

## TRIGONOMETRIK FORMULALAR

#	FORMULA
1	$\sin^2 x + \cos^2 x = 1$
2	$\sin^2 x = 1 - \cos^2 x$
3	$\sin x = \pm \sqrt{1 - \cos^2 x} = \frac{\operatorname{tg} x}{\pm \sqrt{1 + \operatorname{tg}^2 x}} = \frac{1}{\pm \sqrt{1 + \operatorname{ctg}^2 x}}$
4	$\cos^2 x = 1 - \sin^2 x$
5	$\cos x = \pm \sqrt{1 - \sin^2 x} = \frac{\operatorname{ctg} x}{\pm \sqrt{1 + \operatorname{ctg}^2 x}} = \frac{1}{\pm \sqrt{1 + \operatorname{tg}^2 x}}$
6	$\cos^2 x - \sin^2 x = \cos 2x$
7	$1 - \cos 2x = 2\sin^2 x$
8	$\cos 2x = 1 - 2\sin^2 x$
9	$\sin^2 x = \frac{1 - \cos 2x}{2}$
10	$\sin x = \pm \sqrt{\frac{1 - \cos 2x}{2}}$
11	$1 + \cos 2x = 2\cos^2 x$
12	$\cos 2x = 2\cos^2 x - 1$
13	$\cos^2 x = \frac{1 + \cos 2x}{2}$
14	$\cos x = \pm \sqrt{\frac{1 + \cos 2x}{2}}$
15	$\sin^4 x = 1 - 2\cos^2 x + \cos^4 x$
16	$\cos^4 x = 1 - 2\sin^2 x + \sin^4 x$
17	$\cos^4 x + \sin^4 x = 1 - 2\sin^2 x \cos^2 x$
18	$\cos^4 x + \sin^4 x = 1 - \frac{1}{2}\sin^2 2x$
19	$\cos^4 x - \sin^4 x = \cos 2x$
20	$\cos^6 x + \sin^6 x = 1 - 3\sin^2 x \cos^2 x$
21	$\cos^6 x + \sin^6 x = 1 - \frac{3}{4}\sin^2 2x$
22	$\cos^6 x - \sin^6 x = \cos 2x(1 - \frac{1}{4}\sin^2 2x)$

23	$\cos^8 x + \sin^8 x = (1 - \frac{1}{2}\sin^2 2x)^2 - \frac{1}{8}\sin^4 2x$
24	$\cos^8 x - \sin^8 x = \cos 2x(1 - \frac{1}{2}\sin^2 2x)$
25	$\sin(x + y) = \sin x \cdot \cos y + \sin y \cdot \cos x$
26	$\sin(x - y) = \sin x \cdot \cos y - \sin y \cdot \cos x$
27	$\cos(x + y) = \cos x \cdot \cos y - \sin x \cdot \sin y$
28	$\cos(x - y) = \cos x \cdot \cos y + \sin x \cdot \sin y$
29	$\sin^2 x - \sin^2 y = \sin(x + y) \sin(x - y)$
30	$\cos^2 x - \cos^2 y = -\sin(x + y) \sin(x - y)$
31	$\sin x + \cos x = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$
32	$\sin x + \cos x = \sqrt{2} \cos\left(x - \frac{\pi}{4}\right)$
33	$\cos x - \sin x = \sqrt{2} \sin\left(\frac{\pi}{4} - x\right)$
34	$\cos x - \sin x = \sqrt{2} \cos\left(\frac{\pi}{4} + x\right)$
35	$\sin x + \sqrt{3} \cos x = 2 \sin\left(x + \frac{\pi}{3}\right)$
36	$\sin x + \sqrt{3} \cos x = 2 \cos\left(x - \frac{\pi}{6}\right)$
37	$\cos x + \sqrt{3} \sin x = 2 \sin\left(x + \frac{\pi}{6}\right)$
38	$\cos x + \sqrt{3} \sin x = 2 \cos\left(x - \frac{\pi}{3}\right)$
39	$\sin x - \sqrt{3} \cos x = 2 \sin\left(x - \frac{\pi}{3}\right)$
40	$\sin x - \sqrt{3} \cos x = -2 \cos\left(x + \frac{\pi}{6}\right)$
41	$\sqrt{3} \sin x - \cos x = 2 \sin\left(x - \frac{\pi}{6}\right)$
42	$\sqrt{3} \sin x - \cos x = -2 \cos\left(x + \frac{\pi}{3}\right)$
43	$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$
44	$\sin x - \sin y = 2 \sin \frac{x-y}{2} \cos \frac{x+y}{2}$
45	$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$
46	$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$

## TRIGONOMETRIYA

47	$\sin x \cdot \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$	71	$\cos^2 nx = \frac{1 + \cos 2nx}{2}$
48	$\cos x \cdot \cos y = \frac{1}{2}(\cos(x - y) + \cos(x + y))$	72	$\cos^2 x = \frac{1 + \cos 2x}{2}$
49	$\sin x \cdot \cos y = \frac{1}{2}(\sin(x - y) + \sin(x + y))$	73	$\cos^2 2x = \frac{1 + \cos 4x}{2}$
50	$\sin 2x = 2\sin x \cdot \cos x$	74	$\cos^2 3x = \frac{1 + \cos 6x}{2}$
51	$\sin x \cdot \cos x = \frac{1}{2}\sin 2x$	75	$\cos^2 4x = \frac{1 + \cos 8x}{2}$
52	$(\sin x + \cos x)^2 = 1 + \sin 2x$	76	$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$
53	$(\sin x - \cos x)^2 = 1 - \sin 2x$	77	$\cos^2 \frac{x}{4} = \frac{1 + \cos \frac{x}{2}}{2}$
54	$\sin 3x = 3\sin x - 4\sin^3 x$	78	$\cos^2 \frac{x}{6} = \frac{1 + \cos \frac{x}{3}}{2}$
55	$\sin 4x = 4\sin x \cos x - 8\sin^3 x \cos x$	79	$\cos^2 \frac{x}{8} = \frac{1 + \cos \frac{x}{4}}{2}$
56	$\sin 5x = 5\sin x - 20\sin^3 x + 16\sin^5 x$	80	$x + y + z = 180^\circ$ $\sin 2x + \sin 2y + \sin 2z = 4\sin x \sin y \sin z$
57	$\cos 3x = 4\cos^3 x - 3\cos x$	81	$\cos x \cdot \cos \frac{x}{2} \cdot \cos \frac{x}{2^2} \cdots \cos \frac{x}{2^n} = \frac{\sin 2x}{2^{n+1} \sin \frac{x}{2^n}}$
58	$\cos 4x = 8\cos^4 x - 8\cos^2 x + 1$	82	$\sin^2 x + \sin^2 2x + \dots + \sin^2 nx = \frac{n}{2} - \frac{\sin nx \cos(n+1)x}{2 \sin x}$
59	$\cos 5x = 16\cos^5 x - 20\cos^3 x + 5\cos x$	83	$\sin x \sin(60^\circ - x) \sin(60^\circ + x) = \frac{1}{4} \sin 3x$
60	$\cos x = \sin y \text{ agar } x + y = 90^\circ$	84	$\cos x \cos(60^\circ - x) \cos(60^\circ + x) = \frac{1}{4} \cos 3x$
61	$\sin x = \sin y \text{ agar } x + y = 180^\circ$	85	$\sin x + \sin 2x + \sin 3x + \dots + \sin nx = \frac{\sin \frac{nx}{2} \sin \frac{n+1}{2} x}{\sin \frac{x}{2}}$
62	$\sin^2 nx = \frac{1 - \cos 2nx}{2}$	86	$\cos x + \cos 2x + \cos 3x + \dots + \cos nx = \frac{\sin \frac{nx}{2} \cos \frac{n+1}{2} x}{\sin \frac{x}{2}}$
63	$\sin^2 x = \frac{1 - \cos 2x}{2}$	87	$a \sin x + b \cos x = \sqrt{a^2 + b^2} \sin(x + \arctg \frac{b}{a})$
64	$\sin^2 2x = \frac{1 - \cos 4x}{2}$	88	$a \sin x + b \cos x = \sqrt{a^2 + b^2} \cos(x - \arctg \frac{a}{b})$
65	$\sin^2 3x = \frac{1 - \cos 6x}{2}$	89	$\operatorname{tg} x = \frac{\sin x}{\cos x}$
66	$\sin^2 4x = \frac{1 - \cos 8x}{2}$	90	$\operatorname{ctg} x = \frac{\cos x}{\sin x}$
67	$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$	91	$\operatorname{tg} x \cdot \operatorname{ctg} x = 1$
68	$\sin^2 \frac{x}{4} = \frac{1 - \cos \frac{x}{2}}{2}$	92	$1 + \operatorname{tg}^2 x = \frac{1}{\cos^2 x}$
69	$\sin^2 \frac{x}{6} = \frac{1 - \cos \frac{x}{3}}{2}$	93	$1 + \operatorname{ctg}^2 x = \frac{1}{\sin^2 x}$
70	$\sin^2 \frac{x}{8} = \frac{1 - \cos \frac{x}{4}}{2}$		

## TRIGONOMETRIYA

94	$\operatorname{tg}^2 x - \operatorname{tg}^2 y = \frac{\sin(x+y)\sin(x-y)}{\cos^2 x \cos^2 y}$	117	$\cos 2x = \frac{1 - \operatorname{tg}^2 x}{1 + \operatorname{tg}^2 x}$
95	$\operatorname{ctg}^2 x - \operatorname{ctg}^2 y = \frac{\sin(x+y)\sin(y-x)}{\sin^2 x \sin^2 y}$	118	$\operatorname{tg} \frac{x}{2} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$
96	$\operatorname{tg}^2 x - \sin^2 x = \operatorname{tg}^2 x \sin^2 x$	119	$\operatorname{ctg} \frac{x}{2} = \frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$
97	$\operatorname{ctg}^2 x - \cos^2 x = \operatorname{ctg}^2 x \cos^2 x$	120	$\operatorname{tg}^2 \frac{x}{2} = \frac{1 - \cos x}{1 + \cos x}$
98	$\operatorname{tg}(x+y) = \frac{\operatorname{tg} x + \operatorname{tg} y}{1 - \operatorname{tg} x \operatorname{tg} y}$	121	$\operatorname{ctg}^2 \frac{x}{2} = \frac{1 + \cos x}{1 - \cos x}$
99	$\operatorname{tg}(x-y) = \frac{\operatorname{tg} x - \operatorname{tg} y}{1 + \operatorname{tg} x \operatorname{tg} y}$	122	$\operatorname{tg} x \operatorname{tg}(60^\circ - x) \operatorname{tg}(60^\circ + x) = \operatorname{tg} 3x$
100	$\operatorname{tg} x + \operatorname{tg} y = \frac{\sin(x+y)}{\cos x \cos y}$	123	$\operatorname{ctg} x \operatorname{ctg}(60^\circ - x) \operatorname{ctg}(60^\circ + x) = \operatorname{ctg} 3x$
101	$\operatorname{tg} x - \operatorname{tg} y = \frac{\sin(x-y)}{\cos x \cos y}$	124	$(1 + \operatorname{tg} x)(1 + \operatorname{tg} y) = 2 \text{ agar } x + y = 45^\circ$
102	$\operatorname{ctg} x + \operatorname{ctg} y = \frac{\sin(x+y)}{\sin x \sin y}$		
103	$\operatorname{ctg} x - \operatorname{ctg} y = -\frac{\sin(x-y)}{\sin x \sin y}$		
104	$\operatorname{tg} x + \operatorname{ctg} y = \frac{\cos(x-y)}{\cos x \sin y}$		
105	$\operatorname{tg} x - \operatorname{ctg} y = -\frac{\cos(x+y)}{\cos x \sin y}$		
106	$\operatorname{ctg} x - \operatorname{tg} x = 2 \operatorname{ctg} 2x$		
107	$\operatorname{ctg} x + \operatorname{tg} x = \frac{2}{\sin 2x}$		
108	$\operatorname{tg} x \cdot \operatorname{tg} y = \frac{\operatorname{tg} x + \operatorname{tg} y}{\operatorname{ctg} x + \operatorname{ctg} y} = -\frac{\operatorname{tg} x - \operatorname{tg} y}{\operatorname{ctg} x - \operatorname{ctg} y}$		
109	$\operatorname{ctg} x \cdot \operatorname{ctg} y = \frac{\operatorname{ctg} x + \operatorname{ctg} y}{\operatorname{tg} x + \operatorname{tg} y} = -\frac{\operatorname{ctg} x - \operatorname{ctg} y}{\operatorname{tg} x - \operatorname{tg} y}$		
110	$\operatorname{tg} x \cdot \operatorname{ctg} y = \frac{\operatorname{tg} x + \operatorname{tg} y}{\operatorname{ctg} x + \operatorname{ctg} y} = -\frac{\operatorname{tg} x - \operatorname{ctg} y}{\operatorname{ctg} x - \operatorname{tg} y}$		
111	$\operatorname{tg} x = \operatorname{ctg} y \text{ agar } x + y = 90^\circ$		
112	$\operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$		
113	$\operatorname{tg} 3x = \frac{3 \operatorname{tg} x - \operatorname{tg}^3 x}{1 - 3 \operatorname{tg}^2 x}$		
114	$\operatorname{tg} 4x = \frac{4 \operatorname{tg} x - 4 \operatorname{tg}^3 x}{1 - 6 \operatorname{tg}^2 x + \operatorname{tg}^4 x}$		
115	$\operatorname{tg} 5x = \frac{\operatorname{tg}^5 x - 10 \operatorname{tg}^3 x + 5 \operatorname{tg} x}{1 - 10 \operatorname{tg}^2 x + 5 \operatorname{tg}^4 x}$		
116	$\operatorname{sin} 2x = \frac{2 \operatorname{tg} x}{1 + \operatorname{tg}^2 x}$		