

2017-yil matematika variant yechimlari (spectrum)

21-variant

Bizning kanal : @axborotnoma

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Reklama xizmati : @axborotnoma_reklama

21-variant

1. $R = 16$ va $r = 4$ radiusli ikkita aylana bir-biriga va to'g'ri chiziqqa urinadi. Shu to'g'ri chiziqqa va aylanalarga urinadigan kichik aylana radiusini toping.



Berilgan:

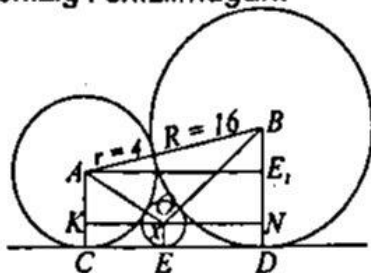
$$AC = 4 = r$$

$$BD = 16 = R$$

$$OE = ?$$

Yechish:

Bu rasmda: 1) O nuqta belgilinmagan; 2) KN chizig'i chizilmagan.



$$OE = x, AO = 4 + x, BO = 16 + x$$

$$AB = 20, CD = AE_1$$

$$E_1B = BD - AC = 16 - 4 = 12, AB = 20$$

$\triangle AE_1B$ to'g'ri burchakli

$$AE_1^2 = AB^2 - E_1B^2 = 20^2 - 12^2 = 16^2$$

$$AE_1 = 16$$

$$\triangle AOK \text{ da } OK = \sqrt{AO^2 - AK^2}$$

$$OK = \sqrt{(4+x)^2 - (4-x)^2} = \sqrt{16x} = 4\sqrt{x}$$

$$\triangle BON \text{ da } ON = \sqrt{OB^2 - BN^2}$$

$$ON = \sqrt{(16+x)^2 - (16-x)^2} = \sqrt{64x} = 8\sqrt{x}$$

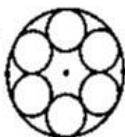
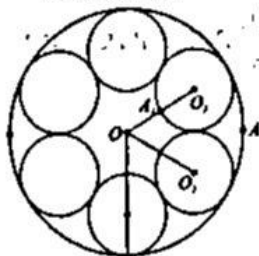
$$KO + ON = 16$$

$$4\sqrt{x} + 8\sqrt{x} = 16$$

$$12\sqrt{x} = 16, \sqrt{x} = \frac{4}{3}, x = \frac{16}{9}$$

$$\text{Javob: } \frac{16}{9}$$

2. Doiraning radiusi 3 dm, unga bir-biriga urinadigan 6 ta teng doira ichki chizilgan. Shu doiralarning diametrlarini toping.

**Yechish:**

$$OA = 3 \text{ dm}, OA = R,$$

$$A_1O_1 = r, OA_1 = x$$

$$\triangle OO_1O_2 \text{ da}$$

$$\angle O_1OO_2 = 60^\circ \text{ va}$$

$$OO_1 = OO_2 \text{ bundan}$$

$$\angle OO_1O_2 = \angle OO_2O_1 =$$

$$= 60^\circ \triangle OO_1O_2$$

$$\text{muntazam uchburchak } R = x + 2r = 3$$

$$x + r = 2r$$

$$x = r$$

$$3r = 3, r = 1, 2r = 2 \cdot 1 = 2.$$

$$\text{Javob: } 2.$$

3. $|\vec{a}| = 3$ va $|\vec{b}| = 4$ hamda vektorlar orasidagi burchak 60° ga teng bo'lsa, $\vec{c} = 3\vec{a} + 2\vec{b}$ vektorining uzunligini toping.

Yechish:

$$|\vec{a}| = 3, |\vec{b}| = 4, (\vec{a}; \vec{b}) = 60^\circ$$

$$\vec{c} = 3\vec{a} + 2\vec{b}, |\vec{c}| = ?$$

$$c^2 = (3\vec{a} + 2\vec{b})^2 = 9\vec{a}^2 + 12\vec{a}\vec{b} + 4\vec{b}^2$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos 60^\circ = 3 \cdot 4 \cdot \frac{1}{2} = 6$$

$$c^2 = 9 \cdot 3^2 + 12 \cdot 6 + 4 \cdot 4^2 = 81 + 72 + 64 = 217$$

$$|\vec{c}| = \sqrt{217}$$

$$\text{Javob: } \sqrt{217}.$$

4. Qarang: 19-variant 25-savol (140-bet).

5. Hisoblang:
$$\frac{25 \cdot (180 \cdot 6^7 - 108 \cdot 6^6)}{216^3 - 36^4}$$

Yechish:

$$\frac{25 \cdot (180 \cdot 6^7 - 108 \cdot 6^6)}{216^3 - 36^4} = \frac{25 \cdot 6^6 (180 \cdot 6 - 108)}{6^9 - 6^8} =$$

$$= \frac{25 \cdot 6^6 (36 \cdot 5 \cdot 6 - 36 \cdot 3)}{6^8 (6 - 1)} =$$

$$= \frac{25 \cdot 6^6 (30 - 3)}{6^8 \cdot 5} = 5 \cdot 27 = 135.$$

$$\text{Javob: } 135.$$

6. Arifmetik progressiyaning ikkinchi va o'n yettinchi hadlari yig'indisi 14 ga, o'n to'qqizinchi va o'n yettinchi hadlari ayirmasi 6 ga teng. Progressiyaning dastlabki yigirmata hadi yig'indisini toping.

Yechish:

$$a_2 + a_{17} = 14, a_{19} - a_{17} = 6,$$

$$S_{20} = ?$$

$$S_{20} = \frac{a_1 + a_{20}}{2} \cdot 20$$

$$a_1 + a_{20} = a_2 + a_{19}$$

$$+ a_2 + a_{17} = 14$$

$$+ a_{19} - a_{17} = 6$$

$$a_2 + a_{19} = 20$$

$$S_{\infty} = \frac{20}{2} \cdot 20 = 200$$

Javob: 200.

7. $\frac{10-4x}{|x+1|} < 2$ tengsizlikning eng kichik butun yechimini toping.

Yechish:

1) aniqlanish sohasi $|x+1| \neq 0, x \neq -1$.

$$2) \frac{10-4x}{|x+1|} < 2, \frac{5-2x}{|x+1|} < 1$$

$$\frac{5-2x-|x+1|}{|x+1|} < 0.$$

$|x+1|$ doimo musbat bo'lganligi sababli $5-2x-|x+1| < 0$.
 $|x+1| > 5-2x$.

$$3) \begin{cases} x+1 > 0 \\ x+1 > 5-2x \end{cases} \Rightarrow \begin{cases} x > -1 \\ x > \frac{4}{3} \end{cases} \Rightarrow x > \frac{4}{3}$$

$$\begin{cases} x+1 < 0 \\ -x-1 > 5-2x \end{cases} \Rightarrow \begin{cases} x < -1 \\ x > 6 \end{cases} \rightarrow \emptyset$$

4) $x > \frac{4}{3}, x \in (\frac{4}{3}; \infty)$ tengsizlikning eng kichik butun yechimi.

Javob: 2.

8. Qarang: 7-variant 8-savol (53-bet).

9. Teng yonli ABCD trapetsiyada AC diagonal CD tomonga perpendikulyar. Agar $AD = 4, AB^2 + BC^2 = 11$ bo'lsa, AB ni toping.

Berilgan:

ABCD – teng yonli

trapetsiya

$AC \perp CD$

$AB = CD$

$AD = 4$

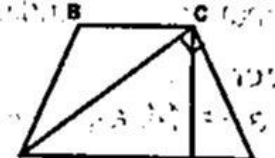
$AB^2 + BC^2 = 11$

$AB = ?$

$$AE = \frac{4+a}{2}, ED = \frac{4-a}{2}$$

$$2) CE = \sqrt{\frac{4+a}{2} \cdot \frac{4-a}{2}} = \sqrt{\frac{16-a^2}{4}}$$

Yechish:



$$1) BC = a, CD = c$$

$$a^2 + c^2 = 11$$

$$3) CD^2 = CE^2 + ED^2$$

$$CE = \sqrt{c^2 - \left(\frac{4-a}{2}\right)^2} = \sqrt{11-a^2 - \left(\frac{4-a}{2}\right)^2}$$

$$4) \frac{16-a^2}{4} = 11-a^2 - \left(\frac{4-a}{2}\right)^2$$

$$16-a^2 = 44-4a^2-16+8a-a^2$$

$$4a^2-8a-12=0, a^2-2a-3=0$$

$$a = -1, a = 3$$

$$5) c^2 = 11-a^2 = 11-3^2 = 11-9 = 2$$

$$c = \sqrt{2}$$

Javob: $\sqrt{2}$.

10. Musbat x, y sonlar uchun $a = 7x + \frac{1}{7}y$

va $b = 2\sqrt{xy}$ bo'lsin. Qaysi tengsizlik har doim o'rinli?

Yechish:

$$x > 0, y > 0, a = 7x + \frac{1}{7}y, b = 2\sqrt{xy}$$

Sonlarning o'рта arifmetigi o'рта geometrigidan kichik emas.

$$\frac{x+y}{2} \geq \sqrt{xy} \Rightarrow x+y \geq 2\sqrt{xy}$$

Shunga asosan

$$a = 7x + \frac{1}{7}y \geq 2\sqrt{7x \cdot \frac{1}{7}y} = 2\sqrt{xy} = b$$

$a \geq b$.

Javob: $a \geq b$.

11. Agar $x = 3$ bo'lsa, $\frac{(x-b)(x-c)}{(a-b)(a-c)} +$

$$+ \frac{(x-a)(x-c)}{(b-a)(b-c)} + \frac{(x-a)(x-b)}{(c-a)(c-b)}$$
 ning qiymatini

toping. (Bu yerda $(a-b)(a-c)(b-c) \neq 0$)

Yechish:

$$1) x = 3 \text{ da}$$

$$\frac{(3-b)(3-c)}{(a-b)(a-c)} + \frac{(3-a)(3-c)}{(b-a)(b-c)} + \frac{(3-a)(3-b)}{(c-a)(c-b)}$$

$$= \frac{(9-3(b+c)+bc) \cdot (b-c) - (9-3(a+c)+ac) \cdot (a-c) + (9-3(a+b)+ab)(a-b)}{(a-b)(a-c)(b-c)}$$

Suratini soddalashtiramiz:

$$2) 9b - 9c - 3b^2 + 3c^2 + b^2c - bc^2 - 9a + 9c + 3a^2 - 3c^2 - a^2c + ac^2 + 9a - 9b - 3a^2 + 3b^2 + a^2b - ab^2 = b^2c - bc^2 - a^2c + ac^2 + a^2b - ab^2 = (a-b)(a-c)(b-c).$$

$$3) \frac{(a-b)(a-c)(b-c)}{(a-b)(a-c)(b-c)} = 1.$$

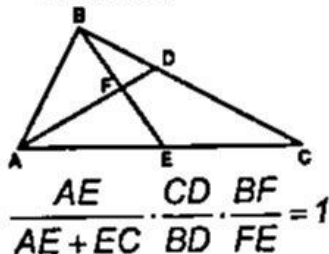
Javob: 1.

12. Berilgan ABC uchburchakda E nuqta AC tomonning o'rtasi. BC tomonda D nuqta shunday olinadiki $2BD = DC$ munosabat o'rinli. AD va BE to'g'ri chiziqlar F nuqtada kesishsin. Agar FDCE to'rtburchakning yuzasi 15 ga teng bo'lsa, BDF uchburchak yuzasini toping.

Berilgan:

$\triangle ABC$
 $E \in AC$
 $AE = EC$
 $D \in BC$
 $2BD = DC$
 $S_{FDCE} = 15$
 $S_{BDF} = ?$

Yechish:



$$AE = y, BD = y, CD = 2x$$

$$BF = m, FE = n$$

$$\frac{x}{2x} \cdot \frac{2y}{y} \cdot \frac{m}{n} = 1, \frac{m}{n} = 1, m = n$$

$$S_{BFD} = \frac{x \cdot m}{2} \cdot \sin \alpha$$

$$S_{BEC} = \frac{3x \cdot 2m}{2} \cdot \sin \alpha$$

$$S_{BEC} = S_{BFD} + 15$$

$$\frac{S_{BFD}}{S_{BFD} + 15} = \frac{1}{6}, 6S_{BFD} = S_{BFD} + 15$$

$$5S_{BFD} = 15, S_{BFD} = 3.$$

Javob: 3.

13. Qarang: 14-variant 22-savol (108-bet).

14. Agar $x = \frac{\sqrt{11}+1}{2}$ bo'lsa,

$\frac{x^3 - 3x^2 + 6,5x - 2}{x^2 - x + 1}$ kasrning qiymatini hisoblang.

Yechish:

$$x = \frac{\sqrt{11}+1}{2}, \frac{x^3 - 3x^2 + 6,5x - 2}{x^2 - x + 1} \text{ ifodani}$$

soddalashtirib olamiz.

$$1) \frac{x^3 - 3x^2 + 6,5x - 2}{x^2 - x + 1} = x - 2 + \frac{3,5x}{x^2 - x + 1} =$$

$$= x - 2 + \frac{3,5x}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$2) x = \frac{\sqrt{11}+1}{2} \text{ da}$$

$$\frac{\sqrt{11}+1}{2} - 2 + \frac{3,5 \cdot \frac{\sqrt{11}+1}{2}}{\left(\frac{\sqrt{11}+1}{2} - \frac{1}{2}\right)^2 + \frac{3}{4}} =$$

$$= \frac{\sqrt{11}-3}{2} + \frac{3,5 \cdot \frac{\sqrt{11}+1}{2}}{3,5} =$$

$$= \frac{\sqrt{11}-3}{2} + \frac{\sqrt{11}+1}{2} = \sqrt{11}-1$$

Javob: $\sqrt{11}-1$.

15. $\left[\frac{1000}{8^2}\right] \cdot 8$ ni hisoblang. Bu yerda

[a] – a sonning butun qismi.

Yechish:

[a] – a sonning butun qismi.

$$\left[\frac{1000}{8^2}\right] \cdot 8 = \left[\frac{1000}{8^2} \cdot 8\right] = \left[\frac{1000}{8}\right] = 125$$

Javob: 125.

16. Qarang: 14-variant 3-savol (106-bet).

17. Beshburchak burchaklari $1:1,5:2:\frac{5}{2}:3$

nisbatda bo'lsa, eng katta burchagini toping.

Yechish:

Beshburchak ichki burchaklari yig'indisi 540° ga teng.

$$a_1 + a_2 + a_3 + a_4 + a_5 = 540^\circ$$

$$a_1 = x, a_2 = 1,5x, a_3 = 2x, a_4 = 2,5x, a_5 = 3x$$

$$x + 1,5x + 2x + 2,5x + 3x = 540^\circ$$

$$10x = 540^\circ, x = 54^\circ$$

Eng katta burchak $3 \cdot x = 3 \cdot 54^\circ = 162^\circ$.

Javob: 162° .

18. Qarang: 14-variant 12-savol (107-bet).

19. Qarang: 14-variant 25-savol (109-bet).

20. $2\sin^2x + \sinx \cdot \cosx = 1$ bo'lsa,
 $\lg 2x$ ning qiymatini toping.

Yechish:

$$2\sin^2x + \sinx \cdot \cosx = 1$$

$$1) \sinx \cdot \cosx = \frac{1}{2} \cdot 2\sinx \cdot \cosx = \frac{1}{2} \sin 2x$$

$$2) 1 - 2\sin^2x = \cos 2x$$

$$3) \sinx \cdot \cosx = 1 - 2\sin^2x$$

$$\frac{1}{2} \sin 2x = \cos 2x,$$

$$\lg 2x = 2.$$

Javob: 2.

21. Qarang: 2-variant 26-savol (17-bet).

22. Arifmetik progressiyada 10-hadi 7 ga,
 7-hadi esa 10 ga teng. Progressiyaning
 3-hadini toping.

Yechish:

$$a_{10} = 7, a_7 = 10, a_3 = ?$$

$$a_{10} - a_7 = 3d, d = \frac{a_{10} - a_7}{3} = \frac{7 - 10}{3} = -1$$

$$a_7 - a_3 = 4d, a_3 = a_7 - 4d = 10 - 4 \cdot (-1) = 14.$$

Javob: 14.

23. a_1, a_2, \dots va b_1, b_2, \dots - arifmetik
 progressiyalar uchun $a_1 = 2,5; b_1 = 7,5;$
 $a_{400} + b_{400} = 10$ bo'lsin. $a_1 + b_1, a_2 + b_2, \dots$
 ketma-ketlikning dastlabki 400 ta hadlar
 yig'indisini toping.

Yechish:

a_1, a_2, \dots va b_1, b_2, \dots - arifmetik progressiya.

$$a_1 = 2,5, b_1 = 7,5$$

$$a_{400} + b_{400} = 10$$

$$a_1 + b_1, a_2 + b_2, \dots$$

$$1) c_1 = a_1 + b_1, c_2 = a_2 + b_2$$

$$S_{400} = \frac{c_1 + c_{400}}{2} \cdot 400$$

$$2) c_1 = a_1 + b_1 = 2,5 + 7,5 = 10$$

$$c_{400} = a_{400} + b_{400} = 10$$

$$3) S_{400} = \frac{10 + 10}{2} \cdot 400 = 4000$$

Javob: 4000.

24. Funksiya nollari yig'indisini toping:

$$f(x) = \left(\frac{x^2}{2} - 3x + 4 \right) \cdot \log_5(8 - 3x).$$

Yechish:

Aniqlanish sohasini topamiz:

$$8 - 3x > 0, x < \frac{8}{3} = 2\frac{2}{3}.$$

Funksiya nollarini topamiz: $\frac{x^2}{2} - 3x + 4 = 0$.

va $\log_5(8 - 3x) = 0$.

$$1) x^2 - 6x + 8 = 0, x = 2, x = 4;$$

$$2) \log_5(8 - 3x) = 0,$$

$$8 - 3x = 5^0$$

$$8 - 3x = 1$$

$$-3x = -7$$

$$x = \frac{7}{3} = 2\frac{1}{3}.$$

Aniqlanish sohasiga $x = 2$ va $x = 2\frac{1}{3}$

yechimlar tegishli:

$$2 + 2\frac{1}{3} = 4\frac{1}{3}.$$

Javob: $4\frac{1}{3}$.

25. Qarang: 18-variant 16-savol (132-bet).

26. $\frac{-9\sin 136^\circ}{\cos 68^\circ \cdot \cos 22^\circ}$ ifodaning qiymatini
 toping.

Yechish:

1) ko'paytmadan yig'indiga o'tish
 formulasiga asosan $\cos x \cdot \cos y =$

$$= \frac{1}{2} (\cos(x - y) + \cos(x + y))$$

$$\begin{aligned} \cos 68^\circ \cdot \cos 22^\circ &= \frac{1}{2} (\cos(68^\circ - 22^\circ) + \\ &+ \cos(68^\circ + 22^\circ)) = \\ &= \frac{1}{2} (\cos 46^\circ + \cos 90^\circ) = \frac{1}{2} \cos 46^\circ \end{aligned}$$

2) keltirish formulasiga asosan
 $\sin 136^\circ = \sin(90^\circ + 46^\circ) = \cos 46^\circ$

$$3) \frac{-9 \sin 136^\circ}{\frac{1}{2} \cos 46^\circ} = \frac{-18 \cos 46^\circ}{\cos 46^\circ} = -18$$

Javob: -18.

27. A(-3; 2; 1) va B(5; -2; 7) bo'lsa, AB kesmani AC:CB = 2:1 nisbatda bo'luvchi C nuqtaning koordinatalarini toping.

Yechish:

A(-3; 2; 1), B(5; -2; 7)
 AC:CB = 2:1, C(x; y; z) = ?
 m = 2, n = 1,

$$x = \frac{mx_2 + nx_1}{m+n}; y = \frac{my_2 + ny_1}{m+n}$$

$$z = \frac{mz_2 + nz_1}{m+n}$$

$$x_1 = -3, x_2 = 5.$$

$$y_1 = 2, y_2 = -2; z_1 = 1, z_2 = 7.$$

$$x = \frac{5 \cdot 2 + (-3)}{2+1} = \frac{7}{3}$$

$$y = \frac{-2 \cdot 2 + 2}{3} = -\frac{2}{3}$$

$$z = \frac{2 \cdot 7 + 1}{3} = 5$$

$$C\left(\frac{7}{3}; -\frac{2}{3}; 5\right).$$

Javob: $\left(\frac{7}{3}; -\frac{2}{3}; 5\right)$

28. Qarang: 7-variant 16-savol (55-bet).

29. $2x - 3\sqrt{2x-1} + 1 = 0$ tenglamaning katta ildizining kichik ildiziga nisbatini toping.

Yechish:

$$2x - 3\sqrt{2x-1} + 1 = 0$$

$$3\sqrt{2x-1} = 2x + 1$$

1) aniqlanish sohasi:

$$\begin{cases} 2x - 1 \geq 0 \\ 2x + 1 \geq 0 \end{cases} \Rightarrow \begin{cases} x \geq \frac{1}{2} \\ x \geq -\frac{1}{2} \end{cases} \Rightarrow x \geq \frac{1}{2}$$

2) tenglikning ikki qismini kvadratga oshiramiz:

$$\begin{aligned} 9(2x - 1) &= (2x + 1)^2 \\ 18x - 9 &= 4x^2 + 4x + 1 \\ 4x^2 - 14x + 10 &= 0 \end{aligned}$$

$$2x^2 - 7x + 5 = 0, x = 1, x = \frac{5}{2}$$

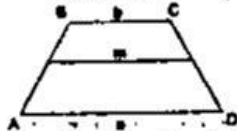
3) $x \geq \frac{1}{2}$ bo'lganligi sababli $x = 1$ va $x = 2,5$ tenglama yechimi bo'ladi.

$$4) \frac{x_{\max}}{x_{\min}} = \frac{2,5}{1} = 2,5$$

Javob: 2,5.

30. Trapetsiyaning parallel tomonlari a va b ga teng. Bu tomonlarga parallel bo'lgan va trapetsiya yuzasini teng ikkiga ajratgan kesma uzunligini toping.

Yechish:



ABCD trapetsiya

AD = a,

BC = b

$$1) S_1 = \frac{b+m}{2} \cdot h_1$$

$$S_2 = \frac{a+m}{2} \cdot h_2$$

$$S_1 = S_2 = \frac{S}{2}$$

$$S = \frac{a+b}{2} \cdot (h_1 + h_2)$$

$$2) \frac{b+m}{2} \cdot h_1 = \frac{a+m}{2} \cdot h_2, \frac{h_1}{h_2} = \frac{a+m}{b+m}$$

$$3) \frac{a+b}{2} \cdot (h_1 + h_2) = 2 \cdot \frac{(b+m) \cdot h_1}{2}$$

$$(a+b) \cdot h_1 - 2(b+m) \cdot h_1 = -(a+b) \cdot h_2$$

$$h_1(a-b-2m) = -(a+b) \cdot h_2$$

$$\frac{h_1}{h_2} = \frac{a+b}{2m+b-a}$$

$$4) \frac{a+m}{b+m} = \frac{a+b}{2m+b-a}$$

$$2am + 2m^2 + ab + bm - a^2 - am =$$

$$= ab + b^2 + am + bm$$

$$2m^2 = b^2 + a^2$$

$$m^2 = \frac{a^2 + b^2}{2}, m = \sqrt{\frac{a^2 + b^2}{2}}$$

Javob: $\sqrt{\frac{a^2 + b^2}{2}}$.

31. Qarang: 20-variant 35-savol (148-bet).

32. «Windows 10» iborasi 12 marta takroran yozilganda, uni saqlash uchun qancha xotira hajmi zarur bo'ladi? (Qo'shtimoq hisobga olinmasin)

Yechish:

Har bir simvolni, shu jumladan probelni ham saqlash uchun 1 bayt, ya'ni 8 bit kerak. «Windows 10» iborasida jami bo'lib 10 ta simvol ishtirok etgan. 12 marta takrorlab yozilganda uni saqlash uchun $12 \cdot 10 \cdot 8 \text{ bit} = 960 \text{ bit}$ xotira hajmi zarur bo'ladi.

Javob: 960 bit.

33. 240, 301, 220, 332 butun sonlarning barchasini yozish mumkin bo'lgan eng kichik asosli sanoq sistemasida shu sonlar yig'indisini aniqlang.

Yechish:

240, 301, 220, 332 butun sonlarda ishtirok etgan raqamlarning eng kattasi 4 ga teng. Demak, ularning barchasini yozish mumkin bo'lgan eng kichik asosli sanoq sistemasi 5-lik sanoq sistemasidir. Yig'indini ketma-ket hisoblaymiz:

$$\begin{array}{r} 240_5 \\ + 301_5 \\ \hline 1041_5 \end{array} \quad \begin{array}{r} 1041_5 \\ + 220_5 \\ \hline 1311_5 \end{array} \quad \begin{array}{r} 1311_5 \\ + 332_5 \\ \hline 2143_5 \end{array}$$

Javob: 2143.

34. Qarang: 2-variant 35-savol (19-bet).

35. Qarang: 3-variant 35-savol (27-bet).

36. "Axborot tizimi" nima?

Yechish:

Keng ma'noda axborot tizimi deyilganda uning ajralmas komponentlari ma'lumotlar, texnik va dasturiy ta'minot, hamda personal va tashkiliy chora-tadbirlar tushuniladi.

Javob: belgilangan maqsadga erishish uchun axborotlarni uzatish, qayta ishlash va saqlashda qo'llaniladigan usullar, shaxslar va vositalarning o'zaro bog'langan majmuasi.