2017-yil matematika variant yechimlari (spectrum)

18-variant

Bizning kanal : @axborotnoma

Adminsratorlar hayati : @axborotnoma_bot

Matematika yordam guruhi : @axborotnomaguruhi

Reklama xizmati : @axborotnoma_reklama

18-variant

1. To'g'ri burchakli teng yonli uchburchakning 16 ga teng gipotenuzasi orqali uchburchak tekisligi bilan 60° li burchak tashkil etuvchi P tekislik o'tkazilgan. Uchburchakning tekisligidagi proyeksiyasi yuzini toping.

Berilgan:

ΔABC – *to'g'ri burchakli*

$\angle C = 90^\circ$

AB – *gipotenuza*

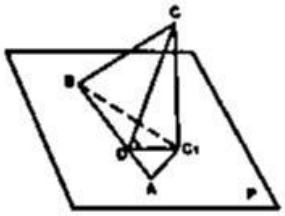
$AB = c = 16$

$BC = AC = a$

$\angle CDC_1 = 60^\circ$

$S_{ABC_1} = ?$

Yechish:



$S_{ABC_1} = S_{ABC} \cdot \cos \angle CDC_1$

$S_{ABC} = \frac{a^2}{2}\ c^2 = 2a^2,\ c = a\sqrt{2},\ a = \frac{c}{\sqrt{2}}$

$S_{ABC} = \frac{a^2}{2} = \frac{c^2}{4}$

$S_{ABC_1} = \frac{c^2}{4} \cdot \cos 60^\circ = \frac{16^2}{4} \cdot \frac{1}{2} = 32.$

Javob: 32.

2. Moddiy nuqta to'g'ri chiziq bo'ylab $x(t) = \frac{1}{2}t^3 - 3t^2 + 2t + 5$ qonun bo'yicha harakatlanmoqda, bu yerda x – koordinatalar boshidan nuqtagacha bo'lgan masofa (metrlarda o'lchanadi), t – vaqt (sekundlarda o'lchanadi). t = 6 sekund bo'lganda nuqtaning tezligini (m/s) toping.

Yechish:

$x(t) = \frac{1}{2}t^3 - 3t^2 + 2t + 5$

$t = 6,\ \upsilon(6) = ?$

$\upsilon(t) = x'(t)$

$x'(t) = \left(\frac{1}{2}t^3 - 3t^2 + 2t + 5\right)' = \frac{3}{2}t^2 - 6t + 2$

$\upsilon(6) = x'(6) = \frac{3}{2} \cdot 6^2 - 6 \cdot 6 + 2 =$

$= 54 - 36 + 2 = 20.$

Javob: 20.

3. $\frac{1 + tg^6\frac{\pi}{3}}{1 - tg^2\frac{\pi}{3} + tg^4\frac{\pi}{3}}$ ifodani soddalashtiring.

Yechish:

1) Qisqa ko'paytirish formulasiga asosan,

$$\frac{1 + (tg^2\frac{\pi}{3})^3}{1 - tg^2\frac{\pi}{3} + tg^4\frac{\pi}{3}} =$$

$$= \frac{(1 + tg^2\frac{\pi}{3})(1 - tg^2\frac{\pi}{3} + tg^4\frac{\pi}{3})}{1 - tg^2\frac{\pi}{3} + tg^4\frac{\pi}{3}} = 1 + tg^2\frac{\pi}{3}$$

2) $tg\frac{\pi}{3} = \sqrt{3}$ *bundan*

$1 + tg^2\frac{\pi}{3} = 1 + (\sqrt{3})^2 = 1 + 3 = 4.$

Javob: 4.

4. Agar $f'(x) = -\frac{2}{e^x}$, $f(\ln 2) = 0$ bo'lsa, $f(x)$ ni toping.

Yechish:

$f'(x) = -\frac{2}{e^x},\ f(\ln 2) = 0,\ f(x) = ?$

$f'(x) = -\frac{2}{e^x} = -2e^{-x}$ *boshlang'ich funksiyasini topamiz.*

$f(x) = 2e^{-x} + C$

$f(\ln 2) = 2e^{-\ln 2} + C = 0$

$\frac{2}{e^{\ln 2}} + C = 0$

$\frac{2}{2} + C = 0, C = -1$

$f(x) = 2e^{-x} - 1.$

Javob: $2e^{-x} - 1.$

5. $y = \frac{|x|}{x}$ funksiyaning aniqlanish sohasini toping.

Yechish:

$y = \frac{|x|}{x}$, *aniqlanish sohasi topamiz.*

$y = \frac{|x|}{x},\ x \neq 0$

$x > 0$ *bo'lganda* $y = 1$

$x < 0$ *bo'lganda* $y = -1$

$D(y) = (-\infty; 0) \cup (0; \infty).$

Javob: $(-\infty; 0) \cup (0; \infty).$

6. a va b natural sonlarning EKUBi 30 ga, ko'paytmasi 36000 ga teng bo'lsa, shu sonlarning EKUKini toping.

Yechish:

$EKUB(a; b) = 30,\ a \cdot b = 36000,$

$EKUK(a; b) = ?$

$EKUB(a; b) \cdot EKUK(a; b) = a \cdot b$
$30 \cdot EKUK(a; b) = 36000$
$EKUK(a; b) = 1200.$

Javob: 1200.

7. Arifmetik progressiyada $a_1 + a_3 = 10$ va $a_1^2 + a_4^2 = 90$ bo'lsa, a_5 ning qiymatlari yig'indisini toping.

Yechish:

1) Xossaga ko'ra $a_2 = \frac{a_1 + a_3}{2}$, $a_2 = \frac{10}{2} = 5$

2) $a_1^2 + a_4^2 = 90$,
$(a_2 - d)^2 + (a_2 + 2d)^2 = 90$
$(5 - d)^2 + (5 + 2d)^2 = 90$
$25 - 10d + d^2 + 25 + 20d + 4d^2 = 90$,
$5d^2 + 10d - 40 = 0$
$d^2 + 2d - 8 = 0$,
$d = 2$, $d = -4$

3) $d = 2$ da $a_5 = a_2 + 3d = 5 + 3 \cdot 2 = 11$
$d = -4$ da $a_5 = a_2 + 3d = 5 + 3 \cdot (-4) = -7$

4) a_5 ning qiymatlari yig'indisi
$11 + (-7) = 4.$

Javob: 4.

8. Qarang: 4-variant 16-savol (31-bet).

9. Qarang: 11-variant 29-savol (87-bet).

10. Qarang: 17-variant 3-savol (124-bet).

11. Uchta idishning har birida turli miqdorda suyuqlik bor. Ularni tenglashtirish uchun 3 marta bir-biriga quyish bajarildi. Oldin 1-idishda bo'lgan suyuqlikning $\frac{1}{3}$ qismi 2-idishga, keyin 2-idishda bo'lgan suyuqlikning $\frac{1}{4}$ qismi 3-idishga quyildi va nihoyat 3-idishda bo'lgan suyuqlikning $\frac{1}{10}$ qismi 1-idishga quyildi. Shundan keyin har bir idishdagi suyuqlik 9 l dan bo'ldi. Oldin har bir idishda qanchadan suyuqlik bo'lgan?

Yechish:
1-idishda x l, 2-idishda y l, 3-idishda z l suyuqlik bo'lsin.

1-idishda

$$x - \frac{x}{3} + \frac{z + \frac{y + \frac{x}{3}}{4}}{10} = 9$$

2-idishda y l

$$y + \frac{x}{3} - \frac{y + \frac{x}{3}}{4} = 9$$

3-idishda z l

$$z + \frac{y + \frac{x}{3}}{4} - \frac{z + \frac{y + \frac{x}{3}}{4}}{10} = 9$$

$y + \frac{x}{3} = 12$, $z + \frac{12}{4} = 10$, $z = 7$,
$x = 12$, $y = 8$
$x = 12$ l, $y = 8$ l, $z = 7$ l.

Javob: 12; 8; 7.

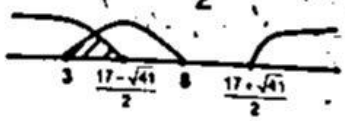
12. $\sqrt{x+2} + |x - 3| \le 5$ tengsizlikning butun sonlardan iborat yechimlari yig'indisini toping.

Yechish:
1) Aniqlanish sohasi $x + 2 \ge 0$, $x \ge -2$
2) $-2 \le x < 3$ da $\sqrt{x+2} + 3 - x \le 5$,
$\sqrt{x+2} \le x + 2$
$\sqrt{x+2}(1 - \sqrt{x+2}) \le 0$
$\sqrt{x+2}(\sqrt{x+2} - 1) \ge 0$
$\sqrt{x+2} \le 0$, $\sqrt{x+2} \ge 1$,
$x = -2$ va $x \ge -1$

$x \in [-1; 3) \cup \{-2\}$

3) $x \ge 3$ da $\sqrt{x+2} + x - 3 \le 5$
$\sqrt{x+2} \le 8 - x$,
$8 - x \ge 0$, $x \le 8$
$x + 2 \le 64 - 16x + x^2$
$x^2 - 17x + 62 \ge 0$

$$x_{1,2} = \frac{17 \pm \sqrt{17^2 - 4 \cdot 62}}{2} = \frac{17 \pm \sqrt{41}}{2}$$



$x \in \left[3; \frac{17-\sqrt{41}}{2}\right]$

4) Tengsizlikning yechimlar oralig'i

$\left[-1; \frac{17-\sqrt{41}}{2}\right] \cup \{-2\}$.

Butun yechimlar yig'indisi:

$-2-1+0+1+2+3+4+5=12$.

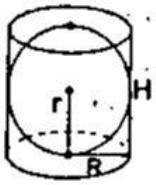
Javob: 12.

13. Qarang: 15-variant 7-savol (112-bet).

14. Qarang: 2-variant 21-savol (16-bet).

15. Silindr sharga tashqi chizilgan. Silindr hajmi 78 ga teng. Shar hajmini toping.

Yechish:



$H = 2R = 2r$

$V = 78$

$V_{sh} = ?$

1) $V = \pi R^2 H = 2\pi R^3 = 78$

$\pi R^3 = 39$

2) $V_{sh} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 = \frac{4}{3} \cdot 39 = 52$.

Javob: 52.

16. Sistemani yeching.

$$\begin{cases} \frac{x+y+4}{5} + \frac{x-y-4}{7} = 9 \\ \frac{x+y+4}{5} - \frac{x-y-4}{7} = 1 \end{cases}$$

Yechish:

$$\begin{cases} \frac{x+y+4}{5} + \frac{x-y-4}{7} = 9 \\ \frac{x+y+4}{5} - \frac{x-y-4}{7} = 1 \end{cases}$$

Sistemada yangi o'zgaruvchi kiritamiz va yangi sistemaga o'tamiz

$$\begin{cases} \frac{x+y+4}{5} = s \\ \frac{x-y-4}{7} = t \end{cases} \Leftrightarrow \begin{cases} s+t=9 \\ s-t=1 \end{cases} \Leftrightarrow \begin{cases} s=5 \\ t=4 \end{cases}$$

$$\begin{cases} \frac{x+y+4}{5} = 5 \\ \frac{x-y-4}{7} = 4 \end{cases} \Leftrightarrow \begin{cases} x+y+4=25 \\ x-y-4=28 \end{cases}$$

Ikkala tenglamani bir biriga qushib,

$2x = 53 \Rightarrow x = 26{,}5$ va

$y = 25 - 26{,}5 - 4 = -5{,}5$ larni topamiz.

Javob: (26,5; –5,5).

17. Qarang: 1-variant 21-savol (7-bet).

18. Agar $f(x) = 3^{\cos x}$ bo'lsa, $f'\left(\frac{\pi}{2}\right)$ ni hisoblang.

Yechish:

$y = 3^{\cos x}$ tenglikning ikkala qismini logarifmlaymiz va hosilasini topamiz.

$\ln y = \ln 3^{\cos x} = \cos x \cdot \ln 3$

$(\ln y)' = \ln 3 \cdot (\cos x)'$

$\frac{y'}{y} = -\ln 3 \cdot \sin x$

$y' = -y \cdot \ln 3 \cdot \sin x = -3^{\cos x} \cdot \ln 3 \cdot \sin x$

$y'\left(\frac{\pi}{2}\right) = -3^{\cos\frac{\pi}{2}} \cdot \ln 3 \cdot \sin\frac{\pi}{2} = -3^0 \cdot \ln 3 \cdot 1 = -\ln 3$.

Javob: –ln3.

19. $y = (x+6) \cdot e^{x-6}$ funksiyaning minimum nuqtasinin toping.

Yechish:

$y = (x+6) \cdot e^{x-6}$ funksiyaning minimum nuqtasini topamiz.

1) $y' = ((x+6) \cdot e^{x-6})' = (x+6)' \cdot e^{x-6} + (x+6) \cdot (e^{x-6})' = e^{x-6} + (x+6) \cdot e^{x-6} = e^{x-6} \cdot (x+7)$

2) $y' = 0$, $e^{x-6} \neq 0$, $x + 7 = 0$, $x = -7$

3) $\overset{-}{\underset{-7}{\longrightarrow}}\overset{+}{}$

$x = -7$ – minimum nuqta.

Javob: –7.

20. Qarang: 17-variant 2-savol (124-bet).

21. Qarang: 9-variant 18-savol (71-bet).

22. 11 nafar ishchilardan 3 ta ishchidan iborat brigada tuzish kerak. Bu ishni nechta usulda amalga oshirsa bo'ladi?

27. $y = 6^{\ln(x^2+5)}$ funksiyaning kamayish oralig'ini toping.

Yechish:

$y = 6^{\ln(x^2+5)}$, $y = a^x$ ko'rsatkichli funksiya $a > 1$ bolganda barcha haqiqiy sonlar to'plamida o'suvchi bo'ladi. $y_1 = \ln(x^2 + 5)$ funksiya juft funksiya va doimo musbat. Demak $y_1 = 6^{\ln(x^2+5)}$ funksiya $y \le 0$ da kamyuvchi bo'ladi

$$y_1' = (6^{\ln(x^2+5)})' = 6^{\ln(x^2+5)} \cdot \ln 6 \cdot \frac{2x}{x^2+5} < 0.$$

$2x \le 0$, $x \le 0$ $[-\infty; 0]$ da funksiya kamayuvchi bo'ladi.

Javob: $(-\infty; 0]$.

28. $(0{,}75\text{tg}^2 30° - \sin^2 60° + \text{tg}^2 225° - \cos 240°)^{-1}$ ifodaning qiymatini toping.

Yechish:

$$1)\ (\text{tg}30°)^2 = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$$

$$(\sin 60°)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

$$(\text{tg}225)^2 = (\text{tg}(180° + 45°))^2 = (-1)^2 = 1,$$

$$\cos 240° = \cos(180° + 60°) = \cos 60° = -\frac{1}{2}$$

$$2)\ \left(0{,}75 \cdot \frac{1}{3} - \frac{3}{4} + 1 + \frac{1}{2}\right)^{-1} = \left(\frac{3}{4} \cdot \frac{1}{3} + \frac{3}{4}\right)^{-1} = 1.$$

Javob: 1.

29. $f(x) = \dfrac{x^2+2x}{x+2}$, $g(x) = \dfrac{x+2}{x^2+2x}$ funksiyalar bir xil qiymat qabul qiladigan x ning haqiqiy qiymtlari yig'indisini toping?

Yechish:

$f(x) = \dfrac{x^2+2x}{x+2}$, $g(x) = \dfrac{x+2}{x^2+2x}$ funksiyalar bir xil qiymat qabul qiladigan x ning haqiqiy qiymatlarini topish uchun $f(x) = g(x)$ yoki $\dfrac{x^2+2x}{x+2} = \dfrac{x+2}{x^2+2x}$ tenglamani yechish kerak.

$$\frac{x^2+2x}{x+2} = \frac{x+2}{x^2+2x},\ x \ne 0,\ x \ne -2$$

$$\frac{x(x+2)}{x+2} = \frac{x+2}{x(x+2)} \Rightarrow x = \frac{1}{x}$$

$x^2 = 1$, $x = \pm 1$.

$x = \pm 1$ da funksiyalar bir xil qiymatga ega bo'ladilar.

$1 + (-1) = 0$.

Javob: 0.

30. $\log_7 x - \log_x \dfrac{1}{7} \ge 2$ tengsizlikni yeching.

Yechish:

$\log_7 x - \log_x \dfrac{1}{7} \ge 2$.

1) aniqlanish sohasi:

$x > 0$, $x \ne 1$

2) $\log_7 x - \log_x 7^{-1} \ge 2$

$\log_7 x + \log_x 7 \ge 2$, $\log_7 x + \dfrac{1}{\log_7 x} \ge 2$

$\log_7 x = a$

$a + \dfrac{1}{a} - 2 \ge 0$, $\dfrac{a^2-2a+1}{a} \ge 0$, $\dfrac{(a-1)^2}{a} \ge 0$.

$a > 0$.

3) $\log_7 x > 0$, $x > 7^0 = 1$, $x > 1$.

Javob: $(1; \infty)$.

31. Rim sanoq sistemasida to'g'ri tenglikni aniqlang.

Yechish:

Rim raqamlarini esga olamiz:

I=1, V=5, X=10, L=50, C=100, D=500, M=1000.

Raqamlar yig'indisi hisoblanadi. Agar katta raqamdan oldin kichigi tursa, kattasidan kichigini ayirish kerak.

Sonlarni Rim sanoq sistemasidan 10-lik sanoq sistemasiga o'tkazib, har bir javobni tekshirib ko'ramiz:

A) CLXXXIII+XXIX=CCXIII
CLXXXIII=183; XXIX=29; CCXIII=213
183+29=212 – noto'g'ri
B) XXX·XXIX=DCCCLXVIII
XXX=30; XXIX=29; DCCCLXVIII=868
30·29=868 – noto'g'ri
C) CCIII:XXIX=VII
CCIII=203; XXIX=29; VII=7
203:29=7 – to'g'ri
D) CCLXXVII-LXXXVIII=CXC
CCLXXVII=277; LXXXVIII=88; CXC=190
277-88=190 – noto'g'ri

Javob: CCIII:XXIX=VII.

32. Qarang: 6-variant 35-savol (51-bet).

33. Qarang: 8-variant 35-savol (67-bet).

34. Quyidagi mulohazalardan rost qiymatga egalarini aniqlang:
1) Axborot ikki turga bo'linadi.
2) Web-sahifalarni aks ettirish vositasi – brauzerlar.
3) Plotter – chizmalarni qog'ozga chiqaruvchi qurilma.
4) www.uz – milliy qidiruv tizimi emas.

Yechish:
Mulohazalarni tahlil qilamiz:
*1) Axborot ikki turga bo'linadi. – **Yolg'on**. Chunki axborot matnli, grafik, ovozli va video ko'rinishida bo'lishi mumkin.*
*2) Web-sahifalarni aks ettirish vositasi – brauzerlar. – **Rost**. Biz Internetdan brauzerlar vositasida foydalanamiz.*
*3) Plotter – chizmalarni qog'ozga chiqaruvchi qurilma. – **Rost**.*
*4) www.uz – milliy qidiruv tizimi emas. – **Yolg'on**. www.uz – O'zbekistonning milliy qidiruv tizimi.*

Javob: 2, 3.

35. Qarang: 4-variant 34-savol (36-bet).

36. A="BIOS dasturi kompyuterning doimiy xotirasida joylashgan." B="Software – qiymati 100% to'langandan keyin o'rnatilib, foydalaniladigan dasturiy ta'minot." C="Windows yo'lboshlovchisining ishlashiga Explorer.exe dasturi javob beradi." Shu mulohazalar asosida quyidagi mantiqiy ifodaning haqiqiyligini toping:
$\neg A \vee (\neg C \wedge B)$

Yechish:
A=BIOS dasturi kompyuterning doimiy xotirada joylashgan = rost (1)
B= Software – qiymati 100% to'langanidan keyin o'rnatilib, foydalaniladigan dasturiy ta'minot = rost
C=Windows yo'lboshlovchisining ishlashiga Explorer.exe dasturi javob beradi = rost
$\neg A \vee (\neg C \wedge B) = \neg 1 \vee (\neg 0 \wedge 1) = 0 \vee (1 \wedge 1) = 0 \vee 1 = 0.$

Javob: yolg'on.