

2017-yil matematika variant yechimlari (spectrum)

17-variant

Bizning kanal : @axborotnoma

Adminsratorlar hayati : @axborotnoma_bot

Matematika yordam guruhi : @axborotnomaguruhi

Reklama xizmati : @axborotnoma_reklama

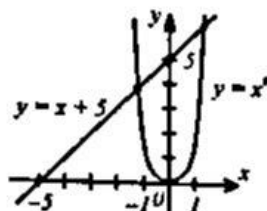
17-variant

1. Ushbu $\begin{cases} y = x^2 \\ y = x + 5 \end{cases}$ tenglamalar

sistemasi nechta yechimga ega?

Yechish:

$\begin{cases} y = x^2 \\ y = x + 5 \end{cases}$ sistemani grafik usulida yechamiz.



Funksiya grafiklari ikkita nuqtada kesishadi. Demak, tenglamalar sistemasi ikkita yechimga ega.

Javob: 2.

2. $8^{18} \cdot 5^{55}$ ko'paytma necha xonali son?

Yechish:

$$8^{18} = (2^3)^{18} = 2^{54}$$

$$8^{18} \cdot 5^{55} = 2^{54} \cdot 5^{55} = (2 \cdot 5)^{54} \cdot 5 = 5 \cdot 10^{54}$$

$5 \cdot 10^{54}$ – 55 xonali son.

Javob: 55.

3. To'p 2 m 43 sm balandlikdan tashlandi va yerga urilib, har gal balandligining $\frac{2}{3}$ qismiga teng balandlikka ko'tarildi. To'p necha marta urilishdan keyin 32 sm balandlikka ko'tariladi? (32 sm dan yuqoriga o'tib ketadigan hollarni qaramang.)

Yechish:

$$H = 2 \text{ m } 43 \text{ sm} = 243 \text{ sm}$$

$$243 \cdot \left(\frac{2}{3}\right)^n = 32, n = ?$$

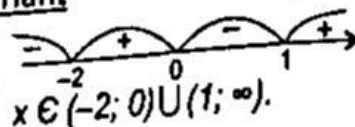
$$\left(\frac{2}{3}\right)^n = \frac{32}{243}, \left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^5, n = 5.$$

Javob: 5.

4. $y = \ln[x(x-1)(x+2)]$ funksiyaning aniqlanish sohasini toping.

Yechish:

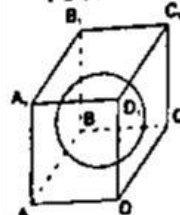
$y = \ln(x(x-1)(x+2))$ aniqlanish sohasini topamiz. Logarifm ostidagi ifoda doimo musbat $x(x-1)(x+2) > 0$, oraliqlar usulida yechamiz.



Javob: $(-2; 0) \cup (1; \infty)$.

5. To'g'ri burchakli parallelepiped 3,5 radiusli sferaga tashqi chizilgan. Parallelepiped hajmini toping.

Yechish:



$R = 3,5$
Parallelepipedga sfera ichki chizilgan bo'lsa, u hq da bu parallelepiped kub bo'ladi.

$$AB = a = 2R$$

$$V = a^3 = (2R)^3 = (2 \cdot 3,5)^3 = 7^3 = 343.$$

Javob: 343.

6. Qarang: 8-variant 28-savol (65-bet).

7. Musbat x, y sonlar uchun $a = x + y$ va $b = 2\sqrt{xy}$ bo'lsin. Qaysi tengsizlik har doim o'rinli?

Yechish:

$x > 0, y > 0$ sonlar uchun $a = x + y, b = 2\sqrt{xy}$.
Sonlarning o'rtta arifmetigi o'rtta geometrigidan kichik emas.

$$\frac{x+y}{2} \geq \sqrt{xy}, x+y \geq 2\sqrt{xy}, a \geq b.$$

Javob: $a \geq b$.

8. Qarang: 8-variant 7-savol (60-bet).

9. ABC uchburchak yuzasi 170 ga teng. $DE \parallel AB$ tomonga parallel o'tkazilgan ABC uchburchakning o'rtta chizig'i, ABCD trapetsiyaning yuzasini toping.

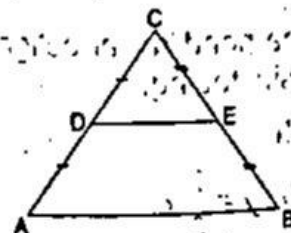
Yechish:

$$S_{ABC} = 170$$

$$DE \parallel AB$$

$$DE = \frac{AB}{2}$$

$$S_{ABED} = ?$$



1) $\triangle ACB$ va $\triangle DCE$ o'xshash.

$$\frac{S_{DCE}}{S_{ACB}} = \left(\frac{DE}{AB}\right)^2 = \left(\frac{2}{AB}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$S_{DCE} = \frac{1}{4} \cdot S_{ACB} = \frac{1}{4} \cdot 170 = \frac{85}{2}$$

$$S_{ADEB} = S_{ACB} - S_{DCE} = 170 - \frac{85}{2} = 127,5$$

Javob: 127,5.

10. Agar $f(x) = \ln(2x + \sqrt{x^2 + 1})$ bo'lsa, $f'(0)$ ni toping.

Yechish:

$$f(x) = \ln(2x + \sqrt{x^2 + 1}), f'(0) = ?$$

$$1) f'(x) = (\ln(2x + \sqrt{x^2 + 1}))' =$$

$$= \frac{(2x + \sqrt{x^2 + 1})'}{2x + \sqrt{x^2 + 1}} = \frac{2 + \frac{2x}{2\sqrt{x^2 + 1}}}{2x + \sqrt{x^2 + 1}}$$

$$2) f'(0) = \frac{2 + \frac{0}{\sqrt{0+1}}}{2 \cdot 0 + \sqrt{0+1}} = \frac{2}{1} = 2$$

Javob: 2.

11. $x^6 - 28x^3 + 27 \leq 0$ tengsizlik nechta butun yechimga ega?

Yechish:

$$x^6 - 28x^3 + 27 \leq 0$$

$$1) x^3 = a$$

$$2) a^2 - 28a + 27 \leq 0, (a - 27)(a - 1) \leq 0$$

$$1 \leq a \leq 27$$

$$3) 1 \leq x^3 \leq 27, 1 \leq x \leq 3$$

4) Tengsizlik butun yechimlari 1, 2, 3.

Demak, uchta butun yechimga ega.

Javob: 3.

12. Qarang: 12-variant 5-savol (89-bet).

13. $y = x^3 - 3x$ funksiyaning $y = \frac{1}{2}x$ to'g'ri

chiziqqa perpendikulyar bo'lgan urinma tenglamasini toping.

Yechish:

$$y = x^3 - 3x, y = \frac{1}{2}x$$

1) $y = k_1x + b_1, y = k_2x + b_2$ to'g'ri chiziqlar perpendikulyar bo'lishi uchun. $k_1 \cdot k_2 = -1$.

$$2) y = \frac{1}{2}x, k_1 = \frac{1}{2}, k_2 = -2.$$

3) $y = x^3 - 3x$ urinma tenglamasini tuzamiz

$$y = y(x_0) + y'(x_0)(x - x_0)$$

$$y(x_0) = x_0^3 - 3x_0$$

$$y' = 3x^2 - 3$$

$$y'(x_0) = 3x_0^2 - 3$$

$$y = x_0^3 - 3x_0 + (3x_0^2 - 3)(x - x_0)$$

$$y'(x_0) = k_2$$

$$3x_0^2 - 3 = -2$$

$$3x_0^2 = 1, x_0 = \pm \frac{1}{\sqrt{3}}$$

$$4) x_0 = \frac{1}{\sqrt{3}} \text{ da } y = \left(\frac{1}{\sqrt{3}}\right)^3 - 3 \cdot \frac{1}{\sqrt{3}} +$$

$$+ 3\left(\left(\frac{1}{\sqrt{3}}\right)^2 - 1\right)\left(x - \frac{1}{\sqrt{3}}\right) =$$

$$= \frac{1}{3\sqrt{3}} - \frac{3}{\sqrt{3}} + 3\left(\frac{1}{3} - 1\right)\left(x - \frac{1}{\sqrt{3}}\right) =$$

$$= \frac{-8}{3\sqrt{3}} - 2\left(x - \frac{1}{\sqrt{3}}\right) = -2x - \frac{6}{3\sqrt{3}} =$$

$$= -2x - \frac{2}{\sqrt{3}} = -2x - \frac{2\sqrt{3}}{3}$$

$$x_0 = -\frac{1}{\sqrt{3}} \text{ da}$$

$$y = 2x + \frac{2\sqrt{3}}{9}$$

$$y = -2x \pm \frac{2\sqrt{3}}{9}$$

$$\text{Javob: } -2x \pm \frac{2\sqrt{3}}{9}$$

14. Tengsizlikni butun yechimlari yig'indisini toping.

$$2 \cdot (x - 3)^2 - (x - 1) \cdot (x + 3) \leq 0$$

Yechish:

$$2 \cdot (x - 3)^2 - (x - 1) \cdot (x + 3) \leq 0 \text{ tengsizlikni oraliqlar usulida yechamiz.}$$

$$1) 2(x^2 - 6x + 9) - (x^2 + 2x - 3) \leq 0$$

$$2x^2 - 12x + 18 - x^2 - 2x + 3 \leq 0$$

$$x^2 - 14x + 21 \leq 0$$

$$x_{1,2} = \frac{14 \pm \sqrt{14^2 - 4 \cdot 21}}{2} = \frac{14 \pm 4\sqrt{7}}{2} =$$

$$= 7 \pm 2\sqrt{7}$$

$$7 - 2\sqrt{7} \leq x \leq 7 + 2\sqrt{7}$$

Butun yechimlar 2, 3, 4, ..., 12.

Butun yechimlar yig'indisi: $\frac{2+12}{2} \cdot 11 = 77.$

Javob: 77.

15. Konusning asos yuzasi yon sirtining yuzasidan 4 marta katta. Yasovchisi bilan asos tekisligi orasidagi burchakni toping.

Yechish:

$$4S_{\text{asos}} = S_{\text{yon}}$$

$$\angle SAO = ?$$

$$1) S_{\text{asos}} = \pi R^2$$

$$S_{\text{yon}} = \pi Rl$$

$$2) \angle SAO = \alpha$$

$$\cos \alpha = \frac{R}{l}$$

$$3) 4\pi R^2 = \pi Rl, 4R = l,$$

$$\cos \alpha = \frac{R}{4R} = \frac{1}{4}$$

$$4) \alpha = \arccos \frac{1}{4}$$

Javob: $\arccos \frac{1}{4}.$

16. $5\frac{1}{2} + 0,5x - \frac{3x}{4} + 2(x + 0,3) - 1$

ifodaning $x = -2$ dagi qiymatini toping.

Yechish:

$$5\frac{1}{2} + 0,5x - \frac{3x}{4} + 2(x + 0,3) - 1,$$

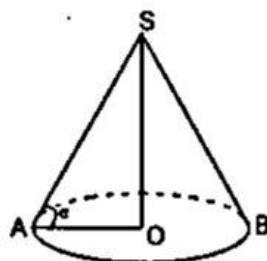
$$x = -2$$

1) soddalashtirib olamiz

$$5\frac{1}{2} + \frac{x}{2} - \frac{3x}{4} + 2x + 2 \cdot \frac{1}{3} - 2 = \frac{7x}{4} + \frac{25}{6}$$

2) $x = -2$ da,

$$\frac{7 \cdot (-2)}{4} + 4 \cdot \frac{1}{6} = -3\frac{1}{2} + 4\frac{1}{6} =$$



$$= (-3 + 4) + (-\frac{1}{2} + \frac{1}{6}) = 1 - \frac{1}{3} = \frac{2}{3} = 0,6.$$

Javob: 0,6.

17. $\frac{2,99 - 3,2}{x} = \frac{5\frac{1}{2} - 3\frac{2}{3}}{7:2}$ proporsiyadan

x ni toping.

Yechish:

$$\frac{2,99 - 3,2}{x} = \frac{5\frac{1}{2} - 3\frac{2}{3}}{7:2}, x = ?$$

$$1) 2,99 - 3,2 = 2\frac{99}{99} - 3\frac{2}{10} = 3 - 3\frac{1}{5} =$$

$$= -\left(3\frac{1}{5} - 3\right) = -\frac{1}{5}$$

$$2) 5\frac{1}{2} - 3\frac{2}{3} = 5 - 3 + \frac{1}{2} - \frac{2}{3} = 2 - \frac{1}{6} = 1\frac{5}{6}$$

$$3) 7:2 = \frac{7}{2}$$

$$4) -\frac{1}{5} : x = 1\frac{5}{6} : \frac{7}{2}$$

$$x = -\frac{1}{5} \cdot \frac{2}{7} \cdot \frac{11}{6} = -\frac{7}{10} \cdot \frac{6}{11} = -\frac{21}{55}$$

Javob: $-\frac{21}{55}.$

18. Qarang: 12-variant 22-savol (93-bet).

19. A(-2; 3) va B(6; 1) nuqtalardan o'tuvchi AB to'g'ri chiziq C(3; 6) va D(0; m) nuqtalardan o'tuvchi CD to'g'ri chiziqqa perpendikulyar m ni toping.

Yechish:

$$A(-2; 3), B(6; 1), C(3; 6), D(0; m)$$

$$AB \perp CD$$

$$m = ?$$

1) $y = k_1x + b_1$ to'g'ri chiziq tenglamasi.

$$A(-2; 3) B(6; 1)$$

$$\begin{cases} 3 = -2k_1 + b_1 \\ 1 = 6k_1 + b_1 \end{cases} \Rightarrow \begin{cases} k_1 = -\frac{1}{4} \\ b_1 = \frac{5}{2} \end{cases}$$

2) $y = k_2x + b_2$
 $C(3; 6) D(0; m)$

$$\begin{cases} 6 = 3k_2 + b \\ m = b_2 \end{cases} \Rightarrow \begin{cases} k_2 = \frac{6-m}{3} \\ b_2 = m \end{cases}$$

3) $CD \perp AB \Rightarrow k_1 \cdot k_2 = -1$.

$$\frac{1}{4} \cdot \frac{6-m}{3} = -1, 6-m = 12.$$

$m = -6$.

Javob: -6.

20. Qarang: 8-variant 16-savol (62-bet).

21. $\frac{(a-3)^2}{a}$ ifoda natural qiymatlar qabul

qiladigan a ning eng katta va eng kichik natural qiymatlari nisbatini toping.

Yechish:

$\frac{(a-3)^2}{a}$ ifoda natural qiymat qabul qilishi

uchun $(a-3)^2$ ifoda a ga qoldiqsiz bo'linishi kerak.

$$\frac{a^2 - 6a + 9}{a} = a - 6 + \frac{9}{a}$$

$a = 1, 3, 9$ da:

$$a = 1, 1 - 6 + \frac{9}{1} = 1 - 6 + 9 = 4 \in \mathbb{N}$$

$$a = 3, 3 - 6 + \frac{9}{3} = 3 - 6 + 3 = 0 \notin \mathbb{N}$$

$$a = 9, 9 - 6 + \frac{9}{9} = 9 - 6 + 1 = 4 \in \mathbb{N}$$

$a = 1$ va $a = 9$ da ifoda natural son bo'ladi.
 $9:1 = 9$.

Javob: 9.

22. Qarang: 11-variant 6-savol (82-bet).

23. Qarang: 11-variant 10-savol (83-bet).

24. Qarang: 13-variant 27-savol (103-bet).

25. To'g'ri burchakli uchburchakka ichki chizilgan va tashqi chizilgan aylanalar radiuslari mos ravishda 4 va 10 ga teng. Uchburchakning perimetrini toping.

Yechish:

$R = 10$

$r = 4$

$P = ?$

1) To'g'ri burchakli uchburchakka tashqi chizilgan aylana radiusi $R = \frac{c}{2}$ bundan

$c = 2R = 2 \cdot 10 = 20$

2) ichki chizilgan aylana radiusi

$r = \frac{a+b-c}{2}$ bundan $a+b = 2r+c$

$a+b = 2 \cdot 4 + 20 = 28$.

3) $P = a+b+c = 28 + 20 = 48$.

Javob: 48.

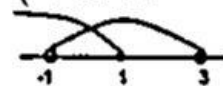
26. $\frac{1-x}{\sqrt{3+2x-x^2}} \geq 0$ tengsizlikni yeching.

Yechish:

$\frac{1-x}{\sqrt{3+2x-x^2}} \geq 0$ tengsizlik quyidagi

tengsizliklar sistemasiga teng kuchli:

$$\begin{cases} 1-x \geq 0 \\ 3+2x-x^2 > 0 \end{cases} \Rightarrow \begin{cases} x \leq 1 \\ x^2 - 2x - 3 < 0 \end{cases} \Rightarrow \begin{cases} x \leq 1 \\ -1 < x < 3 \end{cases}$$



$x \in (-1; 1]$.

Javob: $(-1; 1]$.

27. Qarang: 10-variant 6-savol (76-bet).

28. $f(x) = \sin x + \sin \frac{x}{5} + \cos \frac{x}{3}$ funksiyaning eng kichik musbat davrini toping.

Yechish:

$y = \sin x, y = \cos x$ funksiyalar davri $T = 2\pi$, bo'lganligi uchun, $y_1 = \sin \frac{x}{5}$ uchun

$T_1 = \frac{2\pi}{\frac{1}{5}} = 10\pi, y_2 = \cos \frac{x}{3}$ uchun

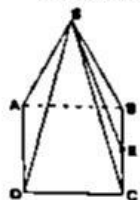
$T_2 = \frac{2\pi}{\frac{1}{3}} = 6\pi.$

Funksiyalarning eng kichik musbat davri
 $T = EKUK(T_1, T_2) = EKUK(10\pi, 6\pi) = 30\pi$.

Javob: 30π .

29. Muntazam to'rtburchakli piramida yon qirrasini 5 sm, to'la sirti 16 sm^2 bo'lsa, asos qirrasini toping.

Yechish:



$SA = 5 \text{ sm}$,
 $AB = a$
 $S_{\text{to'la}} = 16 \text{ sm}^2$
 $AB = a$,
 $SE = h_a$ – apofema

$$S_{\text{to'la}} = a^2 + \frac{4a \cdot h_a}{2} = a^2 + 2ah_a$$

$$SE^2 = SC^2 - EC^2 = 5^2 - \left(\frac{a}{2}\right)^2$$

$$h_a^2 = 25 - \frac{a^2}{4}$$

$$a^2 + 2ah_a = 16$$

$$a^2 + 2a \cdot \frac{\sqrt{100 - a^2}}{2} = 16$$

$$a\sqrt{100 - a^2} = 16 - a^2, \quad 0 < a \leq 4,$$

$$a^2(100 - a^2) = 256 - 32a^2 + a^4$$

$$a^4 - 66a^2 + 128 = 0, \quad a^2 = 2, \quad a^2 = 64$$

$$a = \sqrt{2}, \quad a = 8, \quad 0 < a \leq 4 \text{ bo'lganligi uchun}$$

$$a = \sqrt{2}.$$

Javob: $\sqrt{2}$.

30. Ishorasini aniqlang: $0 < a < 1 < b < c$,
 $\log_a b$; $\log_a c$; $\log_b c$; $\log_c a$.

Yechish:

$$0 < a < 1 < b < c$$

$$a = \frac{1}{2}, \quad b = 2,$$

$$c = 4 \text{ deb olaylik}$$

$$\log_a b = \log_{\frac{1}{2}} 2 = -1, \text{ - manfiy}$$

$$\log_a c = \log_{\frac{1}{2}} 4 = -2, \text{ - manfiy}$$

$$\log_b c = \log_2 4 = 2, \text{ + musbat}$$

$$\log_c a = \log_4 \frac{1}{2} = -\frac{1}{2}, \text{ - manfiy.}$$

Javob: -, -, +, -.

31. Qarang: 11-variant 34-savol (88-bet).

32. Tashqi qurilmalarni boshqarish elektron sxemalar – bu:

Yechish:

Kompyuter tashqi qurilmalarni boshqarish elektron sxemalari (kontrollerlar, adapterlar) ona platasining maxsus bo'linmalari (slotlar)da joylashadi. Ushbu bo'linmalar orqali kontrollerlar kompyuterining ma'lumotlar uzatish tizimli magistrali – shinaga bevosita ulanadi.

Javob: kontrollerlar.

33. Qarang: 9-variant 36-savol (74-bet).

34. Paskal tilida quyidagi dastur lavhasi bajarilgach b o'zgaruvchi qiymatini aniqlang:
 $x := -1$; $y := -1$; $a := 0,1$; IF $(x*x+y>0)$ AND $(a=1/10)$ THEN $b := \text{true}$ else $b := \text{false}$;

Yechish:

Masalaning qo'yilishida

$x = -1$; $y = -1$; $a = 0,1$ ga teng ekan.

IF $(x*x+y>0)$ AND $(a=1/10)$ THEN $b := \text{true}$ else $b := \text{false}$; – tarmoqlanuvchi operatori tahlil qilamiz:

AGAR $(-1*(-1)+(-1)>0)$ va $(a=1/10)$ rost bo'lsa, u holda $b := \text{true}$, aks holda $b := \text{false}$ ekan. Shartini tekshiramiz:

1) $(-1*(-1)+(-1)>0) \rightarrow 1-1 < 0 \rightarrow \text{yolg'on (0)}$

2) $(a=1/10) \rightarrow a=0,1 \rightarrow \text{rost (1)}$

And operandi – kon'yunksiya, ya'ni mantiqiy ko'paytma bo'lgani uchun $0 \cdot 1 = 0$ (yolg'on)ni beradi. Demak, ELSE so'zidan keyingi operator bajariladi: $b := \text{false}$.

Javob: false.

35. IP(Internet Protocol) manzil nima?

Yechish:

IP-manzil–internet tarmog'iga ulangan kompyuterning unikal TCP/IP tugunini anqlovchi 32 razryadli mantiqiy manzil. Har bir IP-manzil ikki qismdan iborat: tarmoq identifikatori va tugun identifikatori.

IP-manzil 32 bit (4 bayt) uzunlikka ega bo'lib, odatda har bir bayt qiymatini aniqlaydigan nuqtalar bilan ajratilgan 4 ta 0 dan 255 gacha bo'lgan o'nlik son tariqasida yoziladi.

Masalan:

128.10.11.31 – manzilning odatdagi o'nlik ko'rinishi.

Javob: internetga ulangan har bir kompyuterning unikal sonli manzili.

36. Ali sakkizlik sanoq sistemasida (73; 100) oraliqdagi barcha butun sonlarni yozib chiqdi. Vali esa shu sonlardan avval 5 raqami, so'ng 6 raqami qatnashgan barcha sonlarni o'chirib tashladi. Qolgan sonlar yig'indisini sakkizlik sanoq sistemasida aniqlang va oltilik sanoq sistemasiga o'tkazing.

Yechish:

Ali yozgan sonlar: 74, 75, 76, 77. Vali shu sonlardan 5 va 6 raqamlari qatnashgan barcha sonlarni o'chirganidan keyin 74, 77 sonlar qoladi. Ularning sakkizlik sanoq sistemasidagi yig'indisini hisoblaymiz: $74_8 + 77_8 = 173_8$. Endi bu sonni avval 10-lik sanoq sistemasiga, keyin 6-likka o'tkazamiz.

$$173_8 = 1 \cdot 8^2 + 7 \cdot 8^1 + 3 \cdot 8^0 = 64 + 56 + 3 = 123_{10}$$

$$\begin{array}{r|l} 123 & 6 \end{array}$$

$$\begin{array}{r|l|l} 120 & 20 & 6 \\ \hline 3 & 18 & 3 \\ \hline & 2 & \end{array}$$

Demak, $123_8 = 323_6$ ekan.

Javob: 323.