, uzunligini va yo‘nalishini aniqlang.

1.14. Koordinata o‘qlari bilan teng o‘tkir burchaklar tashkil etuvchi va moduli $a = 2\sqrt{3}$ ga teng bo‘lgan $\vec{a}$ vektorni toping.

1.15. $\vec{j}$ va $\vec{k}$ ortlar bilan 60° va 120° li burchaklar tashkil etgan va $|\vec{x}| = 5\sqrt{2}$ bo‘lgan $\vec{x}$ vektorni toping.

1.16. $\overline{OA} = 2\vec{i} + 3\vec{j}$ va $\overline{OB} = -2\vec{i} + 4\vec{j}$ vektorlarga parallelogramm yasang va uning diagonallari uzunliklarini toping.

1.17. $\vec{a} = 4\vec{i} - 8\vec{j} + 2\sqrt{5}\vec{k}$ vektor yo‘nalishidagi birlik vektorni toping.

1.18. $A(a; 0; 0)$, $B(0; 0; 2a)$ va $C(a; 0; a)$ nuqtalar berilgan. $\overline{OC}$ va $\overline{AB}$ vektorlarni yasang va uzunliklarini toping.

2- §. Ikki vektorning skalar ko‘paytmasi

Ikki vektorning skalar ko‘paytmasi deb shu vektorlar modullari bilan ular orasidagi burchak kosinusining ko‘paytmasiga aytiladi:

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi. \qquad (1)$$

Skalar ko‘paytmani yana $(\vec{a}, \vec{b})$, $\vec{a}\vec{b}$ kabi ham belgilash mumkin.

Skalar ko‘paytmaning xossalari:

1. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}.$

2. $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}.$

3. $\lambda \cdot \vec{a} \cdot \vec{b} = \lambda \cdot (\vec{a} \cdot \vec{b}).$

4. $a = \sqrt{\vec{a} \cdot \vec{a}} = \sqrt{\vec{a}^2}.$

5. $\vec{a} \cdot \vec{b} = 0 \iff \vec{a} \perp \vec{b}, \quad \varphi = \frac{\pi}{2}.$

Koordinata o‘qlarining birlik vektorlari — ortlarning skalar ko‘paytmalari:

$$\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1; \qquad \vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{i} \cdot \vec{k} = 0.$$

$b \cdot \cos\varphi = pr_{\vec{a}}\vec{b}$ bo‘lganidan (10- rasm)

$$pr_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{a}.$$

$\vec{a}$ va $\vec{b}$ vektorlar to‘g‘ri burchakli bazisdagi koordinatalari bilan berilgan bo‘lsa, ya’ni $\vec{a}\{a_1;\ a_2;\ a_3\}$, $\vec{b}\{b_1;\ b_2;\ b_3\}$ bo‘lsa, u holda skalar ko‘paytma quyidagiga teng bo‘ladi:

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3.$$

Ikkita $\vec{a}$ va $\vec{b}$ vektor orasidagi burchakni topish formulasi:

$$\cos\left(\vec{a}, \widehat{\ } \vec{b}\right) = \frac{\vec{a} \cdot \vec{b}}{a \cdot b}$$

yoki

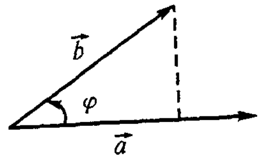
$$\cos\left(\vec{a}, \widehat{\ } \vec{b}\right) = \frac{a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \cdot \sqrt{b_1^2 + b_2^2 + b_3^2}}.$$

Ikki vektorning perpendikularlik sharti:

$\vec{a} \cdot \vec{b} = 0$ yoki $a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3 = 0.$

Ikki vektorning kollinearlik sharti:

$$\vec{a} = \lambda\vec{b} \iff \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}.$$



10- rasm.

$\vec{F}$ kuchning moddiy nuqtani $\vec{S}$ vektor bo'yicha ko'chirishda bajargan ishi quyidagicha hisoblanadi:

$$A = \vec{F} \cdot \vec{S}.$$

1- misol. $\vec{a} = 2\vec{i} + \vec{j}$ va $\vec{b} = -2\vec{j} + \vec{k}$ vektorlarga yasalgan parallelogrammning diagonallari orasidagi burchakni toping.

▶ Diagonallar $\vec{c} = \vec{a} + \vec{b} = 2\vec{i} - \vec{j} + \vec{k}$ va $\vec{d}=\vec{a}-\vec{b}=2\vec{i}+3\vec{j}-\vec{k}$ vektorlar bo'lganligi uchun ular orasidagi burchak quyidagicha topiladi:

$$\cos\left(\vec{c}, \hat{\ } \ \vec{d}\right) = \frac{\vec{c}\cdot\vec{d}}{c\cdot d} = \frac{2\cdot2-1\cdot3+1\cdot(-1)}{\sqrt{2^2+(-1)^2+1^2}\cdot\sqrt{2^2+3^2+(-1)^2}} =$$

$$= \frac{0}{\sqrt{6}\cdot\sqrt{14}} = 0, \quad \left(\vec{c}, \hat{\ } \ \vec{d}\right) = 90°. \ ◀$$

2- misol. Uchlari $A(1;\ 2;\ -4)$, B(4; 2; 0) va C(–3; 2; –1) nuqtalarda bo'lgan uchburchakning perimetrini va burchaklarini toping.

▶ $$\angle A = \left(\overline{AB}, \hat{\ } \ \overline{AC}\right), \quad \angle B = \left(\overline{BA}, \hat{\ } \ \overline{BC}\right),$$

$$\angle C = \left(\overline{CA}, \hat{\ } \ \overline{CB}\right) = 180° - (\angle A + \angle B)$$

ekanligidan foydalanamiz. U holda $\overline{AB} = \overline{AB}\{3;\ 0;\ 4\}$, $\overline{AC} =$ $= \overline{AC}\{-4;\ 0;\ 3\}$,

$$\overline{BC} = \overline{BC}\{-7;\ 0;\ 1\}, \quad \overline{BA} = \overline{BA}\{-3;\ 0;\ -4\},$$

$$\overline{CA} = \overline{CA}\{4;\ 0;\ -3\}, \quad \overline{CB} = \overline{CB}\{7;\ \ 0;\ 1\};$$

$$AB = \sqrt{9+16} = 5, \quad AC = \sqrt{16+9} = 5, \quad BC = \sqrt{49+1} = 5\sqrt{2}.$$

$$\cos\angle A = \frac{\overline{AB}\cdot\overline{AC}}{AB\cdot AC} = \frac{3\cdot(-4)+0\cdot0+4\cdot3}{5\cdot5} = \frac{0}{5} = 0 \ ; \quad \angle A = 90°;$$

$$\cos\angle B = \frac{\overline{BA}\cdot\overline{BC}}{BA\cdot BC} = \frac{-3\cdot(-7)+0\cdot0-4\cdot(-1)}{5\cdot5\sqrt{2}} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}; \quad \angle A = 45°.$$

Unda $\angle C = 180° - (90° + 45°) = 45°$. Uchburchakning perimetri:

$$P = AB + AC + BC = 5 + 5 + 5\sqrt{2}; \quad P = 5\left(2+\sqrt{2}\right). \ ◀$$

3- misol. $\vec{a}$, $\vec{b}$ va $\vec{c}$ komplanar vektorlar uchun $a = 3$, $b = 2$, $c = 5$, $\left(\vec{a}, \hat{\ } \ \vec{b}\right) = 60°$ va $\left(\vec{b}, \hat{\ } \ \vec{c}\right) = 60°$. $\vec{u} = \vec{a} + \vec{b} - \vec{c}$ vektorni yasang va uning modulini toping.

▶ $\vec{u} = \vec{a} + \vec{b} + (-\vec{c})$, ya'ni $\vec{u}$ vektor $\vec{a}$, $\vec{b}$ va $-\vec{c}$ vektorlar yig'indisidan iborat, uni shakldagidek yasaymiz (11- rasm).

Rasmdan ko'rinadiki, $\left(\vec{a}, \hat{\ } -\vec{c}\right) = 120°$. $\vec{u}$ ning modulini $u = \sqrt{\vec{u}^2}$ formula bo'yicha topamiz:

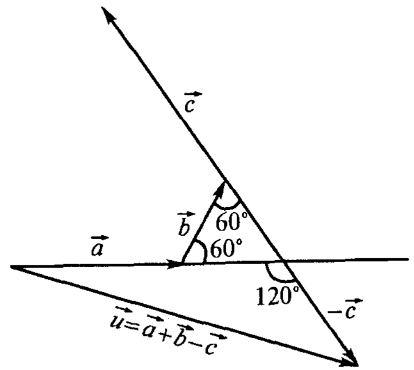
$$u = \sqrt{\vec{u}^2} = \sqrt{\left(\vec{a}+\vec{b}-\vec{c}\right)^2} = \sqrt{\left(\vec{a}+\vec{b}+(-\vec{c})\right)^2} =$$

$$= \sqrt{\vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\cdot\vec{a}\cdot\vec{b} - 2\cdot\vec{b}\cdot\vec{c} - 2\cdot\vec{a}\cdot\vec{c}} =$$

$$= \sqrt{a^2 + b^2 + c^2 + 2\cdot a\cdot b\cdot\cos 60° - 2\cdot b\cdot c\cdot\cos 60° - 2\cdot a\cdot c\cdot\cos 120°} =$$

$$= \sqrt{9+4+25+2\cdot3\cdot2\cdot\tfrac{1}{2} - 2\cdot2\cdot5\cdot\tfrac{1}{2} - 2\cdot3\cdot5\cdot\left(-\tfrac{1}{2}\right)} = \sqrt{49} = 7;$$

$$u = 7. \ ◀$$



11- rasm.

Mustaqil bajarish uchun mashqlar

2.1. $\vec{a}$, $\vec{b}$ vektorlar uchun $a = 2$, $b = 3$, $\left(\vec{a}, \hat{\ } \vec{b}\right) = \frac{2\pi}{3}$ bo'lsa, quyidagilarni toping.
1) $\vec{a} \cdot \vec{b}$; 2) $\left(2\vec{a} + 4\vec{b}\right) \cdot \left(\vec{a} + 2\vec{b}\right)$; 3) $\left(\vec{a} + \vec{b}\right)^2$.

2.2. $\vec{a} = \vec{i} - \vec{j}$ va $\vec{b} = -\vec{i} + 2\vec{j} - 2\vec{k}$ vektorlar orasidagi burchakni toping.

2.3. $c = 3$, $d = 5$ bo'lsa, α ning qanday qiymatlarida $\vec{c} + \alpha\vec{d}$ va $\vec{c} - \alpha\vec{d}$ vektorlar perpendikular bo'ladi?

2.4. $\vec{a} = \vec{e}_1 + 2\vec{e}_2$ va $\vec{b} = 5\vec{e}_1 - 4\vec{e}_2$ vektorlar o'zaro perpendikular bo'lsa, $\vec{e}_1$ va $\vec{e}_2$ birlik vektorlar orasidagi burchakni toping.

2.5. $\vec{a}_1(4;\ -2;\ -4)$ va $\vec{a}_2(6;\ -3;\ 2)$ vektorlar berilgan. Quyidagilarni toping: 1) $\vec{a}_1 \cdot \vec{a}_2$; 2) $\left(2\vec{a}_1 - 3\vec{a}_2\right)\left(\vec{a}_1 + 2\vec{a}_2\right)$; 3) $\left(\vec{a}_1 - \vec{a}_2\right)^2$; 4) $\left|2\vec{a}_1 - \vec{a}_2\right|$; 5) $\text{pr}_{\vec{a}_1} \vec{a}_2$; 6) $\text{pr}_{\vec{a}_2} \vec{a}_1$.

2.6. $A(2;\ 2)$ va $B(5;\ -2)$ nuqtalar berilgan. Abssissalar o'qida shunday P nuqtani topingki, $\angle APB = \frac{\pi}{2}$ bo'lsin.

2.7. Uchlari $A(2;\ -1;\ 3)$, $B(1;\ 1;\ 1)$ va $C(0;\ 0;\ 5)$ nuqtalarda bo'lgan uchburchakning burchaklarini toping.

2.8. Tekislikda uchlari $O(0;\ 0)$, $A(2a;\ 0)$ va $B(a;\ -a)$ nuqtalarda bo'lgan uchburchak berilgan. OB tomon va OM mediana orasidagi burchakni toping.

2.9. $\vec{a} = 3\vec{i} + 4\vec{j}$ va $\vec{b} = 4\vec{i} - 5\vec{j} + 3\vec{k}$ vektorlar berilgan. $\text{pr}_{\vec{a}}\vec{b}$ va $\text{pr}_{\vec{b}}\vec{a}$ ni toping.

2.10. Ifodani hisoblang: $\left(2\vec{i} + 3\vec{j}\right)\vec{j} + \left(3\vec{j} - \vec{k}\right)\vec{k} + \left(2\vec{j} + \vec{k}\right)\left(\vec{i} - \vec{j}\right)$.

2.11. $a = 2\sqrt{2}$, $b = 4$, $\left(\vec{a}, \hat{\ } \vec{b}\right) = 135°$ bo'lsa, $\left(\vec{a} - \vec{b}\right)^2$ ni toping.

2.12. $\vec{m}$ va $\vec{n}$ birlik vektorlar va $\left(\vec{m}, \hat{\ } \vec{n}\right) = 30°$ bo'lsa, $(\vec{m} + \vec{n})^2$ ni toping.

2.13. $\vec{m}$ va $\vec{n}$ birlik vektorlar va $\left(\vec{m}, \hat{\ } \vec{n}\right) = 60°$ bo'lsa, $\vec{a} = 2\vec{m} + \vec{n}$ va $\vec{b} = \vec{m} - 2\vec{n}$ vektorlarga yasalgan parallelogramm diagonallari uzunliklarini toping.

2.14. $ABCD$ parallelogrammning $A(2;\ 1;\ 3)$, $B(5;\ 2;\ -1)$, $C(-3;\ 3,\ -3)$ uchlari berilgan. AC va BD diagonallari orasidagi burchakning kosinusini toping.

2.15. Kvadratning uchidan shu uch yotmagan tomonlar o'rtalari orqali to'g'ri chiziqlar o'tkazilgan. Shu to'g'ri chiziqlar orasidagi burchakni toping.

2.16. Uchlari $A(-3;\ 5;\ 6)$, $B(1;\ -5;\ 7)$, $C(8;\ -3;\ -1)$ va $D(4;\ 7;\ -2)$ nuqtalarda bo'lgan to'rtburchakning kvadrat ekanligini isbotlang.

2.17. Moddiy nuqtani $\vec{F} = \vec{i} + 2\vec{j} + \vec{k}$ kuch ta'sirida $A(-1;\ 2;\ 0)$ nuqtadan $B(2;\ 1;\ 3)$ nuqtaga ko'chirishda bajarilgan ishni toping.

2.18. Harakatdagi nuqta ko'chishining koordinata o'qlaridagi proyeksiyalari $S_x = 2m$, $S_y = 1m$, $S_z = -2m$ va ta'sir etayotgan kuchning proyeksiyalari $F_x = 5\text{N}$, $F_y = 4\text{N}$, $F_z = 3\text{N}$ bo'lsa, $\vec{F}$ kuchning ishini va $\vec{F}$ kuch bilan $\vec{S}$ ko'chish orasidagi burchakni toping.

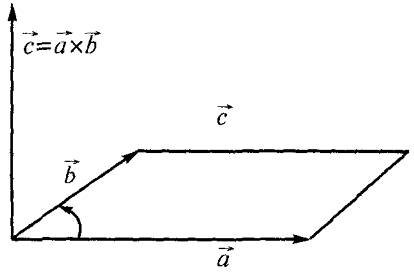
2.19. Tomonlari 6 sm va 4 sm bo'lgan to'g'ri to'rtburchakning uchidan qarama-qarshi tomonlarni teng ikkiga bo'luvchi to'g'ri chiziqlar o'tkazilgan. Shu to'g'ri chiziqlar orasidagi burchakni toping.

2.20. Kubning uchiga shu uchdan chiqib, kub yoqlarining diagonallari bo'ylab yo'nalgan va kattaliklari 1, 2 va 3 ga teng kuchlar qo'yilgan. Shu kuchlar teng ta'sir etuvchisining kattaligini toping.

3- §. Ikki vektorning vektor ko'paytmasi

$\vec{a}$ vektorning $\vec{b}$ vektorga vektor ko'paytmasi deb quyidagicha aniqlanuvchi $\vec{c}$ vektorga aytiladi:

1) $\vec{c}$ ning moduli (uzunligi) son qiymati bo'yicha $\vec{a}$ va $\vec{b}$ ga yasalgan parallelogrammning yuziga teng;



12- rasm.

2) $\vec{c} \perp \vec{a}, \quad \vec{c} \perp \vec{b}$;

3) $\vec{a}$, $\vec{b}$ va $\vec{c}$ *o'ng bog'lamni* tashkil qiladi, ya'ni $\vec{c}$ ning uchidan qaralganda $\vec{a}$ dan $\vec{b}$ ga qarab eng qisqa burilish soat strelkasi yo'nalishiga qarama-qarshi bo'ladi. Agar bu eng qisqa burilish soat strelkasi yo'nalishida bo'lsa $\vec{a}$, $\vec{b}$, $\vec{c}$ lar *chap bog'lamni* tashkil qiladi deyiladi. Vektor ko'paytma $\vec{a}\times\vec{b}$ yoki $\left[\vec{a}, \vec{b}\right]$ kabi belgilanadi. Vektor ko'paytma quyidagi xossalarga ega:

1. $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$;
2. $\vec{a}\times\left(\vec{b}+\vec{c}\right) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$;
3. $\vec{a}\times\vec{a} = 0$;
4. $\vec{a} \parallel \vec{b} \Rightarrow \vec{a}\times\vec{b} = 0$.

Ortlarning vektor ko'paytmalari:

$$\vec{i}\times\vec{i} = \vec{j}\times\vec{j} = \vec{k}\times\vec{k} = 0;$$
$$\vec{i}\times\vec{j} = \vec{k};$$
$$\vec{j}\times\vec{k} = \vec{i};$$
$$\vec{j}\times\vec{i} = -\vec{k};$$
$$\vec{k}\times\vec{j} = -\vec{i},$$
$$\vec{k}\times\vec{i} = \vec{j},$$
$$\vec{i}\times\vec{k} = -\vec{j}.$$

$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ va $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ vektorlarning vektor ko'paytmasi

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

formula bilan hisoblanadi.

$\vec{a}$ va $\vec{b}$ vektorlarga yasalgan parallelogrammning yuzi:

$$S_p = \left|\vec{a}\times\vec{b}\right|,$$

uchburchakning yuzi:

$$S_\Delta = \frac{1}{2}\left|\vec{a}\times\vec{b}\right|$$

formulalar bilan hisoblanadi.

A nuqtaga qo'yilgan $\vec{F}$ kuchning O nuqtaga nisbatan $\vec{M}$ momenti $\vec{M} = \vec{F}\times\vec{AO}$, yoki $\vec{M} = \overline{OA}\times\vec{F}$ formula bilan hisoblanadi:

$$\vec{M} = \overline{OA}\times\vec{F} = -\vec{F}\times\overline{OA} = \vec{F}\times\overline{AO}.$$

1- misol. $\vec{a} = 3\vec{i} - 2\vec{j} + \vec{k}$ va $\vec{b} = 4\vec{i} + 5\vec{j} - \vec{k}$ vektorlarning vektor ko'paytmasini toping.

▶ $\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 1 \\ 4 & 5 & -1 \end{vmatrix} = 2\vec{i} + 4\vec{j} + 15\vec{k} + 8\vec{k} - 5\vec{i} + 3\vec{j} =$

$= -3\vec{i} + 7\vec{j} + 23\vec{k}$.

$$\vec{a}\times\vec{b} = -3\vec{i} + 7\vec{j} + 23\vec{k}. ◀$$

2- misol. Uchlari $A(2; 1; 0)$, $B(1; 3; 4)$ va $C(3; -2; 1)$ nuqtalarda bo'lgan uchburchakning yuzini toping.

▶ ABC uchburchakni $\vec{a} = \overline{AB}$, $\vec{b} = \overline{AC}$ vektorlarga yasalgan uchburchak deb qarasak, uning yuzini

$$S_{ABC} = \frac{1}{2}\left|\vec{a}\times\vec{b}\right|$$

formula bilan topish mumkin. Unda $\overline{AB}\{-1; 2; 4\}$, $\overline{AC}\{1; -3; 1\}$.

$$\overline{AB}\times\overline{AC}=\begin{vmatrix}\vec i & \vec j & \vec k\\ -1 & 2 & 4\\ 1 & -3 & 1\end{vmatrix}=2\vec i+4\vec j+3\vec k-2\vec k+12\vec i+\vec j=$$

$$=14\vec i+5\vec j+\vec k.$$

$$S_{ABC}=\frac{1}{2}\left|14\vec i+5\vec j+\vec k\right|=\frac{1}{2}\sqrt{196+25+1}=\frac{\sqrt{222}}{2}\ \text{kv. birl.}$$

$$S_{ABC}=\frac{1}{2}\sqrt{222}\ \text{kv. birl.}$$

3- misol. $A(3; -2; 1)$ nuqtaga qo'yilgan $\vec F=\vec i+2\vec j-3\vec k$ kuchning $O(2; -1; 0)$ nuqtaga nisbatan momentini toping.

▶ Kuch momentini hisoblash formulasiga ko'ra: $\vec M=\overline{OA}\times\vec F$. Masala shartiga ko'ra

$$\overline{OA}=(3-2)\vec i+(-2+1)\vec j+(1-0)\vec k=\vec i-\vec j+\vec k;\quad \vec F=\vec i+2\vec j-\vec k$$

bo'lganidan

$$\vec M=\overline{OA}\times\vec F=\begin{vmatrix}\vec i & \vec j & \vec k\\ 1 & -1 & 1\\ 1 & 2 & -3\end{vmatrix}=$$

$$=3\vec i+\vec j+2\vec k+\vec k-2\vec i+3\vec j=\vec i+4\vec j+3\vec k;$$

$$\vec M=\vec i+4\vec j+3\vec k.$$ ◀

Mustaqil bajarish uchun mashqlar

3.1. $\vec a=2\vec i+3\vec j+4\vec k$ va $\vec b=-\vec i+\vec j-\vec k$ vektorlarning vektor ko'paytmasini toping.

3.2. Uchlari $A(1; 1; 1)$, $B(2; 3; 4)$ va $C(4; 3; 2)$ nuqtalarda bo'lgan uchburchakning yuzini toping.

3.3. Uchlari $A(1; -1; 2)$, $B(5; -6; 2)$ va $C(1; 3; -1)$ nuqtalarda bo'lgan uchburchakning BD balandligini toping.

3.4. Ifodani soddalashtiring:

1) $\vec i\times\left(\vec j+\vec k\right)-\vec j\times\left(\vec i+\vec k\right)+\vec k\times\left(\vec i+\vec j+\vec k\right)$;

2) $\left(\vec a+\vec b+\vec c\right)\times\vec c+\left(\vec a+\vec b+\vec c\right)\times\vec b+\left(\vec b-\vec c\right)\times\vec a$;

3) $\left(2\vec a+\vec b\right)\times\left(\vec c-\vec b\right)+\left(\vec b+\vec c\right)\times\left(\vec a+\vec b\right)$;

4) $2\vec i\cdot\left(\vec j\times\vec k\right)+3\vec j\cdot\left(\vec i\times\vec k\right)+4\vec k\cdot\left(\vec i\times\vec j\right)$.

3.5. $\left(\vec a-\vec b\right)\times\left(\vec a+\vec b\right)=2\cdot\left(\vec a\times\vec b\right)$ ayniyatni isbotlang va uning geometrik mazmunini tushuntiring.

3.6. $|\vec a_1|=1$, $|\vec a_2|=2$, $\left(\vec a_1,\hat{\ }\vec a_2\right)=\frac{2\pi}{3}$ bo'lsa, $\vec b=(\vec a_1+3\vec a_2)\times$ $\times(3\vec a_1-\vec a_2)$ vektorning modulini toping.

3.7. $|\vec a|=\left|\vec b\right|=5$, $\left(\vec a,\hat{\ }\vec b\right)=\frac{\pi}{4}$ bo'lsa, $\vec c=\vec a-2\vec b$ va $\vec d=3\vec a+2\vec b$ vektorlarga yasalgan parallelogrammning yuzini toping.

3.8. $\vec a=\vec i+\vec j+2\vec k$ va $\vec b=2\vec i+\vec j+\vec k$ vektorlarga perpendikular birlik vektorni toping.

3.9. $\vec a+\vec b+\vec c=0$ bo'lsa, $\vec a\times\vec b=\vec b\times\vec c=\vec c\times\vec a$ bo'lishini isbotlang va uning geometrik ma'nosini tushuntiring.

3.10. $\vec a\{3; -1; 2\}$ va $\vec b\{1; 2; -1\}$ vektorlar berilgan. $\vec c=$ $=\left(2\vec a+\vec b\right)\times\vec b$ va $\vec d=\left(2\vec a-\vec b\right)\times\left(2\vec a+\vec b\right)$ vektorlarni toping.

3.11. $\vec a_1\{4; -2; -3\}$ va $\vec a_2\{0; 1; 3\}$ vektorlarga perpendikular bo'lgan $\vec x$ vektor $\vec j$ ort bilan musbat burchak tashkil qiladi va $|\vec x|=26$. Shu $\vec x$ vektorning koordinatalarini toping.

3.12. $A(4; -2; 3)$ nuqtaga qo'yilgan $\vec F=2\vec i-4\vec j+5\vec k$ kuchning $O(3; 2; -1)$ nuqtaga nisbatan momentini toping.

3.13. $\vec F_1\{2; -1; -3\}$, $\vec F_2\{3; 2; -1\}$ va $\vec F_3\{-4; 1; 3\}$ kuchlar $A(-1; 4; 2)$ nuqtaga qo'yilgan. Shu kuchlar teng ta'sir etuvchisining $O(2; 3; -1)$ nuqtaga nisbatan momentining miqdori va yo'naltiruvchi kosinuslarini toping.

3.14. $\vec{a}=\vec{k}-\vec{j}$ va $\vec{b}=\vec{i}+\vec{j}+\vec{k}$ vektorlarga yasalgan parallelogrammning yuzini toping.

3.15. $A(1;-2;8)$, $B(0;0;4)$ va $C(6;2;0)$ nuqtalar berilgan. $\overrightarrow{AB}$ va $\overrightarrow{AC}$ vektorlarga yasalgan parallelogrammning yuzini va B uchidan tushirilgan balandligini toping.

4- §. Uch vektorning aralash ko'paytmasi

Ikki $\vec{a}$ va $\vec{b}$ vektor vektor ko'paytmasining uchinchi vektorga skalar ko'paytmasi *uch vektorning aralash ko'paytmasi* deyiladi. Aralash ko'paytma $\vec{a}\vec{b}\vec{c}=(\vec{a}\times\vec{b})\cdot\vec{c}$ kabi belgilanadi. Aralash ko'paytma quyidagi xossalarga ega:

1. $(\vec{a}\times\vec{b})\cdot\vec{c}=-(\vec{a}\times\vec{c})\cdot\vec{b}=-(\vec{c}\times\vec{b})\cdot\vec{a}$.

2. Aralash ko'paytmaning istalgan ikkita ko'paytuchisi kollinear bo'lsa, aralash ko'paytma nolga teng.

3. Skalar va vektor ko'paytirish belgilarining o'rinlari almashtirilsa, aralash ko'paytma o'zgarmaydi:

$$\vec{a}\vec{b}\vec{c}=\vec{a}\cdot(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\cdot\vec{c}.$$

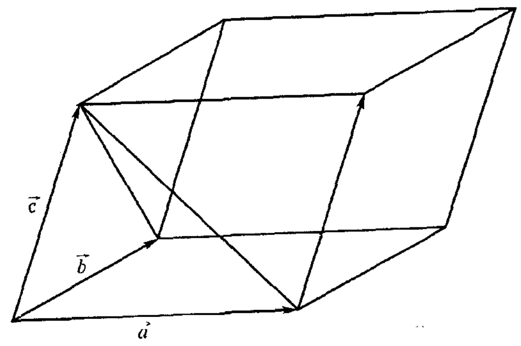
4. $\vec{a}$, $\vec{b}$, $\vec{c}$ lar komplanar bo'lsa, $\vec{a}\vec{b}\vec{c}=0$ bo'ladi. Bu *uch vektorning komplanarlak sharti* ham deyiladi. Noldan farqli vektorlar uchun $\vec{a}\vec{b}\vec{c}=0$ bo'lsa, bu vektorlar komplanar bo'lib, ulardan birini qolganlari orqali ifodalash mumkin.

Agar $\vec{a},\vec{b}$ va $\vec{c}$ vektorlar koordinatalari bilan berilgan, ya'ni:

$$\vec{a}=a_1\vec{i}+a_2\vec{j}+a_3\vec{k},\quad \vec{b}=b_1\vec{i}+b_2\vec{j}+b_3\vec{k},\quad \vec{c}=c_1\vec{i}+c_2\vec{j}+c_3\vec{k}$$

bo'lsa, aralash ko'paytma quidagicha hisoblanadi:

$$\vec{a}\vec{b}\vec{c}=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}.$$



13- rasm.

Bir tekislikda yotmagan uchta $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlarga qurilgan parallelepiped va piramidaning hajmlari (13-rasm)

$$V_{par}=\pm\,\vec{a}\vec{b}\vec{c};\quad V_{pir}=\pm\frac{1}{6}\vec{a}\vec{b}\vec{c}$$

formulalar bilan topiladi, bu yerda $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar o'ng bog'lamni tashkil etsa, «+» ishora, aks holda «–» ishora olinadi.

1- misol. $\vec{a}=3\vec{i}+4\vec{j}$, $\vec{b}=-3\vec{j}+\vec{k}$, $\vec{c}=2\vec{j}+5\vec{k}$ vektorlarga yasalgan parallelepipedning hajmini toping. $(\vec{a},\ \vec{b},\ \vec{c})$ uchlik o'ng bog'lamni hosil qiladimi yoki chap bog'lamnimi, aniqlang.

▶ $\vec{a}\vec{b}\vec{c}=\begin{vmatrix}3 & 4 & 0\\ 0 & -3 & 1\\ 0 & 2 & 5\end{vmatrix}=-45-6=-51;\quad \vec{a}\vec{b}\vec{c}=-51.$

$\vec{a}\vec{b}\vec{c}<0$, demak, $\vec{a},\vec{b},\vec{c}$ uchlik chap bog'lamni tashkil etadi. Unda

$V_{par}=\pm\vec{a}\vec{b}\vec{c}=-(-51)=51;\quad V_{par}=51$ kub birl. ◀

2- misol. Uchlari $A(1;1;1)$, $B(2;0;2)$, $C(2;2;2)$ va $D(3;4;-3)$ nuqtalarda bo'lgan tetraedrning hajmini va $h=DE$ balandligini toping.

▶ Qaralayotgan tetraedrni uning bitta, masalan, A uchidan chiquvchi uchta $\vec{a}=\overline{AB}$, $\vec{b}=\overline{AC}$, $\vec{c}=\overline{AD}$ vektordan hosil bo'lgan piramida deyish mumkin.

$$\vec{a}=\overline{AB}\{1;\,-1;\,1\},\quad \vec{b}=\overline{AC}\{1;\,1;\,1\},\quad \vec{c}=\overline{AD}\{2;\,3;\,-4\};$$

$$\vec{a}\vec{b}\vec{c}=\begin{vmatrix}1 & -1 & 1\\ 1 & 1 & 1\\ 2 & 3 & -4\end{vmatrix}=-4-2+3-2-3-4=-12;$$

U holda

$$V_{\text{pir}}=\pm\frac{1}{6}\vec{a}\vec{b}\vec{c}=\pm\frac{1}{6}(-12)=-\frac{1}{6}(-12)=2;\quad V_{\text{pir}}=2\ \text{ kub birl.}$$

Tetraedr asosi ABC uchburchak, balandligi $h=DE$ bo'lgan uchburchakli piramida bo'lganidan

$$V_{\text{pir}}=\frac{1}{3}\cdot S_{ABC}\cdot h;\quad h=\frac{3V_{\text{pir}}}{S_{ABC}}=\frac{6}{S_{ABC}}.$$

ABC uchburchak $\vec{a}=\overline{AB}$ va $\vec{b}=\overline{AC}$ vektorlarga yasalgan uchburchak bo'lgani uchun

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -1 & 1\\ 1 & 1 & 1\end{vmatrix}=-\vec{i}+\vec{j}+\vec{k}+\vec{k}-\vec{i}-\vec{j}=-2\vec{i}+2\vec{k}:$$

$$S_{ABC}=\frac{1}{2}\cdot\left|\vec{a}\times\vec{b}\right|=\frac{1}{2}\cdot\sqrt{(-2)^2+2^2}=\sqrt{2}.$$

U holda $h=\frac{6}{\sqrt{2}}3\sqrt{2};\ h=3\sqrt{2}.$

Mustaqil bajarish uchun mashqlar

4.1. $\vec{a}=\vec{i}+\vec{j}+4\vec{k}$, $\vec{b}=\vec{i}-\vec{j}$ va $\vec{c}=3\vec{i}-3\vec{j}+4\vec{k}$ vektorlarni yasang. Bu vektorlarning komplanar ekanligini ko'rsating va ular orasidagi chiziqli bog'lanishni toping.

4.2. $A(2;\,-1;\,-2)$, $B(1;\,2;\,1)$, $C(2;\,3;\,0)$ va $D(2;\,3;\,8)$ nuqtalarning bitta tekislikda yotishini ko'rsating.

4.3. Uchlari $A(2;\,0;\,0)$, $B(0;\,3;\,0)$, $C(0;\,0;\,6)$ va $D(2;\,3;\,8)$ nuqtalarda bo'lgan piramidani yasang, uning hajmini va ABC yoqqa tushirilgan balandligini toping.

4.4. $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar o'ng bog'lamni tashkil etadi, o'zaro perpendikular va $|\vec{a}|=4$, $|\vec{b}|=2$, $|\vec{c}|=3$. $\vec{a}\vec{b}\vec{c}$ ni toping.

4.5. $\vec{a}_1\{1;\,-1;\,3\}$, $\vec{a}_2\{-2;\,2;\,1\}$ va $\vec{a}_3\{3;\,-2;\,5\}$ bo'lsa $\vec{a}_1\vec{a}_2\vec{a}_3$ aralash ko'paytmani toping.

4.6. $\overline{OA}=3\vec{i}+4\vec{j}$, $\overline{OB}=-3\vec{j}+\vec{k}$, $\overline{OC}=2\vec{j}+5\vec{k}$ bo'lsa, $OABC$ tetraedrning hajmini toping.

4.7. $\vec{a}=-\vec{i}+3\vec{j}+2\vec{k}$, $\vec{b}=2\vec{i}-3\vec{j}-4\vec{k}$, $\vec{c}=-3\vec{i}+12\vec{j}+6\vec{k}$ vektorlarning komplanar ekanini ko'rsating. $\vec{c}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar orqali chiziqli ifodalang.

4.8. Uchlari $O(0;\,0;\,0)$, $A(5;\,2;\,0)$, $C(1;\,2;\,4)$ nuqtalarda bo'lgan piramidani yasang. Uning hajmini, ABC yog'ining yuzini va bu yoqqa tushirilgan balandligini toping.

4.9. Koordinata burchaklarining bissektrisalari bo'ylab yo'nalgan va uzunliklari 2 ga teng $\overline{OA}$, $\overline{OB}$ va $\overline{OC}$ vektorlarga yasalgan tetraedrning hajmini toping.

4.10. $\vec{a}=\vec{i}+\vec{j}+m\vec{k}$, $\vec{b}=\vec{i}+\vec{j}+(m+1)\vec{k}$ va $\vec{c}=\vec{i}-\vec{j}+m\vec{k}$ vektorlar m ning hech bir qiymatida komplanar bo'la olmasligini ko'rsating.

Mustaqil bajarish uchun berilgan mashqlarning javoblari

1-§. **1.3.** $\frac{bc+cb}{b+c}$. **1.6.** $\sqrt{8+2\sqrt{3}}$. **1.8.** $\overline{ON}=3\vec{m}+\vec{n}$; **1.9.** $\text{pr}_{Ox}\overline{OM}=8$; $\text{pr}_{Oy}\overline{OM}=-2$; $\overline{AC}=2(\vec{n}-\vec{m})$; $\overline{OM}=2\vec{n}+\vec{m}$; $OM=2\sqrt{17}$. **1.10.** $\vec{m}+\vec{n}=\vec{p}$; $\overline{OB}=3(\vec{m}+\vec{n})$; $\overline{BC}=3(\vec{n}-\vec{m})$; $\overline{OE}=3(\vec{m}-\vec{n})$; $\overline{OD}=3(2\vec{n}-\vec{m})$; $\overline{DA}=-6(\vec{m}-\vec{n})$. **1.11.** $r=7$; $\arccos\frac{2}{7}$; $\arccos\frac{3}{7}$; $\arccos\left(-\frac{6}{7}\right)$. **1.12.** $6\sqrt{2}$; $45°$; $90°$; $135°$. **1.13.** 7; $\arccos\frac{2}{7}$; $\arccos\left(-\frac{6}{7}\right)$; $\arccos\frac{3}{7}$. **1.14.** $\vec{a}=2\vec{i}+2\vec{j}+2\vec{k}$. **1.15.** $\vec{x}=\pm5\vec{i}+\frac{5}{\sqrt{2}}\vec{j}-\frac{5}{\sqrt{2}}\vec{k}$. **1.16.** $5;\sqrt{41}$. **1.17.** $\frac{2}{5}\vec{i}-\frac{4}{5}\vec{j}+\frac{\sqrt{5}}{5}\vec{k}$. **1.18.** $\sqrt{2}a;\sqrt{5}a$.

2- §. 2.1. 1) –3; 2) 54; 3) 7. **2.2.** 135° . **2.3.** ±3/5. **2.4.** $\pi/3$. **2.5.** 1) 22; 2) –200; 3) 41; 4) $\sqrt{105}$; 5) 11/3; 6) 22/7. **2.6.** $P_1(1; 0)$ va $P_2(6; 0)$. **2.7.** 90°; 45°; 45°. **2.8.** $\arccos\frac{2\sqrt{5}}{5}$. **2.9.** $-\frac{8}{5}; -\frac{8\sqrt{2}}{10}$. **2.10.** 0. **2.11.** 40. **2.12.** $2+\sqrt{3}$. **2.13.** $\sqrt{7}$; $\sqrt{13}$. **2.14.** $\frac{15}{7\sqrt{85}}$. **2.15.** arccos 0,8. **2.17.** 4. **2.18.** 80; $\arccos\frac{4\sqrt{2}}{15}$. **2.19.** $\arccos\left(0{,}26\sqrt{10}\right)$ **2.20.** 5.

3- §. 3.1. $-7\vec{i}-2\vec{j}+5\vec{k}$. **3.2.** $2\sqrt{6}$. **3.3.** 5. **3.4.** 1) $2\left(\vec{k}-1\right)$; 2) $2\left(\vec{a}\times\vec{b}\right)$; 3) $\vec{a}\times\vec{c}$; 4) 3. **3.6.** $10\sqrt{5}$. **3.7.** $100\sqrt{2}$. **3.8.** $\pm\frac{1}{\sqrt{m}}\left(\vec{i}-3\vec{j}+\vec{k}\right)$. **3.10.** {–6, 10, 14}; {–12, 20, 28}. **3.11.** {–6; –24; 8}. **3.12.** $-4\vec{i}+3\vec{j}+4\vec{k}$. **3.13.** $\sqrt{66}$; $\cos\alpha=\frac{1}{\sqrt{66}}$; $\cos\beta=-\frac{4}{\sqrt{66}}$; $\cos\gamma=-\frac{7}{\sqrt{66}}$. **3.14.** $\sqrt{6}$. **3.15.** $14\sqrt{5}$, $\frac{2\sqrt{21}}{3}$.

4- §. 4.1. $\vec{c}=\vec{a}+2\vec{b}$. **4.3.** $V=14$; $H=\sqrt{14}$. **4.4.** 24. **4.5.** –7. **4.6.** 8,5. **4.7.** $\vec{c}=5\vec{a}+\vec{b}$. **4.8.** $V=14$; $H=\frac{7\sqrt{3}}{3}$. **4.9.** $\frac{2\sqrt{2}}{3}$.

III bob. ISTALGAN CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMALARINI YECHISH

1- §. Arifmetik vektorlar

n ta haqiqiy sonning tartiblangan to'plami *haqiqiy arifmetik vektor* deyiladi. U $x=(x_1,x_2,\dots,x_n)$ kabi belgilanib, $x_1,x_2,\dots,x_n$ lar arifmetik vektorning *komponentalari* deyiladi. Arifmetik vektorlar uchun qo'shish va songa ko'paytirish amallari kiritiladi.

qo'shish: agar $x=(x_1,x_2,\dots,x_n)$, $y=(y_1,y_2,\dots,y_n)$ bo'lsa,

$$x+y=(x_1+y_1,x_2+y_2,\dots x_n+y_n).$$

songa ko'paytirish: agar k haqiqiy son bo'lsa,

$$kx=(kx_1,\ kx_2,\ \dots\ kx_n).$$

Bu kabi qo'shish va songa ko'paytirish amallari aniqlangan arifmetik vektorlar to'plami *arifmetik vektorlar fazosi* deyiladi. Biz n komponentali arifmetik vektorlar fazosini qaraymiz. U R^n deb belgilanadi. Agar hech bo'lmaganda bittasi noldan farqli k_1, k_2, ..., k_m sonlar uchun

$$k_1x_1+k_2x_2+\dots+k_mx_m=0,\quad (\ 0(0,\ 0,\ \dots\ ,0)\ \text{— nol vektor})$$

o'rinli bo'lsa, u holda $(x_1,x_2,\dots,x_n)$ arifmetik vektorlar sistemasi *chiziqli bog'liq*, aks holda *chiziqli erkli* deyiladi.

Q arifmetik vektorlarning biror to'plami bo'lsin. $B=(e_1,\ e_2,\ \dots,\ e_m)$ vektorlar sistemasi Q da *bazis* deyiladi, agar quyidagilar bajarilsa:

1) e_1, e_2, ..., e_m lar Q ga tegishli va chiziqli erkli;

2) Q dagi istalgan x vektor uchun shunday $k_1,k_2,\dots,k_m$ sonlar mavjudki,

$$x = k_1x_1 + k_2x_2 + ... + k_mx_m. \qquad (1)$$

(1) ifoda x vektorning B bazis bo'yicha yoyilmasi, $x_1, x_2, ..., x_m$ sonlar esa x ning B bazisdagi *koordinatalari* deyiladi. $Q \subset R$ bo'lsa, m son Q vektorlar sistemasining *rangi* deyiladi. Butun R^n fazoning rangi n ga teng va u fazoning o'lchami deyiladi. R^n dagi istalgan vektorni biror $(e_1, e_2, ..., e_n)$ bazis bo'yicha yoyish mumkin:

$$x = e_1x_1 + e_2x_2 + ... + e_nx_n.$$

Demak, R^n da istalgan x vektorga uning biror bazisidagi koordinatalaridan iborat ustun-matritsani mos qo'yish mumkin. Ko'pincha bazis sifatida ushbu

$$e_1 = (1, 0, 0, ..., 0),$$
$$e_2 = (0, 1, 0, ..., 0),$$
$$.........................$$
$$e_n = (0, 0, 0, ..., 1)$$

kanonik bazisdan foydalaniladi. Vektorning komponentalari uning koordinatalari bilan faqat kanonik bazisdagina bir xil bo'ladi.

Arifmetik vektorlarni qo'shish va songa ko'paytirish amallari *chiziqli* amallar deyilib, ularni koordinata shaklida quyidagicha yozish mumkin:

1) $z_m = x_m + y_m \quad \Leftrightarrow \quad Z_m = X_m + Y_m;$

2) $y_m = k \cdot x_m \quad \Leftrightarrow \quad Y_m = k \cdot X_m, \qquad m = 1, 2, ..., n.$

1- misol. $a_1 = (1; 2; -3; 2)$, $a_2 = (4; 1; 3; -2)$, $a_3 = (5; -7; 0; 2)$ arifmetik vektorlarning chiziqli kombinatsiyasidan iborat $b = 4a_1 - 3a_2 + 5a_3$ arifmetik vektorni toping.

▶ $b = 4 \cdot (1; \ 2; \ -3; \ 2) - 3 \cdot (4; \ 1; \ 3; \ -2) +$
$+5 \cdot (5; \ -7; \ 0; \ 2) = (4 - 12 + 25; 8 - 3 - 35;$

$-12 - 9 + 0; \ \ 8 + 6 + 10) = (17; \ -30; \ -21; \ 24);$

$b = (17; \ -30; \ -21; \ 24).$ ◀

2- misol. Arifmetik vektorlarning chiziqli bog'liq yoki chiziqli erkli ekanini ko'rsating. $x_1 = (-1; 2; 3)$, $x_2 = (2; 5; 6)$.

▶ $k_1x_1 + k_2x_2 = 0 \Leftrightarrow (-k_1; 2k_1; 3k_1) + (2k_2; 5k_2; 6k_2) = 0 \Leftrightarrow$
$\Leftrightarrow (-k_1 + 2k_2; 2k_1 + 5k_2; 3k_1 + 6k_2) = 0;$

$$\begin{cases} -k_1 + 2k_2 = 0, \\ 2k_1 + 5k_2 = 0, \\ 3k_1 + 6k_2 = 0, \end{cases} \Leftrightarrow \begin{cases} k_1 = 2k_2, \\ k_1 = -2,5k_2, \\ k_1 = -2k_2, \end{cases} \Leftrightarrow k_1 = 0, \ k_2 = 0.$$

ya'ni $k_1x_1 + k_2x_2 = 0$ tenglik faqat $k_1 = k_2 = 0$ dagina o'rinli. Demak, x_1 va x_2 arifmetik vektorlar chiziqli erkli ekan. ◀

3- misol. $e_1 = (1; 1; 1; 1)$, $e_2 = (0; 1; 1; 1)$, $e_3 = (0; 0; 1; 1)$, $e_4 = (0; 0; 0; 1)$ vektorlarning R^4 da bazis tashkil etishini ko'rsating va $x = (5; 4; 3; 2)$ vektorning shu bazisdagi koordinatalarini toping.

▶ Oldin (e_1, e_2, e_3, e_4) sistemaning chiziqli erkli ekanini ko'rsatamiz:

$k_1e_1 + k_2e_2 + k_3e_3 + k_4e_4 = 0 \Leftrightarrow (k_1; k_2; k_3; k_4) +$
$+(0; k_2; k_3; k_4) + (0; 0; k_2; k_3; k_4) + (0; 0; 0; k_3; k_4) +$
$+(0; 0; 0; k_4) = 0(0; 0; 0; 0) \Leftrightarrow$

$$\Leftrightarrow \begin{cases} k_1 = 0, \\ k_1 + k_2 = 0, \\ k_1 + k_2 + k_3 = 0, \\ k_1 + k_2 + k_3 + k_4 = 0 \end{cases} \Leftrightarrow k_1 = k_2 = k_3 = k_4 = 0.$$

Endi $x = (5; 4; 3; 2)$ vektorning bu bazisdagi koordinatalarini topamiz:

$x = e_1k_1 + e_2x_2 + e_3k_3 + e_4k_4 \Leftrightarrow (5; 4; 3; 2) = (x_1; x_1; x_1; x_1) +$
$+(0; x_2; x_2; x_2) + (0; 0; x_3; x_3) + (0; 0; 0; x_4) = 0 \Leftrightarrow$

$$\Leftrightarrow x_1 = 5, x_1 + x_2 = 4 \quad \Leftrightarrow \quad x_1 + x_2 + x_3 = 3, x_1 + x_2 + x_3 + x_4 = 2$$

$$\Leftrightarrow x_1 = 5, \quad x_2 = -1, \quad x_3 = -1, x_4 = -1.$$

Yoki

$$x = 5e_1 - e_2 - e_3 - e_4 \quad \Leftrightarrow \quad x = \begin{pmatrix} 5 \\ -1 \\ -1 \\ -1 \end{pmatrix}. \blacktriangleleft$$

Arifmetik vektorlar sistemasining chiziqli bog'liq yoki erkli ekanini tekshirishda matritsalardan ham foydalanish mumkin. Chunki $(m \times n)$-matritsaning satrlarini (ustunlarini) R^n (R^m) ga tegishli arifmetik vektorlar sistemasi deb qarash mumkin.

Teorema (bazis minor haqida). Matritsaning rangi uning satrlari (ustunlari) sistemasining rangiga teng. Bunda bazis minorni o'z ichiga oluvchi satrlar (ustunlar) sistemasi barcha satrlar (ustunlar) sistemasi uchun bazisni tashkil etadi.

4- misol. $a_1 = (2; -3; 1)$, $a_2 = (3; -1; 5)$, $a_3 = (1; -5; -3)$ arifmetik vektorlar sistemasining chiziqli bog'liq yoki chiziqli erkli ekanini aniqlang. Uning rangini va biriorta bazisini toping.

▶ Ustunlari a_1, a_2, a_3 vektorlarning koordinatalaridan iborat A matritsani tuzamiz:

$$A = (a_1, \ a_2, \ a_3) = \begin{pmatrix} 2 & 3 & 1 \\ -3 & -1 & -5 \\ 1 & 5 & -3 \end{pmatrix}.$$

$r(A) = 2$ ekanini ko'rish ogoh. Bazis minor haqidagi teoremaga ko'ra, sistema chiziqli bog'liq va uning rangi ham ikkiga teng. Noldan farqli istalgan 2- tartibli minorni, masalan,

$$\begin{vmatrix} 2 & 3 \\ -3 & -1 \end{vmatrix}$$

ni bazis deb hisoblash mumkin. Bundan va berilgan sistemaning bazisi ekani kelib chiqadi. ◀

Mustaqil bajarish uchun mashqlar

1.1. $a_1 = (4; 1; 3; -2)$, $a_2 = (1; 2; -3; 2)$, $a_3 = (16; 9; 1; -3)$, $a_4 = (0; 1; 2; 3)$, $a_5 = (1; -1; 15; 0)$ arifmetik vektorlar berilgan. Quyidagi chiziqli kombinatsiyalarni toping:

1) $3a_1 + 5a_2 - a_3$;

2) $a_1 + 2a_2 - a_4 - 2a_5$;

3) $2a_1 + 4a_3 - 2a_5$.

1.2. 1.1-mashqda berilgan arifmetik vektorlar uchun tenglamadan x vektorni toping:

1) $2x + a_1 - 2a_2 - a_5 = 0$;

2) $a_1 - 3a_5 + x + a_3 = 0$;

3) $2(a_1 - x) + 5(a_4 + x) = 0$.

1.3. Berilgan arifmetik vektorlarining chiziqli bog'liq yoki chiziqli erkli ekanini aniqlang:

1) $x_1 = (3; 1; 5)$, $x_2 = (6; -3; 1)$;

2) $x_1 = (1; 2; 3; 0)$, $x_2 = (2; 4; 6; 0)$;

3) $x_1 = (2; -3; 1)$, $x_2 = (3; -1; 5)$, $x_3 = (1; -4; 3)$;

1.4. $e_1 = (1; 1; 1; 1; 1)$, $e_2 = (0; 1; 1; 1; 1)$, $e_3 = (0; 0; 1; 1; 1)$, $e_4 = (0; 0; 0; 1; 1)$, $e_5 = (0; 0; 0; 0; 1)$ arifmetik vektorlar sisitemasining R^5 fazoda bazis tashkil etishini ko'rsating.

1.5. Vektorlar sistemasining chiziqli bog'liq yoki chiziqli erkli ekanini matritsalar yordamida aniqlang:

$x_1 = (1; 1; 1; 1)$, $x_2 = (1; -1; -1; 1)$,

$x_3 = (1; -1; 1; -1)$, $x_4 = (1; 1; -1; -1)$

1.6. Vektorlar sistemasining rangini matritsalar yordamida toping:

$a_1 = (1; -1; 0; 0)$, $a_2 = (0; 1; -1; 0)$, $a_3 = (1; 0; -1; 1)$,

$a_4 = (0; 0; 0; 1)$, $a_5 = (3; -5; 2; -3)$.

1.7. k ning x vektor a_1, a_2, a_3 vektorlar orqali chiziqli ifodalanadigan barcha qiymatlarini matritsalardan foydalanib toping:

1) $a_1 = (2;\ 3;\ 5)$, $a_2 = (3;\ 7;\ 8)$, $a_3 = (1;\ -6;\ 1)$, $x = (7;\ -2;\ k)$;

2) $a_1 = (3;\ 2;\ 5)$, $a_2 = (2;\ 4;\ 7)$, $a_3 = (5;\ 6;\ k)$, $x = (1;\ 3;\ 5)$.

1.8. Vektorlar sistemasining rangini va birorta bazisini toping:

$a_1 = (5;\ 2;\ -3;\ 1)$, $a_2 = (4;\ 1;\ -2;\ 3)$,
$a_3 = (1;\ 1;\ -1;\ -2)$, $a_4 = (3;\ 4;\ -1;\ 2)$.

2- §. Istalgan chiziqli tenglamalar sistemasi

n noma'lumli m ta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases} \qquad (1)$$

berilgan bo'lsin. Quyidagi matritsalarni kiritamiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \ldots \\ b_m \end{pmatrix},$$

$$\bar{A} = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} & b_1 \\ a_{21} & a_{22} & \ldots & a_{2n} & b_2 \\ \ldots & \ldots & \ldots & \ldots & \\ a_{m1} & a_{m2} & \ldots & a_{mn} & b_m \end{pmatrix}$$

A — sistema matritsasi, $\bar{A}$ — sistemaning *kengaytirilgan matritsasi* deyiladi.

(1) sistemani matritsaviy ko'rinishda

$$AX = B \qquad (2)$$

kabi yozish mumkin.

Agar $B = 0$ bo'lsa, sistema *bir jinsli*, aks holda *bir jinslimas* deyiladi. Kamida bitta yechimga ega sistema *birgalikdagi* sistema, yechimga ega bo'lmagan *sistema esa birgalikda bo'lmagan* sistema deb ataladi. Bir xil yechimlar to'plamiga ega bo'lgan sistemalar o'zaro ekvivalent deyiladi.

Kroneker — Kapelli teoremasi. (1) sistemaning birgalikda bo'lishi uchun sistema matritsasining rangi kengaytirilgan matritsaning rangiga teng bo'lishi, y'ani

$$r(A) = r(\bar{A}) \qquad (3)$$

bo'lishi zarur va yetarlidir.

Sistemaning yechimini quyidagi tartibda topish mumkin: $r(A) = r(\bar{A}) = r$ deylik, $r \le \min(m, n)$. Bazis minor matritsaning dastlabki r ta satr va ustunlarida joylashgan desa bo'ladi (aks holda elemantar almashtirishlar yordamida shu shaklga keltirish mumkin). (1) sistemaning dastlabki r ta tenglamasini qoldirib, qisqartirilgan sistema yozamiz:

$$\begin{cases} a_{11}x_1 + \ldots + a_{1r}x_r + a_{1r+1}x_{r+1} + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + \ldots + a_{2r}x_r + a_{2r+1}x_{r+1} + \ldots + a_{2n}x_n = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{r1}x_1 + \ldots + a_{rr}x_r + a_{rr+1}x_{r+1} + \ldots + a_{rn}x_n = b_r. \end{cases} \qquad \textbf{(4)}$$

Bu (1) sistemaga ekvivalentdir. x_1, x_2, x_3, $\ldots, x_r$ larni *bazis noma'lumlar*, qolgan $x_{r+1}, \ldots,$ x_n larni *ozod noma'lumlar* deb olib, bazis noma'lumlarga nisbatan sistema hosil qilamiz:

$$\begin{cases} a_{11}x_1 + \ldots + a_{1r}x_r = b_1 - a_{1r} - x_{r+1} - \ldots - a_{1n}x_n, \\ a_{21}x_1 + \ldots + a_{2r}x_r = b_2 - a_{2r} - x_{r+1} - \ldots - a_{2n}x_n, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{r1}x_1 + \ldots + a_{rr}x_r = b_r - a_{rr} - x_{r+1} - \ldots - a_{rn}x_n. \end{cases}$$

Bu sistemaning determinanti noldan farqli (chunki bazis minor), sistema yagona yechimga ega va bu yechimni, masalan, Kramer usuli bilan topish mumkin. Ozod noma'lumlarning har bir

$$x_{r+1}=c_1,\qquad x_{r+1}=c_2,\ \ldots,\ x_n=c_{n-r}$$

qiymatlari to'plami uchun (1) sistemaning yechimini

$$X(c_1;\ c_2;\ \ldots;\ c_{n-r})=\begin{pmatrix} x(c_1;\ c_2;\ \ldots;\ c_{n-r}) \\ \ldots \\ x_r(c_1,\ c_2,\ \ldots,\ c_{n-r}) \\ c_1 \\ \ldots \\ c_{n-r} \end{pmatrix}=$$

$$=\left(x_1(c_1;\ c_2;\ \ldots;\ c_{n-r}),\ \ldots,\ c_{n-r}\right)^T$$

ko'rinishda yozish mumkin. Bu (1) sistemaning *umumiy yechimi* deyiladi.

1- misol. Sistemaning birgalikdaligini tekshiring va birgalikda bo'lsa, uning umumiy yechimini toping:

$$\begin{cases} 2x-y+z=-2, \\ x+2y+3z=-1, \\ x-3y-2z=3. \end{cases}$$

▶ Sistema asosiy va kengaytirilgan matritsalarining ranglarini topamiz:

$$A=\begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 1 & -3 & -2 \end{pmatrix}\sim\begin{pmatrix} 0 & -1 & 0 \\ 5 & 2 & 5 \\ -5 & -3 & -5 \end{pmatrix}\sim$$

$$\sim\begin{pmatrix} 0 & -1 & 0 \\ 1 & 2 & 0 \\ -1 & -3 & 0 \end{pmatrix}\sim\begin{pmatrix} 0 & -1 \\ 1 & 2 \\ -1 & -3 \end{pmatrix};$$

$r(A)=2;$

$$\overline{A}=\begin{pmatrix} 2 & -1 & 1 & -2 \\ 1 & 2 & 3 & -1 \\ 1 & -3 & -2 & 3 \end{pmatrix}\sim\begin{pmatrix} 2 & 0 & 1 & 0 \\ 1 & 5 & 3 & 0 \\ 1 & -5 & -2 & 4 \end{pmatrix}\sim$$

$$\sim\begin{pmatrix} 0 & 0 & 1 & 0 \\ -5 & 5 & 3 & 0 \\ 5 & -5 & -2 & 4 \end{pmatrix}\sim\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & -1 & 4 \end{pmatrix};$$

$r(\overline{A})=3.$

$r(A)\neq r(\overline{A})$. Sistema birgalikda emas. ◀

2- misol. Sistemaning birgalikdaligini tekshiring va birgalikda bo'lsa, umumiy yechimini toping:

$$\begin{cases} 2x_1+7x_2+3x_3+x_4=6, \\ 3x_1+5x_2+2x_3+2x_4=4, \\ 9x_1+4x_2+x_3+7x_4=2. \end{cases}$$

$$▶\ A=\begin{pmatrix} 2 & 7 & 3 & 1 \\ 3 & 5 & 2 & 2 \\ 9 & 4 & 1 & 7 \end{pmatrix}\sim\begin{pmatrix} 0 & 0 & 0 & 1 \\ -1 & -9 & -4 & 2 \\ -5 & -45 & -20 & 7 \end{pmatrix}\sim$$

$$\sim\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 2 \\ 5 & 5 & 5 & 7 \end{pmatrix}\sim\begin{pmatrix} 0 & 1 \\ 1 & 2 \\ 5 & 7 \end{pmatrix};$$

$r(A)=2.$

$r(A)=r(\overline{A})=r=2$, sistema birgalikda. Bazis minor deb $\begin{vmatrix} 2 & 7 \\ 3 & 5 \end{vmatrix}$ ni olsak, x_1, x_2 — bazis noma'lumlar; x_3, x_4 — ozod noma'lumlar bo'lib, qisqartirilgan sistema

$$\begin{cases} 2x_1+7x_2=6-3x_3-x_4, \\ 3x_1+5x_2=4-2x_3-2x_4 \end{cases}$$

bo'ladi. $x_3 = c_1$, $x_4 = c_2$ desak, bazis noma'lumlarga nisbatan bu sistemaning yechimi:

$$x_1 = -\frac{2}{11} + \frac{c_1}{11} - \frac{9c_2}{11}, \qquad x_2 = \frac{10}{11} - \frac{5c_1}{11} + \frac{c_2}{11}.$$

Sistemaning umumiy yechimi:

$$X(c_1;\ c_2) = \begin{pmatrix} -\frac{2}{11} + \frac{c_1}{11} - \frac{9c_2}{11} \\ \frac{10}{11} - \frac{5c_1}{11} + \frac{c_2}{11} \\ c_1 \\ c_2 \end{pmatrix},$$

yoki $X(c_1;\ c_2) = \left(-\frac{2}{11} + \frac{c_1}{11} - \frac{9c_2}{11};\ \ \frac{10}{11} - \frac{5c_1}{11} + \frac{c_2}{11};\ \ c_1;\ \ c_2\right)^T$. ◄

Mustaqil bajarish uchun mashqlar

2.1. Sistemaning birgalikdaligini tekshiring va umumiy yechimini toping:

1) $\begin{cases} x - \sqrt{3}y = 1, \\ \sqrt{3}x - 3y = \sqrt{3}; \end{cases}$ 2) $\begin{cases} \sqrt{5}x - 5y = \sqrt{5}, \\ x - \sqrt{5}y = 5; \end{cases}$

3) $\begin{cases} x + 2y - 4z = 1, \\ 2x + y - 5z = -1, \\ x - y - z = -2; \end{cases}$ 4) $\begin{cases} 3x - 2y - 5z + t = 3, \\ 2x - 3y + z + 5t = -3, \\ x + 2y - 4t = -3, \\ x - y - 4z + 9t = 22; \end{cases}$

5) $\begin{cases} 2x_1 + x_2 - x_3 - 3x_4 = 2, \\ 4x_1 + x_3 - 7x_4 = 3, \\ 2x_2 - 3x_3 + x_4 = 1, \\ 2x_1 + 3x_2 - 4x_3 - 2x_4 = 3; \end{cases}$ 6) $\begin{cases} 3x_1 - 5x_2 + 2x_3 + 4x_4 = 2, \\ 7x_1 - 4x_2 + x_3 + 3x_4 = 5, \\ 5x_1 + 7x_2 - 4x_3 - 6x_4 = 3; \end{cases}$

7) $\begin{cases} 9x_1 - 3x_2 + 5x_3 + 6x_4 = 4, \\ 6x_1 - 2x_2 + 3x_3 + 4x_4 = 5, \\ 3x_1 - x_2 + 3x_3 + 14x_4 = -8; \end{cases}$

8) $\begin{cases} 3x_1 + 2x_2 + 2x_3 + 2x_4 = 2, \\ 2x_1 + 3x_2 + 2x_3 + 5x_4 = 3, \\ 9x_1 + x_2 + 4x_3 - 5x_4 = 1, \\ 2x_1 + 2x_2 + 3x_3 + 4x_4 = 5, \\ 7x_1 + x_2 + 6x_3 - x_4 = 7; \end{cases}$

9) $\begin{cases} x_1 + 3x_2 + 5x_3 + 7x_4 + 9x_5 = 1, \\ x_1 - 2x_2 + 3x_3 - 4x_4 + 5x_5 = 2, \\ 2x_1 + 11x_2 + 12x_3 + 25x_4 + 22x_5 = 4; \end{cases}$

10) $\begin{cases} 3x + 2y = 4, \\ x - 4y = -1, \\ 7x + 10y = 12, \\ 5x + 6y = 8, \\ 3x - 16y = -5. \end{cases}$

2.2. Sistemaning birgalikdaligini tekshiring va parametrning qiymatlariga bog'liq umumiy yechimni toping:

1) $\begin{cases} 5x_1 - 3x_2 + 2x_3 + 4x_4 = 3, \\ 4x_1 - 2x_2 + 3x_3 + 7x_4 = 1, \\ 8x_1 - 6x_2 - x_3 - 5x_4 = 9, \\ 7x_1 - 3x_2 + 7x_3 + 17x_4 = \lambda; \end{cases}$ 2) $\begin{cases} \lambda x_1 + x_2 + x_3 + x_4 = 1, \\ x_1 + \lambda x_2 + x_3 + x_4 = 1, \\ x_1 + x_2 + \lambda x_3 + x_4 = 1, \\ x_1 + x_2 + x_3 + \lambda x_4 = 1. \end{cases}$

3- §. Bir jinsli chiziqli tenglamalar sistemasi

Ushbu

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = 0, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = 0, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = 0 \end{cases} \qquad (1)$$

yoki matritsaviy shaklda $AX = 0$ bir jinsli sistema har doim birgalikda va *trivial yechim* deb ataluvchi $X(0;\ 0;\ \ldots;\ 0)$ nol yechimga ega. Sistema notrivial yechimga ham ega bo'lishi uchun $r(A) < n$ bo'lishi zarur va yetarli. $m = n$ hol uchun bu $A = 0$ bo'lishi kerakligini bildiradi.

(1) sistemaning umumiy yechimi $X(x_1(c_1, \ldots, c_r);\ \ldots,\ x_r(c_1, \ldots, c_r),\ c_{r+1}, \ldots, c_{n-r})^{\mathrm{T}}$ ustun-vektor bo'lsin. Bundan $c_1, c_2, \ldots, c_{n-r}$ larga navbati bilan bittasiga 1, qolganlariga 0 qiymatlar berib hosil qilinadigan $E_1, E_2, \ldots, E_{n-r}$ ustun-vektorlar sistemasi (1) sistemaning *fundamental yechimlar sistemasi* deyiladi. Umumiy yechimni

$$X = c_1E_1 + \ldots + c_{n-r}E_{n-r}$$

ko'rinishda yozish mumkin, bu yerda $c_1, c_2, \ldots, c_{n-r}$ — ixtiyoriy o'zgarmas sonlar.

Bir jinsli sistema yechimlarining har qanday chiziqli kombinatsiyasi ham yana uning yechimi bo'ladi.

Bir jinslimas $AX = B$ sistemaning umumiy yechimini unga mos bir jinsli $AX = 0$ sistemaning umumiy yechimi bilan bir jinslimas sistemaning biror xususiy yechimining yig'indisi ko'rinishida yozish (topish) mumkin:

$$X = X_0 + c_1E_1 + \ldots + c_{n-k}E_{n-k},$$

bu yerda, X_0 — bir jinslimas sistemaning biror yechimi.

1- misol. Sistemaning fundamental yechimlar sistemasini va umumiy yechimini toping:

$$\begin{cases} 3x_1 + x_2 - 8x_3 + 2x_4 + x_5 = 0, \\ 2x_1 - 2x_2 - 3x_3 - 7x_4 + 2x_5 = 0, \\ x_1 + 11x_2 - 12x_3 + 34x_4 - 5x_5 = 0, \\ x_1 - 5x_2 + 2x_3 - 16x_4 + 3x_5 = 0. \end{cases}$$

▶ Sistema matritsasini tuzamiz va uning rangini topamiz:

$$A = \begin{pmatrix} 3 & 1 & -8 & 2 & 1 \\ 2 & -2 & -3 & -7 & 2 \\ 1 & 11 & -12 & 34 & -5 \\ 1 & -5 & 2 & -16 & 3 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 & 2 & 1 \\ 8 & -4 & -31 & -7 & 2 \\ -32 & 16 & 124 & 34 & -5 \\ 16 & -8 & -62 & -16 & 3 \end{pmatrix} \to \begin{pmatrix} 0 & 1 \\ 1 & 2 \\ -4 & -5 \\ 2 & 3 \end{pmatrix} \Rightarrow r(A) = 2.$$

Qisqartirilgan sistemani quyidagicha olamiz:

$$\begin{cases} 3x_1 + x_2 = 8x_3 - 2x_4 - x_5, \\ 2x_1 - 2x_2 = 3x_3 + 7x_4 - 2x_5. \end{cases}$$

$x_3 = c_1,\ \ x_4 = c_2\ \ \ x_5 = c_3$ deb umumiy yechimni topamiz:

$$X(c_1;\ c_2;\ c_3) = \begin{pmatrix} \dfrac{-19c_1 - 3c_2 + 4c_3}{8} \\ \dfrac{-7c_1 + 25c_2 - 4c_3}{8} \\ c_1 \\ c_2 \\ c_3 \end{pmatrix}.$$

Umuiy yechimdan fundamental yechimlar sistemasini topamiz:

$$E_1 = X(1;\ 0;\ 0) = \begin{pmatrix} -19/18 \\ -7/8 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad E_2 = X(0;\ 1;\ 0) = \begin{pmatrix} -3/8 \\ 25/8 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

$$E_3 = X(0;\ 0;\ 1) = \begin{pmatrix} 1/2 \\ -1/2 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Bu fundamental yechimlar sistemasi yordamida umumiy yechimni

$$X(c_1;\ c_2;\ c_3) = c_1E_1 + c_2E_2 + c_3E_3$$

ko'rinishda yozish mumkin. ◀

2- misol. a parametrning sistema notrivial yechimlarga ega bo'ladigan qiymatlarini va unga mos yechimlarni toping:

$$\begin{cases} a^2x_1 + 3x_2 + 2x_3 = 0; \\ ax_1 - x_2 + x_3 = 0; \\ 8x_1 + x_2 + 4x_3 = 0. \end{cases}$$

▶ Sistema matritsasi

$$A = \begin{pmatrix} a^2 & 3 & 2 \\ a & -1 & 1 \\ 8 & 1 & 4 \end{pmatrix}.$$

Noma'lumlar soni tenglamalar soniga teng bo'lganligi uchun bu sistema determenanti 0 ga teng bo'lganda notrivial yechimga ega bo'ladi:

$$A = \begin{vmatrix} a^2 & 3 & 2 \\ a & -1 & 1 \\ 8 & 1 & 4 \end{vmatrix} = 0; \quad -4a^2 + 24 + 2a + 16 - a^2 - 12a = 0;$$

$$-5a^2 - 10a + 40 = 0; \quad a_1 = -4, \quad a_2 = 2.$$

$a_1 = -4$ bo'lganida:

$$\begin{cases} 16x_1 + 3x_2 + 2x_3 = 0, \\ -4x_1 - x_2 + x_3 = 0, \\ 8x_1 + x_2 + 4x_3 = 0. \end{cases}$$

Bazis minor sifatida $M_2 = \begin{vmatrix} 4 & 3 \\ 2 & -1 \end{vmatrix}$ ni olsak, qisqartirilgan sistemani

$$\begin{cases} 4x_1 + 3x_2 = -2x_3, \\ 2x_1 - x_2 = -x_3 \end{cases}$$

shaklda yozish mumkin. $x_3 = c$ ni ozod noma'lum deb

$$X = \left(-\tfrac{1}{2};\ 0;\ c_1\right) = c_1E_1$$

umumiy yechimni olamiz, $E_1 = \left(-\tfrac{1}{2};\ 0;\ 1\right) = c_1E_1$ – bu yerda fundamental yechimlar sistemasi.

$a_2 = 2$ bo'lgan holda

$$\begin{cases} 4x_1 + 3x_2 + 2x_3 = 0, \\ 2x_1 - x_2 + x_3 = 0, \\ 8x_1 + x_2 + 4x_3 = 0 \end{cases}$$

sistemani hosil qilamiz. $x_3 = c_1$ ni erkli noma'lum deb olsak, bu sistemaning umumiy yechimi

$$X = (-\tfrac{1}{2}c_1;\ 0;\ c_1) = c_1E_1$$

bo'ladi, bu yerda $E_1 = (-\tfrac{1}{2};\ 0;\ 1)$ — fundamental yechimlar sistemasi. ◀

3- misol. Bir jinslimas sistemaning yechimini unga mos bir jinsli sistemaning fundamental yechimlari sistemasidan foydalanib toping:

$$\begin{cases} 2x_1 + x_2 - x_3 + x_4 = 1, \\ x_1 - x_2 + x_3 - 2x_3 = 0, \\ 3x_1 + 3x_2 - 3x_3 + 4x_4 = 2, \\ 4x_1 + 5x_2 - 5x_3 + 7x_4 = 4. \end{cases}$$

▶ Sistema matritsasi va kengaytirilgan matritsasini tuzamiz:

$$A = \begin{pmatrix} 2 & 1 & -1 & 1 \\ 1 & -1 & 1 & -2 \\ 3 & 3 & 3 & 4 \\ 4 & 5 & -5 & 7 \end{pmatrix}, \quad \overline{A} = \left(\begin{array}{cccc|c} 2 & 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -2 & 0 \\ 3 & 3 & 3 & 4 & 2 \\ 4 & 5 & -5 & 7 & 3 \end{array}\right).$$

$r(A) = r(\overline{A}) = 2$, shuning uchun berilgan sistema birgalikda. x_1 va x_2 ni bazis noma'lumlar desak,

$$\begin{cases} 2x_1 + x_2 = 1 - 2x_3 - x_4 \\ x_1 - x_2 = -x_3 + 2x_4 \end{cases}$$

qisqartirilgan sistemani hosil qilamiz. Buning birorta, masalan, $x_3 = x_4 = 0$ dagi yechimini topamiz:

$$\begin{cases} 2x_1 + x_2 = 1 \\ x_1 - x_2 = 0 \end{cases} \Rightarrow x_3 = \tfrac{1}{3}; \quad x_4 = \tfrac{1}{3}.$$

Unda $X_0 = \left(\tfrac{1}{3}; \ \tfrac{1}{3}; \ 0; \ 0\right)^{\mathrm{T}}$ bir jinslimas sistemaning yechimi bo'ladi. Berilgan sistemaga mos

$$\begin{cases} 2x_1 + x_2 - x_3 + x_4 = 1, \\ x_1 - x_2 + x_3 - 2x_3 = 0, \\ 3x_1 + 3x_2 - 3x_3 + 4x_4 = 2, \\ 4x_1 + 5x_2 - 5x_3 + 7x_4 = 4 \end{cases}$$

bir jinsli sistemaning umumiy yechimini topamiz. Qisqartirilgan sistema:

$$\begin{cases} 2x_1 + x_2 = x_3 - x_4, \\ x_1 - x_2 = -x_3 + 2x_4. \end{cases}$$

$x_3 = c_1$, $x_4 = c_2$ ozod noma'lumlar orqali ifodalanuvchi

$$X(c_1; \ c_2) = \left(\tfrac{1}{3}c_2; \ c_1 - \frac{5}{3}c_2; \ c_1; \ c_2\right)$$

umumiy yechimga ega. Fundamental yechimlar sistemasi:

$$E_1 = X(1; 0) = (0; \ 1; \ 1; \ 0)^{\mathrm{T}}, \quad E_2 = X(0;1) = \left(\tfrac{1}{3}; \ -\tfrac{5}{3}; \ 0; 1\right)^{\mathrm{T}}$$

U holda bir jinsli sistemaning umumiy yechimi $X = c_1E_1 + c_2E_2$. Berilgan bir jinslimas sistemaning umumiy yechimi esa

$$X = X_0 + c_1E_1 + c_2E_2$$

bo'ladi. ◀

Mustaqil bajarish uchun mashqlar

3.1. Bir jinsli sistemaning umumiy yechimini va fundamental yechimlar sistemasini toping:

1) $\begin{cases} x_1 + 2x_2 - x_3 = 0, \\ 2x_2 + 9x_2 - 3x_3 = 0. \end{cases}$ 2) $\begin{cases} x_1 - 2x_2 - 3x_3 = 0, \\ -2x_1 + 4x_2 + 6x_3 = 0. \end{cases}$

3) $\begin{cases} 3x_1 + 2x_2 + x_3 = 0, \\ 2x_1 + 5x_2 + 3x_3 = 0, \\ 3x_1 + 4x_2 + 2x_3 = 0. \end{cases}$ 4) $\begin{cases} 2x_1 - 3x_2 + 4x_3 = 0, \\ x_1 + x_2 + x_3 = 0, \\ 3x_1 - 2x_2 + 2x_3 = 0. \end{cases}$

5) $\begin{cases} x_1 + 2x_2 + 4x_3 - 3x_4 = 0, \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0, \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 = 0, \\ 3x_1 + 8x_2 + 24x_3 - 19x_4 = 0. \end{cases}$

6) $\begin{cases} 2x_1 - 4x_2 + 5x_3 + 3x_4 = 0, \\ 3x_1 - 6x_2 + 4x_3 + 2x_4 = 0, \\ 4x_1 - 8x_2 + 17x_3 + 11x_4 = 0. \end{cases}$

3.2. a parametrning sistema notrivial yechimlarga ega bo'ladigan qiymatlarini va bu yechimlarni toping:

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 0, \\ 4x_1 - x_2 + 7x_3 = 0, \\ x_1 + ax_2 + 2x_3 = 0. \end{cases}$$

3.3. Bir jinslimas sistemani yechimining mos bir jinsli sistemaning fundamental yechimlar sistemasidan foydalanib toping:

1) $\begin{cases} 2x_1 + x_2 - x_3 - x_4 + x_5 = 1, \\ x_1 - x_2 + x_3 + x_4 - 2x_5 = 0, \\ 3x_1 + 3x_2 - 3x_3 - 3x_4 + 4x_5 = 2, \\ 4x_1 + 5x_2 - 5x_3 - 5x_4 + 7x_5 = 3. \end{cases}$

2) $\begin{cases} x_1 - x_2 + x_3 - x_4 + x_5 - x_6 = 1, \\ 2x_1 - 2x_2 + 2x_3 + x_4 - x_5 + x_6 = 1. \end{cases}$

3) $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 0, \\ x_1 - 2x_2 - 3x_3 - 4x_4 - 5x_5 = 2, \\ 2x_2 + 3x_3 + 4x_4 + 5x_5 = -1. \end{cases}$

Mustaqil bajarish uchun berilgan mashqlarning javoblari

1- §. 1.1. 1) (1; 4; –7, 7); 2) (4; 6; –35; –1). 3) (70; 40; –20; –16). **1.2.** 1) (–1/2; 1; 3; 3). 2) –17; –13; 41, 5). 3) (–8/5; –7/3; –16/3; –11/3). **1.3.** 1) chiziqli erkli. 2) chiziqli bog'liq. 3) chiziqli erkli. **1.5.** 1) chiziqli erkli. **1.6.** 3. **1.7.** 1) $k = 15$. 2) $k \neq 12$. **1.8.** $r = 3$; $(a_2; a_3; a_4)$. **2.1.** 1) $(1 + \sqrt{3}\, c_1,\ 1 + c_1,\ c_1)^T$. 2) sistema birgalikda emas.

2- §. 2.1. 1) $\left(1 + \sqrt{3}c_1;\ c_1\right)^T$. 2) sistema birgalikda emas. 3) $(-1 + 2c_1;\ 1 + c_1;\ c_1)^T$. 4) $(-1;\ 3;\ -2;\ 2)^T$. 5) $\left(\frac{3}{4} - \frac{1}{4}c_1 + \frac{7}{4}c_2,\ \frac{1}{2} + \frac{3}{2}c_1 - \frac{1}{2}c_2,\ c_1, c_2\right)^T$. 6) sistema birgalikda emas. 7) $(c_1,\ -13 + 3c_1,\ -7,\ 0)^T$. 8) $\left(-\frac{6}{7} + \frac{8}{7}c_1,\ \frac{1}{7} + \frac{13}{7}c_1,\ \frac{15}{7} - \frac{6}{7}c_1,\ c_1\right)^T$. 9) sistema birgalikda emas. 10) sistema birgalikda. $r(A) = r(A_1) = 2$. $x = 1$; $y = \frac{1}{2}$. **2.2.** 1) $\lambda \neq 0$ da sistema birgalikda emas; $\lambda = 0$ bo'lsa,

$X = \left(-\frac{3}{2} - \frac{5}{2}A_1\ \frac{13}{2} - A_1,\ \frac{7}{2} - \frac{7}{2} - - A^9_1 - - A_2,\ A_1,\ A_2\right)^T$. 2) $(\lambda - 1)(\lambda + 3) \neq 0$ da $X = \frac{1}{\lambda+3} \cdot (1,\ 1,\ 1,\ 1\ ;)^T$ $\lambda = 1$da $X = (1 - c_1 - c_2 - c_3,\ c_1,\ c_2,\ c_3)^T$; $\lambda = -3$ da sistema birgalikda emas.

3- §. 3.1. 1) c_1E_1, $E_1 = (3,\ 1,\ 5)^T$. 2) $c_1E_1 + c_2E_2$, $E_1 = (2,\ 1,\ 0)^T$, $E_2 = (3,\ 0,\ 1)^T$. 3) sistema faqat trivial yechimga ega. 4) c_1E_1, $E_1 = (4,\ 1,\ -5)^T$. 5) $c_1E_1 + c_2E_2$, $E_1 = (8,\ -6,\ 1,\ 0)^T$, $E_2 = (-7,\ 5,\ 0,\ 1)^T$. 6) $c_1E_1 + c_2E_2$, $E_1 = (1,\ 0,\ -\frac{5}{2},\ \frac{7}{2})^T$, $E_2 = (0,\ 1,\ 5,\ -7)^T$. **3.2.** $a = -1$, $X = c_1E_1$, $E_1 = \left(-\frac{5}{3}, \frac{1}{3}, 1\right)^T$. **3.3.** 1) $X_0 + c_1E_1 + c_2E_2 + c_3E_3$, $X_0 = \left(\frac{1}{3},\ \frac{1}{3},\ 0,\ 0,\ 0\right)^T$, $E_1 = (0, 1, 1, 0, 0)^T$, $E_2 = (0,\ 1,\ 0,\ 1,\ 0)^T$, $E_3 = \left(\frac{1}{3},\ -\frac{5}{3},\ 0,\ 0,\ 1\right)^T$. 2) $X_0 + c_1E_1 + c_2E_2 + c_3E_3 + c_4E_4$, $X_0 = \left(\frac{1}{3},\ -\frac{1}{3},\ 0,\ 0,\ 0,\ 0\right)^T$ $E_1 = (1,\ 1,\ 0,\ 0,\ 0,\ 0)^T$, $E_2 = (-1, 0, 1, 0, 0, 0)^T$, $E_3 = (0,\ 0,\ 0,\ 1,\ 1,\ 0)^T$, $E_4 = (0,\ 0,\ 0,\ -1,\ 0,\ 1)^T$. 3) $X_0 + c_1E_1 + c_2E_2 + c_3E_3$,

$X_0 = \left(1,\ -\frac{1}{2},\ 0,\ 0,\ 0\right)^T$, $E_1 = (0,\ -\frac{3}{2},\ 1,\ 0,\ 0)^T$, $E_2 = (0,\ -2,\ 0,\ 1,\ 0)^T$, $E_3 = \left(0,\ -\frac{5}{2},\ 0,\ 0,\ 1\right)^T$.

IV bob. TEKISLIKDA ANALITIK GEOMETRIYA

1- §. Tekislikda koordinatalar metodi

Agar tekislikda:

1) har birida musbat yo'nalish tanlab olingan ikkita o'zaro perpendikular to'g'ri chiziq, ya'ni koordinata o'qlari ko'rsatilgan bo'lsa (o'qlardan birinchisi *abssissalar o'qi*, *ikkinchisi ordinatalar* o'qi, o'qlarning kesishgan nuqtasi $O(0;0)$ *koordinatalar boshi* deyiladi);

2) uzunliklarni o'lchash uchun chiziqli birlik ko'rsatilgan bo'lsa, u holda tekislikda *to'g'ri burchakli dekart koordinatalari sistemasi* berilgan deyiladi.

Tekislikning ixtiyoriy nuqtasi M ning to'g'ri burchakli dekart koordinatalari deb x va y sonlarning tartiblangan $(x;\ y)$ juftiga aytiladi, bu yerda x — shu M nuqtaning abssissalar o'qiga proyeksiyasining koordinatasi, y esa ordinatalar o'qiga proyeksiyasining koordinatasi. M nuqta koordinatalari bilan birga $M(x;\ y)$ ko'rinishda yoziladi.

1^0. **Ikkita nuqta orasidagi masofa.** Tekislikda ikkita $A(x_1,\ y_1)$ va $B(x_2;\ y_2)$ *nuqta orasidagi masofa*

$$d = |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

formula bilan hisoblanadi.

1- misol. $A(-2;\ 4)$ va $B(2;\ 1)$ nuqtalar orasidagi masofani toping.

▶ $|AB| = \sqrt{(2+2)^2 + (1-4)^2} = \sqrt{16+9} = \sqrt{25} = 5.$ ◀

2^0. **Kesmani berilgan nisbatda bo'lish.** Tekislikda uchlari $A(x_1;\ y_1)$ va $B(x_1;\ y_1)$ nuqtalarda bo'lgan AB kesmani $\frac{AN}{NB} = \lambda$ *nisbatda bo'luvchi* $N(x;\ y)$ nuqtaning koordinatalari

$$x = \frac{x_1 + \lambda\ x_2}{1+\lambda}, \quad y = \frac{y_1 + \lambda\ y_2}{1+\lambda} \tag{1}$$

formulalar bo'yicha topiladi. Agar N nuqta AB kesmani teng ikkiga bo'lsa, $\lambda = 1$ bo'lib, (1) formulalar

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2} \tag{2}$$

ko'rinishda bo'ladi. (2) — *kesmaning o'rtasini topish formulalari* ham deyiladi.

2- misol. $A(1;\ 4)$ va $B(4;\ -14)$ nuqtalar bilan chegaralangan kesma $C(x_C,\ y_C)$ va $D(x_D,\ y_D)$ nuqtalar orqali uchta teng bo'lakka bo'lingan. C va D nuqtalarning koordinatalarini toping.

▶ C nuqta AB kesmani $\lambda = \frac{AC}{CB} = \frac{1}{2}$ nisbatda bo'ladi. Binobarin, (1) formulaga ko'ra:

$$x_C = \frac{1 + \frac{1}{2} \cdot 4}{1 + \frac{1}{2}} = 2, \ y_C = \frac{4 + \frac{1}{2} \cdot (-14)}{1 + \frac{1}{2}} = -2.$$

Shunday qilib, $C(2;\ -2)$.

D nuqta AB kesmani $\lambda = \frac{AD}{DB} = \frac{2}{1} = 2$ nisbatda bo'ladi. Bu yerdan

$$x_D = \frac{1 + 2 \cdot 4}{1+2} = 3, \quad y_D = \frac{4 + 2 \cdot (-14)}{1+2} = -8.$$

Demak, $D\ (3;\ -8)$. ◀

3^0. **Uchburchak va ko'pburchakning yuzi.** Uchlari $A(x_1;\ y_1)$, $B(x_2;\ y_2)$, $C(x_3;\ y_3)$, ..., $F(x_n;\ y_n)$ nuqtalarda bo'lgan ko'pburchakning yuzi quyidagi formula yordamida hisoblanadi:

$$S = \pm \frac{1}{2} \times \left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \ldots + \begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix} \right]. \tag{3}$$

Xususiy holda, (3) formuladan uchlari $A(x_1,\ y_1)$, $B(x_2,\ y_2)$ va $C(x_3;\ y_3)$ nuqtalarda bo'lgan uchburchak yuzini hisoblash formulasini yozish mumkin:

$$S_{\Delta ABC} = \pm \frac{1}{2} \times \left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} \right]. \tag{4}$$

Bu yerda ishora yuzning musbat ekaniga qarab tanlanadi.

Mustaqil bajarish uchun mashqlar

1.1. $A(2;\ 3)$, $B(5;\ 3)$, $C(0;\ 3)$, $D(-3;\ 0)$, $E(-2;\ 1)$ nuqtalarni yasang.

1.2. Uchlari $A(3;\ 0)$, $B(2;\ 3)$, $C(0;\ 5)$, $D(-2;\ 1)$, $E(-2;-3)$ nuqtalarda bo‘lgan beshburchakni yasang.

1.3. $A(-2;-3)$ va $B(0;-2)$ nuqtalarga abssissalar o‘qiga, ordinatalar o‘qiga va koordinatalar boshiga nisbatan simmetrik bo‘lgan nuqtalarni toping.

1.4. Tomoni 2 birlikka teng bo‘lgan kvadratning diagonallari koordinata o‘qlarida yotadi. Uning uchlarining koordinatalarini toping.

1.5. Agar: 1) $A\ (4;\ -3)$, $B(-11;\ -4)$; 2)$A(2;\ 4)$, $B(-2;\ 1)$ bo‘lsa, A va B nuqtalar orasidagi masofani toping.

1.6. $A(0;\ 1)$, $B(3;\ 3)$ va $C(-4;\ 2)$ nuqtagacha bo‘lgan masofasi 10 ga teng bo‘lgan M nuqtaning koordinatalarini hisoblang.

1.7. Ordinatalar o‘qigacha va $M(1;\ 3)$ nuqtagacha bo‘lgan masofasi 13 ga teng bo‘lgan A nuqtani toping.

1.8. Uchlari: 1) $A(4;2)$, $B(2;-6)$; 2) $A(-2;\ 0)$, $B(6;\ -2)$ nuqtalarda bo‘lgan AB kesmani $\frac{1}{2}$ nisbatda bo‘luvchi nuqtaning koordinatalarini toping.

1.9. Uchlari 1) $A\ (-4;2)$, $B(6;4)$, 2) $A(0;-1)$, $B(6;-3)$ nuqtalarda bo‘lgan AB kesma o‘rtasining koordinatalarini toping.

1.10. Uchlari $A(-4;\ 2)$, $B(6;\ 4)$, $C(-4;\ -1)$ nuqtalarda bo‘lgan uchburchakning medianalari uzunligini toping.

1.11. AB kesmaning uchlaridan biri $A(5;\ -4)$ nuqtada, o‘rtasi esa $C(0;\ -3)$ nuqtada joylashgan. Kesma ikkinchi uchining koordinatalarini toping.

1.12. $ABCD$ parallelogrammning ikkita uchi $A(-6;\ -5)$, $B(2;\ 3)$ va diagonallarining kesishish nuqtasi $M(3;\ 1)$ berilgan. C va D uchlarining koordinatalarini toping.

1.13. Koordinata o‘qlaridan va berilgan $A(4;\ -2)$ nuqtadan teng uzoqlashgan nuqtani toping.

1.14. Shunday M nuqtani topingki, undan abssissalar o‘qigacha va $A\ (-2;\ 4)$ nuqtagacha bo‘lgan masofa 10 ga teng bo‘lsin.

1.15. Uchlari $A(-3;\ 8)$ va $B(1;\ -2)$ nuqtalarda bo‘lgan AB kesmada abssissasi -1 bo‘lgan C nuqtani toping.

1.16. $A(4;\ -2)$ va $B(7;\ 4)$ nuqtalarni tutashtiruvchi kesmada ordinatasi 2 bo‘lgan C nuqtani toping.

1.17. Uchlari $A(2;\ 0)$, $B(5;\ 3)$ va $C(2;\ 6)$ nuqtalarda bo‘lgan uchburchakning yuzini toping.

1.18. $A(1;\ 1)$, $B(-1;\ 7)$ va $C(0;\ 4)$ nuqtalar bitta to‘g‘ri chiziqda *yotishini isbotlang*.

1.19. $A(1;\ 2)$ va $B(4;\ 4)$ nuqtalar berilgan. Absissalar o‘qida shunday C nuqtani topingki, natijada ΔABC ning yuzi 5 kvadrat birlikka teng bo‘lsin.

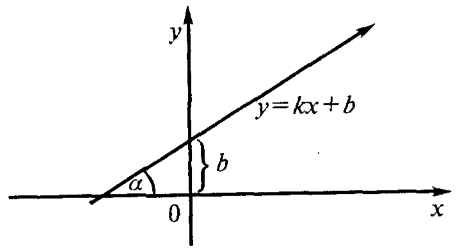
1.20. Uchlari $A(3;\ 1)$, $B(4;\ 6)$, $C(6;\ 3)$ va $D(5;\ -2)$ nuqtalarda bo‘lgan to‘rtburchakning yuzini toping.

2- §. To‘g‘ri chiziq tenglamalari

1⁰. To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi. To‘g‘ri chiziqning burchak *koeffitsiyentli* tenglamasi deb, $y = kx + b$ ko‘rinishdagi tenglamaga aytiladi, bu yerda b — *boshlang‘ich ordinata*, to‘g‘ri chiziqning ordinatalar o‘qidan ajratgan kesmasi; k — to‘g‘ri chiziqning *burchak koeffitsiyenti* deb ataladiva to‘g‘ri chiziq abssissalar o‘qi bilan hosil qiladigan α burchakning tangensiga teng, ya’ni $k = \text{tg}\ \alpha$ (14- rasm).

Agar $b = 0$ bo‘lsa, $y = kx$ tenglama koordinatalar boshidan o‘tuvchi to‘g‘ri chiziq tenglamasi bo‘ladi.

1- misol. Koordinatalar boshidan o‘tuvchi va Oy o‘qi bilan 60° burchak tashkil etuvchi to‘g‘ri chiziqning tenglamasini tuzing.



14- rasm.

▶ Qaralayotgan to'g'ri chiziq koordinatalar boshidan o'tganligi uchun tenglamasini $y = kx$ ko'rinishda qidiramiz. Burchak koeffitsiyent $k = \text{tg}\,\alpha = \text{tg}60° = \sqrt{3}$ bo'lgani uchun $y = \sqrt{3}x$ tenglamani hosil qilamiz. ◀

2- misol. Boshlang'ich ordinatasi $b = 3$, Ox o'qqa og'ish burchagi $\alpha = 30°$ bo'lgan to'g'ri chiziqni yasang va tenglamasini tuzing.

▶ Oy o'qdan $b = 3$ birlik ajratib, bu yerdan Ox o'qqa parallel yordamchi to'g'ri chiziq o'tkazamiz (chizmada shtrixlangan) va bu chiziq bilan 30° burchak tashkil qilib, Oy o'q bilan $b = 3$ birlikda kesishuvchi to'g'ri chiziqni yasaymiz (15- rasm). Bu esa talab qilingan to'g'ri chiziqdir.

To'g'ri chiziqning tenglamasini yozish uchun $b = 3$ va $k = \text{tg}30° = \frac{1}{\sqrt{3}}$ ekanligidan foydalanamiz. U holda $y = kx + b$ tenglamaga ko'ra $y = \frac{1}{\sqrt{3}}x + 3$ izlangan to'g'ri chiziqning tenglamasidir. ◀

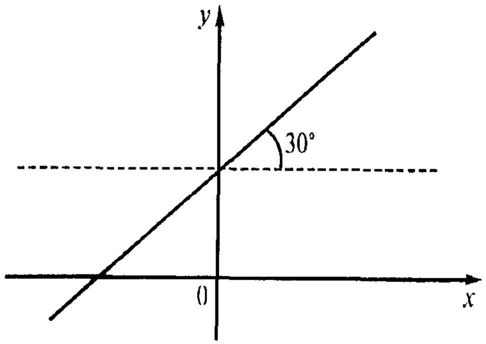
2⁰. To'g'ri chiziqning umumiy tenglamasi. To'g'ri chiziqning umumiy tenglamasi deb quyidagi tenglamaga aytiladi:

$$Ax + By + C = 0,$$

bu yerda: A, B, C — o'zgarmas koeffitsiyentlar.

Xususiy hollari:

1) $C = 0$ bo'lganda, $Ax + By = 0$ yoki $y = kx$, $k = -A/B$, ya'ni koordinatalar boshidan o'tuvchi to'g'ri chiziq tenglamasi;



15- rasm.

2) $B = 0$ bo'lganda $Ax + C = 0$ yoki $x = a$, $a = -C/A$, ya'ni Oy o'qiga parallel bo'lgan to'g'ri chiziq tenglamasi;

3) $A = 0$ bo'lganda $By + C = 0$ yoki $y = b$, $b = -C/B$, ya'ni Ox o'qqa parallel bo'lgan to'g'ri chiziq tenglamasi;

4) $B = 0$, $C = 0$ bo'lganda $Ax = 0$ yoki $x = 0$, ya'ni Oy o'qning tenglamasi;

5) $A = 0$, $C = 0$ bo'lganda $By = 0$ yoki $y = 0$, ya'ni Ox o'qining tenglamasi hosil bo'ladi.

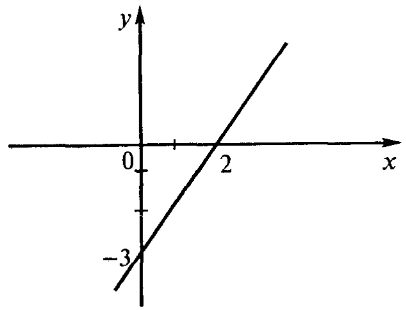
3- misol. To'g'ri chiziq $2x + 3y - 1 = 0$ umumiy tenglamasi bilan berilgan bo'lsa, uning burchak koeffitsiyentli $y = kx + b$ tenglamasini hosil qilib, k va b parametrlarini toping.

▶ To'g'ri chiziqning umumiy tenglamasini y o'zgarvchini x o'zgaruvchiga nisbatan yechib olamiz: $y = -2/3\,x + 1/3$, bu esa berilgan to'g'ri chiziqning burchak koeffitsiyentli tenglamasi bo'lib, undan $k = -3/2$, $b = 1/3$ ekanligini aniqlaymiz. ◀

3⁰. To'g'ri chiziqning koordinata o'qlaridagi kesmalari bo'yicha tenglamasi. To'g'ri chiziqning *koordinata o'qlaridagi kesmalari bo'yicha* tenglamasi deb,

$$\frac{x}{a} - \frac{y}{b} = 1$$

ko'rinishdagi tenglamaga aytiladi, bu yerda a va b — to'g'ri chiziqning Ox va Oy o'qlar bilan kesishish nuqtalarning mos ravishda abssissasi va ordinatasi, ya'ni to'g'ri chiziqning koordinata o'qlaridan ajratgan kesmalarining ma'lum ishora bilan olingan miqdorlari.



16- rasm.

4- misol. $\frac{x}{2}-\frac{y}{3}=1$ tenglama bilan berilgan to‘g‘ri chiziqni yasang.

▶ Tenglamani quyidagicha yozib olamiz: $\frac{x}{2}-\frac{y}{3}=1$, bu yerdan $a=2$, $b=-3$. Ordinatalar o‘qida ordinatasi -3 bo‘lgan, abssissalar o‘qida abssissasi 2 bo‘lgan nuqtalarni belgilaymiz. Ulardan o‘tuvchi to‘g‘ri chiziqni yasaymiz (16- rasm.). Bu esa talab qilingan to‘g‘ri chiziq bo‘ladi. ◀

4°. Berilgan nuqtadan berilgan yo‘nalish bo‘yicha o‘tadigan to‘g‘ri chiziq tenglamasi. $M(x_0;\ y_0)$ nuqta orqali o‘tadigan va k burchak koeffitsiyentiga ega bo‘lgan to‘g‘ri chiziqning tenglamasi ushbu ko‘rinishga ega:

$$y-y_0=k\,(x-x_0) \qquad (1)$$

(1) tenglama tekislikning *bitta nuqtasidan o‘tuvchi to‘g‘ri chiziqlar dastasi tenglamasi* deb ham yuritiladi.

5- misol. (2; −3) nuqtadan o‘tib, Ox o‘qi bilan 45° burchak tashkil qiluvchi to‘g‘ri chiziq tenglamasini tuzing.

▶ Izlanayotgan to‘g‘ri chiziqning burchak koeffitsiyenti $k=\operatorname{tg}45°=1$ ga teng. Shu sababli (1) tenglamadan foydalanib topamiz:

$$y+3=1\cdot(x-2) \text{ yoki } x-y-5=0. \blacktriangleleft$$

5°. Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi. Berilgan $M_1(x_1;\ y_1)$ va $M_2(x_2;\ y_2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi ushbu ko‘rinishga ega:

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}. \qquad (2)$$

(2) *tenglama ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi* deyiladi.

6- misol. $A(2;\ 3)$ va $B(1;\ -1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

▶ (2) tenglamadan foydalanamiz:

$$\frac{x-2}{1-2}=\frac{y-3}{-1-3} \text{ yoki } \frac{x-2}{-1}=\frac{y-3}{-4}$$

bu yerdan $4x-y-5=0$. ◀

Mustaqil bajarish uchun mashqlar

2.1. Koordinatalar boshidan o‘tuvchi va Ox o‘qi bilan:
1) 30°; 2) 45°; 3) 120°;
4) 135°; 5) arctg 2; 6) arctg (−3)
burchak tashkil etuvchi to‘g‘ri chiziqlarning tenglamalarini tuzing.

2.2. Quyidagi to‘g‘ri chiziqlar Ox o‘qi bilan qanday burchak tashkil etishini aniqlang.

1) $y=\frac{\sqrt{3}}{3}x$; 2) $y=-\sqrt{3}x$;
3) $y=4x$; 3); 4) $y=-3x$.

2.3. Boshlang‘ich ordinatasi $b=3$, Ox o‘qiga og‘ish burchagi
1) $\alpha=45°$; 2) $\alpha=60°$; 3) $\alpha=135°$
bo‘lgan to‘g‘ri chiziqlarni yasang va tenglamalarini tuzing.

2.4. Boshlang‘ich ordinatasi $b=-2$, Ox o‘qiga og‘ish burchagi
1) $\alpha=\pi/6$; 2) $\alpha=\pi/3$; 3) $\alpha=120°$
bo‘lgan to‘g‘ri chiziqlarni yasang va tenglamalarini tuzing.

2.5. Abssissalar o‘qi bilan 45° burchak tashkil qilib, M (2; 3) nuqta orqali o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing hamda k va b parametrlarini aniqlang.

2.6. Quyidagi berilgan to‘g‘ri chiziq tenglamalarini burchak koeffitsiyentli tenglamaga keltiring va har birida k va b parametrlarni aniqlang:
1) $2x-3y=6$; 2) $2x+3y=0$;
3) $2y=-4$; 4) $3x+6=0$,
5) $x/3+y/4=1$.

2.7. To‘g‘ri chiziqlarni yasang:
1) $3x+2y=6$; 2) $2x+3y=0$;
3) $4y-2=0$; 4) $3-x=0$.

2.8. To‘g‘ri chiziq tenglamalarini kesmalar bo‘yicha tenglamasiga keltiring va yasang.
1) $3x+4y=12$; 2) $3x-4y=12$;
3) $2x-3y=6$.

2.9. Koordinata o‘qlari va $2x-5y+20=0$ to‘g‘ri chiziq bilan chegaralangan uchburchakning yuzini toping.

2.10. (2; 3) nuqtadan o'tuvchi va koordinata burchagidan yuzi 12 kv birlikga teng bo'lgan uchburchak ajratuvchi to'g'ri chiziq tenglamasini tuzing.

2.11. Rombning diagonallari 8 va 3 birlikka teng. Rombning katta diagonalini Ox o'q uchun, kichkina diagonalini Oy o'q uchun qabul qilib, romb tomonlarining tenglamalarini yozing.

2.12. (–2; 5) nuqtadan o'tib Ox o'q bilan: 1) 30°; 2) 45°; 3) 60°; 4) 135°; 5) 0° burchaklar tashkil qiluvchi to'g'ri chiziqlar tenglamalarini tuzing.

2.13. (–3; 6) nuqtadan o'tuvchi to'g'ri chiziqlar dastasidan koordinata o'qlarining musbat yarimo'qlaridan teng kesmalar ajratadiganining tenglamasini yozing.

2.14. 1) $A(4; -1)$ va $B(-2; -9)$; 2) $C(0; 2)$ va $D(-2; 4)$; 3) $E(-2; 1)$ va $F(-4; 0)$ nuqtalardan o'tuvchi to'g'ri chiziqlar tenglamalarini yozing.

2.15. Uchlari $A(-1; 3)$, $B(4; -2)$, $C(0; -5)$ nuqtalarda bo'lgan uchburchak tomonlarining tenglamalarini tuzing.

2.16. A(2; 8) nuqtada hamda uchlari $M(6; -5)$ va $N(-2; 1)$ nuqtalarda bo'lgan MN kesmaning o'rtasidan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

2.17. $A(6; 2)$ va $B(-3; 8)$ nuqtalardan o'tuvchi to'g'ri chiziqning koordinata o'qlaridan ajratgan kesmalarini toping.

3- §. Ikki to'g'ri chiziq orasidagi burchak

Tenglamalari $y = k_1x + b_1$ va $y = k_2x + b_2$ bilan berilgan ikkita to'g'ri chiziq orasidagi φ burchakning tangensi

$$\mathrm{tg}\varphi = \pm\left|\frac{k_2-k_1}{1+k_1\cdot k_2}\right| \qquad (1)$$

formula bo'yicha hisoblanadi, bunda « + » ishora o'tkir burchakka, «–» ishora esa o'tmas burchakka mos keladi.

(1) formuladan ikki to'g'ri chiziqning

— *parallellik*: $k_1 = k_2$;

— *perpendikularlik*: $k_1k_2 = -1$

shartlarini olish mumkin. Ikki to'g'ri chiziq umumiy $A_1x + B_1y + C_1 = 0$ va $A_2x + B_2y + C_2 = 0$ tenglamalari bilan berilgan bo'lsa, ular orasidagi φ burchakning tangensi

$$\mathrm{tg}\varphi = \pm\left|\frac{A_1B_2-A_2B_1}{A_1A_2+B_1B_2}\right| \qquad (2)$$

formula bo'yicha hisoblanadi. (2) formuladan to'g'ri chiziqlarning

— *parallellik*: $A_1/A_2 = B_1/B_2$;

— *perpendikularlik*: $A_1A_2 + B_1B_2 = 0$

shartlarini olish mumkin.

1- misol. $x - 3y + 5 = 0$ va $2x + 4y - 7 = 0$ to'g'ri chiziqlar orasidagi o'tkir burchakni toping.

▶ $A_1 = 1$, $B_1 = -3$, $A_2 = 2$, $B_2 = 4$ bo'lganligi uchun

$$\mathrm{tg}\varphi = \pm\left|\frac{1\cdot4-2\cdot(-3)}{1\cdot2+(-3)\cdot4}\right| = \frac{10}{10} = 1,\ \mathrm{tg}\varphi = 1,\ \varphi = 45°.$$ ◀

2- misol. $y = 2x - 3$, $y = 1/2\,x + 1$ to'g'ri chiziqlar orasidagi o'tkir burchakni toping.

▶ $k_1 = 2$, $k_2 = 1/2$ bo'lganligi uchun

$$\mathrm{tg}\varphi = \pm\left|\frac{\frac{1}{2}-2}{1+\frac{1}{2}\cdot2}\right| = \frac{3}{4};\ \varphi = \mathrm{arctg}\ 3/4.$$ ◀

Mustaqil bajarish uchun mashqlar

3.1. Quyidagi to'g'ri chiziqlar orasidagi o'tkir burchakni toping:

1) $5x - y + 7 = 0$; $2x - 3y + 1 = 0$;
2) $2x + y = 0$; $y = 3x + 4$;
3) $3x + 2y = 0$; $6x + 3y + 9 = 0$;
4) $3x - 4y = 6$; $8x + 6y = 11$.

3.2. Quyidagi tenglamalar bilan berilgan to'g'ri chiziqlar orasidan o'zaro parallel va perpendikular bo'lganlarini ajrating: $3x - 2y + 7 = 0$, $6x - 4y - 7 = 0$, $6x + 4y + 4 = 0$, $2x + 3y - 1 = 0$

3.3. $A(2; 3)$ nuqtadan o'tuvchi va $2x - y = 2$ to'g'ri chiziqqa 1) *parallel*; 2) *perpendikular* bo'lgan to'g'ri chiziqlar tenglamlarini yozing.

3.4. Tomonlarining tenglamalari mos ravishda $x + 2y = 0$, $x + 4y = 6$, $x - 4y - 6 = 0$ bo'lgan uchburchakning ichki burchaklarni toping.

3.5. Koordinata boshidan o'tuvchi va $y = 4 - 2x$ tenglama bilan berilgan to'g'ri chiziq bilan 45° burchak ostida kesishuvchi to'g'ri chiziq tenglamasini tuzing.

3.6. Uchlari $A(0; 7)$, $B(6; -1)$, $C(2; 1)$ nuqtalarda bo'lgan uchburchakning burchaklarini toping.

3.7. Uchlari $A(-4; 2)$, $B(2; -5)$, $C(5; 0)$ nuqtalarda bo'lgan uchburchakning B uchidan tushirilgan balandligi tenglamasini tuzing.

3.8. Parallelogrammning $x - y + 1 = 0$ va $2x + 3y - 6 = 0$ tomonlarini hamda uning uchlaridan biri $C(7; 1)$ ni bilgan holda qolgan ikkita tomonining tenglamasini tuzing.

3.9. Parallelogrammning uchta uchi $A(-1; 3)$, $B(4; 6)$, $C(2; -5)$ berilgan. Uning tomonlari tenglamalarini tuzing.

3.10. $M(-1; 7)$ va $N(3; -1)$ nuqtalarini tutashtiruvchi kesma o'rtasiga o'tkazilgan perpendikularning tenglamasini tuzing.

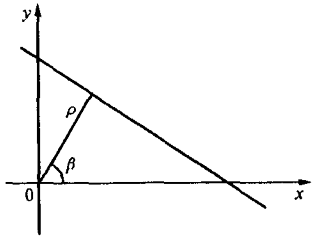
3.11. Rombning ikkita qarama-qarshi $M(-3; 2)$, $N(7; -6)$ uchlari ma'lum. Rombning diagonallari tenglamasini tuzing.

3.12. $A(3; 4)$ nuqtadan $2x + 5y + 3 = 0$ to'g'ri chiziqqa tushirilgan perpendikularning asosini toping.

3.13. *Kvadratning qarama-qarshi uchlari* $B(-2; 2)$ va $D(0; -3)$ nuqtalarda. Kvadrat tomonlarining tenglamalarini tuzing.

3.14. Teng yonli to'g'ri burchakli ABC uchburchakda o'tkir burchak uchi $A(1; 3)$ va qarshi tomondagi katet tenglamasi $2x - y + 4 = 0$ berilgan. Uchburchakning qolgan ikkita tomoni tenglamalarini tuzing.

4- §. To'g'ri chiziqning normal tenglamasi



17- rasm.

1°. Nuqtadan to'g'ri chiziqqacha bo'lgan masofa. To'g'ri chiziqning *normal tenglamasi* deb

$$x \cos\beta + y \sin\beta - \rho = 0 \qquad (1)$$

ko'rinishdagi tenglamaga aytiladi. Bu yerda ρ — koordinata boshidan to'g'ri chiziqqa tushirilgan perpendikular (*normal*) ning uzunligi; β — bu normalning Ox o'qiga og'ish burchagi (17- rasm.). Agar to'g'ri

chiziq umumiy $Ax + By + C = 0$ tenglamasi bilan berilgan bo'lsa, uning tenglamasini normal ko'rinishdagi tenglamaga keltirish uchun tenglamaning har bir hadi normallovchi ko'paytuvchi $M = \pm\frac{1}{\sqrt{A^2+B^2}}$ ga ko'paytiriladi. Normallovchi ko'paytuvchining ishorasi ozod had C ning ishorasiga qarama-qarshi qilib olinadi.

Berilgan $(x_0; y_0)$ nuqtadan to'g'ri chiziqqacha bo'lgan masofa

$$d = |x_0 \cos\beta + y_0 \sin\beta - p| \qquad (2)$$

yoki

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2+B^2}} \qquad (3)$$

formulalar bilan hisoblanadi.

1- misol. Umumiy tenglamasi $x + y - 3 = 0$ bilan berilgan to'g'ri chiziqning normal tenglamasini yozing.

▶ Normallovchi ko'paytuvchini tuzamiz:

$$M = \pm\frac{1}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2};$$

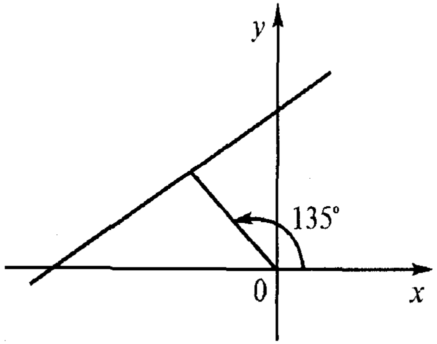
berilgan tenglamani $M = \frac{\sqrt{2}}{2}$ — normallovchi ko'paytuvchiga ko'paytiramiz:

$$\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - \frac{\sqrt{2}\cdot 3}{2} = 0 \text{ yoki } x\cos 45^\circ + y \sin 45^\circ - \frac{3}{\sqrt{2}} = 0,$$

bu yerda $\beta = 45°$; $\rho = \frac{3}{\sqrt{2}}$. ◀

2- misol. Normal uzunligi $\rho = 2$, normalning Ox o'qqa og'ish burchagi 135° bo'lgan to'g'ri chiziqni yasang va tenglamasini yozing.

▶ To'g'ri burchakli dekart koordinatalari sistemasini quramiz. Koordinatalar boshidan ikki birlik uzunlikka ega bo'lgan va Ox o'qi bilan 135° burchak tashkil etuvchi normalni yasaymiz. Bu normalning uchidan unga perpendikular qilib to'g'ri chiziq yasaymiz (18- rasm). Yasalgan to'g'ri chiziq talab qilingan to'g'ri chiziqni beradi. ◀



18- rasm.

To‘g‘ri chziqning tenglamasini yozish uchun esa $\beta = 135°$, $\rho = 2$ ekanligini e’tiborga olsak, $x \cos 135° + y \sin 135° - 2 = 0$ — normal ko‘rinishdagi yoki

$$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 2 = 0,$$

$$x - y + 2\sqrt{2} = 0$$

— umumiy ko‘rinishdagi tenglamasini yozish mumkin. ◀

3- misol. $A(2; -3)$ nuqtadan $2x - 3y - 1 = 0$ to‘g‘ri chiziqgacha bo‘lgan masofani toping.

▶ (3) formuladan foydalanamiz:

$$d = \frac{|2 \cdot 2 - 3(-3) - 1|}{\sqrt{2^2 + (-3)^2}} = \frac{|4+9-1|}{\sqrt{4+9}} = \frac{12}{\sqrt{13}};\ d = \frac{12}{\sqrt{13}}.\ ◀$$

Mustaqil bajarish uchun mashqlar

4.1. To‘g‘ri chiziq tenglamalarini normal ko‘rinishga keltiring: 1) $3x - 4y - 20 = 0$; 2) $x - y - 1 = 0$; 3) $x + y + 1 = 0$; 4) $y = 2x + 5$.

4.2. Normal uzunligi $\rho = 3$, normalning Ox o‘qiga og‘ish burchagi: 1) 45°; 2) 225°; 3) 315° bo‘lgan to‘g‘ri chiziqni yasang va uining tenglamasini yozing.

4.3. $A(2; 3)$, $B(3; 2)$ va $C(0; 1)$ nuqtalardan $3x + 4y - 10 = 0$ to‘g‘ri chiziqqacha bo‘lgan masofalarni toping. Nuqtalar va to‘g‘ri chiziqni yasang.

4.4. O‘zaro parallel $2x - 3y = 6$, $4x - 6y = 25$ to‘g‘ri chiziqlar orasidagi masofani toping.

4.5. Koordinatalar boshidan $a = \sqrt{5}$ birlik masofadan o‘tuvchi $y = kx + 5$ to‘g‘ri chiziq tenglamasidagi k parametrni toping.

4.6. $4x - 3y = 0$ to‘g‘ri chiziqdan $d = 4$ birlik masofada yotuvchi nuqtalarning geometrik o‘rni tenglamasini tuzing.

4.7. $8x - 15y = 0$ to‘g‘ri chiziqqa parallel bo‘lib $A(4;-2)$ nuqtadan $d = 4$ birlik masofadan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

4.8. $2x - y = 4$ to‘g‘ri chiziqqa nisbatan $2x + y = 4$ to‘g‘ri chiziqdan ikki barobar uzoqda joylashgan nuqtalarning geometrik o‘rni tenglamasini tuzing.

4.9. Uchlari $A(-3; 0)$, $B(2; 5)$, $C(3; 2)$ nuqtalarda bo‘lgan uchburchakning BD balandligini aniqlang.

4.10. $A(2; 4)$ nuqtadan o‘tuvchi va koordinatalar boshidan 2 birlik uzoqlikdan o‘tadigan to‘g‘ri chiziq tenglamasini tuzing.

4.11. $A(-4; -3)$, $B(-5; 0)$, $C(5; 6)$, $D(1; 0)$ nuqtalar trapetsiyaning uchlari bo‘lishini tekshiring va uning balandligini toping.

4.12. Koordinatalar boshidan o‘tuvchi to‘g‘ri chiziq $A(2; 2)$ va $B(4; 0)$ nuqtalardan bir xil masofadan o‘tishi ma’lum bo‘lsa, bu masofani toping.

5-§. Ikkinchi tartibli chiziqlar. Aylana

Ikkinchi tartibli chiziq deb tenglamasi x va y o‘zgaruvchilarga nisbatan ikkinchi tartibli algebraik tenglama bo‘lgan chiziqqa aytiladi. Uning tenlamasi, umumiy holda,

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$$

ko‘rinishda yoziladi. Xususiy hollarda , bu tenglama aylana, ellips, giperbola, parabolani, biror nuqtani ifodalashi yoki hech qanday geometrik shaklni ifodalamasligi ham mumkin.

Aylana deb berilgan nuqtadan (markazdan) teng uzoqlikda yotuvchi nuqtalarning geometrik o‘rnidan iborat chiziqqa aytiladi. Markazi $C(a; b)$ nuqtada va radiusi r bo‘lgan aylana tenglamasi

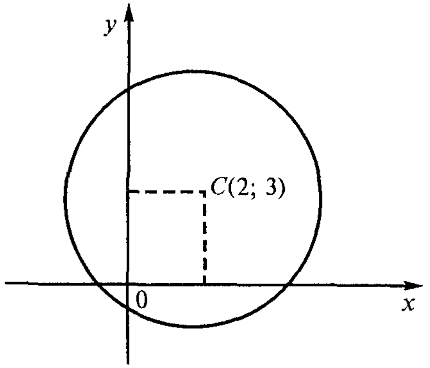
$$(x - a)^2 + (y - b)^2 = r^2 \qquad (1)$$

ko‘rinishiga ega. Bu aylananing kanonik tenglamasidir.

Aylananing umumiy tenglamasi deb

$$Ax^2 + Ay^2 + 2Dx + 2Ey + F = 0 \qquad (2)$$

ko‘rinishidagi tenglamaga aytiladi.



19- rasm.

1- misol. Markazi $C(2;\ 3)$ nuqtada bo'lgan, $r = 4$ radiusli aylana tenglamasini tuzing va uni yasang.

▶ Masala shartiga ko'ra: $a = 2,\ b = 3,\ r = 4$.

Bu qiymatlarni (1) tenglamaga qo'yib aylananing $(x-2)^2+(y-3)^2=16$ tenglamasni hosil qilamiz. Aylanani yasash uchun to'g'ri burchakli dekart koordinatalar sistemasini qurib, bu sistemada aylana markazining o'rnini aniqlaymiz. Markazdan 4 birlik radius bilan aylanani yasaymiz (19- rasm). ◀

2- misol. Umumiy tenglamasi bilan berilgan aylana markazi C ning koordinatalarini va r radiusni toping:

$$9x^2+9y^2+36x-18y+20=0.$$

▶ Berilgan tenglamani 9 ga hadlab bo'lamiz va o'zgaruvchilarni alohida guruhlaymiz:

$$\left(x^2+4x\right)+\left(y^2-2y\right)+\frac{20}{9}=0.$$

Qavsdagi ifodalarni to'la kvadratga to'ldiramiz:

$$(x+2)^2-4+(y-1)^2-1+20/50=0$$
$$\text{yoki } (x+2)^2+(y-1)^2=(5/3)^2.$$

Shunday qilib, berilgan aylana markazi C(–2;1) nuqtada bo'lib, radiusi $r = 5/3$. ◀

Mustaqil bajarish uchun mashqlar

5.1. Markazi C nuqtada bo'lgan va radiusi r berilgan quyidagi aylanalarning tenglamalarini tuzing va yasang:

1) $C(4;-7),\ r=5$; 2) $C-(-3;\ 3),\ r=1$; 3) $C(-1;\ 0),\ r=\sqrt{5}$; 4) $C(-1;\ 0),\ r=3$.

5.2. Markazi C(–5; 7) nuqtada, radiusi 10 ga teng aylana $M(-11;15)$ nuqtadan o'tadimi?

5.3. Markazi $C(12;\ -5)$ *nuqtada bo'lgan va koordinatalar boshidan* o'tuvchi aylana tenglamasini tuzing.

5.4. Diametrli $M(2;\ -7)$ va N(–4; 3) nuqtalarda bo'lgan aylana tenglamasini tuzing.

5.5. Diametri $12x + 5y - 60 = 0$ to'g'ri chiziqning koordinata o'qlari orasidagi kesmasidan iborat bo'lgan aylana tenglamasini tuzing.

5.6. Ox o'qqa koordinatalar boshida urinuvchi va Oy o'qini (0; 10) nuqtada kesib o'tuvchi aylana tenglamasini tuzing.

5.7. $A(3;\ +1)$ va $B(-4;\ 8)$ nuqtalardan o'tuvchi, $r = 13$ radiusli aylana tenglamasini tuzing.

5.8. Koordinatalar o'qiga urinuvchi va $M(-2;\ -4)$ nuqtadan o'tuvchi aylana tenglamasini tuzing.

5.9. Uchlari $A(-2;\ 9)$, $B(-4;\ 5)$, $C(5;\ 8)$ nuqtalarda bo'lgan uchburchakka tashqi chizilgan aylana tenglamasini tuzing.

5.10. $M(-8;\ -10)$, $N(-1;\ 7)$ nuqtalardan o'tuvchi aylana ordinatalar o'qiga urinadi. Uning tenglamasini tuzing.

5.11. Quyidagi aylanalarning markazi C ning koordinatalarini va radiusi r ni toping:
1) $x^2 + y^2 - 8x + 12y - 29 = 0$;
2) $x^2 + y^2 - 6x - 4y - 17 = 0$.

5.12. Quyidagi aylanalarning koordinata o'qlari bilan kesishish nuqtalarini toping:
1) $x^2 + y^2 - 4x + 4y + 3 = 0$;
2) $x^2 + y^2 + 6x + 11y + 10 = 0$.

5.13. $x^2 + y^2 + 6x - 14y - 6 = 0$ va $x^2 + y^2 - 24x + 2y - 51 = 0$ aylanalar markazlari orasidagi masofani toping.

5.14. $x - y + 1 = 0$ to'g'ri chiziqning $x^2 + y^2 - 4x + 16y - 5 = 0$ aylana bilan kesishish nuqtalarini toping.

5.15. Markazi $C(8;\ 6)$ nuqtada bo'lgan va $5x - 12y = 46$ to'g'ri chiziqqa urinadigan aylana tenglamasini tuzing.

5.16. $x^2 + y^2 - 2x + 2y - 23 = 0$ aylananing $A(4;\ -5)$ nuqtasiga o'tkazilgan urinma tenglamasini tuzing.

6- §. Ellips

Ellips deb *fokuslar* deb ataluvchi ikkita tayinlangan nuqtagacha bo'lgan masofalari yig'indisi o'zgarmas ($2a$) bo'lib, fokuslar orasidagi masofa ($2c$) dan katta bo'lgan nuqtalarning geometrik o'rniga aytiladi. Fokuslari F_1 va F_2 nuqtalar Ox o'qida joylashgan, koordinata o'qlariga nisbatan simmetrik ellipsning *kanonik* (sodda) tenglamasi (20- rasm)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (1)$$

ko'rinishda bo'ladi.

Ellipsning o'z simmetriya o'qlari (koorinata o'qlari) bilan kesishish nuqtalari A_1 va A_2, B_1 va B_2 *ellipsning uchlari* deyiladi.

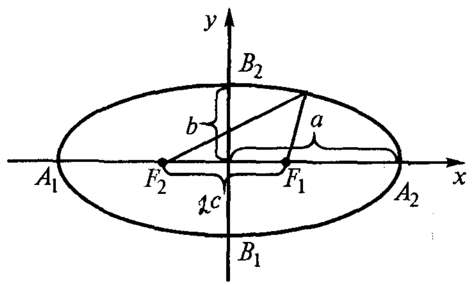
$A_1A_2 = 2a$ — katta o'q, $B_1B_2 = 2b$ — kichik o'q, jumladan, a — *katta yarim* o'q, b — *kichik yarim* o'q deb aytiladi. $F_1(-c;\ 0)$, $F_2\ (c;\ 0)$ fokus nuqtalarining koordinatalarini topishda

$$a^2 - b^2 = c^2 \qquad (2)$$

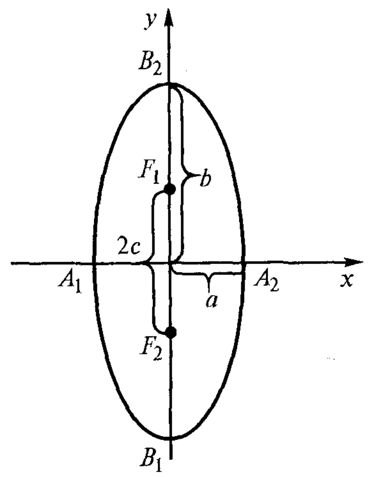
tenglikdan foydalaniladi, bu yerda c — fokus nuqtalar orasidagi masofaning yarmi. Fokus nuqtalar orasidagi $2c$ masofaning katta $2a$ o'qqa nisbati ellipsning *ekssentrisiteti* deb yuritiladi. Ekssentrisitet

$$\varepsilon = \frac{c}{a} \qquad (3)$$

formula bilan hisoblanadi. Ravshanki, $\varepsilon < 1$.



20- rasm.



21- rasm.

Agar koordinata o'qlariga nisbatan simmetrik ellipsning fokuslari Oy o'qida yotadigan bo'lsa (21- rasm), u holda $b > a$ bo'ladi va $B_1B_2 = 2b$ — katta o'q, $A_1A_2 = 2a$ kichik o'q bo'ladi. Bunday ellipsning ekssentrisiteti

$$\varepsilon = \frac{c}{b} \qquad (4)$$

formula bilan hisoblanadi, bu yerda $c^2 = b^2 - a^2$.

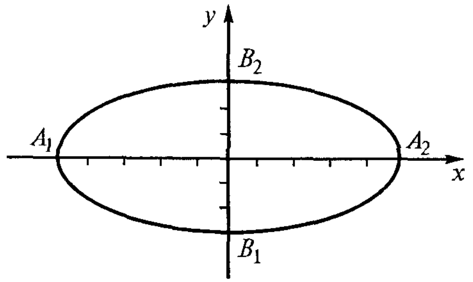
Ellipsning ixtiyoriy $M(x;\ y)$ nuqtasidan fokuslargacha masofalari ellipsninig *fokal radiuslari* deyiladi. F_1 va F_2 — fokuslargacha bo'lgan fokal radiuslarni mos ravishda r_1 va r_2 orqali belgilasak, ular quyidagi formulalar yordamida hisoblanadi.

$$r_1 = |a - \varepsilon x|, \quad r_2 = |a + \varepsilon x|. \qquad (5)$$

1- misol. $9x^2 + 25y^2 - 225 = 0$ ellipsning uchlarini, o'qlarini, fokuslarini va ekssentrisitetini toping hamda ellipsni yasang.

▶ Berilgan tenglamani (1) ko'rinishidagi kanonik ko'rinishga keltiramiz, buning uchun ozod hadni o'ng tomonga o'tkazamiz va tenglamaning barcha hadlarini unga bo'lamiz. Natijada

$$\frac{x^2}{25}+\frac{y^2}{9}=1 \text{ yoki } \frac{x^2}{5^2}+\frac{y^2}{3^2}=1.$$



22- rasm.

Hosil qilingan tenglikdan $a = 5$, $b = 3$ ni aniqlaymiz. Bu yerda ellipsning o'qlari $2a = 10$, $2b = 6$, uchlarining koordinatalari esa $A_1(-5;\ 0)$, $A_2(5;\ 0)$, $B_1(0;\ -3)$, $B_2(0;\ 3)$.

Nihoyat, $c = \sqrt{a^2 - b^2} = \sqrt{5^2 - 3^2} = 4$ bo'lganligi uchun fokuslari $F_1(-4;\ 0)$, $F_2(4;\ 2)$ nuqtalarda joylashgan ekan. Ellipsning ekssentrisiteti esa $\varepsilon = 4/5 = 0{,}8$.

Ellipsni yasash uchun to'g'ri burchakli dekart koordinatalari sistemasida ellipsning uchlarini aniqlaymiz va bu nuqtalar orqali silliq egri chiziq yordamida ellipsning shaklini yasaymiz (22-rasm). ◀

Mustaqil bajarish uchun mashqlar

6.1. Ellipslarning uchlari koordinatalarini, o'qlarini, fokuslarini toping hamda ellipsni yasang:

1) $16x^2 + 25y^2 = 400$;
2) $4x^2 + 9y^2 = 36$;
3) $16x^2 + 9y^2 = 144$;
4) $25x^2 + 9y^2 = 900$.

6.2. Fokuslari *Ox* o'qda bo'lib, yarim o'qlari 4 va $\sqrt{5}$ ga teng ellipsning tenglamasini tuzing.

6.3. Ellipsning katta yarim o'qi $a = 4$ bo'lib, $M\left(-2;\ 3\sqrt{\frac{5}{2}}\right)$ nuqtadan o'tadi. Ellipsning kanonik tenglamasini tuzing.

6.4. Kichik yarim o'qi 24 ga teng va fokuslaridan biri $A(-5;0)$ nuqtada bo'lgan ellipsning kanonik tenglamasini tuzing.

6.5. Ellipsning fokuslari orasidagi masofa 30 ga, *Ox* o'qida yotuvchi katta o'qi 34 ga teng. Ellipsning kanonik tenglamasini tuzing va uning ekssentrisitetini toping.

6.6. *Ellipsning fokuslaridan biri $A(6;0)$ nuqtada va ekssentrisiteti* $\varepsilon = \frac{1}{2}$ bo'lsa, uning kanonik tenglamasini tuzing.

6.7. Fokuslari *Ox* o'qida bo'lgan ellipsning yarim o'qlari yig'indisi 8 ga, fokuslari orasidagi masofa esa 8 ga teng bo'lsa, uning tenglamasini tuzing.

6.8. Fokuslari *Ox* o'qida bo'lgan ellips $M\left(\sqrt{3};\ \sqrt{6}\right)$ va $N\left(3;\ \sqrt{2}\right)$ nuqtalardan o'tadi. Ellipsning kanonik tenglamasini tuzing.

6.9. $M(6;\ 4)$ va $N(8;\ 3)$ nuqtalardan o'tuvchi ellipsning fokuslari *Ox* o'qida yotadi. Ellipsning kanonik tenglamasini tuzing.

6.10. $2x^2 + 4y^2 = 8$ ellips fokuslarining koordinatalari, ekssentrisiteti va $M\left(1 : \sqrt{3/2}\right)$ nuqtasining fokal radiuslarini toping.

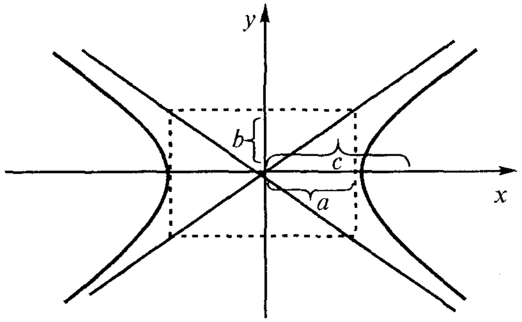
6.11. Koordinata o'qlariga nisbatan simmetrik ellipsning fokuslari *Ox* o'qida joylashgan bo'lib, ekssentrisiteti $\varepsilon = 3/4$. Ellipsning $M(-4;\sqrt{21})$ nuqtasidan fokuslarigacha bo'lgan masofalarni toping.

6.12. Ellips fokuslarining biridan katta o'qi uchlarigacha bo'lgan masofalari mos ravishda 1 va 5. *Ellipsning kanonik tenglamasini* tuzing.

6.13. Yer shari fokuslaridan birida Quyosh turgan ellips bo'yicha harakat qiladi. Agar Yerning Quyoshdan eng uzoqlashgan masofasi 152,5 million kilometr, eng yaqinlashgan masofasi 147,5 million kilometr bo'lsa, Yer orbitasining katta o'qi va ekssentrisitetini toping.

7- §. Giperbola

Fokuslar deb ataluvchi berilgan ikki nuqtagacha masofalari *ayirmasining absolut qiymati o'zgarmas* $(2a)$ *bo'lgan va fokuslar* orasidagi masofa $(2c)$ dan kichik bo'lgan nuqtalarning geometrik o'rni *giperbola* deb ataladi. Koordinata o'qlariga nisbatan simmetrik



23- rasm.

bo'lgan, fokuslari Ox o'qida joylashgan (23- rasm) *giperbolaning kanonik* sodda tenglamasi quyidagi ko'rinishga ega:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \quad (1)$$

$A_1(-a;\ 0)$ va $A_2(a;\ 0)$ nuqtalar giperbolaning uchlari orasidagi $2a$ masofa — giperbolaning *haqiqiy* o'qi, $B_1(0;\ -b)$, $B_2(0;\ b)$ nuqtalar orasidagi $2b$ masofa giperbolaning *mavhum o'qi* deb yuritiladi. Koordinatalar boshidan fokus nuqtagacha bo'lgan masofa

$$c = \sqrt{b^2 + a^2} \quad (2)$$

formula yordamida hisoblanadi.

Giperbolaning *ekssentrisiteti* deb, fokuslar orasidagi masofaning uning haqiqiy o'qiga nisbatiga aytiladi:

$$\varepsilon = \frac{A}{a}. \quad (3)$$

Ravshanki, $\varepsilon > 1$.

Giperbola ikkita asimptotaga ega, ularning tenglamalari

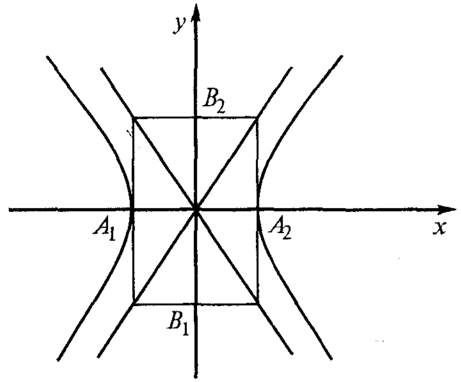
$$y = \frac{b}{a}x,\ y = -\frac{b}{a}x. \quad (4)$$

Giperbolaning $M(x;\ y)$ nuqtasidan F_1 va F_2 fokuslarigacha bo'lgan r_1 va r_2 masofalar *fokal radiuslari* deb atalib, quyidagicha topiladi.

$$r_1 = |\varepsilon x + a|,\quad r_2 = |\varepsilon x - a|. \quad (5)$$

1- misol. $16x^2 - 9y^2 - 144 = 0$ giperbolaning o'qlarini, uchlarini, ekssentrisitetini toping, asimptotalarining tenglamalarini yozing hamda yasang.

▶ Ozod hadni o'ng tomonga o'tkazamiz va berilgan tenglamaning barcha hadlarini unga bo'lamiz. Natijada giperbolaning kanonik tenglamasini hosil qilamiz:



24- rasm.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1 \text{ yoki } \frac{x^2}{3^2} - \frac{y^2}{4^2} = 1.$$

Bu yerda $a = 3$, $b = 4$ yoki haqiqiy o'qi $2a = 6$, mavhum o'qi $2b = 8$ ekan. Uchlari $A_1(-3;\ 0)$ va $A_2(3;\ 0)$ va $B_1(0;\ -4)$, $B_2(0;\ 4)$ nuqtalarda. (2) formulaga asosan $c = \sqrt{a^2 + b^2} = \sqrt{9 + 16} = 5$ bo'lgani uchun, giperbolaning fokuslari $F_1(-5;\ 0)$ va $F_2(5;\ 0)$ nuqtalarda bo'ladi. Giperbolaning ekssentrisiteti esa (3) formulaga asosan $\varepsilon = c/a$, $\varepsilon = 5/3$. Nihoyat, giperbola asimptotalari tenglamalari (4) formulaga ko'ra $y = -4/3\ x$, $y = 4/3x$ bo'ladi.

Yasash uchun to'g'ri burchakli dekart koordinatalar sistemasini quramiz va bu sistemada dastlab asimptotalarni yasaymiz. Shundan keyin giperbola uchlari va fokuslarini aniqlab, silliq chiziq bilan giperbolaning grafigini yasaymiz (24- rasm). ◀

Mustaqil bajarish uchun mashqlar

7.1. Giperbolalar uchlarining koordinatalarini, o'qlarini, fokuslarini, ekssentrisitetini toping va yasang:

1) $4x^2 - 5y^2 = 100$;
2) $9x^2 - 4y^2 - 144 = 0$;
3) $9x^2 - 16y^2 - 144 = 0$;
4) $9x^2 - 7y^2 - 252 = 0$.

7.2. Giperbolaning kanonik tenglamasini tuzing, agar:

1) fokuslari orasidagi masofa 10, uchlari orasidagi masofa 8 bo'lsa;

2) haqiqiy yarim o'qi $2\sqrt{5}$ va ekssentrisiteti $\varepsilon = \sqrt{1,2}$ bo'lsa.

7.3. $M(6;\ -2\sqrt{2}\ )$ nuqtadan o'tuvchi, mavhum yarim o'qi 2 bo'lgan giperbola koordinata o'qlariga nisbatan simmetrik. Giperbolaning kanonik tenglamasini tuzing va M nuqtadan fokuslargacha bo'lgan masofani toping.

7.4. Koordinata o'qlariga nisbatan simmetrik bo'lgan giperbolaning fokuslari Ox o'qida joylashgan. Giperbolaning fokuslaridan bitta uchigacha bo'lgan masofalar 1 va 9 ga tengligini bilgan holda uning tenglamasini tuzing.

7.5. Giperbolaning yarim o'qlari yig'indisi 17 ga, ekssentrisiteti 13/12 ga teng. Giperbolaning kanonik tenglasimani tuzing va fokuslarini aniqlang.

7.6. Giperbolaning ekssentrisiteti $\sqrt{3}$ ga teng, fokuslari (–6; 0) va (6; 0) nuqtalarda joylashgan. Giperbolaning kanonik tenglamasini tuzing va asimptotalarining tenglamalarini yozing.

7.7. Asimptotalari $y = \pm\frac{2}{3}x$ bo'lgan giperbolaning $M(6;\sqrt{2})$ nuqtadan o'tishi ma'lum. Giperbolaning kanonik tenglamasini tuzing.

7.8. Fokuslaridan biri (–10; 0) nuqtada bo'lgan va $y = \pm\frac{3}{4}x$ asimptotalarga ega giperbolaning kanonik tenglamasini tuzing.

7.9. Ekssentrisiteti 1,2 ga teng giperbola $\frac{x^2}{64} + \frac{y^2}{28} = 1$ ellips bilan umumiy fokuslarga ega. Giperbolaning kanonik tenglamasini tuzing.

7.10. Giperbola $\frac{x^2}{289} + \frac{y^2}{225} = 1$ ellipsning fokuslaridan o'tishi ma'lum, fokuslari esa bu ellipsning uchlarida joylashgan. Giperbolaning kanonik tenglamasini tuzing.

7.11. Agar giperbola yarim o'qlarining nisbati $b/a = 3/2$ va bu giperbolada yotgan $M\left(4;-3\sqrt{6}\right)$ nuqta ma'lum bo'lsa, fokuslari Ox o'qda yotuvchi giperbolaning kanonik tenglamasini tuzing.

7.12. Uchlari (–2; 0) va (2; 0) nuqtalarda bo'lgan giperbolaning $M\left(2\sqrt{5};\ 1\right)$ nuqtasidan fokuslarigacha bo'lgan masofalarni toping.

7.13. $x^2 - 9y^2 = 36$ giperbolaning $x + 5y = 0$ to'g'ri chiziq bilan kesishish nuqtalaridan fokuslarigacha bo'lgan masofalarni toping.

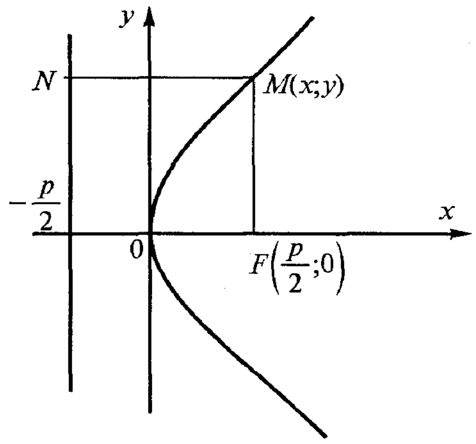
8-§. Parabola

Fokus deb ataluvchi berilgan nuqtadan va *direktrisa* deb ataluvchi berilgan to'g'ri chiziqdan baravar uzoqlashgan nuqtalarning geometrik o'rni *parabola* deyiladi.

Ox o'qiga nisbatan simmetrik bo'lib, uchi koordinatalar boshida, fokusi $F\left(\frac{p}{2},0\right)$ nuqtada bo'lgan parabolaning kanonik tenglamasi (25- rasm)

$$y^2 = 2px \qquad (1)$$

ko'rinishda bo'ladi. Parabolaning direktrisasi $x = -\frac{p}{2}$ tenglama bilan ifodalanadi. Ixtiyoriy $M(x; y)$ nuqtasidan fokusgacha bo'lgan masofa — *fokal radiusi* $r = x + \frac{p}{2}$ formula yordamida aniqlanadi.



25- rasm.

Oy o'qiga nisbatan simmetrik bo'lib, uchi koordinatalar boshida, fokusi $F\left(0,\frac{p}{2}\right)$ nuqtada bo'lgan parabolaning kanonik tenglamasi (26- rasm)

$$x^2 = 2py \qquad (2)$$

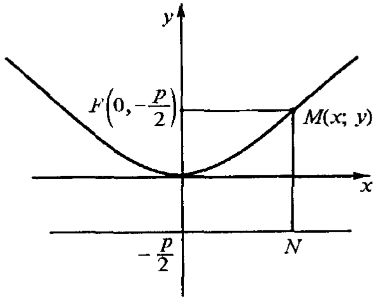
ko'rinishda bo'ladi. Parabolaning direkrisasi $y = -\frac{p}{2}$, tenglama bilan ifodalanadi. Ixtiyoriy $M(x; y)$ nuqtadan fokusigacha bo'lgan masofa — *fokal radiusi* $r = y + \frac{p}{2}$ formula yordamida aniqlanadi.

1- misol. $y^2 = 4x$ parabola fokusi koordinatalarini aniqlang va direktrisa tenglamasini tuzing.

▶ Parabola tenglamasining berilishiga ko'ra $2p = 4$, yoki $\frac{p}{2} = 1$. Shunday qilib, $F(1; 0)$ — nuqta parabola fokusi, $x + 1 = 0$ to'g'ri chiziq uning direktrisasi bo'ladi. ◀

2- misol. Uchi koordinatalar boshida va fokusi $F(0; -10)$ nuqtada bo'lgan parabolaning tenglamasini tuzing.

▶ Parabolaning fokusi Oy o'qida, uchi esa koordinatalar boshida yotadi, shu sababli va fokus nuqtasining abssissasi manfiy son bo'lganligi uchun bu parabolaning tenglamasini $x^2 = -2py$ ko'rinishda izlash kerak. Parabola uchidan fokusgacha bo'lgan masofa $\frac{p}{2} = 10$ bo'lgani uchun $p = 20$, $2p = 40$ bo'ladi. U holda parabola tenglamasi $x^2 = -40y^2$ ko'rinishda bo'ladi. ◀



26- rasm.

Mustaqil bajarish uchun mashqlar

8.1. Parabolalarning fokusi koordinatalarini toping va direktrisa tenglamalarini yozing.
1) $y^2 = 8x$; 2) $y^2 = -12x$;
3) $x^2 = 10y$; 4) $x^2 = -16y$.

8.2. Uchi koordinatalar boshida va fokusi:
1) $F(0; 2)$; 2) $F(0; -2)$;
3) $F(0; -5)$; 4) $F(-3,5; 0)$.
nuqtalarda bo'lgan parabola tenglamasini tuzing.

8.3. Uchi koordinatalar boshida va fokusi Ox o'qda bo'lgan parabolaning uchidan fokusigacha bo'lgan masofa 12 ga teng. Parabola tenglamasini tuzing.

8.4. Fokusi Oy o'qda bo'lgan parabola $O(0; 0)$, $A(6; -2)$ nuqtalardan o'tadi. Parabola tenglamasini tuzing va fokusining koordinatalarini aniqlang.

8.5. Uchi koordinatalar boshida bo'lgan va Ox o'qiga nisbatan simmetrik parabolaning direktrisasi

$$2x - 5 = 0$$

to'g'ri chiziqdan iborat. Parabola tenglamasini tuzing va uning fokusini aniqlang.

8.6. $F(0; 2)$ nuqtadan va $y = 4$ to'g'ri chiziqdan bir xil masofada yotgan nuqtalarning geometrik o'rni tenglamasini tuzing.

8.7. Tenglamalar bilan berilgan parabolalarni yasang:
1) $y^2 = 8x$;
2) $y^2 = -8x$;
3) $x^2 = 4y$;
4) $x^2 = 1-4y$;

8.8. Fokusi absssisalar o'qida va uchi koordinatalar boshida joylashgan parabolaning $M(1; 2)$ nuqtasidan fokusgacha bo'lgan masofani toping.

8.9. Tenglamasi

$$y^2 = 6x$$

bo'lgan parabolaning shunday nuqtasini topingki, bu nuqtaga mos keluvchi fokal radiusi uzunligi 4,5 bo'lsin.

9-§. Dekart koordinatalar sistemasini almashtirish. Qutb koordinatalar sistemasi

1°. Dekart koordinatalar sistemasini almashtirish. Nuqtaning bir sistemadagi koordinatalari bilan shu nuqtaning boshqa sistemadagi koordinatalari orasidagi bog'lanishni qaraymiz.

a) *koordinatalar boshi ko'chirilib, koordinata o'qlari eski o'qlarga parallel bo'lgan hol.* xOy koordinatalar sistemasi va unda$(a;\ b)$ nuqta fiksirlangan tayinlangan bo'lsin. Berilgan xOy koordinatalar sistemasining boshini $(a;\ b)$ nuqtaga ko'chirib, koordinata o'qlarini eski o'qlarga parallel qilib quramiz. Yangi qurilgan XOY koordinatalar sistemasidagi ixtiyoriy $(X;\ Y)$ nuqtaning eski xOy koordinatalar sistemasidagi $(x;\ y)$ nuqtasi koordinatalari orasidagi munosabat

$$\left.\begin{aligned} x &= a + X \\ y &= b + Y \end{aligned}\right\} \text{ yoki } \left.\begin{aligned} X &= x - a, \\ Y &= y - b \end{aligned}\right\} \qquad (1)$$

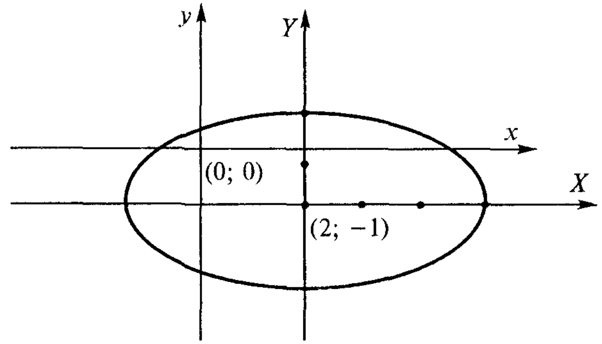
formula orqali ifodalanadi.

1- misol. Koordinatalar boshini ko'chirish yordamida $\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} = 1$ ko'rinishdagi berilgan chiziq tenglamasini soddalashtiring. Eski va yangi koordinata sistemalarida chiziqni yasang.

▶ $\left.\begin{aligned} X &= x - 2 \\ Y &= y + 1 \end{aligned}\right\}$ munosabatga ko'ra eski koordinatalar boshi (0; 0) ni (2; 1) nuqtaga ko'chirib, so'ngra bu nuqtadan eski koordinata o'qlariga parallel o'qlar yasaymiz va yangi koordinatalar XOY sistemasini hosil qilamiz. Yangi sistemaga nisbatan berilgan chiziq tenglamasi $\frac{X^2}{9} = \frac{Y^2}{4} = 1$ ko'rinishni oladi. Bu ellipsning kanonik tenglamasidir. ◀

Ellipsning grafigini yangi sistemaga nisbatan yasaymiz (27- rasm).

b) *koordinatalar boshini o'zgartirmay, koordinata o'qlarini α burchakka burilgan hol.*



27- rasm.

xOy koordinatalar sistemasi (0; 0) koordinatalar boshini siljitmasdan, Ox o'qini α burchakka burib, yangi XOY sistemani hosil qilaylik. Eski sistemadagi ixtiyoriy $(x;\ y)$ nuqtaning yangi sistemadagi $(X;\ Y)$ nuqtasi koordinatalari orasidagi munosabat

$$\left.\begin{aligned} x &= X\cos\alpha - Y\sin\alpha, \\ y &= X\sin\alpha + Y\cos\alpha \end{aligned}\right\} \qquad (2)$$

ko'rinishda bo'ladi.

2- misol. Chiziq $x^2 - y^2$ ko'rinishdagi tenglama bilan berilgan. Koordinatalar sistemasini shunday almashtirish kerakki, yangi sistemada bu tenglama $X \cdot Y = 2$ ko'rinishda bo'lsin (28- rasm).

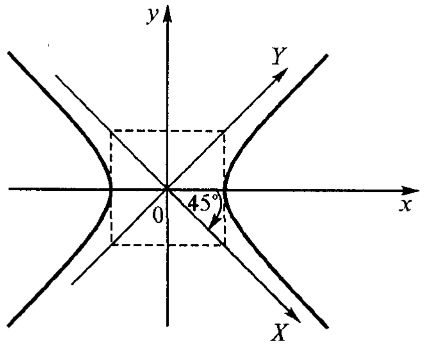
▶ Koordinatalar boshini siljitmasdan, sistemani $\alpha = 45°$ burchakka buramiz. Yuqorida keltirilgan formulaga muvofiq,

$$\left.\begin{aligned} x &= X\cos(-45°) - Y\sin(-45°), \\ y &= X\sin(-45°) - Y\cos(-45°) \end{aligned}\right\}$$

yoki $x = \frac{\sqrt{2}}{2}(X + Y)$, $y = \frac{\sqrt{2}}{2}(Y - X)$; topilgan x va y ning bu ifodalarini berilgan tenglamaga qo'yib, so'ngra ixchamlasak, quyidagi tenglikni hosil qilamiz:

$$X \cdot Y = 2. ◀$$

2°. Qutb koordinatalari sistemasi. Tekislikda biror l son o'qini, ya'ni sanoq boshiga, musbat yo'nalish va masshtab birligiga ega



28- rasm.

bo'lgan to'g'ri chiziqni qaraymiz (29- rasm). Bu o'qni *qutb o'qi*, uning O sanoq boshini esa *qutb* deb ataymiz.

M tekislikdagi qutbdan boshqa biror nuqta bo'lsin. Bu nuqta va qutb orqali l_1 o'qni o'tkazamiz. l va l_1 o'qlar orasidagi φ burchak —*qutb burchagi*, M nuqtadan qutbgacha bo'lgan masofa $OM = r$ esa nuqtaning *qutb radiusi* deyiladi. (30-rasm). φ va r lar nuqtaning qutb koordinatalari deyiladi va $M(\varphi; r)$shaklda yoziladi.

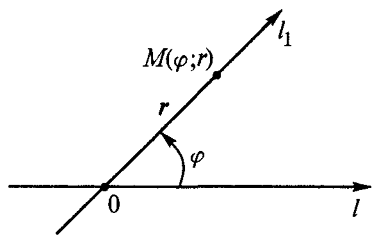
Qutb koordinatalar sistemasida chiziq tenglamasini qarayotganda φ va r istalgan musbat va manfiy qiymatlarni qabul qilishi mumkin.

Bunda manfiy burchaklar soat strelkasi yo'nalishi bo'ylab hisoblanadi, manfiy qutb radiusi esa qaralayotgan nur bo'ylab emas, balki qutbdan davomida olinadi.

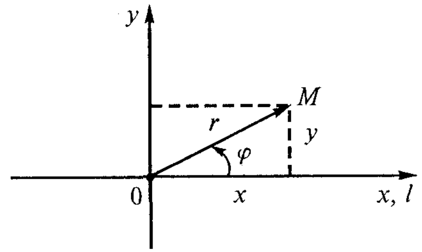
Agar O qutbni dekart koordinatalar boshi, Ol qutb o'qini esa Ox abssissalar o'qi deb qabul qilsak (30-rasm), unda M nuqtaning $(x; y)$ dekart koordinatalari bilan $(\varphi;\ r)$ qutb koordinatari orasidagi bog'lanishlarni topish mumkin:

$$x = r\cos\varphi \quad , \quad y = r\sin\varphi; \tag{3}$$

$$r = \sqrt{x^2 + y^2} \qquad \mathrm{tg}\varphi = \frac{y}{x} \text{ yoki } \varphi = \mathrm{arctg}\frac{y}{x}. \tag{4}$$



29- rasm.



30- rasm.

Eslatma: (4) formula orqali topilgan $\mathrm{tg}\varphi$ qiymatga, masalan, $0 < \varphi \le 2\pi$ shartda φ ning ikkita qiymati mos keladi. Ulardan (3) tenglikni qanoatlantiradiganini olish kerak.

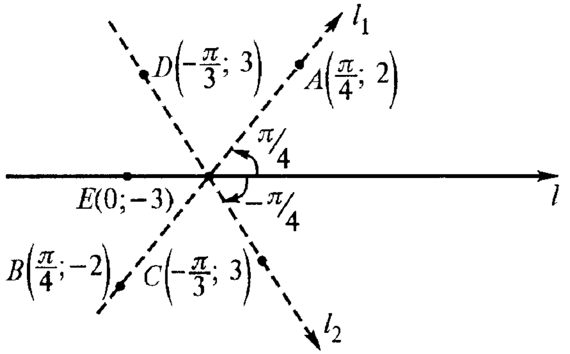
1- misol. Qutb koordinatalar sistemasida $A\left(\frac{\pi}{4};2\right), B\left(\frac{\pi}{4};-2\right),$ $C\left(-\frac{\pi}{3};3\right) D\left(-\frac{\pi}{4};-3\right),\ E(0;-3)$ nuqtalarni yasang.

Qutb O dan chiqib, qutb o'qi bilan $\varphi = \frac{\pi}{4}$ burchak tashkil etuvchi l_1 nurni o'tkazamiz. $A\left(\frac{\pi}{4};2\right)$ nuqta bu nur bo'ylab qutbdan 2 birlik masofadagi, $B\left(\frac{\pi}{4};-2\right)$nuqta esa nurning qutb davomi bo'ylab qutbdan 3 birlik masofada yotadi. $E(0;-3),\ C\left(\frac{\pi}{3};3\right)$va $D\left(-\frac{\pi}{3};\ -3\right)$ nuqtalar ham shu kabi yasaladi. Faqat bu narsa $\varphi = -\frac{\pi}{3} = -60°$ burchak manfiy bo'lganligi uchun u soat strelkasi, ya'ni manfiy yo'nalish bo'ylab olinadi (31-rasm). ◀

2- misol. Qutb koordinatalari sistemasida $r = \frac{2}{1-\cos\varphi}$ tenglama bilan berilgan chiziqni chizing va bu chiziqning tenglamasini dekart koordinatalarida yozing.

▶ φ ga qiymatlar berib, r ning unga mos qiymatlarini hisoblaymiz:

$$\varphi = \frac{\pi}{4}, \quad r = \frac{2}{1-\cos\frac{\pi}{4}} = \frac{2}{1-\frac{\sqrt{2}}{2}} = \frac{4}{2-\sqrt{2}} \approx 6{,}828; \quad M_1\left(\frac{\pi}{4};\ 6{,}828\right);$$



31- rasm.

$$\varphi = \frac{\pi}{2}, \quad r = \frac{2}{1-\cos\frac{\pi}{2}} = \frac{2}{1-0} = 2; \quad M_2\left(\frac{\pi}{2};\ 2\right);$$

$$\varphi = \frac{3\pi}{4}, \quad r = \frac{2}{1-\cos\frac{3\pi}{4}} = \frac{2}{1+\frac{\sqrt{2}}{2}} = \frac{4}{2+\sqrt{2}} \approx 1{,}172; \quad M_3\left(\frac{3\pi}{4};\ 1{,}172\right);$$

$$\varphi = \pi, \quad r = \frac{2}{1-\cos\pi} = \frac{2}{1+1} = \frac{2}{2} = 1, \quad M_4(\pi;\ 1);$$

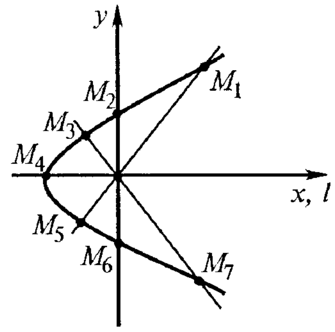
$$\varphi = \frac{5\pi}{4}, \quad r = \frac{2}{1-\cos\frac{5\pi}{4}} = \frac{2}{1+\frac{\sqrt{2}}{2}} = \frac{4}{2+\sqrt{2}} \approx 1{,}172; \quad M_5\left(\frac{5\pi}{4};\ 1{,}172\right);$$

$$\varphi = \frac{3\pi}{2}, \quad r = \frac{2}{1-\cos\frac{3\pi}{2}} = \frac{2}{1-0} = 1; \quad M_6\left(\frac{3\pi}{2};\ 1\right);$$

$$\varphi = \frac{7\pi}{4}, \quad r = \frac{2}{1-\cos\frac{7\pi}{4}} = \frac{2}{1-\frac{\sqrt{2}}{2}} = \frac{4}{2-\sqrt{2}} \approx 6{,}828; \quad M_7\left(\frac{7\pi}{4};\ 6{,}828\right);$$

$$\varphi = 2\pi, \quad r = \frac{2}{1-\cos 2\pi} = \frac{2}{1-1} = \frac{2}{0} = +\infty; \quad M_8(2\pi;\ +\infty).$$

Topilgan qiymatlarga mos nuqtalarni 1- misoldagi kabi yasaymiz. Ularni tutashtirsak, berilgan tenglamaga mos chiziqni hosil qilamiz. Ko‘rinib turibdiki, u paraboladan iborat (32- rasm).



32- rasm.

Endi chiziqning berilgan tenglamasini dekart koordinatalarida yozamiz, buning uchun (3) va (4) formulalardan foydalanamiz:

$$r = \sqrt{x^2+y^2}, \quad \cos\varphi = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}.$$

Bularni chiziq tenglamasiga qo‘ysak:

$$r = \frac{2}{1-\cos\varphi}; \quad \sqrt{x^2+y^2} = \frac{2}{1-\frac{x}{\sqrt{x^2+y^2}}};$$

$$\left(\sqrt{x^2+y^2} - x\right)\sqrt{x^2+y^2} = 2\sqrt{x^2+y^2};$$

$$\sqrt{x^2+y^2} - x = 2; \qquad \sqrt{x^2+y^2} = 2 + x;$$

$$x^2 + y^2 = 4 + 4x + x^2; \; y^2 = 4(x+1).$$

Bu uchi $(-1; 0)$ nuqtada bo‘lib, abssissalar o‘qiga nisbatan simmetrik parabolaning tenglamasidir. ◀

Mustaqil bajarish uchun mashqlar

9.1. $A(5;\ 5)$, $B(2;\ -3)$, $C(-2;\ 3)$ nuqtalar berilgan. Koordinata o‘qlari yo‘nalishlari o‘zgarmay qolib, koordinatalar boshi: 1) A nuqtagacha; 2) B nuqtagacha; 3) C nuqtagacha ko'chirilgan. A, B, C nuqtalarning koordinatalarini yangi sistemaga nisbatan aniqlang.

9.2. Koordinata o'qlari $\alpha = 30°$ ga burilgan bo'lib, yangi koordinatalar sistemasidagi: 1) $A(1;\ 1)$; 2) $B\left(\sqrt{3};\ 2\right)$; 3) $C\left(0;\ 2\sqrt{3}\right)$ nuqtaning koordinatalarini eski sistemaga nisbatan aniqlang.

9.3. Koordinatalar boshini ko'chirish yordamida quyidagi chiziq tenglamalarini soddalashtiring va eski, yangi koordinatal sistemalarni hamda chiziqni yasang:

1) $\frac{(x-1)^2}{9} + \frac{(y-1)^2}{4} = 1$; 2) $\frac{x^2}{9} + \frac{(y-1)^2}{4} = 1$;

3) $\frac{(x+1)^2}{16} - \frac{(y-3)^2}{4} = 1$; 4) $\frac{(x-4)^2}{4} - (y+1)^2 = 1$;

5) $x^2 + 4y^2 - 6x + 8y = 3$; 6) $y^2 - 8y = 4x$.

9.4. Koordinatalar boshini siljitmasdan, koordinata o'qlarini $\alpha = 45°$ga burish yordamida quyidagi chiziqlar tenglamasini soddalashtiring:

1) $5x^2 - 6xy + 5y^2 = 32$;

2) $3x^2 - 10xy + 3y^2 - 32 = 0$.

9.5. Tenglamasi qutb koordinatalar sistemasida berilgan chiziqni: a) yasang; b) chiziq tenglamasini dekart koordinatalar sistemasida yozing:

1) $r = \frac{a}{\cos\varphi}$; 2) $r = 2a\sin\varphi$;

3) $r = a\left(1 + \cos\varphi\right)$; 4) $r = \frac{9}{5-4\cos\varphi}$;

5) $r = \frac{9}{4-5\cos\varphi}$; 6) $r = \frac{3}{1-\cos\varphi}$.

9.6. Tenglamasi dekart koordinatalari sistemasida berilgan chiziq tenglamasini qutb koordinatalarda yozing.

1) $x^2 + y^2 = a^2$; 2) $x^2 - y^2 = a^2$;

3) $x^2 + y^2 = ax$; 4) $x^2 + y^2 = ay$;

5) $\left(x^2 + y^2\right)^2 = a^2\left(x^2 - y^2\right)$; 6) $y = x$.

Mustaqil bajarish uchun berilgan mashqlarning javoblari

1- §. 1.4. $(-1;\ 0)$, $(0;\ 1)$, $(1;\ 0)$, $(0;\ -1)$. **1.5.** 17. **1.6.** $(13;\ -2)$, $(13;\ 8)$. **1.8.** 1) $\left(4;-\frac{2}{3}\right)$, 2) $\left(\frac{2}{3};\ -\frac{2}{3}\right)$. **1.9.** 1) $(1;\ 3)$, 2) $(3;\ -2)$. **1.10.** $\sqrt{41}$, $0{,}5\sqrt{13}$, $0{,}5\sqrt{449}$. **1.11.** $(-5;\ -2)$. **1.12.** $C(12;\ 7)$, $D(4;\ -1)$. **1.13.** $(2;\ -2)$. **1.14.** $(-10;\ 10)$, $(6,\ 10)$. **1.15** $C(-1;\ 3)$. **1.16.** $C(6;\ 2)$. **1.17.** 9. **1.19.** $C(3;\ 0)$, $C(-7;\ 0)$. **1.20.** 13.

2- §. 2.1. 1) $y = \frac{1}{\sqrt{3}}x$, 2) $y = x$, 3) $y = -\sqrt{3}x$, 4) $y = -x$, 5) $y = 2x$, 6) $y = -3x$. **2.2.** 1) $30°$, 2) $120°$, 3) arctg4, 4) arctg(–3). **2.3.** 1) $y = x + 3$, 2) $y = \sqrt{3}x + 3$, 3) $y = -x + 3$. **2.4.** 1) $y = \frac{1}{\sqrt{3}}x - 2$. 2) $y = \sqrt{3}x - 2$. 3) $y = -\sqrt{3}x - 2$. **2.5.** $y = x + 1$, $k = 1$, $b = 1$. **2.6.** 1) $y = \frac{2}{3}x + 2$, $k = \frac{2}{3}$, $b = 2$. 2) $y = -\frac{2}{3}x$, $k = -\frac{2}{3}$, $b = 0$. 3) $y = -2$, $k = 0$, $b = -2$. 4) $x = -2$, $k = \infty$, $b = 0$. 5) $y = -\frac{4}{3}x + 4$, $k = -\frac{4}{3}$, $b = 4$. **2.8.** 1) $\frac{x}{4} + \frac{y}{3} = 1$. 2) $\frac{x}{4} + \frac{y}{-3} = 1$. 3) $\frac{x}{3} + \frac{y}{-2} = 1$. **2.9.** 20. **2.10.** $3x + 2y - 12 = 0$. **2.11.** $3x + 4y - 12 = 0$, $3x + 4y + 12 = 0$, $3x - 4y + 12 = 0$, $3x - 4y - 12 = 0$. **2.12.** 1) $y = \frac{1}{\sqrt{3}}x + \frac{2+5\sqrt{3}}{\sqrt{3}}$. 2) $y = x + 7$. 3) $y = \sqrt{3}x + 2\sqrt{3} + 5$. 4) $y = -x + 3$. 5) $y = 5$. **2.13.** $y = -x + 3$. **2.14.** 1) $4x - 3y - 19 = 0$ 2) $y = -x + 2$ 3) $y = \frac{1}{2}x + 2$. **2.15.** AB: $x + y - 2 = 0$, AC: $8x + y + 5 = 0$, BC: $3x - 4y - 20 = 0$. **2.16.** $5x + 2y - 6 = 0$. **2.17.** $a = 9$, $b = 6$.

3- §. 3.1. 1) $\text{arctg}\frac{13}{3}$. 2) $\text{arctg}\frac{5}{7}$. 3) $\text{arctg}\frac{1}{4}$. 4) $\text{arctg}\frac{25}{24}$. **3.3.** 1) $y = 2x - 1$. 2) $y = \frac{1}{2}x + 4$. **3.4.** $\text{arctg}\frac{1}{3}$, $\pi - \text{arctg}\frac{2}{7}$, $\text{arctg}\frac{8}{17}$. **3.5.** $y = 3x$, $y = \frac{1}{3}x$. **3.6.** $45°$, $45°$, $90°$. **3.7.** $9x - 2y - 28 = 0$. **3.8.** $x - y - 6 = 0$. $2x + 3y - 17 = 0$. **3.9.** $3x - 5y + 18 = 0$, $3x - 5y - 28 = 0$, $11x + 3y - 29 = 0$, $11x + 3y + 2 = 0$. **3.10.** $x - 2y + 5 = 0$. **3.11.** $5x - 4y - 18 = 0$. **3.12.** $(1;\ -1)$. **3.13.** $y = 2$, $y = 5$, $x = -5$, $x = -2$. **3.14.** $x + 2y - 5 = 0$, $x - 3y + 8 = 0$.

4- §. 4.1. 1) $\frac{3}{5}x - \frac{4}{5}y - 4 = 0$. 2) $\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y - \frac{\sqrt{2}}{2} = 0$. 3) $-\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y - \frac{\sqrt{2}}{2} = 0$. 4) $\frac{-2}{\sqrt{5}}x + \frac{1}{\sqrt{5}}y - \sqrt{5} = 0$. **4.2.** 1) $\sqrt{2}x + \sqrt{2}y - 6 = 0$. 2) $\sqrt{2}x + \sqrt{2}y + 6 = 0$. 3) $\sqrt{2}x - \sqrt{2}y - 6 = 0$. **4.3.** 1,6; 1,4; 1,2; **4.4.** $\frac{23}{2\sqrt{3}}$. **4.5.** $k = \pm 2$. **4.6.** $4x - 3y - 20 = 0$, $4x - 3y + 20 = 0$. **4.7.** $8x - 15y + 6 = 0$, $8x - 15y - 130 = 0$. **4.8.** $2x - 3y - 4 = 0$, $6x - y - 12 = 0$. **4.9.** $\sqrt{10}$. **4.10.** $3x - 4y + 10 = 0$. **4.11.** $\sqrt{10}$. **4.12.** $2\sqrt{2}$.

5- §. 5.1. 1) $(x-4)^2 + (y+7)^2 = 25$. 2) $(x+3)^2 + (y-3)^2 = 1$. 3) $(x+1)^2 + y^2 = 5$. 4) $(x+1)^2 + y^2 = 9$. **5.2.** o'tadi. **5.3.** $(x-12)^2 + (y+5)^2 = 169$. **5.4.** $(x+1)^2 + (y+2)^2 = 34$. **5.5.** $\left(x - \frac{5}{2}\right)^2 + (y-6)^2 = 169$. **5.6.** $x^2 + (y-5)^2 = 25$. **5.7.** $(x+9)^2 + (y+4)^2 = 169$. **5.8.** $(x+2)^2 + (y+2)^2 = 4$. **5.9.** $(x-2)^2 + (y-6)^2 = 25$. **5.10.** $(x+5)^2 + (y-2)^2 = 25$.

5.11. 1) $C(4;-6), r=9$. 2) $C(-3;-2)$ $r=\sqrt{30}$. **5.12.** 1) $(0;-1)$, $(0;-3)$, $(1;0)$, $(3,0)$. 2) $(0;-1)$, $(0;-10)$. **5.13.** 17. **5.14.** $(-1;0)$, $(-6;-5)$. **5.15.** $(x-8)^2+$ $+(y-6)^2=36$. **5.16.** $3x-4y-32=0$.

6- §. 6.1. 1) $(\pm 5;0)$, $(0;\pm 4)$, $(\pm 3;0)$. 2) $(\pm 3;0)$, $(0;\pm 2)$, $(\pm\sqrt{5};0)$. 3) $(\pm 3;0)$, $(0;\pm 4)$, $(0;\pm\sqrt{7})$. 4) $(\pm 6;0)$, $(0;\pm 10)$, $(0;\pm 8)$. **6.2.** $\frac{x^2}{16}+$ $+\frac{y^2}{5}=1$. **6.3.** $\frac{x^2}{16}+\frac{y^2}{30}=1$. **6.4.** $\frac{x^2}{49}+\frac{y^2}{24}=1$. **6.5.** $\frac{x^2}{289}+\frac{y^2}{64}=1$, $\varepsilon=\frac{15}{17}$. **6.6.** $\frac{x^2}{144}+$ $+\frac{y^2}{108}=1$. **6.7.** $\frac{x^2}{25}+\frac{y^2}{9}=1$. **6.8.** $\frac{x^2}{12}+\frac{y^2}{8}=1$. **6.9.** $\frac{x^2}{100}+\frac{y^2}{25}=1$. **6.10.** $(\pm\sqrt{2};0)$, $\varepsilon=\frac{\sqrt{2}}{2}$, $r_1=\frac{4-\sqrt{2}}{2}$, $r_2=\frac{4+\sqrt{2}}{2}$. **6.11.** $r_1=11$, $r_2=5$. **6.12.** $\frac{x^2}{9}+\frac{y^2}{5}=1$. **6.13.** $a=150$, $\varepsilon=\frac{1}{60}$.

7- §. 7.1. 1) $(\pm 5;0)$, $2a=10$, $2b=4\sqrt{5}$, $(\pm 3\sqrt{5};0)$, $\varepsilon=\frac{3\sqrt{5}}{5}$. 2) $(\pm 4;0)$, $2a=8$, $2b=12$, $(\pm\sqrt{62};0)$, $\varepsilon=\frac{\sqrt{62}}{4}$. 3) $(\pm 4;0)$, $2a=8$, $2b=6$, $(\pm 5;0)$ $\varepsilon=\frac{5}{4}$. 4) $(\pm 2\sqrt{7};0)$, $2a=4\sqrt{7}$, $2b=12$, $(\pm 8;0)$, $\varepsilon=\frac{4}{\sqrt{7}}$. **7.2.** 1) $\frac{x^2}{16}-\frac{y^2}{9}=1$. 2) $\frac{x^2}{20}-\frac{y^2}{4}=1$. **7.3.** $\frac{x^2}{12}-\frac{y^2}{4}=1$, $r_1=2\sqrt{3}$, $r_2=6\sqrt{3}$. **7.4.** $\frac{x^2}{16}-\frac{y^2}{9}=1$. **7.5.** $\frac{x^2}{144}-\frac{y^2}{25}=1$, $(\pm 13;0)$. **7.6.** $\frac{x^2}{12}-\frac{y^2}{24}=1$, $y=\pm\sqrt{2}x$. **7.7.** $\frac{x^2}{18}-\frac{y^2}{8}=1$. **7.8.** $\frac{x^2}{64}-\frac{y^2}{36}=1$. **7.9.** $\frac{x^2}{25}-\frac{y^2}{21}=1$. **7.10.** $\frac{x^2}{64}-\frac{y^2}{225}=1$. **7.11.** $\frac{x^2}{12}-\frac{y^2}{27}=1$. **7.12.** $r_1=3$, $r_2=7$. **7.13.** $r_1=6+\frac{5\sqrt{10}}{2}$, $r_2=6-\frac{5\sqrt{10}}{2}$.

8- §. 8.1. 1) $F(2;0)$, $x+2=0$; 2) $F(-6;0)$, $x-6=0$; 3) $F(0;5)$, $y+$ $+5=0$; 4) $F(0;-8)$, $y-8=0$. **8.2.** 1) $x^2=8y$, 2) $x^2=-8y$, 3) $x^2=-20y$, 4) $y^2=-14x$. **8.3.** $y^2=48x$, $y^2=-48x$. **8.4.** $x^2=-18y$, $F(0;-4{,}5)$. **8.5.** $x^2=-10y$. **8.6.** $x^2=-4y+12$, **8.8.** $r=2$. **8.9.** $M(3;\pm 3\sqrt{2})$.

9- §. 9.1. 1) $A(0;0)$, $B(-3;-8)$, $C(-7;-2)$. 2) $A(3;8)$, $B(0;0)$, $C(-4;6)$. 3) $A(7;2)$, $B(4;6)$, $C(0;0)$. **9.2.** 1) $\left(\frac{\sqrt{3}-1}{2};\frac{\sqrt{3}+1}{2}\right)$; 2) $\left(\frac{1}{2};\frac{5}{2}\right)$; 3) $(-\sqrt{3};\sqrt{3})$. **9.3.** 1) $\frac{X^2}{9}+\frac{Y^2}{4}=1$; 2) $\frac{X^2}{9}+\frac{Y^2}{4}=1$; 3) $\frac{X^2}{16}-\frac{Y^2}{4}=1$; 4) $\frac{X^2}{4}-Y^2=1$; 5) $X^2+4Y^2=16$; 6) $Y^2-4X=16$. **9.4.** 1) $X^2+4Y^2=8$; 2) $-X^2+4Y^2=8$. **9.5.** 1) $x=a$; 2) $x^2+y^2=2ay$; 3) $x^2+y^2=$ $=a\left(x+\sqrt{x^2+y^2}\right)$; 4) $5\sqrt{x^2+y^2}=9+4x$; 5) $4\sqrt{x^2+y^2}=9+5x$; 6) $y^2=9+6x$. **9.6.** 1) $r=a$; 2) $r=\frac{a}{\sqrt{\cos 2\varphi}}$; 3) $r=a\cos\varphi$; 4) $r=a\cos\varphi$; 5) $r=a\sqrt{\cos 2\varphi}$; 6) $\cos\varphi-\sin\varphi=0$.

V bob. FAZODA ANALITIK GEOMETRIYA

1- §. Tekislik. Tekislikka doir asosiy masalalar

Bu paragrafda tekislikka doir asosiy masalalar qaraladi. Asosiy formulalar keltiriladi.

1⁰. Tekislikning umumiy tenglamasi

$$Ax+By+Cz+D=0 \qquad (1)$$

ko'rinishda bo'lib, u:

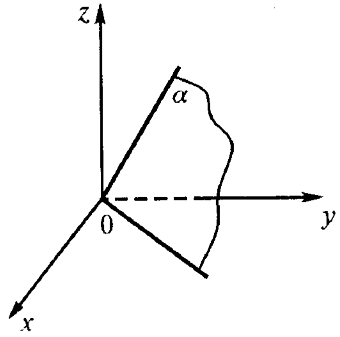
1) $D=0$ da

$$Ax+By+Cz=0 \qquad (2)$$

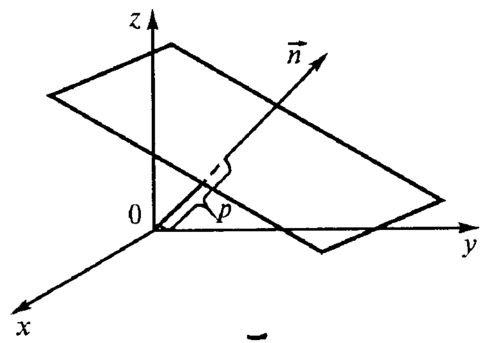
ko'rinishni oladi (33- rasm). Bu koordinata boshidan o'tadigan tekislik tenglamasi;

2) $C=0$ da

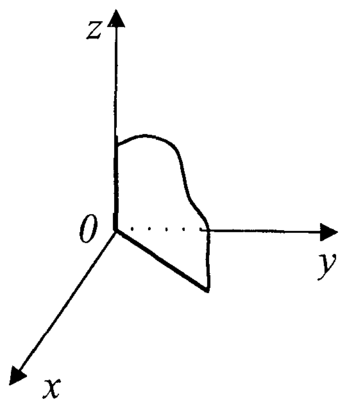
$$Ax+By+D=0 \qquad (3)$$



33- rasm.



34- rasm.



35- rasm.

ko‘rinishni oladi (34- rasm). Bu Oz o‘qiga parallel bo‘lgan tekislik tenglamasi;

3) $B = 0$ da

$$Ax + Cz + D = 0 \tag{4}$$

ko‘rinishni oladi. Bu Oy o‘qiga parallel tekislik tenglamasi;

4) $A = 0$ da tekislik

$$By + Cz + D = 0 \tag{5}$$

tenglamaga ega bo‘lib, u Ox o‘qiga parallel bo‘ladi.

Umuman olganda, tekislikning umumiy tenglamasida koordinatalardan qaysi biri qatnashmasa, tekislik o‘sha koordinata o‘qiga paralleldir. Agar (3), (4), (5) tenglamalarda $D = 0$ bo‘lsa, u holda tenglamalar

$$Ax + By = 0, \tag{6}$$

$$Ax + Cz = 0, \tag{7}$$

$$By + Cz = 0 \tag{8}$$

ko‘rinishni oladi. (6) tenglama Oz o‘qidan o‘tuvchi tekislik tenglamasi (35- rasm), (7) tenglama Oy o‘qidan o‘tuvchi tekislik tenglamasi, (8) tenglama Ox o‘qidan o‘tuvchi tekislik tenglamasidir. Agar (1) tenglamada $A = 0$ va $B = 0$ bo‘lsa, u holda tenglamasi $Cz + D = 0$ bo‘lgan tekislik Oz o‘qiga perpendikular va Oxy tekislikka parallel bo‘ladi.

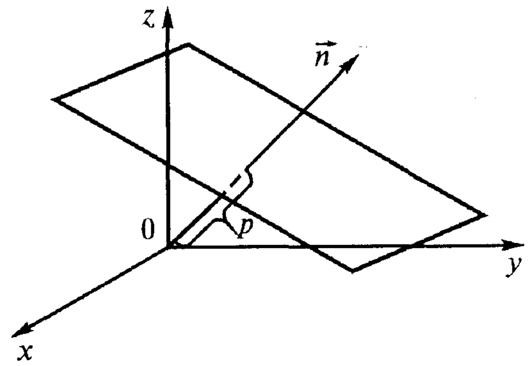
Yuqoridagidek, $By + D = 0$ tenglama Oxz tekislikka parallel tekislikni, $Ax + D = 0$ tenglama esa Oyz tekislikka parallel tekislikni aniqlaydi.

Nihoyat, (1) tenglamada uchta koeffitsiyent nolga teng bo‘lsa, masalan, $B = 0$, $C = 0$, $D = 0$, bo‘lsa, $Ax = 0$ yoki $x = 0$ tenglama koordinatalar boshidan o‘tkazilgan Oyz koordinata tekisligini aniqlaydi. Shuningdek, $By = 0$ yoki $y = 0$ tenglama Oxz koordinata tekisligini, $Cz = 0$ yoki $z = 0$ tenglama esa Oxy tekislikni aniqlaydi.

2^0. Tekislikning normal tenglamasi

$$x\cos\alpha + y\cos\beta + z\cos\gamma - p = 0 \tag{9}$$

ko‘rinishda bo‘ladi, bu yerda α, β va γ — mos ravishda koordinata o‘qlari bilan koordinatalar boshidan tekislikka o‘tkazilgan perpendikular — *normal* orasidagi burchaklar, p — bu perpendikularning (normalning) uzunligi (36- rasm).



36- rasm.

3^0. Tekislik tenglamasini normal tenglamaga keltirish. $Ax + By + Cz + D = 0$ tekislikning umumiy tenglamsi bo‘lsin. Ushbu

$$N = \pm\frac{1}{\sqrt{A^2+B^2+C^2}} \tag{10}$$

son *normallovchi ko‘paytuvchi* deyiladi. Bu yerda ishora (1) tenglamadagi ozod had ishorasiga teskari qilib olinadi. Tekislikning umumiy tenglmasini (9) ko‘rinishidagi normal holga keltirish uchun uning ikkala tomonini normallashtiruvchi ko‘paytuvchiga ko‘paytirish lozim.

4^0. Tekislikning koordinata o‘qlaridan ajratgan kesmalari bo‘yicha tenglamasi

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \tag{11}$$

ko'rinishda bo'ladi, bu yerda a, b va c — tekislikning koordinata o'qlaridan ajratgan kesmalari qiymatlari.

Tekislikning (1) umumiy tenglamasini (11) ko'rinishga keltirish mumkin. Buning uchun D ni tenglikning o'ng tomoniga o'tkazib, ikkala tomonini D ga bo'lamiz: $\frac{A}{-D}x + \frac{B}{-D}y + \frac{C}{-D}z = 1$ va $a = -\frac{D}{A}$, $b = -\frac{D}{B}$, $c = -\frac{D}{C}$ deb olamiz. Natijada (11) hosil bo'ladi.

5⁰. Berilgan nuqta orqali o'tuvchi va berilgan normal vektorga ega tekislik tenglamasi

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0 \qquad (12)$$

ko'rinishda bo'lib, bu yerda $M(x_0;\ y_0;\ z_0)$ tekislikning berilgan nuqtasi, $\vec{N}\{A;B;C\}$ tekislikka perpendikular vektor. (12) tenglamada A, B va C koeffitsiyentlarga har xil qiymatlar berib, $M(x_0;\ y_0;\ z_0)$ nuqtadan o'tuvchi turli xil tekisliklarni hosil qilamiz. $\vec{N}\{A;B;C\}$ tekislikning *normal vektori* deyiladi.

6⁰. Ikki tekislik orasidagi burchak. Ikki tekislik

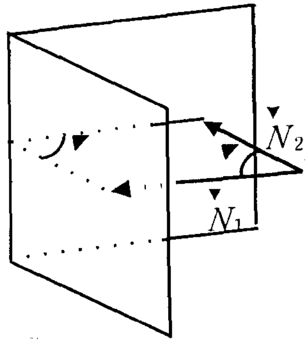
$$A_1x + B_1y + C_1z + D_1 = 0,\quad A_2x + B_2y + C_2z + D_2 = 0$$

tenglamalar bilan, yoki $\vec{N}_1\{A_1;\ B_1;\ C_1\}$, $\vec{N}_2\{A_2;\ B_2;\ C_2\}$ ni hisobga olgan holda

$$\left(\vec{N}_1,\ \vec{r}\right) + D_1 = 0,\quad \left(\vec{N}_2,\ \vec{r}\right) + D_2 = 0$$

tenglamalar bilan berilgan bo'lsin, bu yerda $\vec{N}_1\{A_1;B_1;C_1\}$, $\vec{N}_2\{A_2;B_2;C_2\}$ lar mos ravishda berilgan tekisliklarga perpendukular vektorlardir (37- rasm). Bu tekisliklar tashkil etuvchi ikki yoqli burchaklardan ixtiyoriy birini φ deb belgilaymiz. $\vec{N}_1$ va $\vec{N}_2$ vektorlar orasidagi burchakni θ bilan belgilaymiz. U holda

$$\cos\theta = \frac{\left(\vec{N}_1\cdot\vec{N}_2\right)}{\left|\vec{N}_1\right|\cdot\left|\vec{N}_2\right|};$$



37- rasm.

φ va θ burchaklar, geometrik tushunchalarga asosan, $\varphi = 0$ yoki $\varphi = \pi - \theta$ tenglik bilan bog'lanadi, shuningdek, $\cos\varphi = \cos\theta$ yoki $\cos\varphi = \cos(\pi - \theta) = \cos\theta$. Bu yerdan $\cos\varphi = \pm\cos\theta$, ya'ni

$$\cos\theta = \pm\frac{\left(\vec{N}_1\cdot\vec{N}_2\right)}{\left|\vec{N}_1\right|\cdot\left|\vec{N}_2\right|}$$

Bu tenglik yordamida tenglamasi vektor shaklda berilgan tekisliklar orasidagi burchakni topamiz.

Umumiy tenglamalari $A_1x + B_1y + C_1z + D_1 = 0$ va $A_2x + B_2y + C_2z + D_2 = 0$ bilan berilgan tekisliklar orasidagi burchak

$$\cos\varphi = \pm\frac{(\vec{N}_1\cdot\vec{N}_2)}{|\vec{N}_1||\vec{N}_2|} = \pm\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\cdot\sqrt{A_2^2+B_2^2+C_2^2}} \qquad (13)$$

formula bilan hisoblanadi. Bu yerda $\vec{N}_1\{A_1,B_1,C_1\}$ va $\vec{N}_2\{A_2,B_2,C_2\}$ — tekisliklarga o'tkazilgan normal vektorlar.

Ikki tekislikning perpendikularlik sharti:

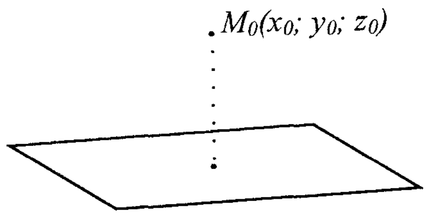
$$A_1A_2 + B_1B_2 + C_1C_2 = 0;$$

Ikki tekislikning parallellik sharti:

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}.$$

7°. Nuqtadan tekislikkacha bo‘lgan masofa. $M(x_0;\ y_0;\ z_0)$ nuqtadan $Ax + By + Cz + D = 0$ tekislikkacha bo‘lgan masofa (38-rasm)

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}} \qquad (14)$$



38- rasm.

formula bilan hisoblanadi.

8°. Tekislikka doir masalalarni yechishda uch noma’lumli ikkita bir jinsli tenglama sistemasi

$$\begin{cases} a_1x + b_1y + c_1z = 0, \\ a_2x + b_2y + c_2z = 0 \end{cases}$$

ni yechish tez-tez uchrab turadi. Bu kabi sistemalarni yechish III bobda qaralgan edi. Uning yechimi formulasini keltiramiz:

$$x = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \cdot k, \quad y = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix} \cdot k, \quad z = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \cdot k, \qquad (15)$$

bu yerda k —ixtiyoriy son hamda determinantlarning hech bo‘lmaganda bittasi noldan farqli.

9°. Berilgan uch nuqtadan o‘tuvchi tekislik tenglamasi. Berilgan $M_1(x_1;\ y_1;\ z_1)$, $M_2(x_2;\ y_2;\ z_2)$ va $M_3(x_3;\ y_3;\ z_3)$ nuqtalardan o‘tuvchi tekislik tenglamasi

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \qquad (16)$$

ko‘rinishda bo‘ladi.

1- misol. $M_1(2;\ 3;\ 2)$ va $M_2(7;\ 1;\ 0)$ nuqtalardan o‘tuvchi va Ox o‘qiga parallel bo‘lgan tekislik tenglamasini yozing.

▶ Ox o‘qiga parallel bo‘lgan tekislik tenglamasi $By + Cz + D = 0$ ni olamiz. Agar tekislik berilgan nuqtadan o‘tsa, u holda uning koordinatalari tekislik tenglamasini qanoatlantiradi. M_1 va M_2 nuqtalarning koordinatalarini tekislik tenglamasiga qo‘ysak,

$$\begin{cases} -3B + 2C + D = 0, \\ B + D = 0 \end{cases}$$

tenglamalar sistemasi hosil bo‘ladi. B, C va D koeffitsiyentlarni aniqlash uchun, uch noma’lumli ikkita bir jinsli tenglama sistemasiga ega bo‘ldik. Bu tenglamalar koeffitsiyentlari yordamida

$$\begin{pmatrix} -3 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

matritsani tuzamiz. 8°-bandda qaralgan formuladan foydalanib,

$$B = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} \cdot k, \quad C = \begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} \cdot k, \quad D = \begin{vmatrix} -3 & 2 \\ 1 & 0 \end{vmatrix} \cdot k,$$

$B = 2k$, $C = 4k$, $D = -2k$ larni topamiz. B, C va D ning topilgan qiymatlarini tekislik tenglamasiga qo‘yib, $2ky + 4kz - 2k = 0$ yoki $y + 2z - 1 = 0$ ni hosil qilamiz. Bu tekislik tenglamasi. ◀

2- misol. $2x + y - z + 6 = 0$ tekislik koordinata o‘qlarini qanday birliklarda kesib o‘tadi?

▶ Masalani ikki usul bilan yechamiz.

I usul. Ma’lumki, Ox o‘qida yotuvchi nuqtaning y va z koordinatalari nolga teng. Tekislik tenglamasida $y = 0$, $z = 0$ desak, $2x + 6 = 0$ bo‘lib, bundan $x = -3$. Bu tekislikning Ox o‘qidan kesib o‘tuvchi kesmasi miqdori (birligi).

Xuddi shunday, $x = 0$, $z = 0$ desak, $y + 6 = 0$ yoki $y = -6$ kesma o‘qidan kesgan bo‘lagi (birligi), $x = 0$, $y = 0$ desak, $z + 6 = 0$, $z = 6$ kesma Oz o‘qidan kesgan bo‘lagi (birligi).

II usul. Tekislik tenglamasidagi ozod hadni tenglikning o‘ng tomoniga o‘tkazamiz: $2x + y - z = -6$. Tenglikning har ikkala tomonini — 6 ga bo‘lamiz. $\frac{x}{-3} + \frac{y}{-6} + \frac{z}{6} = 1$. Bu yerdan $a = -3$, $b = -6$, $c = 6$ kelib chiqadi. ◀

3- misol. $5x+7y-34z+5=0$ tekislik tenglamasini normal ko'rinishga keltiring.

▶ Tekislik tenglamasini normal ko'rinishga keltirish uchun (10) formula yordamida normallashtiruvchi ko'paytuvchini topamiz. Qaralayotgan hol uchun ko'paytuvchining minus ishorasi olinadi. Berilgan tenglamada $A=5,\quad B=7,\quad C=-34$. Demak,

$$N=-\frac{1}{\sqrt{5^2+7^2+(-34)^2}}=-\frac{1}{\sqrt{1230}}.$$

Endi berigan tenglamani shu songa ko'paytiramiz. Natijada tenglama ushbu ko'rinishni oladi:

$$-\frac{5}{\sqrt{1230}}x-\frac{7}{\sqrt{1230}}y+\frac{34}{\sqrt{1230}}z-\frac{5}{\sqrt{1230}}=0.$$ ◀

4- misol. Koordinatalar boshidan $15x-10y+6z-190=0$ tekislikka tushirilgan perpendikular uzunligini va bu perpendikular bilan koordinata o'qlari orasidagi burchaklarni toping.

▶ Tekislik tenglamasini normal ko'rinishga keltiramiz. (10) formula bilan $N=\frac{1}{19}$ normallashtiruvchi ko'paytuvchini topamiz. Berilgan tekislik tenglamasining ikki tomonini $\frac{1}{19}$ ga ko'paytirib, tekislikning normal ko'rinishdagi tenglamasini olamiz:

$$\frac{15}{19}x-\frac{10}{19}y+\frac{6}{19}z-10=0$$

bu yerda $p=10,\quad \cos\alpha=\frac{15}{19},\quad \cos\beta=-\frac{10}{19}\quad \cos\gamma=\frac{6}{19}$. Bu tengliklarning o'ng tomonidagi oddiy kasrlarni o'nli kasrga aylantirib, $\alpha,\ \beta,\ \gamma$ larning qiymatini topamiz:

$$\cos\alpha=0,7894,\quad \alpha=37^{\circ}10',$$
$$\cos\beta=0,5263,\quad \beta=58^{\circ}44',$$ ◀
$$\cos\gamma=0,3157,\quad \gamma=71^{\circ}24'.$$

5- misol. $M(5;\ 1;\ -1)$ nuqtadan $x-2y-2z+4=0$ tekislikkacha bo'lgan masofani toping.

▶ Nuqtadan tekislikkacha bo'lgan masofa (14) formula bilan topiladi. Bu yerda $A=1,\quad B=-2,\quad C=-2,\quad x_0=5,\ y_0=1,\ z_0=-1,$. Bu qiymatlarni (14) ga qo'ysak,

$$d=\left|\frac{1\cdot5+(-2)\cdot1+(-2)\cdot(-1)+4}{\sqrt{1^2+(-2)^2+(-2)^2}}\right|=\frac{|5-2+2+4|}{3}=\frac{9}{3}=3$$

hosil bo'ladi.

6- misol. $M(2;\ 3;\ -1)$ nuqtadan $2x-3y+5z-4=0$ tekislikka parallel tekislik o'tkazing.

▶ M nuqtadan o'tuvchi tekislik tenglamasini (12) formulaga asosan yozamiz:

$$A(x-2)+B(y-3)+C(z+1)=0.$$

Ikki tekislikning parallellik shartiga ko'ra, $A=2k,\ B=3k,\ C=5k$ bo'ladi. Bularni oxirgi tenglikka qo'ysak, $2k(x-2)-3k(y-3)+5k(z+1)=0$ yoki $2x-3y+5z+10=0$ kelib chiqadi. Bu izlanayotgan tekislik tenglamasi.

Masalani boshqa usul bilan ham yechish mumkin. Parallel tekisliklar bir-biridan faqat ozod hadlari bilan farq qilishi mumkin. Shunga asosan, berilgan tekislikka parallel tekisliklar oilasi $2x-3y+5z+D=0$ ko'rinishda bo'ladi.

Bu tenglamaga M nuqtaning koordinatalarini qo'yamiz va D ning qiymatini topamiz: $2\cdot2-3\cdot3+5\cdot(-1)+D=0\ \Rightarrow\ D=10$,bu qiymatni oxirgi tenglikka qo'yib, $2x-3y+5z+10=0$ tenglamani olamiz. ◀

7- misol. $M_1(-1;\ -2;\ 0)$ va $M_2(1;\ 1;\ 2)$ nuqtalardan o'tadigan $x+2y+2z-4=0$ tekislikka perpendikular bo'lgan tekislik tenglamasini yozing.

▶ Berligan nuqtadan o'tib, berilgan normal vektorga ega tekislik tenglamasi (12) ko'rinishda bo'ladi. (12) formuladagi $x_0,\ y_0,\ z_0$ lar o'rniga M_1 nuqtaning koordinatalarini qo'yib quyidagini olamiz:

$$A(x+1)+B(y+2)+C(z-0)=0. \qquad (*)$$

Xuddi shunday, bu tekislik M_2 nuqtadan ham o'tadi, u holda bu nuqtaning koordinatalari tekislik tenglamasini qanoatlantiradi:

$$A(1+1)+B(1+2)+C(2-0)=0$$

bundan

$$2A+3B+2C=0.$$

Izlanayotgan tekislik berilgan tekislikka perpendikular bo'lishi kerak. Shuning uchun ikki tekislikning perpendikularlik shartiga asosan,

$$1\cdot A+2\cdot B+2\cdot C=0$$

bo'ladi. Oxirgi ikki tenglikni birlashtirib, uch noma'lumli ikkita bir jinsli tenglama sistemasini hosil qilamiz:

$$\begin{cases} 2A+3B+2C=0, \\ A+2B+2C=0. \end{cases}$$

Bu sistemani (15) formula bilan yechib, $A=2k,\ B=-2k,$ $C=k$ larni topamiz. A, B va C larning qiymatini (*) ga qo'yib va k ga qisqartirib $2(x+1)-2(y+2)+z=0$ ni hosil qilamiz. Buni soddalashtirsak, izlanayotgan tekislik tenglamasi kelib chiqadi: $2x-2y+z-2=0$. ◀

8- misol. $\begin{cases} 5x-3y+4z-4=0, \\ 3x-4y-2z+5=0 \end{cases}$ tekisliklar orasidagi o'tkir burchakni toping.

▶ Ikki tekislik orasidagi o'tkir burchak (13) formula bilan topiladi. Birinchi tenglamadan $A_1=5,\ B_1=-3,\ C_1=4$. Ikkinchi tenglamadan $A_2=3,\ B_2=-4, C_2=-2,$

$$\cos\varphi=\frac{15+12-8}{\sqrt{50}\cdot\sqrt{29}};\quad \cos\varphi=\frac{19}{5\sqrt{58}};\quad \cos\varphi=0,49;\quad \varphi=60°04'. ◀$$

9- misol. $M_1(1;\ -1;\ 2)$, $M_2(2;\ 1;\ 2)$ va $M_3(1;\ 1;\ 4)$ nuqtalardan o'tuvchi tekislik tenglamasini yozing.

▶ Izlanayotgan tekislik tenglamasi (16) formulaga asosan:

$$\begin{vmatrix} x-1 & y+1 & z-2 \\ 2-1 & 1+1 & 2-2 \\ 1-1 & 1+1 & 4-2 \end{vmatrix}=0 \text{ yoki } \begin{vmatrix} x-1 & y+1 & z-2 \\ 1 & 2 & 0 \\ 0 & 2 & 2 \end{vmatrix}=0.$$

Determinantni hisoblaymiz: $4(x-1)+2(z-2)-2(y+1)=0.$ Bu yerdan $4x-2y+2z-10=0$ yoki $2x-y+z-5=0$. Bu izlangan tekislik tenglamasi.

Mustaqil bajarish uchun mashqlar

1.1. $M_1(2;\ 1;-2)$ va $M_2(-7;\ -2;\ 1)$ nuqtalardan o'tuvchi va Oy o'qiga parallel bo'lgan tekislik tenglamasini tuzing.

1.2. $M(1;\ 2;-4)$ nuqtadan o'tuvchi va xOy tekislikka parallel bo'lgan tekislik tenglamasini tuzing.

1.3. $M(3;7;-1)$ nuqta orqali o'tuvchi va xOz o'qiga perpendikular bo'lgan tekislik tenglamasini tuzing.

1.4. $M(2;-3;4)$ nuqtadan o'tuvchi va xOz tekislikka parallel bo'lgan tekislik tenglamasini toping.

1.5. $M(0;\ -2;\ 3)$ nuqtadan va Ox o'qidan o'tuvchi tekislik tenglamasini yozing. Tekislikni yasang.

1.6. $M(2;-4;3)$ nuqtadan va Oz o'qidan o'tuvchi tekislik tenglamasini yozing. Tekislikni yasang.

1.7. Ox va Oy o'qlaridan a va c birlikda kesib o'tuvchi hamda Oy o'qiga parallel bo'lgan tekislik tenglamasini yozing. Tekislikni yasang.

1.8. $M(-2;\ 4;-4)$ nuqtadan va Oz o'qidan o'tuvchi tekislik tenglamasini yozing.

1.9. $M(2;\ -5;\ 4)$ nuqtadan va Oy o'qidan o'tuvchi tekislik tenglamasini yozing.

1.10. $x-10y+2z-12=0$ tekislikning koordinata o'qlaridan kesib o'tgan kesmalarini toping.

1.11. $2x+3y-4z+24=0$ tekislik tenglamasini o'qlardan ajratgan kesmalar bo'yicha tenglamasiga keltiring.

1.12. $3x-4y+5z-24=0$ tekislik tenglamasini o'qlardan ajratgan kesmalar bo'yicha tenglamasiga keltiring.

1.13. $2x+9y-6z+33=0$ tekislik tenglamasini normal ko'rinishga keltiring.

1.14. Tekislik tenglamalarini normal ko'rinishga keltiring:

1) $2x-9y+6z-22=0$;

2) $10x+2y-11z+60=0$;

3) $6x-6y-7z+33=0$.

1.15. $3x-4y+5z-14=0$ tekislik tenglamasini normal ko'ri nishga keltiring.

1.16. Koordinatalar boshidan $5x-y+3z+12=0$ tekislikka perpendikular tushirilgan. Bu perpendikularning uzunligini va uning koordinata o'qlari bilan tashkil qilgan burchaklarini toping.

1.17. $M_0(2;3;-1)$ nuqtadan $7x-6y-6z+42=0$ tekislikkacha bo'lgan masofani toping.

1.18. $M_0(2;-4;2)$ nuqtadan $2x+11y+10z-10=0$ tekislikkacha bo'lgan masofani toping.

1.19. $A(3;4;-1)$ nuqtadan $3x+4y-5=0$ tekislikkacha bo'lgan masofani toping.

1.20. $5x+3y-4z+15=0$, $15x+9y-12z-5=0$ parallel tekisliklar orasidagi masofani toping.

Ko'rsatma: Birinchi tekislikdan ixtiyoriy nuqtani, masalan, $(-3;0;0)$ ni olib, bu nuqtadan ikkinchi tekislikkacha bo'lgan masofa topiladi.

1.21. $\begin{cases}2x-3y+6z-14=0,\\ 2x-3y+6z+28=0\end{cases}$ parallel tekisliklar orasidagi masofani toping.

1.22. $\begin{cases}4x+3y-5z-8=0,\\ 4x+3y-5z+12=0\end{cases}$ parallel tekisliklar orasidagi masofani toping.

1.23. $x-2y+2z-5=0$ tekislikka parallel bo'lib, undan 2 birlik uzoqlikda joylashgan tekislik tenglamasini yozing.

1.24. $M(-4;-1;2)$ nuqta orqali o'tib, $3x+4y-z-8=0$ tekislikka parallel bo'lgan tekislik tenglamasini tuzing.

1.25. $(2;5;-1)$ nuqta orqali o'tib, $x+3y-4z+5=0$ tekislikka parallel bo'lgan tekislik tenglamasini tuzing.

1.26. $(1;-3;2)$ nuqta orqali o'tib, $7x-4y+z-4=0$ tekislikka parallel bo'lgan tekislik tenglamasini tuzing.

1.27. $M_1(1;2;3)$ va $M_{12}(-2;-1;3)$ nuqtalar orqali o'tib, $x+4y-2z+5=0$ tekislikka perpendikular bo'lgan tekislik tenglamasini tuzing.

1.28. $M(-1;2;-3)$ va $N(1;4;-5)$ nuqtalardan o'tib, $3x+5y-6z+1=0$ tekislikka perpendikular bo'lgan tekislik tenglamasini yozing.

1.29. $(-1;-1;2)$ nuqtadan o'tib, $x-2y+z-4=0$ va $x+2y-2z+4=0$ tekisliklarga perpendikular bo'lgan tekislik tenglamasini yozing.

1.30. $(0;0;a)$ nuqtadan o'tib, $x-y-z=0$ va $2y=x$ tekisliklarga perpendikular bo'lgan tekislik tenglamasini yozing.

1.31. $5x-3y+5z+5=0$ va $x-2y+3z-5=0$ tekisliklar orasidagi burchakni toping.

1.32. Berilgan tekisliklar orasidagi burchakni toping:

1) $4x-5y+3z-1=0$ va $x-4y-z+9=0$;

2) $3x-y+2z+15=0$ va $5x+9y-3z-1=0$;

3) $6x+2y-4z+17=0$ va $9x+3y-6z-4=0$.

1.33. $M_1(1;2;-1)$, $M_2(-1;0;4)$, $M_3(-2;-1;1)$ nuqtalardan o'tuvchi tekislik tenglamasini tuzing.

1.34. $M_1(1;-3;4)$, $M_2(0;-2;-1)$, $M_3(1,1,-1)$ nuqtalardan o'tuvchi tekislik tenglamasini tuzing.

1.35. $M_1(1;\ -2;\ -1/2)$, $M_2(2;\ 1;\ 43)$, $M_3(0;\ -1;\ -1)$ nuqtalardan o‘tuvchi tekislik tenglamasini tuzing.

1.36. $M_1(1;\ 3;\ 0)$, $M_2(4;\ -1;\ 2)$, $M_3(3;\ 0;\ 1)$ nuqtalardan o‘tuvchi teksilikdan $N(4;\ 3;\ 0)$ nuqtagacha bo‘lgan masofani toping.

2- §. Fazodagi to‘g‘ri chiziq. Fazodagi to‘g‘ri chiziqqa doir asosiy masalalar

Bu paragrafda fazodagi to‘g‘ri chiziqqa doir asosiy formulalar va misol-masalalar keltirilgan.

1°. To‘g‘ri chiziqning kanonik tenglamalari.

$A(a;b;c)$ nuqtadan o‘tuvchi va $\vec{p}\{m,\ n,\ p\}$ vektorga perpendikular bo‘lgan to‘g‘ri chiziq tenglamasini tuzamiz. $B(x;\ y;\ z)$ to‘g‘ri chiziqda yotuvchi ixtiyoriy nuqta bo‘lsin, u holda $\overrightarrow{AB}$ va $\vec{o}$ vektorlarning $\left(\overrightarrow{AB} \parallel \vec{p}\right)$ parallellik shartiga asosan

$$\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-c}{p} \tag{1}$$

tenglamalarni hosil qilamiz. Bu tenglamalar *to‘g‘ri chiziqning* kanonik tenglamalari deyiladi. $\vec{p}\{m,\ n,\ p\}$ vektor to‘g‘ri chizining *yo‘naltiruvchi vektori* deyiladi. m, n va p — to‘g‘ri chiziqning *yo‘naltiruvchi koeffitsiyentlari* yo‘naltiruvchi vektorning Ox, Oy, Oz koordinata o‘qlaridagi proyeksiyalari hisoblanadi.

Agar α, β va γ — to‘g‘ri chiziq bilan mos ravishda Ox, Oy, Oz koordinata o‘qlari orasidagi burchaklar bo‘lsa, u holda

$$\cos\alpha = \pm\frac{m}{\sqrt{m^2+n^2+p^2}};\quad \cos\beta = \pm\frac{n}{\sqrt{m^2+n^2+p^2}};$$
$$\cos\gamma = \pm\frac{p}{\sqrt{m^2+n^2+p^2}} \tag{2}$$

bo‘ladi. $\cos\alpha$, $\cos\beta$ va $\cos\gamma$ lar to‘g‘ri chiziqning *yo‘naltiruvchi kosinuslari* deyiladi. m, n va p yo‘naltiruvchi koeffitsiyentlarni to‘g‘ri chiziqqa parallel bo‘lgan vektorning koordinata o‘qlaridagi proyeksiyalari deb qarash mumkin. m, n va p lar bir vaqtda nolga teng bo‘lmaydi. (1) tenglamalarni

$$\frac{x-a}{\cos\alpha} = \frac{y-b}{\cos\beta} = \frac{z-c}{\cos\gamma} \tag{3}$$

ko‘rinishda ham yozish mumkin.

2°. To‘g‘ri chiziqning parametrik tenglamasi (1) nisbatning har birini t parametrga tenglashtirib hosil qilinadi:

$$x = mt + a,\quad y = nt + b,\quad z = pt + c, \tag{4}$$

bu yerda t — parametr.

3°. To‘g‘ri chiziqning umumiy tenglamasi. Ikkita kesishuvchi tekislik

$$\left(\vec{N}_1,\ \vec{r}\right) + D_1 = 0 \quad \text{va} \quad \left(\vec{N}_2,\ \vec{r}\right) + D_2 = 0$$

tenglamalari bilan berilgan bo‘lsin, bu yerda $\vec{N}_1\{A_1,\ B_1,\ C_1\}$; $\vec{N}_2\{A_2,\ B_2,\ C_2\}$; $\vec{r}\{x,\ y,\ z\}$.

U holda

$$\begin{cases}\left(\vec{N}_1,\ \vec{r}\right) + D_1 = 0,\\ \left(\vec{N}_2,\ \vec{r}\right) + D_2 = 0\end{cases}$$

tenglamalar sistemasini ikki tekislikning kesishish chizig‘idan iborat to‘g‘ri chiziq tenglamasi deb qarash mumkin. Bu tenglamalar sistemasi *fazodagi to‘g‘ri chiziqning vektor shaklida berilgan umumiy tenglamasi* deb ataladi. Koordinatalaridan foydalanib ushbuni hosil qilamiz:

$$\begin{cases}A_1x + B_1y + C_1z + D_1 = 0,\\ A_2x + B_2y + C_2 + D_2 = 0.\end{cases} \tag{5}$$

Bu yerda A_1, B_1, C_1 koeffitsiyentlar A_2, B_2, C_2 koeffi-t-siyentlar bilan proporsional emas. (5) — qaralayotgan to'g'ri chiziq ikkita tekislikning kesishish chizig'i ekanini bildiradi.

4⁰. **Ikki to'g'ri chiziq orasidagi burchak.** *Fazoda ikki to'g'ri chiziq orasidagi burchak* deb, bu to'g'ri chiqlarga parallel bo'lgan yo'naltiruvchi vektorlari orasidagi burchakka aytiladi.

Ikki to'g'ri chiziq quyidagi tenglamalari bilan berilgan bo'lsin:

$$\vec{r} = \vec{r}_1 + \vec{s}_1 t \quad (l_1),$$

bu yerda $\vec{r}\{x;\ y;\ z\}, r_1\{x_1;\ y_1;\ z_1\}, \vec{s}_1\{m_1;\ n_1;\ p_1\}$ va

$$\vec{r} = \vec{r}_2 + \vec{s}_2 t \quad (l_2), \tag{11}$$

bu yerda $\vec{r}\{x;\ y;\ z\}, \vec{r}_2\{x_2;\ y_2;\ z_2\}, \vec{s}_2\{m_2;\ n_2;\ p_2\}$.

(l_1) va (l_2) to'g'ri chiziqlar orasidagi burchakni φ bilan, ularning $\vec{s}_1$ va $\vec{s}_2$ yo'naltiruvchi vektorlari orasidagi burchakni θ bilan belgilaymiz. Unda

$$\cos\theta = \frac{\left(\vec{s}_1 \cdot \vec{s}_2\right)}{\left|\vec{s}_1\right| \cdot \left|\vec{s}_2\right|};$$

$\varphi = \theta$ yoki $\varphi = \pi - \theta$ bo'lganidan $\cos\varphi = \pm\cos\theta$. Bularga asosan,

$$\cos\varphi = \pm\frac{\left(\vec{s}_1 \cdot \vec{s}_2\right)}{\left|\vec{s}_1\right| \cdot \left|\vec{s}_2\right|}$$ kelib chiqadi. Agar to'g'ri chiziqlar

$$\frac{x-x_1}{m} = \frac{y-y_1}{n} = \frac{z-z_1}{p}, \quad (l_1)$$

$$\frac{x-x_2}{m_1} = \frac{y-y_2}{n_1} = \frac{z-z_2}{p_1}, (l_2)$$

kanonik tenglamalari bilan berilgan bo'lsa, u holda bu to'g'ri chiziqlar orasidagi burchak

$$\cos\varphi = \pm\frac{m\ m_1 + n\ n_1 + p\ p_1}{\sqrt{m^2+n^2+p^2}\cdot\sqrt{m_1^2+n_1^2+p_1^2}} \tag{6}$$

formula bilan aniqlanadi.

5⁰. **Fazodagi ikki to'g'ri chiziqning parallellik va perpendikularlik shartlari.** Ushbu

$$\frac{x-x_0}{m} = \frac{y-y_0}{n} = \frac{z-z_0}{p},$$

$$\frac{x-x_1}{m_1} = \frac{y-y_1}{n_1} = \frac{z-z_1}{p_1} \tag{7}$$

tenglamalar bilan berilgan to'g'ri chiziqlarning *parallellik sharti:*

$$\frac{m}{m_1} = \frac{n}{n_1} = \frac{p}{p_1}; \tag{8}$$

perpendikularlik sharti:

$$mm_1 + nn_1 + pp_1 = 0 \tag{9}$$

bo'ladi.

6⁰. **Berilgan ikki nuqtadan o'tuvch to'g'ri chiziq tenglamasi.** Berilgan ikki $A(x_1; y_1;\ z_1)$ va $B(x_2;\ y_2;\ z_2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \tag{10}$$

ko'rinishda yoziladi.

7⁰. **To'g'ri chiziqning proeksiyalar bo'yicha tenglamasi.** (5) tenglamalar sistemasida bir marta y ni, ikkinchi marta x ni yo'qotib, to'g'ri chiziqning *proyeksiyalar bo'yicha tenglamasini* hosil qilamiz:

$$x = mz + a, \quad y = nz + b. \tag{11}$$

(11) tenglamalarni kanonik shaklda

$$\frac{x-a}{m}=\frac{y-b}{n}=\frac{z-0}{1}$$

ko'rinishda yozish mumkin.

1- misol. $\frac{x-1}{4}=\frac{y-5}{-3}=\frac{z+2}{12}$ to'g'ri chiziq bilan koordinata o'qlari orasidagi burchakni toping.

▶ $m=4,\quad n=-3,\quad p=12$ larni (2) formuladagi o'rniga qo'yib topamiz:

$$\cos\alpha=\pm\frac{4}{\sqrt{16+9+144}}=\pm\frac{4}{13}\quad\text{yoki}$$

$$\cos\alpha=\frac{4}{13},\ \cos\beta=\mp\frac{3}{13},\cos\gamma=\pm\frac{12}{13}.$$

To'g'ri chiziqning koordinata o'qlari bilan tashkil etgan o'tkir burchaklari $\alpha=72°55',\beta=76°20',\gamma=22°22'$ bo'ladi. ◀

2- misol. Ikki to'g'ri chiziq orasidagi o'tkir burchakni toping:

$$\frac{x-2}{3}=\frac{y-1}{-1}=\frac{z-3}{2}\quad\text{va}\quad\frac{x-1}{2}=\frac{y+2}{4}=\frac{z+1}{-2}.$$

▶ Ikki to'g'ri chiziq orasidagi burchak (9) formula bilan topiladi. Bu yerda $m=3,\quad n=-1,\quad p=2$ va $m=2,\quad n=4,\quad p=-2;$

$$\cos\varphi=\pm\frac{3\cdot2+(-1)\cdot4+2\cdot(-2)}{\sqrt{3^2+(-1)^2+2^2}\cdot\sqrt{2^2+4^2+(-2)^2}}=\pm\frac{-2}{\sqrt{14}\cdot\sqrt{24}};$$

$$\cos\varphi=\mp\frac{1}{2\sqrt{27}}=\mp\ 0,1091.$$

Masalaning shartiga asosan o'tkir burchakni topish kerak, shuning uchun $\cos\varphi$ ning musbat qiymatini olamiz:

$$\cos\varphi=0,1091\ \text{yoki}\ <\ \varphi=88°44'$$ ◀

Mustaqil bajarish uchun mashqlar

2.1. 1) $\frac{x-5}{2}=\frac{y+1}{3}=\frac{z-4}{6}$; 2) $\frac{x}{12}=\frac{y-7}{9}=\frac{z+3}{20}$ to'g'ri chiziqlarning yo'naltiriuvchi kosinuslarini toping.

2.2. $A(1;-5;3)$ nuqtadan o'tuvchi va koordinata o'qlari bilan mos ravishda 60°, 45°, 120° burchaklar tashkil qiluvchi to'g'ri chiziq tenglamasini tuzing.

2.3. To'g'ri chiziqlar umumiy tenglamalari bilan berilgan. Bu to'g'ri chiziqlar uchun kanonik tenglama va to'g'ri chiziqning proyeksiyalar tenglamasini yozing:

$$1)\begin{cases}2x-y+2z-3=0,\\x+2y-z-1=0,\end{cases}$$

$$2)\begin{cases}x+2y-3z-5=0,\\2x-y+z+2=0;\end{cases}$$

2.4. $A(4;\ 3;\ 0)$ nuqtadan o'tuvchi va $\vec{p}\{-1;\ 1;\ 1\}$ vektorga parallel to'g'ri chiziq tenglamasini yozing. To'g'ri chiziqning yOz tekislikdagi izini toping.

2.5. $x=4,\quad y=3$ to'g'ri chiziqni yasang va uning yo'naltiruvchi vektorlarini toping.

2.6. 1) $y=3,\quad z=2;$ 2) $y=2,\quad z=x+1;$ 3) $x=4,\quad z=y$ to'g'ri chiziqlarni yasang va ularning yo'naltiruvchi vektorlarini aniqlang.

2.7. To'g'ri chiziq umumiy tenglamasining kanonik ko'rinishga keltiring:

$$\begin{cases}2x-3y+2z-9=0,\\x-2y+z+3=0.\end{cases}$$

2.8. $\begin{cases}5x-6y+2z-9=0,\\x-z+3=0\end{cases}$ to'g'ri chiziqning yo'naltiruvchi kosinuslarini toping.

2.9. $M_0(2;\ 0;\ -3)$ nuqtadan o'tuvchi va $\vec{q}(2;\ -3;\ 5)$ vektorga:

1) $\frac{x-1}{5}=\frac{y+2}{2}=\frac{z+1}{-1}$ to'g'ri chiziqqa; 2) Ox o'qiga;

3) Oz o'qiga; 4) $\begin{cases} 3x - y + 2z - 7 = 0, \\ x + 3y - 2z - 3 = 0 \end{cases}$ to'g'ri chiziqqa;

5) $x = -2 + t, \quad y = 2t, \quad z = 1 - \frac{t}{2}$

to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziqning kanonik tenglamasini yozing.

2.10. $A(2;-5;\ 3)$ nuqtadan o'tuvchi va:

1) Oz o'qiga parallel;

2) $\frac{x-1}{4} = \frac{y-2}{-6} = \frac{z+3}{9}$ to'g'ri chiziqqa parallel;

3) $\begin{cases} 2x - y + 3z - 1 = 0, \\ 5x + 4y - z - 7 = 0 \end{cases}$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini yozing.

2.11. Quyidagi to'g'ri chiziqlarning kesishishini tekshiring:

1) $\frac{x-1}{2} = \frac{y-7}{1} = \frac{z-5}{4}$; va $\frac{x-6}{3} = \frac{y+1}{-2} = \frac{z}{1}$;

2) $\begin{cases} 4x + z + 1 = 0, \\ x - 2y + 3 = 0 \end{cases}$ va $\begin{cases} 3x + y - z + 4 = 0, \\ y + 2z - 8 = 0. \end{cases}$

2.12. $A(2;\ 3;\ 1)$ nuqtadan $\frac{x+1}{2} = \frac{y}{-1} = \frac{z-2}{3}$ to'g'ri chiziqqa o'tkazilgan perpen-dikular tenglamasini yozing.

2.13. $\frac{x-2}{1} = \frac{y+3}{2} = \frac{z-4}{4}$ to'g'ri chiziqning koordinata tekislik laridagi izining koordinatalarini toping.

2.14. $\frac{x-1}{3} = \frac{y+2}{6} = \frac{z-5}{2}$ va $\frac{x}{2} = \frac{y-3}{9} = \frac{x+1}{6}$ to'g'ri chiziqlar orasidagi burchakni toping.

2.15. Quyida berilgan to'g'ri chiziqlar orasidagi burchakni toping:

1) $\begin{cases} 2x + 3y - 4z + 5 = 0, \\ x - y + z = 0 \end{cases}$ va

$\begin{cases} x - y + 2z - 4 = 0, \\ 2x + y - z - 5 = 0; \end{cases}$

2) $\begin{cases} x - y + z - 4 = 0, \\ 2x + y - 2z + 5 = 0 \end{cases}$ va

$\begin{cases} x + y + z - 4 = 0, \\ 2x + 3y - z - 6 = 0; \end{cases}$

3) $\begin{cases} 3x - 4y - 2z = 0, \\ 2x + y - 2z = 0 \end{cases}$ va

$\begin{cases} 4x + y - 6z - 4 = 0, \\ y - 3z + 2 = 0. \end{cases}$

2.16. $A(3;\ -1;\ 4)$ nuqtadan o'tuvchi va Oz o'qiga parallel bo'lgan to'g'ri chiziq tenglamasini yozing.

2.17. $A(1;\ -1;\ 2)$ nuqtadan o'tuvchi va $\frac{x-2}{1} = \frac{y-3}{3} = \frac{z+1}{2}$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziq tenglamasini yozing.

2.18. $A(-1;\ 2;\ 3)$ va $B(2;\ 6;\ -2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglmasini yozing.

2.19. $A(2;\ -1;\ 3)$ va $B(2;\ 3;\ 3)$ nuqtalardan o'tuvchi to'g'ri chiziqni yasang va uning tenglmamasini yozing.

2.20. $A(4;\ -3;\ 1)$ nuqtadan chiqib, $v(2;\ 3;\ 1)$ tezlik bilan harakatlanuvchi nuqta trayektoriyasi tenglamasini yozing.

2.21. Berilgan M_1 va M_2 nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini yozing:

1) $M_1(1;\ -2;\ 1)$, $M_2(3;\ 1;\ -1)$;

2) $M_1(3;\ -1;\ 0)$, $M_2(1;\ 0;\ -3)$

2.22. 1) $(-2;\ 1;\ -1)$ nuqtadan o'tib $\vec{p}\{1;\ -2;\ 3\}$ vektorga parallel bo'lgan;

2) $A(3;\ -1;\ 4)$ va $B(1;\ 1;\ 2)$ nuqtalardan o'tuvchi to'g'ri chiziqning parametrik tenglamasini yozing.

2.23. $x = 2z - 1,\ \ y = -2z + 1$ to‘g‘ri chiziq bilan koordinatalar boshi va $(1; -1; -1)$ nuqtadan o‘tuvchi to‘g‘ri chiziqlar orasidagi burchakni toping.

2.24. $\frac{x}{2} = \frac{y}{3} = \frac{z}{1}$ to‘g‘ri chiziq bilan $x = z + 1,\ \ y = 1 - z$ to‘g‘ri chiziqning perpendikular ekanligini isbotlang.

2.25. $(-4; 3; 0)$ nuqtadan o‘tuvchi va

$$\begin{cases} x - 2y + z = 0, \\ 2x + y - z = 0 \end{cases}$$

to‘g‘ri chiziqqa parallel to‘g‘ri chiziq tenglamasini yozing.

2.26. $(2; -3; 4)$ nuqtadan o‘tib, Oz o‘qiga perpendikular bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

2.27. $N(2; -1; 3)$ nuqtadan

$$\frac{x+1}{3} = \frac{y+2}{4} = \frac{z-1}{5}$$

to‘g‘ri chiziqqacha bo‘lgan masofani toping.

Ko‘rsatma. *$A(-1; -2; 1)$ nuqta to‘g‘ri chiziqda yotadi; $\vec{p}\{3;\ 4;\ 5\}$ to‘g‘ri chiziqning yo‘naltiruvchi vektori. U holda, nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa*

$$d = |AN| \cdot \sin\alpha = \frac{|AN| \cdot \left|\left[\vec{p} \times \overline{AN}\right]\right|}{|\vec{p}| \cdot |AN|} = \frac{\left|\left[\vec{p} \times \overline{AN}\right]\right|}{|\vec{p}|}$$

formula bilan topiladi.

3- §. Fazoda to‘g‘ri chiziq va tekislik

Bu paragrafda tekislik va to‘g‘ri chiziq orasidagi munosabatlarga doir asosiy formulalar keltiriladi, misol-masalalar qaraladi.

1°. $\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-c}{p}$ **to‘g‘ri chiziq bilan** $Ax + By + Cz + D = 0$ **tekislik orasidagi o‘tkir burchak**

$$\sin\varphi = \left|\frac{A\ m + Bn + Cp}{\sqrt{A^2 + B^2 + C^2} \cdot \sqrt{m^2 + n^2 + p^2}}\right| \quad (1)$$

formula bilan topiladi.

To‘g‘ri chiziq va tekislikning parallellik sharti:

$$Am + Bn + Cp = 0. \quad (2)$$

To‘g‘ri chiziq va tekislikning perpendikularlik sharti:

$$\frac{A}{m} = \frac{B}{n} = \frac{C}{p}. \quad (3)$$

2°. Berilgan $\begin{cases} Ax + By + Cz + D = 0, \\ Ax_1 + By_1 + Cz_1 + D_1 = 0 \end{cases}$ **to‘g‘ri chiziqdan o‘tuvchi tekisliklar dastasining tenglamasi**

$$Ax + By + Cz + D + \lambda(A_1x + B_1y + C_1z + D_1) = 0 \quad (4)$$

ko‘rinishda bo‘ladi, bu yerda λ — ixtiyoriy haqiqiy son.

3°. To‘g‘ri chiziq bilan tekislikning kesishish nuqtasi. To‘g‘ri chiziqning parametrik tenglamasi $x = mt + a,\ \ y = nt + b,$ $z = pt + c$ larni tekislikning umumiy tenglamasidagi $x,\ y,\ z$ lar o‘rniga qo‘yib, $Ax + By + Cz + D = 0$ dan t_0 ning qiymatini, so‘ngra $x_0,\ y_0,\ z_0$ larni topamiz. Bu esa to‘g‘ri chiziq bilan tekislikning kesishish nuqtasi bo‘ladi.

4°. Ikki to‘g‘ri chiziqning bir tekislikda yotish sharti:

$$\begin{vmatrix} a - a_1 & b - b_1 & c - c_1 \\ m & n & p \\ m_1 & n_1 & p_1 \end{vmatrix} = 0. \quad (5)$$

1- misol. $\begin{cases} x+y+z-4=0, \\ 2x-y+4z+5=0 \end{cases}$ to'g'ri chiziq bilan

$$x+y+3z-1=0$$

tekislik orasidagi burchakni toping.

▶ To'g'ri chiziq tenglamasini kanonik ko'rinishga keltirmasdan ham to'g'ri chiziq bilan tekislik orasidagi burchakni topish mumkin. Buning uchun to'g'ri chiziqning yo'naltiruvchi kosinuslarini topish yetarli.

To'g'ri chiziq tenglamasining koeffsiyentlaridan

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 4 \end{pmatrix}$$

matritsani tuzib, $t=1$ desak, 1- § dagi (15) formula yordamida

$$m=\begin{vmatrix} 1 & 1 \\ -1 & 4 \end{vmatrix}=5, \quad n=\begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix}=-2, \quad p=\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}=-3$$

larni topamiz. Tekislik tenglamasidan $A=1$, $B=1$, $C=3$ ni topib, (1) formula yordamida o'tkir burchakni topamiz:

$$\sin\varphi=\left|\frac{-6}{\sqrt{11}\cdot\sqrt{38}}\right|=\frac{6}{\sqrt{418}}=0,2935, \quad \varphi=17^\circ 04'.$$ ◀

2- misol. $M(1;-1;2)$ nuqtadan va $\begin{cases} 3x+y-4z+5=0, \\ x-y+2z-1=0 \end{cases}$ to'g'ri chiziqdan o'tuvchi tekislik tenglamasini yozing.

▶ Berilgan to'g'ri chiziqdan o'tuvchi tekisliklar dastasining tenglamasi (4) formulaga asosan quyidagicha bo'ladi:

$$3x+y+-4z+5+\lambda(x-y+z-1)=0.$$

Bu tekisliklar dastasidan $M(1;-1;2)$ nuqtadan o'tuvchi tekislikni ajratib olish talab qilinadi. Agar tekislik bu nuqtadan o'tsa, u holda bu nuqtaning koordinatalari tekislik tenglamasini qanoatlantiradi. Tenglamaga M nuqtaning koordinatalarini qo'yib, λ ning qiymatini topamiz:

$$5\lambda-1=0, \quad \lambda=1/5.$$

λ ning qiymatini tekislik teglamasiga qo'yib, ushbu tenglamani topamiz:

$$8x+2y-9z+12=0.$$ ◀

3- misol. $\begin{cases} 3x-y+z-5=0, \\ x+2y-z+2=0 \end{cases}$ to'g'ri chiziqdan o'tib,

$$\frac{x-1}{-1}=\frac{y+2}{2}=\frac{z-1}{2}$$

to'g'ri chiziqqa parallel bo'lgan tekislik tenglamasini yozing.

▶ Birinchi to'g'ri chiziqdan o'tuvchi tekisliklar dastasining tenglamasi

$$3x-y+z-5+\lambda\ \ (x+2y-z+2)=0$$

yoki

$$(3+\lambda)x+(2\lambda-1)y+(1-\lambda)z-5+2\lambda=0.$$

Bu tekisliklar dastasidan ikkinchi to'g'ri chiziqqa parallel bo'lgan tekislik tenglamasini ajratib olamiz, buning uchun to'g'ri chiziq va tekislikning parallellik sharti (2) bajarilishi kerak. (*) tenglikdan $A=3+\lambda$, $B=2\lambda-1$, $C=1-\lambda$. Ikkinchi to'g'ri chiziq tenglamasidan $m=-1$, $n=2$, $p=2$. U holda to'g'ri chiziq va tekislikning parallelik shartiga asosan:

$$(3+\lambda)\cdot(-1)+(2\lambda-1)\cdot 2+(1-\lambda)\cdot 2=0$$

yoki

$$-3-\lambda+4\lambda-2+2-2\lambda=0, \quad \lambda=3.$$

λ ning bu qiymatini (*) ga qo'yib, $6x+5y-2z+1=0$ tenglamani hosil qilamiz. ◀

Mustaqil bajarish uchun mashqlar

3.1. $\frac{x-1}{2}=\frac{y+2}{1}=\frac{z-1}{2}$ to‘g‘ri chiziq bilan $2x+y-z+4=0$ tekislik orasidagi burchakni toping.

3.2. $y=3x-1,\quad 2z=-3x+2$ to‘g‘ri chiziq bilan $2x+y+z-4=0$ tekislik orasidagi burchakni toping.

3.3. $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z-3}{3}$ to‘g‘ri chiziq bilan $2x+y-z=0$ tekislikning parallelligini, $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z+3}{3}$ to‘g‘ri chiziqning bu tekislikda yotishini ko‘rsating.

3.4. $P(1;\,2;-1)$ nuqtadan o‘tuvchi va $\frac{x-3}{1}=\frac{y-2}{-3}=\frac{z+1}{4}$ to‘g‘ri chiziqqa perendikular bo‘lgan tekislik tenglamasini yozing.

3.5. $(-1;\,2;-3)$ nuqtadan o‘tuvchi va $x=2,\quad y-z=1$ to‘g‘ri chiziqqa perendikular bo‘lgan tekislik tenglamasini yozing.

3.6. $P(2;\,-4;\,-2)$ nuqtadan o‘tuvchi va $\begin{cases} x-4y+5z-1=0, \\ 2x+y+3=0 \end{cases}$

to‘g‘ri chiziqqa perpendikular bo‘lgan tekislik tenglamasini yozing.

3.7. $(2;\,1;\,6)$ nuqtadan o‘tuvchi va $x-4y+5z=0$ tekislikka perpendikular bo‘lgan to‘g‘ri chiziq tenglamasini yozing va uning yo‘naltiruvchi kosinuslarini aniqlang.

3.8. $(1;\,-1;\,2)$ nuqtadan o‘tuvchi va $3x-y-5z-8=0$ tekislikka perendikular bo‘lgan to‘g‘ri chiziq tenglamasini yozing.

3.9. $\frac{x-1}{3}=\frac{y+1}{-1}=\frac{z-2}{5}$ to‘g‘ri chiziq bilan $x+y-2z-4=0$ tekislikning kesishish nuqtasini toping.

3.10. Kesishish nuqtasini toping:

1) $\frac{x+1}{2}=\frac{y-3}{4}=\frac{z}{3}$ to‘g‘ri chiziq bilan $3x-3y+2z-5=0$ tekislikning;

2) $\frac{x-13}{8}=\frac{y-1}{2}=\frac{z-4}{3}$ to‘g‘ri chiziq bilan $x+2y-4z+1=0$ tekislikning;

3) $\frac{x-7}{5}=\frac{y-4}{1}=\frac{z-5}{4}$ to‘g‘ri chiziq bilan $3x-y+2z-5=0$ tekislikning.

3.11. $(2;\,-1;\,3)$ nuqtadan o‘tuvchi va $x+3y-4z-13=0$ tekislikka perpendikular bo‘lgan to‘g‘ri chiziq tenglamasini yozing.

3.12. $(3;\,4;\,0)$ nuqtadan va $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z+1}{3}$ to‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasini yozing.

3.13. $\frac{x-1}{1}=\frac{y+1}{2}=\frac{z+2}{2}$ to‘g‘ri chiziqdan o‘tuvchi va $2x+3y-z-4=0$ tekislikka perpendikular bo‘lgan tekislik tenglamasini yozing.

3.14. $\frac{x-3}{2}=\frac{y}{1}=\frac{z-1}{2}$ va $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z}{2}$ parallel to‘g‘ri chiziqlar orqali o‘tuvchi tekislik tenglamasini yozing.

3.15. $2x+y-3z+1=0$ tekislik bilan $\frac{x-3}{1}=\frac{y-5}{-5}=\frac{z+1}{2}$ va $\frac{x-5}{2}=\frac{y-3}{2}=\frac{z+4}{-6}$ to‘g‘ri chiziqlarning kesishish nuqtalaridan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

3.16. A ning qanday qiymatida $Ax+3y-5z+1=0$ tekislik bilan $\frac{x-1}{4}=\frac{y+5}{-4}=\frac{z+1}{3}$ to‘g‘ri chiziq parallel bo‘ladi?

3.17. A va B ning qanday qiymatida $Ax+By+6z-7=0$ tekislik bilan $\frac{x-2}{2}=\frac{y+5}{-4}=\frac{z+1}{3}$ to‘g‘ri chiziq o‘zaro perpendikular bo‘ladi?

3.18. $(3;-2;\,4)$ nuqtadan $5x+3y-7z+1=0$ tekislikka perpendikular o‘tkazing.

3.19. Koordinatalar boshidan $\frac{x+2}{4}=\frac{y-3}{5}=\frac{z-1}{-2}$ to‘g‘ri chiziqqa perpendikular o‘tkazing.

3.20. $M(2;\,-1;\,0)$ nuqtadan va $\begin{cases} x-y+3z-1=0, \\ 2x+y-z+2=0 \end{cases}$ to‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasini tuzing.

3.21. $(1;\,1;\,-2)$ nuqtadan va $\frac{x-1}{2}=\frac{y-3}{1}=\frac{z}{5}$ to‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasini tuzing.

3.22. $\frac{x-1}{1}=\frac{y+2}{1}=\frac{z}{2}$ to'g'ri chiziqdan o'tib, $3x-y+2z-2=0$ tekislikka perpendikular bo'lgan tekislik tenglamasini yozing.

3.23. $\begin{cases}3x+2y+3z-5=0,\\ x+y+z-4=0\end{cases}$ to'g'ri chiziqdan o'tib, $\begin{cases}x-y+2z+1=0,\\ 2x+y-3z+4=0\end{cases}$ to'g'ri chiziqqa parallel bo'lgan tekislik tenglamasini yozing.

3.24. $\frac{x-4}{5}=\frac{y-3}{1}=\frac{z+1}{2}$ to'g'ri chiziqdan o'tib, $x+4y-$ $-3z+7=0$ tekislikka perpendikular bo'lgan tekislik tenglamasini yozing.

3.25. $\begin{cases}x-2y+3z-1=0,\\ x-y+z+5=0\end{cases}$ to'g'ri chiziqdan o'tib, $2x+2y-$ $-z+5=0$ tekislikka perpendikular bo'lgan tekislik tenglamasini yozing.

3.26. $\frac{x-1}{2}=\frac{y-3}{3}=\frac{z}{4}$ va $\frac{x+2}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ parallel to'g'ri chiziqlardan o'tuvchi tekislik tenglamasini yozing.

3.27. $\frac{x+2}{4}=\frac{y-1}{-1}=\frac{z}{3}$ va $\frac{x-1}{4}=\frac{y}{-1}=\frac{z+1}{3}$ parallel to'g'ri chiziqlardan o'tuvchi tekislik tenglamasini yozing.

3.28. $\frac{x}{4}=\frac{y-4}{8}=\frac{z+1}{-2}$ to'g'ri chiziqning $x-y-3z+8=0$ tekislikdagi proyeksiyasini toping.

3.29. $\frac{x}{7}=\frac{y+2}{3}=\frac{z-1}{5}$ va $\frac{x-1}{7}=\frac{y-3}{3}=\frac{z+2}{5}$ parallel to'g'ri chiziqlardan o'tuvchi tekislik tenglamasini yozing.

3.30. $\frac{x}{6}=\frac{y}{2}=\frac{z}{-3}$ va $\frac{x+1}{5}=\frac{y-3}{4}=\frac{z-4}{2}$ parallel to'g'ri chiziqlardan va $P(4;-3;1)$ nuqtadan o'tuvchi tekislik tenglamasini yozing.

3.31. $\frac{x-3}{2}=\frac{y+4}{1}=\frac{z-2}{-3}$ to'g'ri chiziqdan o'tib, $\frac{x+5}{4}=\frac{y-2}{7}=\frac{z-1}{2}$ to'g'ri chiziqqa parallel bo'lgan tekislik tenglamasini yozing.

3.32. $\frac{x+5}{3}=\frac{y-2}{1}=\frac{z}{4}$ to'g'ri chiziqdan o'tib, $x+y-z+15=0$ tekislikka parallel bo'lgan tekislik tenglamasini yozing.

3.33. $P(7;9;7)$ nuqtadan $\frac{x-2}{4}=\frac{y-1}{3}=\frac{z}{2}$ to'g'ri chiziqqacha bo'lgan masofani toping.

4- §. Ikkinchi tartibli sirtlar

Bu paragrafda ikkinchi tartibli sirtlar tenglamalari bayon qilinadi, ular yordamida misol-masalalar yechish qaraladi.

Koordinatalari $F(x, y, z)=0$ ko'rinishdagi tenglamani qanoatlantiradigan nuqtalarning geometrik o'rni *sirt* deb ataladi. Agar bu tenglama z ga nisbatan yechilsa, u holda sirt tenglamasi $z=f(x, y)$ ko'rinishda bo'ladi. Sirt tenglamasida har doim ham uchala o'zgaruvchi bir vaqtda qatnashavermasligi ham mumkin.

1°. Sferik sirt. *Markaz* deb ataluvchi nuqtadan bir xil uzoqlikda joylashgan nuqtalarning geometrik o'rni *sfera* deb ataladi. *Sferaning kanonik (sodda) tenglamasi:*

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2 \qquad (1)$$

ko'rinishda bo'ladi, bu yerda a, b, c — sfera markazining koordinatalari, R — uning radiusi.

Sferaning markazi koordinatalar boshida bo'lsa, uning tenglmasi

$$x^2+y^2+z^2=R^2 \qquad (2)$$

ko'rinishda bo'ladi.

Sferaning umumiy tenglamasi:

$$Ax^2 + Ay^2 + Az^2 + 2Bx + 2Cy + 2Dz + E = 0\,, \ (A \neq 0).$$

1- misol. $x^2 + y^2 + z^2 - x + 2y + 1 = 0$ tenglama bilan berilgan sferaning markazi koordinatalarini va radiusini toping.

▶ Berilgan tenglamani, x, y, z o'zgaruvchilarga nisbatan to'la kvadrat ajratib, sferaning kanonik ko'rinishdagi tenglamasiga keltiramiz:

$$\left(x^2 - x + \tfrac{1}{4}\right) - \tfrac{1}{4} + \left(y^2 + 2y + 1\right) - 1 + z^2 + 1 = 0$$

yoki

$$\left(x - \tfrac{1}{2}\right)^2 + (y + 1)^2 + z^2 = \tfrac{1}{4}.$$

Bu yerdan ko'rinadiki, sferaning markazi $C(1/2; -1; 0)$ nuqtada, radiusi $R = 1/2$ ga teng. ◀

2- misol. Markazi nuqtada $C(1; 1; -1)$ va radiusi $R = 8$ ga teng bo'lgan sfera tenglamasini yozing.

▶ (1) formulada $a = 1$, $b = 1$, $c = -1$ va $R = 8$ bo'lsa, sfera tenglamasi

$$(x - 1)^2 + (y - 1)^2 + (z + 1)^2 = 64$$

yoki $x^2 + y^2 + z^2 - 2x - 2y + 2z - 61 = 0$ ko'rinishda bo'ladi. ◀

3- misol. $\begin{cases} (x-3)^2 + (y+2)^2 + (z-1)^2 = 100, \\ 2x - 2y - z + 9 = 0 \end{cases}$ aylana markazi ning koordinatalari va radiusini toping.

▶ Sfera markazi $C(3; -2; 1)$ nuqtadan tekislikka perpendikular o'tkazamiz, uning tenglamasi

$$\frac{x-3}{2} = \frac{y+2}{-2} = \frac{z-1}{-1} \qquad (*)$$

ko'rinishda bo'ladi. Tekislikning normal vektorini perpendikularning yo'naltiruvchi vektori deb qabul qilish mumkin.

Endi (*) to'g'ri chiziq bilan $2x - 2y - z + 9 = 0$ tekislikning kesishish nuqtasi koordinatalarini topamiz. Bu nuqta koordinatalari sfera bilan berilgan tekislikning kesishishidan hosil bo'lgan aylana markazining koordinatalari bo'ladi. To'g'ri chiziq tenglamasini

$$x = 2t + 3, \quad y = -2t - 2, \quad z = -t + 1$$

parametrik shaklda yozib, tekislik tenglamasidagi x, y, z lar o'rniga ularning t orqali qiymatini qo'ysak,

$$2(2t + 3) - 2(-2t - 2) - (-t + 1) + 9 = 0,$$

ya'ni $t = -2$ ni olamiz. Bunga asosan aylana markazining koordinatalari

$$x = 2(-2) + 3 = -1, \quad y = -2(-2) - 2 = 2, \quad z = -(-2) + 1 = 3$$

yoki $C(-1; 2; 3)$ bo'ladi.

Endi sfera markazi $C(3; -2; 1)$ nuqtadan $2x - 2y - z + 9 = 0$ tekislikkacha bo'lgan masofani topamiz:

$$d = \frac{|2\cdot3 - 2\cdot(-2) - 1 + 9|}{\sqrt{4+4+1}} = \frac{18}{3} = 6.$$

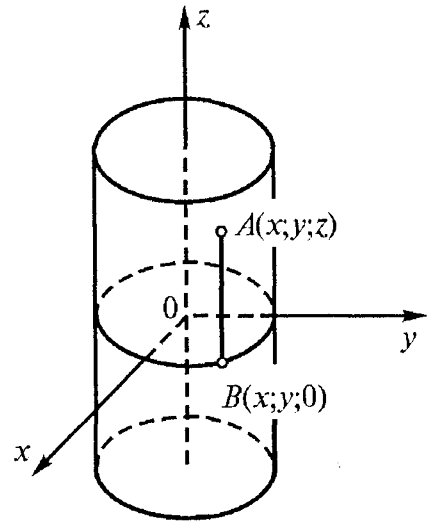
Aylana radiusi r ushbu $r^2 = R^2 - d^2$ tenglikdan topiladi, bu yerda R – sfera radiusi. Shunday qilib, $r^2 = 100 - 36 = 64$, ya'ni $R = 8$. ◀

2°. Silindrik sirt. *Yasovchi* deb ataluvchi to'g'ri chiziqning *yo'naltiruvchi* deb ataluvchi biror egri chiziq bo'ylab berilgan yo'nalishga parallell holda harakatlanishidan hosil bo'lgan sirt *silindrik sirt* deyiladi.

Yasovchi Oz o'qiqa parallel, yo'naltiruvchi chiziq esa xOy tekislikda yotadigan va

$$F(x; y) = 0$$

tenglama bilan aniqlanadigan holni qaraymiz (39- rasm). Sirtning yasovchisida ixtiyoriy $A(x; y; z)$ nuqta olamiz, uning birinchi ikkita koordinatasi $B(x; y; 0)$ nuqta koordinatalari bilan bir xil bo'ladi.



39- rasm.

Shu sababli silindrik sirtning $A(x;\ y;\ z)$ nuqtasining koordinalari yo'naltiruvchi chiziq tenglamasi $F(x;\ y) = 0$ ni qanoatlantiradi. Demak, bu tenglama yasovchilari Oz o'qiqa parallel bo'lgan silindrik sirtning tenglamasidir. Shunday qilib, z koordinatani o'z ichiga olmagan $F(x;\ y) = 0$ tenglama fazoda yasovchilari Oz o'qiqa parallel va yo'naltiruvchisi xOy tekislikda o'sha tenglama bilan aniqlanadigan silindrik sirtni ifodalaydi.

Shunga o'xshash, x koordinatani o'z ichiga olmagan

$$F(y;\ z) = 0$$

tenglama va y koordinatani o'z ichiga olmagan

$$F(x;\ z) = 0$$

tenglama yasovchilari mos ravishda Ox va Oy o'qlarga parallel bo'lgan silindrik sirtlarni aniqlaydi.

1- misol. $x^2 + y^2 = R^2$ tenglama qanday sirtni aniqlaydi?

▶ Berilgan tenglama bilan aniqlanadigan sirt silindrik sirt bo'lib, u *doiraviy silindr* deb ataladi. Uning yasovchilari Oz o'qiqa parallel,

xOy tekislikdagi yo'naltiruvchisi esa radiusi R va markazi koordinatalar boshida bo'lgan $x^2 + y^2 = R^2$ aylana tenglamasidir. ◀

3⁰. Fazoda chiziq. Ikkita sirtning kesishishi fazoda chiziqni ifodalaydi. Agar bu sirtlarning tenglamalari $F(x;\ y;\ z) = 0$ va $F_1(x;\ y;\ z) = 0$ bo'lsa, u holda bu ikki tenglama sistemasi

$$\begin{cases} F(x, y,\ z) = 0, \\ F_1(x,\ y,\ z) = 0 \end{cases}$$

fazodagi chiziqning tenglamasi bo'ladi. Shunday qilib, bu tenglamalar sistemasini qanoatlantiruvchi nuqtalarning geometrik o'rni chiziq bo'ladi.

1- misol. $\begin{cases} x^2 + y^2 + (z-7)^2 = 16, \\ z = 6 \end{cases}$ tenglamalar sistemasi qanday chiziqni ifodalaydi?

▶ Birinchi tenglik sferani, ikkinchi tenglik xOy tekislikka parallel bo'lgan tekislikni ifodalaydi. Berilgan sferani berilgan tekislik bilan kesilganda aylana hosil bo'ladi. Demak, masalada qaralayotgan chiziq $z = 6$ tekislikda yotuvchi aylanadan iborat ekan.

Bu aylana tenglamasini tuzamiz. $z = 6$ qiymatni birinchi tenglikka qo'yib ushbuni hosil qilamiz:

$$\begin{cases} x^2 + y^2 + (6-7)^2 = 16, \\ z = 6 \end{cases} \quad \text{yoki} \quad \begin{cases} x^2 + y^2 + 1 = 16. \\ z = 6. \end{cases}$$

Natijada

$$\begin{cases} x^2 + y^2 = 15, \\ z = 6. \end{cases}$$

Birinchi tenglik o'qi Oz o'qidan iborat bo'lgan doiraviy silindrdan, ikkinchisi xOy tekislikka parallel tekislikdan iborat. Bu sistemaning birinchi tenglamasi $x^2 + y^2 = 15$ shu xOy tekislikdagi aylana tenglamasi bo'ladi. ◀

4⁰. Aylanish sirtlari. yOz tekislikdagi $F(y,\ z) = 0$ tenglama bilan berilgan L chiziqni qaraylik. Bu chiziqning Oy o'qi atrofida aylanishidan hosil bo‘lgan sirtning tenglamasini topamiz. Bu sirtda ixtiyoriy $M(x;\ y;\ z)$ nuqtani olamiz va u orqali aylanish o‘qiga perpendikular tekislik o‘tkazamiz. Kesimda markazi aylanish o‘qidagi $N(0;\ y;\ 0)$ nuqtada bo‘lgan aylana hosil bo‘ladi. Bu aylana radiusi $\sqrt{x^2+z^2}$ ga teng. Lekin, ikkinchi tomondan, bu radius berilgan L chiziq $M_1(0;\ y;\ z)$ nuqtasi applikatasining absolut qiymatiga teng. Demak, berilgan tenglamada

$$Y = y, \quad Z = \pm\sqrt{x^2+z^2}$$

(M nuqtaning koordinatalari) deb, izlanayotgan aylanish sirtning ushbu $F(y,\ \pm\sqrt{x^2+z^2}) = 0$ tenglamasini hosil qilamiz.

Shunday qilib, L chiziqning Oy o‘qi atrofida aylanishidan hosil bo‘lgan sirt tenglamasini olish uchun bu chiziq tenglamasida z ni $\pm\sqrt{x^2+z^2}$ ga almashtirish kerak. Shunga o‘xshash qoida chiziqlarning boshqa koordinata o‘qlari atrofida aylanishidan hosil bo‘lgan sirtlar uchun ham o‘rinlidir.

Aylanish sirtlari tenglamalarini quyidagi jadvalda keltiramiz:

Chiziq tenglamasi	*Aylanish o‘qi*	*Aylanish sirti tenglamasi*
$\begin{cases} F(x,\ y) = 0, \\ z = 0. \end{cases}$	Ox Oy	$F(x,\ \sqrt{y^2+z^2}) = 0$ $F(\sqrt{x^2+z^2}, y) = 0$
$\begin{cases} F(x,\ z) = 0, \\ z = 0. \end{cases}$	Ox Oz	$F(x,\ \sqrt{y^2+z^2}) = 0$ $F(\sqrt{x^2+y^2}, z) = 0$
$\begin{cases} F(x,\ z) = 0, \\ z = 0. \end{cases}$	Oy Oz	$F(y,\ \sqrt{x^2+z^2}) = 0$ $F(\sqrt{x^2+y^2}, z) = 0$

5⁰. Konussimon (konik) sirtlar. *Konussimon sirt* deb, *konusning uchi* deb ataladigan berilgan nuqtadan o‘tuvchi va *konusning yo‘naltiruvchisi* deb ataladigan berilgan chiziqni kesuvchi barcha to‘g‘ri chiziqlardan tashkil topgan sirtga aytiladi. Konussimon sirt tashkil etadigan to‘g‘ri chiziqlarning har biri *konusning yasovchisi* deb ataladi.

Konussimon sirtning uchi koordinatalar boshida, yasovchisi $F(x;\ y) = 0$ esa $z = h$ tekislikda bo‘lsin. U holda yasovchi tenglamasi ushbu ko‘rinishda bo‘ladi: $\frac{x}{x_0} = \frac{y}{y_0} = \frac{z}{h}$, bu yerda $(x_0;\ y_0;\ h)$ yo‘naltiruvchi nuqta. Bu yerdan x_0 va y_0 ni topib, $F(x;\ y) = 0$ tenglikka qo‘ysak, uchi koordinatalar boshida bo‘lgan konussimon sirt tengamasini olamiz:

$$F\left(\frac{xh}{z},\ \frac{yh}{z}\right) = 0. \tag{3}$$

Agar konusning uchi $(a;\ b;\ c)$ nuqtada bo‘lsa, u holda uning tenglamasi ushbu ko‘rinishda bo‘ladi:

$$F\left[\frac{(x-a)(h-c)}{z-c} + a,\ \frac{(y-b)(h-c)}{z-c} + b\right] = 0. \tag{4}$$

(3) tenglama x, y va z o‘zgaruvchiga nisbatan bir jinsli, (4) tenglama esa $x - a$, $y - b$, $z - c$ o‘zgaruvchiga nisbatan bir jinsli. Tenglamaning bir jinsliligidan uning konussimon sirt ekanligini bilish mumkin.

Mustaqil bajarish uchun mashqlar

4.1. Markazi koordinatalar boshida bo'lgan va radiusi $R = 5$ bo'lgan sfera teglamasini tuzing.

4.2. Markazi $C(-1;\ 2;\ -3)$ nuqtada va radiusi $R = 3$ ga bo'lgan sfera tenglamasini tuzing.

4.3. Markazi $C(-1;\ -2;\ -4)$ nuqtada va radiusi $R = 6$ bo'lgan sfera tenglamasini tuzing.

4.4. $x^2 + y^2 + z^2 - 6x + 8y + 10z + 25 = 0$ sfera markazining koordinatalarini va radiusini toping.

4.5. $4x^2 + 4y^2 + 4z^2 - 4x + 12y - 16z + 1 = 0$ sfera markazining koordinatalarini va radiusini toping.

4.6. $x^2 + y^2 + z^2 + x - y + z = 0$ sfera markazining koordinatalarini va radiusini toping.

4.7. Tenglamasi bilan berilgan sferaning markazi koordinatalarini va radiusini toping:

1) $(x+1)^2 + (y+2)^2 + z^2 = 25;$
2) $x^2 + y^2 + z^2 - 4x + 6y + 2z + 2 = 0;$
3) $2x^2 + 2y^2 + 2z^2 + 4y - 3z + 2 = 0;$
4) $x^2 + y^2 + z^2 = 2x;$ 5) $x^2 + y^2 + z^2 = 4z - 3.$

4.8. Agar $M(4;\ -1;\ -3)$ va $N(0;\ 3;\ -1)$ nuqtalar sferaning birorta diametrining oxirlari bo'lsa, uning tenglamasini tuzing.

4.9. $\begin{cases} x^2 + y^2 + z^2 = 100, \\ 2x + 2y - z = 18 \end{cases}$ aylananing markazi koordinatalari va radiusini toping.

4.10. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ tenglama qanday sirtni aniqlaydi?

4.11. $y^2 = 2px$ tenglama qanday sirtni aniqlaydi?

4.12. $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$ tenglama qanday sirtni aniqlaydi?

4.13. Quyidagi tenglamalar qanday sirtni aniqlaydi:

1) $x^2 + z^2 = 16;$ 2) $\dfrac{x^2}{6} + \dfrac{z^2}{4} = 1;$
3) $x = 2z^2;$ 4) $\dfrac{z^2}{5} - \dfrac{x^2}{7} = 1.$

4.14. Quyida berilgan tenglamalar qanday sirtni ifodalashini aniqlang va ularni yasang:

1) $x^2 + y^2 = 4;$ 2) $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1;$
3) $x^2 - y^2 = 1;$ 4) $y^2 = 2px;$
5) $z^2 = y;$ 6) $z + x^2 = 0;$
7) $x^2 + y^2 = 2y;$ 8) $x^2 + y^2 = 0;$
9) $x^2 - z^2 = 0;$ 10) $y^2 = xy.$

4.15. $x^2 + y^2 + z^2 - 2ax = 0$ sferaga tashqi chizilgan va yasovchilari: 1) Ox o'qiga, 2) Oy o'qiga, 3) Oz o'qiga parallel bo'lgan silindrik sirt tenglamasini yozing.

4.16. Yo'naltiruvchisi $y^2 = 4x,\ z = 0$ va yasovchisi $\vec{p}\{1;\ 1;\ 1\}$ vektorga parallel bo'lgan silindrik sirt tenglamasini yozing.

4.17. Yo'naltiruvchisi $x^2 + z^2 = 4x,\ z = 0$ va yasovchisi $\vec{p}\{1;\ 2;\ 3\}$ vektorga parallel bo'lgan silindrik sirt tenglamasini yozing.

4.18. $y^2 = 4x,\ z = 0,\ z = 4,\ x = 4$ sirtlar bilan chegaralangan jismni yasang va $x = 4$ tekislikda yotuvchi yog'ining diagonali tenglamasini yozing.

4.19. $\begin{cases} \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1, \\ z = 5 \end{cases}$ tenglamalar sistemasi qanday chiziqni ifodalaydi?

4.20. $\begin{cases} y^2 = z, \\ x = 5 \end{cases}$ tenglamalar sistemasi qanday chiziqni ifodalaydi?

4.21. $\begin{cases} z = x^2 + y^2, \\ z = 9 \end{cases}$ tenglamalar sistemasi bilan qanday chiziqni aniqlasa bo'ladi?

4.22. $x^2 + y^2 = R^2$ aylana Ox o'qi atrofida aylanadi.Aylanish sirti tenglamasini yozing.

4.23. $x = z$ to'g'ri chiziq Oz o'qi atrofida aylanadi. Aylanish sirti tenglamasini yozing.

4.24. $y = z$ to'g'ri chiziq Oy o'qi atrofida aylanadi. Aylanish sirti tenglamasini yozing.

4.25. $y = 3x$ to'g'ri chiziqning Ox o'qi atrofida aylanishdan hosil bo'lgan aylanish sirti tenglamasini yozing.

4.26. $z = x^2$, $y = 0$ chiziqning: 1) Ox o'qi atrofida; 2) Oz o'qi atrofida aylanishidan hosil bo'lgan sirt tenglamalarini yozing.

Bu sirtlarni yasang.

4.27. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipsning: 1) Ox o'qi atrofida; 2) Oy o'qi atrofida aylanishidan hosil bo'lgan aylanish sirti tenglamasini yozing.

4.28. $\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$ ellipsning: 1) Ox o'qi atrofida; 2) Oz o'qi atrofida aylanishidan hosil bo'lgan sirt tenglamasini yozing.

4.29. $\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$ giperbolaning: 1) Ox o'qi atrofida; 2) Oz o'qi atrofida aylanishidan hosil bo'lgan sirt tenglamasini yozing.

4.30. $y^2 = 2pz$ parabola Oz o'qi atrofida aylanadi. Aylanish sirti tenglamasini yozing.

4.31. $y^2 = x$ parabolaning Ox o'qi atrofida aylanishidan hosil bo'lgan sirt tenglamasini yozing.

4.32. Yo'naltiruvchisi $x^2 + y^2 = a^2$, $z = c$ bo'lib, uchi koordinatalar boshida bo'lgan konussimon sirt tenglamasini yozing. Sirt tasvirini yasang.

4.33. Uchi $A(0; -a; 0)$ nuqtada va yo'naltiruvchisi $x^2 = 2py$, $z = h$ bo'lgan konussimon sirt tenglamasini yozing. Sirt tasvirini yasang.

4.34. Yo'naltiruvchisi $z = a$ tekislikda bo'lgan $x^2 + (y - a)^2 - -z^2 = 0$ konusning uchini toping va uni yasang.

4.35. Yo'naltiruvchisi $z = h$ tekislikda bo'lgan $x^2 = 2yz$ konusning uchini toping va uni yasang.

4.36. Uchi $O(0; 0; 0)$ nuqtada, yo'naltiruvchisi $x^2 + (y - 6)^2 + +z^2 = 25$, $y = 3$ bo'lgan konussimon sirt tenglamasini yozing va sirtni chizing.

4.37. Uchi $C(0; -a; 0)$ nuqtada, yo'naltiruvchisi $x^2 + y^2 + +z^2 = 25$, $y = 3$ bo'lgan konussimon sirt tenglamasini yozing va sirtni chizing.

4.38. $z^2 = xy$ konus bilan $x + y = 2a$ tekislikning kesishish chizig'i ellips ekanligini ko'rsating va uning yarim o'qlarini toping.

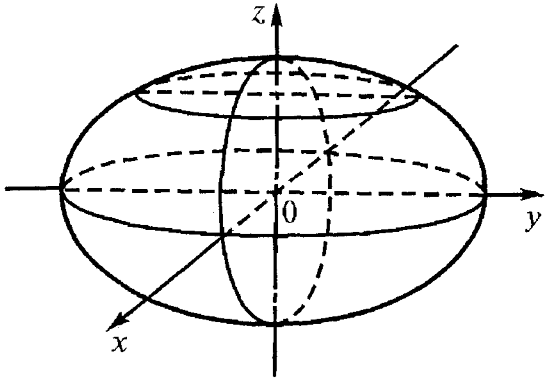
5- §. Asosiy ikkinchi tartibli sirtlar tenglamalarining kanonik shakli

Asosiy ikkinchi tartibli sirtlar tenglamalarining kanonik shakllarini qaraymiz. Bu sirtlarning xususiyati shundaki, koordinata o'qlari ular uchun simmetriya o'qlari bo'ladi, ularning uchi yoki simmetriya markazi esa koordinatalar boshi bilan ustma-ust tushadi.

1°. Ellipsoid. Kanonik tenglamasi

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

ko'rinishda bo'lgan ikkinchi tartibli sirt *ellipsoid* deb ataladi, bu yerda a, b, c — berilgan o'zgarmas musbat sonlar bo'lib, ular *ellipsoidning yarim o'qlari* deb ataladi. Agar a, b, c sonlar orasida tenglari bo'lmasa, ellipsoid *uch yoqli ellipsoid* deb ataladi.



40- rasm.

Agar a, b, c sonlar orasida qandaydir ikkitasi o'zaro teng bo'lsa, u holda *aylanish ellipsoidiga* ega bo'lamiz. Ellipsoidning $z=0$, $y=0$, $x=0$, ya'ni xOy, xOz yOz koordinata tekisliklari bilan kesimlari ellipslardan iborat (40- rasm).

2⁰. Bir pallali giperboloid. Kanonik tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

bo'lgan sirt *bir pallali giperboloid* deb ataladi, bu yerda a, b, c — berilgan musbat sonlar.

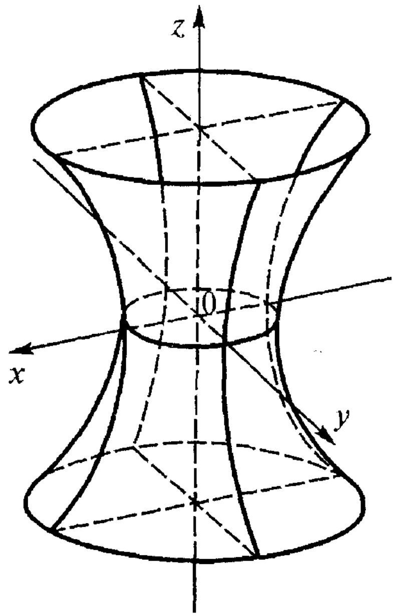
Giperboloidning koordinata tekisliklari bilan kesishishi natijasida quyidagi chiziqlar hosil bo'ladi (41- rasm):

1) $xOy(z=0)$ tekislik bilan: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellips;

2) $xOz(y=0)$ tekislik bilan: $\frac{x^2}{a^2}-\frac{z^2}{c^2}=1$ giperbola;

3) $yOz(x=0)$ tekislik bilan: $\frac{x^2}{a^2}-\frac{z^2}{c^2}=1$ giperbola.

Berilgan giperboloidning Oxy koordinata tekisligiga parallel $z=h$ tekislik bilan kesimida ellips hosil bo'ladi. $a=b$ da *bir pallali aylanma giperboloid* hosil bo'ladi:



41- rasm.

$$\frac{x^2+y^2}{a^2}-\frac{z^2}{c^2}=1.$$

3⁰. Ikki pallali giperboloid. Kanonik tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

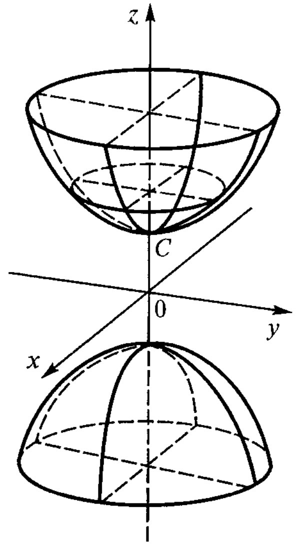
bo'lgan ikkinchi tartibli sirt *ikki pallali giperboloid* deb ataladi, bu yerda a, b, c — berilgan o'zgarmas musbat sonlar.

Ikki pallali giperboloid xOy tekislik bilan kesishmaydi. Giperboloid bilan xOz va yOz teki sliklar kesishuvidan, mos ravishda,

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}=-1 \text{ va } \frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

giperbolalar hosil bo'ladi (42- rasm). $a=b$ da *ikki pallali aylanma giperboloid* hosil bo'ladi:

$$\frac{x^2+y^2}{a^2}-\frac{z^2}{c^2}=-1.$$



42- rasm.

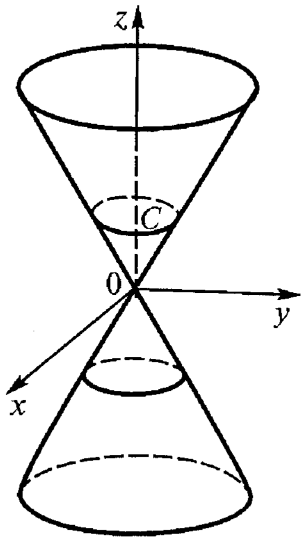
4⁰. Ikkinchi tartibli konus. Kanonik tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

bo'lgan ikkinchi tartibli sirt *konus* deb ataladi. Bu konusning uchi koordinatalar boshida joylashgan bo'lib, u uchining ikki tomonida joylashgan ikki qismdan iborat bo'ladi. Bu konusning yo'naltiruvchilaridan biri (43- rasm)

$$\begin{cases}\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\\ z=c\end{cases}$$

ellipsdan iborat bo'ladi.



43- rasm.

5⁰. Elliptik paraboloid. Kanonik tenglamasi

$$\frac{x^2}{p}+\frac{y^2}{q}=2z$$

bo'lgan ikkinchi tartibli sirt *elliptik paraboloid* deb ataladi, bu yerda p va q bir xil ishorali berilgan sonlar. (Masalan $p>0,\ q>0$). Buning o'qi Oz o'qidan iborat. Xuddi shunday,

$$\frac{x^2}{2p}+\frac{z^2}{2q}=y$$

elliptik paraboloidning o'qi Oy o'qi;

$$\frac{y^2}{2q}+\frac{z^2}{2p}=x$$

elliptik paraboloidning o'qi Ox o'qi bo'ladi.

Elliptik paraboloidning kanonik tenglamasida:

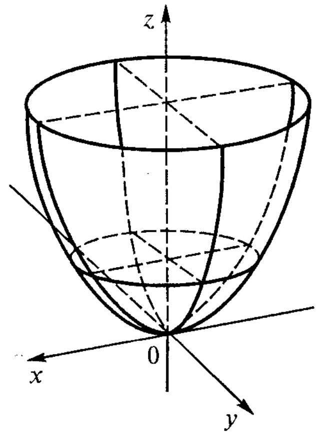
$x=0$ bo'lsa, $y^2=2qz$ parabola;

$y=0$ bo'lsa, $x^2=2pz$ parabola,

$z=h$ bo'lsa, $\frac{x^2}{2ph}+\frac{y^2}{2qh}=1$ ellips

hosil bo'ladi.

$p=q$ bo'lsa, $z=h,\ h>0$ tekislikdagi kesimi markazi Oz o'qidan iborat bo'lgan aylanadan iborat bo'ladi (44- rasm).



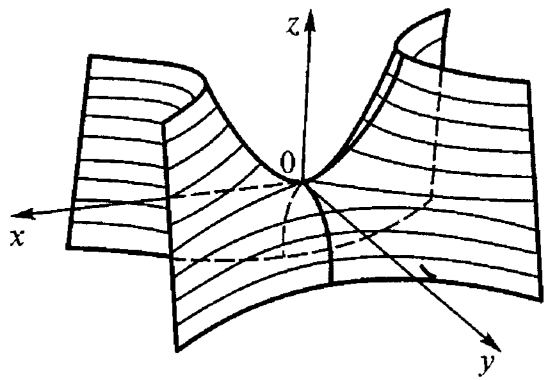
44- rasm.

6⁰. Giperbolik paraboloid. Kanonik tenglamasi

$$\frac{x^2}{p}-\frac{y^2}{q}=2z$$

bo'lgan ikkinchi tartibli sirt *giperbolik paraboloid* deb ataladi, bu yerda p va q bir xil ishorali berilgan sonlar. (Masalan $p>0,\ q>0.$)

Giperbolik paraboloidning $y=0$ tekislik bilan kesimida (45- rasm)



45- rasm.

$$\left.\begin{aligned}\frac{x^2}{p}-\frac{y^2}{q}&=2z,\\ y&=0\end{aligned}\right\}\ \text{yoki}\ \left.\begin{aligned}x^2&=2pz,\\ y&=0\end{aligned}\right\};$$

$x = 0$ tekislik bilan kesimida

$$\left.\begin{aligned}\frac{x^2}{p}-\frac{y^2}{q}&=2z,\\ x&=0\end{aligned}\right\}\ \text{yoki}\ \left.\begin{aligned}y^2&=-2qz,\\ x&=0\end{aligned}\right\}$$

parabolalar hosil bo'ladi.

Giperbolik paraboloidning $z = h$ tekislik bilan kesimida

$$\left.\begin{aligned}\frac{x^2}{p}-\frac{y^2}{q}&=2z,\\ z&=h\end{aligned}\right\}\ \text{yoki}\ \left.\begin{aligned}\frac{x^2}{p}-\frac{y^2}{q}&=2h,\\ z&=h\end{aligned}\right\}$$

chiziqlar hosil bo'ladi.

Agar $h > 0$ bo'lsa, u holda markazi (0; 0; h) nuqtada va haqiqiy o'qi Ox o'qiga parallel bo'lgan giperbola hosil bo'ladi. $h = 0$ bo'lsa, kesimda giperbolik paraboloidning to'g'ri chiziqli yasovchisi deb ataluvchi to'g'ri chiziqlar hosil bo'ladi:

$$\left.\begin{aligned}\frac{x^2}{p}-\frac{y^2}{q}&=0,\\ z&=0\end{aligned}\right\},\quad \left.\begin{aligned}\frac{x}{\sqrt{p}}+\frac{y}{\sqrt{q}}&=0,\\ z&=0\end{aligned}\right\}$$

yoki

$$\left.\begin{aligned}\frac{x}{\sqrt{p}}-\frac{y}{\sqrt{q}}&=0,\\ z&=0\end{aligned}\right\}\quad \left.\begin{aligned}\frac{x}{\sqrt{p}}+\frac{y}{\sqrt{q}}&=0,\\ z&=0.\end{aligned}\right\}.$$

Agar $h < 0$ bo'lsa, kesimda haqiqiy o'qi Oy o'qiga parallel bo'lgan giperbola hosil bo'ladi. Giperbolik paraboloidning yOz tekislikka parallel kesimini topamiz.

$x = h$ tekislik bilan kesimida

$$\left.\begin{aligned}\frac{x^2}{p}-\frac{y^2}{q}&=2z,\\ x&=h\end{aligned}\right\}\ \text{yoki}\ \left.\begin{aligned}y^2&=-2q\left(z-\frac{h^2}{2p}\right),\\ x&=h\end{aligned}\right\},$$

— uchi $\left(h;\ 0;\ \frac{h^2}{2p}\right)$ nuqtada, simmetriya o'qi Oz o'qiga parallel bo'lgan parabola hosil bo'ladi. Parabolaning tarmoqlari pastga yo'nalgan.

Qolgan tekisliklarga parallel kesimlari ham xuddi shunday parabolalar bo'ladi.

Mustaqil bajarish uchun mashqlar

5.1. $\frac{x^2}{a^2}+\frac{z^2}{c^2}=1,\ \ y=0$ ellipsning o'qi atrofida aylanishidan hosil bo'lgan sirt tenglamasini yozing.

5.2. $\frac{x^2}{a^2}-\frac{z^2}{c^2}=1,\ \ y=0$ chiziqning: 1) Oz o'qi atrofida; 2) Ox o'qi atrofida aylanishidan hosil bo'lgan sirt tenglamasini yozing. Sirtni yasang.

5.3. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ giperboloid bilan koordinata tekisliklarining va $z = 2$, $x = 3$ tekisliklarning kesishish chiziqlarini toping.

5.4. Quyidagilar qanday sirt tenglamalari:

1) $\frac{x^2+z^2}{6}-\frac{y^2}{15}=-1;$

2) $\frac{x^2}{6}-\frac{y^2}{5}+\frac{z^2}{1}-1=0;$

3) $-x^2+\frac{y^2}{5}+\frac{z^2}{7}=0;$

4) $z=-(x^2+y^2)$; 5) $z=1-x^2-y^2$?

5.5. Quyidagi tenglamalar qanday sirtni ifodalaydi:

1) $2x^2-5y^2-8=0;$

2) $4x^2-8y^2+16z^2=0;$

3) $8x^2-4y^2+24z^2-48=0;$

4) $y^2=6x-4;$

5) $2x^2 - y^2 - z^2 = 0;$

6) $3x^2 + 5z^2 = 12z;$

7) $x^2 + 4y^2 - 8 = 0;$

8) $z^2 - 4x = 0;$

9) $2x^2 - 3z^2 = -12y;$

10) $4x^2 - 12y^2 - 6z^2 = 12$?

5.6. Sirtni yasang:

1) $x^2 + y^2 - z^2 = 4;$

2) $x^2 - y^2 + z^2 + 4 = 0.$

5.7. $\frac{x^2}{16} + \frac{y^2}{4} - \frac{z^2}{36} = 1$ giperboloidni yasang va uning (4; 1; −3) nuqtadan o'tuvchi yasovchisini toping.

5.8. Sirtni yasang:

1) $2z = x^2 + \frac{y^2}{2}$; 2) $z = c\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)$.

5.9. $x^2 - y^2 = 4z$ sirtni yasang va uning (3; 1; 2) nuqtadan o'tuvchi yasovchisini toping.

5.10. Har bir nuqtasidan $x = a$ tekislikkacha bo'lgan masofaning $F(a; 0; 0)$ nuqtagacha bo'lgan masofaga nisbati 2 ga teng bo'lgan nuqtalarning geometrik o'rni tenglamasini yozing. Sirtni yasang.

5.11. $F(-a; 0; 0)$ nuqtadan va $x = a$ tekislikdan bir xil uzoqlikda joylashgan nuqtalarning geometrik o'rni tenglamasini yozing. Sirtni yasang.

5.12. $\frac{x^2}{169} + \frac{y^2}{25} + \frac{z^2}{9} = 1$ ellipsoidning eng katta doiraviy kesimini toping.

Mustaqil bajarish uchun berilgan mashqlarning javoblari

1- §. 1.1. $x + 3z + 4 = 0$. **1.2.** $z + 4 = 0$. **1.3.** $x - y = 0$. **1.4.** $y + 3 = 0$. **1.5.** $3y + 2x = 0$. **1.6.** $2x + y = 0$. **1.7.** $\frac{x}{a} + \frac{z}{c} = 1$ **1.8.** $2x + y = 0$. **1.9.** $2x - z = 0$. **1.10.** $a = 12$, $b = -\frac{6}{5}$, $c = -6$. **1.11.** $\frac{x}{-12} + \frac{y}{-8} + \frac{z}{6} = 1$. **1.12.** $\frac{x}{8} + \frac{y}{-6} + \frac{z}{4{,}8} = 1$. **1.13.** $-\frac{2}{11}x - \frac{9}{11}y + \frac{6}{11}z - 3 = 0$. **1.14.** 1) $\frac{2}{11}x - \frac{9}{11}y + \frac{6}{11}z - 2 = 0$. 2) $-\frac{2}{3}x - \frac{2}{15}y + \frac{7}{11}z - 3 = 0$. 3) $-\frac{6}{11}x + \frac{6}{11}y + \frac{7}{11}z - 3 = 0$. **1.15.** $\frac{3}{\sqrt{50}}x - \frac{4}{\sqrt{50}}y + \frac{5}{\sqrt{50}}z - \frac{14}{\sqrt{50}} = 0$. **1.16.** $p = \frac{12}{\sqrt{35}}$, $\cos\alpha = -\frac{5}{\sqrt{35}}$, $\cos\beta = \frac{1}{\sqrt{35}}$, $\cos\gamma = -\frac{3}{\sqrt{35}}$. **1.17.** $d = 4$. **1.18.** $d = 2$. **1.19.** $d = 4$. **1.20.** $d = \frac{5}{3}\sqrt{2}$. **1.21.** $d = 6$. **1.22.** $d = 2\sqrt{2}$. **1.23.** $x - 2y + 2z = 1$, $x - 2y + 2z = -1$. **1.24.** $3x + 4y - z + 18 = 0$. **1.25.** $x + 3y - 4z - 21 = 0$. **1.26.** $7x - 4y + z - 21 = 0$. **1.27.** $2x - 2y - 3z + 11 = 0$. **1.28.** $x - 3y - 2z + 1 = 0$. **1.29.** $2x + 3y + 4z = 3$. **1.30.** $2x + y + z = a$. **1.31.** $\cos\varphi = 0{,}9046$; $\varphi = 25°14'$. **1.32.** 1) $\varphi = \arccos 0{,}7$. 2) 1. 3) 11. **1.33.** $x - y + 1 = 0$. **1.34.** $15x - 5y - 4z - 14 = 0$. **1.35.** $5x - 3y - 4z - 1 = 0$. **1.36.** $d = \sqrt{6}$.

2- §. 2.1. 1) $\alpha = 73°24'$, $\beta = 64°37'$, $\gamma = 31°1'$. 2) $\cos\alpha = \frac{12}{25}$, $\cos\beta = \frac{9}{25}$, $\cos\gamma = \frac{20}{25}$. **2.2.** $x - 1 = \frac{y+5}{\sqrt{2}} = -(z-3)$. **2.3.** 1) $\frac{x-\frac{7}{3}}{-\frac{3}{5}} = \frac{y+\frac{1}{5}}{\frac{5}{5}} = \frac{z-0}{1}$ – *kanonik*, $\begin{cases} 5x + 3y - 7 = 0 \\ 4z - 5y - 1 = 0. \end{cases}$ *proyeksiya.*

2) $\frac{x-\frac{1}{5}}{\frac{1}{5}} = \frac{y-\frac{12}{5}}{\frac{7}{5}} = \frac{z-0}{1}$ – *kanonik*, $\begin{cases} 5x - z - 1 = 0 \\ 7z - 5y + 12 = 0. \end{cases}$ *proyeksiya.*

2.4. $\frac{x-4}{-1} = \frac{y-3}{1} = \frac{z}{1}$. **2.5.** $P\{0;\ 0;\ 1\}$. **2.6.** 1) $p = i$. 2) $p + i + k$. 3) $p = j + k$. **2.7.** $\frac{x}{9} = \frac{y}{5} = \frac{z+3}{1}$. **2.8.** $\cos\alpha = \frac{6}{11}$, $\cos\beta = \frac{7}{11}$, $\cos\gamma = \frac{6}{11}$. **2.9.** 1) $\frac{x-2}{2} = \frac{y}{-2} = \frac{z+3}{5}$. 2) $\frac{x-2}{5} = \frac{y}{2} = \frac{z+3}{-1}$. 3) $\frac{x-2}{-1} = \frac{y}{0} = \frac{z+3}{0}$. 4) $\frac{x-2}{0} = \frac{y}{0} = \frac{z+3}{1}$. 5) $\frac{x-2}{1} = \frac{y}{2} = \frac{z+3}{-\frac{1}{2}}$. **2.10.** 1) $\begin{cases} x - 2 = 0, \\ y + 5 = 0. \end{cases}$ 2) $\frac{x-2}{4} = \frac{y+5}{-6} = \frac{z-3}{9}$. 3) $\frac{x-2}{-11} = \frac{y+5}{17} = \frac{z-3}{13}$.

2.11. 1) *kesishadi.* 2) *kesishadi.* **2.12.** $\frac{x-2}{3} = \frac{y-3}{3} = \frac{z-1}{-1}$. **2.13.** (1; −5; 0), $\left(\frac{7}{4};\ 0;\ 10\right)$, (0; −7; −4). **2.14.** $\cos\varphi = \frac{72}{77}$. **2.15.** 1) $\cos\varphi = 0{,}9445$; $\varphi = 19°11'$. 2) $\cos\varphi = \frac{11}{26}$; 3) $\cos\varphi = \frac{98}{195}$. **2.16.** $x - 3 = 0$, $y + 1 = 0$. **2.17.** $\frac{x-1}{1} =$

$= \frac{y+1}{3} = \frac{z-2}{2}$. **2.18.** $\frac{x+1}{3} = \frac{y-2}{4} = \frac{z-3}{-5}$. **2.19.** $\varphi = 24°5'$. **2.20.** *t vaqt o'tgandan so'ng M nuqtasining koordinatalari* $x = 4 + 2t,\ y = -3 + 2t,\ z = 1 - t$; $\frac{x-4}{2} = \frac{y+3}{3} = \frac{z-1}{1}$. **2.21.** 1) $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-1}{-2}$. 2) $\frac{x-3}{-2} = \frac{y+1}{1} = \frac{z}{-3}$. **2.22.** 1) $x = -2 + t,\ y = 1 - 2t,\ z = -1 + 3t$. 2) $x = 1 + t,\ y = 1 - t,\ z = 2 + t$; **2.23.** $\cos\varphi = \frac{1}{\sqrt{3}}$. **2.24.** $p = N_1 \times N_2 = \vec{i} = 3\vec{j} = 5\vec{k}$ *yo'naltiruvchi vektorlar.* $\frac{x+4}{1} = \frac{y-3}{3} = \frac{z}{5}$. **2.27.** $0,3\sqrt{38}$.

3- §. 3.1. $\varphi = 24°5'$. **3.2.** $\sin\theta = \frac{1}{\sqrt{6}}$. **3.4.** $x - 3y + 4z + 9 = 0$. **3.5.** $y + z + 1 = 0$. **3.6.** $5x - 10y - 9z - 68 = 0$. **3.7.** $\frac{x-2}{1} = \frac{y-1}{-4} = \frac{z-6}{5}$; $\cos\alpha = \pm\frac{1}{\sqrt{42}}$, $\cos\beta = \mp\frac{4}{\sqrt{42}}$, $\cos\gamma = \pm\frac{5}{\sqrt{42}}$. **3.8.** $\frac{x-1}{3} = \frac{y+1}{-1} = \frac{z-2}{-5}$. **3.9.** $(-2;\ 0;\ 3)$. **3.10.** 1) *To'g'ri chiziq va tekislik parallel.* 2) *Kesishish nuqtasi aniqlanmagan. To'g'ri chiziq tekislikda yotmaydi.* **3.11.** $\frac{x-2}{1} = \frac{y+1}{3} = \frac{z-3}{-4}$. **3.12.** $x - 2y + z + 5 = 0$. **3.13.** $8x - 5y + z - 11 = 0$. **3.14.** $x + 2y - 2z = 1$. **3.15.** $\frac{x-3}{5} = \frac{y+1}{-7} = \frac{z-2}{1}$. **3.16.** $A = \frac{27}{4}$. **3.17.** $A = 4,\ B = -8$. **3.18.** $\frac{x-3}{5} = \frac{y+2}{3} = \frac{z-4}{-7}$. **3.19.** $4x + 5y - 2z = 0$. **3.20.** $x - 7y + 17z - 9 = 0$. **3.21.** $2x + y - z - 5 = 0$. **3.22.** $4x + 2y - 5z = 0$. **3.23.** $7x - 4y + 7z + 49 = 0$. **3.24.** $11x - 17y - 19z + 10 = 0$. **3.25.** $4x - 3y + 2z + 26 = 0$. **3.26.** $19x - 14y + z + 23 = 0$. **3.27.** $4x + 13y - z - 5 = 0$. **3.28.** $\frac{x+9}{7} = \frac{y+1}{4} = \frac{z}{-1}$. **3.29.** $17x - 13y - 16z - 10 = 0$. **3.30.** $16x - 27y + 14z - 159 = 0$. **3.31.** $23x - 16y + 10z - 153 = 0$. **3.32.** $x + y - z + 3 = 0$. **3.33.** $d = \sqrt{22}$.

4- §. 4.1. $x^2 + y^2 + z^2 = 25$. **4.2.** $x^2 + y^2 + z^2 + 2x - 4y + 6z + 5 = 0$. **4.3.** $x^2 + y^2 + z^2 + 2x + 4y + 8z - 15 = 0$. **4.4.** $C(3;\ -4;\ -5),\ R = 5$. **4.5.** $C\left(\frac{1}{2};\ -\frac{3}{2};\ 2\right),\ R = \frac{5}{2}$. **4.6.** $C\left(-\frac{1}{2};\ \frac{1}{2};\ -\frac{1}{2}\right), R = \frac{\sqrt{3}}{2}$. **4.7.** 1) $C(-1;\ -2;\ 0)$, 2) $C(2;\ -3;\ -1),\ R = 4$. 3) $C(0;\ -1;\ 3),\ R = \frac{3}{4}$. 4) $C(1;\ 0;\ 3;),\ R = 1$. 5) $C(0;\ 0;\ 2),\ R = 1$. **4.8.** $(x-2)^2 + (y-1)^2 + (z+2)^2 = 9$. **4.9.** $C(4;\ 4;\ -2)$, $R = 8$. **4.15.** 1) $x^2 + y^2 = 2ax$. 2) $x^2 + z^2 = 2ax$. 3) $y^2 + z^2 = a^2$. **4.16.** $(3y - 2z)^2 = 12(3x - z)$. **4.17.** $(x - z)^2 + (y - z)^2 = 4(x - z)$. **4.18.** $x = 4,\ z \pm y = 2$. **4.22.** $x^2 + y^2 + z^2 = R^2$ (*Sfera*). **4.23.** $x^2 + y^2 - z^2 = 0$ (*Konus*). **4.24.** $x^2 + y^2 - z^2 = 0$. **4.25.** $y^2 + z^2 - 9x^2 = 0$. **4.26.**

1) $z = x^2 + y^2$, 2) $\sqrt{y^2 + z^2} = 16y^2$. **4.27.** 1) $\frac{x^2}{a^2} + \frac{y^2+z^2}{b^2} = 1$. 2) **4.28.** 1) $\frac{x^2}{a^2} + \frac{z^2+y^2}{c^2} = 1$. (*Aylanma ellipsoid*). 2) $\frac{x^2+y^2}{a^2} + \frac{z^2}{c^2} = 1$. (*Aylanma ellipsoid*). **4.29.** 1) $\frac{y^2+z^2}{c^2} - \frac{x^2}{a^2} = 1$ (*Ikki pallali giperboloid*). 2) $\frac{x^2+y^2}{a^2} - \frac{z^2}{c^2} = 1$. (*Bir pallali giperboloid*). **4.30.** $z = a(x^2 + y^2)$, $\frac{1}{2p} = a$. **4.31.** $x = y^2 + z^2$ (*Aylanma paraboloid*). **4.32.** $\frac{x^2+y^2}{a^2} = \frac{z^2}{c^2}$. **4.33.** $h^2x^2 = 2pz\left[h(h + a) - az\right]$. **4.34.** $A(0;\ a;\ 0),\ z = a,\ x^2 + (y - a)^2 = a^2$. **4.36.** $9(x^2 + z^2) = 16y^2$. **4.37.** $x^2 + z^2 = z(y + a)$. **4.38.** *Ox va Oy o'qlarni Oz o'qi atrofida 45° ga burib,* $2z^2 = x^2 - y^2$ *sirt va* $x = a\sqrt{2}$ *tekislik tenglamasini olamiz. Bu yerda kesim: yarim o'qlari* $a\sqrt{2}$ *va* a *lardan iborat bo'lgan ellips:* $x = a\sqrt{2},\ \frac{y^2}{2a^2} + \frac{z^2}{a^2} = 1$.

5- §. 5.1. $\frac{x^2+y^2}{a^2} + \frac{z^2}{c^2} = 1$. **5.2.** 1) $\frac{x^2+y^2}{a^2} - \frac{z^2}{c^2} = 1$ (*bir pallali giperboloid*) 2) $\frac{x^2}{a^2} - \frac{y^2+z^2}{c^2} = 1$ (*Ikki pallali giperboloid*). **5.10.** $\frac{x^2}{2a^2} + \frac{y^2+z^2}{a^2} = 1$. **5.11.** $x = -\frac{z^2+y^2}{4a}$. **5.12.** $9x = \pm 13z$.

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