2017-yil matematika variant yechimlari (spectrum)

20-variant

Bizning kanal : @axborotnoma

Adminsratorlar hayati : @axborotnoma_bot

Matematika yordam guruhi : @axborotnomaguruhi

Reklama xizmati : @axborotnoma_reklama

20-variant

1. Qarang: 19-variant 21-savol (139-bet).

2. Balandligi 6 ga va asosining radiusi 2 ga teng bo'lgan konus sharga ichki chizilgan. Shar sirtining yuzini toping.

Berilgan:
$SO_1 = H = 6$
$AO_1 = R = 2$
$S = ?$

Yechish:

$S = 4\pi r^2$, $r = OA$, $SA = SB = b$

$$r = \frac{b^2 \cdot 2R}{4 \cdot \frac{2R \cdot H}{2}} = \frac{b^2}{2H}$$

$$b^2 = H^2 + R^2 = 36 + 4 = 40$$

$$r = \frac{40}{2 \cdot 6} = \frac{10}{3}$$

$$S = 4\pi \cdot \left(\frac{10}{3}\right)^2 = \frac{400\pi}{9} = 44\frac{4}{9}\pi$$

Javob: $44\frac{4}{9}\pi$

3. Natural n sonning kvadrati 3 ga bo'linganda qanday qoldiqlarga ega bo'lishi mumkin.

Yechish:

Natural n sonning kvadratini 3 ga bo'lganda qoldiqlar 1 va 2 bo'lishi mumkin.

Javob: 1 va 2.

4. $y = 5\sin^2 x + 4\sin x \cos x + 2\cos^2 x$ funksiyaning eng katta va eng kichik qiymatlarini toping.

Yechish:

1) $\sin^2 x = \frac{1-\cos 2x}{2}$, $\cos^2 x = \frac{1+\cos 2x}{2}$

$4\sin x \cos x = 2\sin 2x$

2) $5\frac{1-\cos 2x}{2} + 2\sin 2x + 2\frac{1+\cos 2x}{2} =$

$= \frac{5}{2} - \frac{5}{2}\cos 2x + 2\sin 2x + 1 + \cos 2x =$

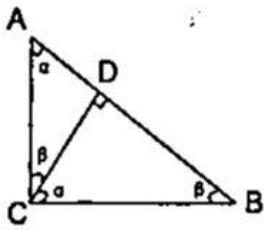
$= 2\sin 2x - 1{,}5\cos 2x + 3{,}5$

3) $-\sqrt{4+2,25} \le 2\sin 2x - 1,5\cos 2x \le$
$\le \sqrt{4+2,25}$
$-2,5 \le 2\sin 2x - 1,5\cos 2x \le 2,5$
4) $y_{min} = -2,5 + 3,5 = 1$
$y_{max} = 2,5 + 3,5 = 6.$

Javob: 6 va 1.

5. O'tkir burchagi 75° ga, balandligi esa 2 ga teng bo'lgan to'g'ri burchakli uchburchakning gipotenuzasini toping.

Yechish:



ΔACB to'g'ri burchakli
$\angle C = 90°$,
$\angle A = 75°$,
$CD = 2$
$AB = ?$

1) $\alpha = 75°, \beta = 15°$
ΔADC va ΔCDB to'g'ri burchakli va o'xshash.
2) $AB = AD + DB = CD \cdot ctg75° + CD \cdot ctg15° =$
$= CD(ctg75° + ctg15°)$
3) $ctg75° + ctg15° = tg15° + ctg15° =$
$= \dfrac{1}{\sin 15° \cdot \cos 15°} = \dfrac{2}{\sin 30°} = 4.$
4) $AB = CD \cdot 4 = 2 \cdot 4 = 8.$

Javob: 8.

6. Qarang: 12-variant 26-savol (94-bet).

7. Qarang: 15-variant 17-savol (113-bet).

8. Hisoblang: $\dfrac{2:4^{-2} + (3^{-2})^3 \cdot (\frac{1}{9})^{-3}}{5^{-3} \cdot 25^2 + (0,7)^0 \cdot (\frac{1}{2})^{-2}}$.

Yechish:

$$\frac{2:4^{-2} + (3^{-2})^3 \cdot (\frac{1}{9})^{-3}}{5^{-3} \cdot 25^2 + (0,7)^0 \cdot (\frac{1}{2})^{-2}} = \frac{2 \cdot 4^2 + 3^{-6} \cdot 9^3}{5^{-3} \cdot 5^4 + 1 \cdot 2^2} =$$

$$= \frac{2^5 + 3^0}{5+4} = \frac{33}{9} = \frac{11}{3} = 3\frac{2}{3}.$$

Javob: $3\frac{2}{3}$.

9. Qarang: 7-variant 14-savol (54-bet).

10. Qarang: 13-variant 22-savol (102-bet).

11. x, y butun sonlar uchun $-5 \le x < 6$ va $-4 \le y < 5$ bo'lsa, $x^3 - y^2$ ning eng kichik qiymatini toping.

Yechish:
$x, y \in Z$
$-5 \le x < 6$
$-4 \le y < 5$
$x^3 - y^2 \to min$
x minimum qiymati -5.
$x^3 = (-5)^3 = -125$
$x^3 - y^2$ ayirma eng kichik bo'lishi uchun y^2 eng katta qiymatga ega bo'lishi kerak.
$y^2 = 5^2 = 25$
$x^3 - y^2 = -125 - 25 = -150.$

Javob: -150.

12. Qarang: 17-variant 5-savol (124-bet).

13. Qarang: 16-variant 11-savol (119-bet).

14. $f(x) = \cos(\sin((3x+5) \cdot \dfrac{\pi}{16}))$ funksiyaning $x_0 = 1$ nuqtadagi hosilasini toping.

Yechish:

$$f'(x) = (\cos(\sin((3x+5) \cdot \frac{\pi}{16})))' =$$

$$= -\sin(\sin((3x+5) \cdot \frac{\pi}{16})) \cdot \cos((3x+5) \cdot \frac{\pi}{16}) \cdot \frac{3\pi}{16}$$

$$f'(1) = -\sin(\sin 8 \cdot \frac{\pi}{16}) \cdot \cos(8 \cdot \frac{\pi}{16}) \cdot \frac{3\pi}{16} =$$

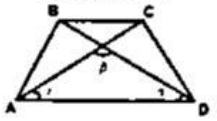
$$= -\sin 1 \cdot \cos\frac{\pi}{2} \cdot \frac{3\pi}{16} = 0.$$

Javob: 0.

15. ABCD teng yonli trapetsiyani AC diagonali 4 teng va u AD katta asos bilan 22,5° li burchak tashkil etadi. Trapetsiyaning yuzini toping.

Berilgan:
$ABCD$ – teng yonli trapetsiya
$AB = CD$

Yechish:



$AD \| BC$ | $AC = BD$
$AC = 4$ | $S = \frac{AC \cdot BD}{2} \cdot \sin\beta$
$\angle CAD = 22,5°$
$S = ?$

$\angle OAD = \angle ODA = 22,5°$
$\alpha = 22,5°$, $\angle AOD = \beta = 180° - 2\alpha =$
$= 180° - 2 \cdot 22,5° = 180° - 45° = 135°.$

$S = \frac{4 \cdot 4}{2} \cdot \sin 135° = 8\sin(\pi - 45°) =$

$= 8 \cdot \sin 45° = 8 \cdot \frac{\sqrt{2}}{2} = 4\sqrt{2}.$

Javob: $4\sqrt{2}$.

16. m, n natural sonlar $m^2 = n^2 + 229$ tenglikni qanoatlantirsa $2m - n$ ni toping.

Yechish:
$m, n \in N$, $m^2 = n^2 + 229$,
$2m - n = ?$
$m^2 - n^2 = 229$
229 tub son. $229 = 1 \cdot 229$
$(m - n)(m + n) = 1 \cdot 229$

$$\begin{cases} m - n = 1 \\ m + n = 229 \end{cases} \Rightarrow \begin{matrix} m = 115 \\ n = 114 \end{matrix}$$

$2m - n = 2 \cdot 115 - 114 = 116.$

Javob: 116.

17. Qarang: 11-variant 9-savol (83-bet).

18. $x^2 - (m - 1)x - 5 = 0$ tenglamaning x_1 va x_2 ildizlari orasida $x_1 + \frac{1}{x_2} = 2$ munosabat o'rinli. m ning qiymatini toping.

Yechish:
$x^2 - (m - 1)x - 5 = 0$, $x_1 + \frac{1}{x_2} = 2$, $m = ?$

1) Viyet teoremasiga ko'ra

$$\begin{cases} x_1 + x_2 = m - 1 \\ x_1 \cdot x_2 = -5 \end{cases}$$

2) $x_1 + \frac{1}{x_2} = 2$
$x_1 \cdot x_2 + 1 = 2x_2$,
$-5 + 1 = 2x_2$, $x_2 = -2$

$x_1 = \frac{-5}{x_2} = \frac{-5}{-2} = 2,5$

3) $x_1 + x_2 = m - 1$,
$m - 1 = 2,5 - 2 = 0,5$, $m = 1,5$.

Javob: 1,5.

19. $x^2 + 2017x + 2016 = 0$ tenglamaning yechimlari yig'indisini toping.

Yechish:
$x^2 + 2017x + 2016 = 0$ ildizlari yig'indisini topamiz. Viyet teoremasiga ko'ra

$$\begin{cases} x_1 + x_2 = -2017 \\ x_1 \cdot x_2 = 2016 \end{cases}$$

Javob: –2017.

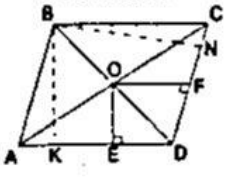
20. $3x^5 - x^4 - 3x + 1 =$
$= (x^2 + 1)(3x^3 + Ax^2 + Bx + C)$ bo'lsa,
$A + B + C$ ni toping.

Yechish:
1) $(x^2 + 1)(3x^3 + Ax^2 + Bx + C) =$
$= 3x^5 + Ax^4 + Bx^3 + Cx^2 + 3x^3 + Ax^2 + Bx + C =$
$= 3x^5 + Ax^4 + x^3(B + 3) + x^2(A + C) + Bx + C.$
2) $3x^5 - x^4 - 3x + 1 = 3x^5 + Ax^4 +$
$+ x^3(B + 3) + x^2(A + C) + Bx + C$
x^4: $A = -1$.
x^3: $B + 3 = 0$ $A = -1$
x^2: $A + C = 0$ $B = -3$ $\Rightarrow$
x^1: $B = -3$ $C = 1$
x^0: $C = 1$
$\Rightarrow A + B + C = -1 - 3 + 1 = -3.$

Javob: –3.

21. Parallelogrammning o'tkir burchagi α ga teng. Diagonallarning kesishish nuqtasidan teng bo'lmagan tomonlarigacha masofasi mos ravishda m va p ga teng. Parallelogramm yuzasini toping.

Yechish:



$ABCD$ – parallelogramm.
$\angle A = \angle C = \alpha$
$OE \perp AD$, $OE = m$
$OF \perp CD$, $OF = p$
$S = ?$

1) $OE = \frac{BK}{2}$, $BK = 2m$. $AB = a$.

ΔAKB dan $\sin\alpha = \frac{BK}{AB} = \frac{2m}{a}$, $a = \frac{2m}{\sin\alpha}$

2) $OF = \frac{BN}{2}$, $BN = 2p$, $BC = b$.

ΔBNC dan $\sin\alpha = \frac{BN}{BC} = \frac{2p}{b}$, $b = \frac{2p}{\sin\alpha}$

3) $S = a \cdot b \cdot \sin\alpha = \frac{2m}{\sin\alpha} \cdot \frac{2p}{\sin\alpha} \cdot \sin\alpha = \frac{4mp}{\sin\alpha}$.

Javob: $\frac{4mp}{\sin\alpha}$.

22. $y = \frac{\sqrt{2x-1}+\sqrt{x-1}}{x^2-5x+8}$ funksiyaning aniqlash sohasini toping.

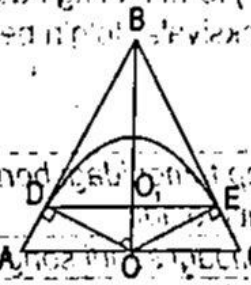
Yechish:

$$\begin{cases} 2x-1 \ge 0 \\ x-1 \ge 0 \\ x^2-5x+8 \ne 0 \end{cases} \Rightarrow \begin{cases} x \ge \frac{1}{2} \\ x \ge 1 \Rightarrow x \ge 1 \\ D < 0 \end{cases}$$

Javob: $[1; \infty)$.

23. Teng yonli uchburchakga markazi asosida joylashgan ichki yarim aylana chizilgan. Yon tomonining uzunligi 10 ga, asosi esa 12 ga teng. Shu aylananing yon tomonlari bilan urinish nuqtalar orasidagi masofani toping.

Yechish:



ΔABC teng yonli
$OD = OE = r$
$AB \perp OD$,
$BC \perp OE$
$AB = BC = 10$
$AC = 12$
$DE = ?$
1) $BD = BE$.
ΔAOB to'g'ri burchakli.

$$BO = \sqrt{AB^2 - AO^2} = \sqrt{AB^2 - \left(\frac{AC}{2}\right)^2} = \sqrt{10^2 - 6^2} = 8.$$

2) $OD = \frac{BO \cdot AO}{AB} = \frac{8 \cdot 6}{10} = 4{,}8$

3) $\angle BAO = \angle DOO_1 \Rightarrow \sin\alpha = \frac{8}{10} = \frac{4}{5}$.

$\sin\alpha = \frac{DO_1}{OD}$, $DO_1 = OD \cdot \sin\alpha = 4{,}8 \cdot \frac{4}{5}$

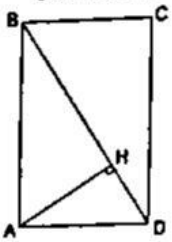
4) $DE = 2 \cdot DO_1 = 2 \cdot 4{,}8 \cdot 0{,}8 = 7{,}68$.

Javob: 7,68.

24. Qarang: 16-variant 1-savol (117-bet).

25. ABCD to'g'ri to'rtburchakda BD – diagonal, $H \in |BD|$, $|AH| \perp |BD|$, $|AH| = 12$, $|BC| = 15$ bo'lsa, uning yuzini toping.

Yechish:



ABCD to'g'ri to'rtburchak
$AH \perp BD$,
$|AH| = 12$
$|BC| = |AD| = 15$.
$S = AH \cdot BD$

1) ΔAHD to'g'ri burchakli
$HD^2 = AD^2 - AH^2$

$$HD = \sqrt{15^2 - 12^2} = 9$$

2) ΔBAD to'g'ri burchkli.
$AH^2 = BH \cdot HD$

$$BH = \frac{AH^2}{HD} = \frac{12^2}{9} = \frac{144}{9} = 16$$

$BD = BH + HD = 16 + 9 = 25$
$S = 12 \cdot 25 = 300$.

Javob: 300.

26. $y = \sin(\cos(\mathrm{tg}2x))$ funksiyaning hosilasini toping.

Yechish:

$y = \sin(\cos(\mathrm{tg}2x))$
$y' = (\sin(\cos(\mathrm{tg}2x)))' =$
$= \cos(\cos(\mathrm{tg}2x)) \cdot (\cos(\mathrm{tg}2x))' =$
$= -\cos(\cos(\mathrm{tg}2x)) \cdot \sin(\mathrm{tg}2x) \cdot \frac{2}{\cos^2 2x} =$
$= \frac{-2\cos(\cos(\mathrm{tg}2x)) \cdot \sin(\mathrm{tg}2x)}{\cos^2 2x}$

Javob: $\frac{-2\cos(\cos(\mathrm{tg}2x)) \cdot \sin(\mathrm{tg}2x)}{\cos^2 2x}$

27. Qarang: 2-variant 28-savol (17-bet).

28. $(x^3 + 2x - 4)^{18} \cdot (x^2 - 3x + 1)^6$ ko'paytma koeffisiyentlari yig'indisini toping.

Yechish:

Ko'phadning barcha koeffitsentlari yig'indisi uning $x = 1$ nuqtadagi qiymatiga teng.

$(x^3 + 2x - 4)^{18} \cdot (x^2 - 3x + 1)^6$

$x = 1$ da

$(1 + 2 - 4)^{18} \cdot (1 - 3 + 1)^6 = (-1)^{18} \cdot (-1)^6 = 1.$

Javob: 1.

29. Ildizlari $-\frac{2}{3}$ va $\frac{13}{6}$ ga teng, oxirgi koeffitsiyent esa -26 ga teng bo'lgan ikkinchi darajali ko'phadni toping.

Yechish:

$x_1 = -\frac{2}{3}$, $x_2 = \frac{13}{6}$, $c = -26$.

$ax^2 + bx + c = ?$

1) Viyet teoremasiga ko'ra $\begin{cases} x_1 + x_2 = -\frac{b}{a} \\ x_1 \cdot x_2 = \frac{c}{a} \end{cases}$

2) $x_1 = -\frac{2}{3}$, $x_2 = \frac{13}{6}$, $c = -26$.

$-\frac{2}{3} \cdot \frac{13}{6} = \frac{-26}{a}$, $a = 18$.

$-\frac{2}{3} + \frac{13}{6} = \frac{-b}{18}$, $\frac{-b}{18} = \frac{9}{6}$, $b = -27$

3) $ax^2 + bx + c = 18x^2 - 27x - 26.$

Javob: $18x^2 - 27x - 26$.

30. $ABCDA_1B_1C_1D_1$ parallelepipedda $AB = 3a$, $BC = 3a$, $AA_1 = a$ bo'lsa, AB va CD_1 chiziqlar orasidagi burchakni toping.

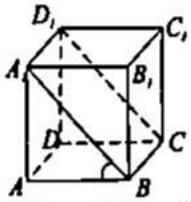
Berilgan:

$ABCDA_1B_1C_1D_1$ – *parallelepiped*

$AB = 3a$, $BC = 3a$, $AA_1 = a$

$(AB, CD_1) - ?$

Yechish:



AB va CD_1 chiziqlar orasidagi burchak DD_1C_1C tekislikka parallel bo'lgan AA_1B_1B tekislikda yotgan A_1B va AB chiziklar orasidagi burchakka teng. Izlanayotgan $\angle A_1BA$ ΔAA_1B dan:

$tgB = \frac{1}{3} \Rightarrow B = arctg\frac{1}{3}$.

Javob: $arctg\frac{1}{3}$.

31. Qarang: 5-variant 33-savol (44-bet).

32. MS Excel. =?(Остат(23;9)+??(Сцепить(23;9);2) formulaning natijasi 24 bo'lishi uchun ? va ?? belgilarining o'rniga qo'yish bo'lgan funksiyalar to'g'ri berilgan javobni aniqlang.

Yechish:

ЛЕВСИМВ(matn; belgi soni)	*Matnning chap tomonidagi berilgan sondagi belgilarni ajratib oladi*
ЗНАЧЕН(matn)	*Matn ko'rinishidagi sonni songa o'tkazadi*
ДЛСТР(matn)	*Matndagi belgilar sonini aniqlaydi*
СТЕПЕНЬ(son; daraja ko'rsatkichi)	*Sonni darajaga ko'taradi*
СРЗНАЧ(son1; son2;...)	*son1, son2, ... larning o'rta arifmetik qiymatini aniqlaydi*
ЗНАК(son)	*Son manfiy bo'lsa –1, 0 bo'lsa 0, musbat bo'lsa 1 qiymatga teng*
СЦЕПИТЬ(matn1; matn2;...)	*Bir nechta matnlarni ketma-ket ulaydi*
ОСТАТ(son; bo'luvchi)	*Sonni bo'luvchiga bo'lgandagi qoldiqni hisoblaydi*

=?(ocmam (23;9)+??(Сценить (23;9));2) formulani 2 bo'lakka bo'lib qaraymiz:
=Ocmam (23;9) (1)
=Сценить (23;9) (2)
(1) formula natijasini hisoblaymiz: 23 ni 9 ga bo'lganda qoldiq 5 ga teng.
(2) formulaning javobi matnli ifoda bo'lib, '239' ga teng. Matnli ifodani songa aylantirish uchun ?? o'rnida ЗНАЧЕН(matn) funksiyasidan foydalanamiz: =ЗНАЧЕН('239')=239.
(1) va (2) ni qo'shsak, 5 + 239 = 244. Matnning chap tomonidagi 2 ta belgilarni ajratib olsak, 24 hosil bo'ladi. Demak, ? o'rniga ЛЕВСИМВ('244';2) funksiyasini qo'yish kerak.

Javob: Левсимв, Значен.

33. Ikkilik sanoq sistemasidagi 1100111_2 sonini o'nlik sanoq sistemasiga o'tkazing:

Yechish:

$$\overset{6\,5\,4\,3\,2\,1\,0}{1100111_{(2)}} - X_{10}$$

$$1\cdot2^6 + 1\cdot2^5 + 0\cdot2^4 + 0\cdot2^3 + 1\cdot2^2 + 1\cdot2^1 + 1\cdot2^0 = 64 + 32 + 0 + 0 + 4 + 2 + 1 = 103_{10}.$$

Javob: 103.

34. Ali sakkizlik sanoq sistemasida (54; 67) oraliqdagi barcha butun sonlarni yozib chiqdi. Vali esa shu sonlardan 2 raqami, so'ng 6 raqami qatnashgan barcha sonlarni o'chirib tashladi. Qolgan sonlar yig'indisini sakkizlik sanoq sistemasida aniqlang va o'n birlik sanoq sistemasiga o'tkazing.

Yechish:

Ali yozgan sonlar: 55, 56, 57, 60, 61, 62, 63, 64, 65, 66. Vali shu sonlardan 2 va 6 raqamlari qatnashgan barcha sonlarni o'chirganidan keyin 55, 57 sonlar qoladi. Ularning sakkizlik sanoq sistemasidagi yig'indisini hisoblaymiz: $55_8+57_8=144_8$. Endi bu sonni avval 10-lik sanoq sistemasiga, keyin 11-likka o'tkazamiz.

$$144_8 = 1\cdot8^2 + 4\cdot8^1 + 4\cdot8^0 = 64 + 32 + 4 = 100_{10}$$

$$\begin{array}{r|l} 100 & 11 \\ \hline 99 & 9 \\ 1 & \end{array}$$

Demak, $100_8=91_{11}$ ekan.

Javob: 91.

35. Paskal tilida yozilgan dastur natijasini aniqlang.

```
Var k:byte; s, N:string; F:array[1..11] of byte;
Begin Randomize; S:='INFORMATIKA';N:='';
F[1]:=Random(1); F[2]:=Random(2)+1;
For k:=3 To 5 Do F[k]:=F[k-2]+F[k-1];
For k:=1 To 5 Do N:=N+s[F[k]+1],
Write(N): Readln; End.
```

Yechish:

Dasturda k – 0..255 diapazondagi butun o'zgaruchi; s, N – satrlar va F – 11 ta 0..255 diapazondagi butun sondan iborat massivdan foydalanilgan.
Randomize – tasodifiy sonlar generatori.
s:='INFORMATIKA';
N:='' – bo'sh satr.

F[1]:=Random(1)+1; F[2]:=Random(2)+1 {random(1) 0 qiymatni, random(2) esa (0; 2) oraliqdan, ya'ni 1 qiymatni qabul qiladi. Demak, F[1]=0, F[2]=2 bo'ladi}
For k:=3 To 5 Do F[k]:=F[k-2]+ F[k-1];
k=3 dan 5 gacha ketma-ket o'zgarganda har bir k uchun F[k-2]ga F[k-1] ni qo'shib qiymatini F[k] ta'minlaydi.

k	F[k]
1	0
2	2
3	2
4	4
5	6

For k:=1 To 5 Do N:=N+s[F[k]+1],
k=1 dan 5 gacha ketma-ket o'zgarganda har bir k uchun N satrga s satrning (F[k]+1)-pozitsiyasidagi belgini qo'shadi.

k	F[k]+1	N
1	1	I
2	3	IF
3	3	IFF
4	5	IFFR
5	7	IFFRA

Write (N) – N ning qiymatini, ya'ni 'IFFRA' ni ekranda aks ettiradi.

Javob: IFFRA.

36. Ali sakkizlik sanoq sistemasida (57; 72) oraliqdagi barcha butun sonlarni yozib chiqdi. Vali esa shu sonlardan 6 raqami qatnashgan barcha sonlarni o'chirib tashladi. Qolgan sonlar yig'indisini sakkizlik sanoq sistemasida aniqlang va o'n uchlik sanoq sistemasiga o'tkazing.

Yechish:
Ali yozgan sonlar: 60, 61, 62, 63, 64, 65, 66, 67, 70, 71. Vali shu sonlardan 6 raqami qatnashgan barcha sonlarni o'chirganidan keyin 70, 71 sonlar qoladi. Ularning sakkizlik sanoq sistemasidagi yig'indisini hisoblaymiz: $70_8+71_8=161_8$. Endi bu sonni avval 10-lik sanoq sistemasiga, keyin 16-likka o'tkazamiz.

$161_8 = 1\cdot 8^2 + 6\cdot 8^1 + 1\cdot 8^0 = 64 + 48 + 1 = 113_{10}$

$$\begin{array}{r|l} 113 & 13 \\ \hline 104 & 8 \\ 9 & \end{array}$$

Demak, $113_8=89_{13}$ ekan.

Javob: 89.