2017-yil matematika variant yechimlari (spectrum)
27-variant

Bizning kanal : @axborotnoma

Adminsratorlar hayati : @axborotnoma_bot

Matematika yordam guruhi : @axborotnomaguruhi

Reklama xizmati : @axborotnoma_reklama

1. Qarang: 25-variant 27-savol (178-bet).

2. $\bar{c}(-1; 0)$ va $\bar{a}(1; 2)$ vektorlar berilgan. $2\bar{c} + 3\bar{a}$ vektorni toping.

Yechish:

$\bar{c}(-1; 0)$, $\bar{a}(1; 2)$, $2\bar{c} + 3\bar{a} = ?$

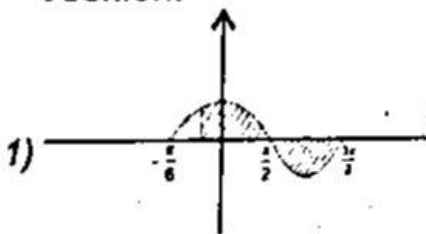1) $2\bar{c} = (-2; 0)$

2) $3\bar{a} = (3; 6)$

3) $2\bar{c} + 3\bar{a} = (-2; 0) + (3; 6) = (1; 6)$.

Javob: $(1; 6)$.

3. Integralni hisoblang:

$$\int\limits_{-\frac{\pi}{6}}^{\frac{3\pi}{2}} \left(2\cos^2\frac{x}{2} - |\cos x|\right)dx.$$

Yechish:

1)

$-\frac{\pi}{6} \le x < \frac{\pi}{2}$, $\cos x > 0$

$\frac{\pi}{2} < x < \frac{3\pi}{2}$, $\cos x < 0$.

2) $\int\limits_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (2\cos^2\frac{x}{2} - \cos x)dx +$

$+\int\limits_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (2\cos^2\frac{x}{2} + \cos x)dx$.

3) $2\cos^2\frac{x}{2} = 2\frac{1+\cos x}{2} = 1 + \cos x$.

4) $\int\limits_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (1 + \cos x - \cos x)dx +$

$+\int\limits_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (1 + \cos x + \cos x)dx = \int\limits_{-\frac{\pi}{6}}^{\frac{\pi}{2}} dx +$

$+\int\limits_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (1 + 2\cos x)dx = x\Big|_{-\frac{\pi}{6}}^{\frac{\pi}{2}} + (x + 2\sin x)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} =$

$= \frac{\pi}{2} - (-\frac{\pi}{6}) + (\frac{3\pi}{2} + 2\sin\frac{3\pi}{2}) - (\frac{\pi}{2} + 2\sin\frac{\pi}{2}) =$

$= \frac{\pi}{2} + \frac{\pi}{6} + \frac{3\pi}{2} - 2 - \frac{\pi}{2} - 2 = \frac{5\pi}{3} - 4$.

Javob: $\frac{5\pi}{3} - 4$.

4. $\frac{|x+3|+x}{x+2} > 1$ tengsizlikning manfiy butun yechimlari nechta?

Yechish:

$\frac{|x+3|+x}{x+2} > 1$, $\frac{|x+3|+x}{x+2} - 1 > 0$

$$\frac{|x+3|+x-x-2}{x+2} > 0$$

$$\frac{|x+3|-2}{x+2} > 0 \text{ tengsizlik quyidagi}$$

tengsizliklar sistemasiga teng kuchli.

1) $\begin{cases} |x+3|-2>0 \\ x+2>0 \end{cases}$ va 2) $\begin{cases} |x+3|-2<0 \\ x+2<0 \end{cases}$

1)

$\begin{cases} |x+3|-2>0 \\ x+2>0 \end{cases} \Rightarrow \begin{cases} x+3<-2, x+3>2 \\ x>-2 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x<-5, x>-1 \\ x>-2 \end{cases} \Rightarrow x>-1$

$x \in (-1; \infty)$

2) $\begin{cases} |x+3|<2 \\ x<-2 \end{cases} \Rightarrow \begin{cases} -2<x+3<2 \\ x<-2 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} -5<x<-1 \\ x<-2 \end{cases} \Rightarrow -5<x<-2$

Tengsizlikning yechimlari oraliqlari $(-5; -2) \cup (-1; \infty)$. Butun manfiy yechimlari $-4, -3$. 2 ta.

Javob: 2.

5. Qarang: 6-variant 16-savol (48-bet).

6. $P(x) = (3x+1)^{2017} \cdot (8x+1)^{2016} + (4x-1)^3 \cdot (2x-1)^2 + x - 1$ ko'phadning ozod hadini toping.

Yechish:
Ozod hadini topish uchun P(x) ko'phadning 0 dagi qiymatini topish kerak.
$P(0) = (3 \cdot 0 + 1)^{2017} \cdot (8 \cdot 0 + 1)^{2016} + (4 \cdot 0 - 1)^3 \cdot (2 \cdot 0 - 1)^2 + 0 - 1 = 1 + (-1) + (-1) = -1$. Ozod hadi -1.

Javob: -1.

7. $2x - 3\sqrt{2x-1} + 1 = 0$ tenglamani yeching.

Yechish:

Aniqlanish sohasi: $2x - 1 \geq 0$, $x \geq \frac{1}{2}$

$3\sqrt{2x-1} = 2x + 1$
$9(2x-1) = 4x^2 + 4x + 1$
$4x^2 - 14x + 10 = 0$

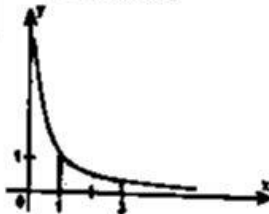$2x^2 - 7x + 5 = 0$
$x = 1, x = 2,5$
$x = 1, x = 2,5$ aniqlanish sohasiga tegishli.

Javob: 1 va 2,5.

8. $y = \frac{1}{x^2}$, $y = 0$, $x = 1$, $x = 3$ chiziqlar bilan chegaralangan shaklning yuzini toping.

Yechish:

$$S = \int_1^3 \frac{1}{x^2}dx = \int_1^3 x^{-2}dx = -\frac{1}{x}\Big|_1^3 =$$

$$= -\frac{1}{3} - (-1) = -\frac{1}{3} + 1 = \frac{2}{3}$$

Javob: $\frac{2}{3}$.

9. Ketma-ket x, y, z natural sonlar uchun $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + \frac{y}{x} + \frac{x}{z} + \frac{z}{y}$ son butun bo'lsa, $x + y + z$ ni toping.

Yechish:
x, y, z – ketma-ket natural sonlar.
$x = n, y = n + 1, z = n + 2$.

$$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + \frac{y}{x} + \frac{x}{z} + \frac{z}{y} = \frac{n}{n+1} + \frac{n+1}{n+2} +$$

$$+ \frac{n+2}{n} + \frac{n+1}{n} + \frac{n}{n+2} + \frac{n+2}{n+1} =$$

$$= \frac{n+n+2}{n+1} + \frac{n+1+n}{n+2} + \frac{n+2+n+1}{n} =$$

$$= 2 + 2 - \frac{3}{n+2} + 2 + \frac{3}{n} = 6 - \frac{3}{n+2} + \frac{3}{n}$$

$6 - \frac{3}{n+2} + \frac{3}{n}$ butun son bo'lishi uchun $n = 1$ bo'lishi kerak.
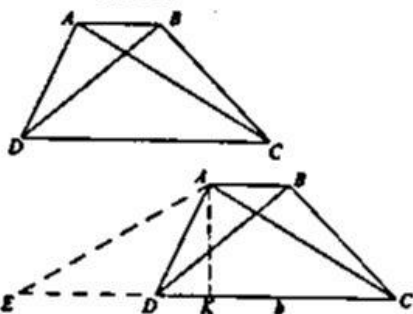$n = 1$ da $x = 1, y = 2, z = 3$
$x + y + z = 6$.

Javob: 6.

10. Qarang: 24-variant 29-savol (174-bet).

11. Trapetsiya asoslari 5 va 15, diagonallari 12 va 16 ga teng bo'lsa, uning yuzini toping.

Yechish:

ABCD trapetsiya

$AB = 5$ sm

$CD = 15$ sm

$BD = 12$ sm

$AC = 16$ sm.

$S = ?$

1) ABCE parallelogramm

$AB \| ED, AB = ED = 5$

$AE \| BD, AE = BD = 12$.

2) ACE uchburchak yuzini topamiz:

$a = AE = 12$

$b = AC = 16$

$c = CE = 20$

$\triangle ACE$ to'g'ri burchakli, chunki

$c^2 = a^2 + b^2$

$S_\triangle = \dfrac{a \cdot b}{2} = \dfrac{12 \cdot 16}{2} = 96$.

3) ACE uchburchak balandligi AK, ABCD trapetsiyaning ham balandligi bo'ladi:

$S_\triangle = \dfrac{CE \cdot AK}{2} = \dfrac{AB + CD}{2} \cdot AK = 96$

$S_{ABCD} = \dfrac{AB + CD}{2} \cdot AK = 96$.

Javob: 96.

12. Hisoblang:

$7,5 + 9,8 + 12,1 + \ldots + 53,5 + 55,8 + 58,1$.

Yechish:

$7,5 + 9,8 + 12,1 + \ldots + 53,5 + 55,8 + 58,1$.

1) $a_1 = 7,5$

$a_2 = 9,8$

$d = a_2 - a_1 = 98 - 7,5 = 2,3$.

2) $a_n = a_1 + d(n - 1)$,

$58,1 = 7,5 + 2,3(n - 1)$

$50,6 = 2,3(n - 1)$

$n - 1 = 22, n = 23$.

3) $S_{23} = \dfrac{a_1 + a_{23}}{2} \cdot 23 =$

$= \dfrac{7,5 + 58,1}{2} \cdot 23 = 754,4$.

Javob: 754,4.

13. Qarang: 8-variant 10-savol (61-bet).

14. Asosi 2 ga teng bo'lgan to'rtta sonning logarifmlari arifmetik progressiyani tashkil etadi. Birinchi va to'rtinchi hadlari ko'paytmasi – 8 ga, o'rta hadlar ko'paytmasi 0 ga teng. Shu sonlarni toping.

Yechish:

a_1, a_2, a_3, a_4 – arifmetik progressiya hadlari.

a, b, c, d sonlar.

$a_1 = \log_2 a$

$a_2 = \log_2 b$

$a_3 = \log_2 c$

$a_4 = \log_2 d$

1) $\begin{cases} a_1 \cdot a_4 = -8 \\ a_2 \cdot a_3 = 0 \end{cases} \Rightarrow \begin{cases} \log_2 a \cdot \log_2 d = -8 \\ \log_2 b \cdot \log_2 c = 0 \end{cases}$

2) $\log_2 b = 0$ yoki $\log_2 c = 0$

$b = 1$ yoki $c = 1$

$a_2 = 0$ yoki $a_3 = 0$

$(a_2 - d)(a_2 + 2d) = -8$

$-2d^2 = -8, d^2 = 4, d = \pm 2$.

3) $d = 2, a_2 = 0$ bo'lsa $a_1 = -2, a_3 = 2, a_4 = 4$

$d = -2, a_2 = 0$ bo'lsa $a_1 = 2$,

$a_3 = -2, a_4 = -4$.

4) $a_1 = -2, a_2 = 0, a_3 = 2, a_4 = 4$

$\log_2 a = -2, a = \dfrac{1}{4}$

$\log_2 b = 0, b = 1$

$\log_2 c = 2, c = 4$

$\log_2 d = 4, d = 16$.

5) $a_1 = 2, a_2 = 0, a_3 = -2, a_4 = -4$

$\log_2 a = 2, a = 4$

$\log_2 b = 0, b = 1$

$\log_2 c = -2, c = \dfrac{1}{4}$

$log_2 d = -4$, $d = \dfrac{1}{16}$.

Javob: 4; 1; $\dfrac{1}{4}$; $\dfrac{1}{16}$ va $\dfrac{1}{4}$; 1; 4; 16.

15. Qarang: 16-variant 9-savol (118-bet).

16. Qarang: 7-variant 12-savol (54-bet).

17. Qarang: 26-variant 29-savol (183-bet).

18. Musbat sonlardan tashkil topgan a_1, a_2, a_3 ... ketma-ketlik uchun $a_1 = a_2 = 1$ va barcha natural n larda $a_{n+2} = a_n a_{n+1}$ shartlar bajarilsin. Ketma-ketlikning 200-hadini toping.

Yechish:

$a_1 = a_2 = 1$, $n \in N$

$a_{n+2} = a_n \cdot a_{n+1}$, $a_{200} = ?$

$a_3 = a_1 \cdot a_2 = 1$

$a_4 = a_3 \cdot a_2 = 1$

Demak $a_{200} = a_{198} \cdot a_{199} = 1$.

Javob: 1.

19. Shar bilan tekislikning kesishishida hosil bo'lgan kesim yuzasi 25π ga teng. Sharning hajmi $\dfrac{8788\pi}{3}$ ga teng bo'lsa, shu kesimdan shar markazigacha bo'lgan masofani toping.
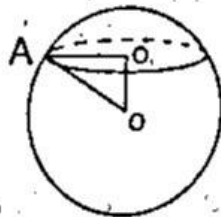
Yechish:

$S_{kesim} = 25\pi$

$V = \dfrac{8788}{3}\pi$,

$OA = R$, $O_1 A = r$

$OO_1 = ?$

1) $S_{kesim} = \pi r^2 = 25\pi$, $r = 5$.

2) $V = \dfrac{4}{3}\pi R^3 = \dfrac{8788}{3}\pi$,

$R^3 = 2197 = 13^3$, $R = 13$.

3) $OO_1 = \sqrt{R^2 - r^2} = \sqrt{13^2 - 5^2} =$

$= \sqrt{169 - 25} = \sqrt{144} = 12$.

Javob: 12.

20. Qarang: 11-variant 24-savol (86-bet).

21. Qarang: 19-variant 11-savol (137-bet).

22. Qarang: 25-variant 4-savol (175-bet).

23. Qarang: 21-variant 9-savol (151-bet).

24. Agar $x^2 + mx + m^2 + a = 0$ tenglamaning ildizlari a va b bo'lsa, $a^2 + ab + b^2 + a$ ning qiymatini toping.

Yechish:

$x^2 + mx + m^2 + a = 0$.

Viyet teoremasiga ko'ra $\begin{cases} a + b = -m \\ a \cdot b = m^2 + a \end{cases}$

$a^2 + ab + b^2 + a = (a + b)^2 - ab + a =$
$= (-m)^2 - (m^2 + a) + a = m^2 - m^2 - a + a = 0$.

Javob: 0.

25. Qarang: 12-variant 13-savol (91-bet).

26. Qarang: 12-variant 30-savol (95-bet).

27. ABCD parallelogramm berilgan. M nuqta BD diagonalda yotadi, bunda MD:BM = 2:1. Agar ADCM to'rtburchak yuzi 24 ga teng bo'lsa, ABCD parallelogramm yuzini toping.

Berilgan: **Yechish:**

ABCD – parallelogramm

AC, BD – diagonallari
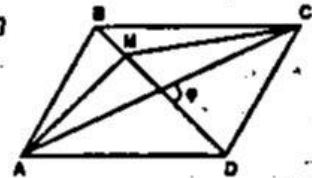
MD:BM = 2:1

BD = 3x

$S_{ADCM} = 24$

$S_{ABCD} = ?$

$S_{ABCD} = \dfrac{AC \cdot BD}{2} \cdot \sin\varphi = \dfrac{AC \cdot 3x}{2} \cdot \sin\varphi$

$S_{ADCM} = \dfrac{AC \cdot 2x}{2} \cdot \sin\varphi$

$\dfrac{S_{ABCD}}{S_{ADCM}} = \dfrac{AC \cdot 3x}{2} \cdot \sin\varphi : \dfrac{AC \cdot 2x}{2} \cdot \sin\varphi = \dfrac{3}{2}$
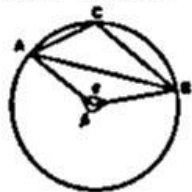
$S_{ABCD} = \dfrac{3}{2} S_{ADCM} = \dfrac{3}{2} \cdot 24 = 36$

Javob: 36.

28. Aylanaga o'tkazilgan vatar uni 4:8 nisbatda bo'ladi. Ushbu vatarga tiralgan, aylanaga ichki chizilgan katta burchakni toping.

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Yechish:

AB – vatar

$\alpha : \beta = 4:8$, $\alpha = 4x$, $\beta = 8x$
$\alpha + \beta = 360°$, $12x = 360°$,
$x = 30°$
$\alpha = 4 \cdot 30° = 120°$,
$\beta = 8 \cdot 30° = 240°$

$$\angle ACB = \frac{\beta}{2} = \frac{240°}{2} = 120°$$

Javob: 120°.

34. A="IO.SYS – ma'lumotlani kiritish-chiqarish tizimining kengaytirish moduli."
B="Free and Open Source Software – mutlaqo bepul, birlamchi kodi ochiq dasturiy ta'minot."
C="FoxPro – operatsion sistemadir." Shu mulohazalar asosida quyuагi mantiqiy ifodaning natijasini toping: (A or B) and (not B or C).

Yechish:

Mulohazalarni tahlil qilamiz:
A="IO.SYS – ma'lumotlani kiritish-chiqarish tizimining kengaytirish moduli". – rost (1)
B="Free and Open Source Software – mutloqo bepul, birlamchi kodi ochiq dasturiy ta'minot". – rost (1)
C="FoxPro –operatsion sistemadir". – yolg'on (0)
Shu mulohazalar qiymatlarini mantiqiy ifodaga qo'yamiz:
(A or B) and (not B or C) = (1 or 1) and (not 1 or 0) = 1 and 0 = 0.

Javob: yolg'on.