2017-yil matematika variant yechimlari (spectrum)
29-variant

Bizning kanal : @axborotnoma

Adminsratorlar hayati : @axborotnoma_bot
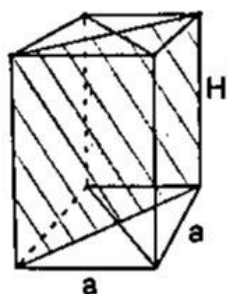
Matematika yordam guruhi : @axborotnomaguruhi

Reklama xizmati : @axborotnoma_reklama

1. Qarang: 24-variant 30-savol (175-bet).

2. To'g'ri parallelepiped asosi romb. Asos yuzi 1 m², diagonal kesimlari yuzalari 3 m² va 6 m² bo'lsa, hajmini toping.

Yechish:

$ABCDA_1B_1C_1D_1$ parallelepiped
ABCD romb
$S_{asos} = 1, S_1 = 3, S_2 = 6$
$S_1 = d_1 \cdot H$
$S_2 = d_2 \cdot H$
$V = S_{asos} \cdot H$

$$S_{asos} = \frac{d_1 \cdot d_2}{2}$$

1) $\dfrac{S_1}{S_2} = \dfrac{d_1 \cdot H}{d_2 \cdot H} = \dfrac{d_1}{d_2} = \dfrac{3}{6} = \dfrac{1}{2}, \quad d_2 = 2d_1$

2) $S_{asos} = 1 = \dfrac{d_1 \cdot d_2}{2} = \dfrac{d_1 \cdot 2d_1}{2} = d_1^2,$

$d_1 = 1,$
$d_2 = 2.$

3) $H = \dfrac{S_1}{d_1} = \dfrac{3}{1} = 3$

4) $V = S_{asos} \cdot H = 1 \cdot 3 = 3.$

Javob: 3.

3. $\left((a^2 + b^2 + ab) \cdot \left(b - \dfrac{b^2}{a+b} \right) \right) : \dfrac{a^3 - b^3}{a^2 - b^2}$

ifodaning a = 15, b = 17 dagi qiymatini toping.

Yechish:
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
$a^2 - b^2 = (a - b)(a + b)$

$$(a^2 + b^2 + ab)\cdot\left(b - \frac{b^2}{a+b}\right):\frac{a^3 - b^3}{a^2 - b^2} =$$

$$= (a^2 + ab + b^2)$$

$$\frac{(ab + b^2 - b^2)}{a+b}\cdot\frac{(a-b)(a+b)}{(a-b)(a^2 + ab + b^2)} =$$

$$= ab = 15\cdot17 = 255.$$

Javob: 255.

4. Soddalashtiring:

$$5\sqrt{\frac{7}{5}}\cdot\left(2\sqrt{\frac{5}{7}} - 8\sqrt{\frac{7}{20}} + 4\sqrt{\frac{7}{5}}\right).$$

Yechish:

$$5\cdot\left(2\cdot\sqrt{\frac{7}{5}\cdot\frac{5}{7}} - 8\cdot\sqrt{\frac{7}{20}\cdot\frac{7}{5}} + 4\sqrt{\frac{7}{5}\cdot\frac{7}{5}}\right) =$$

$$= 5\cdot\left(2\cdot1 - 8\cdot\frac{7}{10} + 4\cdot\frac{7}{5}\right) =$$

$$= 5\cdot\left(2 - \frac{28}{5} + \frac{28}{5}\right) = 5\cdot2 = 10.$$

Javob: 10.

5. Uchburchakning ikki tomoni mos ravishda 17 sm va 28 sm, yuzi 210 ga teng bo'lsa, uchinchi tomonini toping.

Yechish:
a = 17 sm, b = 28 sm,
S = 210,
c = ?

1. $S = \dfrac{a\cdot b}{2}\cdot\sin\gamma$, $\sin\gamma = \dfrac{2S}{a\cdot b}$

$$\sin\gamma = \frac{2\cdot210}{28\cdot17} = \frac{15}{17}.$$

2. Kosinuslar teoremasidan uchinchi tomonini topamiz.

$$c^2 = a^2 + b^2 - 2ab\cdot\cos\gamma$$

$$\cos\gamma = \pm\sqrt{1 - \sin^2\gamma} = \pm\sqrt{1 - \left(\frac{15}{17}\right)^2} = \pm\frac{8}{17}$$

1) $c^2 = 17^2 + 28^2 - 2\cdot17\cdot28\cdot\dfrac{8}{17} = 625,$

c = 25.

2) $c^2 = 17^2 + 28^2 - 2\cdot17\cdot28\cdot\left(-\dfrac{8}{17}\right) = 1521,$

c = 39.

Javob: 25 va 39.

6. Agar $f(x) = x^{\sin2x}$ bo'lsa, $f'\left(\dfrac{\pi}{4}\right)$ ni toping.

Yechish:
1) $y = x^{\sin2x}$ tenglikning ikkala qismini logarifmlaymiz.

$\ln y = \ln x^{\sin2x}$

$\ln y = \sin2x\cdot\ln x$

$(\ln y)' = (\sin2x\cdot\ln x)' =$

$= (\sin2x)'\cdot\ln x + \sin2x\cdot(\ln x)'$

$$\frac{y'}{y} = 2\cos2x\cdot\ln x + \frac{\sin2x}{x},$$

$$y' = y\cdot\left(2\cos2x\cdot\ln x + \frac{\sin2x}{x}\right)$$

$$y' = x^{\sin2x}\cdot\left(2\cos2x\cdot\ln x + \frac{\sin2x}{x}\right)$$

2) $y'\left(\dfrac{\pi}{4}\right) = \dfrac{\pi}{4}^{\sin2\cdot\frac{\pi}{4}}\cdot\left(2\cos2\cdot\dfrac{\pi}{4}\cdot\ln\dfrac{\pi}{4} + \right.$

$$\left. + \frac{4}{\pi}\sin2\cdot\frac{\pi}{4}\right) = \frac{\pi}{4}^{\sin\frac{\pi}{2}}\cdot\left(2\cos\frac{\pi}{2}\cdot\ln\frac{\pi}{4} + \right.$$

$$\left. + \frac{4}{\pi}\sin\frac{\pi}{2}\right) = \frac{\pi}{4}\cdot\left(2\cdot0\cdot\ln\frac{\pi}{4} + \frac{4}{\pi}\right) = \frac{\pi}{4}\cdot\frac{4}{\pi} = 1$$

Javob: 1.

7. Qarang: 27-variant 28-savol (187-bet).

8. Qarang: 25-variant 28-savol (179-bet).

9. Balandligi 5 sm, yon qirrasining asos qirrasiga nisbati 2:3 kabi bo'lgan muntazam uchburchakli piramidaga tashqi chizilgan shar radiusini toping.

Berilgan:
Muntazam uchburchakli piramidagi tashki chizilgan shar.
SO = 5 sm
SA:AB = 2:3
R – ?

http://bit.ly/Docufy

Yechish:

Muntazam uchburchakli piramidaga tashqi chilgan shar radiusni topish uchun

$$R = \frac{3h^2 + a^2}{6h} \quad (1)$$

formulasidan foydalanamiz.
Bu yerda h – balandlik,
a – piramida asos tomoni:
1) shartga ko'ra:

$$\frac{SA}{AB} = \frac{2}{3}$$

$$AB = a$$

$$\Rightarrow SA = \frac{2}{3}a.$$

SAO – to'g'ri burchakli uchburchak;
2) ΔSAO dan: SA² = AO² + SO² (Pifagor teoremasi)
AO = R – ΔABC ga tashqi chizilgan aylana radiusi

$$AO = \frac{a\sqrt{3}}{3};$$

3) $\dfrac{4}{9}a^2 = \dfrac{3a^2}{9} + 25 \Rightarrow \dfrac{a^2}{9} \Rightarrow a^2 = 25 \cdot 9$

a = 15;
4) a qiymatini (1) formulaga qo'yib, R qiymatini topamiz:

$$R = \frac{3 \cdot 25 + 225}{30} = \frac{300}{30} = 10 \text{ sm.}$$

Javob: 10 sm.

10.

11. ABCD teng yonli trapetsiyaning AC diagonali 2 ga teng va AD katta asos bilan 30° li burchak tashki etadi. Trapetsiyaning yuzini toping.
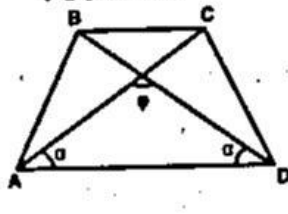
Berilgan:
ABCD – teng yonli trapetsiya
AC = 2
∠ACD = 30°
S_{ABCD} = ?

$$S_{ABCD} = \frac{AC \cdot BD}{2} \cdot \sin\varphi$$

Yechish:

$AC = BD, \ \angle CAD = \angle ADB = 30°$
$\alpha = 30°$
$\varphi = 180° - 2\alpha = 180° - 2 \cdot 30° = 120°$

$$S_{ABCD} = \frac{2 \cdot 2}{2} \cdot \sin 120° = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$$

Javob: $\sqrt{3}$.

12.

13. Arifmetik progressiyada
$a_2 + a_8 + a_{10} + a_{12} + a_{14} + a_{20} = 102$ bo'lsa,
$a_3 + a_{19}$ ni toping.

Yechish:
$a_2 + a_8 + a_{10} + a_{12} + a_{14} + a_{20} = 102$
$a_3 + a_{19} = ?$
Arifmetik progressiya xossasiga ko'ra:
$a_n + a_k = a_m + a_e, \ n + k = m + e$
$a_2 + a_{20} = a_8 + a_{14} = a_{10} + a_{12}$
$3(a_2 + a_{20}) = 102, \ a_2 + a_{20} = 34$
$a_3 + a_{19} = a_2 + a_{20} = 34.$

Javob: 34.

14. Agar x = –2 bo'lsa,
$$\frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-a)(x-c)}{(b-a)(b-c)} + \frac{(x-a)(x-b)}{(c-a)(c-b)}$$ ning qiymatini toping.
(Bu yerda (a – b)(a – c)(b – c) ≠ 0)

Yechish:

1) $\dfrac{(-2-b)(-2-c)}{(a-b)(a-c)} + \dfrac{(-2-a)(-2-c)}{(b-a)(b-c)} +$

$+\dfrac{(-2-a)(-2-b)}{(c-a)(c-b)};$

2) $\dfrac{(2+b)(2+c)(b-c) - (2+a)(2+c)(a-c) +}{(a-b)(a-c)(b-c)}$

$\dfrac{+(2+a)(2+b)(a-b)}{} =$

$= \dfrac{4(b-c) + 2(b^2 - c^2) + bc(b-c) - 4(a-c) -}{}$

$\dfrac{-2(a^2 - c^2) - ac(a-c) + 4(a-b) +}{(a-b)(a-c)(b-c)}$

$\dfrac{+2(a^2 - b^2) + ab(a-b)}{} = \dfrac{4b - 4c +}{}$

$$\frac{+2b^2 - 2c^2 + b^2c - bc^2 - 4a + 4c - 2a^2 +}{(a-b)(a-c)(b-c)}$$

$$\frac{+2c^2 - a^2c + ac^2 + 4a - 4b + 2a^2 - 2b^2 +}{}$$

$$\frac{+a^2b - ab^2}{} = \frac{b^2c - bc^2 - a^2c + ac^2 + a^2b - ab^2}{(a-b)(a-c)(b-c)} =$$

$$= \frac{(a-b)(a-c)(b-c)}{(a-b)(a-c)(b-c)} = 1.$$

Javob: 1.

15. Qarang: 10-variant 30-savol (80-bet).

16. Qarang: 19-variant 13-savol (138-bet).

17. Qarang: 6-variant 9-savol (47-bet).

18. $\dfrac{\left(2\frac{1}{6} + 4\frac{1}{2}\right) \cdot 0,375}{2\frac{3}{4} - 1\frac{1}{2}}$ ifoda qiymatining

35% ini toping.

Yechish:

1) $\dfrac{\left(2\frac{1}{6} + 4\frac{1}{2}\right) \cdot 0,375}{2\frac{3}{4} - 1\frac{1}{2}} = \dfrac{6\frac{1}{6} + \frac{3}{6}}{1\frac{3}{4} - \frac{2}{4}} \cdot \dfrac{375}{1000} =$

$$= \frac{6\frac{4}{6}}{1\frac{1}{4}} \cdot \frac{3}{8} = 6\frac{2}{3} : 1\frac{1}{4} \cdot \frac{3}{8} = \frac{20}{3} \cdot \frac{4}{5} \cdot \frac{3}{8} = 2.$$

2) 2 ning 35% ini topamiz.

2·0,35 = 0,7.

Javob: 0,7.

19. 3 – 4 + 5 – 6 + ... + 2017 – 2018 + 2019 ni hisoblang.

Yechish:

3 – 4 + 5 – 6 + ... + 2017 – 2018 +
+ 2019 = 3 + (5 – 4) + (7 – 6) + ... +
+ (2019 – 2018) = 3 + 1008 = 1011.

Javob: 1011.

20. $y = \cos^2 x$ funksiya grafigi berilgan bo'lib, uni parallel ko'chirish yordamida $y = \cos^2(x - a) + b$ funksiya grafigi hosil

qilingan. Bunday parallel ko'chirishda koordinata boshi qanday nuqtaga ko'chadi?

Yechish:

$y = \cos^2 x$ funksiya grafigini parallel ko'chirganda $y = \cos^2(x - a) + b$ grafik hosil bo'ladi.
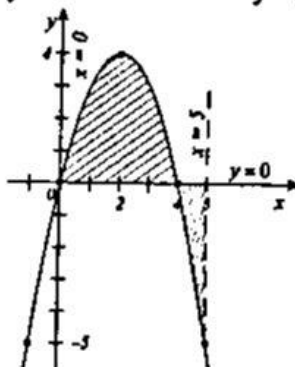
Koordinata boshi O(0; 0) nuqta N(a; b) nuqtaga ko'chadi.

Javob: N(a; b).

21. Qarang: 24-variant 21-savol (172-bet).

22. $y = 4x - x^2$, $y = 0$, $x = 0$, $x = 5$ chiziqlar bilan chegaralangan shakl yuzini toping.

Yechish:

$y = 4x - x^2$ funksiya grafigini yasaymiz.

$$S = \int_0^4 (4x - x^2)\,dx - \int_4^5 (4x - x^2)\,dx =$$

$$= \left(2x^2 - \frac{x^3}{3}\right)\Big|_0^4 - \left(2x^2 - \frac{x^3}{3}\right)\Big|_4^5 =$$

$$= 32 - \frac{64}{3} - \left(50 - \frac{125}{3} - 32 + \frac{64}{3}\right) =$$

$$= 14 - \frac{128}{3} + \frac{125}{3} = 14 - 1 = 13.$$

Javob: 13.

23. To'g'ri javobni ko'rsating. Bu yerda [a] – a sonning butun qismi.

Yechish:

[a] – sonning butun qismi.
Sonning butun qismi xossasiga ko'ra:
1) a, b ∈ Z bo'lganda [a + b] = [a] + [b] bo'ladi.
2) a, b ∈ R bo'lganda [a + b] ≥ [a] + [b] bo'ladi.

Javob: Agar a, b ∈ R bo'lsa,
[a + b] ≥ [a] + [b] bo'ladi.

24. [1; 10] kesmada berilgan $f(x)$ funksiyaning eng katta va eng kichik qiymatlari teng bo'lsin. U holda $f(9) - f(5)$ ifodaning katta qiymatini toping.
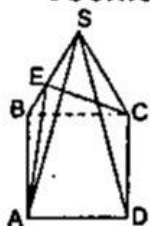
Yechish:

[1; 10] kesmada $f_{max}(x) = f_{min}(x)$

$f(9) - f(5)$ maksimum qiymatini toping.

Agar [1; 10] kesmada $y = f(x)$ funksiyaning eng katta va eng kichik qiymatlari teng bo'lsa, u holda bu funksiya o'zgarmas funksiya bo'ladi, ya'ni $f(x) = c$. Bundan $f(9) = c$, $f(5) = c$.

$f(9) - f(5) = 0$ bo'ladi.

Javob: 0.

25. Tomoni a ga teng bo'lgan ABCD kvadrat α tekislikda yotadi. S nuqta ABCD tekislikga tegishli emas. S nuqtadan kvadratning har bir uchigacha bo'lgan masofa a ga teng bo'lsa, SAB va SBC tekisliklar orasidagi burchak kosinusni toping.

Yechish:

ABCD kvadrat
$SA = SB = SC = SD = l$
$AB = a$
$\cos \angle (SAB, SBC)$
$SABCD$ piramida

1) SAB va SBC tekisliklar orasidagi burchak $\angle AEC$

$AE \perp SB$, $CE \perp SB$,

$AE = EC = \dfrac{a\sqrt{3}}{2}$

$\cos \angle AEC = \cos\varphi$

$AC^2 = AE^2 + EC^2 - 2AE \cdot EC \cdot \cos\varphi$

$AC = a\sqrt{2}$.

$2a^2 = \dfrac{3a^2}{4} + \dfrac{3a^2}{4} - 2\dfrac{a\sqrt{3}}{2} \cdot \dfrac{a\sqrt{3}}{2}\cos\varphi$.

$\dfrac{3}{2}\cos\varphi = -\dfrac{1}{2}$, $\cos\varphi = -\dfrac{1}{3}$.

Javob: $-\dfrac{1}{3}$.

26.

27.

28. Bir guruh bolalarning o'rtacha og'irligi 40 kg ga teng. Qiz bolalarning o'rtacha og'irligi 35 kg, o'g'il bolalarning o'rtacha og'irligi esa 50 kg ligi ma'lum. Agar guruh a'zolarining 15 nafari o'g'il bolalar bo'lsa, qiz bolalar sonini toping.

Yechish:

$\dfrac{a_1 + a_2 + \ldots + a_n}{n} = 35$ – qiz bolalarning o'rtacha og'irligi.

$\dfrac{b_1 + b_2 + \ldots + b_k}{k} = 50$ – o'g'il bolalarning o'rtacha og'irligi.

$\dfrac{a_1 + a_2 + \ldots + a_n + b_1 + b_2 + \ldots + b_k}{n + k} = 40$ – guruh o'quvchilari o'rtacha og'irligi.

$k = 15$ nafar o'g'il bolalar, $n = ?$

$b_1 + b_2 + \ldots + b_k = 50 \cdot 15$

$a_1 + a_2 + \ldots + a_n = 35 \cdot n$

$a_1 + a_2 + \ldots + a_n + b_1 + b_2 + \ldots + b_k = 40(n + k)$

$50 \cdot 15 + 35 \cdot n = 40(n + 15)$

$50 \cdot 15 - 40 \cdot 15 = 40n - 35n$

$5n = 10 \cdot 15$, $n = 30$

Qizlar 30 nafar.

Javob: 30.

29.

30. ABC to'g'ri burchakli uchburchakning katta AC katetini diametr qilib yarim aylana chizilgan. AB kateti 30 ga teng. Yarim aylananing gipotenuzani kesgan nuqtasi bilan A to'g'ri burchakni tutashtiruvchi kesma 24 ga teng. Yarim aylana uzunligini toping.

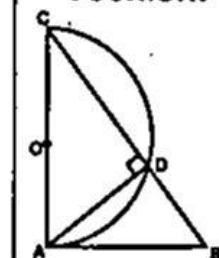**Berilgan:**

$\triangle ABC$ to'g'ri burchakli

$AC = 2R$

$AB = 30$

$AD = 24$

$L = 2\pi R = ?$

Yechish:

Yarim aylana uzunligi

$\dfrac{L}{2} = ?$

Diametrga tiralgan burchak 90° ga teng.
Demak $\angle ADC = 90°$, $\triangle ADC$ to'g'ri burchakli.
$\triangle ADB$ to'g'ri burchakli.
$DB^2 = AB^2 - AD^2 = 30^2 - 24^2 = 54 \cdot 6 = 9 \cdot 6^2$
$DB = 3 \cdot 6 = 18$

$\dfrac{AB^2}{DB} = CB = \dfrac{30^2}{18} = 50$

$AC^2 = CB^2 - AB^2 = 50^2 - 30^2 = 40^2$

$AC = 40 = 2R$

$L = 2\pi R = 40\pi$

$\dfrac{L}{2} = \dfrac{40\pi}{2} = 20\pi$

Javob: 20π.

31. Qarang: 8-variant 34-savol (67-bet).

32. Qarang: 24-variant 35-savol (175-bet).

33. Qarang: 7-variant 35-savol (59-bet).

34. Qarang: 4-variant 32-savol (35-bet).

35. A="IO.SYS – ma'lumotlarni kiritish-chiqarish sistemasini kengaytirish moduli."
B="Free and Open Source Software – muloqot bepul, birlamchi kodi ochiq dasturiy ta'minot."
C="FoxPro – operatsion sistemadir." Shu mulohazalar asosida quyidagi mantiqiy ifodaning natijasini toping:
$(A \vee B) \wedge (\neg B \vee C)$

Yechish:

Mulohazalarni tahlil qilamiz:
A="IO.SYS – ma'lumotlarni kiritish-chiqarish sistemasini kengaytirish moduli." – rost (1)
B="Free and Open Source Software – muloqot bepul, birlamchi kodi ochiq dasturiy ta'minot." – rost (1)
C="FoxPro – operatsion sistemadir." – yolg'on (0).
Shu mulohazalar asosida quyidagi mantiqiy ifodaning natijasini topamiz:
$(A \vee B) \wedge (\neg B \vee C) = (1 \vee 1) \wedge (\neg 1 \vee 0) = 1 \wedge 0 = 0$
Javob: yolg'on.

36. Qarang: 9-variant 33-savol (74-bet).