

2017-yil matematika variant yechimlari (spectrum)

28-variant

Bizning kanal : @axborotnoma

Adminsratorlar hayati : @axborotnoma_bot

Matematika yordam guruhi : @axborotnomaguruhi

Reklama xizmati : @axborotnoma_reklama

1. Qarang: 1-variant 7-savol (4-bet).
 2. Qarang: 19-variant 3-savol (136-bet).
 3. $\int_0^1 \frac{2x}{x+1} dx$ integralni hisoblang.

Yechish:

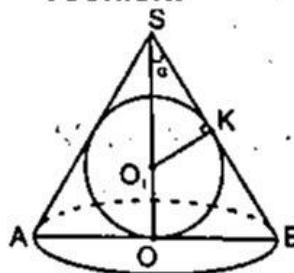
$$\begin{aligned} \int_0^1 \frac{2x}{x+1} dx &= 2 \int_0^1 \frac{x}{x+1} dx = 2 \int_0^1 \frac{x+1-1}{x+1} dx = \\ &= 2 \int_0^1 \left(1 - \frac{1}{x+1}\right) dx = 2(x - \ln|x+1|) \Big|_0^1 = \\ &= 2(1 - \ln 2 - (0 - \ln 1)) = 2(1 - \ln 2) = \\ &= 2(\ln e - \ln 2) = 2 \ln \frac{e}{2}. \end{aligned}$$

Javob: $2 \ln \frac{e}{2}$.

4. Qarang: 12-variant 10-savol (91-bet).

5. Konusga chizilgan ichki sharning hajmi $\frac{4}{5}$ ga teng. Konus hajmi 1,8 ga teng. O'q kesimi uchidagi burchagini toping.

Yechish:



$$SA = SB = l$$

$$SO = H$$

$$OB = AO = R$$

$$OO_1 = r, V_{sh} = \frac{4}{5}$$

$$V_k = 1,8$$

$$\angle ASB = ?$$

$$1) V_{sh} = \frac{4}{3} \pi r^3, OO_1 = OK = r.$$

$$V_k = \frac{1}{3} \pi R^2 H.$$

$$2) \operatorname{tg} \alpha = \frac{r}{R}, H = \frac{R}{\operatorname{tg} \alpha}.$$

$$3) \frac{V_m}{V_k} = \frac{\frac{4}{3} \pi r^3}{\frac{1}{3} \pi R^2 \cdot H} = \frac{4r^3}{R^2 H}$$

$$\frac{4r^3}{R^2 H} = \frac{4}{5} \Rightarrow \frac{r^3}{R^2 H} = \frac{1}{9}$$

$$4) \frac{r^3}{R^2 \cdot \frac{R}{\operatorname{tg} \alpha}} = \frac{1}{9}$$

$$\left(\frac{r}{R}\right)^3 \cdot \operatorname{tg} \alpha = \frac{1}{9}$$

$$\operatorname{tg}^3 \alpha \cdot \operatorname{tg} \alpha = \frac{1}{9}, \operatorname{tg}^4 \alpha = \frac{1}{9}, \operatorname{tg} \alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = 30^\circ$$

$$5) \angle ASB = 2\alpha = 2 \cdot 30^\circ = 60^\circ$$

Javob: 60° .

6. Qarang: 5-variant 4-savol (37-bet).

7. Dastlabki to'qqizta natural sonlar yig'indisining kvadrati 2025 ga teng bo'lsa, shu sonlar kublarining yig'indisini toping.

Yechish:

$$(1 + 2 + \dots + 9)^2 = 2025$$

$$1^3 + 2^3 + \dots + 9^3 = ?$$

$$1^3 + 2^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2 \text{ tenglikka}$$

asosan

$$(1 + 2 + \dots + 9)^2 = \left(\frac{9(9+1)}{2}\right)^2 =$$

$$= (9 \cdot 5)^2 = 45^2 = 2025$$

$$(1 + 2 + \dots + 9)^2 = 1^3 + 2^3 + \dots + 9^3 = 2025.$$

Javob: 2025.

8. $x = 4$ dagi

$$\frac{7x+2}{2} - 1,5 - \frac{4x-1}{3} - \frac{0,75x}{6} \text{ ning qiymatini}$$

toping.

Yechish:

$$x = 4.$$

$$\frac{7x+2}{2} - 1,5 - \frac{4x-1}{3} - \frac{0,75x}{6}$$

$$1) \frac{7x+2}{2} - \frac{3}{2} - \frac{4x-1}{3} - \frac{3x}{4 \cdot 6} =$$

$$= \frac{7x-1}{2} - \frac{4x-1}{3} - \frac{x}{8} =$$

$$= \frac{84x-12-32x+8-3x}{24} = \frac{49x-4}{24}$$

$$2) \frac{49 \cdot 4 - 4}{24} = \frac{49-1}{6} = \frac{48}{6} = 8.$$

Javob: 8.

9. Qarang: 3-variant 17-savol (23-bet).

$$10. y = 3 - \sqrt{16 - \sqrt{4x^2 - 4\sqrt{3}x + 3}}$$

funksiyaning qiymatlar sohasiga tegishli bo'lmagan eng katta manfiy butun sonni toping.

Yechish:

1) aniqlanish sohasi:

$$16 - \sqrt{4x^2 - 4\sqrt{3}x + 3} \geq 0$$

$$16 - \sqrt{(2x - \sqrt{3})^2} \geq 0$$

$$16 - |2x - \sqrt{3}| \geq 0$$

$$-16 \leq 2x - \sqrt{3} \leq 16$$

$$-16 + \sqrt{3} \leq 2x \leq 16 + \sqrt{3}$$

$$-8 + \frac{\sqrt{3}}{2} \leq x \leq 8 + \frac{\sqrt{3}}{2}$$

2) qiymatlar sohasi:

$$y_{\max} = 3, y_{\min} = 3 - \sqrt{16} = -1$$

$$E(y) = [-1; 3]$$

3) qiymatlar sohasiga tegishli bo'lmagan eng katta butun manfiy son -2 .

Javob: -2 .

11. Agar $f(x) = 2^x \cdot x$ bo'lsa, $f'(x) \geq 0$ tengsizlikni yeching.

Yechish:

$$1) f'(x) = (2^x \cdot x)' = (2^x)' \cdot x + 2^x \cdot (x)' = 2^x \cdot \ln 2 \cdot x + 2^x = 2^x (x \cdot \ln 2 + 1)$$

2) $f'(x) > 0$, $2^x(x \cdot \ln 2 + 1) > 0$, $2^x > 0$ doimo musbat.

$$x \cdot \ln 2 + 1 > 0, x \ln 2 > -1, x > -\frac{1}{\ln 2} = -\log_2 e$$

$$x \in (-\log_2 e; \infty).$$

Javob: $(-\log_2 e; \infty)$.

12. Qarang: 24-variant 12-savol (170-bet).

13. ABCD parallelogramm uchta uchining koordinatalari ma'lum: A(0; 1), B(1; 3), C(10; 3). ABCD parallelogram yuzini toping.

Berilgan:

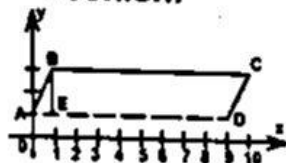
$$A(0; 1)$$

$$B(1; 3)$$

$$C(10; 3)$$

$$S_{ABCD} = ?$$

Yechish:



$$S_{ABCD} = AD \cdot BE$$

$$BC = AD = 9, BE = 2$$

$$S_{ABCD} = 9 \cdot 2 = 18.$$

Javob: 18.

14. Qarang: 24-variant 3-savol (169-bet).

$$15. \begin{cases} 3x + 2y = k \\ x^2 + y^2 = 117 \end{cases} \text{ tenglamalar sistemasi}$$

k ning qanday qiymatida bitta yechimga ega bo'ladi?

Yechish:

$$\begin{cases} 3x + 2y = k \\ x^2 + y^2 = 117 \end{cases} \Rightarrow \begin{cases} y = \frac{k - 3x}{2} \\ x^2 + \left(\frac{k - 3x}{2}\right)^2 = 117 \end{cases}$$

$$4x^2 + k^2 - 6xk + 9x^2 = 117 \cdot 4$$

$$13x^2 - 6xk + k^2 - 468 = 0 \text{ tenglama bitta}$$

yechimga ega bo'lishi uchun

$D = 0$ bo'lishi kerak.

$$D = (6k)^2 - 4 \cdot 13 \cdot (k^2 - 468) = 0$$

$$9k^2 - 13k^2 + 13 \cdot 468 = 0,$$

$$-4k^2 + 13 \cdot 468 = 0$$

$$k^2 - 13 \cdot 117 = 0,$$

$$k^2 = 13 \cdot 13 \cdot 9,$$

$$k = \pm 39.$$

Javob: ± 39 .

16. Qarang: 15-variant 11-savol (112-bet).

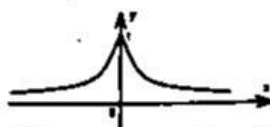
17. Qarang: 2-variant 20-savol (16-bet).

$$18. y = \frac{5}{x^2 + 5} \text{ funksiyaning qiymatlar}$$

sohasiga kirmaydigan eng katta manfiy butun sonni toping.

Yechish:

Aniqlanish sohasi $(-\infty; \infty)$.



Qiymatlar sohasi $(0; 1]$.

Qiymatlar sohasiga kirmaydigan eng katta manfiy butun son -1 .

Javob: -1 .

19. Konusga shar ichki chizilgan. Konus asosining yuzi shar sirti yuziga teng. Konus o'q kesimi uchidagi burchakning kosinusini toping.

Berilgan:

Konusga shar ichki chizilgan

R - konus radiusi

r - shar radiusi

$$S_{asos} = S_{sh}$$

$$S_{asos} = \pi R^2$$

$$S_{sh} = 4\pi r^2$$

$$\angle ASB = ?$$

$$1) \pi R^2 = 4\pi r^2, R^2 = 4r^2, R = 2r.$$

$$2) \Delta ASB \text{ teng yonli. } AS = SB = \ell.$$

Uchburchakka ichki chizilgan aylana

$$\text{radiusi: } r = \frac{2 \cdot \frac{2R \cdot H}{2}}{2\ell + 2R} = \frac{RH}{\ell + R}.$$

$$3) R = 2r$$

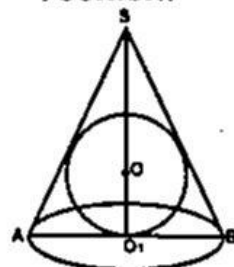
$$r = \frac{2rH}{\ell + 2r}, \ell + 2r = 2H.$$

$$4) \ell^2 = R^2 + H^2, \ell = 2H - 2r$$

$$4H^2 - 8Hr + 4r^2 = 4r^2 + H^2, H = \frac{8r}{3}.$$

$$5) \operatorname{tg} \alpha = \frac{R}{H} = \frac{2r \cdot 3}{8r} = \frac{3}{4}.$$

Yechish:



6) $\angle ASB = 2\alpha$

$$\operatorname{tg} 2\alpha = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha} = \frac{2 \cdot \frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} = \frac{24}{7}$$

$$\cos 2\alpha = \frac{1-\operatorname{tg}^2\alpha}{1+\operatorname{tg}^2\alpha} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{7}{25}$$

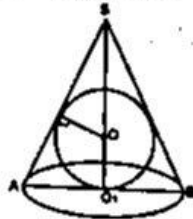
Javob: $\frac{7}{25}$.

20. Konusning yasovchisi $3\sqrt{3}$ ga teng. Konusning uchidan unga ichki chizilgan shar markazigacha bo'lgan masofa 3 ga teng. Konus yasovchisi va asos tekisligi orasidagi burchakni toping.

Berilgan:

$SA = SB = 3\sqrt{3}$
 $SO = 3$
 $\angle SAB = ?$

Yechish:



$H = SO + OO_1 = 3 + r$

$$r = \frac{2S \cdot H \cdot R}{2l + 2R} = \frac{H \cdot R}{l + R}$$

$r(3\sqrt{3} + R) = (3 + r) \cdot R$

$3\sqrt{3} \cdot r + rR = 3R + rR$

$R = \sqrt{3} r$

$l^2 = H^2 + R^2$

$(3\sqrt{3})^2 = (3 + r)^2 + (\sqrt{3} r)^2$

$27 = 9 + 6r + r^2 + 3r^2$

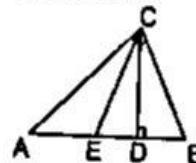
$2r^2 + 3r - 9 = 0, r = 1,5, H = 4,5$

$\sin \alpha = \frac{H}{l} = \frac{4,5}{3\sqrt{3}} = \frac{\sqrt{3}}{2}, \alpha = \frac{\pi}{3}$

Javob: $\frac{\pi}{3}$.

21. Gipotenuzaga tushirilgan balandlik va mediananing nisbati 40:41 ga teng bo'lsa, ushbu to'g'ri burchakli uchburchak katetlarining nisbatini toping.

Yechish:



$\triangle ABC$ to'g'ri burchakli

$CD = h$ balandlik.

$CE = m$ mediana.

$CE = AE = EB$

$h:m = 40:41$

$\frac{a}{b} = ?$

$h = 40x, m = 41x, c = 82x.$

$AD = b_c; DB = a_c$ - proeksiyalar.

$\frac{a_c}{b_c} = \left(\frac{a}{b}\right)^2,$

$ED = \sqrt{(41x)^2 - (40x)^2} = 9x$

$a_c = 41x - 9x = 32x$

$b_c = 41x + 9x = 50x$

$\left(\frac{a}{b}\right)^2 = \frac{32x}{50x} = \frac{16}{25}, \frac{a}{b} = \frac{4}{5}$

Javob: 4:5.

22. Uchburchakning ikkita burchakning tangenslari mos ravishda 1,5 va 5 ga teng. Ushbu uchburchakning uchinchi burchagini toping.

Yechish:

$\operatorname{tg} \alpha = 1,5$

$\operatorname{tg} \beta = 5$

$\gamma = ?$

1) uchburchakning ichki burchaklari

yig'indisi $\alpha + \beta + \gamma = 180^\circ,$

$\alpha + \beta = 180^\circ - \gamma.$

2) $\operatorname{tg}(\alpha + \beta) = \operatorname{tg}(180^\circ - \gamma) = -\operatorname{tg} \gamma$

$\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta} = \frac{1,5 + 5}{1 - 1,5 \cdot 5} =$

$\frac{6,5}{1 - 7,5} = \frac{6,5}{-6,5} = -1.$

$-\operatorname{tg} \gamma = \operatorname{tg}(\alpha + \beta) = -1.$

$\operatorname{tg} \gamma = 1, \gamma = 45^\circ$

Javob: 45° .

23. Agar $\vec{a}(6; 2; 1)$ va $\vec{b}(0; -1; 2 + 2\sqrt{14})$ bo'lsa, $\vec{c} = 2\vec{a} - \vec{b}$ vektor uzunligini toping.

Yechish:

$$\bar{a}(6; 2; 1),$$

$$\bar{b}(0; -1; 2 + 2\sqrt{14}),$$

$$\bar{c} = 2\bar{a} - \bar{b}, |\bar{c}| = ?$$

$$1) 2\bar{a} = (12; 4; 2).$$

$$2) 2\bar{a} - \bar{b} = (12; 4; 2) - (0; -1; 2 + 2\sqrt{14}) = (12; 5; -2\sqrt{14})$$

$$3) \bar{c} = (12; 5; -2\sqrt{14})$$

$$|\bar{c}| = \sqrt{12^2 + 5^2 + (-2\sqrt{14})^2} =$$

$$= \sqrt{144 + 25 + 56} = \sqrt{225} = 15.$$

Javob: 15.

24. Qarang: 23-variant 24-savol (166-bet).

25. $f(x) = 3\cos x - 4\sin x + 3$ funksiyaning qiymatlar sohasini toping.

Yechish:

$f(x) = 3\cos x - 4\sin x + 3$ qiymatlar sohasini topamiz.

$$y = a\cos kx + b\sin kx$$

$$-\sqrt{a^2 + b^2} \leq y \leq \sqrt{a^2 + b^2}$$

$$1) y = 3\cos x - 4\sin x$$

$$-\sqrt{3^2 + (-4)^2} \leq y \leq \sqrt{3^2 + (-4)^2}$$

$$-5 \leq y \leq 5.$$

$$2) f_{\min} = -5 + 3 = -2$$

$$f_{\max} = 5 + 3 = 8.$$

$$3) E(f) = [-2; 8].$$

Javob: $[-2; 8]$.

26. Qarang: 10-variant 18-savol (78-bet).

27. Arifmetik progressiyada $a_1 = 1$, $S_{20} - S_{12} = 380$ bo'lsa, ayirmasini toping.

Yechish:

$$a_1 = 1$$

$$S_{20} - S_{12} = a_1 + a_2 + \dots + a_{20} -$$

$$-(a_1 + a_2 + \dots + a_{12}) = a_{13} + a_{14} + a_{15} +$$

$$+ a_{16} + a_{17} + a_{18} + a_{19} + a_{20} =$$

$$= 4(a_{13} + a_{20}) = 4(2a_1 + 31d)$$

$$4(2a_1 + 31d) = 380$$

$$2 \cdot 1 + 31d = 95, 31d = 93, d = 3.$$

Javob: 3.

28. $-6(2 - 0,2x) + 11 = -4(3 - 0,3x) - 1$ tenglamaning yechimi x ga teng bo'lsa, $\frac{2x+1}{4}$ ning qiymatini toping.

Yechish:

$$\frac{2x+1}{4} = ?$$

$$-6(2 - 0,2x) + 11 = -4(3 - 0,3x) - 1$$

$$-12 + 1,2x + 11 = -12 + 1,2x - 1$$

$$11 = -1 \rightarrow \emptyset$$

Tenglama yechimga ega emas.

Javob: \emptyset .

29. $y = \sqrt{4x^2 - 4x + 1} + \sqrt{x^2 - 6x + 9}$ funksiyaning eng kichik qiymatini toping.

Yechish:

$$1) \sqrt{4x^2 - 4x + 1} = \sqrt{(2x-1)^2} = |2x-1|;$$

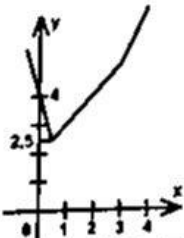
$$2) \sqrt{x^2 - 6x + 9} = \sqrt{(x-3)^2} = |x-3|;$$

$$3) y = |2x-1| + |x-3|$$

$$x < \frac{1}{2}, y = 1 - 2x + 3 - x = 4 - 3x$$

$$\frac{1}{2} \leq x < 3, y = 2x - 1 + 3 - x = x + 2$$

$$x \geq 3, y = 2x - 1 + x - 3 = 3x - 4$$



$$y_{\min} = 2,5, E(y) = [2,5; \infty).$$

Javob: 2,5.

30. To'g'ri burchakli uchburchakning gipotenuzasi 25 ga, unga ichki chizilgan aylana radiusi 4 ga teng. Uchburchakning perimetrini toping.

Berilgan:

ΔABC to'g'ri burchakli

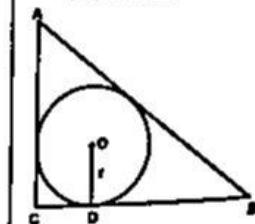
$AB = 25$

r - ichki chizilgan

aylana radiusi

$P = ?$

Yechish:



1) $AB = c, AC = b, BC = a$

$c = 25, r = \frac{a+b-c}{2}$ to'g'ri burchakli

uchburchakka ichki chizilgan aylana radiusi.

$4 = \frac{a+b-25}{2}, a+b = 33;$

2) $P = a + b + c = 33 + 25 = 58.$

Javob: 58.

31. Qarang: 2-variant 32-savol (18-bet).

32. Qarang: 13-variant 34-savol (104-bet).

33. Ali sakkizlik sanoq sistemasida (43; 67) oraliqdagi barcha butun sonlarni yozib chiqdi. Vali esa shu sonlardan avval 6 raqami, so'ng 5 raqami qatnashgan barcha sonlarni o'chirib tashladi. Qolgan sonlar yig'indisini sakkizlik sanoq sistemasida aniqlang va uchlik sanoq sistemasiga o'tkazing.

Yechish:

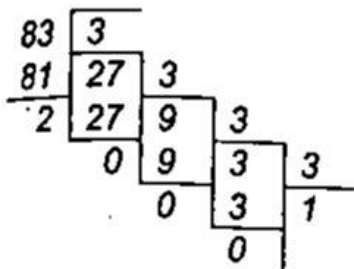
Ali yozgan sonlar: 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 65, 66.

Vali shu sonlardan 6 va 5 raqamlari qatnashgan barcha sonlarni o'chirganidan keyin 44,

47 sonlar qoladi. Ularning sakkizlik sanoq sistemasidagi yig'indisini hisoblaymiz:

$44_8 + 47_8 = 123_8$. Endi bu sonni avval 10-lik sanoq sistemasiga, keyin 3-likka o'tkazamiz.

$123_8 = 1 \cdot 8^2 + 2 \cdot 8^1 + 3 \cdot 8^0 = 64 + 16 + 3 = 83_{10}$



Demak, $123_8 = 10002_3$ ekan.

Javob: 10002.

34. Ma'lumotlar omborida ma'lumotlarni saqlash fayli kengaytmasini aniqlang:

Yechish:

Fayl kengaytmasi faylning turini, qaysi dasturda yaratilganligini bildiradi.

Ma'lumotlar omborining fayllarida .dbf (Data Base file) kengaytmasi ishlatiladi.

Javob: dbf.

35. Qarang: 17-variant 34-savol (128-bet).

36. Paskal tilida yozilgan dastur natijasini aniqlang.

Var N, k:integer;

b:array[1..21] of Integer;

Begin Randomize; N:=1;

b[1]:=1+random(1)+random(1); k:=1;

While k<b[k]+2 Do begin k:=k+3;

b[k]:=b[k-3]+2;

N:=N+1; end;

Write(N); Readln; End.

Yechish:

Dasturda N , k –butun o'zgaruvchilar va b – 21 ta butun sondan iborat massivdan foydalanilgan.

Randomize – tasodifiy sonlar generatori.

$N:=1$;

$b[1]:=1+\text{Random}(1)+\text{Random}(1)$; {random(1) 0 qiymatni, qabul qiladi. Demak, $b[1]=1$ bo'lad}

While $k < b[k]+2$ Do {Toki $k < b[k]+2$ bajar:}

begin $k:=k+3$;

$b[k]:=b[k-3]+2$;

$N:=N+1$;

end;

Write(N); { N qiymatini ekranda aks ettir}

Tamom.

Batafsilroq while operatorini ko'rib chiqamiz:

$k=1$, $b[1]=1$ – boshlang'ich qiymatlar.

1-qadam. $1 < (1+2)$ (rost)

$k=k+3=1+3=4$,

$b[4]=b[1]+2=1+2=3$.

$N=N+1=2$ {2 ni ekranga chiqaradi}

2-qadam. $4 < (3+2)$ (rost)

$k=k+3=4+3=7$,

$b[7]=b[1]+2=3+2=5$.

$N=N+1=3$ {3 ni ekranga chiqaradi}

3-qadam. $7 < (5+2)$ (yolg'on) – sikl tugaydi va dastur bajarilishi to'xtaydi.

Javob: 3.