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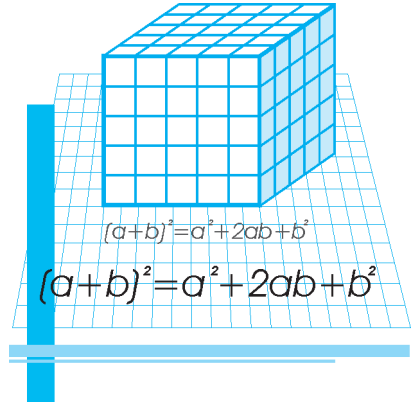
ALGEBRA

Umumiy ta'lim maktablarining
7- sinfi uchun darslik

Tuzatilgan 3- nashri

O'zbekiston Respublikasi Xalq ta'limi vazirligi tasdiqlagan



„O'QITUVCHI" NASHRIYOT-MATBAA IJODIY UYI
TOSHKENT — 2009

BBK 22.14 ya 72

Aziz o‘quvchim!

Ona yurtimiz mustaqil O‘zbekiston jahon ilm-u faniga, madaniyatiga yuzlab buyuk olimlarni, shoirlarni, davlat arboblarini, musavvirlarni yetishtirib bergan. Bilingki, siz ular ezgu ishlarining davomchisisiz!

Yoshlik bilim olish davridir.

Allomalar aytadi: „Yoshlikda olingan bilim toshga bitilgan yozuv kabi o‘chmasdir“. Algebrani, umuman, matematikani o‘rganish qunt va izchillikni, ko‘plab masala va misollarni tushunib, idrok qilib yechishni talab etadi. Meni yaxshi o‘rganib olsangiz, sizga umrbod do‘st bo‘lib qolaman!

Xulq-u odobingiz barkamol, ilmingiz ziyoda bo‘lishini istab

„*Algebra*“ darsligingiz.

Darslikdagi shartli belgilar:

 — asosiy qoidalar va xossalar.

○ — matematik tasdiqni asoslash yoki formulani keltirib chiqarish boshlandi.

● — asoslash yoki formulani keltirib chiqarish tugadi.

△ — masalani yechish boshlandi.

▲ — masalani yechish tugadi.

№ — qiziqarli masalalar.

 — sinov mashqlari.

 — asosiy material bo‘yicha bilimni tekshirish uchun mustaqil ish.

 — tarixiy masalalar.

 — tarixiy ma’lumotlar.

A $\frac{\textbf{4306020502 — 26}}{\textbf{354(04) — 2009}}$ **buyurt.var. — 2009**

5—6- SINFLARDA O‘RGANILGAN MAVZULARNI TAKRORLASH

Aziz o‘quvchi! Siz 5—6- sinflarda natural sonlar, oddiy va o‘nli kasrlar, ratsional sonlar ustida to‘rt amalga doir misol va masalalarni yechgansiz. 5—6- sinflarda matematikadan olgan bilimlaringizni yodga solish maqsadida Sizga bir nechta mashqlar taklif etamiz.

1. Amallarni bajaring:

1) $10\frac{1}{2}\cdot 1\frac{5}{7}+4\frac{4}{5}\cdot\left(10\frac{2}{3}+3\frac{11}{12}-6\frac{23}{24}\right)$;

2) $\left(6\frac{15}{29}+7\frac{14}{29}\right):1\frac{2}{5}-12\frac{6}{7}:3\frac{6}{7}\cdot 2\frac{1}{5}$;

3) $1\frac{5}{22}\cdot 7\frac{1}{3}+3\frac{1}{2}:\left(6\frac{1}{4}-3\frac{3}{4}+3\frac{1}{2}\right)$;

4) $4\frac{3}{8}:2\frac{3}{16}\cdot 2\frac{3}{4}-\left(3\frac{10}{17}+10\frac{7}{17}\right):2\frac{4}{5}$.

2. Amallarni bajaring:

1) $\frac{31,2\cdot 58,4-27,2}{31,2+58,4\cdot 30,2}$;

2) $\frac{28,4\cdot 40,3-11,9}{28,4+40,3\cdot 27,4}$.

3) $\frac{26\frac{2}{3}\cdot 17,8+8\frac{13}{15}}{26\frac{2}{3}+17\frac{4}{5}\cdot 25\frac{2}{3}}$;

4) $\frac{8,75\cdot 19\frac{2}{5}-10,65}{8,75+19\frac{2}{5}\cdot 7,75}$.

3. Tenglamani yeching:

1) $5x+48:4=20:10+2\cdot 10$;

2) $7x+32:2=(72+18):3$;

3) $4\frac{1}{2}x+3\frac{3}{10}\cdot 5=7\frac{6}{13}+18\frac{7}{13}$;

4) $6\frac{1}{2}x+3\frac{1}{2}\cdot 3=11\frac{4}{17}+5\frac{13}{17}$.

4. Ahmad velosipedda soatiga $10\frac{2}{5}$ km tezlik bilan $1\frac{1}{4}$ soat yo‘l yurdi. So‘ngra soatiga $12\frac{4}{5}$ km tezlik bilan $2\frac{1}{2}$ soat yo‘l yurdi. Ahmad jami necha kilometr yo‘l yurgan?

5. To‘g‘ri to‘rtburchakning bo‘yi 8 sm ga teng. Eni bo‘yidan 1,5 sm qisqa. To‘g‘ri to‘rtburchakning yuzini toping.

6. To‘g‘ri to‘rtburchakning yuzi 20,25 dm^2 ga, eni 3,24 dm ga teng. Shu to‘g‘ri to‘rtburchakning perimetrini toping.

7. „Tiko“ rusumli avtomobil 100 km masofaga 5 *l* benzin sarflaydi. Bu avtomobil: 50 km; 60 km; 70 km; 80 km; 120 km; 250 km; 360 km yo‘lga qancha benzin sarflaydi?

8. Sayyoh yo‘lning qismini $\frac{2}{7}$ o‘tdi. Hisoblab ko‘rsa, yo‘lning yarmiga yetishi uchun yana 9 km yurishi kerak ekan. Sayyoh jami necha kilometr yo‘l yurishni mo‘ljallagan?

9. „Matiz“ rusumli avtomobil 100 km masofaga 8 *l,* „Jiguli“ rusumli avtomobil esa shuncha masofaga 10 *l* benzin sarflaydi. Agar har bir avtomobil bakida 32 *l* dan benzin bo‘lsa, bu yonilg‘i ular uchun necha kilometr yo‘lga yetadi?

10. 1) Matoning narxi 20 % pasaytirildi. Ma’lum vaqtdan so‘ng, yangi narx ham 25 % pasaytirildi. Matoning narxi jami necha foiz kamaygan?

2) Gazlamaning narxi 20 % ortdi. Ma’lum vaqtdan so‘ng, yangi narx ham 25 % ortdi. Gazlamaning narxi jami necha foiz ortdi?

11. 1- va 2- nav mahsulotlarning birgalikdagi narxi 10 800 so‘m. Savdogar 1- nav mahsulotni 24 % foydasiga sotib, 7 812 so‘mlik bo‘ldi. 2- nav mahsulotidan foyda ko‘rmadi: olgan narxiga sota oldi, xolos. Savdogar natijada jami necha foiz foyda ko‘rdi? Uning foydasi necha so‘m bo‘lgan?

12. Chinni idishning narxi 200 so‘mga arzonlashdi, so‘ngra yangi narx ham 10 % ga arzonlashdi va endi idish 900 so‘mdan sotila boshladi. Idishning dastlabki narxi necha so‘m edi?

13. Tadbirkor 1- va 2- nav mollarni sotib, jami 5 400 so‘m foyda qildi. 1- nav molning narxi 12 000 so‘m edi, tadbirkor uni 15 % foydasiga sotdi. 2- nav moldan 20 % foyda ko‘rdi. 2- nav molning narxi necha so‘m? Ikkala nav molni sotib, tadbirkor necha foiz foyda ko‘rgan?

14. To‘g‘ri to‘rtburchak asosining uzunligi 20%, balandligi 25% orttirilsa, uning yuzi necha foiz ortadi?

15. To‘g‘ri to‘rtburchak asosining uzunligi 10%, balandligi 20% kamaytirilsa, uning yuzi necha protsent kamayadi?

16. Amallarni bajaring:

1) $(-120):\big((-8)\cdot(-3)+12:(-3)\big)-(-48):(-16)$;

2) $(-75)\cdot 4-204:(-3)+(-210):(-7)$;

3) $(-20,25):(-3,6)+90,72:(-4,5)-7,5\cdot 3,2$;

4) $5\frac{5}{19}\cdot(-0,95)+2\frac{16}{17}\cdot(-0,34)-8\frac{4}{7}:2\frac{1}{7}$.

17. Tenglamani yeching:

1) $3x+2x=17+(-27)$;

2) $6x-7x=3,5\cdot(-1)+4$;

3) $1,3x-3,5x=11\cdot(-0,5)$;

4) $4x-2\frac{1}{3}x=3\frac{1}{3}:(-2)$.

18. 5 ta sonning o‘rta arifmetigi 18,4 ga teng. Bu sonlarga yana bitta son qo‘shib, o‘rta arifmetik qiymat hisoblangan edi, u 20 ga teng chiqdi. Qo‘shilgan sonni toping.

19. Karim ota 90 yoshda. Uning nabiralarining o‘rtacha yoshi 20 da. Nabiralar yoshlariga Karim ota yoshini ham qo‘shib, o‘rta arifmetik qiymat hisoblangan edi, u 22 ga teng chiqdi. Karim otaning nechta nabirasi bor?

20. Avtomobil 72 km/soat tezlik bilan 3,5 soat, 60 km/soat tezlik bilan 2,5 soat yurdi. Avtomobil jami necha kilometr yo‘l yurgan? Uning o‘rtacha tezligini toping.

21. Proporsiyaning noma’lum hadini toping:

1) $3,5:x=2,4:4,8$;

2) $x:2\frac{1}{3}=9,2:2,3$;

3) $7,2:2,4=x:4\frac{1}{3}$;

4) $4\frac{2}{7}:2\frac{1}{7}=3,2:x$.

I BOB
ALGEBRAIK IFODALAR

1-§ Sonli ifodalar

Algebra so‘zi buyuk o‘zbek matematigi va astronomi, vatandoshimiz Abu Abdulloh Muhammad ibn Muso al-Xorazmiyning „Kitob al-muxtasar fi hisob al-jabr val-muqobala“ („Al-jabr val-muqobala“) asaridagi *al-jabr* (lotinchasiga *algebra*) so‘zidan olingan. Bu asarda al-Xorazmiy dunyoda birinchi marta algebra fanini izchillik bilan bayon qilgan.

Algebraning asosiy masalasi algebraik ifodalar ustida matematik amallarni o‘rganishdir. Algebraik ifodalarning eng sodda ko‘rinishi bo‘lgan sonli ifodalar V—VI sinf matematika kurslarida qaralgan edi.

Sonli ifoda sonlardan tuzilib, amallar belgilari bilan birlashtirilgan yozuv ekanligini eslatib o‘tamiz. Masalan,

$$2\cdot 3+7;\ \ 10:2-3;\ \ \frac{4\cdot 0,5+3}{5};\ \ \frac{1}{3}-\frac{1}{2}$$

yozuvlar sonli ifodalardir.

Sonli ifodaning qiymati deb, shu sonli ifodada ko‘rsatilgan amallarni bajarish natijasida hosil bo‘lgan sonni aytiladi.

Masalan, $2\cdot 3+7$ sonli ifodaning qiymati 13 soni, $\frac{1}{3}-\frac{1}{2}$ sonli ifodaning qiymati $-\frac{1}{6}$ sonidir.

Sonli ifoda bitta sondan iborat bo‘lishi ham mumkin. Uning qiymati shu sonning o‘zi bo‘ladi.

Ba’zan sonli ifodada sonlar va amallar belgilaridan tashqari amallarning ma’lum tartibda bajarilishini ko‘rsatuvchi qavslardan foydalaniladi. Masalan,

$$(2,5+3,5)\cdot 2,1$$

Abu Abdulloh Muhammad ibn Muso al-Xorazmiy (783—850) buyuk o‘zbek matematigi va astronomi.

sonli ifodaning qiymatini hisoblashda avval qavs ichidagi qo‘shish amali, keyin ko‘paytirish amali bajariladi.

$(2,5 + 3,5) \cdot 2,1$ ifodaning qiymatini hisoblab, 12,6 sonini hosil qilamiz. Shuning uchun

$$(2,5 + 3,5) \cdot 2,1 = 12,6$$

tenglikni yozish mumkin.

„=“ belgisi bilan birlashtirilgan ikkita sonli ifoda sonli tenglikni tashkil qiladi.

Agar tenglikning chap va o‘ng qismlarining qiymatlari bir xil son bo‘lsa, u holda tenglik to‘g‘ri tenglik deyiladi.

Masalan, $\frac{15-1}{2} = 8-1$ to‘g‘ri tenglik, chunki uning ikkala qismining ham qiymati 7 soniga teng.

Sonli ifodalar va sonli tengliklardan, hisoblashlar bilan bir qatorda, sonlarning xossalarini yozishda ham foydalaniladi.

Masalan, $\frac{3}{4} = \frac{6}{8}$ tenglik kasrlarning asosiy xossasini, $35 + 21 = 21 + 35$ tenglik esa qo‘shishning o‘rin almashtirish qonunini ifodalaydi.

Endi $6 + 12 \cdot 3$ sonli ifodani qaraylik. $6 + 12 \cdot 3 = 6 + 36 = 42$ dan iborat bo‘lgan to‘g‘ri natija amallarni qabul qilingan bajarish tartibiga rioya qilingan holdagina hosil bo‘ladi.

Agar qabul qilingan hisoblash tartibi buzilsa va avval 6 bilan 12 ni qo‘shib, so‘ngra 3 ga ko‘paytirilsa, u holda 54 dan iborat noto‘g‘ri natija hosil qilinadi. Bu natija dastlabki ifoda

$$(6 + 12) \cdot 3$$

kabi yozilsa to‘g‘ri bo‘lar edi.

Demak, hisoblashning to‘g‘riligi sonli ifodadagi amallarning bajarilish tartibiga bog‘liq ekan.

Sonlar ustida amallarning bajarilish tartibi algebraik ifodalarning son qiymatlarini topishga oid masalalarni bajarishda ham saqlanib qoladi.

Qo‘shish va ayirish *birinchi bosqich amallar*, ko‘paytirish va bo‘lish esa *ikkinchi bosqich amallar* deyilishini eslatib o‘tamiz. Kvadrat va kubga ko‘tarish *uchunchi bosqich amallar* deyiladi.

Sonli ifodaning son qiymatini topishda amallar bajarilishining quyidagi tartibi qabul qilingan:

1) Agar ifodada qavslar bo‘lmasa, u holda avval uchinchi bosqich amallar, keyin ikkinchi bosqich amallar va, nihoyat, birinchi bosqich amallar bajariladi, shu bilan birga, bir xil bosqich amallar ular qanday tartibda yozilgan bo‘lsa, xuddi shu tartibda bajariladi.

Masalan,

$$3\cdot 5^2\cdot 4-5\cdot 4+7=3\cdot 25\cdot 4-5\cdot 4+7=300-20+7=280+7=287.$$

2) Agar ifodada qavslar bo‘lsa, u holda avval qavslar ichidagi sonlar ustida barcha amallar, so‘ngra esa qolgan barcha amallar bajariladi, bunda qavs ichidagi va undan tashqaridagi barcha amallar 1-bandda ko‘rsatilgan tartibda bajariladi.

Masalan,

$$(2^3\cdot 4-5)\cdot 6+(2+2\cdot 4)=(8\cdot 4-5)\cdot 6+(2+2\cdot 4)=$$

$$=(32-5)\cdot 6+(2+8)=27\cdot 6+10=162+10=172.$$

3) Agar kasrning qiymati hisoblanadigan bo‘lsa, u holda kasrning suratidagi va maxrajidagi amallar bajariladi, so‘ngra birinchi natija ikkinchisiga bo‘linadi.

Masalan,

$$\frac{2\cdot 3^3-3\cdot 5}{3+5^2}=\frac{2\cdot 27-3\cdot 5}{3+25}=\frac{54-15}{28}=\frac{39}{28}=1\frac{11}{28}.$$

4) Agar ifodada qavslar ichida boshqa qavslar bo‘lsa, u holda avval eng ichkaridagi qavslar ichidagi amallar bajariladi.

Masalan,

$$2\cdot(8-(5^2-4))=2\cdot(8-(25-4))=2\cdot(8-21)=2\cdot(-13)=-26.$$

1. Amallarni bajaring:

1) $2{,}17+(3{,}2-0{,}17)$;

2) $9{,}49-(1{,}5+0{,}99)$;

3) $13\frac{7}{9}-\left(2{,}64+2\frac{7}{9}\right)$;

4) $6\frac{7}{8}-\left(3{,}14-2\frac{1}{8}\right)$.

2. Sonli ifodaning qiymatini toping:

1) $\left(\frac{1}{2}+\frac{1}{3}\right)\cdot\left(\frac{1}{5}-\frac{1}{4}\right)$;

2) $\left(\frac{2}{7}-\frac{3}{4}\right)\cdot\left(\frac{2}{13}-\frac{1}{2}\right)$;

3) $\left(0{,}3-\frac{1}{20}\right):\left(\frac{3}{4}-1{,}25\right)$;

4) $\left(2{,}7-\frac{1}{5}\right):\left(\frac{1}{2}+4{,}5\right)$.

3. Qiymati: 1) 8; 2) 0; 3) 1; 4) —14 ga teng bir nechta sonli ifoda yozing.

4. Tenglik to‘g‘rimi:

1) $\frac{12{,}5-4{,}1}{4}=1{,}7+0{,}4$;

2) $\frac{0{,}75-0{,}15}{2}=0{,}15+0{,}25$;

3) $\frac{2{,}13+4{,}33}{7{,}58-4{,}35}=1\frac{5}{12}+\frac{1}{3}+\frac{1}{4}$;

4) $\frac{8{,}92-6{,}61}{5{,}38-1{,}55}=2\frac{1}{9}-\frac{1}{2}-\frac{1}{3}$?

Sonli tenglik shaklida yozing **(5—6):**

5. 1) $\frac{1}{3}$ va $\frac{1}{5}$ sonlarining yig‘indisi $\frac{2}{3}$ va $\frac{2}{15}$ sonlarining ayirmasiga teng;

2) 40 va 0,03 sonlarining ko‘paytmasi 6 sonini 5 ga bo‘linmasiga teng.

6. 1) 10 va –2 sonlari ayirmasining ikkilangani shu sonlar yig‘indisidan uch marta katta;

2) 2 va 6 sonlari yig‘indisining uchlangani shu sonlar ko‘paytmasidan ikki marta ortiq.

7. Amallar tartibini ko‘rsating va hisoblang:

1) $1{,}7\cdot 3^2+\frac{2}{3}\cdot 12-15$;

2) $27{,}7-\left(\frac{1}{2}\right)^2\cdot 100+6{,}4:0{,}8$;

3) $48\cdot 0{,}05-\left(\frac{1}{3}\right)^2\cdot 54+1{,}7$;

4) $(2{,}5)^2+15\cdot\frac{3}{5}-0{,}24:0{,}6$.

8. Sonli ifodaning qiymatini toping:

1) $\left(\frac{1}{4}+\frac{1}{6}\right)\cdot\left(\frac{2}{5}-\frac{1}{2}\right)$;

2) $\left(\frac{4}{7}-\frac{3}{2}\right)\cdot\left(\frac{1}{13}-\frac{1}{4}\right)$;

3) $4\frac{2}{3}+\frac{1}{4}\cdot\left(1\frac{7}{9}-\frac{1}{9}\right)$;

4) $5\frac{1}{7}-\frac{1}{7}\cdot\left(1\frac{3}{4}+\frac{1}{4}\right)$.

9. Amallarni bajaring:

1) $\frac{0,3\cdot 5^2-15}{3,5+2^2}$;

2) $\frac{4,2:6-3\frac{1}{3}\cdot 0,3}{7,5:0,5}$;

3) $13\frac{1}{3}\cdot(18,1-(3^2+6,1))$;

4) $((7,8:0,3-3^3)+3,1):0,7$.

2-§ *Algebraik ifodalar*

Quyidagi masalani qaraymiz.

1-masala. Biror son o‘ylang, uni 3 ga ko‘paytiring, hosil bo‘lgan natijaga 6 ni qo‘shing, topilgan yig‘indini 3 ga bo‘ling va o‘ylangan sonni ayiring. Qanday son hosil bo‘ladi?

△ Aytaylik, o‘ylangan son 8 bo‘lsin. Barcha amallarni masala shartida ko‘rsatilgan tartibda bajaramiz:

1) $8\cdot 3=24$; 2) $24+6=30$; 3) $30:3=10$; 4) $10-8=2$.

2 soni hosil bo‘ldi.

Bu yechimni qiymati 2 ga teng bo‘lgan $(8\cdot 3+6):3-8$ sonli ifoda shaklida yozish mumkin.

Bordi-yu, agar 5 soni o‘ylangan bo‘lsa, u holda qiymati yana 2 ga teng bo‘lgan $(5\cdot 3+6):3-5$ sonli ifoda hosil qilingan bo‘lar edi.

Biz qanday sonni o‘ylamaylik, natijada 2 soni hosil bo‘laverar ekan-da, degan faraz tug‘iladi. Buni tekshirib ko‘ramiz. O‘ylangan sonni a harfi bilan belgilaymiz va amallarni yana masala shartida ko‘rsatilgan tartibda yozamiz:

$$(a\cdot 3+6):3-a.$$

Arifmetik amallarning bizga ma’lum bo‘lgan xossalaridan foydalanib, bu ifodani soddalashtiramiz:

$$(a\cdot 3+6):3-a=a+2-a=2.\ \blacktriangle$$

Masalani yechishda istagan sonni bildiruvchi a harfi, 3 va 6 sonlari, amallar belgilari va qavslardan iborat

$$(a \cdot 3 + 6) : 3 - a$$

ifoda hosil qilindi. Bu algebraik ifodaga misoldir.

Yana algebraik ifodalarga misollar keltiramiz:

$$2(m+n), \quad 3a + 2ab - 7, \quad (a+b)(a-b), \quad \frac{x+y}{a}.$$

Algebraik ifoda sonlar va harflardan tuzilib, amallar belgilari bilan birlashtirilgan ifodadir.

Agar algebraik ifodaga kirgan harflar o‘rniga biror sonni qo‘yilsa va ko‘rsatilgan amallar bajarilsa, u holda natijada hosil qilingan sonni *berilgan algebraik ifodaning son qiymati* deyiladi.

Masalan, $a = 2$, $b = 3$ bo‘lganda

$$3a + 2b - 7$$

algebraik ifodaning qiymati 5 ga teng, chunki $3 \cdot 2 + 2 \cdot 3 - 7 = 5$; shu algebraik ifodaning qiymati $a = 1$; $b = 0$ bo‘lganda -4 ga teng, chunki

$$3 \cdot 1 + 2 \cdot 0 - 7 = -4.$$

a ning istalgan qiymatida

$$(a \cdot 3 + 6) : 3 - a$$

algebraik ifodaning qiymati 2 ga teng.

2-masala. $\frac{(3a+7)b}{a-b}$ ifodaning qiymatini $a = 10$, $b = 5$ bo‘lganda toping.

$$\triangle \frac{(3 \cdot 10 + 7) \cdot 5}{10 - 5} = \frac{37 \cdot 5}{5} = 37. \ \blacktriangle$$

Mashqlar

10. Algebraik ifodaning qiymatini toping:

1) $3a - 2b$, bunda $a = \frac{1}{3}$, $b = 1$;

2) $2a + 3b$, bunda $a = 3$, $b = -2$;

3) $0{,}25a - 4c^2$, bunda $a = 4$, $c = 3$;

4) $\left(2a^2 - \frac{1}{3}b\right)$, bunda $a = 2$, $b = 9$.

11. Algebraik ifodaning qiymatini toping:

1) $\frac{1}{4}x-\frac{3}{7}y$, bunda $x=8, y=-14$;

2) $\frac{2}{3}x+\frac{4}{5}y$, bunda $x=9,\ y=-10$;

3) $\frac{a-3b}{a+3b}$, bunda $a=4,\ b=-2$;

4) $\frac{a+3c}{2a-c}$, bunda $a=3,\ c=-1$.

12. Neft quvuridan 1 soatda 7 t neft oqadi, m soatda quvurdan necha tonna neft oqib o‘tadi? Bir sutkada-chi?

13. 1) m soatda; 2) p sekundda; 3) m soat l minut va p sekundda necha minut bor?

14. x va y sonlar ayirmasining uchlanganini yozing. Shu ifodaning:

1) $x=-0,37,\ y=-0,42$; 2) $x=-2,98,\ y=-4,48$;

3) $x=-\frac{5}{6},\ y=-\frac{9}{4}$; 4) $x=\frac{2}{15},\ y=-0,7$

bo‘lgandagi son qiymatini toping.

15. x va y sonlar yig‘indisi bilan ular ayirmasining ko‘paytmasini yozing. Hosil bo‘lgan algebraik ifodaning:

1) $x=-\frac{1}{8},\ y=\frac{1}{4}$; 2) $x=-\frac{5}{8}, y=\frac{3}{4}$;

3) $x=0,15,\ y=-0,75$; 4) $x=1,32,\ y=-1,28$

bo‘lgandagi son qiymatini toping.

Algebraik ifodalarning son qiymatini toping **(16—17):**

16. 1) $\frac{2mn(n+k)}{n-k}$, bunda $m=k=\frac{1}{3},\ n=\frac{1}{2}$;

2) $\frac{(3p+1)\cdot 2p}{p-l}+\frac{1}{3}$, bunda $p=\frac{1}{3},\ l=1$.

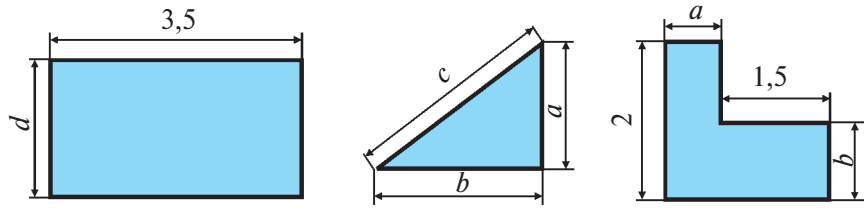
17. 1) $\frac{3(x-y)}{2p+q}$, bunda $x=8,31;\ y=2,29;\ p=2,01;\ q=2$;

2) $\frac{5(bc+m)}{2q+4\frac{1}{4}}$, bunda $b=\frac{2}{3};\ c=6;\ q=\frac{1}{2},\ m=\frac{1}{5}$.

18. Toq son formulasi $n = 2k + 1$ dan foydalanib, $k = 0$, $k = 1$, $k = 7$, $k = 10$ bo‘lganda n ning qiymatini toping.

19. Algebraik ifoda shaklida yozing:
1) kichigi n ga teng bo‘lgan ikkita ketma-ket natural sonning yig‘indisi; 2) kattasi m ga teng bo‘lgan ikkita ketma-ket natural sonning ko‘paytmasi; 3) kichigi $2k$ ga teng bo‘lgan uchta ketma-ket juft natural sonning yig‘indisi; 4) kichigi $2p + 1$ ga teng bo‘lgan uchta ketma-ket toq natural sonning ko‘paytmasi.

20. Shakllarning perimetri va yuzini algebraik ifoda shaklida yozing (1-rasm):



1-rasm.

21. Uyni isitish uchun p tonna ko‘mir g‘amlandi; shu zaxiradan q tonna sarf qilindi. Necha tonna ko‘mir qoldi? 1) $p = 20$, $q = 15$ bo‘lganda hisoblang. 2) q son p sondan katta bo‘lishi mumkinmi? p ga teng bo‘lishi-chi?

22. Kurash musobaqasida har biri 400 so‘mdan n ta chipta va har biri 500 so‘mdan m ta chipta sotildi. Hamma chiptalar uchun qancha pul olingan? $n = 200$, $m = 150$; $n = 100$, $m = 230$ bo‘lganda hisoblang.

23. Bitta albomning bahosi 200 so‘m, bitta daftarning bahosi 40 so‘m, bitta ruchkaning bahosi 60 so‘m. c ta albom, a ta daftar va b ta ruchkaning umumiy (so‘mlardagi) bahosini p harfi bilan belgilab, uni formula shaklida yozing. Agar $c = 9$, $a = 21$, $b = 4$ bo‘lsa, bu formula bo‘yicha p ni hisoblang.

24. Issiqlik uzatish stansiyasi uchun mo‘ljallangan gaz quvuri orqali har minutda 26 kub metr gaz o‘tadi. 5 sutkada; m sutkada quvurdan necha kub metr gaz o‘tadi?

25. Geologlar o‘z yo‘nalishi bo‘yicha harakat qilib, otda soatiga c kilometr tezlik bilan 3 soat-u 10 minut yurishdi; oqimining tezligi soatiga a kilometr bo‘lgan daryoda oqim bo‘yicha 1 soat-u

40 minut solda suzishdi va soatiga b kilometr tezlik bilan 2 soat-u 30 minut piyoda yurishdi. Yo‘nalishning (km lardagi) uzunligini s harfi bilan belgi-lab, geologlar bosib o‘tgan yo‘l formulasini yozing. Agar $a = 3{,}3$ km/soat, $b = 5{,}7$ km/soat, $c = 10{,}5$ km/soat bo‘lsa, yo‘nalishning uzunligini hisoblang.

3- §. *Algebraik tengliklar, formulalar*

Ko‘pgina amaliy masalalarni yechishda sonlarni belgilash uchun harflardan foydalanish qulaydir.

Masalan, agar a va b to‘g‘ri to‘rtburchak tomonlarining uzunliklari bo‘lsa, u holda $a \cdot b$ — uning yuzi, $2 \cdot (a + b)$ — uning perimetri. Bu yerda a va b harflari bilan musbat sonlar — to‘g‘ri to‘rtburchak tomonlarining uzunliklari belgilangan. Agar to‘g‘ri to‘rtburchak yuzini S harfi bilan, perimetrini esa P bilan belgilasak, u holda quyidagi formulalarni hosil qilamiz:

$$S = a \cdot b, \qquad P = 2 \cdot (a + b).$$

Agar tomonlar uzunliklari santimetrlarda o‘lchangan bo‘lsa, u holda S yuz kvadrat santimetrlarda, P perimetr esa santimetrlarda ifodalanadi.

Yozuvni qisqartirish uchun ko‘paytirish belgisi „nuqta“ ko‘pincha tushirib qoldiriladi. Masalan, $S = ab$, $P = 2(a + b)$ deb yoziladi.

Harflar bilan, shuningdek, tenglamalardagi noma’lum sonlar ham belgilanadi. Masalan:

$$x + 12{,}3 = 95{,}1$$

tenglamadagi noma’lum son x harfi bilan belgilangan,

$$2y + 3 = 7$$

tenglamadagi noma’lum son esa y harfi bilan belgilangan.

Harflar bilan arifmetik amallar qonunlari va xossalarini yozish ham qulaydir. Masalan:

$$a - (b + c) = (a - b) - c = a - b - c, \quad (1)$$
$$(a + b) \cdot c = a \cdot c + b \cdot c, \quad (2)$$
$$(a + b) : c = a : c + b : c. \quad (3)$$

XVI asrning taniqli matematigi Fransua Viyet (1540—1603) algebraga harfiy simvolikani kiritishning asoschisi hisoblanadi.

Algebrada birgina harfning o‘zi har xil sonli qiymatlar qabul qilishi mumkin. Jumladan, (1) va (2) tengliklarda a, b, c — ixtiyoriy sonlar; (3) tenglikda esa a, b — istalgan sonlar, lekin $c \neq 0$, chunki nolga bo‘lish mumkin emas.

Harflar yordamida juft va toq natural sonlar formulasini yozish mumkin.

Agar a juft son bo‘lsa, u holda bu son 2 ga bo‘linadi va uni bunday yozish mumkin:

$$a = 2n,$$

bu yerda n — natural son.

Agar b toq son bo‘lsa, u holda uni 2 ga bo‘lgandagi qoldiq 1 ga teng, binobarin, b sonni bunday yozish mumkin:

$$b = 2n + 1,$$

bu yerda n — natural son yoki nol.

Ba’zan, toq natural sonlar formulasini quyidagicha ham yozishadi:

$$b = 2k - 1,$$

bu yerda k — natural son.

Harflardan foydalanish bir xil toifadagi ko‘pgina masalalarni yechish yo‘lini yozishga imkon beradi. Shunga doir masalalar qaraylik.

1-masala. Fermerning bog‘ maydoni to‘g‘ri to‘rtburchak shaklida bo‘lib, uning bo‘yi a kilometrga, eni esa b kilometrga teng. Yangi yer o‘zlashtirilgandan keyin maydonning yuzi 0,88 km^2 ga ortdi. Bog‘ maydonining yuzi qancha bo‘ldi? Hisoblashlarni 1) $a = 2{,}2$ va $b = 0{,}8$; 2) $a = 1{,}4$ va $b = 4{,}3$ uchun bajaring.

△ Dastlab bog‘ning yuzi $a \cdot b$ km^2 ga teng edi, yangi yer ochilgandan keyin u $(ab + 0{,}88)km^2$ ga teng bo‘ldi.

1) $a = 2{,}2$ va $b = 0{,}8$ bo‘lganda, $2{,}2 \cdot 0{,}8 + 0{,}88 = 2{,}64$.

2) $a = 1{,}4$ va $b = 4{,}3$ bo‘lganda, $1{,}4 \cdot 4{,}3 + 0{,}88 = 6{,}9$. ▲

2-masala. Sayyoh qishloqdan chiqib, shahar tomon jo‘nadi. U a kilometr piyoda yurganidan keyin avtobusga o‘tirdi va avtobusda t soatda shaharga yetib keldi. Agar avtobus 60 km/soat tezlik bilan harakat qilgan bo‘lsa: 1) $a = 5$ va $t = 0{,}5$ bo‘lganda qishloq bilan shahar orasidagi s masofani hisoblang; 2) $s = 70$, $a = 10$ bo‘lganda t ni toping.

△ Sayyoh avtobusda t soatda 60 t kilometr yo‘l bosgan. Shuning uchun qishloq bilan shahar orasidagi masofa

$$s = a + 60t$$

formula bilan ifodalanadi.

1) $a = 5$ va $t = 0{,}5$ bo‘lganda, $s = 5 + 60 \cdot 0{,}5 = 35$ km bo‘ladi;

2) $s = a + 60t$ formuladan t ni topamiz: $t = \frac{s - a}{60}$. Bu yerdan $s = 70$, $a = 10$ bo‘lganda, $t = (70 - 10) : 60 = 1$ soat. ▲

Mashqlar

26. Yozing:

1) m va n sonlarning yig‘indisini;
2) a va b sonlarning ayirmasini;
3) a va b sonlar ayirmasining ikkilanganini;
4) m va n sonlar ko‘paytmasining ikkilanganini;
5) n va m sonlar yig‘indisining ular ayirmasiga bo‘linmasini;
6) a va b sonlar yig‘indisining ular ayirmasiga ko‘paytmasini.

27. Quyidagi ifodalarda harflar qanday sonlarni ifodalashi mumkin:

1) tanaffus n minut davom etadi;
2) sinfimizda y nafar o‘quvchi bor;
3) VII sinfda x ta o‘quv fani o‘qitiladi;
4) bir oyda k kun bor?

28. Yerning sun’iy yo‘ldoshi 9 km/s tezlik bilan harakat qiladi. Ushbu jadvalni to‘ldiring:

Bosib o‘tilgan masofa, km	45 000	1 350 000
Harakat vaqti, s		

29. „Matiz“ avtomobili 100 km yo‘lga a litr yonilg‘i sarf qiladi. Ushbu jadvalni to‘ldiring:

Bosib o‘tilgan masofa, km	300	800	1000			s
Yonilg‘i sarfi, l				$5a$	$4a$	

30. Birinchi qopda m kilogramm, ikkinchi qopda esa birinchi qopdagidan n kilogramm kam un bor. Ikkinchi qopda necha kilogramm un bor? Masalani 1) $m = 50$ va $n = 12$; 2) $m = 45$ va $n = 15$ hollar uchun yeching.

31. Piyoda 1 soatda 5 km yo‘l bosadi. U: 1) 3 soatda necha kilometr yo‘l bosadi? 2) k soatda-chi?

32. Do‘konga har birida 50 kg dan un bo‘lgan a ta qop keltirildi. Do‘konga necha kilogramm un keltirilgan?

33. Bog‘bonlar 1 kunda 15 gektar bog‘ga ishlov berishdi. Ular a kunda necha gektar bog‘ga ishlov berishadi?

34. Har biri x so‘mdan 6 ta daftar va har biri y so‘mdan 3 o‘ram qog‘oz sotib olindi. Hamma xarid qancha turadi?

35. Yuk mashinasi do‘konga ombordan har biri a kilogrammdan 15 yashik olxo‘ri va har biri b kilogrammdan 20 yashik olma keltirdi. Do‘konga necha kilogramm meva keltirilgan?

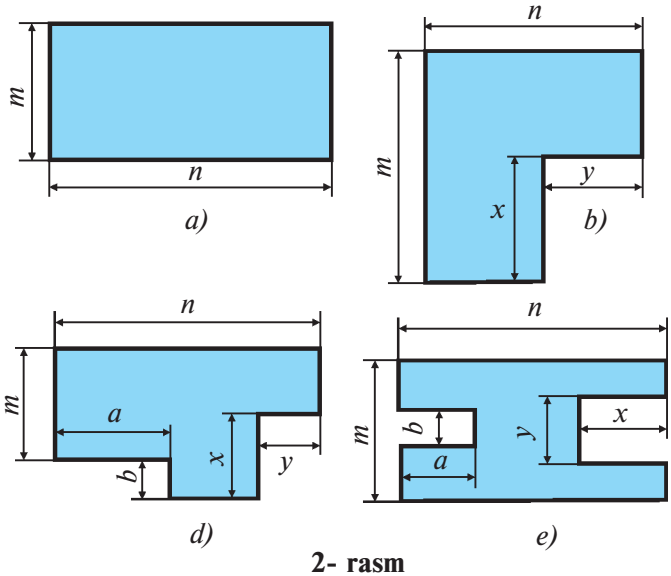
36. Mashinaga har biri m kilogrammdan k qop bug‘doy va har biri n kilogrammdan c qop arpa yuklandi. Mashinaga necha kilogramm don yuklangan?

37. To‘g‘ri to‘rtburchak shaklidagi tajriba maydonining bo‘yi a metrga teng, eni esa bo‘yidan b metr qisqa. Shu maydonning yuzi S ning formulasini yozing.

38. Kinoteatrda har biri n ta o‘rindiqqa ega bo‘lgan m ta qator va yana k ta qo‘shimcha o‘rindiq bor. Kinoteatrda hammasi bo‘lib nechta o‘rindiq bor? Masalani yechish formulasini tuzing va $m = 30$, $n = 25$, $k = 60$ bo‘lganda hisoblashlarni bajaring.

39. Dars jadvalida 5 ta dars, ikkita 15 minutlik va ikkita 10 minutlik tanaffus bo‘lgan kuni o‘quvchi maktabda necha soat bo‘ladi? (1 dars — 45 minut).

40. O‘lchamlari 2- rasmda ko‘rsatilgan shakllarning perimetrlarini va yuzlarini hisoblash uchun formulalar yozing:



2- rasm

41. To‘g‘ri to‘rtburchakning bo‘yi kvadratning tomonidan 8 m uzun, eni esa shu kvadrat tomonidan 4 m qisqa. Kvadrat tomonini biror harf bilan belgilab, to‘g‘ri to‘rtburchak uchun:
1) tomonlarning uzunligini; 2) perimetrini; 3) yuzini yozing.

42. Avtobus t soatda s kilometr yo‘l bosadi. Avtomobil xuddi shu yo‘lni avtobusdan 1 soat oldin bosib o‘tishi uchun qanday tezlikka ega bo‘lishi kerak?

43. $x = 2a + 3b$ (km) formula avtobusning harakati haqidagi masala yechilishini bildiradi. Masala shartini tuzing.

44. Maktab tajriba maydoni a kvadrat metr yuzga ega. Bog‘ yuzi 1500 m^2 bo‘lgan joyni egallagan, qolgan maydon 20 ta bir xil maydonchaga bo‘lingan. Shu maydonchalarning har biri qanday yuzga ega?

45. Bankka 50 000 so‘m pul qo‘yildi. Bir yildan so‘ng jamg‘arma p % ko‘paydi. Bir yildan keyin jamg‘armaning miqdori necha so‘mga yetdi?

46. Asosi a detsimetr, perimetri esa 42 dm bo‘lgan to‘g‘ri to‘rtburchakning yuzini hisoblash uchun ifoda tuzing. a ning ushbu jadvalda keltirilgan

qiymatlari uchun to‘g‘ri to‘rtburchak yuzi S ning qiymatini (dm^2 larda) hisoblang:

a	5	6	7,5	10	12	12,5	15
S							

№ 1 Faqat 4 ta 9 va arifmetik amal belgilari yordamida qiymati 100 ga teng bo‘lgan sonli ifoda tuzing.

47. Velosipedchi soatiga v kilometr tezlik bilan harakat qilmoqda. U jo‘nash joyidan s kilometr uzoqlikda bo‘lgan qishloqqa borishi kerak. Agar u 3 km yo‘lni bosib o‘tgan bo‘lsa, unga qishloqqa yetib borishi uchun yana qancha vaqt talab qilinadi? Agar u 3 km yurgan va $s = 36$, $v = 12$ bo‘lsa, qishloqqa 2,5 soatda yetib bora oladimi?

48. „Tiko“ rusumli avtomobil 100 km yo‘lga o‘rtacha 5 l, „Jiguli“ rusumli avtomobil esa 100 km yo‘lga o‘rtacha 10 l benzin sarflaydi. Har bir avtomobil bakida a l benzin bo‘lsa, ular qanday masofaga bora oladi? Agar $a = 20$ l va avtomobillar Toshkentdan bir vaqtda Samarqandga qarab yo‘lga chiqishgan bo‘lsa, qaysi mashina Samarqandga yetib kela oladi? (Toshkent va Samarqand orasidagi masofa 300 km).

4-§ *Arifmetik amallarning xossalari*

Algebrani puxta o‘rganish uchun arifmetik amallarning xossalarini yaxshi bilish lozim. Eslatib o‘taylik, arifmetik amallar deb qo‘shish, ayirish, ko‘paytirish va bo‘lish amallarini aytiladi. Sonlar ustidagi bu amallarning xossalarini qisqacha formulalar ko‘rinishida yozamiz. Amallarning asosiy xossalari odatda *qonunlar* deb ataladi. Qonunlardan foydalanib, amallarning boshqa xossalarini ham asoslash mumkin.

1. Qo‘shish va ko‘paytirish.

Qo‘shish va ko‘paytirishning asosiy qonunlarini sanab o‘tamiz.

1. O‘rin almashtirish qonuni:

$$a + b = b + a,\ ab = ba.$$

2. Guruhlash qonuni:

$$(a+b)+c=a+(b+c),\ \ (ab)c=a(bc).$$

3. Taqsimot qonuni:

$$a(b+c)=ab+ac.$$

Bu tengliklarda *a, b, c* — ixtiyoriy sonlar. Masalan,

$$1,2+3,5=3,5+1,2;\quad \frac{3}{4}\cdot\left(-\frac{2}{7}\right)=\left(-\frac{2}{7}\right)\cdot\frac{3}{4};$$

$$(-8)\cdot(125+7)=(-8)\cdot 125+(-8)\cdot 7.$$

Qo‘shish va ko‘paytirish qonunlari yordamida amallarning boshqa xossalarini ham hosil qilish mumkin. Masalan:

$$a+b+c+d=a+(b+c+d),\ (abc)d=(ab)(cd),$$
$$(a+b+c)d=ad+bd+cd.$$

1-masala. Hisoblang: 75 + 37 + 25+ 13.

△Hisoblashlarni ko‘rsatilgan tartibda olib borish mumkin: 75 ga 37 ni qo‘shib, natijaga 25 ni qo‘shish va oxirgi natijaga 13 ni qo‘shish. Lekin qo‘shishning xossalaridan foydalanib, hisoblashlarni soddalashtirish mumkin:

$$75+37+25+13=(75+25)+(37+13)=100+50=150.\ ▲$$

Bu misol shuni ko‘rsatadiki, amallarning xossalaridan foydalanib, hisoblashlarni eng sodda (oqilona) usulda bajarish mumkin.

Amallarning xossalari algebraik ifodalarni soddalashtirish maqsadida bajariladigan almashtirishlarda ham qo‘llaniladi.

2-masala. Ifodani soddalashtiring:

$$3(2a+4b)+5(7a+b).$$

$$△3(2a+4b)+5(7a+b)=3\cdot 2a+3\cdot 4b+5\cdot 7a+5\cdot b=6a+12b+35a+5b=$$
$$=(6a+35a)+(12b+5b)=(6+35)a+(12+5)b=41a+17b.\ ▲$$

Bu masalani yechish jarayonida quyidagi ifoda hosil bo‘ldi:

$$6a+12b+35a+5b.$$

Bu ifodada $6a$ va $35\ a$ qo‘shiluvchilar o‘xshashdir, chunki ular bir-biridan faqat koeffitsiyentlari bilangina farq qiladi. $12\ b$ va $5\ b$ qo‘shiluvchilar ham o‘xshash. Shu sababli $6a + 12b + 35a + 5b$ ifoda o‘rniga $41a + 17b$ ifodani yozish, ya’ni o‘xshash hadlarni ixchamlash mumkin bo‘ladi.

Oraliq hisoblashlarni og‘zaki bajarib, almashtirishlar yozuvini qisqartirish mumkin. Masalan,

$$6(3x+4)+2(x+1)=18x+24+2x+2=20x+26.$$

2. Ayirish.

3-masala. Toshkent va Samarqand shaharlari orasida Jizzax shahri joylashgan. Toshkentdan Samarqandgacha bo‘lgan masofa 300 km, Toshkentdan Jizzaxgacha bo‘lgan masofa esa 180 km. Jizzaxdan Samarqandgacha bo‘lgan masofani toping.

△ Jizzaxdan Samarqandgacha bo‘lgan masofa x kilometr bo‘lsin. U holda

$$180+x=300,\ \text{bu yerdan}\ \ x=300-180=120.$$

Javob. 120 km. ▲

$180 + x = 300$ tenglikdan x qo‘shish amaliga teskari deb ataluvchi ayirish amali yordamida topiladi.

Ayirishni qarama-qarshi sonni qo‘shish bilan almashtirish mumkin:

$$a-b=a+(-b).$$

Shu sababli ayirish amalining xossalarini qo‘shish amalining xossalari orqali asoslash mumkin. Masalan:

$$251+(49-13)=251+49-13=287, \qquad a+(b-c)=a+b-c,$$
$$123-(23+39)=123-23-39=61, \qquad a-(b+c)=a-b-c,$$
$$123-(83-77)=123-83+77=117, \qquad a-(b-c)=a-b+c.$$

4-masala. Ifodaning qiymatini hisoblang:

$$4(3x-5y)+6(x-y),$$

bunda $x=\frac{1}{2},\ \ y=\frac{1}{13}$.

△ Avval berilgan ifodani soddalashtiramiz:

$$4(3x-5y)+6(x-y)=12x-20y+6x-6y=18x-26y.$$

Hosil bo‘lgan ifodaning $x=\frac{1}{2}$, $y=\frac{1}{13}$ dagi qiymatini hisoblaymiz:

$$18\cdot\frac{1}{2}-26\cdot\frac{1}{13}=9-2=7.\ \blacktriangle$$

Shunday qilib, amallarning xossalaridan foydalanish algebraik ifodani avval soddalashtirib, so‘ngra uning qiymatini oson yo‘l bilan hisoblash imkonini beradi.

3. Bo‘lish.

5-masala. To‘g‘ri to‘rtburchakning yuzi 380 sm^2, tomonlaridan biri 95 sm. To‘g‘ri to‘rtburchakning ikkinchi tomoni uzunligini toping.

$\triangle$ $S=ab$ formuladan $b=\frac{S}{a}$ ni topamiz. $S=380$, $a=95$ bo‘lgani uchun

$$b=\frac{380}{95}=4.$$

Javob. 4 sm. $\blacktriangle$

$ab=S$ tenglikdan b ko‘paytirish amaliga teskari deb ataluvchi bo‘lish amali yordamida topiladi.

Bo‘lish bo‘luvchiga teskari bo‘lgan songa ko‘paytirish bilan almashtirilishi mumkin:

$$\frac{a}{b}=a\cdot\frac{1}{b}.$$

Shu sababli bo‘lishning xossalarini ko‘paytirishning xossalaridan keltirib chiqarish mumkin.

6-masala. Tenglikni isbotlang:

$$\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c},$$

bu yerda $c\neq 0$.

$\triangle$ Bo‘lishni ko‘paytirish bilan almashtirib, quyidagini hosil qilamiz:

$$\frac{a+b}{c}=(a+b)\cdot\frac{1}{c}.$$

Taqsimot qonunini qo‘llab,

$$(a+b)\cdot\frac{1}{c}=a\cdot\frac{1}{c}+b\cdot\frac{1}{c}$$

ni topamiz. Ko‘paytirishni bo‘lish bilan almashtirib,

$$a\cdot\frac{1}{c}+b\cdot\frac{1}{c}=\frac{a}{c}+\frac{b}{c}$$

ni hosil qilamiz. ▲

Mashqlar

49. Arifmetik amallar qonunlari va xossalarini qo‘llab, sonli ifodaning qiymatini toping:

1) $29\cdot 0,45+0,45\cdot 11$;

2) $(51,8+44,3+48,2-24,3)\cdot\frac{1}{3}$;

3) $4,07-5,49+8,93-1,51$;

4) $-11,401-23,17+4,401-10,83$.

50. O‘xshash hadlarni ixchamlang:

1) $4a+2b+a-b$;

2) $x-2y-3x+5y$;

3) $0,1c-0,3+d-c-2,1d$;

4) $8,7-2m+n-\frac{1}{3}m+\frac{2}{3}n$.

51. O‘xshash hadlarni ixchamlang:

1) $2,3a-0,7a+3,6a-1$;

2) $0,48b+3+0,52b-3,7b$;

3) $\frac{1}{3}x+\frac{1}{2}x-\frac{1}{6}a-\frac{5}{6}a+2$;

4) $\frac{5}{6}y-\frac{1}{3}b-\frac{1}{6}y+\frac{2}{3}b-3$;

5) $2,1m+n-3,2n+2m+1,1m-n$;

6) $5,7p-2,7q+0,3p+0,8q+1,9q-p$

52. Ifodani soddalashtiring:

1) $3(2x+1)+5(1+3x)$;

2) $4(2+x)-3(1+x)$;

3) $10(n+m)-4(2m+7n)$;

4) $11(5c+d)+3(d+c)$.

53. Ifodani soddalashtiring va son qiymatini toping:

1) $5(3x-7)+2(1-x)$, bunda $x=\frac{1}{26}$;

2) $7(10-x)+3(2x-1)$, bunda $x=-0,048$;

3) $\frac{1}{3}(6x-3)+\frac{2}{5}(5x-15)$, bunda $x=3,01$;

4) $0,01(2,2x-0,1)+0,1(x-100)$, bunda $x=-10$.

54. Arifmetik amallarning xossalaridan foydalanib hisoblang:

1) $\frac{1}{7}(0,14+2,1-3,5)$;

2) $\frac{1}{12}(4,8-0,24-1,2)$;

3) $(18\frac{6}{7}+21\frac{3}{4}):3$;

4) $(15\frac{5}{7}+20\frac{15}{16})\cdot\frac{1}{5}$.

5- §. Qavslarni ochish qoidalari

1. Algebraik yig‘indi.

1 - m a s a l a . Yigirma qavatli binoda lift ishlamoqda. U 8-qavatdan 6 qavat pastga tushdi, so‘ngra 12 qavat yuqoriga ko‘tarildi. 4 qavat pastga tushdi. 7 qavat yuqoriga ko‘tarildi. 13 qavat pastga tushdi. Lift qaysi qavatda turibdi?

△ Liftning qaysi qavatda turganligini topish uchun $8-6+12-4+7-13$ ifodaning qiymatini hisoblash kerak. Bu qiymat 4 ga teng. Demak, lift 4- qavatda turibdi. ▲

Siz VI sinf matematika kursidan

$$8-6+12-4+7-13$$

ifoda algebraik yig‘indi deb atalishini bilasiz, chunki uni yig‘indi shaklida bunday yozish mumkin:

$$8+(-6)+12+(-4)+7+(-13).$$

Algebraik yig‘indilarga oid yana misollar keltiramiz:

$$3-(-7)+(-2),\quad a-b+c-d,\quad a+(-b)-(-c).$$

$(-c)$ sonni ayirish $(-c)$ songa qarama-qarshi sonni, ya’ni c sonni qo‘shishni bildirishini eslatib o‘tamiz. Shuning uchun oxirgi algebraik yig‘indini bunday yozish mumkin:

$$a+(-b)+c.$$

Algebraik yig‘indi — bu „+“ va „—“ ishoralari bilan birlashtirilgan bir nechta algebraik ifodalardan tuzilgan yozuvdir.

Odatda, $3-(-7)+(-2)$, $a+(-b)-(-c)$ ko‘rinishidagi algebraik yig‘indilarni qisqacha bunday yoziladi:

$$3-(-7)+(-2)=3+7-2;\quad a+(-b)-(-c)=a-b+c.$$

$3+7-2$ algebraik yig‘indida qo‘shiluvchilar 3, 7 va –2 sonlari bo‘ladi, chunki $3+7-2=3+7+(-2)$; $a-b+c$ algebraik yig‘indida qo‘shiluvchilar a, $-b$, c sonlar bo‘ladi, chunki $a-b+c=a+(-b)+c$.

2. Qavslarni ochish va qavs ichiga olish.

$a+(b+c)$ ifodani qaraymiz: qo‘shishning guruhlash qonunini qo‘llab, uni bunday yozish mumkin:

$$a+(b+c)=a+b+c.$$

Bu tenglikda c ni — d bilan almashtiramiz:

$$a+(b-d)=a+b-d.$$

Qavs oldida „+“ ishorasi turgan ifodalarda almashtirishlar bajarish shu tengliklarga asoslangan. Bu tengliklar qavslarni ochishning quyidagi birinchi qoidasiga olib keladi:

Agar algebraik ifodaga qavs ichiga olingan algebraik yig‘indi qo‘shiladigan bo‘lsa, u holda shu algebraik yig‘indidagi har bir qo‘shiluvchining ishorasini saqlagan holda qavslarni tushirib qoldirish mumkin.

Masalan:

1) $14+(7-13+2)=14+7-13+2$;
2) $a+(b+c-d)=a+b+c+d$;
3) $(a-b)+c=a-b+c$.

Qavs oldida „—“ ishorasi turgan ifodalarda almashtirishlar bajarish ayirish amalining quyidagi xossalariga asoslangan:

$$-(-a)=a,\quad -(a+b)=-a-b,$$
$$a-(b+c)=a-b-c,$$
$$a-(b-c)=a-b+c.$$

Bu tengliklardan *qavslarni ochishning* quyidagi *ikkinchi qoidasi* kelib chiqadi:

Agar algebraik ifodadan qavs ichiga olingan algebraik yig‘indi ayirilsa, u holda shu algebraik yig‘indidagi har bir qo‘shiluvchining ishorasini qarama-qarshisiga o‘zgartirib, qavslarni tushirib qoldirish mumkin.

Masalan:

1) $14-(7-13+2)=14-7+13-2$;
2) $a-(b+c-d)=a-b-c+d$;
3) $-(a-b)+c=-a+b+c$.

2 - m a s a l a . Qavslarni ochib soddalashtiring:

$$3x+(5-(8x+3)).$$

△ $3x+(5-(8x+3))=3x+5-(8x+3)=3x+5-8x-3=2-5x.$ ▲

Ba’zan bir necha qo‘shiluvchini qavs ichiga olish foydali bo‘ladi. Masalan:

1) $108-137+37=108-(137-37)=108-100=8$;

2) $a+b-c+d=a+(b-c+d).$

Bu yerda qavs oldiga „+“ belgisi qo‘yilgan, shuning uchun qavs ichidagi barcha qo‘shiluvchilarning ishoralari saqlanib qoladi.

3) $a-b-c+d=a-(b+c-d).$

Bu yerda qavs oldiga „–“ belgisi qo‘yilgan, shuning uchun qavs ichiga olingan barcha qo‘shiluvchilarning ishoralari qarama-qarshisiga o‘zgartirildi.

55. Algebraik yig‘indini qavslarsiz yozing:

1) $(+4)+(-3)-(+7)$;
2) $(-4)+(-9)-(-11)$;
3) $(-a)+(-7b)+\frac{1}{3}c$;
4) $2a+(-3b)-4c$.

56. Algebraik yig‘indining qo‘shiluvchilarini ayting:

1) $15-c$; 2) $m-7$; 3) $-a+47$; 4) $-13-b$.

57. Algebraik yig‘indi shaklida yozing:

1) $a-b+c$; 2) $2+b-c$; 3) $a-2-b$; 4) $3+a-b-c$.

Qavslarni oching **(58—59)**:

58. 1) $a+(2b-3c)$;
2) $a-(2b-3c)$;
3) $a-(2b+3c)$;
4) $-(a-2b+3c)$.

59. 1) $a+(b-(c-d))$;
2) $a-(b-(c-d))$;
3) $a-((b-c)-d)$;
4) $a-(b+(c-(d-k)))$.

60. Qavslarni oching va soddalashtiring:

1) $3a-(a+2b)$;
2) $5x-(2y-3x)$;
3) $3m-(5m-(2m-1))$;
4) $4a+(2a-(3a+3))$.

61. m yoki $(-m)$ sonlaridan boshlab, barcha qo‘shiluvchilarni qavs oldiga „+“ ishorasini qo‘ygan holda qavs ichiga oling:

1) $a+2b+m-c$;
2) $a-2b+m+c$;
3) $a-m+3c+4d$;
4) $a-m+3b^2-2a^3$.

62. m yoki $(-m)$ sonlaridan boshlab, barcha qo‘shiluvchilarni qavs oldiga „–“ ishorasini qo‘ygan holda qavs ichiga oling:

1) $2a+3b+m-c$;
2) $2a+b+m+3c$;
3) $c-m-2a+3b^2$;
4) $a-m+3b^2-2a^3$.

63. 1) $a+b-1$ ifodani biri a ga teng bo‘lgan ikkita qo‘shiluvchining yig‘indisi shaklida yozing;
2) $a-b+1$ ifodani kamayuvchisi a bo‘lgan ayirma shaklida yozing;
3) $2a-b+4$ ifodani kamayuvchisi $2a$ bo‘lgan ayirma shaklida yozing;

4) $a-2b+8$ ifodani biri 8 ga teng bo‘lgan ikkita qo‘shiluvchining yig‘indisi shaklida yozing.

64. $2x^2+5x^2y-4xy^2-y^3$ ifodada:

1) oxirgi uchta qo‘shiluvchi oldiga „–“ ishorasini qo‘yib, qavslar ichiga oling;
2) oxirgi ikkita qo‘shiluvchi oldiga „+“ ishorasini qo‘yib, qavslar ichiga oling;
3) ikkinchi va uchinchi qo‘shiluvchilar oldiga „–“ ishorasini qo‘yib, qavslar ichiga oling;
4) birinchi va ikkinchi qo‘shiluvchilar oldiga „–“ ishorasini qo‘yib, qavslar ichiga oling.

65. Ko‘p nuqtalar o‘rniga „+“ va „–“ ishoralarini shunday qo‘yingki, natijada to‘g‘ri tenglik hosil bo‘lsin:

1) $a-(b+c)=a+(...b\ ...c)$;
2) $c-(a-b)=c+(...a\ ...b)$;
3) $m-(n-a)=m+(...n\ ...a)$;
4) $n-(d-l)=n+(...d\ ...l)$.

O‘zingizni tekshirib ko‘ring!

1. Hisoblang:

1) $(17,2\cdot 4,01+4,01\cdot 32,8):1\frac{2}{3}$;

2) $\frac{1}{2}-\left(\frac{1}{2}\right)^2\cdot 2\left(\frac{2}{3}\right)-25\cdot 0,03\cdot 4$.

2. Ifodani soddalashtiring va $x=-\frac{2}{9}$, $y=0,25$ bo‘lganda uning son qiymatini toping:

$$3(2y-x)-2(y-3x).$$

3. Bolalar oromgohi uchun 10 ta shaxmat va 15 ta koptok sotib olishdi. Bitta shaxmat a so‘m, bitta koptok b so‘m turadi. Jami xarid uchun qancha pul to‘langan?

66. Soddalashtiring:

1) $(5a-2b)-(3b-5a)$;
2) $(6a-b)-(2a+3b)$;
3) $7x+3y-(-3x+3y)$;
4) $8x-(3x-2y)-5y$.

67. Tenglamani yeching:

1) $(2x+1)+3x=16$;
2) $(x-4)+(x+6)=4$;
3) $(x-5)-(5-3x)=2$;
4) $23-(x+5)=13$.

68. Ifodani avval soddalashtirib, keyin uning son qiymatini toping:

1) $(2c+5d)-(c+4d)$, bunda $c=0,4$, $d=0,6$;
2) $(3a-4b)-(2a-3b)$, bunda $a=0,12$, $b=1,28$;
3) $(7x+8y)-(5x-2y)$, bunda $x=-\frac{3}{4}$, $y=0,025$;
4) $(5c-6b)-(3c-5b)$, bunda $c=-0,25$, $b=2\frac{1}{2}$.

I bobga doir mashqlar

Algebraik ifodaning son qiymatini hisoblang **(69—75)**:

69. 1) $a+bc$, bunda $a=-1$, $b=3$, $c=0$;
2) $a-bc$, bunda $a=2$, $b=-1$, $c=-3$;
3) $(a+b)c$, bunda $a=1$, $b=-3$, $c=2$;
4) $(a-b)c$, bunda $a=3$, $b=1,2$, $c=5$;
5) $(a-b)+(c-d)$, bunda $a=4$, $b=2$, $c=3$, $d=-1$;
6) $(a-b)-(c-d)$, bunda $a=0$, $b=-4$, $c=-2$, $d=3$;
7) $a-(b-c)$, bunda $a=0,5$, $b=\frac{1}{2}$, $c=-1,2$;
8) $a-(b-c)-d$, bunda $a=5,2$, $b=1,3$, $c=2,8$, $d=2,8$.

70. 1) $5(x-y)^2$; 2) $3(x+y)^2$; 3) $(5x-y)^2$; 4) $(3x+y)^2$, bunda $x=2,5$, $y=4,5$.

71. 1) $2((a-b)^2+1)$;
2) $4(3-(a-b)^2)$;
3) $((a-b)a-8):2$;
4) $(5a-(a+b)):3$, bunda $a=5$, $b=-1$.

72. 1) $3(a+b)-2ab$;
2) $3a+b-2ab$;
3) $3(a-b)+2ab$;
4) $3a-b+2ab$, bunda $a=1,2$, $b=1,8$.

73. 1) $\frac{1}{2}b^3-3c^2$, bunda $b=-2$, $c=-\frac{1}{3}$;

2) $-0,75a^2+1\frac{2}{3}b^2$, bunda $a=-2$, $b=3$;

3) $(a^2-26)^2$, bunda $a=-5$; 4) $(a^3+26)^3$, bunda $a=-3$.

74. 1) $7x^2-2ax$, bunda $x=-\frac{3}{7}$, $a=1,5$;

2) $3ax-5x^2$, bunda $x=\frac{2}{5}$, $a=-\frac{1}{3}$;

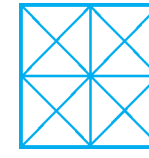
3) $2m^3(3m^2-k)^2$, bunda $m=-\frac{1}{2}$, $k=0,75$;

4) $3m^2(2m^2-n)^2$, bunda $n=\frac{1}{4}$, $m=-0,5$.

75. Hisoblang:

1) $\frac{(t-2)}{3t^2+4t-1}$, bunda $t=-2$; 2) $\frac{2a^2-4a-1}{(a+1)^2}$, bunda $a=-3$.

№ 2 | *Shu rasmda nechta uchburchak, kvadrat va to‘g‘ri to‘rtburchak bor?*



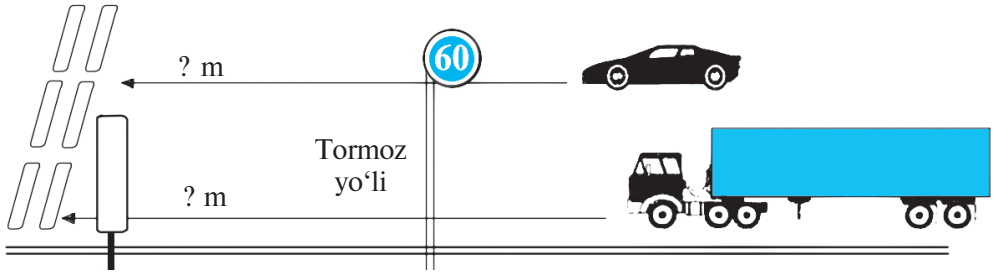
76. Bir gektar ko‘kalamzor bir yil davomida havoni 70 t changdan tozalashga qodir. 10 ga; 100 ga; m gektar ko‘kalamzor bir yilda havoni necha tonna changdan tozalaydi? Umumiy maydoni 16 000 ga bo‘lgan ko‘kalamzor havoni necha tonna changdan tozalaydi?

77. Avtomobilning harakat tezligi ikki marta ortishi bilan uning tormozlanish yo‘li to‘rt marta ortishi ma’lum. Jadvallardan foydalanib, harakat tezligi 30 km/soatdan 60 km/soatga ortganda tormozlanish yo‘lining uzunligini toping:

Yengil mashina uchun:	
v (km/soat)	t (m)
30	7,25

Yuk mashinasi uchun:	
v (km/soat)	t (m)
30	9,5

78. *(Abu Rayhon Beruniy masalasi.)* Agar 10 dirham pul ikki oyda 5 dirham foyda keltirgan bo‘lsa, 8 dirham puldan uch oyda qancha foyda olish mumkin?



I bobga doir sinov mashqlari — testlar

Quyida sinov (test) mashqlari keltirilgan bo‘lib, ularning har biriga 5 tadan „javob“ berilgan. 5 ta „javob“ning faqat bittasi to‘g‘ri, qolganlari esa noto‘g‘ri. Sizdan sinov mashqlarini bajarish yoki boshqa mulohazalar yordamida ana shu to‘g‘ri javobni topish (uni belgilash) talab qilinadi.

1. $a=2,4$, $b=3,6$, $h=1,6$ bo‘lsa, $s=\frac{h}{2}(a+b)$ ifodaning son qiymatini toping.

A) 48; B) 3,18; C) 6,36; D) 0,48, E) 4,8.

2. $a=12,5$, $h=6,4$ bo‘lsa, $s=\frac{1}{2}ah$ ifodaning son qiymatini toping.

A) 40; B) 400; C) 4; D) 36,1; E) 37,1.

3. $a=5,1$, $b=4,7$ bo‘lsa, $P=2(a+b)$ ifodaning son qiymatini toping.

A) 196; B) 19,6; C) 1,96; D) 18,16; E) 18,14.

4. To‘g‘ri to‘rtburchakning yuzi S ga, asosi a ga teng. Uning perimetrini topish uchun ifoda tuzing.

A) $\frac{S}{2a}+a$; B) $\frac{S}{a}+2a$; C) $2\left(\frac{S}{a}+a\right)$; D) $\frac{S}{a}+a$; E) $\frac{2S}{a}$.

5. Teng yonli uchburchakning perimetri P ga, asosining uzunligi a ga teng. Uchburchakning yon tomoni uzunligini topish uchun ifoda tuzing.

A) $2a-P$; B) $2P-a$; C) $P-a$; D) $\frac{1}{2}(P-a)$; E) $P-2a$.

6. $a=2,5$, $b=2,4$ va $c=3,5$ bo‘lsa, $V=abc$ ifodaning son qiymatini toping.

A) 18,3; B) 21; C) 2,1; D) 12,1; E) 121.

7. $a=5, b=6,4, c=4,5$ bo‘lsa, $S=2(ab+ac+bc)$ ifodaning son qiymatini toping:

A) 50,45; B) 83,3; C) 166,6; D) 109; E) 54,5.

8. Ona farzandlari uchun a so‘mdan 8 ta rasm daftar, b so‘mdan 5 ta ruchka, c so‘mdan 20 ta daftar sotib oldi. Jami xaridni hisoblash uchun ifoda tuzing.

A) $33(a+b+c)$; B) $8a+25(b+c)$; C) $800abc$;
D) $8a+100ba$; E) $8a+5b+20c$.

9. Qavslarni oching va soddalashtiring: $5a+(3a-(4a+3))$.

A) $8a+3$; B) $4a-3$; C) $-4a-3$; D) $3-4a$; E) $4a+3$.

10. Ifodani soddalashtiring va uning $a=2,4$; $b=1,5$ bo‘lgandagi qiymatini toping: $0,5\cdot(2a-3b)-(4b+2,5a)$.

A) 17,4; B) –17,4; C) –1,4; D) –11,85; E) 0,6.

Ifodaning son qiymatini toping **(11—14):**

11. $(64,2\cdot 7,02+17,9\cdot 14,04):4\frac{2}{13}$.

A) 169; B) 16,9; C) 159; D) 15,9; E) 149.

12. $3\frac{13}{29}\cdot 0,87-2,34\cdot 1,8+3\frac{1}{3}\cdot 1\frac{1}{2}$.

A) 67; B) 3,788; C) –6,2; D) –3,788; E) 9,3.

13. $(35,7\cdot 12,24-21,4\cdot 6,12):1\frac{8}{9}$.

A) 578; B) 306; C) 162; D) 16,2; E) –16,2.

14. $(-1,5)\cdot(2,7:(-0,9)-(-7,2):3,6)+2,4\cdot(-2,5)$.

A) –6; B) –7,5; C) 7,5; D) –4,5; E) 4,5.

15. To‘g‘ri to‘rtburchakning perimetri p ga, asosi a ga teng. Uning balandligini hisoblash uchun ifoda tuzing.

A) $0{,}5 \cdot (p - a)$; B) $p - 2a$; C) $\frac{2a - p}{2}$;

D) $p - 2a$; E) $\frac{p - 2a}{2}$.

16. Uchburchak bir tomonining uzunligi a ga teng bo‘lib, u ikkinchi tomonidan 2 sm qisqa, uchinchi tomonidan esa 3 sm uzun. Shu uchburchakning perimetrini hisoblash uchun ifoda tuzing.

A) $3a - 1$; B) $3a - 5$; C) $3a + 5$;

D) $1 - 3a$; E) $3a + 2$.

17. Ifodani soddalashtiring va uning $a = 2{,}7$, $b = 42$ bo‘lgandagi son qiymatini toping: $3(2a - b) - 2(a - 2b)$.

A) 24,36; B) 27,6; C) 8,7; D) 15; E) 14.

18. Uchburchak bir tomonining uzunligi a ga teng. Ikkinchi tomoni uzunligi bu tomonning 80 % ini tashkil qiladi. Uchinchi tomoni esa birinch va ikkinchi tomonlar yig‘indisining yarmiga teng bo‘lsa, shu uchburchakning perimetrini toping.

A) $1{,}8a$; B) $2{,}7a$; C) $3a$;

D) $3a + 0{,}8$; E) $2a + 0{,}8$.

Tarixiy masalalar

① *Al-Xorazmiy masalasi.* Sondan uning uchdan biri va to‘rtdan biri ayirilsa, 8 qoladi. Sonning o‘zini toping.

② *Al-Xorazmiy masalasi.* Sen o‘nni ikki qismga ajratding, keyin ulardan birini boshqasiga bo‘lding, bo‘linmada to‘rt chiqdi. Sen o‘nni qanday qismlarga ajratding?

③ *Geron masalasi* (eramizning I asri). Hovuzga ikkita quvurdan suv keladi. Birinchi quvurdan 1 soatda 1 m^3, ikkinchi quvurdan 1 soatda 4m^3 suv tushadi. Hovuzning hajmi 12 m^3. Ikkala quvur baravar ochib qo‘yilsa, bo‘sh hovuz qancha vaqtda to‘ladi?

④ *Nyuton masalasi.* Oralaridagi masofa 59 mil bo‘lgan ikki qishloqdan *A* va *B* kishilar bir-biriga qarab yo‘lga chiqdi. *B* kishi *A* ga qaraganda 1 soat kech yo‘lga chiqdi. *A* kishi 2 soatda 7 mil, *B* esa 3 soatda 8 mil bosadi. *A* kishi *B* bilan uchrashguncha necha mil yo‘l yuradi? (Mil uzunlik o‘lchov birliklaridan bo‘lib, 1 mil ≈ 1,852 km.)

Tarixiy ma’lumotlar

Yurtdoshimiz buyuk matematik va astronom olim Abu Abdulloh Muhammad ibn Muso al-Xorazmiy (783—850) ning arifmetik („Algorizmi hind hisobi haqida“) va algebraik („Al-jabr val-muqobala“) asarlari matematikaning rivojiga nihoyatda kuchli ta’sir ko‘rsatdi. Bu asarlar ko‘p tillarga tarjima qilindi, asrlar davomida matematikadan asosiy qo‘llanma bo‘lib xizmat qildi.

„Algorizmi hind hisobi haqida“ risolasining XII asr boshidagi lotincha tarjimasi Angliyaning Kembrij universitetida saqlanadi. Al-Xorazmiyning bu asari tufayli Yevropaga o‘nli sanoq sistemasi kirib borgan.

Xorazmiy algebrasi — „Al-jabr val-muqobala hisobi haqida qisqacha kitob“ asarining arabcha nusxasi Oksford universitetining Bodleyan kutubxonasida saqlanadi. Risola uch qismdan iborat:

1) algebraik qism; 2) geometrik qism; 3) vasiyatlar haqidagi qism (Xorazmiy uni „Vasiyatlar kitobi“ deb atagan). Al-Xorazmiy risolasida barcha masalalarning bayoni va yechimlari so‘zlar bilan beriladi, hech qanday belgilashlar, harfiy ifodalar ishlatilmaydi. Al-Xorazmiy yozadi: „... Men arifmetikaning oddiy va murakkab masalalarini o‘z ichiga oluvchi „Al-jabr val-muqobala hisobi haqida qisqacha kitob“ni ta’lif qildim, chunki meros taqsim qilishda, vasiyatnoma tuzishda, mol taqsimlashda va adliya ishlarida, savdoda va har qanday bitimlarda va, shuningdek, yer o‘lchashda, ariqlar o‘tkazishda, muhandislikda va boshqa shunga o‘xshash turlicha ishlarda kishilar uchun bu zarurdir“. Binobarin, olim o‘zining bu asarini kundalik hayot talabi va ehtiyojlarini hisobga olgan holda yozgan.

II BOB
BIR NOMA'LUMLI BIRINCHI DARAJALI TENGLAMALAR

6-§ Tenglama va uning yechimlari

Ushbu masalani yechaylik.

Masala. Qalam va chizg'ich birgalikda 370 so'm turadi. Qalam chizg'ichdan 90 so'm arzon. Chizg'ichning bahosini toping.

△ Aytaylik, chizg'ich x so'm tursin, u holda qalam $(x-90)$ so'm turadi. Masalaning shartiga ko'ra

$$x+(x-90)=370,$$

bundan $2x-90=370$, $2x=460$, $x=230$.

Javob. Chizg'ich 230 so'm turadi. ▲

$x+(x-90)=70$, tenglikda x harfi noma'lum sonni yoki qisqacha noma'lumni bildiradi.

Harf bilan belgilangan noma'lum son qatnashgan tenglik tenglama deyiladi.

Tenglik belgisidan chap va o'ngda turgan ifodalar tenglamaning chap va o'ng qismlari deyiladi. Tenglamaning chap yoki o'ng qismidagi har bir qo'shiluvchi tenglamaning hadi deyiladi.

$2x-90=370$ tenglamada chap qism $2x-90$, o'ng qism esa 370. So'ngra $x=230$ bo'lganda shu tenglamaning chap qismi 370 ga teng, chunki $2\cdot 230-90=370$; o'ng qismi ham 370 ga teng. Demak, $x=230$ bo'lganda bu tenglama to'g'ri tenglikka aylanadi: $2\cdot 230-90=370$. Shu 230 soni berilgan *tenglamaning ildizi* deyiladi.

Tenglamaning ildizi deb, noma'lumning shu tenglamani to'g'ri tenglikka aylantiradigan qiymatiga aytiladi.

Masalan, 1 soni

$$2x+3=5$$

tenglamaning ildizi, chunki $2\cdot 1+3=5$ — to'g'ri tenglik.

Tenglama ikkita, uchta va hokazo ildizlarga ega bo‘lishi mumkin. Masalan,

$$(x-1)(x-2)=0$$

tenglama ikkita ildizga ega: 1 va 2, chunki $x=1$ va $x=2$ da tenglama to‘g‘ri tenglikka aylanadi.

$$(x-3)(x+4)(x-5)=0$$

tenglama esa uchta ildizga ega: 3, – 4 va 5.

Tenglama ildizlarining soni cheksiz ko‘p bo‘lishi mumkin. Masalan,

$$2(x-1)=2x-2$$

tenglamaning ildizlari soni cheksiz ko‘p: x ning istalgan qiymati tenglamaning ildizi bo‘ladi, chunki har bir x da tenglamaning chap qismi o‘ng qismiga teng.

Tenglama ildizlarga ega bo‘lmasligi ham mumkin. Masalan, $2x+5=2x+3$ tenglamaning ildizlari yo‘q, chunki x ning istalgan qiymatida bu tenglamaning chap qismi o‘ng qismidan katta bo‘ladi.

Tenglamani yechish — uning barcha ildizlarini topish yoki ularning yo‘qligini ko‘rsatish demakdir.

Sodda hollarda x ning tenglamaning ildizi bo‘ladigan qiymatini tanlash oson bo‘ladi. Masalan, $2x+1=3$ tenglamaning ildizi 1 soni ekanligini osongina ko‘rish mumkin. Biroq murakkab holda ildizni birdaniga topish oson bo‘lmaydi. Masalan,

$$\frac{x-6}{5}+\frac{4(x+3)}{2}-1=\frac{x-1}{2}+3x-\frac{7x-1}{10}$$

tenglama $x=7$ bo‘lganda to‘g‘ri tenglikka aylanishini bilish ancha qiyin. Shuning uchun tenglamalarni yechishni o‘rganish muhim.

Ko‘pgina amaliy masalalarni yechish

$$\boxed{ax=b} \qquad (1)$$

ko‘rinishdagi tenglamaga keltiriladi, bunda a va b — berilgan sonlar, x — noma’lum son. (1) tenglama *chiziqli tenglama* deb ataladi. Masalan, $3x=1$, $-2x=3$, $\frac{3}{5}x=-\frac{1}{2}$ — chiziqli tenglamalardir.

79. Tenglik shaklida yozing:

1) 34 soni x sondan 18 ta ortiq;
2) 56 soni 14 sonidan x marta ortiq;
3) x va 3 sonlari ayirmasining ikkilangani 4 ga teng;
4) x va 5 sonlari yig‘indisining yarmi ularning ko‘paytmasiga teng.

80. 3; –2; 1 sonlaridan qaysi biri tenglamaning ildizi bo‘ladi:

1) $3x=-6$;
2) $x+3=6$;
3) $4x-4=x+5$;
4) $5x-8=2x+4$?

81. (Og‘zaki.) x ning qanday qiymatlarida tenglama to‘g‘ri tenglikka aylanadi:

1) $x+5=-6$; 2) $4-x=-1$; 3) $2x-1=0$; 4) $3x+2=0$?

82. -1; $\frac{1}{2}$; 1 sonlari orasida tenglamaning ildizi bormi:

1) $4(x-1)=2x-3$;
2) $7(x+1)-6x=10$;
3) $3(x+2)=4+2x$;
4) $5(x+1)-4x=4$.

83. Ildizi:

1) 5 soni; 2) 3 soni; 3) –6 soni; 4) –4 soni bo‘lgan tenglama tuzing.

84. a sonni shunday tanlangki, $4x-3=2x+a$ tenglama

1) $x=1$; 2) $x=-1$; 3) $x=\frac{1}{2}$; 4) $x=0{,}3$

ildizga ega bo‘lsin.

7-§ *Bir noma’lumli birinchi darajali tenglamalarni yechish*

Al-Xorazmiyning „Kitob al-muxtasar fi hisob al-jabr val-muqobala“ asaridagi al-jabr musbat hadlarni tiklash, ya’ni manfiy hadlarni tenglamaning bir qismidan ikkinchi qismiga musbat qilib o‘tkazishni, val-muqobala esa tenglamaning ikkala qismidan teng hadlarni tashlab yuborishni bildirgan.

Bu bir noma'lumli tenglamalarni yechish to'g'ri tengliklarning sizlarga ma'lum xossalariga asoslangan ekanini ko'rsatadi. Shu xossalarni eslatib o'tamiz.

Xossaning so'z bilan ifodalanishi	Xossaning umumiy ko'rinishda yozilishi	Misol
1. Agar to'g'ri tenglikning ikkala qismiga bir xil son qo'shilsa yoki ikkala qismidan bir xil son ayirilsa, u holda yana to'g'ri tenglik hosil bo'ladi.	Agar $a = b$ bo'lib, l ixtiyoriy son bo'lsa, u holda $a + l = b + l$, $a - l = b - l$ bo'ladi.	$7 = 7$ $7 + 2 = 7 + 2$ $7 - 2 = 7 - 2$
2. Agar to'g'ri tenglikning ikkala qismini nolga teng bo'lmagan ayni bir songa ko'paytirilsa yoki bo'linsa, u holda yana to'g'ri tenglik hosil bo'ladi.	Agar $a = b$ bo'lib, $m \neq 0$ bo'lsa, u holda $a \cdot m = b \cdot m$ va $a : m = b : m$ bo'ladi.	$27 = 27$ $27 \cdot 3 = 27 \cdot 3$ $27 : 3 = 27 : 3$

Birinchi xossadan qo'shiluvchilarni, ularning ishoralarini qarama-qarshisiga almashtirib, tenglikning bir qismidan ikkinchi qismiga olib o'tish mumkinligi kelib chiqadi.

○ Aytaylik, $a = b + m$ bo'lsin, u holda

$$a + (-m) = b + m + (-m);\ \ a - m = b.\ \bullet$$

Tengliklarning bu xossalari tenglamalarni yechishda qanday qo'llanishini ko'raylik.

1-masala. $9x - 23 = 5x - 11$ tenglamani yeching.

△ x son berilgan tenglamaning ildizi, ya'ni x shunday sonki, uni tenglamaga qo'yilganda tenglama to'g'ri tenglikka aylanadi, deb faraz qilamiz.

Noma'lum qatnashgan $5x$ hadni „–" ishora bilan tenglikning chap qismiga, -23 hadni „+" ishora bilan o'ng qismiga olib o'tamiz.

Natijada yana to'g'ri tenglik hosil bo'ladi:

$$9x - 5x = 23 - 11.$$

Tenglamaning ikkala qismidagi o'xshash hadlarni ixchamlab,

$$4x = 12$$

tenglamani hosil qilamiz. Bu tenglamaning ikkala qismini 4 ga bo'lib, $x = 3$ ekanini topamiz.

Shunday qilib, tenglama ildizga ega deb faraz qilib, bu ildiz faqat 3 soniga teng bo‘lishi mumkinligini ko‘rdik. $x=3$ haqiqatan ham berilgan tenglamaning ildizi bo‘lishini tekshiramiz: $9\cdot 3-$ $-23=5\cdot 3-11$. Bu to‘g‘ri tenglik, chunki uning chap va o‘ng qismlari birgina 4 soniga teng.

Demak, berilgan tenglama faqat bitta ildizga ega: $x=3$. ▲

Tekshirishni bajarmaslik ham mumkinligini ta’kidlaymiz, chunki tenglikning foydalanilgan xossalari bir to‘g‘ri tenglikni ikkinchi to‘g‘ri tenglik bilan almashtirishga imkon beradi. Yechishning bu usulida har doim to‘g‘ri natija hosil qilinadi (agar hisoblashlarda xatolikka yo‘l qo‘yilmasa, albatta).

$5x\underset{-3x}{-7}=\underset{-3x}{3x}+11$ $\underset{+7}{}$ $5x-3x=11+7$	**AL-JABR:** $3x$, chapga $-3x$ bo‘lib o‘tasan! -7, sen o‘ngga $+7$ bo‘lib o‘tasan!
$\cancel{4x}-\cancel{5}+2x=\cancel{4x}+8-\cancel{5}$ $2x=8$	**VAL-MUQOBALA:** chap va o‘ng qismdagi -5 lar-u, $4x$ lar, sizlar bilan xayrlashamiz!

Tenglama yechilishini yozishda 1- masalani yechishdagidek batafsil yozma tushuntirishlarni bajarish shart emas.

Masalan, $5x+17=2x+5$ tenglamaning yechilishini bunday yozish mumkin:

$$5x-2x=5-17,\ \ 3x=-12,\ \ x=-4.$$

Javob: $x=-4$.

2-masala. $2(x+3)-3(x+2)=5-4(x+1)$ tenglamani yeching.

△ Tenglamaning chap va o‘ng qismlarini soddalashtiramiz: qavslarni ochamiz va o‘xshash hadlarni ixchamlaymiz. Natijada $2x+6-3x-$ $-6=5-4x-4$, $-x=-4x+1$ tenglamani hosil qilamiz.

Demak, $3x=1$, bundan $x=\frac{1}{3}$. ▲

3-masala. $\frac{5x}{2}-\frac{x-3}{3}=1+\frac{x-5}{6}$ tenglamani yeching.

△ Tenglamaning ikkala qismini kasrlarning umumiy maxrajiga, ya'ni 6 ga ko'paytiramiz, u holda

$$\frac{5x}{2}\cdot 6-\frac{x-3}{3}\cdot 6=1\cdot 6+\frac{x-5}{6}\cdot 6; \qquad 15x-2(x-3)=6+(x-5).$$

Qavslarni ochamiz va o'xshash hadlarni ixchamlaymiz:

$$15x-2x+6=6+x-5; \qquad 13x+6=x+1,$$

bundan $12x=-5$, $x=-\frac{5}{12}$. ▲

Shunday qilib, tenglamani yechishda *tenglamaning* quyidagi *asosiy xossalaridan* foydalaniladi.

1-xossa. *Tenglamaning istalgan hadi ishorasini qarama-qarshisiga o'zgartirib, uning bir qismidan ikkinchi qismiga o'tkazish mumkin.*

2-xossa. *Tenglamaning ikkala qismini nolga teng bo'lmagan bir xil songa ko'paytirish yoki bo'lish mumkin.*

Bu xossalar bir noma'lumli istalgan tenglamani yechish imkonini beradi. Buning uchun:

1) noma'lum qatnashgan hadlarni tenglikning chap qismiga, noma'lum qatnashmagan hadlarni esa o'ng qismiga o'tkazish lozim (1-xossa);

2) o'xshash hadlarni ixchamlash kerak;

3) tenglamaning ikkala qismini noma'lum oldida turgan koeffitsiyentga (agar u nolga teng bo'lmasa) bo'lish (2-xossa) kerak.

Ko'rib chiqilgan misollarda har bir tenglama bitta ildizga ega bo'ldi. Ammo ba'zi hollarda bir noma'lumli tenglama ildizlarga ega bo'lmasligi mumkin yoki cheksiz ko'p ildizlarga ega bo'lishi mumkin. Shunday tenglamalarga misollar keltiramiz.

4-masala. $2(x+1)-1=3-(1-2x)$ tenglama ildizlarga ega emasligini ko‘rsating.

△ Tenglamaning ikkala qismini soddalashtiramiz:

$$2x+2-1=3-1+2x,\ \ 2x+1=2+2x,$$

bundan

$$2x-2x=2-1,\ \ 0\cdot x=1.$$

Bu tenglama ildizlarga ega emas, chunki uning $0\cdot x$ dan iborat chap qismi nolga teng va, demak, 1 ga teng emas. ▲

5-masala. $3(1-x)+2=5-3x$ tenglama cheksiz ko‘p yechimlarga ega ekanligini ko‘rsating.

△ Tenglamani soddalashtiramiz: $3-3x+2=5-3x;\ \ 5-3x=5-3x.$ Oxirgi tenglik x ning istagan qiymatida to‘g‘ri bo‘ladi. Demak, x ning istalgan qiymati bu tenglamaning ildizi bo‘ladi. ▲

Mashqlar

Tenglamani yeching **(85 — 96)**:

85. 1) $11x=50$; 2) $-9x=243$; 3) $4x=0,24$; 4) $7x=7,063$.

86. 1) $9x=\frac{2}{5}$; 2) $3x=2\frac{1}{7}$; 3) $\frac{1}{2}x=3$; 4) $\frac{3}{4}x=\frac{1}{2}$.

87. 1) $0,3x=6$; 2) $1,3x=-1,69$; 3) $0,7x=49$; 4) $10x=0,5$.

88. 1) $8x=8$; 2) $\frac{1}{4}x=16$; 3) $3^2x=243$; 4) $16x=16$.

89. 1) $5x=\left(\frac{5}{7}\right)^2$; 2) $4x=-\left(\frac{4}{5}\right)^2$; 3) $-0,1x=10^3$; 4) $0,3x=10^2$.

90. 1) $25x-1=9$; 2) $7x+8=11$; 3) $3x-5=10-x$; 4) $4x+4=x+5$.

91. 1) $5x+3(3x+7)=35$; 2) $8x-(7x+8)=9$; 3) $8y-9-4y+5=12y-4-5y$; 4) $4+8y+8=2y-10-7y+9$.

92. 1) $\frac{11}{7}=\frac{2-x}{5}$; 2) $\frac{3x}{5}=\frac{6+x}{3}$; 3) $\frac{x}{3}+\frac{x}{5}=8$; 4) $\frac{y}{3}+\frac{y}{4}=14$.

93. 1) $3y+5=4\left(9-\frac{y}{2}\right)$; 3) $3\left(5+\frac{x}{2}\right)=4+2x$;

2) $8\left(11-\frac{3}{4}z\right)=16z-44$; 4) $2\left(3-\frac{x}{3}\right)=5+x$.

94. 1) $0,71x+1,98=0,37x-1,76$;

2) $0,18y-7,4=0,05y-5,71$;

3) $5(5x-1)-2,7x+0,2x=6,5-0,5x$;

4) $0,36x-0,6=0,3(0,4x-1,2)$.

95. 1) $11\frac{2}{3}x-5\frac{1}{6}=3\frac{3}{4}+2\frac{3}{4}x$; 3) $\frac{6x+7}{7}=3-\frac{5x-3}{8}$;

2) $12\frac{3}{4}+\frac{3}{7}y=\frac{y}{2}-10\frac{1}{28}$; 4) $10-\frac{3x-1}{2}=\frac{6x+3}{11}$.

96. 1) $\frac{4x-51}{3}-\frac{17-3x}{4}=\frac{x+5}{2}$; 3) $\frac{9x-5}{2}-\frac{3+5x}{3}-\frac{8x-2}{4}=2$

2) $\frac{3x-7}{4}-\frac{9x+11}{8}=\frac{3-x}{2}$; 4) $\frac{4x-3}{2}-\frac{5-2x}{3}=\frac{3x-4}{3}$

№ 3 | — Buvijon, nabirangiz necha yoshda?
— Mening yoshim nechada bo‘lsa, nabiram shuncha oylik.
— Buvijon, sizning yoshingiz nechada?
— Nabiram yoshi bilan mening yoshimni qo‘shsang, 65 chiqadi. Nabiramning yoshini endi o‘zing topa qol.

97. Tenglamaning ildizlarga ega emasligini ko‘rsating:

1) $28 - 20x = 2x + 25 - 16x - 12 - 6x$;

2) $25x - 17 = 4x - 5 - 13x + 14 + 34x$;

3) $\frac{x-1}{3}+\frac{5x+2}{12}=\frac{5+3x}{4}$; 4) $\frac{2x+1}{3}-\frac{7x+5}{15}=\frac{x-2}{5}$.

98. x ning istalgan qiymati tenglamaning ildizi bo‘la olishini ko‘rsating:

1) $10-4x+3=9x-2-6x+9-7x+6$;

2) $9x+4-5x=8+7x-9-3x+5$;

3) $6(1,2x-0,5)-1,3x=5,9x-3$;

4) $8(1,3x+0,25)-6,6x=3,8x+2$.

99. Tenglamani yeching:

1) $3(x-1)-2(x+2)=4x+8$;

2) $4(x+1,5)+3(1-x)=10$;

3) $4(3x+2)-7(x+1)=3(x-1)$;

4) $2,5(2x+3)-2(x+2,5)=3,5+2x$.

100. Tenglamani yeching:

1) $\frac{96}{7,2}=\frac{4x+300}{21}$;

2) $\frac{3x+14,7}{20,4}=\frac{7,5}{10}$;

3) $4,2:(2x-7)=10:7\frac{1}{7}$;

4) $4\frac{1}{11}:10=4,5:(3x-1)$.

8-§ *Masalalarni tenglamalar yordamida yechish*

Tenglamalarni qo‘llash ko‘pgina masalalarni yechishni osonlashtiradi. Bunda masalani yechish odatda ikki bosqichdan iborat bo‘ladi:

1) masalaning sharti bo‘yicha tenglama tuzish;

2) hosil bo‘lgan tenglamani yechish.

Ushbu masalani yechaylik.

Masala. Sayyohlar tushgan kema sohildagi bekatdan daryo oqimi bo‘yicha jo‘nab, 5 soatdan keyin qaytib kelishi kerak. Daryo oqimining tezligi 3 km/soat; kemaning turg‘un suvdagi tezligi 18 km/soat. Agar sayyohlar qaytishdan oldin qirg‘oqda 3 soat dam olgan bo‘lsalar, ular sohildagi bekatdan qancha masofaga suzib borganlar?

△ 1) izlanayotgan masofa x kilometr bo‘lsin. Kema bu masofani oqim bo‘yicha $18+3=21$ (km/soat) tezlik bilan o‘tadi va bunga $\frac{x}{21}$ soat sarf qiladi. Kema $18-3=15$ (km/soat) tezlik bilan orqasiga

qaytadi va bunga $\frac{x}{15}$ soat sarf qiladi. Sayyohlar qirg‘oqda 3 soat dam oldilar. Demak, sayohat $\left(\frac{x}{21}+\frac{x}{15}+3\right)$ soat davom etdi, bu esa masala shartiga ko‘ra 5 soatga teng. Shunday qilib, biz noma’lum x masofani aniqlash uchun quyidagi tenglamani hosil qildik:

$$\frac{x}{21}+\frac{x}{15}+3=5;$$

2) endi

$$\frac{x}{21}+\frac{x}{15}=2$$

tenglamani yechamiz. Bu tenglamaning ikkala qismini 105 ga (21 va 15 sonlarining eng kichik umumiy bo‘linuvchisiga) ko‘paytirib, $5x + 7x = 210$, $12x = 210$ tenglikni hosil qilamiz, bundan $x = 17{,}5$.

Shunday qilib, kema sohildagi bekatdan 17,5 km masofaga suzib boradi. ▲

Masalani yechishning birinchi bosqichida (ya’ni tenglama tuzishda) kema bilan daryo oqimi tezliklari oqim bo‘yicha harakatda qo‘shilishi, oqimga qarshi harakatda esa ayirilishi va yo‘lning tezlikka nisbati harakat vaqti ekanligini bilish zarur bo‘ldi.

Ikkinchi bosqichda (ya’ni hosil bo‘lgan tenglamani yechishda) tenglamalarning bundan oldingi paragrafda o‘rganilgan xossalarini qo‘llash talab etildi.

Masalaning shartidan foydalanib, yechimning to‘g‘riligini tekshirish mumkin. Bunda topilgan natijani ma’lum deb qarab, berilgan biror boshqa kattalik topiladi. Xususan, masala yechimining to‘g‘riligini bunday tekshirish mumkin.

Sayyohlar sohildagi bekatdan 17,5 km ga suzib bordilar. Demak, ular daryo oqimi bo‘yicha $17{,}5 : 21 = \frac{5}{6}$ soat suzdilar. Sayyohlar qaytish uchun $17{,}5 : 15 = 1\frac{1}{6}$ soat vaqt sarfladilar.

Ular qirg‘oqda 3 soat dam olganliklari e’tiborga olinsa, sayohatga ketgan umumiy vaqt $\frac{5}{6}+3+1\frac{1}{6}=5$ soat, ya’ni masala shartidagi kabi bo‘ladi.

101. 1) O‘quvchi bir son o‘yladi. Agar uni 4 ga ko‘paytirilsa, ko‘paytmaga esa 8 soni qo‘shilsa va hosil bo‘lgan yig‘indini 2 ga bo‘linsa, u holda 10 hosil bo‘ladi. O‘quvchi qanday sonni o‘ylagan?
2) Bir bola bir son o‘yladi va unga 5 ni qo‘shdi, so‘ngra yig‘indini 3 ga bo‘ldi, hosil bo‘lgan bo‘linmaga 5 ni qo‘shdi va o‘ylagan sonini hosil qildi. U qanday sonni o‘ylagan?

102. 1) Uchta sinfda hammasi bo‘lib 119 nafar o‘quvchi bor. Birinchi sinfda ikkinchisidagidan 4 ta o‘quvchi ko‘p, uchinchisidan esa 3 ta kam. Har bir sinfda nechtadan o‘quvchi bor?
2) Poyezd tarkibida sisternalar, platformalar va yuk vagonlari bor. Sisternalar platformalardan 4 ta kam, yuk vagonlaridan esa 8 ta kam. Agar sisterna, platforma va yuk vagonlarining jami soni 60 ta bo‘lsa, poyezd tarkibida ularning har biridan nechtadan bor?

103. 1) Uchta firmada 624 nafar ishchi bor. Ikkinchi firmada birinchisidagiga qaraganda ishchilar 5 marta ko‘p, uchinchi firmada esa birinchi va ikkinchi firmalarda birgalikda nechta ishchi bo‘lsa, shuncha ishchi bor. Har bir firmada nechtadan ishchi bor?
2) Uchta kichik korxonada 869 ta mahsulot tayyorlandi. Ikkinchi kichik korxonada birinchi kichik korxonaga qaraganda 3 marta ko‘p, uchinchi kichik korxonada esa ikkinchisidagidan 2 marta kam mahsulot tayyorlandi. Har bir kichik korxonada nechtadan mahsulot tayyorlangan?

104. 1) Teng yonli uchburchakning perimetri 25 sm ga teng. Agar uning yon tomoni asosidan 5 sm ortiq bo‘lsa, uchburchak tomonlari uzunliklarini toping.

2) Teng yonli uchburchakda asos yon tomonning $\frac{3}{4}$ qismini tashkil etadi. Agar uchburchakning perimetri 22 sm ga teng bo‘lsa, uning tomonlari uzunliklarini toping.

105. 1) Eni 200 m bo‘lgan to‘g‘ri to‘rtburchak maydonning chegarasi bo‘ylab ariq qazildi. Ariqning uzunligi 1 km. Maydonning bo‘yini toping.
2) Bo‘yi enidan 2 marta uzun bo‘lgan to‘g‘ri to‘rtburchak may-

donni uzunligi 120 m bo‘lgan panjara bilan o‘rashdi. Maydonning bo‘yi va enini toping.

106. Yig‘indisi 81 ga teng bo‘lgan uchta ketma-ket toq sonni toping.

107. To‘rtta ketma-ket juft son berilgan. Agar chetki sonlar yig‘indisining ikkilanganidan o‘rtadagi sonlar musbat ayirmasining uchlangani ayirilsa, 22 hosil bo‘ladi. Shu sonlarni toping.

108. 1) Fermer xo‘jaligi har kuni belgilangan rejani 5 sr ga ortiq bajarib, haftalik (6 ish kuni) topshiriqni 4 kunda bajardi. Xo‘jalik bir kunda necha sentner paxta topshirgan?

2) Fabrikaga avtomat o‘rnatildi. U bir soatda ishchiga qaraganda 8 ta ortiq mahsulot ishlab chiqaradi. 2 soatdan keyin avtomat ishchining 6 soatlik rejasini bajardi. Avtomat bir soatda nechta mahsulot ishlab chiqaradi?

109. 1) Onasi 50 yoshda, qizi esa 28 yoshda. Necha yil oldin qizi onasidan 2 marta yosh bo‘lgan?

2) Otasi 40 yoshda, o‘g‘li esa 16 yoshda. Necha yildan keyin otasi o‘g‘lidan 2 marta katta bo‘ladi?

110. 1) Birinchi qopda 50 kg, ikkinchisida esa 80 kg shakar bor edi. Ikkinchi qopdan birinchi qopdan olinganidan 3 marta ko‘p shakar olishdi va natijada birinchi qopda ikkinchidagiga qaraganda ikki marta ko‘p shakar qoldi. Har bir qopdan necha kilogrammdan shakar olishgan?

2) Bir omborda ikkinchisiga qaraganda 2 marta ko‘p don bor edi. Birinchi ombordan 750 t donni olib ketishdi, ikkinchisiga esa 350 t don olib kelishdi, natijada ikkala ombordagi don miqdori bir xil bo‘lib qoldi. Dastlab har bir omborda qanchadan don bo‘lgan?

111. 1) Uzumni har bir yashikka 9,2 kg dan solish mo‘ljallangan edi. Bu yashiklar o‘rniga har biriga 13,2 kg uzum sig‘adigan boshqa yashiklar olishdi va shunda mo‘ljaldagidan 50 ta kam yashik talab qilindi. Yashiklarga hammasi bo‘lib necha kilogramm uzum joylangan?

2) A va B bekatlar orasidagi masofani yo‘lovchi poyezdi yuk poyezdiga nisbatan 45 minut tez bosib o‘tadi. Agar yo‘lovchi poyezdining tezligi 48 km/soat, yuk poyezdiniki esa

36 km/soat ekanligi ma'lum bo'lsa, shu bekatlar orasidagi masofani toping.

112. 1) Neft omborida 6340 t benzin bor edi. Ikkinchi kuni ombor birinchi kundagidan 423 t ko'p, uchinchi kuni esa ikkinchi kundagidan 204 t kam benzin tarqatdi. Shundan so'ng omborda 3196 t benzin qoldi. Ombor birinchi kuni necha tonna benzin tarqatgan?

2) Do'konda uch kunda 110 kg yog' sotildi. Ikkinchi kuni birinchi kundagining $\frac{3}{8}$ qismicha, uchinchi kuni esa dastlabki ikki kunda qancha yog' sotilgan bo'lsa, shuncha sotildi. Do'konda birinchi kuni necha kilogramm yog' sotilgan?

113. 1) Usta buyurtmani 10 kunda bajarishi kerak edi. U har kuni rejadan tashqari 27 ta mahsulot tayyorlab, 7 kunda topshiriqni bajaribgina qolmasdan, balki ortiqcha yana 54 ta mahsulot tayyorladi. Usta bir kunda nechta mahsulot tayyorlagan?

2) Zavod mashina ishlab chiqarish bo'yicha buyurtmani 15 kunda bajarishi kerak edi. Lekin zavod har kuni rejadan tashqari 2 ta ortiq mashina ishlab chiqarib, muddatga 2 kun qolganda faqat rejani bajaribgina qolmasdan, rejadan ortiq yana 6 ta mashina ishlab chiqardi. Zavod reja bo'yicha nechta mashina ishlab chiqarishi kerak edi?

O'zingizni tekshirib ko'ring!

1. 1; 0; –4 sonlari ichida $3(x-7)+4=7x-1$ tenglamaning ildizi bormi?

2. Tenglamani yeching:

1) $2x-3(x-1)=4+2(x-1)$;

2) $\frac{x}{3}+\frac{x+1}{4}=2$.

3) 1 kg uzum 300 so'm, 1 kg anjir 400 so'm turadi. Jami sakkiz kilogramm uzum va anjir uchun birgalikda 2700 so'm to'landi. Necha kilogramm uzum va necha kilogramm anjir xarid qilingan?

114. Tenglamani yeching:

1) $5(x-3)-2(x-7)+7(2x+6)=7;$

2) $11(y-4)+10(5-3y)-3(4-3y)=-6;$

3) $5(8z-1)-7(4z+1)+8(7-4z)=9;$

4) $10(3x-2)-3(5x+2)+5(11-4x)=25.$

115. 1) $\frac{x-4}{5}=9+\frac{2x+4}{9};$

2) $2-\frac{3x-7}{4}+\frac{x+17}{5}=0;$

3) $\frac{8-y}{6}+\frac{5-4y}{3}=\frac{y+6}{2};$

4) $\frac{4x+7}{5}+\frac{3x-2}{2}-\frac{5x-2}{2}=32.$

116. Yerning birinchi ikkita sun'iy yo'ldoshi massasi 592,4 kg ni tashkil qildi. Birinchi sun'iy yo'ldosh uchinchisidan 1243,4 kg yengil, ikkinchisi esa 818,2 kg yengil. Yerning birinchi uchta sun'iy yo'ldoshining har birining massasini toping.

117. Qayiq daryo oqimi bo'yicha 2,4 soat va oqimga qarshi 3,2 soat suzdi. Qayiqning oqim bo'yicha bosib o'tgan yo'li oqimga qarshi bosib o'tgan yo'lidan 13,2 km ortiq bo'ldi. Agar daryo oqimining tezligi 3,5 km/soat bo'lsa, qayiqning turg'un suvdagi tezligini toping.

118. Suzish bo'yicha maktab musobaqalarida o'quvchi ma'lum masofani daryo oqimi bo'yicha 24 sekundda va shu masofani oqimga qarshi 40 sekundda suzib o'tdi. Agar daryo oqimining tezligi 25 sm/s bo'lsa, suzuvchining tezligini suzishning boshidan oxirigacha bir xil deb hisoblab, uning o'z tezligini aniqlang.

№ 4 | Xodani 3 bo'lakka arralash uchun 12 minut kerak. Shu xodani 4 bo'lakka arralash uchun necha minut kerak bo'ladi?

119. Bir sabzavot omboriga 145 t 480 kg, ikkinchisiga esa 89 t 7 sr kartoshka keltirishdi. Birinchi ombordan do'konga har kuni 4 t 40 kg dan, ikkinchisidan esa 2 t 550 kg dan kartoshka jo'natiladi. Necha kundan keyin ikkinchi omborda birinchidagidan 2 marta kam kartoshka qoladi?

120. Oralaridagi masofa 230 km bo‘lgan A va B shaharlardan bir vaqtda bir-birlariga qarab ikki mototsiklchi yo‘lga chiqdi. Harakat boshlanganidan 3 soat o‘tgandan keyin ularning oralaridagi masofa 20 km bo‘ldi. Agar mototsiklchilardan birining tezligi ikkinchi-sinikidan 10 km/soat kam bo‘lsa, mototsiklchilarning tezliklarini toping.

II bobga doir sinov mashqlari — testlar

1. $\frac{5x-3}{8}=\frac{x}{2}+3+\frac{11-3x}{4}$ tenglamaning ildizi x_0 bo‘lsa, x_0^2+1 ifodaning son qiymatini toping.

A) 50; B) 10; C) 5; D) 37; E) 26.

2. $\frac{2x+1}{3}+2=\frac{3x-2}{2}+\frac{x+1}{3}$ tenglamaning ildizi x_0 bo‘lsa, $18:x_0$ ifodani hisoblang:

A) 6; B) 7; C) –7; D) $46\frac{2}{7}$; E)

3. $(x+3):(x-2)=5:3$ tenglamaning ildizi x_0 bo‘lsa, $2x_0+61$ ifodaning son qiymatini toping.

A) –80; B) 70; C) 80; D) 81; E) 90.

4. $4:(2x+5)=2:(3x-2)$ tenglamaning ildizi x_0 bo‘lsa, $4x_0+11$ ifodaning son qiymatini toping.

A) –18; B) –20; C) 19; D) 20; E) 21.

5. $0{,}8\cdot(1{,}5x-2)-0{,}4x=0{,}3\cdot(6x-5)-2{,}6$ tenglamaning ildizi x_0 bo‘lsa, $x_0{}^2-0{,}5\,x_0$ ifodaning son qiymatini toping.

A) –6,25; B) 1,25; C) 6,25; D) –5; E) 5.

6. Uchta javonda hammasi bo‘lib 385 ta kitob bor. Birinchi javonda ikkinchisiga qaraganda 8 ta ko‘p, ammo uchunchi javondagidan 9 ta kam kitob bor. Har bir javonda nechtadan kitob bor?

A) 128; 120; 137; B) 127; 119; 139;
C) 127; 122; 136; D) 126; 134; 125;
E) 130; 117; 138.

7. Teng yonli uchburchakning perimetri 51 sm ga teng. Asos yon tomondan 6 sm uzun. Shu uchburchak yon tomonining asosiga nisbatini toping.

A) 1,4; B) $\frac{5}{7}$; C) $\frac{2}{3}$; D) $\frac{10}{7}$; E) 0,7.

8. Teng yonli uchburchakning perimetri 42 sm ga teng. Yon tomon asosning $\frac{2}{3}$ qismini tashkil qiladi. Shu uchburchakning asosi yon tomonidan necha santimetr uzun?

A) 7,5 sm. B) 6,5 sm; C) 6 sm; D) 7 sm; E) 5 sm.

9. Birinchi to‘pda 75 m, ikkinchi to‘pda 120 m atlas bor edi. Ikkinchi to‘pdan birinchidan sotilganiga qaraganda 3 marta ko‘p atlas sotildi. Natijada birinchi to‘pda ikkinchisiga qaraganda 2 marta ko‘p atlas qoldi. Har bir to‘pdan necha metrdan atlas sotilgan?

A) 24 m; 72 m; B) 30 m; 90 m; C) 15 m 45 sm;
D) 33 m; 99 m; E) 22 m; 66 m.

10. Usta buyurtmani 8 kunda bajarishi kerak edi. U har kuni rejadan tashqari 6 ta mahsulot tayyorlab, buyurtmani 5 kunda bajaribgina qolmasdan, balki ortiqcha yana 12 ta mahsulot tayyorladi. Usta reja bo‘yicha bir kunda nechta mahsulot tayyorlashi kerak edi?

A) 8; B) 4; C) 5; D) 7; E) 6.

Tenglamani yeching **(11—19):**

11. $8(x+2)-5x=-2(x+4,5)$.

A) -5; B) 5; C) 6;
D) $-4,5$; E) to‘g‘ri javob berilmagan.

12. $3(x+2)-2(x+3)=7-5(x+1)$.

A) $-\frac{1}{3}$; B) $\frac{1}{3}$; C) -1; D) 2; E) $-\frac{2}{3}$;

13. $\frac{x-4}{6}+2=1,5x-\frac{x-2}{3}$.

A) 2; B) 4; C) $\frac{2}{3}$; D) $-\frac{2}{3}$; E) yechimga ega emas.

14. $8(x+2)-6=7-(5-8x)$.

A) -2; B) $0{,}5$; C) $1{,}6$; D) ildizga ega emas; E) $\frac{5}{8}$.

15. $6\cdot(2{,}3x-1)-3{,}5x+0{,}7x=0{,}5(x-14)$.

A) $-10{,}5$; B) $10{,}5$; C) $\frac{2}{21}$; D) 7; E) $-\frac{2}{21}$.

16. $1{,}5\cdot(2-x)+1=2\left(2-\frac{3}{4}x\right)$.

A) cheksiz ko‘p yechimga ega; B) $\frac{8}{3}$; C) 2; D) $\frac{3}{8}$; E) -2.

17. $(1-3x):5=(2-x):2$.

A) 8; B) -8; C) 2; D) -1; E) $\frac{1}{2}$.

18. $3(4{,}5+5{,}5x)=4(11{,}5-4x)$.

A) 3; B) -2; C) 1; D) -1; E) $\frac{3}{2}$.

19. $7\frac{5}{7}:10=5{,}4:(4x-1)$.

A) -1; B) $-\frac{1}{4}$; C) $\frac{3}{4}$; D) 2; E) -2.

Tarixiy masalalar

Quyida keltirilgan 1—12- tenglamalar al-Xorazmiyning „Al-jabr val-muqobala hisobi haqida qisqacha kitob“ asarining „Vasiyatlar kitobi“ bobidagi masalalar mazmunini aks ettiruvchi tenglamalardir. Shu tenglamalarni yeching:

① $\frac{10-x}{x}=4$.

② $\frac{x}{x+2}=\frac{1}{2}$.

③ 1) $10:6=x:4$; 2) $10:8=4:x$; 3) $30:10=6:x$.

④ $\frac{10+x}{3}=x.$

⑤ 1) $\frac{\frac{4}{5}\cdot(10+x)+1}{3}=x;$ 2) $\frac{\frac{4}{5}\cdot(10+x)-1}{2}=x.$

⑥ $2\frac{1}{2}+20-x=\frac{2}{3}\cdot 50.$

⑦ 1) $110-x+\frac{1}{3}(20+x)-x=4x;$

2) $90-x+\frac{1}{2}(10+x)-x=4-x;$

3) $90-x+\frac{x}{3}=2x.$

⑧ 1) $300-x+\frac{x}{2}=2x;$ 2) $300-x+\frac{12}{33}\cdot(100-10-x)-20=2x.$

⑨ $300-x+\frac{1}{2}[500-(300-x)]+300-x=4x$

⑩ 1) $500-x+100-\frac{x}{5}=2(100+x);$

2) $500-x+100-\frac{x}{5}-\frac{3}{4}x=2\cdot(100+x+\frac{3}{4}x).$

⑪ 1) $300-x+100-\frac{x}{3}=2x;$ 2) $300-x-\frac{x}{3}+100-\frac{x}{3}=2x.$

⑫ $300-x-\frac{x}{3}+100-\frac{x}{3}-x-\frac{x}{3}=4\left(x+\frac{x}{3}\right).$

G‘iyosiddin Jamshid al-Koshiyning „Hisob ilmi kaliti“ asaridan olingan masalalarni yeching **(13—14).**

⑬ Oltin va durdan yasalgan bezakning massasi 3 misqol, bahosi 24 dinor. 1 misqol oltin 5 dinor, 1 misqol dur 15 dinor tursa, bezakda necha misqoldan oltin va dur bor?

⑭ O‘ylangan sonni 2 ga ko‘paytirib, hosil bo‘lgan songa 1 qo‘shilsa, yig‘indini 3 ga ko‘paytirib, ko‘paytmaga 2 qo‘shilsa, so‘ng hosil bo‘lgan son 4 ga ko‘paytirilib, bu ko‘paytmaga 3 qo‘shilsa, 95 hosil bo‘ladi. O‘ylangan sonni toping.

Tarixiy ma’lumotlar

Muhammad ibn Muso al-Xorazmiy „Al-jabr val-muqobala hisobi haqida qisqacha kitob“ asarida kiritilgan „al-jabr“, „val-muqobala“ qoidalarini biz 7- § da tenglamaning asosiy xossalari sifatida bayon qildik, xolos.

Algebrada uch xil sonlar bilan ish ko‘riladi, deydi al-Xorazmiy. Ular:

— ildiz yoki narsa (tenglamadagi noma’lum son x);

— kvadrat (mol) (noma’lumning kvadrati — x^2);

— oddiy son (bunda natural son nazarda tutiladi).

Xorazmiy shu uch xil miqdorlar orasidagi turli bog‘lanishlarni tahlil qiladi va ushbu ko‘rinishdagi tenglamalarni yechish usullarini ko‘rsatadi:

1) $cx^2 = bx$ — kvadratlar ildizlarga teng;
2) $cx^2 = a$ — kvadratlar sonlarga teng;
3) $bx = a$ — ildizlar songa teng;
4) $cx^2 + bx = a$ — kvadratlar va ildizlar sonlarga teng;
5) $cx^2 + a = bx$ — kvadratlar va son ildizlarga teng;
6) $bx + a = cx^2$ — ildizlar va son kvadratlarga teng.

Biz 7- sinfda faqat chiziqli tenglamalarni o‘rganamiz [3) banddagi $bx = a$ tenglama]. Qolganlari 8-sinfda o‘rganiladi. Har qanday chiziqli yoki kvadrat tenglama „al-jabr“, „val-muqobala“ almashtirishlari natijasida yuqoridagi 6 ta tenglamaning biriga keltirilishi mumkin.

III BOB
BIRHADLAR VA KO‘PHADLAR

9-§ Natural ko‘rsatkichli daraja

Teng sonlarni qo‘shishni ko‘paytirish bilan almashtirish mumkin:

$$\underbrace{3+3+3+3+3}_{5\text{ marta}}=3\cdot 5 \qquad \underbrace{a+a+a+a+\ldots+a}_{n\text{ marta}}=a\cdot n$$

Bir xil sonlarning ko‘paytmasini ham ko‘p hollarda ixchamroq yozuv bilan almashtirish maqsadga muvofiq bo‘ladi. Tomonining uzunligi 5 birlikka teng kvadratni qaraylik (3- rasm). U $5\cdot 5=25$ ta birlik kvadratdan iborat. Tomonining uzunligi 5 birlikka teng kub (4- rasm) esa $5\cdot 5\cdot 5=125$ ta birlik kubni o‘z ichiga oladi.

Sizga ma’lumki, $5\cdot 5$ ko‘paytmani 5^2 (o‘qilishi: «beshning kvadrati»); $5\cdot 5\cdot 5$ ko‘paytmani esa 5^3 (o‘qilishi: „beshning kubi“) kabi belgilanadi:

$$5\cdot 5=5^2, \quad 5\cdot 5\cdot 5=5^3.$$

Xuddi shu kabi, ko‘paytuvchilari bir xil sonlardan iborat ko‘paytmani yangi amal — *darajaga ko‘tarish* amali bilan almashtirish mumkin:

$$\underbrace{3\cdot 3\cdot 3\cdot 3\cdot 3}_{5\text{ marta}}=3^5, \qquad \underbrace{\frac{1}{7}\cdot\frac{1}{7}\cdot\frac{1}{7}\cdot\ldots\cdot\frac{1}{7}}_{9\text{ marta}}=\left(\frac{1}{7}\right)^9,$$

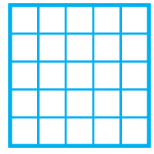
$$0{,}4=(0{,}4)^1.$$

Umuman, n ta teng ko‘paytuvchining ko‘paytmasini belgilash uchun a^n yozuvidan foydalaniladi:

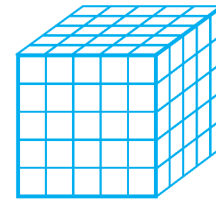
$$\underbrace{a\cdot a\cdot a\cdot a\cdot\ldots\cdot a}_{n\text{ marta}}=a^n.$$

U bunday o‘qiladi: „a sonning n ko‘rsatkichli darajasi“. Odatda, qisqacha qilib: „a ning n- darajasi“ deb aytiladi.

a sonning n natural ko‘rsatkichli darajasi deb, har biri a ga teng bo‘lgan n ta ko‘paytuvchining ko‘paytmasiga aytiladi:



3- rasm.



4- rasm.

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ marta}}$$

a sonni (takrorlanuvchi ko‘paytuvchini) *darajaning asosi, n* sonni (ko‘paytuvchi necha marta takrorlanishini ko‘rsatuvchi sonni) *daraja ko‘rsatkichi* deyiladi.

Masalan,

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81,$$

bu yerda 3 — darajaning asosi, 4 — daraja ko‘rsatkichi, 81 esa 3^4— darajaning qiymati.

Xususan, sonning birinchi darajasi deb, shu sonning o‘zini aytiladi:

$$a^1 = a.$$

Masalan, $5^1 = 5, 25^1 = 25, \left(\frac{1}{7}\right)^1 = \frac{1}{7}.$

Darajaning asosi istalgan son bo‘lishi mumkinligini aytib o‘tamiz, masalan,

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32; \quad \left(\frac{2}{5}\right)^3 = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} = \frac{8}{125};$$

$$(-2)^5 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = -32;$$

$$\left(-\frac{2}{3}\right)^4 = \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) = \frac{16}{81};$$

$$0,2^3 = 0,2 \cdot 0,2 \cdot 0,2 = 0,008;$$

$$(-1)^6 = (-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) = 1;$$

$$0^3 = 0 \cdot 0 \cdot 0 = 0; \quad 10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10\,000.$$

Darajaga ko‘tarish amali — uchinchi bosqich amal. Agar ifodada qavslar bo‘lmasa, u holda avval uchinchi bosqich amallar, keyin

ikkinchi bosqich amallar (ko‘paytirish va bo‘lish), va nihoyat, birinchi bosqich amallar (qo‘shish va ayirish) bajarilishini eslatib o‘tamiz.

Masala. Hisoblang: $7 \cdot 2^4 - 5 \cdot 3^2$.

$$7 \cdot 2^4 - 5 \cdot 3^2 = 7 \cdot 16 - 5 \cdot 9 = 112 - 45 = 67.$$

Sonlarni daraja yordamida yozishdan juda ko‘p hollarda, masalan, natural sonlarni xona birliklari bo‘yicha yig‘indisi shaklida yozish uchun foydalaniladi:

$$\triangle\ 3245 = 3 \cdot 1000 + 2 \cdot 100 + 4 \cdot 10 + 5 = 3 \cdot 10^3 + 2 \cdot 10^2 + 4 \cdot 10 + 5. \ \blacktriangle$$

Katta sonlarni yozish uchun ko‘pincha 10 sonining darajalari qo‘llaniladi. Masalan, Yerdan Quyoshgacha bo‘lgan masofa taxminan 150 mln km ga teng bo‘lib, uni $1{,}5 \cdot 10^8$ km shaklida yoziladi: Yer sharining radiusi taqriban 6,37 mln m ga teng, u $6{,}37 \cdot 10^6$ m kabi yoziladi; Yerdan eng yaqin yulduz (Sentavrning α si)gacha bo‘lgan masofani $4 \cdot 10^{13}$ km shaklida yoziladi.

10 dan katta bo‘lgan har bir sonni $a \cdot 10^n$ shaklida yozish mumkin, bunda $1 \le a < 10$ va n – natural son. Bunday yozuv *sonning standart shakli* deyiladi.

Masalan,

$4578 = 4{,}578 \cdot 10^3$, $45{,}78 = 4{,}578 \cdot 10$, $103000 = 1{,}03 \cdot 10^5$.

Fizika va kimyo fanlarini o‘rganishda, mikrokalkulatorda hisoblashlarda va boshqa ko‘p hollarda sonning standart shakldagi yozuvidan foydalaniladi.

Mashqlar

Yig‘indini ko‘paytma shaklida yozing **(121—122):**

121. 1) $4+4+4+4+4$;
2) $6+6+6+6$;
3) $c+c+c$;
4) $a+a+a+a+a$.

122. 1) $2m+2m+2m$;
2) $17ab+17ab+17ab$;
3) $(c-2d)+(c-2d)$;
4) $(3b-a)+(3b-a)+(3b-a)$;

5) $\underbrace{3+3+\ldots+3}_{\text{21 marta}}$;

6) $\underbrace{5+5+\ldots+5}_{\text{17 marta}}$;

7) $\underbrace{m+m+\ldots+m}_{n\text{ marta}}$;

8) $\underbrace{b+b+\ldots+b}_{k\text{ marta}}$.

Ko‘paytmani daraja shaklida yozing **(123—125):**

123. 1) $2\cdot2\cdot2\cdot2\cdot2$; 2) $\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}$; 3) $\left(\frac{3}{4}\right)\cdot\left(\frac{3}{4}\right)\cdot\left(\frac{3}{4}\right)$;
4) $(-2,7)\cdot(-2,7)\cdot(-2,7)\cdot(-2,7)$.

124. 1) $x\cdot x\cdot x\cdot x\cdot x$; 3) $(2a)\cdot(2a)\cdot(2a)$;
2) $m\cdot m\cdot m\cdot m\cdot m$; 4) $(-3b)\cdot(-3b)\cdot(3b)\cdot(3b)$.

125. 1) $(x-y)\cdot(x-y)\cdot(x-y)$; 3) $\frac{3x}{2}\cdot\frac{3x}{2}$;
2) $(a+b)\cdot(a+b)$; 4) $\frac{m}{n}\cdot\frac{m}{n}\cdot\frac{m}{n}\cdot\frac{m}{n}\cdot\frac{m}{n}$.

Ko‘paytmaning daraja shaklidagi yozuvidan foydalanib, ifodani soddalashtiring **(126—128):**

126. 1) $2\cdot2\cdot2\cdot15$; 3) $5\cdot5\cdot8\cdot8\cdot8\cdot2\cdot2$;
2) $4\cdot4\cdot4\cdot4\cdot21$; 4) $6\cdot6\cdot7\cdot7\cdot3\cdot3\cdot3$.

127. 1) $1{,}2\cdot1{,}2\cdot2\cdot2\cdot2\cdot5\cdot5$; 2) $0{,}5\cdot0{,}5\cdot0{,}5\cdot2\cdot2\cdot4\cdot4$;
3) $0{,}3\cdot0{,}3\cdot\frac{1}{7}\cdot\frac{1}{7}\cdot\frac{1}{7}\cdot\frac{1}{7}$; 4) $\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot2{,}3\cdot2{,}3$.

128. 1) $9\cdot9\cdot9\cdot a\cdot a\cdot a$; 3) $\frac{x}{y}\cdot\frac{x}{y}\cdot\frac{x}{y}(x-y)\cdot(x-y)$;
2) $x\cdot x\cdot x\cdot x\cdot3\cdot3$; 4) $\frac{a}{b}\cdot\frac{a}{b}\cdot(8a-b)\cdot(8a-b)\cdot(8a-b)$.

Ifodani soddalashtiring **(129—130):**

129. 1) $p\cdot p\cdot p\cdot p+q\cdot q$; 3) $a\cdot a+a\cdot a+a\cdot a$;
2) $a\cdot a+b\cdot b\cdot b\cdot b$; 4) $x\cdot x\cdot x+x\cdot x\cdot x$.

130. 1) $\underbrace{c\cdot c+c\cdot c+\ldots+c\cdot c}_{k\text{ marta}}$; 3) $\underbrace{a\cdot a\cdot\ldots\cdot a}_{n\text{ marta}}+\underbrace{b\cdot b\cdot\ldots\cdot b}_{m\text{ marta}}$;
2) $\underbrace{a\cdot a\cdot a+a\cdot a\cdot a+\ldots+a\cdot a\cdot a}_{n\text{ marta}}$; 4) $\underbrace{5\cdot5\cdot\ldots\cdot5}_{k\text{ marta}}+\underbrace{a\cdot a\cdot\ldots\cdot a}_{17\text{ marta}}$.

131. Ifodani o‘qing, darajaning asosini, daraja ko‘rsatkichini ayting:

1) 3^2; 3) $\left(-\frac{2}{9}\right)^{41}$; 5) $(4m+n)^{15}$;

2) $\left(1\frac{3}{8}\right)^3$; 4) $(-1,2)^{39}$; 6) $\left(\frac{2a}{3b}\right)^7$.

Hisoblang **(132—139)**:

132. 1) 2^3; 2) 3^2; 3) 4^4; 4) 5^3.

133. 1) 1^5; 2) $(-1)^7$; 3) 0^{15}; 4) 0^5.

134. 1) $\left(\frac{2}{3}\right)^3$; 2) $\left(\frac{3}{5}\right)^2$; 3) $\left(1\frac{2}{7}\right)^2$; 4) $\left(2\frac{1}{3}\right)^3$.

135. 1) $(2,5)^2$; 2) $(1,7)^2$; 3) $(-0,2)^3$; 4) $(-0,2)^4$.

136. 1) $(-5)^3$; 2) -5^3; 3) $\left(-2\frac{1}{4}\right)^2$; 4) $-\left(2\frac{1}{4}\right)^2$.

137. 1) $\frac{(-0,2)^4}{(0,1)^5}$; 2) $\frac{(0,3)^3}{(-0,1)^4}$; 3) $\frac{(3,2)^2}{(1,6)^2}$; 4) $\frac{(2,6)^2}{(1,3)^2}$.

138. 1) $2(-3)^2$; 2) $-5(-2)^3$; 3) $-\frac{1}{2}(-4)^2$; 4) $-\frac{2}{3}(-3)^2$.

139. 1) $(-5)^2+\left(\frac{3}{5}\right)$; 2) $(-3)^3\left(-\frac{2}{3}\right)$;

3) $-(-3)^2 2^3$; 4) $-(-3)^2(-2)^3$.

140. $-x^2; (-x)^2; (-x)^3$ ifodaning qiymatini $x=1\frac{1}{2}; -5$ da toping.

141. x^2 ifodaning qiymatini x ning jadvalda keltirilgan qiymatlari uchun hisoblang:

x	0	1	–1	2	–2	3	–3	4	–4	5	–5	6	–6
x^2													

142. x^3 ifodaning qiymatini x ning jadvalda ko‘rsatilgan qiymatlari uchun hisoblang:

x	0	1	–1	2	–2	3	–3	4	–4	5	–5	6	–6
x^3													

143. Ikki xonali son xona qo‘shiluvchilari yig‘indisi shaklida quyidagicha yozilishi mumkin: $a \cdot 10 + b$, bu yerda a — o‘nliklar soni, b — birliklar soni; uch xonali sonni $a \cdot 10^2 + b \cdot 10 + c$ ko‘rinishida yozish mumkin, bu yerda a — yuzliklar soni, b — o‘nliklar soni; c — birliklar soni. Aytaylik, to‘rt xonali sonda a — mingliklar soni, b — yuzliklar soni; c — o‘nliklar soni, d — birliklar soni bo‘lsin. Shu sonni xona qo‘shiluvchilari yig‘indisi shaklida yozing.

144. Sonni xona qo‘shiluvchilari yig‘indisi shaklida yozing:
1) 127359; 2) 5432135; 3) 1027305; 4) 12350107.

145. Xona qo‘shiluvchilari yig‘indisi shaklida tasvirlangan sonni yozing:
1) $2 \cdot 10^5 + 3 \cdot 10^4 + 5 \cdot 10^3 + 1 \cdot 10^2 + 2 \cdot 10 + 1$;
2) $3 \cdot 10^6 + 5 \cdot 10^5 + 3 \cdot 10^4 + 2 \cdot 10^3 + 3 \cdot 10 + 7$;
3) $7 \cdot 10^5 + 1 \cdot 10^3 + 5 \cdot 10^2 + 8$;
4) $1 \cdot 10^5 + 1 \cdot 10^3 + 1$.

10-§. *Natural ko‘rsatkichli darajaning xossalari*

Darajaga ko‘tarish bir nechta muhim xossalarga ega.

1- xossa.

$$a^m \cdot a^n = a^{m+n}.$$

Bir xil asosli darajalarni ko‘paytirishda asos o‘zgarmasdan qoladi, daraja ko‘rsatkichlari esa qo‘shiladi.

○ Natural ko‘rsatkichli darajaning ta’rifiga ko‘ra

$2^2 \cdot 2^3 = \underbrace{(2 \cdot 2)}_{2 \text{ marta}} \cdot \underbrace{(2 \cdot 2 \cdot 2)}_{3 \text{ marta}} =$ | $a^m \cdot a^n = \underbrace{(a \cdot a \cdot a \cdot \ldots \cdot a)}_{m \text{ marta}} \times \underbrace{(a \cdot a \cdot a \cdot \ldots \cdot a)}_{n \text{ marta}} =$

ko‘paytirishning guruhlash qonuniga ko‘ra

$= \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{5 \text{ marta}} =$ | $= \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{(m+n) \text{ marta}} =$

natural ko‘rsatkichli darajaning ta’rifiga ko‘ra

$= 2^5.$ | $= a^{m+n}.$

Shunday qilib,

$2^2 \cdot 2^3 = 2^{2+3}.$ | $a^m \cdot a^n = a^{m+n}.$

2- xossa.

$$a^m : a^n = a^{m-n},\ m > n,\ a \neq 0.$$

Bir xil asosli darajalarni bo‘lishda asos o‘zgarmasdan qoladi, daraja ko‘rsatkichlari esa ayiriladi.

○ Shartga ko‘ra

$5 > 3.$ | $m > n,\ a \neq 0.$

Darajaning birinchi xossasiga ko‘ra

$2^{5-3} \cdot 2^3 = 2^5.$ | $a^{m-n} \cdot a^n = a^m.$

Shuning uchun

$2^{5-3} = 2^5 : 2^3.$ | $a^{m-n} = a^m : a^n.$

Shunday qilib,

$2^5 : 2^3 = 2^{5-3}.$ | $a^m : a^n = a^{m-n},\ m > n,\ a \neq 0.$ ●

$\frac{a^n}{a^n} = 1,\ a \neq 0$ ekanligini ta’kidlaymiz.

3- xossa.

$$(a^m)^n = a^{mn}.$$

Darajani darajaga ko‘tarishda asos o‘zgarmasdan qoladi, daraja ko‘rsatkichlar esa o‘zaro ko‘paytiriladi.

○ Natural ko‘rsatkichli darajaning ta’rifiga ko‘ra

$(2^3)^2 = 2^3 \cdot 2^3 =$ | $(a^m)^n = \underbrace{a^m \cdot a^m \cdot a^m \cdot \ldots \cdot a^m}_{n \text{ marta}} =$

darajaning birinchi xossasiga ko‘ra

$= 2^{3+3} =$ | $= \overbrace{a^{m+m+\ldots+m}}^{n \text{ marta}} =$

ko‘paytirishning ta’rifiga ko‘ra

$= 2^{3 \cdot 2}.$ | $= a^{mn}.$

Shunday qilib,

$(2^3)^2 = 2^{3 \cdot 2}.$ | $(a^m)^n = a^{mn}.$ ●

4- xossa.

$$(ab)^n = a^n b^n .$$

Ko‘paytmani darajaga ko‘tarishda har bir ko‘paytuvchi shu darajaga ko‘tariladi.

○ $(2\cdot 3)^3 = \underbrace{(2\cdot 3)\cdot(2\cdot 3)\cdot(2\cdot 3)}_{3 \text{ marta}} =$ | $(ab)^n = \underbrace{(ab)(ab)\ldots(ab)}_{n \text{ marta}} =$

ko‘paytirishning guruhlash va o‘rin almashtirish qonuniga ko‘ra

$= \underbrace{(2\cdot 2\cdot 2)}_{3 \text{ marta}} \cdot \underbrace{(3\cdot 3\cdot 3)}_{3 \text{ marta}} =$ | $= \underbrace{(a\cdot a\cdot \ldots \cdot a)}_{n \text{ marta}} \underbrace{(b\cdot b\cdot \ldots \cdot b)}_{n \text{ marta}} =$

natural ko‘rsatkichli darajaning ta’rifiga ko‘ra

$= 2^3 \cdot 3^3.$ | $= a^n \cdot b^n.$

Shunday qilib,

$(2\cdot 3)^3 = 2^3 \cdot 3^3.$ | $(ab)^n = a^n b^n.$ ●

5- xossa.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}; \quad b \neq 0 .$$

Kasrni darajaga ko‘tarishda uning surat va maxraji xuddi shu darajaga ko‘tariladi.

○ Natural ko‘rsatkichli darajaning ta’rifiga ko‘ra

$\left(\frac{2}{3}\right)^3 = \underbrace{\left(\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\right)}_{3 \text{ marta}} =$ | $\left(\frac{a}{b}\right)^n = \underbrace{\left(\frac{a}{b}\cdot\frac{a}{b}\cdots\frac{a}{b}\right)}_{n \text{ marta}} =$

kasrlarni ko‘paytirish qoidasiga ko‘ra

$= \frac{\overbrace{2\cdot 2\cdot 2}^{3 \text{ marta}}}{\underbrace{3\cdot 3\cdot 3}_{3 \text{ marta}}} =$ | $= \frac{\overbrace{a\cdot a\ldots\cdot a}^{n \text{ marta}}}{\underbrace{b\cdot b\ldots\cdot b}_{\text{n marta}}} =$

natural ko‘rsatkichli darajaning ta’rifiga ko‘ra

$= \frac{2^3}{3^3}.$ | $= \frac{a^n}{b^n}.$

Shunday qilib,

$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}.$ | $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad b \neq 0.$

1 - masala. Hisoblang: $\dfrac{11^7 \cdot 7^3 \cdot 3^4}{11^6 \cdot 7 \cdot 3^4}$.

△ $\dfrac{11^7 \cdot 7^3 \cdot 3^4}{11^6 \cdot 7 \cdot 3^4} = 11^{7-6} \cdot 7^{3-1} \cdot 1 = 11 \cdot 49 = 539.$ ▲

2- masala. Yorug‘likning tarqalish tezligi $3 \cdot 10^8$ m/s ga yaqin, Yerdan Quyoshgacha bo‘lgan o‘rtacha masofa $1{,}5 \cdot 10^{11}$ m. Yorug‘lik nuri Quyoshdan Yergacha bo‘lgan masofani qancha vaqtda bosib o‘tadi?

△ Tekis harakatda bosib o‘tilgan yo‘lning $s = vt$ formulasiga asosan:

$$1{,}5 \cdot 10^{11} = 3 \cdot 10^8 \cdot t,$$

bu yerdan $t = \dfrac{1{,}5 \cdot 10^{11}}{3 \cdot 10^8} = 0{,}5 \cdot 10^3 = 500$.

Javob: 500 s = 8 min 20 s. ▲

Mashqlar

Ko‘paytmani daraja shaklida yozing **(146—152):**

146. 1) $3^5 \cdot 3^4$; 2) $7^2 \cdot 7^4$; 3) $6^3 \cdot 6$; 4) $5 \cdot 5^5$.

147. 1) $c^3 \cdot c^2$; 2) $a^3 \cdot a^4$; 3) $\left(\frac{1}{2}a\right)^7\left(\frac{1}{2}a\right)$; 4) $(3b)(3b)^6$.

148. 1) $(-2)^2 \cdot (-2)^3$; 2) $(-3)^2 \cdot (-3)^2$; 3) $(-0{,}5)^4 \cdot (-0{,}5)^2$; 4) $(-1{,}2)^3 \cdot (-1{,}2)^4$.

149. 1) $2^3 \cdot 2^2 \cdot 2^4$; 2) $3^2 \cdot 3^5 \cdot 3^3$; 3) $(-5)^6 \cdot (-5)^3 \cdot (-5)^4$; 4) $(-6)^3 \cdot (-6)^2 \cdot (-6)^7$.

150. 1) $(1{,}3)^2 \cdot (1{,}3) \cdot (1{,}3)^5$; 2) $\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)^3 \cdot \left(\frac{2}{3}\right)^4$; 3) $y^4 y^3 y^7$; 4) $b^6 b^8 b$.

151. 1) $(-2{,}5a)^3 \cdot (-2{,}5a)^8$; 2) $\left(-\frac{5x}{6}\right)^5 \cdot \left(-\frac{5x}{6}\right)^7$; 3) $(x-a)^7(x-a)^{10}$; 4) $(n+m)^{15}(n+m)^5$.

152. 1) $4^4 \cdot 4^5$; 2) $3^8 \cdot 3^n$; 3) $c^{28} c^n$; 4) $a^n a^{13}$ (n — natural son).

153. Darajani bir xil asosli ikkita darajaning ko‘paytmasi shaklida yozing:

1) 3^4; 2) $\left(\frac{5}{9}\right)^5$; 3) y^3; 4) c^{10}; 5) $(-x)^{17}$; 6) $(-11b)^{43}$.

Sonlarni asosi 2 bo‘lgan daraja shaklida yozing **(154—157):**

154. 1) 32; 2) 4; 3) 2; 4) 128.

155. 1) 16; 2) 64; 3) 256; 4) 1024.

156. 1) $2 \cdot 2^6$; 2) $2^4 \cdot 2^3 \cdot 2^7$; 3) $8 \cdot 2^7$; 4) $16 \cdot 2^5$.

157. 1) $2^7 \cdot 128$;
2) $2^{10} \cdot 32 \cdot 256$;
3) $2^n \cdot 8$;
4) $16 \cdot 2^n$ (n — natural son).

Sonlarni asosi 3 bo‘lgan daraja shaklida yozing **(158—161):**

158. 1) 9; 2) 3; 3) 27; 4) 81.

159. 1) 729; 2) 243; 3) $3 \cdot 3^4$; 4) $3^6 \cdot 3$.

№ 5 | Sonning o‘nli yozuvidagi oxirgi raqam nechaga teng:
1) 846^{847}; 2) 1987^{1987}; 3) 1998^{1998}; 4) 2009^{2009}?

160. 1) $3^5 \cdot 3^{17} \cdot 3$; 2) $3^2 \cdot 3^{11} \cdot 3^5$; 3) $3^5 \cdot 27$; 4) $81 \cdot 3^2$.

161. 1) $3^n \cdot 3^2$;
2) $3 \cdot 3^n$;
3) $3^{n+1} \cdot 81$;
4) $27 \cdot 3^n$ (n — natural son).

Bo‘linmani daraja shaklida yozing **(162—164):**

162. 1) $7^{10} : 7^8$; 2) $4^3 : 4$; 3) $(0{,}2)^4 : (0{,}2)^3$; 4) $10^{12} : 10^4$.

163. 1) $\left(-\frac{9}{7}\right)^8 : \left(-\frac{9}{7}\right)^5$; 2) $\left(\frac{1}{17}\right)^{18} : \left(\frac{1}{17}\right)^{17}$; 3) $x^{21} : x^7$; 4) $d^{24} : d^{12}$.

164. 1) $\left(\frac{3y}{4}\right)^6 : \left(\frac{3y}{4}\right)^2$;
2) $(2a)^5 : (2a)^3$;
3) $(a - b)^7 : (a - b)^5$;
4) $(m + n)^{10} : (m + n)^5$.

Sonlarni asosi 2 bo‘lgan daraja shaklida yozing **(165—166):**

165. 1) $2^3 : 2$; 2) $2^4 : 4$; 3) $64 : 4$; 4) $32 : 2^3$.

166. 1) $8 : 2^2$; 2) $256 : 32$; 3) $\frac{2^7}{2^5}$; 4) $\frac{2^{10}}{2}$.

Sonlarni asosi 3 bo‘lgan daraja shaklida yozing **(167—168):**

167. 1) $3^5 : 3^2$; 2) $3^4 : 3$; 3) $3^4 : 9$; 4) $27 : 3^2$.

168. 1) $243 : 27$; 2) $81 : 9$; 3) $\frac{3^{15}}{3}$; 4) $\frac{3^8}{3^4}$.

Hisoblang **(169—171):**

169. 1) $\frac{2 \cdot 3^3}{3^2}$; 2) $\frac{2^4 \cdot 3^2}{2^3 \cdot 3}$; 3) $\frac{3^5 \cdot 3^{10}}{3^6 \cdot 3^7}$; 4) $\frac{5^8 \cdot 5^7}{5^4 \cdot 5^9}$.

170. 1) $\frac{8 \cdot 3^3}{2 \cdot 3^2}$; 2) $\frac{11^3 \cdot 4^2}{11^2 \cdot 4}$; 3) $\frac{2^4 \cdot 2^6 \cdot 2^3}{2^5 \cdot 2^7}$; 4) $\frac{3^6 \cdot 3^3}{3^5 \cdot 3 \cdot 3}$.

171. 1) $\frac{(-5)^9}{5^7}$; 2) $\frac{6^8}{(-6)^7}$; 3) $\frac{6^6}{3^4 \cdot 2^3}$; 4) $\frac{3^6 \cdot 2^7}{6^5}$.

Tenglamani yeching **(172—174):**

172. 1) $x : 3^2 = 3^3$; 2) $x : 2^4 = 2^2$; 3) $x \cdot 2^6 = 2^8$; 4) $x \cdot 3^5 = 3^8$.

173. 1) $5^5 x = 5^7$; 2) $4^6 x = 4^8$; 3) $3^8 : x = 3^8$; 4) $2^{11} : x = 2^9$.

174. 1) $\frac{x}{2^3} = 2^2$; 2) $\frac{x}{3^2} = 3^3$; 3) $\frac{2^8}{x} = 2^5$; 4) $\frac{3^9}{x} = 3^7$.

Ifodani asosi a bo‘lgan daraja shaklida yozing **(175—177):**

175. 1) $(a^5)^6$; 2) $(a^8)^7$; 3) $(a^2)^5 a^8$; 4) $a^5 (a^2)^8$.

176. 1) $a^7 a^5 (a^2)^4$; 2) $a^3 (a^3)^3 a^3$; 3) $(a^3)^2 a^4 (a^4)^3$; 4) $a^5 (a^3)^4 (a^2)^3$.

177. 1) $(a^7)^5 : (a^3)^4$; 2) $(a^6)^4 : (a^3)^5$; 3) $\frac{(a^3)^5 a^4}{a^{12}}$; 4) $\frac{a^8 (a^4)^4}{(a^3)^4}$.

178. n ning qanday qiymatida tenglik to‘g‘ri bo‘ladi:

1) $3^n = 9$; 2) $128 = 2^n$; 3) $(2^2)^n = 16$; 4) $(3^n)^2 = 81$?

Sonlarni ko‘rsatkichi 2 bo‘lgan daraja shaklida yozing **(179—181):**

179. 1) $0{,}01$; 2) $\frac{25}{36}$; 3) $1\frac{9}{16}$; 4) $0{,}0004$.

180. 1) 5^4; 2) 7^6; 3) $(-0{,}7)^{14}$; 4) $\left(-\frac{2}{3}\right)^{24}$.

181. 1) a^4; 2) b^6; 3) c^{10}; 4) x^{20}.

Ko‘paytmani darajaga ko‘taring **(182—187):**

182. 1) $(3 \cdot 5)^4$; 2) $(7 \cdot 6)^5$; 3) $(1{,}3 \cdot 8)^5$; 4) $\left(4 \cdot \frac{1}{7}\right)^3$.

183. 1) $(2a)^3$; 2) $(3x)^4$; 3) $(-4x)^5$; 4) $(-8b)^2$.

184. 1) $(ax)^7$; 2) $(6y)^6$; 3) $(2{,}5cd)^2$; 4) $(3nm)^3$.

185. 1) $(abc)^4$; 2) $(xuz)^7$; 3) $(3 \cdot 5 \cdot 11)^8$; 4) $(2 \cdot 4 \cdot 9)^9$.

186. 1) $(xy^3)^2$; 2) $(a^2b)^3$; 3) $(2b^4)^5$; 4) $(0{,}1c^3)^2$.

187. 1) $(10n^2m^3)^3$; 2) $(8a^4b^7)^3$; 3) $(-2{,}3a^3b^4)^2$; 4) $(-2nm^3)^4$.

Ko‘paytmani $3^2b^2 = (3b)^2$ namunaga qarab daraja shaklida yozing **(188—190):**

188. 1) 4^5x^5; 2) 2^3a^3; 3) 5^47^4; 4) 2^53^5.

189. 1) $\left(\frac{2}{5}\right)^2 a^2$; 2) $(3{,}4)^4b^4$; 3) $(-1{,}2)^3y^3$; 4) $\left(-\frac{2}{3}\right)^2 a^2$.

190. 1) $16a^2$; 2) $81r^2$; 3) $9^7n^7m^7$; 4) $15^3a^3b^3$.

Ifodani ko‘rsatkichi 2 bo‘lgan daraja shaklida yozing **(191—193):**

191. 1) c^2d^{10}; 2) a^4b^6; 3) $25a^4$; 4) $81m^2$.

192. 1) $a^4b^6c^2$; 2) $x^2y^4z^8$; 3) $49x^8y^6$; 4) $100c^8x^6$.

193. 1) $0{,}25a^{10}b^6$; 2) $0{,}49n^2m^{10}$; 3) $\frac{49}{81}x^{12}y^{14}$; 4) $\frac{16}{625}a^{10}b^{16}$.

Ifodani ko‘rsatkichi 3 bo‘lgan daraja shaklida yozing **(194—197):**

194. 1) a^6; 2) b^9; 3) 5^{15}; 4) 4^6.

195. 1) $(-0{,}2)^{12}$; 2) $\left(-\frac{2}{3}\right)^{15}$; 3) $-0{,}125$; 4) $-0{,}001$.

196. 1) x^3y^9; 2) a^6b^3; 3) $b^9c^{12}d^3$; 4) $x^{12}y^9z^6$.

197. 1) $-27a^3$; 2) $-1000b^6$; 3) $-125n^6m^6$; 4) $-0{,}008x^3y^9$.

Hisoblang **(198—202)**:

198. 1) $(0,25)^7 4^7$; 2) $\left(\frac{4}{5}\right)^{17} \cdot \left(\frac{5}{4}\right)^{17}$; 3) $(-0,125)^{11} 8^{11}$; 4) $(-0,2)^5 5^5$.

199. 1) $(-0,25)^9 (-4)^9$; 3) $\left(\frac{6}{11}\right)^3 \cdot (8,5)^3$;

2) $\left(-\frac{2}{7}\right)^7 \cdot (-3,5)^7$; 4) $\left(\frac{1}{9}\right)^5 \cdot (4,5)^5$.

200. 1) $\frac{2^8 \cdot 3^8}{6^5}$; 2) $\frac{4^5 \cdot 3^5}{12^3}$; 3) $\frac{10^5}{2^5 \cdot 5^5}$; 4) $\frac{14^4}{2^3 \cdot 7^3}$.

201. 1) $\frac{6^{12} \cdot 4^{12}}{3^{12} \cdot 8^{12}}$; 2) $\frac{4^{10} \cdot 3^{10}}{2^{10} \cdot 6^{10}}$; 3) $\frac{15^4}{3^4 \cdot 5^2 \cdot 25}$; 4) $\frac{4^{16}}{8^{10}}$.

202. 1) $\frac{8 \cdot 27^3}{3^8}$; 2) $\frac{2^8 \cdot (7^2)^4}{14^7}$; 3) $\frac{16^2 \cdot 3^5}{12^4}$; 4) $\frac{2^9 \cdot (2^2)^5}{(2^5)^3}$.

Kasrni darajaga ko‘taring **(203—206):**

203. 1) $\left(\frac{2}{3}\right)^2$; 2) $\left(\frac{5}{7}\right)^2$; 3) $\left(\frac{3}{a}\right)^2$; 4) $\left(\frac{b}{8}\right)^3$.

204. 1) $\left(-\frac{m}{11}\right)^2$; 2) $\left(-\frac{13}{n_4}\right)^2$; 3) $\left(\frac{d}{-2}\right)^3$; 4) $\left(\frac{-4}{c}\right)^3$.

205. 1) $\left(\frac{a}{2b}\right)^4$; 2) $\left(\frac{3b}{5c}\right)^7$; 3) $\left(\frac{2^3}{3^2}\right)^7$; 4) $\left(\frac{5^2}{7^4}\right)^3$.

206. 1) $\left(\frac{a+b}{3}\right)^3$; 2) $\left(\frac{7}{2+c}\right)^2$; 3) $\left(\frac{m+n}{m-n}\right)^5$; 4) $\left(\frac{a+b}{a-b}\right)^7$.

Kasrni daraja shaklida yozing **(207—209)**:

207. 1) $\frac{3^7}{4^7}$; 2) $\frac{2^5}{5^5}$; 3) $\frac{m^3}{2^3}$; 4) $\frac{5^7}{a^7}$.

208. 1) $\frac{x^6}{y^6}$; 2) $\frac{a^3}{b^3}$; 3) $\frac{25}{36}$; 4) $\frac{49}{100}$.

209. 1) $\frac{(2b)^2}{(3b)^2}$; 2) $\frac{(4x)^4}{(3y)^4}$; 3) $\frac{1}{-8}$; 4) $\frac{-1}{27}$.

Hisoblang **(210—211):**

210. 1) $\left(\frac{3}{5}\right)^4 \cdot \frac{5^3}{3^2}$; 2) $\frac{7^5}{5^7} \cdot \left(\frac{5}{7}\right)^6$; 3) $\left(\frac{2}{3}\right)^3 \cdot \left(\frac{3}{2}\right)^4$; 4) $\left(\frac{3}{4}\right)^6 \cdot \left(\frac{4}{3}\right)^8$.

211. 1) $\left(\frac{35}{48}\right)^2 \cdot \left(\frac{6}{7}\right)^3 \cdot \left(1\frac{3}{5}\right)^2$; 3) $\left(\frac{5^3}{6^2}\right)^4 \cdot \left(\frac{2}{5}\right)^5 \cdot \left(\frac{3}{5}\right)^7$;

2) $\left(\frac{14}{15}\right)^4 \cdot \left(\frac{3}{7}\right)^4 \cdot (2,5)^3$; 4) $\left(\frac{7^4}{15^2}\right)^3 \cdot \left(\frac{5}{7}\right)^6 \cdot \left(\frac{3}{7}\right)^5$.

212. 1) Yerning massasi $6 \cdot 10^{24}$ kg ga teng. Quyoshning massasi $2 \cdot 10^{30}$ kg. Yerning massasi Quyoshning massasidan necha marta kam?

2) Yerdan Sirius deb nomlanuvchi yulduzgacha bo‘lgan masofa 83 000 000 000 000 km. Yorug‘lik nuri Yerdan Siriusgacha necha yilda yetib borishini taqriban hisoblang.

213. Ifodaning qiymatini toping:

1) $\frac{2-b^2}{2b}$, bunda $b=-2$; 2) $\frac{3a}{a^3-3}$, bunda $a=-3$.

214. Ifodani daraja shaklida yozing:

1) $5^{3n+4} \cdot 5^{2n-1} : 5^{n+2}$; 3) $\frac{a^{6n-4}a^{4n+1}}{a^{5n-2}}$;

2) $3^{4n+3} \cdot 3^{3n-2} : 3^{2n-1}$; 4) $\frac{b^{5n-3}b^{3n+2}}{b^{4n-1}}$ (n—natural son).

215. n ning qanday qiymatida tenglik to‘g‘ri bo‘ladi:

1) $(4^4)^n = 4^{12}$; 2) $(5^n)^2 = 5^{14}$; 3) $2^{2n} = 4^5$; 4) $3(3^2)^n = 3^{11}$?

216. Ko‘paytmani darajaga ko‘taring:

1) $(8a^2b^4c^3)^3$; 2) $(9x^4y^3z^7)^2$; 3) $(-1,2x^5y^7z^7)^2$; 4) $(-1,2a^3b^2c^4)^5$.

217. Ifodani asosi a bo‘lgan daraja shaklida yozing:

1) $\frac{a^8a^5}{a^3a^6}$; 2) $\frac{a^9a^6}{a^5a^8}$; 3) $\frac{(a^3)^4(a^4)^3}{a^6a^9}$; 4) $\frac{a^6(a^3)^5}{(a^4)^2a^9}$.

218. Sonlardan qaysi biri katta:

1) 54^4 mi yoki 21^{12} mi; 3) 100^{20} mi yoki 9000^{10} mi;

2) 10^{20} mi yoki 20^{10} mi; 4) 6^{20} mi yoki 3^{40} mi?

219. Hisoblang:

1) $\dfrac{2\cdot 5^{22}-9\cdot 5^{21}}{25^{10}}$;

2) $\dfrac{5\cdot 2^{32}-4\cdot 2^{30}}{4^{16}}$;

3) $\dfrac{(4\cdot 3^{22}+7\cdot 3^{21})\cdot 57}{(19\cdot 27^{4})^{2}}$;

4) $\dfrac{5(3\cdot 7^{15}-19\cdot 7^{14})}{7^{16}+3\cdot 7^{15}}$.

220. Tenglamani yeching:

1) $x:1{,}75=7{,}125-3\frac{1}{8}$;

2) $\frac{5}{12}+\frac{1}{18}=\frac{17}{12}x$;

3) $18{,}9:x=0{,}021\cdot 100$;

4) $754{,}5:(37{,}1+x)=15$.

221. Sonni standart shaklda yozing:

1) 26 000; 2) 8 647 000;

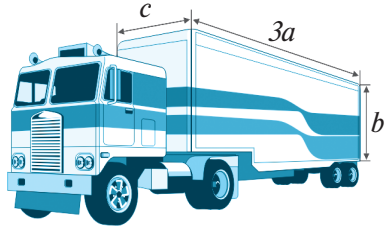
3) Yerdan Quyoshgacha bo‘lgan masofa 149 500 000 km.

11-§ Birhad va uning standart shakli

Turli masalalarni yechishda ko‘pincha ab, $\frac{1}{2}abc$, $3a^2b$ ko‘rinishdagi algebraik ifodalarga duch kelinadi. Masalan, o‘lchamlari 5- rasmda ko‘rsatilgan sovitkichli mashina sig‘imi $3abc$ ga teng.

$3abc$ ifoda birinchisi raqam bilan, qolgan uchtasi a, b, c harflari bilan belgilangan to‘rtta ko‘paytuvchining ko‘paytmasidir.

Raqamlar bilan yozilgan ko‘paytuvchilar *sonli ko‘paytuvchilar*, harflar bilan belgilangan ko‘paytuvchilar esa *harfiy ko‘paytuvchilar* deyiladi. *Sonli va harfiy ko‘paytuvchilar ko‘paytmasidan iborat algebraik ifoda birhad* deyiladi.



5- rasm.

Masalan, ushbu ifodalar birhadlardir:

$$abc,\ (-4)a\cdot 3ab,\ \frac{1}{4}a(-0{,}3)bab.$$

Teng ko‘paytuvchilar ko‘paytmasini natural ko‘rsatkichli daraja shaklida yozish mumkin bo‘lganligi uchun sonning darajasi va sonlar darajalarining ko‘paytmasi ham birhadlar deyiladi. Masalan, ushbu ifodalar birhadlar bo‘ladi:

$$\left(\frac{3}{4}\right)^2,\ (-7),\ c^5,\ 4a^2,\ \left(-\frac{1}{2}\right)a^2b.$$

Har bir sonni shu son bilan birning ko‘paytmasi shaklida yozish mumkin bo‘lgani uchun a, 2, $\frac{3}{8}$ ko‘rinishdagi ifodalar ham birhadlar deb hisoblanadi.

Masala. Birhadning qiymatini hisoblang:

$$16ac\cdot(0,5)\,a\cdot(0,25)\,b,$$

bunda $a=\frac{1}{3}$, $b=34$, $c=\frac{9}{17}$.

△ Harflarning qiymatlarini birhadga qo‘yib, uning qiymatini topamiz, ya’ni yettita sonning ko‘paytmasini hisoblaymiz:

$$16\cdot\frac{1}{3}\cdot\frac{9}{17}\cdot0,5\cdot\frac{1}{3}\cdot0,25\cdot34.$$

Sonlarning birinchisini ikkinchisiga, ular qanday yozilgan bo‘lsa, xuddi shu tartibda ko‘paytirish mumkin:

$$16\cdot\frac{1}{3}=\frac{16}{3};\ \frac{16}{3}\cdot\frac{9}{17}=\frac{48}{17};\ \frac{48}{17}\cdot0,5=\frac{24}{17};$$
$$\frac{24}{17}\cdot\frac{1}{3}=\frac{8}{17};\ \frac{8}{17}\cdot\frac{1}{4}=\frac{2}{17};\ \frac{2}{17}\cdot34=4.$$

Ko‘paytirishning o‘rin almashtirish va guruhlash qonunlarini qo‘llab, hisoblashni qisqacha bajarish ham mumkin:

$$16ac\,(0,5)\,a\,(0,25)\,b=(16\cdot0,5\cdot0,25)\,(a\cdot a)\,bc=2a^2bc.$$

Endi $a=\frac{1}{3}$, $b=34$, $c=\frac{9}{17}$ bo‘lganda $2a^2bc$ birhadning qiymatini topamiz:

$$2\cdot\left(\frac{1}{3}\right)^2\cdot34\cdot\frac{9}{17}=\frac{2\cdot34\cdot9}{9\cdot17}=4.\ \blacktriangle$$

Masalani ikkinchi usul bilan yechishda berilgan birhad ancha sodda ko‘rinishda yozilgan edi: $2a^2bc$. Bu — birhadning *standart shakliga* misol.

Umuman, birinchi o‘rinda turgan faqat bitta son ko‘paytuvchidan va har xil asosli harfiy darajalardan tuzilgan birhadni *standart shakldagi birhad* deyiladi.

Har qanday birhadni standart shaklda yozish mumkin. Buning uchun barcha son ko‘paytuvchilarni o‘zaro ko‘paytirish va ularning ko‘paytmasini birinchi o‘ringa yozish kerak. So‘ngra bir xil harfiy ko‘paytuvchilar ko‘paytmasini daraja shaklida yozish kerak. Harfiy ko‘paytuvchilar ko‘pincha, shart bo‘lmasa ham, alifbo tartibida joylashtiriladi.

Birhadning standart shaklida bir xil harflar yo‘qligini eslatib o‘tamiz.

Standart shaklda yozilgan birhadning son ko‘paytuvchisini shu *birhadning koeffitsiyenti* deyiladi.

Masalan, $2a$ birhadning koeffitsiyenti 2 ga teng; $\frac{5}{6}ab^2$ birhadning koeffitsiyenti $\frac{5}{6}$ ga teng, $(-7)a^2b^3c$ birhadning koeffitsiyenti (-7) ga teng. Oxirgi holda birhadni qavssiz yozish mumkin:

$$(-7)\,a^2b^3c = -7a^2b^3c\,.$$

1 ga teng bo‘lgan koeffitsiyent odatda yozilmaydi, chunki birga ko‘paytirgan bilan son o‘zgarmaydi. Masalan, $1\cdot abc^2=abc^2$, ya’ni abc^2 birhadning koeffitsiyenti birga teng.

Agar koeffitsiyent (-1) ga teng bo‘lsa, bu holda ham birni va qavslarni yozmasdan, faqat «—» ishorasini qoldirish mumkin. Masalan, $(-1)abc = -abc$, ya’ni $-abc$ birhadning koeffitsiyenti -1 ga teng.

Mashqlar

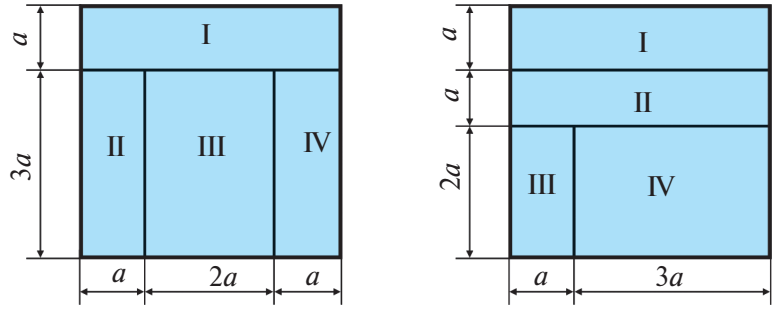
So‘z orqali aytilgan fikrni algebraik ifoda yordamida yozing **(222—224):**

222. 1) a va b sonlar ko‘paytmasining ikkilangani;
2) b va c sonlar ko‘paytmasining uchlangani;
3) x va y sonlar kvadratlarining ko‘paytmasi;
4) a son bilan b son kvadratining ko‘paytmasi.

223. 1) m son kubi bilan p sonning ko‘paytmasi;
2) a son kvadrati bilan b son ko‘paytmasining uchlangani.

224. 1) t soatdagi sekundlar soni;
2) n metrdagi santimetrlar soni.

225. Kvadrat o‘lchamlari 6- rasmda ko‘rsatilganidek to‘rtta to‘g‘ri to‘rtburchakka bo‘lingan. Shu to‘rtburchaklarning yuzlarini toping:



6- rasm.

226. Birhadning son qiymatini toping.

1) $\frac{3}{4}a^3$, bunda $a=-2$;

2) $0,5b^2$, bunda $b=-4$;

3) $3abc$, bunda $a=2,\ b=\frac{1}{2},\ c=\frac{1}{3}$;

4) $4pqr$, bunda $p=\frac{1}{2},\ q=3,\ r=\frac{1}{6}$;

5) $\frac{1}{7}m^2(-0,2)n$, bunda $m=3,\ n=-35$;

6) $\frac{1}{9}y(-0,3)x^2$, bunda $y=-15,\ x=6$.

227. Birhadni standart shaklda yozing:

1) $3m^2m$; 3) $ab\cdot 0,5$; 5) $5^2pq^2(-4)pq$;

2) z^5z^5z; 4) $(-m)(-m^3)$; 6) $2^3qp^2(-3)^2pq$.

228. Birhadni standart shaklda yozing va son qiymatini toping:

1) $ac12c$, bunda $a=-\frac{1}{3},\ c=4$;

2) $\frac{1}{6}a8b^2\ \frac{3}{4}ba^3$, bunda $a=-2,\ b=\frac{1}{2}$;

229. *(Qadimiy masala.)* Hovuzga 4 ta quvur o‘tkazilgan bo‘lib, birinchi quvur hovuzni bir kunda, ikkinchi quvur ikki kunda, uchinchi quvur uch kunda, to‘rtinchi quvur to‘rt kunda to‘ldiradi. To‘rtala quvur birgalikda hovuzni qancha vaqtda to‘ldiradi?

12-§. Birhadlarni ko‘paytirish

Quyidagi masalani yechaylik.

Masala. To‘g‘ri burchakli parallelepipedning hajmi $V = abc$ formula bo‘yicha hisoblanadi, bu yerda a — parallelepipedning bo‘yi, b — eni va c — balandligi. Agar shu parallelepipedning bo‘yini 5 marta, enini $2n$ marta, balandligini $3n$ marta uzaytirilsa, yangi parallelepipedning hajmi qanday bo‘ladi?

△ Yangi parallelepipedning o‘lchamlarini topamiz: bo‘yi $5a$, eni $2nb$, balandligi $3nc$. Bu holda uning hajmi

$$V_1 = (5a)\cdot(2nb)\cdot(3nc)$$

bo‘ladi. ▲

$(5a)\cdot(2nb)\cdot(3nc)$ ifoda quyidagi uchta birhadning ko‘paytmasidir: $5a$, $2nb$, $3nc$. Sonlarni ko‘paytirish qoidalariga ko‘ra bunday tenglikni yozish mumkin:

$$(5a)\cdot(2nb)\cdot(3nc) = 5a\cdot 2nb\cdot 3nc = (5\cdot 2\cdot 3)(annbc) = 30n^2abc.$$

Birhadlarni ko‘paytirish natijasida yana birhad hosil bo‘ladi va uni standart shaklda yozib, soddalashtirish lozim, masalan,

$$(3a^2b^3c)\cdot(4ab^2) = 3a^2b^3c\cdot 4ab^2 = 12a^3b^5c.$$

Ikki yoki bir nechta bir xil birhadlarning ko‘paytmasini, ya’ni birhadning darajasini qaraymiz, masalan: $(5a^3b^2c)^2$. Bu birhad 5, a^3b^2c ko‘paytuvchilarning ko‘paytmasi bo‘lgani uchun ko‘paytmani darajaga ko‘tarish xossasiga ko‘ra:

$$(5a^3b^2c)^2 = 5^2(a^3)^2(b^2)^2c^2 = 25a^6b^4c^2.$$

Xuddi shu kabi:

$$(2pq^2)^3 = 2^3p^3(q^2)^3 = 8p^3q^6.$$

Birhadni natural ko‘rsatkichli darajaga ko‘tarish natijasida yana birhad hosil bo‘ladi.

Birhadlarni ko‘paytiring **(230—237):**

230. 1) $(2a)(3b)$; 2) $(3a)(2b)$; 3) $b^2(-3b^3)$; 4) $(-2a)a^2$;

231. 1) $(2p)(-3c^2)$;
2) $(-5m^2)(-7n)$;
3) $(4a^2)(6a^3)$;
4) $\left(-\frac{1}{2}b^3\right)(8b^2)$.

232. 1) $\left(0,3a^2\right)\left(\frac{1}{4}b^3\right)$;
2) $(-8m^3)(0,25n)$;
3) $(0,2p)(-1,3q^2)$;
4) $\left(-\frac{3}{7}c^2\right)\left(-\frac{5}{6}b^3\right)$.

233. 1) $(3ab)\left(-2a^2b\right)$;
2) $(-4x^2y)(-7xy^2)$;
3) $\left(8ab^2\right)\left(\frac{1}{4}ac^2\right)$;
4) $(6a^2b)\left(\frac{1}{3}bc^2\right)$.

234. 1) $(3a^2b^5c)(6a^3bc^2)$;
2) $(7a^5b^2c)(-3ab^4c)$;
3) $\left(\frac{2}{3}a^2b^3x\right)\left(\frac{3}{4}a^3bx^2\right)$;
4) $\left(-\frac{3}{2}a^3xy^3\right)\left(\frac{3}{4}ax^2y\right)$.

235. 1) $(-0,4x^5y^6z^2)(-1,2xyz^3)$;
2) $(-2,5n^4m^5r^2)(3nm^2r^5)$;
3) $\left(-1\frac{1}{3}x^2y^3z\right)\left(-1\frac{1}{2}xy^2z^3\right)$;
4) $\left(2\frac{1}{4}a^2b^5c^3\right)\left(-3\frac{1}{3}a^3b^2c^4\right)$.

236. 1) $\left(-\frac{1}{3}m^2\right)(-24n)(4mn)$; 2) $(-18n)\left(-\frac{1}{6}m^2\right)(-5mn)$;
3) $\left(\frac{1}{3}ay^3\right)\left(\frac{3}{4}x^2y\right)(0,2a^3x)$; 4) $(-13a^2bc)(-5ab^2c)(-0,4abc^3)$.

237. 1) $(-a)(3b)\left(4a^2b\right)\left(5ab^2\right)$; 3) $(-1,5ab)\left(\frac{1}{4}bc\right)(2ac)(24ab)$;
2) $(5a)\left(a^2b^2\right)(-2b)(-3a)$; 4) $(1,2a^2)\left(-\frac{1}{3}ab\right)(-5bc)\left(2c^2\right)$.

Birhadni darajaga ko‘taring **(238—241):**

238. 1) $(2a)^3$; 2) $(5b)^2$; 3) $(3b^2)^4$; 4) $(2a^3)^2$.

239. 1) $(-3ab)^4$; 2) $(-4ab)^2$; 3) $(-abc)^5$; 4) $(-2xyz)^3$.

240. 1) $(-2a^2b)^3$; 2) $(-a^2bc)^5$; 3) $(-3x^3y)^2$; 4) $(-2x^2y^3)^4$.

241. 1) $\left(\frac{1}{2}nm^2\right)^3$; 2) $\left(\frac{1}{3}n^2m^2\right)^4$; 3) $(-0,1a^3b^3)^3$; 4) $(0,4a^3b^2)^2$.

Amallarni bajaring **(242—243):**

242. 1) $(-2a)^3\left(-3a\right)$;

2) $(-a)^3\left(2a\right)$;

3) $(-0,2bc^2)^2\left(20cx^2\right)$;

4) $(-0,1ab^2c)^2\left(100by^2\right)$.

243. 1) $\left(-1\frac{3}{5}x^3y^2\right)\left(-\frac{1}{2}c^2x^2\right)^3$;

2) $\left(2\frac{1}{4}x^3y\right)\left(\frac{2}{3}xy^2\right)^2$;

3) $(-3bc^2)^3\left(2ab^2\right)^2$;

4) $(-2a^2b)^2\left(-a^2b^3\right)^3$.

244. Birhadni boshqa birhadning kvadrati shaklida yozing:

1) $9a^2$; 2) $16x^4$; 3) $25a^2b^4$;

4) $81x^6y^2$; 5) $36x^{10}y^4$; 6) $1,21a^8b^4$.

245. Birhadlarni ko‘paytiring va hosil bo‘lgan ifodaning qiymatini toping:

1) $\frac{1}{3}a^2\cdot 3a^2b$, bunda $a=-2,\ b=\frac{5}{7}$;

2) $\frac{2}{5}mn\cdot 10n^2$, bunda $m=0,8,\ n=4$;

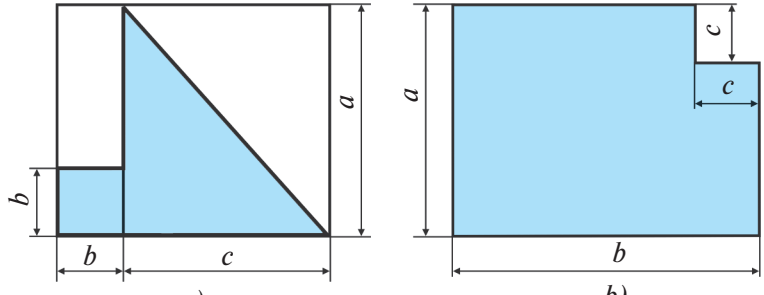
3) $4a\cdot\frac{1}{16}a^2b^2c$, bunda $a=4,\ b=\frac{1}{4};\ c=3$;

4) $0,7m^2n\cdot 100np$, bunda $m=0,3,\ n=-0,2,\ p=4$.

246. *(Qadimiy masala.)* Baliqning uchdan bir qismi loyda, to‘rtdan bir qismi suv tagida va uch qarichi suv ustida. Baliqning uzunligi necha qarich?

13-§ Ko‘phadlar

Algebrada ko‘pincha birhadlarning yig‘indisi yoki ayirmasidan iborat bo‘lgan algebraik ifodalar qaraladi. Masalan, 7- *a* rasmda tasvirlangan shaklning shtrixlangan qismining yuzi $\frac{1}{2}ac + b^2$ ga teng, 7- *b* rasmda tasvirlangan shaklning yuzi esa $ab - c^2$ ga teng. $\frac{1}{2}ac + b^2$ ifoda ushbu ikkita birhadning yig‘indisi: $\frac{1}{2}ac$ va b^2; $ab - c^2$ ifoda ab va c^2 birhadlarning ayirmasi yoki ab va $(-c^2)$ birhadlarning yig‘indisi. Bu ifodalar birhadlarning algebraik yig‘indisi bo‘ladi. Bunday ifodalar *ko‘phadlar* deyiladi.



7-rasm.

Bir nechta birhadning algebraik yig‘indisi ko‘phad deyiladi. Ko‘phadni tashkil qiluvchi birhadlar shu *ko‘phadning hadlari* deyiladi.

Masalan, $5nm^2 - 3m^2k - 7nk^2 + 4nm$ ko‘phadning hadlari $5nm^2$, $-3m^2k$, $-7nk$, $4nm$ bo‘ladi.

Ikkita haddan tuzilgan ko‘phad *ikkihad* deyiladi, uchta haddan tuzilgan ko‘phad *uchhad* deyiladi va hokazo.

Ikkihadga misollar: $a^2 - b^2$, $5ab + 4c$.

Uchhadga misollar: $a + 2b - 3c$, $\frac{1}{2} - bc + 3ab$.

Birhadni ham ko‘phad deb hisoblaymiz.

Agar ko‘phadning ba’zi hadlari standart shaklda yozilmagan bo‘lsa, u holda shu ko‘phadning barcha hadlarini standart shaklda yozib, uni soddalashtirish mumkin.

Masala. $2a4b-5abac+9bc\frac{1}{3}c$ ko‘phadni soddalashtiring.

△ Berilgan ko‘phadning barcha hadlarini standart shaklda yozamiz:

$$2a4b=8ab;\ -5abac=-5a^2bc;\ 9bc\frac{1}{3}c=3bc^2.$$

Demak, $2a4b-5abac+9bc\frac{1}{3}c=8ab-5a^2bc+3bc^2$. ▲

Mashqlar

247. Ko‘phadni tashkil qiluvchi birhadlarni ayting:

1) $-2x^2+3x-1$;
2) $4x^2-3x+6$;
3) $7a^2-\frac{1}{3}b-\frac{2}{5}c$;
4) $-3a+0,5x-2x^2$.

248. Ko‘phadni birhadlarning yig‘indisi shaklida yozing:

1) $7a^4-9a^3-2a+11$;
2) $-6x^5+3x^4-12x^2+5$;
3) $1,6a^3b-4a^2b^2+13ab^3-b^4$;
4) $2,5x^4-18x^3y-16x^2y-3xy^2$.

249. Birhadlardan ko‘phad tuzing:

1) $6x^2$, $7x$ va 9;
2) $2x^2$, $-11x$ va 3;
3) $-x^4$, x^3 va $-x$;
4) a^5, $-a^4$ va a;
5) $8a$, $4a^2b$, $-2ab^2$ va b^3;
6) $4a^3b$, $-2a^2b^2$, $-5ab^3$.

250. Ko‘phadni, uning har bir hadini standart shaklga keltirib, soddalashtiring:

1) $12a^23ba-2ab3ab^2+11aba$;
2) $2ab^24ab-3a^28aba-2ababb^2$;
3) $1,5xy^2(-4)xyz-4mnk5m^2nk$;
4) $4cc^2c\left(-\frac{1}{4}\right)bc+5xy^2xy^2$.

251. Ifodani, uning har bir qo‘shiluvchisini standart shaklga keltirib, soddalashtiring:

1) $3aaa\left(-1\frac{2}{3}ab\right)+4xxx3xy$;

2) $1,5yyy(-4xyz)-4mnk\cdot 5m^2nk^2$;

3) $(2ab)\left(\frac{1}{4}a^2b^2\right)-(3a^2b)\left(\frac{1}{9}b\right)$;

4) $(3a)\left(\frac{1}{9}ab^2\right)-(4b^2)\left(\frac{1}{2}a^2b\right)$.

252. Ko‘phadning son qiymatini toping:

1) $2a^3+3ab+b^2$, bunda $a=0,5$, $b=\frac{2}{3}$;

2) $2a^4-ab+2b^2$, bunda $a=-1$, $b=-0,5$;

3) $x^2-2xy+y^2$, bunda $x=y=-4,2$;

4) $x^2+2xy+y^2$, bunda $x=1,2$, $y=-1,2$.

253. Ko‘phadni soddalashtiring va uning son qiymatini toping:

1) $-aba+a^2b2ab^2+4$, bunda $a=2$, $b=\frac{1}{2}$;

2) $b^25ab-5a5a^2b$, bunda $a=\frac{1}{5}$, $b=-2$;

3) $x^2yxy-xy^2xy+xy$, bunda $x=-3$, $y=2$;

4) $xy^2x^2y-xyxy$, bunda $x=-2$, $y=3$.

14- §. O‘xshash hadlarni ixchamlash

Ushbu masalani yechaylik.

1-masala. Har bir sahifasida bir xil sondagi harflar bo‘lgan ikkita kitob bor; har bir sahifadagi satrlar soni n ta va har bir satrdagi harflar soni m ta. Birinchi kitob 300 sahifalik, ikkinchisi 500 sahifalik. Ikkala kitobda hammasi bo‘lib nechta harf bor?

1- usul. Har bir sahifadagi harflar soni mn ta. Birinchi kitobda 300 nm ta harf, ikkinchisida 500nm ta harf, ikkalasida esa

$$300nm+500\ nm=800nm$$

ta harf bor.

2- usul. Har bir sahifadagi harflar soni mn ga teng. Ikkala kitobdagi sahifalar soni 300+500=800 ga, ulardagi harflar soni 800nm ga teng.

Ikkala javob ham to‘g‘riligi ko‘rinib turibdi, shuning uchun

$$300nm+500\ nm=800nm.$$

Ammo hisoblashlarda ikkinchi usul ancha qulay bo‘ladi. Masalan, agar $n = 40$, $m = 50$ bo‘lsa, u holda $nm = 2\,000$ va $300nm+500nm$ ifodani hisoblash uchun yana uchta hisoblashni bajarish kerak:

$$300 \cdot 2000 + 500 \cdot 2000 = 600\,000 + 1\,000\,000 = 1\,600\,000.$$

$800nm$ ifodani hisoblash uchun esa bor-yo‘g‘i bitta amalni bajarish kerak, xolos: $800 \cdot 2000 = 1\,600\,000$.

Mana shuning uchun ham algebraik ifodalarni soddalashtirishni bilish muhim ahamiyatga ega.

$300nm+500nm$ ikkihad ikkita birhadning yig‘indisidan iborat:

$$300nm \text{ va } 500nm.$$

Bu birhadlar bir-biridan faqat koeffitsiyentlari bilan farq qiladi. Bunday birhadlarni *o‘xshash birhadlar* deyiladi. Masalan, abc va $3abc$ birhadlar o‘xshash, $2pq^2$ va $5q^2p$ birhadlar ham o‘xshash, lekin a^2b va ab^2 birhadlar o‘xshash emas.

Bir xil birhadlarni ham o‘xshash deb hisoblaymiz. Masalan, $2a^2b$ va $2a^2b$ birhadlar o‘xshash.

2 - masala. $3ab - 2bc + 4ac - ab + 3bc + 4ab$ ko‘phadni soddalashtiring.

△ O‘xshash birhadlarni ajratamiz: $3ab$, $-ab$, $4ab$ birhadlar o‘xshash, ularning tagiga bittadan chiziq chizamiz, $-2bc$ va $3bc$ o‘xshash birhadlarning tagiga ikkitadan chiziq chizamiz. $4ac$ birhadga o‘xshash had yo‘q, uning tagiga chizmaymiz, ya’ni

$$\underline{3ab} - \underline{\underline{2bc}} + 4ac - \underline{ab} + \underline{\underline{3bc}} + \underline{4ab}.$$

Ko‘phad hadlarining o‘rinlarini o‘xshash hadlar yonma-yon turadigan qilib almashtiramiz va o‘xshash hadlarni qavs ichiga olamiz:

$$(3ab - ab + 4ab) + (-2bc + 3bc) + 4ac.$$

Ammo

$$3ab - ab + 4ab = (3 - 1 + 4)ab = 6ab,$$
$$-2bc + 3bc = (-2 + 3)bc = bc$$

bo‘lgani uchun

$$3ab - 2bc + 4ac - ab + 3bc + 4ab = 6ab + bc + 4ac. \blacktriangle$$

Ko‘phadlarni o‘xshash birhadlar algebraik yig‘indisi bitta birhad bilan almashtiriladigan bunday soddalashtirish *o‘xshash hadlarni ixchamlash* deyiladi.

$6ab + bc + 4ac$ ko‘phadda har bir had standart shaklda yozilgan va ular orasida o‘xshash hadlar yo‘q. Ko‘phadning bunday shakli *standart shakl* deyiladi.

Har qanday ko‘phadni standart shaklda yozish mumkin. Buning uchun avval ko‘phadning har bir hadini standart shaklda yozish va so‘ngra o‘xshash hadlarni ixchamlash kerak.

3 - masala. Ko‘phadni standart shaklga keltiring:

$$6ab\frac{1}{3}ac - 3aca - 8a^2\frac{1}{2}b + 25a^2\frac{1}{5}c + aba - a^2bc.$$

$$\triangle\ 6ab\frac{1}{3}ac - 3aca - 8a^2\frac{1}{2}b + 25a^2\frac{1}{5}c + aba - a^2bc =$$

$$= \underline{2a^2bc} - \underline{\underline{3a^2c}} - \underline{\underline{\underline{4a^2b}}} + \underline{\underline{5a^2c}} + \underline{\underline{\underline{a^2b}}} - \underline{a^2bc} =$$

$$= (2a^2bc - a^2bc) + (-3a^2c + 5a^2c) + (-4a^2b + a^2b) =$$

$$= a^2bc + 2a^2c - 3a^2b. \blacktriangle$$

Mashqlar

O‘xshash hadlarni ixchamlang **(254—255):**

254. 1) $\frac{1}{3}x + \frac{1}{2}x + \frac{1}{6}x$;

2) $\frac{5}{6}y - \frac{1}{3}y - \frac{1}{6}y$;

3) $\frac{3}{2}y^4 - \frac{1}{16}y^4 + \frac{1}{32}y^4 - \frac{1}{4}y^4$;

4) $\frac{3}{2}a^2b - \frac{5}{8}a^2b + \frac{1}{8}a^2b - \frac{3}{16}a^2b$.

255. 1) $2m + q + q - 4m$;

2) $3a + 2b - b - a$;

3) $x^2 + 3y^2 + 4x - y^2$;

4) $5a^2 - 4b^2 - 3a^2 + b^2$.

Ko‘phadni standart shaklga keltiring **(256—261):**

256. 1) $11x^2 + 4x - x^2 - 4x$;

2) $2y^2 - 3y + 2y - 2y^2$;

3) $0,3c^2 - 0,1c^2 - 0,5c^3$;

4) $1,2a^2 + 3,4a^2 - 0,8a^2$.

257. 1) $\frac{1}{3}x^2-\frac{1}{3}y+\frac{2}{3}x^2+\frac{1}{3}y$; 2) $\frac{1}{5}a^2+\frac{3}{4}b^2+\frac{4}{5}a^2-\frac{3}{4}b^2$;

3) $2ab+0,7b^2-5ab+1,2b^2+8ab$;

4) $5xy-3,5y^2-2xy+1,3y^2-xy$.

258. 1) $-\frac{3}{4}xy+\frac{2}{3}x^2y+xy-\frac{5}{6}x^2y-\frac{1}{2}xy$;

2) $\frac{1}{2}ab^2-\frac{7}{8}ab^2+\frac{3}{4}a^2b-\frac{3}{8}a^2b-\frac{1}{2}ab^2$;

3) $-9,387a-3,89b+8,197a-1,11b-0,81a$;

4) $8,53x-4,73y-5,12x+2,27y+0,59x$.

259. 1) $2a^2b-8b^2+5a^2b+5c^2-3b^2+4c^2$;

2) $3xy^2+4x^3-5x^2y-3x^3+4x^2y-9xy^2$;

3) $\frac{1}{7}ab+\frac{3}{8}a^2-\frac{2}{5}b^3+\frac{6}{7}ab-\frac{3}{8}a^2+\frac{3}{5}b^3$;

4) $\frac{3}{5}ab^2-\frac{2}{3}ab+\frac{1}{4}a^3+\frac{8}{3}ab+\frac{2}{5}ab^2-\frac{3}{4}a^3+\frac{1}{2}a^3$.

260. 1) $5b3b-4c3b-5b2c-4c\left(-2\right)c$;

2) $b8b-3c8b+5cb-3c5c$;

3) $6a^22a^2+5b^22a^2-6a^24b^2-5b^24b^2$;

4) $2x^2\frac{1}{2}y-\frac{1}{3}ab3a+1\frac{1}{4}y\frac{4}{5}x^2+aab$.

261. 1) $-9a^2\frac{1}{3}b+a^2b+24a^2\frac{1}{4}c$;

2) $2ab\frac{1}{3}ac-4aca-a^2bc$;

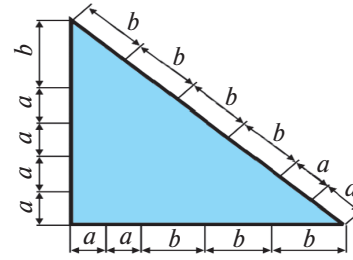
3) $4x^2\frac{1}{2}y-\frac{1}{3}ab9a+4y\frac{4}{5}x^2+aba$;

4) $5a\frac{1}{2}b+\frac{2}{3}a\left(\frac{1}{4}b^2\right)-5b\left(0,5a\right)-\frac{1}{3}a^2\left(\frac{1}{15}ab\right)$.

15-§ Ko‘phadlarni qo‘shish va ayirish

O‘lchamlari 8- rasmda ko‘rsatilgan uchburchakni qaraymiz. Uning P perimetri tomonlar uzunliklarining yig‘indisiga teng:

$$P = (2a + 3b) + (4a + b) + (2a + 4b).$$



8- rasm.

Bu ifoda quyidagi uchta ko‘phadning yig‘indisidir:

$$2a + 3b,\quad 4a + b,\quad 2a + 4b.$$

Qavslarni ochish qoidasiga ko‘ra bunday yozish mumkin:

$$P = 2a + 3b + 4a + b + 2a + 4b.$$

O‘xshash hadlarni ixchamlasak,

$$P = 8a + 8b$$

tenglik hosil bo‘ladi.

Ko‘phadlarning istalgan algebraik yig‘indisi ham xuddi shunga o‘xshash soddalashtiriladi, masalan,

$$(2n^2 - m^2) - (n^2 - m^2 + 3q^2) = 2n^2 - m^2 - n^2 + m^2 - 3q^2 = n^2 - 3q^2;$$

$$(3ab - 4bc) + (bc - ab) - (ac - 3bc) =$$

$$= 3ab - 4bc + bc - ab - ac + 3bc = 2ab - ac.$$

Bir nechta ko‘phadlarni qo‘shish va ayirish natijasida yana ko‘phad hosil bo‘ladi.

Bir nechta ko‘phadning algebraik yig‘indisini standart shakldagi ko‘phad ko‘rinishida yozish uchun qavslarni ochish va o‘xshash hadlarni ixchamlash kerak.

Ba’zi ko‘phadlarning yig‘indisi yoki ayirmasini sonlarni qo‘shish va ayirishga o‘xshash «ustun» usulida topish qulay bo‘ladi. Bunda o‘xshash hadlar birining ostiga ikkinchisi turadigan qilib yoziladi, masalan,

1) $+\begin{array}{r} 5a-4bc+3ac \\ 3bc-7ac \\ \hline 5a-bc-4ac \end{array}$;

2) $-\begin{array}{r} 5abc-2ab+4ac-bc \\ 3abc-3ab-ac+3bc \\ \hline 2abc+ab+5ac-4bc \end{array}$.

Mashqlar

Ko‘phadlarning algebraik yig‘indisini toping **(262—267):**

262. 1) $8a+(-3b+5a)$;
2) $5x-(2x-3y)$;
3) $(6a-2b)-(5a+3b)$;
4) $(4x+2)+(-x-1)$.

263. 1) $3x^2-(4x^2+2y)$;
2) $2a^2-(b^2-3a^2)$;
3) $0,6a^2-(0,5a^2-0,4a)$;
4) $1\frac{1}{2}b^2-\left(2b^2-1\frac{1}{4}\right)$.

264. 1) $\left(2\frac{3}{5}b-\frac{3}{4}b^2\right)+\left(\frac{1}{4}b^2-1\frac{3}{5}b\right)$;
2) $(0,1c-0,4c^2)-(0,1c-0,5c^2)$;
3) $(13x-11y+10z)-(-15x+10y-15z)$;
4) $(17a+12b-14c)-(11a-10b-14c)$.

265. 1) $(7m^2-4mn-n^2)-(2m^2-mn+n^2)$;
2) $(5a^2-11ab+8b^2)+(-2b^2-7a^2+5ab)$;
3) $11ac+13bc+17b^2-(10ac+10bc-3b^2)$;
4) $41z+13az+26az^2-(16z+13az-4az^2)$.

266. 1) $\left(\frac{1}{2}a+\frac{1}{3}b\right)-\left(\frac{5}{2}a-\frac{2}{3}b\right)+(a+b)$;
2) $(0,3a-1,2b)+(a-b)-(1,3a-0,2b)$;
3) $11p^3-2p^2-(p^3-p^2)+(-5p^2-3p^3)$;
4) $5x^2+6x^3+(x^3-x^2)-(-2x^3+4x^2)$.

267. 1) $\left(-2x^3+xy^2\right)+\left(x^2y-1\right)+\left(x^2y-xy^2+3x^3\right)$;

2) $\left(3x^2+5xy+7x^2y\right)-\left(5xy+3x^2\right)-\left(7x^2y-3x^2\right)$;

3) $\left(8a^2-10ab-b^2\right)+\left(-6a^2+2ab-b^2\right)-\left(a^2-8ab+4b^2\right)$;

4) $\left(4a^2-2ab-b^2\right)-\left(-a^2+b^2-2ab\right)+\left(3a^2+b^2-ab\right)$.

268. Ko‘phadlarning yig‘indisi va ayirmasini toping:

1) $0,1x^2+0,02y^2$ va $0,17x^2-0,08y^2$;

2) $0,1x^2-0,02y^2$ va $-0,17x^2+0,08y^2$;

3) $a^3-0,12b^3$ va $0,39a^3-b^3$;

4) $a^3+0,12b^3$ va $-0,39a^3+b^3$.

269. Ko‘phadlarning yig‘indisini «ustun» usulida toping:

1) $3ab+a^2-2b^2$ va $2a^2-3ab$;

2) $3x^2+2xy-4y^2$ va $4y^2-2xy+3x^2y^2-x^3$.

270. Ko‘phadlarning ayirmasini «ustun» usulida toping:

1) $3a^2+8a-4$ va $3+8a-5a^2$;

2) b^3-3b^2+4b va $b+2b^2+b^3$.

271. 1) Agar $P=5a^2+b$, $Q=-4a^2-b$ bo‘lsa, $P+Q$ ifoda nimaga teng?

2) Agar $P=2p^2-3q^3$, $Q=2p^2-4q^3$ bo‘lsa, $P-Q$ ifoda nimaga teng?

3) Agar $A=a^2-b^2+ab$, $B=2a^2+3ab-5b^2$, $C=-4a^2+2ab-3b^2$ bo‘lsa, $A+B+C$ ni toping;

4) Agar $A=2a^2-3ab+4b^2$, $B=3a^2+4ab-b^2$, $C=a^2+2ab+3b^2$ bo‘lsa, $A-B+C$ ni toping.

272. Isbotlang:

1) beshta ketma-ket natural sonning yig‘indisi 5 ga bo‘linadi;
2) to‘rtta ketma-ket natural sonning yig‘indisi 4 ga bo‘linmaydi;
3) to‘rtta ketma-ket toq natural sonning yig‘indisi 8 ga bo‘linadi;
4) to‘rtta ketma-ket juft natural sonning yig‘indisi 4 ga bo‘linadi.

273. Avtobusda n nafar yo‘lovchi bor edi. Dastlabki ikki bekatning har birida m nafardan yo‘lovchi avtobusdan tushdi, uchinchi

bekatda esa hech kim tushmadi, lekin bir necha kishi avtobusga chiqdi, shundan so‘ng avtobusdagi yo‘lovchilar soni k nafar bo‘ldi. Uchinchi bekatda avtobusga necha kishi chiqqan?

16- §. Ko‘phadni birhadga ko‘paytirish

9-rasm

O‘lchamlari 9- rasmda ko‘rsatilgan to‘g‘ri burchakli parallelepipedni qaraymiz. Uning hajmi asosining yuzi bilan balandligining ko‘paytmasiga teng:

$$(a + 2b + c)(3ab).$$

Bu ifoda $a + 2b + c$ ko‘phad bilan $3ab$ birhadning ko‘paytmasi bo‘ladi.

Ko‘paytirishning taqsimot qonunini qo‘llab, bunday yozish mumkin:

$$(a + 2b + c)(3ab) = a(3ab) + 2b(3ab) + c(3ab) = 3a^2b + 6ab^2 + 3abc.$$

Istalgan ko‘phadni birhadga ko‘paytirish ham xuddi shunday bajariladi, masalan:

$$\left(2n^2m - 3nm^2\right)\left(-4nm\right) = \left(2n^2m\right)\left(-4nm\right) + \left(-3nm^2\right)\left(-4nm\right) = -8n^3m^2 + 12n^2m^3;$$

$$\left(3a^2 - 4ab + 5c^2\right)\left(-5bc\right) = 3a^2\left(-5bc\right) - 4ab\left(-5bc\right) + 5c^2\left(-5bc\right) = -15a^2bc + 20ab^2c - 25bc^3.$$

Ko‘phadni birhadga ko‘paytirish uchun ko‘phadning har bir hadini shu birhadga ko‘paytirish va hosil bo‘lgan ko‘paytmalarni qo‘shish kerak.

Ko‘phadni birhadga ko‘paytirish natijasida yana ko‘phad hosil bo‘ladi. Hosil bo‘lgan ko‘phadni uning barcha hadlarini standart shaklda yozib, soddalashtirish kerak. Oraliqdagi natijalarni yozmasdan, birhadlarni og‘zaki ko‘paytirib, birdaniga javobni yozish ham mumkin, masalan,

$$\left(-3ab+2a^2-4b^2\right)\left(-\frac{1}{2}ab\right)=\frac{3}{2}a^2b^2-a^3b+2ab^3.$$

Birhadni ko‘phadga ko‘paytirish ham shunga o‘xshash bajariladi, chunki ko‘paytuvchilarning o‘rinlarini almashtirish bilan ko‘paytma o‘zgarmaydi, masalan, $4pq(3p^2-q+2)=12p^3q-4pq^2+8pq$.

Mashqlar

Ko‘phad va birhad ko‘paytmasini toping **(274—278):**

274. 1) $-5(10+m)$;
2) $-\frac{1}{2}(-2+x)$;
3) $(2y-5)\left(-\frac{1}{7}\right)$;
4) $(-2m+3n)(-10)$;
5) $2\left(3a^2-4a+8\right)$;
6) $\left(-\frac{1}{3}\right)(m-n+p)$;
7) $(3a-5b+bc)(-3)$;
8) $(-5)\left(3x^3+7x^2-x\right)$.

275. 1) $(a-b)n$;
2) $(-5x+4y)2z$;
3) $-6x(5y-2x)$;
4) $\left(x^2-x+1\right)x$.

276. 1) $7ab(2a+3b)$;
2) $5a^2b(15b+3)$;
3) $12p^2q\left(q^2p-q^2\right)$;
4) $3xy^2\left(xy-2x^3\right)$.

277. 1) $17a(5a+6b-3ab)$;
2) $8ab\left(2b-3ac+c^2\right)$;
3) $3x^2y(5x+6y+7z)$;
4) $xyz\left(x^2+2y^2+3z^2\right)$.

278. 1) $\left(\frac{1}{2}a^3b^2-\frac{3}{4}ab^4\right)\frac{4}{3}a^3b$;
2) $\left(\frac{2}{3}a^2b^4+\frac{1}{2}a^3b\right)\frac{3}{2}ab^3$;
3) $\left(1\frac{4}{7}a^3x^3-2\frac{3}{4}a^2x^3-11ax^4\right)\left(-2\frac{6}{11}ax^6\right)$.

Ifodani soddalashtiring **(279—281):**

279. 1) $6(2t-3n)-3(3t-2n)$;
2) $5(a-b)-4(2a-3b)$;
3) $-2(3x-2y)-5(2y-3x)$;
4) $7(4p+3)-6(5+7p)$.

280. 1) $(x^2-1)3x-(x^2-2)2x$;

2) $(4a^2-3b)2b-(3a^2-4b)3b$;

3) $2(3a+4)+3(a-7)-7(2a-7)$;

4) $3(2x-1)-5(x-3)+6(3x-4)$.

281. 1) $5(0,8y-0,1)-0,7(4y+1)+8(0,7-0,4y)$;

2) $3\left(\frac{1}{2}x-1\frac{1}{2}\right)+2\left(\frac{1}{4}x+\frac{1}{2}\right)$; 3) $\frac{5}{4}\left(\frac{1}{5}x-\frac{1}{5}\right)-\frac{4}{5}\left(\frac{1}{4}x-\frac{3}{4}\right)$;

4) $0,2(5y+6)-4(0,25y-1,3)+5(0,1y-1,62)$.

282. Algebraik ifodaning qiymatini toping:

1) $7(4a+3b)-6(5a+7b)$, bunda $a=2, b=-3$;

2) $a(2b+1)-b(2a-1)$, bunda $a=10,\ b=-5$;

3) $3ab(4a^2-b^2)+4ab(b^2-3a^2)$, bunda $a=10, b=-5$;

4) $4a^2(5a-3b)-5a^2(4a+b)$, bunda $a=-2, b=-3$.

17-§ Ko‘phadni ko‘phadga ko‘paytirish

Ushbu masalani qaraylik.

Masala. O‘lchamlari 10- rasmda ko‘rsatilgan shkaflar bilan band devor sirtining yuzini toping.

△ Shkaflar bilan band bo‘lgan devorning sirti tomonlari

$$2a+c+2a=4a+c \text{ va } a+b+a=2a+b$$

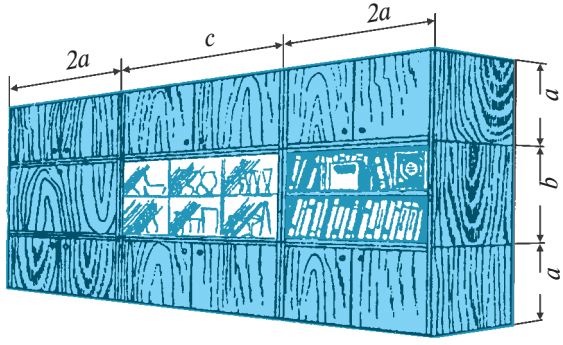
bo‘lgan to‘g‘ri to‘rtburchakdan iborat. Bu to‘g‘ri to‘rtburchakning yuzi $S=(4a+c)(2a+b)$ ga teng. ▲

$(4a+c)(2a+b)$ ifoda $(4a+c)$ va $(2a+b)$ ko‘phadlarning ko‘paytmasidir.

Sonlarni ko‘paytirishning taqsimot qonunini qo‘llab,

$$S = (4a + c)(2a + b) = 4a(2a + b) + c(2a + b)$$

kabi yozish mumkin. So‘ngra, $4a(2a + b) = 8a^2 + 4ab$ va $c(2a + b) = 2ac + bc$ bo‘lgani uchun $S = 8a^2 + 4ab + 2ac + bc$.



10- rasm.

Shunday qilib, mazkur ko‘phadlarning ko‘paytmasini topish uchun $4a + c$ ko‘phadning har bir hadini $2a + b$ ko‘phadning har bir hadiga ko‘paytirish va natijalarni qo‘shishga to‘g‘ri keldi. Ixtiyoriy ikkita ko‘phadni ko‘paytirish ham xuddi shunday bajariladi, masalan,

$$(7n - 2m)(3n - 5m) = (7n) \cdot (3n) + (7n) \cdot (-5m) + (-2m) \cdot (3n) +$$

$$+ (-2m) \cdot (-5m) = 21n^2 - 35nm - 6mn + 10m^2 = 21n^2 - 41nm + 10m^2.$$

Ko‘phadni ko‘phadga ko‘paytirish uchun birinchi ko‘phadning har bir hadini ikkinchi ko‘phadning har bir hadiga ko‘paytirish va hosil bo‘lgan ko‘paytmalarni qo‘shish kerak.

Ko‘phadni ko‘phadga ko‘paytirish natijasida yana ko‘phad hosil bo‘ladi. Hosil qilingan ko‘phadni standart shaklda yozish kerak. Bunda birhadlarni ko‘paytirishni og‘zaki bajarib, oraliq natijalarni yozmaslik mumkin, masalan,

$$(2a - 4b + 3c)(5b - c) = 10ab - 2ac - 20b^2 + 4bc +$$

$$+ 15bc - 3c^2 = 10ab - 2ac - 20b^2 + 19bc - 3c^2.$$

Bir nechta ko‘phadni ko‘paytirishni navbatma-navbat bajarish kerak, masalan,

$$(a+b)(a+2b)(a-3b)=\left(a^2+3ab+2b^2\right)(a-3b)=$$

$$=a^3-3a^2b+3a^2b-9ab^2+2ab^2-6b^3=a^3-7ab^2-6b^3.$$

Mashqlar

Ko‘phadlarni ko‘paytiring **(283—291):**

283. 1) $(a+2)(a+3)$; 3) $(m+6)(n-1)$;
2) $(z-1)(z+4)$; 4) $(b+4)(c+5)$.

284. 1) $(c-4)(d-3)$; 3) $(x+y)(x+1)$;
2) $(a-10)(-a-2)$; 4) $(-p+q)(-1-q)$.

285. 1) $(2x+1)(x+4)$; 3) $(3m-2)(2m-1)$;
2) $(2a+3)(5a-4)$; 4) $(5p-3q)(4p-q)$.

286. 1) $\left(\frac{1}{2}a+3b\right)\left(\frac{1}{2}a-3b\right)$; 3) $\left(\frac{1}{3}a-2b\right)\left(\frac{1}{3}a+2b\right)$;
2) $(0,3-m)(m+0,3)$; 4) $(0,2a+0,5x)(0,2a-0,5x)$.

287. 1) $\left(a^2+b\right)\left(a+b^2\right)$; 3) $\left(a^2+2b\right)\left(2a+b^2\right)$;
2) $\left(5x^2-6y^2\right)\left(6x^2-5y^2\right)$; 4) $\left(x^2+2x+1\right)(x+3)$.

288. 1) $(2a-b)\left(4a^2+2ab+b^2\right)$;
2) $(3a-2b)\left(9a^2+6ab+4b^2\right)$;
3) $(5x+3y)\left(25x^2-15xy+9y^2\right)$;
4) $(3a+2b)\left(9a^2-6ab+4b^2\right)$.

289. 1) $(5c-4y)(-8c-2x+6y)$; 3) $(4x-3y+2z)(3x-3y)$;
2) $(4b-c)(-5b+3c-4y)$; 4) $(3a-3b+4c)(3a-5b)$.

290. 1) $(0,2x+0,2y-z)(x-y)$; 3) $\left(\frac{1}{3}m-\frac{1}{3}n+\frac{1}{5}p\right)(60m+12)$;

2) $(0,3x-0,3y+z)(x+y)$; 4) $(0,1a^2-0,3a+1)(3a^2-10)$.

291. 1) $(a-b)(a+b)(a-3b)$; 3) $(x+3)(2x-1)(3x+2)$;

2) $(a+b)(a-b)(a+3b)$; 4) $(x-2)(3x+1)(4x-3)$.

292. 1) $(5x-1)(x+3)-(x-2)(5x-4)$ ifodaning qiymati $x=2\frac{1}{7}$ bo‘lganda 49 ga tengligini ko‘rsating;

2) $(a+3)(9a-8)-(2+a)(9a-1)$ ifodaning qiymati $a=-3,5$ bo‘lganda -29 ga tengligini ko‘rsating.

293. Ifodaning qiymatini hisoblang:

1) $\left(n+\frac{1}{2}\right)\left(n^2-\frac{1}{2}n+\frac{1}{4}\right)$, bunda $n=-2\frac{1}{2}$;

2) $\left(n-\frac{1}{3}\right)\left(n^2+\frac{1}{3}n+\frac{1}{9}\right)$, bunda $n=\frac{7}{3}$.

294. 1) $ABCD$ to‘g‘ri to‘rtburchakning (11- rasm) yuzi

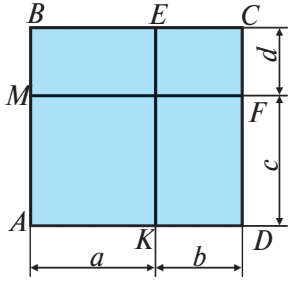
$$(a+b)(c+d)=ac+bc+ad+bd$$

ekanligini ko‘rsating.

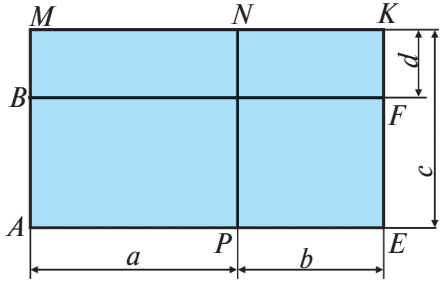
2) $ABFE$ to‘g‘ri to‘rtburchakning (12- rasm) yuzi

$$(a+b)(c-d)=ac+bc-ad-bd$$

ekanligini ko‘rsating.



11- rasm.



12- rasm.

18-§ Birhad va ko‘phadni birhadga bo‘lish

Bir nechta birhad va ko‘phadlarni qo‘shish, ayirish, ko‘paytirish va natural ko‘rsatkichli darajaga ko‘tarish natijasida yana ko‘phad hosil bo‘lishi oldingi paragraflarda ko‘rsatildi. Sanab o‘tilgan bu amallar ichida bo‘lish amali uchramadi. Bo‘lish amalini o‘z ichiga olgan ifodalar V bobda batafsil qaraladi. Ba’zan bo‘lish natijasida ham ko‘phad hosil bo‘ladi.

1. Birhadni birhadga bo‘lish

Masala. $32a^3b^2$ birhadni $4a^2$ birhadga bo‘ling.

Sonni sonlar ko‘paytmasiga bo‘lish xossasidan foydalanamiz: sonni ko‘paytmaga bo‘lishda shu sonni ko‘paytmaning birinchi ko‘paytuvchisiga bo‘lish kerak, so‘ngra hosil bo‘lgan natijani ikkinchi ko‘paytuvchiga bo‘lish kerak va hokazo. Natijada

$$\left(32a^3b^2\right) : \left(4a^2\right) = \left(\left(32a^3b^2\right) : 4\right) : a^2.$$

Endi ushbu qoidani qo‘llaymiz: *ko‘paytmani songa bo‘lishda ko‘paytmaning ko‘paytuvchilaridan birini shu songa bo‘lish kerak*. U holda

$$\left(32a^3b^2\right) : 4 = (32 : 4)\, a^3b^2 = 8a^3b^2;$$

$$\left(8a^3b^2\right) : a^2 = \left(8a^3 : a^2\right) b^2 = 8ab^2.$$

Shunday qilib,

$$\left(32a^3b^2\right) : \left(4a^2\right) = 8ab^2.$$

Birhadlar boshqa hollarda ham xuddi shunday bo‘linadi, masalan,

$$4a^2b^3 : \left(4a^2b^3\right) = 1;$$

$$\left(66a^4b^2c\right) : \left(22a^2b\right) = 3a^2bc;$$

$$\left(9k^2n^2m^2\right) : \left(-3kn^2m^2\right) = -3k.$$

Bo‘lish natijasini ko‘paytirish bilan tekshirish mumkin: *bo‘linuvchi bo‘luvchining bo‘linmaga ko‘paytmasiga teng bo‘lishi kerak*.

Masalan, $(56a^5b^3c) : (7a^2b^2c) = 8a^3b$ — bo‘lish to‘g‘ri bajarilgan, chunki $56a^5b^3c = (7a^2b^2c)\,8a^3b$.

2. Ko‘phadni birhadga bo‘lish

Masala. $2a^2b + 4ab^2 + 8abc$ ko‘phadni $2ab$ birhadga bo‘ling.

△ Ushbu qoidadan foydalanamiz: *yig‘indini songa bo‘lishda har bir qo‘shiluvchini shu songa bo‘lish kerak,* ya’ni

$$(2a^2b + 4ab^2 + 8abc) : (2ab) = (2a^2b) : (2ab) +$$
$$+ (4ab^2) : (2ab) + (8abc) : (2ab) = a + 2b + 4c. \ ▲$$

Ko‘phadni birhadga boshqa hollarda ham xuddi shunday bo‘linadi, masalan,

$$(9a^3b^2 - 3a^2b^3 + a^2b^2) : (3a^2b^2) =$$
$$= (9a^3b^2) : (3a^2b^2) + (-3a^2b^3) : (3a^2b^2) + (a^2b^2) : (3a^2b^2) = 3a - b + \frac{1}{3}.$$

Ko‘phadni birhadga bo‘lish uchun ko‘phadning har bir hadini shu birhadga bo‘lish va hosil bo‘lgan natijalarni qo‘shish kerak.

Ko‘phadni birhadga bo‘lish natijasini ko‘paytirish bilan tekshirish mumkin. Masalan, $(36n^4m^2 - 45n^2m^4) : (9n^2m^2) = 4n^2 - 5m^2$ — bo‘lish to‘g‘ri bajarilgan, chunki

$$36n^4m^2 - 45n^2m^4 = (4n^2 - 5m^2)\,(9n^2m^2).$$

Ko‘rilgan misollarda birhad (ko‘phad)ni birhadga bo‘lish natijasida birhad (ko‘phad) hosil bo‘ladi. Bunday hollarda ko‘phad birhadga qoldiqsiz bo‘linadi, deyiladi. Ammo, ko‘phadni birhadga qoldiqsiz (butun) bo‘lish hammavaqt ham mumkin bo‘lavermaydi. Masalan, $ab + ac$ ko‘phad ab birhadga qoldiqsiz (butun) bo‘linmaydi.

Birhad (ko‘phad)ni birhadga bo‘lishda harflar bo‘luvchi nolga teng bo‘lmaydigan qiymatlarni qabul qiladi, deb faraz qilinadi.

Bo‘lishni bajaring **(295–305):**

295. 1) $b^5 : b^2$; 2) $y^{11} : y^7$; 3) $a^7 : a^7$; 4) $b^9 : b^9$.

296. 1) $12x : 4$; 2) $-15a : 5$; 3) $-18y : 6$; 4) $10c : (-2)$.

297. 1) $(8c) : (-2)$; 2) $\frac{2}{3}a : 5$; 3) $-\frac{1}{2}b : 2$; 4) $3c : \left(-\frac{1}{3}\right)$.

298. 1) $\frac{2}{5}x : (-2)$; 2) $-7m : \left(-\frac{7}{9}\right)$; 3) $-\frac{3}{4}a : \left(-\frac{8}{9}\right)$; 4) $\frac{16}{25}b : \frac{4}{5}$.

299. 1) $5a : a$; 2) $8x : x$; 3) $5a : (-a)$; 4) $(-7y) : (-y)$.

300. 1) $(-6x) : (2x)$; 3) $(-6xy) : (-3xy)$;
2) $15z : (5z)$; 4) $12ab : (-4ab)$.

301. 1) $3a : \left(\frac{1}{2}a\right)$; 3) $(-5c) : \left(\frac{1}{3}c\right)$;
2) $\frac{2}{3}b : \left(-\frac{2}{5}b\right)$; 4) $(-1{,}69n) : (1{,}3n)$.

302. 1) $8abc : (-4a)$; 3) $-6{,}4xy : (-4x)$;
2) $(-10pq) : (6q)$; 4) $(-0{,}24abc) : (-0{,}6ab)$.

303. 1) $14a^5 : (7a^2)$; 3) $-0{,}2a^{10} : (-a^{10})$;
2) $(-42m^7) : (6m)$; 4) $\left(-2\frac{1}{3}a^{17}\right) : (-2a^{17})$.

304. 1) $\frac{1}{3}m^3n^2p^2 : \left(-\frac{2}{3}m^2n^2p^2\right)$; 3) $(28{,}9p^2q^2y^3) : (-1{,}7p^2y^3)$;
2) $\left(-1\frac{1}{2}a^4b^3c^2\right) : \left(-\frac{2}{3}a^3bc^2\right)$; 4) $-6a^3b^2c : (-2a^2bc)$.

305. 1) $20m^4n^3 : \left(-5m^2n^3\right)$;

2) $-1,3a^3x^2y^3 : \left(16,9a^2xy\right)$;

3) $\left(-\frac{2}{5}a^4x^3y^2\right) : \left(-\frac{1}{2}a^3xy^2\right)$;

4) $\left(-\frac{3}{4}a^5b^3c\right) : \left(-1\frac{1}{2}a^2b^2c\right)$.

306. Ifodani soddalashtiring:

1) $\left(4a^3b^2\right)^3 : \left(2a^2b\right)^2$;

2) $\left(9x^2y\right)^3 : (3xy)^2$;

3) $\left(-abc^2\right)^5 : \left(-a^2bc^3\right)^2$;

4) $\left(-x^2y^3z\right)^4 : (xyz)$.

Bo‘lishni bajaring **(307—310):**

304. 1) $(12a+6) : 3$;

2) $(10b-5) : 5$;

3) $(14m-8) : (-2)$;

4) $(-6+3x) : (-3)$.

308. 1) $(5mn-6np) : n$;

2) $\left(4a^2-3ab\right) : a$;

3) $(x-xy) : x$;

4) $(cd-d) : (-d)$.

309. 1) $\left(3a^2b-4ab^3\right) : (5ab)$;

2) $\left(2c^5b^4+3c^4b^3\right) : \left(-3c^4b^3\right)$;

3) $\left(-27k^4l^5+21k^3l^2\right) : \left(-10k^3l^2\right)$;

4) $\left(-a^5b^3+3a^6b^2\right) : \left(4a^4b^2\right)$.

310. 1) $(6a-8b+10) : 2$;

2) $(8x+12y-16) : (-4)$;

3) $\left(10a^2-12ab+8a\right) : 2a$;

4) $\left(2ab+6a^2b^2-4b\right) : (2b)$.

311. Ifodani soddalashtiring:

1) $\left(6a^3-3a^2\right) : a^2+\left(12a^2+9a\right) : (3a)$;

2) $\left(8x^3-4x^2\right) : \left(2x^2\right)-\left(4x^2-3x\right) : x$;

3) $\left(3x^3-2x^2y\right) : x^2-\left(2xy^2+x^2y\right) : \left(\frac{1}{3}xy\right)$;

4) $\left(a^2b-3ab^2\right) : \left(\frac{1}{2}ab\right)+\left(6b^3-5ab^2\right) : b^2$.

312. Algebraik ifodaning qiymatini toping:

1) $\left(15a^3+25a^2\right) : (5a)-9a^4 : \left(3a^2\right)$, bunda $a=7$;

2) $\left(18a^4-27a^3\right):\left(9a^2\right)-10a^3:\left(5a\right)$, bunda $a=-8$;

3) $\left(3x^3+4x^2y\right):x^2-\left(10xy+15y^2\right):\left(5y\right)$, bunda $x=2, y=-5$;

4) $\left(2xy^2-5y^3\right):y^2+\left(12xy+6x^2\right):\left(3x\right)$, bunda $x=-3, y=-12$.

O‘zingizni tekshirib ko‘ring!

1. Ifodani daraja ko‘rinishida tasvirlang:

$5^3\cdot 5^2$; $3^8:3^6$; $(2^3)^4$; $3^5\cdot 2^5$.

2. Ifodani soddalashtiring: $(3b+c^2-d)-(c^2-2d)$.

3. Amallarni bajaring:

$(-0,25a^3b^2c)\cdot(5abc)$; $(7m^2-20mn-10m):10m$.

4. Ifodani soddalashtiring va uning $m=-0,25$ bo‘lgandagi son qiymatini toping:

$2m(m-1)+(m-2)(m+2)+2m$.

III bobga doir mashqlar

313. Yozing:
1) m sonning kvadratini;
2) a sonning kubini;
3) c va 3 sonlar yig‘indisining kvadratini;
4) c va 3 sonlar kvadratlarining yig‘indisini.

314. Yozing:
1) n va m sonlar ayirmasining kvadratini;
2) n va m sonlar kvadratlarining ayirmasini;
3) n va m sonlar ayirmasining kubini;
4) $\frac{1}{2}$ va b sonlar kublarining ayirmasini.

315. Kvadratning tomoni c metrga teng. Uning perimetri va yuzini yozing.

316. Kubning qirrasi k santimetrga teng. Uning sirti yuzini va hajmini yozing.

317. Bir tomoni ikkinchi tomonidan 3 marta katta bo‘lgan to‘g‘ri to‘rtburchakning bir tomonini x bilan belgilab, uning yuzi formulasini yozing.

318. Agar bir kub metr kub santimetrlarga ajratilsa va ular ustma-ust qo‘yilsa, qanday balandlikdagi ustun hosil bo‘ladi?

319. Agar odamning yuragi 1 minutda o‘rtacha 75 marta ursa, uning yuragi bir sutka davomida necha marta uradi?

320. O‘quvchi 1 m^3 po‘kakni ko‘tara oladimi? (1 sm^3 po‘kakning massasi 0,2 g).

321. Quyidagi sonlarni standart shaklda yozing:

1) 0° C va 760 mm sim. ust. bosimli 1 sm^3 gazdagi molekulalar soni 27 000 000 000 000 000 000 ga teng;
2) parsek (astronomiyada qabul qilingan uzunlik birligi) 30 800 000 000 000 km ga teng;
3) elektron hisoblash mashinasi 1 sekundda 1 000 000 ta amal bajarishi mumkin.

322. Yer shari sirti 510 mln km^2 dan ortiq. Yer hajmi 1000 mlrd km^3 dan ortiq. Bu sonlarni standart shaklda yozing.

323. 1 l dengiz suvida o‘rtacha 0,00001 mg oltin bor. 1 km^3 dengiz suvida qancha oltin bor?

324. Ko‘phadni standart shaklga keltiring:

1) $(2m)(4n)-3a(2b)-(0,2n)(5m)+b(5a)-5nm+8ab$;

2) $13ab-0,2xy-(2a)(5b)+(6x)(0,2y)+a(-3)b$;

3) $2abc5a+1\frac{5}{7}a^2\frac{7}{12}bc-2\frac{2}{3}ab\left(-\frac{3}{8}\right)a$;

4) $3nmk4n-\frac{3}{8}nm\left(2\frac{2}{3}\right)nk+\frac{2}{9}n^2m\left(-4\frac{1}{2}\right)k$.

325. Ko‘phadning qiymatini toping:

1) $-0,08x+73xy^2+27xy^2$, bunda $x=4, y=0,2$;

2) $-2a^2b+4b+11a^2b$, bunda $a=-\frac{1}{3}, b=2\frac{3}{4}$;

3) $5p^3-3p^2+11p-7p-6p^2-7p^2+p$, bunda $p=-1$;

4) $8x^2-7x^3+6x-5x^2+2x^3+3x^2-8x$, bunda $x=1$.

326. Ko‘phadlarning algebraik yig‘indisini toping

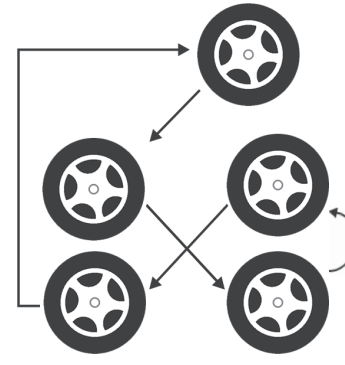
1) $\left(-2x^3+xy^2\right)+\left(x^2y-1\right)+\left(x^2y-xy^2+3x^3\right)$;

2) $\left(3x^2+5xy+7x^2y\right)-\left(5xy+3x^2\right)-\left(7x^2y-3x^2\right)$;

3) $\left(8a^2-10ab-b^2\right)+\left(-6a^2+2ab-b^2\right)-\left(a^2-8ab+4b^2\right)$;

4) $\left(4a^2-2ab+b^2\right)-\left(-a^2+b^2-2ab\right)+\left(3a^2+b^2-ab\right)$.

№ 6 Yangi „Matiz“ avtomobilining egasi yurib turgan va zahiradagi g‘ildiraklarni rasmda ko‘rsatilgan tartibda almashtirib turdi. 30 000 km yo‘l yurilgach, „Matiz“ egasi hamma g‘ildiraklar bir xil yedirilganini sezib qoldi. Har bir g‘ildirak necha kilometr yo‘l bosgan?



Ko‘phadlarni ko‘paytiring **(327—328):**

327. 1) $(0,3x+0,3y-z)(x-z)$;

2) $(0,5x-0,5y+z)(x+y)$;

3) $\left(\frac{1}{4}m-\frac{1}{4}n+\frac{1}{5}p\right)(20m+8)$;

4) $\left(0,2a^2-0,4a+1\right)\left(5a^2-10\right)$.

328. 1) $(a-b)(a+b)(2a-3b)$;

2) $(a+b)(a-b)(2a+3b)$;

3) $(x+2)(3x+1)(2x-1)$;

4) $(x-3)(2x+1)(3x-1)$.

329. Bo‘lishni bajaring:

1) $(0,01a^4 - 0,2a^3 + 0,04a^2 + 0,002a) : (0,01a)$;

2) $(-0,05x^5 - 0,08x^4 - 0,09x^3 + 0,01x^2) : (-0,01x^2)$;

3) $\left(-4m^5n^2 - \frac{4}{9}m^4n^5 + \frac{2}{3}m^3n^6\right) : \left(\frac{2}{3}m^3n^2\right)$;

4) $\left(\frac{3}{4}a^6x^3 + \frac{6}{5}a^3x^4 - \frac{9}{10}ax^5\right) : \left(\frac{3}{5}ax^3\right)$.

III bobga doir sinov mashqlari — testlar

1. Hisoblang: $\frac{3^3 \cdot 9^5}{81^3}$.

A) 3; B) $\frac{1}{3}$; C) $\frac{1}{9}$; D) $\frac{1}{27}$; E) 9.

2. Hisoblang: $\frac{a^8(b^4)^4}{(b^2)^6 \cdot (a^2)^3 \cdot (ab)^2}$.

A) a^2b^2; B) b^2; C) a^2; D) $\frac{1}{b^2}$; E) $\frac{a}{b}$.

3. Birhadning son qiymatini toping:

$\frac{1}{5}a^2b^3c$, bunda $a = -2, b = -1, c = 10$.

A) $-\frac{4}{5}$; B) $\frac{4}{5}$; C) -8; D) 8; E) -40.

4. Birhadni standart shaklda yozing: $2^4ab^2\left(-\frac{1}{2}\right)^3a^2b$

A) $2aa^2b^2b$; B) $\frac{4}{3}a^3b^3$; C) $-\frac{4}{3}b^3a^3$;

D) $4a^3b^3$; E) $-2a^3b^3$.

5. Birhadlarni ko‘paytiring: $\left(-\frac{7}{15}a^3b^2c^3\right)\left(\frac{9}{14}ab^2c\right)$.

A) $0,3a^3b^4c^4$; B) $-0,3(abc)^4$; C) $-\frac{9}{15}a^4b^2c^3b^2$;

D) $\frac{9}{15}a^4c^4b^3$; E) to‘g‘ri javob berilmagan.

6. Ko‘phadni uning har bir hadini standart shaklga keltirib, soddalashtiring: $3b^2a5ab-6b^24aba+ab4ab^2$.

A) 43 a^3b^3; B) $43a^2b^3$; C) $-5a^3b^2$;
D) $-5a^2b^3$; E) $5a^2b^3$.

7. Ko‘phadlarning algebraik yig‘indisini toping:

$$\left(0,5a+\frac{2}{3}b\right)-\left(\frac{7}{2}a-\frac{1}{3}b\right)+2(a+b).$$

A) $a+3b$; B) $-a+3b$; C) $-a-3b$; D) $a-3b$;

E) $6a+2\frac{1}{2}b$.

8. Ko‘phadni birhadga ko‘paytiring: $\left(4a-\frac{1}{3}x\right)\cdot(-3x)$.

A) $-12ax-3x^2$; B) $3x^2-12ax$; C) $3x^2+12ax$;

D) x^2-12ax; E) $-x^2+12ax$.

9. Soddalashtiring: $5a(0,4a-b)-4a\left(\frac{1}{4}a-b\right)$.

A) $a(a-b)$; B) $a(a+b)$; C) a^2+9ab;
D) $3a^2+9ab$; E) $3a^2-ab$.

10. Ko‘phadni standart shaklga keltiring:

$$4abc3a-2\frac{5}{7}b^21\frac{2}{19}ac-3\frac{2}{3}ab\left(-1\frac{4}{11}\right)ac.$$

A) $12a^2bc-8ab^2c$; B) $17a^2bc-3ab^2c$; C) $-12a^2bc-3ab^2c$;

D) $12a^2bc-3ab^2c$; E) $17a^2bc+3ab^2c$.

11. Ko‘phadlarni ko‘paytiring: $(a-b)(a+b)(a^2+b^2)$.

A) a^3-b^4; B) a^4+b^3; C) a^3-b^3;
D) a^4-b^4; E) a^4+b^4.

12. Bo‘lishni bajaring: $(16a^3b^2 - 4a^2b^3 + a^2b^2) : (4a^2b^2)$.

A) $4a + b + \frac{1}{4}$; B) $4a + b + 4$; C) $4ab - \frac{1}{6} + 4$;

D) $4a - 4b + 4$; E) $4a - b + \frac{1}{4}$.

13. Algebraik ifodaning son qiymatini toping:

$$(10a^4 + 15a^3) : 5a^2 - 18a^3 : (-6a), \text{ bunda } a = -\frac{3}{2}.$$

A) $-15{,}75$; B) $6{,}75$; C) $15{,}75$; D) -12; E) 18.

14. Ifodani soddalashtiring: $\left(18a^4 + 21a^2\right) : 3a^2 - 5a\left(2a + \frac{1}{a}\right)$.

A) $4a^2 + 2$; B) $16a^2 + 12$; C) $-4a^2 + 2$;
D) $16a^2 + 2$; E) $-4a^2 + 12$.

15. Ko‘phadlarni ko‘paytiring: $(a + 2b)(a - 2b)(a^2 + 4b^2)$.

A) $a^3 - 16b^3$; B) $a^4 - 8b^3$; C) $a^3 - 8b^3$;
D) $a^4 + 16b^4$; E) $a^4 - 16b^4$.

Hisoblang: **(16—20):**

16. $\frac{(-0{,}2)^5}{(-0{,}1)^4}$.

A) $-3{,}2$; B) $3{,}2$; C) $0{,}00032$;
D) $-0{,}00032$; E) -1.

17. $-(-3)^3 \cdot \left(-\frac{1}{3}\right)^2$.

A) -3; B) 3; C) $-2{,}7$; D) $\frac{1}{9}$; E) -9.

18. $\frac{(5{,}2)^3}{(1{,}3)^2}$.

A) 832; B) $8{,}32$; C) $83{,}2$; D) $5{,}2$; E) 16.

19. $\frac{10^4}{2^5 \cdot 5^4}$.

A) 1; B) $\frac{1}{5}$; C) 10; D) $\frac{1}{2}$; E) 2.

20. $\dfrac{18^5}{9^4 \cdot 2^4}$.

A) $\dfrac{1}{18}$; B) $\dfrac{9}{2^3}$; C) $\dfrac{1}{2}$; D) $\dfrac{1}{9}$; E) 18.

21. Birhadning son qiymatini toping:

$\dfrac{1}{18} \cdot x \cdot (-0,9)y^2$, bunda $x = -\dfrac{1}{2}$, $y = 10$

A) –45; B) 45; C) –2,5; D) 2,5; E) 5.

22. Birhadni standart shaklda yozing:

$\left(-\dfrac{1}{3}\right)^3 \cdot x^2y \cdot (-18)y^2x.$

A) $\dfrac{2}{3}x^3y^3$; B) $18x^3y^3$; C) $-18x^3y^3$;

D) $-\dfrac{2}{3}x^2yy^2x$; E) $2x^3y^3$.

Tarixiy masalalar

① *Nyuton masalasi.* Bitta xattot 8 kunda 15 varaq yoza oladi. 405 varaqni 9 kunda yozib tugatish uchun nechta xattot kerak bo‘ladi?

② Yem otning o‘ziga 14 kunga, ot bilan toychoqqa esa 10 kunga yetadi. Shu yem toychoqning o‘ziga necha kunga yetadi?

③ Asalarilarning beshdan biri oq gullarga, uchdan biri esa qizil gullarga qo‘ndi. Ular ayirmasining uch baravari esa sariq gullardan bol yig‘moqda. Faqat bittagina asalari gullar iforidan rohatlanib, uchib yuribdi. Qani menga ayt-chi, gulzorda qancha asalari bor?

Tarixiy ma’lumotlar

Noma’lum kattaliklarni harflar bilan belgilash mashhur yunon matematigi Diofant (III asr) asarlaridayoq uchraydi. Koeffitsiyentlarni ham, ma’lum miqdorlarni ham harflar bilan belgilashni F.Viyet (1540—1603) birinchilardan bo‘lib qo‘llagan. Algebraik tenglamalarni umumiy

holda tadqiq qilish harfiy koeffitsiyentlar kiritilgandan keyingina mumkin bo‘ldi. F. Viyet undosh bosh lotin harflari — *B, G, D,* ... bilan koeffitsiyentlarni, unli harflari — *A, E, I,* ... bilan esa noma’lumlarni belgilagan. Mashhur fransuz matematigi va faylasufi R. Dekart (1596—1650) koeffitsiyentlarni belgilash uchun lotin alifbosining dastlabki (kichik) harflari *a, b, c, d*, ... dan, noma’lumlarni belgilash uchun esa alifboning oxirgi harflari *x, y, z* lardan foydalangan. Darajaning hozirgi zamonaviy belgilanishi a^2, a^3, ..., a^n (n— natural son)ni ham Dekart kiritgan (1637- yil).

„Al-jabr val muqobala“ asarining „Ko‘paytirish haqida bob“ida al-Xorazmiy birhadlarni ko‘paytirishga, ikkihadni ikkihadga ko‘paytirishga hamda soddalashtirishga doir misollarni qaraydi. Al-Xorazmiy misollaridan ba’zilarini keltiramiz:

1) $(10-x)x$;
2) $(10+x)(10+x)$;
3) $(10-x)(10-x)$;
4) $(10-x)(10+x)$;
5) $\left(10+\frac{x}{2}\right)\cdot\left(\frac{1}{2}-5x\right)$;
6) $(10+x)(x-10)$;
7) $(100+x^2-20x)-(50+10x-2x^2)$;
8) $(100+x^2-20x)+(50+10x-2x^2)$.

Al-Xorazmiy, Ahmad Farg‘oniy, Beruniy, G‘iyosiddin al-Koshiy asarlarida algebraik simvolika bo‘lmagan. Matematik Abu Hasan Ali ibn Muhammad al-Kalasadiy (XV asr) asarida algebraik simvolika elementlarini uchratish mumkin. Al-Kalasadiy tenglamalarda noma’lumning birinchi darajasini „shay“ so‘zining birinchi harfi bilan, kvadratini „mol“ so‘zining, kubini „ka’b“ so‘zining birinchi harflari bilan belgilagan. Tenglik „=“ belgisi o‘rniga „adala“ (tenglik) so‘zidagi *a* harfini ishlatgan. Biz o‘rganayotgan „Algebra“ kursining simvolikasi (belgilashlar tizimi) XIV—XVII asrlarda shakllangan.

IV BOB
KO‘PHADNI KO‘PAYTUVCHILARGA AJRATISH

19-§ Umumiy ko‘paytuvchini qavsdan tashqariga chiqarish

Aytaylik, $a^2 - b^2$ ifodaning qiymatini $a = 573$ va $b = 427$ bo‘lganda topish talab qilinayotgan bo‘lsin.

Agar son qiymatlarni qo‘yib, hisoblashlarni bajarilsa, u holda 573 va 427 sonlarini kvadratga ko‘tarishga, so‘ngra esa ayirishni bajarishga to‘g‘ri keladi.

Agar $a^2 - b^2$ ifodani unga teng bo‘lgan $(a + b)(a - b)$ ifoda bilan almashtirilsa, hisoblashni ancha sodda yo‘l bilan olib borish mumkin.

$a = 573$ va $b = 427$ bo‘lganda:

$a^2 - b^2 = (a + b)(a - b) = (573 + 427)(573 - 427) = 1\,000 \cdot 146 = 146\,000.$

Hisoblashlarni soddalashtirish uchun $a^2 - b^2$ ko‘phad $(a + b)(a - b)$ ko‘paytma bilan almashtirildi.

Ko‘phadni ikkita yoki bir nechta ko‘phadlar ko‘paytmasi shaklida ifodalash ko‘phadni *ko‘paytuvchilarga ajratish (yoyish)* deyiladi.

Bunga o‘xshash almashtirish bilan natural sonlarni ko‘paytuvchilarga yoyishda (ajratishda) duch kelingan edi. Masalan, murakkab son 60 ni tub sonlarning ushbu ko‘paytmasi shaklida ifodalash mumkin:

$$60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5.$$

Sonlarni ko‘paytuvchilarga ajratishdan kasrlarni qisqartirishda, ularni umumiy maxrajga keltirishda va boshqa masalalarni yechishda foydalaniladi. Ko‘phadni ko‘paytuvchilarga ajratish algebraik ifodalar ustida amallar bajarishda ham keng qo‘llaniladi.

1-masala. $ab + ac - ad$ ifodaning $a = 43$, $b = 26$, $c = 17$, $d = 23$ bo‘lganda son qiymatini toping.

△ Hisoblashlarni quyidagicha olib boramiz:

$43 \cdot 26 + 43 \cdot 17 - 43 \cdot 23 = 43 \cdot (26 + 17 - 23) = 43 \cdot 20 = 860.$ ▲

Bu yerda ko‘paytirishning taqsimot qonuni qo‘llanilgan:

$$ab + ac - ad = a(b + c - d).$$

$43 \cdot 26 + 43 \cdot 17 - 43 \cdot 23$ sonli ifodada umumiy ko‘paytuvchi 43 soni bo‘ladi; $ab + ac - ad$ algebraik ifodada esa umumiy ko‘paytuvchi a bo‘ladi.

Agar ko‘phadning barcha (son yoki harfiy) hadlari umumiy ko‘paytuvchiga ega bo‘lsa, u holda shu ko‘paytuvchini qavsdan tashqariga chiqarish mumkin.

Qavs ichida berilgan ko‘phadni shu umumiy ko‘paytuvchiga bo‘lish natijasida hosil qilingan ko‘phad qoladi.

2 - m a s a l a . Ushbu ko‘phadni ko‘paytuvchilarga ajrating:

$$6ab + 3b - 12bc.$$

△ Berilgan ko‘phadning barcha hadlari $3b$ umumiy ko‘paytuvchiga ega, chunki $6ab = 3b \cdot 2a$, $3b = 3b \cdot 1$, $-12bc = 3b \cdot (-4c)$.

Demak, $6ab + 3b - 12bc = 3b(2a + 1 - 4c)$. ▲

Ko‘phadning umumiy hadini masala mazmuniga qarab, qavsdan tashqariga «+» ishorasi bilan ham, «–» ishorasi bilan ham chiqarish mumkin. Misollar keltiramiz:

1) $ab - b = b(a - 1) = -b(1 - a)$;
2) $4a^2b^3 - 6a^3b^2 = 2a^2b^2\ (2b - 3a)$ yoki
$4a^2b^3 - 6a^3b^2 = -2a^2b^2\ (-2b + 3a) = -2a^2b^2\ (3a - 2b)$.

Shunday qilib, *ko‘phadni umumiy ko‘paytuvchini qavsdan tashqariga chiqarish yo‘li bilan ko‘paytuvchilarga ajratish uchun:*

1) *shu umumiy ko‘paytuvchini topish;*
2) *uni qavsdan tashqariga chiqraish kerak.*

Agar ko‘phad hadlarining koeffitsiyentlari natural sonlar bo‘lsa, u holda umumiy ko‘paytuvchini topish uchun ko‘phad hadlari koeffitsiyentlarining eng katta umumiy bo‘luvchisini topish, bir xil asosli darajalar orasidan esa eng kichik ko‘rsatkichli darajani topish lozimligini ta’kidlab o‘tamiz. Masalan, $28x^2b^3 - 21x^3b^2$ ko‘phadni ko‘paytuvchilarga ajratib, quyidagini hosil qilamiz:

$$7x^2b^2\ (4b - 3x).$$

Bu yerda 7 soni 28 va 21 sonlarining eng katta umumiy bo‘luvchisi, x^2 va b^2 esa x va b ning eng kichik ko‘rsatkichli darajalaridir.

Ko‘phadni ko‘paytuvchilarga ajralganligining to‘g‘riligini hosil bo‘lgan ko‘phadlarni ko‘paytirish yo‘li bilan tekshirish mumkin. Masalan, ko‘paytirishni bajarib, hosil qilamiz:

$$7x^2b^2(4b - 3x) = 28x^2b^3 - 21x^3b^2.$$

Umumiy ko‘paytuvchi ko‘phad bo‘lishi ham mumkin, masalan:

1) $5(a + b) + x(a + b) = (a + b)(5 + x)$;
2) $3x(a - 2b) + 5y(a - 2b) + 2(a - 2b) = (a - 2b)(3x + 5y + 2)$.

Ba’zan umumiy ko‘paytuvchini qavsdan tashqariga chiqarishdan oldin $a - b = -(b - a)$ tenglikni qo‘llash foydali bo‘ladi, masalan:

1) $(a - 3)x - (3 - a)y = (a - 3)x + (a - 3)y = (a - 3)(x + y)$;
2) $15a^2b(x^2 - y) - 20ab^2(x^2 - y) + 25ab(y - x^2) = 15a^2b(x^2 - y) - 20ab^2(x^2 - y) - 25ab(x^2 - y) = 5ab(x^2 - y)(3a - 4b - 5)$.

Mashqlar

330. Sonlarni tub ko‘paytuvchilarga ajrating: 70, 121, 240, 168, 225.

331. Kasrlarni qisqartiring: $\frac{45}{60}$; $\frac{18}{24}$; $\frac{75 \cdot 15}{25 \cdot 24}$; $\frac{40 \cdot 14}{7 \cdot 15}$.

332. Ko‘paytirishning taqsimot qonunini qo‘llang va hisoblang:

1) $81 \cdot 17 - 15 \cdot 81$;
2) $24 \cdot 2{,}78 + 41 \cdot 2{,}78$;
3) $15 \cdot 17 + 15 \cdot 67$;
4) $14\frac{3}{8} \cdot 1\frac{1}{4} - 4\frac{3}{8} \cdot 1\frac{1}{4}$.

333. Ko‘paytmani ko‘phad shaklida yozing:

1) $(a + 2)(a + 3)$;
2) $2x(x - 1)$;
3) $3c^3(2c^3 - 5)$;
4) $(a^2 + b)(a - b^2)$.

334. A bekatdan B bekatga tomon motorli qayiq 20 km/soat tezlik bilan jo‘nadi. Oradan ikki soat o‘tgandan keyin A dan B ga tomon ikkinchi motorli qayiq 24 km/soat tezlik bilan yo‘lga chiqdi. Ikkala qayiq ham B ga bir vaqtda yetib keldi. A dan B gacha bo‘lgan masofani toping.

335. Hisoblang:

1) $13 \cdot 512 + 13 \cdot 488$;
2) $125 \cdot 375 + 275 \cdot 375$;
3) $25 \cdot 734 - 25 \cdot 726$;
4) $26 \cdot 1\frac{1}{3} - 1\frac{1}{3} \cdot 23$.

Umumiy ko‘paytuvchini qavsdan tashqariga chiqaring **(336—344):**

336. 1) $2m+2n$; 2) $3a-3x$; 3) $8-4x$; 4) $6a+12$.

337. 1) $9a+12b+3$; 3) $-10x+15y-5z$;
2) $8a-4b-2$; 4) $9x-3y+12z$.

338. 1) $ax-ay$; 2) $cd+bc$; 3) $xy+2x$; 4) $3x-xy$.

339. 1) $9mn+9n$; 2) $3bd-3ab$; 3) $11z-33yz$; 4) $6pk-3p$.

340. 1) $ab-ac+a^2$; 3) $6a^2-3a+12ba$;
2) $xy-x^2+xz$; 4) $4b^2+8ab-12a^2b$.

341. 1) a^4+2a^2; 3) $a^4b^2+ab^3$;
2) a^4-3a^3; 4) $x^2y^3-x^3y^2$.

342. 1) $18y^7+12y^4$; 3) $15x^5-5x^3$;
2) $6x^4-24x^2$; 4) $6a^5+3a^2$.

343. 1) $9a^2b^2-12ab^3$; 3) $7a^2bc+14ab^2c$;
2) $20x^3y^2+4x^2y$; 4) $9xyz^2-12xy^2z$.

344. 1) $6y^5+12y^4-3y^3$; 3) $4a^2b^2+36a^2b^3+6ab^4$;
2) $20a^4-5a^3+15a^5$; 4) $2x^2y^4-2x^4y^2+6x^3y^3$.

345. Hisoblang:

1) $137^2+137\cdot 63$; 3) $0,7^3+0,7\cdot 9,51$;
2) $187^2-187\cdot 87$; 4) $0,9^3-0,81\cdot 2,9$.

Ko‘paytuvchilarga ajrating **(346—349):**

346. 1) $a(m+n)+b(m+n)$; 3) $a(b-5)-(b-5)$;
2) $b(a+5)-c(a+5)$; 4) $(y-3)+b(y-3)$.

347. 1) $2a(a-b)+3b(a-b)$; 3) $5a(x+y)-4b(x+y)$;
2) $3n(m-3)+5m(m-3)$; 4) $7a(c-d)-2b(c-d)$.

349. 1) $a^2(x-y)+b^2(x-y)$; 3) $a\left(x^2+y^2\right)-b\left(x^2+y^2\right)$;

2) $a^2(x+y)-b^2(x+y)$; 4) $x\left(a^2-2b^2\right)+y\left(a^2-2b^2\right)$.

349. 1) $2b(x-1)-3a(x-1)+c(x-1)$;

2) $c(p-q)-a(p-q)+d(p-q)$;

3) $x\left(a^2+b^2\right)+y\left(a^2+b^2\right)-z\left(a^2+b^2\right)$;

4) $m\left(x^2+1\right)-n\left(x^2+1\right)-l\left(x^2+1\right)$.

Ko‘paytuvchilarga ajrating **(350—352):**

350. 1) $c(a-b)+b(b-a)$; 3) $(x-y)+b(y-x)$;

2) $a(b-c)-c(c-b)$; 4) $2b(x-y)-(y-x)$.

351. 1) $7(y-3)-a(3-y)$; 3) $b^2(a-1)-c(1-a)$;

2) $6(a-2)+a(2-a)$; 4) $a^2(m-2)+b(2-m)$.

352. 1) $a(b-c)+b^2(b-c)-7(c-b)$;

2) $x(x-y)+y(y-x)-3(x-y)$;

3) $x(a-2)+y(2-a)+(2-a)$;

4) $a(b-3)+(3-b)-b(3-b)$.

353. Tenglamani yeching:

1) $8-(x-3)(x+3)=10-(x-1)^2$; 3) $x:15=2\frac{1}{12}:14{,}5$;

2) $(2x+1)^2-(2x-3)^2=4(7x-5)$; 4) $\frac{x}{2{,}3}=\frac{2{,}1}{9\frac{6}{7}}$.

354. It tulkining orqasidan quvdi. It sekundiga 8 m, tulki esa 6 m tezlik bilan chopmoqda. Ularning orasidagi masofa dastlab 360 m bo‘lgan, tulkining o‘z uyasiga yetib olishi uchun esa 1 km qolgan edi. Tulki o‘z uyasiga yetib olishga ulguradimi?

20-§ Guruhlash usuli

Guruhlash usuli hamma hadlari uchun umumiy ko‘paytuvchi mavjud bo‘lmagan ko‘phadlarga qo‘llaniladi.

Ba’zan, berilgan ko‘phadning bir nechta hadlarini qavs ichiga olib, umumiy ko‘paytuvchini aniqlash mumkin. Ko‘phadni guruhlash usuli qo‘shish va ko‘paytirishning guruhlash, o‘rin almashtirish va taqsimot qonunlariga asoslangan.

Misollar qaraymiz:

1) $a(b+c)+b+c=a(b+c)+(b+c)=(b+c)(a+1)$;

2) $a(b-c)-b+c=a(b-c)-(b-c)=(b-c)(a-1)$.

Birinchi misolda ko‘phadning oxirgi ikkita hadini «+» ishorasi bilan, ikkinchi misolda ko‘phadning oxirgi ikkita hadini «—» ishorasi bilan qavs ichiga olish yetarli bo‘ldi.

3) $m(3x-y)+3nx-ny=m(3x-y)+(3nx-ny)=$
$=m(3x-y)+n(3x-y)=(3x-y)(m+n)$;

4) $-mx^2-my^2+n(x^2+y^2)=(-mx^2-my^2)+n(x^2+y^2)=$
$=-m(x^2+y^2)+n(x^2+y^2)=(x^2+y^2)(n-m)$.

Uchinchi va to‘rtinchi misollarda ko‘phadning ikkita hadini qavs ichiga olishdan tashqari hosil qilingan har bir guruhda umumiy ko‘paytuvchi qavsdan tashqariga: birinchi holda «+» ishorasi bilan, ikkinchisida esa «–» ishorasi bilan chiqarildi.

Ba’zan ko‘phad hadlarini turli usullar bilan guruhlash mumkin. Masalan, $2am+2an-3bm-3bn$ ko‘phadni ko‘paytuvchilarga ikki usul bilan ajratish mumkin:

I usul	II usul
$2am+2an-3bm-3bn=$	$2am+2an-3bm-3bn=$
$=(2am+2an)-(3bm+3bn)=$	$=(2am-3bm)+(2an-3bn)=$
$=2a(m+n)-3b(m+n)=$	$=m(2a-3b)+n(2a-3b)=$
$=(m+n)(2a-3b)$.	$=(2a-3b)(m+n)$.

Oltita haddan iborat ko‘phadni ko‘paytuvchilarga ajratishga doir misol qaraymiz:

$$ax+bx-ay-by+az+bz=(ax+bx)-(ay+by)+(az+bz)=$$

$$=x(a+b)-y(a+b)+z(a+b)=(a+b)(x-y+z).$$

Bu yerda ko‘phadlar ikkitadan guruhlarga ajratilgan; ularni uchtadan guruhlash ham mumkin edi:

$$ax+bx-ay-by+az+bz=(ax-ay+az)+(bx-by+bz)=$$

$$=a(x-y+z)+b(x-y+z)=(a+b)(x-y+z).$$

Shunday qilib, ko‘phadni guruhlash usuli bilan ko‘paytuvchilarga ajratish uchun:

1) ko‘phadning hadlarini, ular ko‘phad shaklidagi umumiy ko‘paytuvchiga ega bo‘ladigan qilib, guruhlarga birlashtiriladi;

2) bu umumiy ko‘paytuvchini qavsdan tashqariga chiqariladi.

Mashqlar

Ko‘paytuvchilarga ajrating **(355—360)**:

355. 1) $a+b+c(a+b)$;
2) $m-n+p(m-n)$;
3) $x+3a(x+y)+y$;
4) $x+2a(x-y)-y$.

356. 1) $(x+y)+(x+y)^{2;}$
2) $(a-b)^2+a-b$;
3) $2m(m-n)+(m-n)^2$;
4) $4q(p-1)+(p-1)^2$.

357. 1) $2m(m-n)+m-n$;
2) $4q(p-1)+p-1$;
3) $2m(m-n)-n+m$;
4) $4q(p-1)+1-p$.

358. 1) $a(x-c)+bc-bx$;
2) $a(b+c)+db+dc$;
3) $3a(2b+c)+8b+4c$;
4) $2x(3x-4y)-6x+8y$.

359. 1) $ac+bc-2ad-2bd$;
2) $ac-3bd+ad-3bc$;
3) $2bx-3ay-6by+ax$;
4) $5ay-3bx+ax-15by$.

360. 1) $xy^2-by^2-ax+ab+y^2-a$; 2) $ax^2-ay-bx^2+cy+by-cx^2$.

361. Hisoblang:

1) $139\cdot 15+18\cdot 139+15\cdot 261+18\cdot 261$;

2) $125\cdot 48-31\cdot 82-31\cdot 43+125\cdot 83$;

3) $14,7\cdot 13-2\cdot 14,7+13\cdot 5,3-2\cdot 5,3$;

4) $3\frac{1}{3}\cdot 4\frac{1}{5}+4,2\cdot\frac{2}{3}+3\frac{1}{3}\cdot 2\frac{4}{5}+2,8\cdot\frac{2}{3}$.

362. Ifodaning son qiymatini toping:

1) $5a^2-5ax-7a+7x$, bunda $x=-3$, $a=4$;

2) $m^2-mn-3m+3n$, bunda $m=0,5$, $n=0,25$;

3) $a^2+ab-5a-5b$, bunda $a=6,6$, $b=0,4$;

4) $a^2-ab-2a+2b$, bunda $a=\frac{7}{20}$, $b=0,15$.

363. Hisoblang:

1) $287^2-287\cdot 48+239\cdot 713$; 2) $73,4^2+73,4\cdot 17,2-90,6\cdot 63,4$.

364. Tenglamani yeching:

1) $x(x-4)+x-4=0$; 2) $t(t+7)-4t-28=0$.

№ 7 Ali bilan Valining massasi birgalikda 5 ta tarvuz massasiga teng. Valining massasi 1 ta qovun massasidan 4 marta ko‘p. Vali bilan 2 ta qovunning birgalikdagi massasi 3 ta tarvuz massasiga teng. Alining massasi nechta qovunning massasiga teng?

21-§ Yig‘indining kvadrati. Ayirmaning kvadrati

O‘rta Osiyo xalqlari madaniyatini o‘rta asrlarda dunyo madaniyatining oldingi qatoriga olib chiqqan buyuk mutafakkirlardan biri Abu Ali ibn Sinoning matematikaga oid ishlarida sonlarni kvadrat va kubga ko‘tarish amallari o‘rganilgan.

Ikkita son yig‘indisining kvadrati $(a+b)^2$ni qaraymiz. Ko‘phadni ko‘phadga ko‘paytirish qoidasidan foydalanib, hosil qilamiz:

Abu Ali ibn Sino (980—1037) buyuk mutafakkir, vatandoshimiz

$$(a+b)^2=(a+b)\,(a+b)=a^2+ab+ab+b^2=a^2+2ab+b^2,$$

ya'ni

$$\boxed{(a+b)^2=a^2+2ab+b^2.} \qquad (1)$$

Ikki son yig'indisining kvadrati birinchi son kvadrati, qo'shuv birinchi son bilan ikkinchi son ko'paytmasining ikkilangani, qo'shuv ikkinchi son kvadratiga teng.

(1) formulani 13- rasmda tasvirlangan kvadratning yuzini ko'zdan kechirib, osongina hosil qilish mumkinligini aytib o'tamiz.

Endi ikki son ayirmasining kvadratini qaraymiz:

$$(a-b)^2=(a-b)(a-b)=a^2-ab-ab+b^2=a^2-2ab+b^2,$$

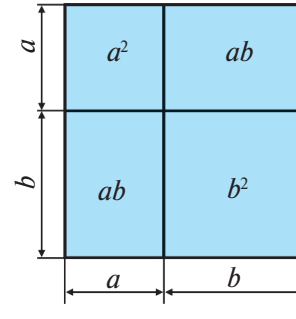
ya'ni

$$\boxed{(a-b)^2=a^2-2ab+b^2.} \qquad (2)$$

Ikki son ayirmasining kvadrati birinchi son kvadrati, ayiruv birinchi son bilan ikkinchi son ko'paytmasining ikkilangani, qo'shuv ikkinchi son kvadratiga teng.

(1) va (2) tengliklarda a va b istalgan sonlar yoki algebraik ifodalardir.

(1) va (2) formulalarni qo'llashga doir misollar:



13- rasm.

1) $(2m+3k)^2=(2m)^2+2\cdot 2m\cdot 3k+(3k)^2=4m^2+12mk+9k^2$;

2) $(5a^2-3)^2=(5a^2)^2-2\cdot 5a^2\cdot 3+3^2=25a^4-30a^2+9$;

3) $(-a-3b)^2=((-1)(a+3b))^2=(-1)^2(a+3b)^2=$
$=(a+3b)^2=a^2+2a\cdot 3b+(3b)^2=a^2+6ab+9b^2$.

Zaruriy hisoblashlarni og‘zaki bajarib, oraliq natijalarni yozmaslik mumkin. Masalan, birdaniga bunday yozish mumkin:

$$(5a^2-7b^2)^2=25a^4-70a^2b^2+49b^4.$$

Yig‘indi yoki ayirmaning kvadrati formulalari *qisqa ko‘paytirish formulalari* deyiladi va ba’zi hollarda hisoblashlarni soddalashtirish uchun qo‘llaniladi. Masalan:

1) $99^2=(100-1)^2=10\,000-200+1=9801$;

2) $52^2=(50+2)^2=2500+200+4=2704$.

(1) formula $(1+a)^2$ ifodaning qiymatlarini taqribiy hisoblashlarda ham qo‘llaniladi. a son musbat yoki manfiy son bo‘lib, uning moduli 1ga nisbatan kichik bo‘lsa (masalan, $a=0{,}0032$ yoki $a=-0{,}0021$), u holda a^2 son yanada kichik bo‘ladi va shu sababli

$$(1+a)^2=1+2a+a^2$$

tenglikni $(1+a)^2\approx 1+2a$ taqribiy tenglik bilan almashtirish mumkin. Masalan:

1) $(1{,}002)^2=(1+0{,}002)^2\approx 1+2\cdot 0{,}002=1{,}004$;
2) $(0{,}997)^2=(1-0{,}003)^2\approx 1-2\cdot 0{,}003=0{,}994$.

Yig‘indining kvadrati va ayirmaning kvadrati formulalari ko‘phadni ko‘paytuvchilarga ajratishda ham qo‘llaniladi, masalan:

1) $x^2+10x+25=x^2+2\cdot 5\cdot x+5^2=(x+5)^2$;

2) $a^4-8a^2b^3+16b^6=(a^2)^2-2\cdot a^2\cdot 4b^3+(4b^3)^2=(a^2-4b^3)^2$.

Masala. Formulani isbotlang:

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3. \qquad (3)$$

$\triangle\ (a+b)^3 = (a+b)(a+b)^2 = (a+b)(a^2+2ab+b^2) =$

$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3 = a^3 + 3a^2b + 3ab^2 + b^3.$ ▲

Xuddi shunga o‘xshash,

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \qquad (4)$$

formulani ham isbotlash mumkin.

(3) va (4) formulalar mos ravishda *yig‘indining kubi va ayirmaning kubi* deb ataladi.

(3) va (4) formulalar ham *qisqa ko‘paytirish formulalari* hisoblanadi.

Mashqlar

Quyidagi mashqlarda ikkihadning kvadratini ko‘phad shaklida tasvirlang **(365—372)**:

365. 1) $(c+d)^2$; 3) $(2+x)^2$; 5) $(y+3)^2$;
2) $(x-y)^2$; 4) $(x+1)^2$; 6) $(7+m)^2$.

366. 1) $(m-2)^2$; 3) $(7-m)^2$; 5) $\left(a+\frac{1}{3}\right)^2$;
2) $(x-3)^2$; 4) $(y-6)^2$; 6) $\left(b+\frac{1}{2}\right)^2$.

367. 1) $(q+2p)^2$; 2) $(3x+2y)^2$; 3) $(6a-4b)^2$; 4) $(5z-t)^2$.

368. 1) $(3a^2+1)^2$; 2) $(a^2+1)^2$; 3) $(2x^2+3n^2)^2$; 4) $(x^2+y^2)^2$.

369. 1) $\left(m-\frac{1}{5}\right)^2$; 2) $\left(a-\frac{1}{3}\right)^2$; 3) $\left(\frac{a}{2}-\frac{b}{3}\right)^2$; 4) $\left(\frac{x}{3}+\frac{y}{4}\right)^2$.

370. 1) $(0,2x+0,3y)^2$; 3) $\left(\frac{2}{3}x^3-\frac{3}{4}\right)^2$;
2) $(0,4b-0,5c)^2$; 4) $\left(\frac{1}{4}a^3-\frac{4}{5}\right)^2$.

371. 1) $\left(\frac{1}{2}a^3+\frac{2}{3}a\right)^2$; 3) $\left(-8p^3+5p^2\right)^2$;

2) $\left(\frac{1}{3}x^2+\frac{1}{2}x\right)^2$; 4) $\left(10x^2-3xy^3\right)^2$.

372. 1) $\left(-4ab-5a^2\right)^2$; 3) $\left(0,2x^2+5xy\right)^2$;

2) $\left(-3b^2-2ab\right)^2$; 4) $\left(4xy+0,5y^2\right)^2$.

Qisqa ko‘paytirish formulalaridan foydalanib, amallarni bajaring **(373—375)**:

373. 1) $(90-1)^2$; 2) $(40+1)^2$; 3) 101^2; 4) 98^2.

374. 1) 999^2; 2) 1003^2; 3) 51^2; 4) 39^2.

375. 1) 72^2; 2) 57^2; 3) 997^2; 4) 1001^2.

Ifodani soddalashtiring **(376—377):**

376. 1) $(x-y)^2+(x+y)^2$; 3) $(2a+b)^2-(2a-b)^2$;

2) $(x+y)^2-(x-y)^2$; 4) $(2a+b)^2+(2a-b)^2$.

377. 1) $(3a-1)^2+2(1+a)^2$; 3) $(x-1)^2-(x+1)^2$;

2) $3(2-a)^2+4(a-5)^2$; 4) $-(3+x)^2+5(1-x)^2$.

Tenglamani yeching **(378—379):**

378. 1) $16x^2-(4x-5)^2=15$; 3) $-5x(x-3)+5(x-1)^2=-20$;

2) $64x^2-(3-8x)^2=87$; 4) $(2x-3)^2-(2x+3)^2=12$.

379. 1) $(3x-1)^2-(3x-2)^2=0$;

2) $(y-2)(y+3)-(y-2)^2=5$;

3) $(x+3)(x+7)-(x+4)^2=0$;

4) $(y+8)^2-(y+9)(y-5)=117$.

380. Ifodaning qiymatini toping:

1) $9a^3 - a(3a+2)^2 + 4a(3a+7)$, bunda $a = -1\frac{1}{6}$;

2) $(2y-5)^2 - 4(y-3)^2 - 4y$, bunda $y = -\frac{2}{7}$;

3) $25m(m-1) - (5m-3)^2 - 6m$, bunda $m = -0,3$;

4) $24x^2 - (7x-2)^2 + (5x-3)(5x+1)$, bunda $x = -\frac{5}{9}$.

381. x ni shunday birhadga almashtiringki, natijada tenglik bajarilsin:

1) $(x - 4b^7)^2 = 25a^4b^2 - 40a^2b^8 + 16b^{14}$;

2) $(x+7c)^2 = 25b^6 + 70b^3c + 49c^2$;

3) $(10m^5 + x)^2 = 100m^{10} + 120m^7n^3 + 36m^4n^6$;

4) $(5b^2 - x)^2 = 25b^4 - 30a^2b^3 + 9a^4b^2$.

382. Ifodani ikkihadning kvadrati shaklida tasvirlang:

1) $a^2 - 10ab + 25b^2$;
2) $25 + 10x + x^2$;
3) $k^4 + 2k^2 + 1$;
4) $p^2 - 1,6p + 0,64$.

x ni shunday birhadga almashtiringki, natijada ikkihadning kvadrati hosil bo‘lsin **(383—384):**

383. 1) $a^2 + 4a + x$;
2) $p^2 - 0,5p + x$;
3) $36a^2 - x + 49b^2$;
4) $a^2 - 6ab + x$.

384. 1) $m^4 - 3m^2 + x$;
2) $a^2 + ab + x$;
3) $4a^2 - 5a + x$;
4) $x + 6a + 9a^2$.

385. Isbot qiling:

1) $(a-b)^2 = (b-a)^2$;
2) $(-a-b)^2 = (b+a)^2$;
3) $(-a-b)(a+b) = -(a+b)^2$;
4) $(a-b)^3 = -(b-a)^3$;
5) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$;
6) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

22-§ Kvadratlar ayirmasi formulasi

Ikki son yig‘indisini ularning ayirmasiga ko‘paytiramiz:

$$(a+b)(a-b)=a^2-ab+ab-b^2=a^2-b^2,$$

ya’ni

$$(a+b)(a-b)=a^2-b^2. \quad (1)$$

$$a^2-b^2=(a-b)(a+b). \quad (2)$$

Ikki son kvadratlarining ayirmasi shu sonlar ayirmasi bilan ular yig‘indisining ko‘paytmasiga teng.

(1) va (2) tenglikda a, b istalgan sonlar yoki algebraik ifodalardir, masalan:

1) $(nm+3k)(nm-3k)=n^2m^2-9k^2$;

2) $4a^4b^2-25a^2b^4=(2a^2b+5ab^2)(2a^2b-5ab^2)$;

3) $(a+b)^2-16=(a+b-4)(a+b+4)$.

(1) formulani ham qisqa ko‘paytirish formulasi deyiladi. Uni hisoblashlarni soddalashtirish uchun qo‘llaniladi.

Masalan:

1) $63\cdot 57=(60+3)(60-3)=3\,600-9=3\,591$;

2) $98\cdot 102=(100-2)(100+2)=100^2-2^2=10\,000-4=9\,996$.

(2) tenglikni kvadratlar ayirmasi formulasi deyiladi. U ko‘phadlarni ko‘paytuvchilarga ajratishda qo‘llaniladi.

Masalan:

1) $a^2-9=(a-3)(a+3)$;

2) $4b^4-0,64c^2=(2b^2)^2-(0,8c)^2=(2b^2-0,8c)(2b^2+0,8c)$;

3) $(a-b)^2-1=(a-b-1)\,(a-b+1)$;

4) $(a+b)^2-(a-c)^2=(a+b-a+c)(a+b+a-c)=$

$=(b+c)(2a+b-c)$.

Mashqlar

(1) formuladan foydalanib, ko‘paytirishni bajaring **(386—394):**

386. 1) $(c+d)(c-d)$;
2) $(p+q)(p-q)$;
3) $(a+c)(c-a)$;
4) $(m-n)(m+n)$;
5) $(a-b)(-a-b)$;
6) $(2-m)(-2-m)$.

387. 1) $(x+5)(x-5)$;
2) $(a+3)(a-3)$;
3) $(a-4)(4+a)$;
4) $(7+x)(x-7)$;
5) $(1+a)(1-a)$;
6) $(b-1)(1+b)$.

388. 1) $(2b+a)(2b-a)$;
2) $(c+3d)(c-3d)$;
3) $(y+6x)(6x-y)$;
4) $(3m-2n)(2n+3m)$.

389. 1) $\left(4d-\frac{1}{2}\right)\left(\frac{1}{2}+4d\right)$;
2) $\left(\frac{5}{6}a-b\right)\left(b+\frac{5}{6}a\right)$;
3) $\left(\frac{1}{2}y-\frac{1}{3}x\right)\left(\frac{1}{2}y+\frac{1}{3}x\right)$;
4) $\left(\frac{2}{3}m+\frac{3}{4}n\right)\left(\frac{2}{3}m-\frac{3}{4}n\right)$.

390. 1) $(c^2+d^2)(c^2-d^2)$;
2) $(a^2+b^3)(a^2-b^3)$;
3) $(x^4-y^3)(y^3+x^4)$;
4) $(m^3-n^3)(m^3+n^3)$.

391. 1) $(3a^2+4b^3)(3a^2-4b^3)$;
2) $(2m^4-5n^2)(5n^2+2m^4)$;
3) $(0,2t^3+0,5p^4)(0,5p^4-0,2t^3)$;
4) $(1,2a^2-0,3b^2)(1,2a^2+0,3b^2)$.

392. 1) $\left(\frac{3}{4}a^2-\frac{1}{2}b^3\right)\left(\frac{1}{2}b^3+\frac{3}{4}a^2\right)$;
2) $\left(\frac{2}{3}x^4-\frac{4}{5}y^5\right)\left(\frac{2}{3}x^4+\frac{4}{5}y^5\right)$;
3) $\left(0,5q+\frac{1}{3}p^2\right)\left(0,5q-\frac{1}{3}p^2\right)$;
4) $\left(1,5c^2-\frac{3}{4}b\right)\left(\frac{3}{4}b+1,5c^2\right)$.

393. 1) $(3x^2y-4xy^2)(3x^2y+4xy^2)$; 3) $(7ab+x^2y^3)(7ab-x^2y^3)$;

2) $(5ab^2+2a^2b)(5ab^2-2a^2b)$; 4) $(ab^3-4xy)(ab^3+4xy)$.

394. 1) $(3+x)(3-x)(9+x^2)$; 3) $(4x^2+y^2)(2x+y)(2x-y)$;

2) $(x^2+1)(x+1)(x-1)$; 4) $(3a-2b)(3a+2b)(9a^2+4b^2)$.

Qisqa ko‘paytirish formulalaridan foydalanib hisoblang **(395—396):**

395. 1) $48\cdot52$; 2) $68\cdot72$; 3) $43\cdot37$; 4) $47\cdot53$.

396. 1) $47\cdot33$; 2) $44\cdot36$; 3) $84\cdot76$; 4) $201\cdot199$.

397. Soddalashtiring:

1) $(c-3)^2-(c+3)(3-c)$;

2) $(a+2)^2-(a+2)(2-a)$;

3) $(2x+3y)(2x-3y)+(2x+3y)^2$;

4) $(3a-4b)(3a+4b)-(3a-4b)^2$;

5) $(-b-a)(a+b)+a^2+b^2$;

6) $(b-a)(-a-b)+2b^2$.

398. Ifodaning qiymatini toping:

1) $4m-(m+3)^2+(m-3)(m+3)$, bunda $m=-2,4$;

2) $(3x+4)^2-10x-(x-4)(4+x)$, bunda $x=-0,1$;

3) $2(k-7)(k+5)-(k-5)^2-(k-7)(7+k)$, bunda $k=-\frac{1}{2}$;

4) $(a+3)^2+(a-3)(3+a)-2(a+2)(a-4)$, bunda $a=-\frac{1}{5}$.

399. Tenglamani yeching:

1) $(2x+3)^2-4(x-1)(x+1)=49$;

2) $(3x+4)^2-(3x-1)(1+3x)=49$;

3) $(3x+2)(3x-2)-(3x-4)^2=28$;

4) $(3x+1)^2-(3x-2)(2+3x)=17$.

400. Kvadratning ikki qarama-qarshi tomonining har biri 8 sm ga uzaytirildi, qolgan ikki tomoni esa shuncha qisqartirildi. Shaklning yuzi qanday o‘zgardi?

401. Hisoblang: $\dfrac{5^4\cdot 0,128-5^3\cdot 0,628\cdot 5}{125\cdot 0,25}$.

23-§ *Ko‘phadni ko‘paytuvchilarga ajratishning bir necha usullarini qo‘llash*

Ko‘phadni ko‘paytuvchilarga ajratishda ba’zan bir emas, balki bir necha usullar qo‘llaniladi. Misollar keltiramiz:

1) a^3-a ko‘phadni ko‘paytuvchilarga ajrating:

$$\triangle\ a^3-a=a\left(a^2-1\right)=a\,(a-1)\,(a+1).\ \blacktriangle$$

Bu yerda ikkita usuldan foydalanilgan: umumiy ko‘paytuvchini qavsdan tashqariga chiqarish va kvadratlar ayirmasi formulasini qo‘llash.

2) $(a^2+1)-4a^2$ ko‘phadni ko‘paytuvchilarga ajrating:

$$\triangle\ \left(a^2+1\right)^2-4a^2=\left(\left(a^2+1\right)-2a\right)\left(\left(a^2+1\right)+2a\right)=$$
$$=\left(a^2+1-2a\right)\left(a^2+1+2a\right)=\left(a^2-2a+1\right)\left(a^2+2a+1\right)=$$
$$=(a-1)^2\,(a+1)^2.\ \blacktriangle$$

Bu yerda qo‘shiluvchilar umumiy ko‘paytuvchiga ega emasligi sababli, avval kvadratlar ayirmasi formulasidan foydalanildi, so‘ngra yig‘indi va ayirma kvadratlarining formulalaridan foydalanildi.

3) $4x^2-y^2+4x+2y=\left(4x^2-y^2\right)+(4x+2y)=$
$$=(2x-y)(2x+y)+2\,(2x+y)=(2x+y)(2x-y+2).$$

Birhadlar umumiy ko‘paytuvchiga ega bo‘lmagani va biror formulani qo‘llash mumkin bo‘lmagani uchun, avval guruhlash usulidan foydalanildi, so‘ngra esa kvadratlar ayirmasi formulasi qo‘llanildi.

Ko‘rib chiqilgan bu misollar ko‘phadni ko‘paytuvchilarga ajratishga doir topshiriqlarni bajarishda quyidagi tartibga rioya qilish foydali ekanligini ko‘rsatadi:

1) umumiy ko‘paytuvchini (agar u bor bo‘lsa) qavsdan tashqariga chiqarish;

2) ko‘phadni qisqa ko‘paytirish formulalari bo‘yicha ko‘paytuvchilarga ajratishga urinib ko‘rish;

3) agar oldingi usullar maqsadga olib kelmasa, guruhlash usulini qo‘llashga harakat qilish.

Masala. Tenglikni isbotlang:

$$a^3+b^3=(a+b)\left(a^2-ab+b^2\right). \qquad (1)$$

△ Tenglikning o‘ng tomonidagi qavslarni ochamiz:

$$(a+b)\left(a^2-ab+b^2\right)=a^3-a^2b+ab^2+a^2b-ab^2+b^3=a^3+b^3.$$

Tenglikning o‘ng tomoni chap tomoniga tengligi kelib chiqdi, ya’ni (1) tenglik isbot qilindi. ▲

Xuddi shu kabi

$$a^3-b^3=(a-b)\left(a^2+ab+b^2\right) \qquad (2)$$

tenglikning to‘g‘riligi isbotlanadi.

(1) va (2) tengliklar mos ravishda *kublar yig‘indisi va ayirmasi* deb ataladi. Bu formulalar ham ko‘phadni ko‘paytuvchilarga ajratishda qo‘llaniladi.

Masalan:

1) $27+b^3=(3+b)\left(9-3b+b^2\right)$;

2) $x^4-8xy^3=x\left(x^3-8y^3\right)=x(x-2y)\left(x^2+2xy+4y^2\right)$.

Mashqlar

402. Hisoblang:

1) 47^2-37^2;
2) 54^2-44^2;
3) $50,7^2-50,6^2$;
4) $29,4^2-29,3^2$.

403. (Og‘zaki.) Ko‘paytuvchilarga ajrating:

1) $36-x^2$; 2) a^2-25; 3) y^2-1; 4) $1-b^2$.

404. (Og‘zaki.)Ifodani birhadning kvadrati shaklida tasvirlang:

$100a^2$; $0,01b^2$; $\frac{9}{16}m^2n^2$; $0,25x^6$; $1\frac{9}{16}x^2$; x^4y^6.

Ko‘paytuvchilarga ajrating **(405—416):**

405. 1) $25x^2-9$; 2) $4a^2-9$; 3) $64y^2-36x^2$; 4) $81a^2-16b^2$.

406. 1) c^2d^2-9; 2) a^2b^2-16; 3) $4a^2-9b^2$; 4) $16x^2-25y^2$.

407. 1) $\frac{1}{9}y^2-\frac{16}{25}x^2$; 3) $0,25a^2-49b^2$;

2) $\frac{4}{9}a^2-\frac{1}{16}b^2$; 4) $0,09x^2-16y^2$.

408. 1) $36x^2y^2-1$; 2) x^2y^4-16; 3) $81a^6-49b^4$; 4) $25a^2-9b^6$.

409. 1) a^4-b^4; 2) a^4-b^8; 3) a^4-16; 4) b^4-81.

410. 1) $(a+b)^2-c^2$; 3) $(a+2b)^2-9a^2$;

2) $(m-n)^2-k^2$; 4) $(3x-y)^2-4y^2$.

411. 1) $(a+b)^2-(a-c)^2$; 3) $(2a+b)^2-(2b+a)^2$;

2) $(a+b)^2-(b+c)^2$; 4) $(a-3b)^2-(3a+b)^2$.

412. 1) $9a^2-6a+1$; 3) $36b^2+12b+1$;

2) $1+2c+c^2$; 4) $81-18x+x^2$.

413. 1) $9x^2+24x+16$; 3) $36m^2+12mn+n^2$;

2) $100-60a+9a^2$; 4) $a^2+10ab+25b^2$.

414. 1) $x^4+2x^2y+y^2$; 3) $4c^4+12c^2b^3+9b^6$;

2) $p^4-2p^2q+q^2$; 4) $25a^6+30a^3b+9b^2$.

415. 1) a^4-8a^2+16; 3) $25a^4-10a^2b+b^2$;

2) b^4-18b^2+81; 4) $16-8a^2b^2+a^4b^4$.

416. 1) $-a^2-2a-1$; 3) $-2a^2+8ab-8b^2$;

2) $-9+6b-b^2$; 4) $-12ab-3a^2-12b^2$.

417. Ifodaning son qiymatini toping:

1) $5m^2-10mn+5n^2$, bunda $m=142,\ n=42$;

2) $6m^2+12mn+6n^2$, bunda $m=56,\ n=44$;

3) $-36a^3+4a^2b-\frac{1}{9}ab^2$, bunda $a=4,\ b=48$;

4) $-64a^3-8a^2b-\frac{1}{4}ab^2$, bunda $a=-6,\ b=64$.

418. Tenglamani yeching:

1) $x^2-36=0$; 3) $4x^2+4x+1=0$;

2) $\frac{1}{4}-x^2=0$; 4) $25-10x+x^2=0$.

419. Hisoblang:

1) $101^2-202\cdot 81+81^2$; 3) $\frac{48^2+2\cdot 48\cdot 18+18^2}{48^2-18^2}$;

2) $37^2+126\cdot 37+63^2$; 4) $\frac{85^2-17^2}{85^2+2\cdot 85\cdot 17+17^2}$.

420. Tushirib qoldirilgan shunday uchhadni topingki, tenglik bajarilsin:

1) $x^3+y^3=(x+y)(\ldots)$; 3) $x^3-y^3=(x-y)(\ldots)$;

2) $(x+y)^3=(x+y)(\ldots)$; 4) $(x-y)^3=(x-y)(\ldots)$.

421. Ko‘paytuvchilarga ajrating:

1) x^3-y^3; 3) x^3+27; 5) n^3-64; 7) $1-p^3$;

2) c^3+d^3; 4) a^3-27; 6) a^3+1; 8) $125-b^3$.

Ko‘paytuvchilarga ajrating **(422—424):**

422. 1) $27m^3-8$; 2) $64-125y^3$; 3) $125+\frac{1}{8}b^3$; 4) $64y^3+\frac{1}{27}$.

423. 1) $8a^3+1$; 3) $\frac{1}{27}a^3+64b^6$;

2) $1+27b^3$; 4) $\frac{1}{8}a^6+125b^3$.

424. 1) a^9-b^3; 2) a^6-b^6; 3) x^6-729; 4) $64-y^6$.

Ifodani qisqa ko‘paytirish formulalaridan foydalanib, ikkihad shaklida yozing **(425—426):**

425. 1) $(z+5)\left(z^2-5z+25\right)$; 3) $(2x+3y)\left(4x^2-6xy+9y^2\right)$;

2) $(y+2)\left(y^2-2y+4\right)$; 4) $(4c-5d)\left(16c^2+20cd+25d^2\right)$.

426. 1) $\left(10a^2-1\right)\left(100a^4+10a^2+1\right)$;

2) $\left(a^2b^2-5a\right)\left(a^4b^4+5a^3b^2+25a^2\right)$;

3) $\left(\frac{1}{5}m-n\right)\left(\frac{1}{25}m^2+\frac{1}{5}mn+n^2\right)$;

4) $\left(\frac{1}{2}x-\frac{1}{3}y\right)\left(\frac{1}{4}x^2+\frac{1}{6}xy+\frac{1}{9}y^2\right)$.

427. Ko‘paytuvchilarga ajrating:

1) $\left(8a^3-27b^3\right)-2a\left(4a^2-9b^2\right)$; 3) $\left(a^3+b^3\right)+(a+b)^2$;

2) $\left(64a^3+125b^3\right)+5b\left(16a^2-25b^2\right)$; 4) $\left(a^3-b^3\right)+(a-b)^2$.

428. Hisoblang:

1) $\frac{258^3-147^3}{258^2+258\cdot147+147^2}$; 2) $\frac{17{,}98^2-17{,}98\cdot32{,}02+32{,}02^2}{17{,}98^3+32{,}02^3}$.

429. Ifodaning qiymatini toping:

1) $(x+2)\left(x^2-2x+4\right)-x(x-3)(x+3)$, bunda $x=2$;

2) $(2x-1)\left(4x^2+2x+1\right)-4x\left(2x^2-3\right)$, bunda $x=0{,}5$;

3) $(4x+1)\left(16x^2-4x+1\right)-16x\left(4x^2-5\right)$, bunda $x=\frac{1}{5}$;

4) $x(x+2)(x-2)-(x-3)\left(x^2+3x+9\right)$, bunda $x=\frac{1}{4}$.

430. Tenglamani yeching:

1) $(x+2)\left(x^2-2x+4\right)-x(x-3)(x+3)=26$;

2) $(x-3)\left(x^2+3x+9\right)-x(x+4)(x-4)=21$;

3) $(2x-1)\left(4x^2+2x+1\right)-4x\left(2x^2-3\right)=23$;

4) $(4x+1)\left(16x^2-4x+1\right)-16x\left(4x^2-5\right)=17$.

Ko‘paytuvchilarga ajrating **(431—434):**

431. 1) $3a^3-3$; 2) y^3-y; 3) m^3n-mn^3; 4) $2a^3-2ab^2$.

432. 1) $x^4y^2-x^2y^4$;
2) $7c^2d^2-63c^2b^2$;
3) $8-72x^6y^2$;
4) $32a^4b-2a^2b$.

433. 1) $2a^2+4ab+2b^2$;
2) $2m^2+2n^2-4mn$;
3) $5x^2+10xy+5y^2$;
4) $8p^2-16p+8$;
5) $27a^2b^2-18ab+3$;
6) $12m^5n+24m^4n+12m^3n$.

434. 1) $2c^3+2d^3$;
2) $54x^3-16$;
3) $2cd^3-16c^4$;
4) $\frac{1}{8}a^2-a^5$;
5) $7x^2-56x^2y^3$;
6) $4a^2b+32a^5b$.

435. Hisoblang: $19,7^2-8,3^2+28\cdot 8,6$.

Ko‘paytuvchilarga ajrating **(436—438):**

436. 1) $\left(x^2+1\right)^2-4x^2$;
2) $\left(x^2+2x\right)^2-1$;
3) $4y^2-(y-c)^2$;
4) $81-\left(y^2+6y\right)^2$.

437. 1) $\left(a^2+2ab+b^2\right)-c^2$;
2) $1-\left(x^2-2xy+y^2\right)$;
3) $1-a^2-2ab-b^2$;
4) $4+\left(-x^2-2xy-y^2\right)$.

438. 1) a^2-b^2+a+b;
2) a^2-b^2-a-b;
3) $x-y-x^2+y^2$;
4) x^3+x^2-x-1;
5) $m^5-m^3+m^2-1$;
6) x^4+x^3+x+1.

439. 27^2-14^2 soni 13 ga bo‘linishini isbotlang.

440. n istalgan butun son bo‘lganda $(7n-2)^2-(2n-7)^2$ ifodaning qiymati 5 ga bo‘linishini; 9 ga bo‘linishini isbot qiling.

441. Tenglamani yeching:

1) $(x-3)\left(x^2+3x+9\right)-(3x-17)=x^3-12;$

2) $5x-\left(4-2x+x^2\right)(x+2)+x(x-1)(x+1)=0.$

442. Motorli qayiqning oqim bo‘yicha tezligi 18 km/soat, oqimga qarshi tezligi esa 14 km/soat. Daryo oqimining tezligini va qayiqning turg‘un suvdagi tezligini toping.

O‘zingizni tekshirib ko‘ring!

1. Ifodani standart ko‘phad ko‘rinishida tasvirlang:

$$(a-3)^2+(a-3)(a+3)+6a.$$

2. Ko‘paytuvchilarga ajrating:

1) $xy-2y;$ 2) $16a^2-81;$ 3) $3x^2-6x^3;$

4) $x^2-10x+25;$ 5) $3(x-1)+y(x-1);$ 6) $2a^2-4ab+2b^2.$

3. Ko‘phadni ko‘paytuvchilarga ajrating va uning $a=1,\ b=-\frac{1}{3}$ bo‘lgandagi son qiymatini toping:

$$a^2-3ab+3a-9b.$$

IV bobga doir mashqlar

Ko‘paytuvchilarga ajrating **(443—447)**:

443. 1) $6(a+b)+(a+b)^2;$ 3) $(a-b)+(b-a)^2;$

2) $4(x-y)+3(x-y)^2;$ 4) $(a-b)^2-(b-a).$

444. 1) $3(x+y)(x-y)+(x+y)^2;$ 3) $5(a-b)^2-(a+b)(b-a);$

2) $(x+y)^3-x(x+y)^2;$ 4) $a(a-b)^2-(b-a)^2.$

445. 1) $(y+z)(12x^2+6x)+(y-z)(12x^2+6x)$;

2) $(y-z)(12x^2-6x)+(y-z)(12x^2+6x)$;

3) $(6x^2-3)+7x(6x^2-3)-4y(6x^2-3)$;

4) $2x(8x-4y)-3y(8x-4y)-(8x-4y)$.

446. 1) $18a^2-27ab+14ac-21bc$; 2) $10x^2+10xy+5x+5y$;

3) $35ax+24xy-20ay-42x^2$; 4) $48xz^2+32xy^2-15yz^2-10y^3$.

447. 1) $16ab^2-5b^2c-10c^3+32ac^2$;

2) $6mnk^2+15m^2k-14n^3k-35mn^2$;

3) $-28ac+35c^2-10cx+8ax$;

4) $-24bx-15c^2+40bc+9cx$.

448. Ifodani soddalashtiring:

1) $(2x-1)^2-2(2x-3)^2+17$;

2) $(3x+2)^2-2(x-1)^2-7x^2$;

3) $24y^2-(7y-2)^2+(5y-3)(5y+1)$;

4) $(3y+1)(2y-3)+(2y-3)^2-10y^2$.

449. Ikkita ketma-ket natural son kvadratlari ayirmasining moduli toq son bo‘lishini isbotlang.

450. Kasrni qisqartiring:

1) $\dfrac{53^2-27^2}{79^2-51^2}$; 3) $\dfrac{49^2-2\cdot 49\cdot 29+29^2}{49^2-19^2}$;

2) $\dfrac{38^2-17^2}{47^2-19^2}$; 4) $\dfrac{47^2-3^2}{27^2+2\cdot 27\cdot 13+13^2}$.

451. x va y ning istalgan qiymatlarida $(x+y)(x^2-y^2)=(x-y)(x+y)^2$ tenglik to‘g‘ri bo‘lishini isbotlang.

№ 8 | 1) Oiladagi 6 ta qizning har birining akasi bor. Shu oilada nechta farzand bor?
2) Muhammadjonning akalari qancha bo‘lsa, opalari ham shuncha. Katta opasining ukalari soni singillari sonidan 2 marta ko‘p. Shu oilada nechta o‘g‘il, nechta qiz bor?

IV bobga doir sinov mashqlari — testlar

1. Umumiy ko‘paytuvchini qavsdan tashqariga chiqaring:
$24a^3b^2 - 30a^2b^3$.
A) $6a^2b^2(4a-5b)$; B) $6ab(4a^2b-5ab^2)$; C) $6a^2(4ab^2-5b^3)$;
D) $6b^2(4a^3-5a^2)$; E) $3a^2b(8ab-10b^2)$.

2. Ko‘paytuvchilarga ajrating: $5(a-b)+a^2(a-b)-3(b-a)$.
A) $(a-b)(a^2+2)$; B) $(a-b)(a^2-8)$; C) $(a-b)(8-a^2)$;
D) $(a-b)(a^2+8)$; E) $(b-a)(a^2-8)$.

3. Ko‘paytuvchilarga ajrating: $4a(x-y)+4az+7b(y-x-z)$.
A) $(x-y+z)(7b-4a)$; B) $(y-x-z)(7b+4a)$;
C) $(x-y-z)(4a-7b)$; D) $-(x-y+z)(4a+7b)$;
E) $(x-y+z)(4a-7b)$.

4. Hisoblang: $16,9^2-16,9\cdot3,7-16,9\cdot3,2$.
A) 169; B) 1,69; C) 16,9; D) –1,69; E) –16,9.

5. Hisoblang: $47,8\cdot1,5+1,8\cdot52,2+52,2\cdot1,5+1,8\cdot47,8$.
A) 300; B) 330; C) 150; D) 180; E) 230.

6. Ko‘paytuvchilarga ajrating: $ax+bx-3ay-3by$.
A) $(a+b)(x+3y)$; B) $(a-b)(x+3y)$; C) $(a-b)(x-3y)$;
D) $(a+b)(x-3y)$; E) to‘g‘ri javob berilmagan.

7. Ko‘paytuvchilarga ajrating: $7a(5a-3b)-10a+6b$.
A) $(5a+3b)(7a-2)$; B) $(3b-5a)(7a+2)$; C) $(3b+5a)(7a+2)$;
D) $(5a-3b)(7a+2)$; E) $(5a-3b)(7a-2)$.

8. Ifodaning son qiymatini toping:

$a^3 - a^2b - 3a + 3b$, bunda $a = 2{,}5$; $b = -1{,}5$.

A) 3,25; B) 13; C) –13; D) 5; E) –3,25.

9. Tenglamani yeching: $(3x+2)^2 - (3x-4)^2 = 132$.

A) 4; B) 3; C) –5; D) –4; E) ildizi yo‘q.

10. Tenglamani yeching: $81x^2 - (4-9x)^2 = 56$.

A) –1; B) 1; C) 2; D) –2; E) 3.

11. Ko‘paytuvchilarga ajrating: $8a^3 - 27b^3$.

A) $(2a-3b)^2(2a+3b)$; B) $(2a+3b)^2 \cdot (2a-3b)$;

C) $(2a)^3 - (3b)^3$; D) $(2a-3b)(4a^2+6ab+9b^2)$;

E) $(2a+3b)(4a^2+6ab+9b^2)$.

12. Ko‘paytuvchilarga ajrating: $(a^2+25)^2 - 100a^2$.

A) $(a-5)^3(a+5)$; B) $(a-5)(a+5)^3$; C) $(a+5)^4 - (10a)^2$.

D) $(a-5)^2 \cdot (a+5)$; E) $(a-5)^2 \cdot (a+5)^2$.

13. Hisoblang: $(53^3 + 47^3) : (53^2 - 53 \cdot 47 + 47^2)$.

A) 6; B) 100; C) 600; D) $53^2 + 47^2$; E) 110.

Tarixiy masalalar

① *Abu Ali ibn Sino masalalaridan:*

1) Agar sonni 9 ga bo‘lganda 2 yoki 7 qoldiq qolsa, bunday sonning kvadratini 9 ga bo‘lganda 4 qoldiq chiqadi;

2) Agar sonni 9 ga bo‘lganda 4 yoki 5 qoldiq qolsa, bunday sonning kvadratini 9 ga bo‘lganda 7 qoldiq chiqadi.

3) Agar sonni 9 ga bo‘lganda 1 yoki 8 qoldiq qolsa, bunday sonning kvadratini 9 ga bo‘lganda 1 qoldiq chiqadi.

4) Agar sonni 9 ga bo‘lganda 3 yoki 6 qoldiq qolsa, bunday sonning kvadrati 9 ga qoldiqsiz bo‘linadi.

5) Agar sonni 9 ga bo‘lganda qoldiq 1, 4 yoki 7 bo‘lsa, u holda bunday son kubini 9 ga bo‘lganda qoldiq 1 bo‘ladi.

6) Agar sonni 9 ga bo‘lganda qoldiq 2, 5 yoki 8 bo‘lsa, u holda bunday son kubini 9 ga bo‘lganda qoldiq 8 bo‘ladi.

7) Agat sonni 9 ga bo‘lganda qoldiq 3 yoki 6 bo‘lsa, u holda bunday sonning kubi 9 ga qoldiqsiz bo‘linadi.

8) Kubdan qirra ayirilsa, bu 6 ga karrali son bo‘ladi, ya’ni $n^3–n$ shaklidagi son 6 ga qoldiqsiz bo‘linadi, bunda n—natural son.

② *(Diofant masalasi).* Quyidagi tenglikning to‘g‘riligini ko‘rsating:

$$(a^2+b^2)(c^2+d^2)=(ac\pm bd)^2+(bc\mp ad)^2.$$

③ *L. Eyler masalasi.* Quyidagi tenglikning to‘g‘riligini ko‘rsating:

$$(a^2+b^2+c^2+d^2)(m^2+n^2+p^2+q^2)=(an+bm+cq+dp)^2+$$

$$+(am-bn+cp-dq)^2+(-ap-bq+cm+dn)^2+(aq-bp-cn+dm)^2.$$

Tarixiy ma’lumotlar

Al-Koshiyning „Arifmetika kaliti“ asarida ikkihadni ixtiyoriy natural darajaga ko‘tarish qoidalari berilgan.

Turli algebraik formulalarni isbotlashda, tenglamalarni yechishda geometrik mulohazalardan foydalanish qadimgi Xitoy, Yunoniston, Hindiston, O‘rta Osiyo matematiklari asarlarida uchraydi.

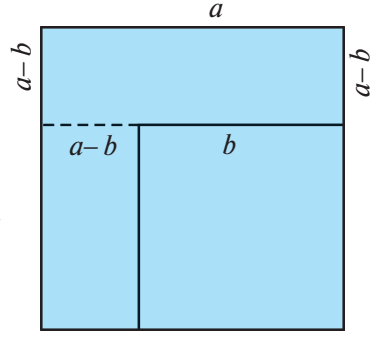
Ular $(a+b)^2=a^2+2ab+b^2$, $(a-b)^2=a^2-2ab+b^2$, $a^2-b^2=(a-b)\times$ $\times(a+b)$ (yoki $(a^2-b^2)=(a-b)^2+2b(a-b)$) kabi ayniyatlarni geometrik usulda isbotlaganlar. Masalan, $a^2-b^2=(a-b)(a+b)$ formulani isbotlashga shunday yondoshilgan: tomoni a ga teng kvadratdan tomoni b ga teng kvadratni qirqib olinsa, qolgan shaklning yuzi: $a(a-b)+b(a-b)=(a-b)(a+b)$ ga, yoki baribir, $(a-b)^2+2b(a-b)$ ga teng bo‘lishi 14- rasmdan ravshan ko‘rinib turibdi.

Demak, $a^2-b^2=(a-b)(a+b)$ formula to‘g‘ri.

To‘g‘ri burchakli uchburchakning tomonlarini butun (yoki ratsional) sonlarda ifodalash uchun xitoy matematiklari miloddan avvalgi birinchi ming yillardayoq

$$\left(\frac{p^2-q^2}{2}\right)^2+(pq)^2=\left(\frac{p^2+q^2}{2}\right)^2$$

tenglikdan foydalanganlar.



14- rasm.

V BOB
ALGEBRAIK KASRLAR

24-§ Algebraik kasr. Kasrlarni qisqartirish

1- masala. Katerning turg‘un suvdagi tezligi soatiga a kilometrga, daryo oqimining tezligi soatiga b kilometrga teng. Katerning daryo oqimi bo‘yicha harakat tezligi uning daryo oqimiga qarshi harakat tezligidan necha marta ortiq?

△ Katerning daryo oqimi bo‘yicha tezligi soatiga $(a+b)$ kilometrga teng; oqimga qarshi tezligi soatiga $(a-b)$ kilometrga teng. Shuning uchun daryo oqimi bo‘yicha harakat tezligi oqimga qarshi harakat tezligidan

$$\frac{a+b}{a-b}$$

marta ortiq bo‘ladi. ▲

$\frac{a+b}{a-b}$ ifoda *algebraik kasr* deyiladi. Bu kasrning surati $a+b$, maxraji esa $a-b$.

Umuman, *surat va maxraji algebraik ifodalar bo‘lgan kasr algebraik kasr deyiladi.*

Algebraik kasrlarga doir yana bir necha misollar keltiramiz:

$$\frac{a}{b};\quad \frac{2}{x+y};\quad \frac{a-b}{c};\quad \frac{x(b+c)}{y(a-c)}.$$

Agar algebraik kasrga kiruvchi harflar o‘rniga biror sonlar qo‘yilsa, u holda zarur hisoblashlar bajarilgandan keyin shu algebraik kasrning *son qiymati* hosil bo‘ladi.

Masalan, $a=10$, $b=8$ bo‘lganda $\frac{a+b}{a-b}$ algebraik kasrning son qiymati $\frac{10+8}{10-8}=\frac{18}{2}=9$ ga teng bo‘ladi.

$\frac{a+b}{a-b}$ algebraik kasrda a va b o‘rniga o‘zaro teng bo‘lmagan $(a\neq b)$

istalgan sonlarni qo‘yish mumkin, chunki $a = b$ bo‘lganda kasrning maxraji nolga aylanadi, nolga bo‘lish esa mumkin emas.

Bundan keyin algebraik kasrga kiruvchi harflar yo‘l qo‘yiladigan (joiz) qiymatlarnigina, ya’ni shu kasrning maxraji nolga teng bo‘lmaydigan qiymatlarnigina qabul qiladi, deb shartlashamiz.

Masalan, $\frac{a}{a(a-1)}$ kasr uchun joiz qiymatlar a ning $a = 0$ va $a = 1$ dan boshqa barcha qiymatlari bo‘ladi.

Kasrning asosiy xossasini bunday yozish mumkin:

$$\frac{a}{b} = \frac{ma}{mb},$$

bu yerda $b \neq 0$, $m \neq 0$.

Bu xossa kasrning surat va maxrajini bir xil algebraik ifodaga ko‘paytirilsa yoki bo‘linsa, unga teng kasr hosil bo‘lishini bildiradi, masalan:

$$\frac{3}{4} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20}, \quad \frac{a+b}{b} = \frac{(a+b) \cdot c}{bc}.$$

Kasrning asosiy xossasidan foydalanib, algebraik kasrni surat va maxrajga bir vaqtda kiruvchi umumiy ko‘paytuvchiga qisqartirish mumkin, masalan:

$$\frac{a(b+c)}{a(b-c)} = \frac{b+c}{b-c}, \quad \frac{(a+b)c}{(a+b)d} = \frac{c}{d}.$$

Kasrlarni soddalashtirish uchun avval ularning surat va maxrajining umumiy ko‘paytuvchisini ajratib olish kerakligiga doir misollar keltiramiz.

2- m a s a l a . Kasrlarni qisqartiring:

1) $\frac{12a^2b}{4ab^2}$; 2) $\frac{m^2 - n^2}{m^2 + mn}$.

△ 1) $12a^2b$ va $4ab^2$ birhadlar $4ab$ umumiy ko‘paytuvchiga ega. Kasrning surat va maxrajini $4ab$ ga bo‘lamiz:

$$\frac{12a^2b}{4ab^2} = \frac{4ab \cdot 3a}{4ab \cdot b} = \frac{3a}{b}.$$

2) m^2-n^2 va m^2+mn ko‘phadlar $m+n$ umumiy ko‘paytuvchiga ega, chunki $m^2-n^2=(m+n)(m-n)$, $m^2+mn=m(m+n)$. Kasrning surat va maxrajini $m+n$ ga bo‘lamiz:

$$\frac{m^2-n^2}{m^2+mn}=\frac{(m+n)(m-n)}{m(m+n)}=\frac{m-n}{m}. \quad \blacktriangle$$

Shunday qilib, kasrlarni qisqartirish uchun bu kasrlarning surat va maxrajini ularning umumiy ko‘paytuvchisiga bo‘lish kerak.

Agar $\frac{a}{b}$ kasrning surat yoki maxrajidagi ishorani qarama-qarshisiga o‘zgartirilsa, u holda berilgan kasrga qarama-qarshi kasr hosil bo‘lishini ta’kidlab o‘tamiz:

$$\frac{-a}{b}=-\frac{a}{b};\quad \frac{a}{-b}=-\frac{a}{b}.$$

Masalan, $\frac{-3}{7}=-\frac{3}{7}$; $\frac{-a}{1-a}=-\frac{a}{1-a}=\frac{a}{a-1}$.

3- masala. $\frac{3a(y-x)}{a^2(x-y)}$ kasrni qisqartiring:

$$\frac{3a(y-x)}{a^2(x-y)}=\frac{-3a(x-y)}{a^2(x-y)}=\frac{-3}{a}=-\frac{3}{a}.$$

Mashqlar

452. Surati x va y sonlarning ko‘paytmasiga, maxraji esa ularning yig‘indisiga teng algebraik kasrni yozing.

453. Surati p va q sonlarning ayirmasiga, maxraji esa ularning ko‘paytmasiga teng bo‘lgan algebraik kasrni yozing.

454. Surati a va b sonlar kvadratlarining ayirmasiga, maxraji esa shu sonlar ayirmasining kvadratiga teng bo‘lgan algebraik kasrni yozing.

455. Surati c va d sonlar kublarining yig‘indisiga, maxraji esa shu sonlar ko‘paytmasining ikkilanganiga teng bo‘lgan algebraik kasrni yozing.

456. Algebraik kasrning son qiymatini toping:

1) $\frac{1}{a}$, bunda $a = 2\frac{3}{5}$;

2) $\frac{b+1}{b-1}$, bunda $b = 1{,}5$;

3) $\frac{a^2+1}{2a}$, bunda $a = -3$;

4) $\frac{a-b}{a+2b}$, bunda $a = 16, b = -3$;

5) $\frac{5a+b^2}{a^2-5b}$, bunda $a = 2, b = 8$;

6) $\frac{-7ab}{3b^2-a^3}$, bunda $a = 3, b = -4$.

457. 1) $S = vt$ formuladan v ni; 2) $p = \frac{m}{V}$ formuladan V ni;

3) $C = 2\pi R$ formuladan R ni;

4) $P = 2(a + b)$ formuladan a ni toping.

458. Har bir yuk mashinasiga t tonnadan kartoshka yuklash mumkin bo‘lsa, har birida p kilogrammdan kartoshka bo‘lgan n qop kartoshkani tashib ketish uchun nechta yuk mashinasi (x) kerak bo‘ladi? x ni $n = 90$, $p = 50$, $t = 1{,}5$ bo‘lganda toping.

459. Mashina soatiga o‘rtacha c metr linoleum ishlab chiqaradi. Agar mashina kuniga n soatdan ishlasa, u a metr linoleumni necha kunda ishlab chiqaradi? Izlanayotgan vaqtni t bilan belgilab, t ni $c = 47$, $a = 11280$ va $n = 16$ bo‘lganda toping.

460. Berilgan ikkita kasrning tengligini ko‘rsating:

1) $\frac{6}{7}$ va $\frac{18}{21}$;

2) $\frac{-3}{5}$ va $\frac{27}{-45}$;

3) $\frac{2}{3}$ va $\frac{2a}{3a}$;

4) $\frac{2a}{7b}$ va $\frac{2a^2b}{7ab^2}$;

5) $\frac{m-n}{m+n}$ va $\frac{m^2-n^2}{(m+n)^2}$;

6) $\frac{a+3b}{c}$ va $\frac{(a+3b)c}{c^2}$.

Kasrni qisqartiring **(461—463):**

461. 1) $\frac{-48}{-56}$; 2) $\frac{-64}{-80}$; 3) $\frac{-121}{55}$; 4) $\frac{28}{-14}$.

462. 1) $\frac{12a}{20}$; 2) $\frac{2c}{3c}$; 3) $\frac{7b}{21b}$; 4) $\frac{4ab}{8ac}$; 5) $\frac{a^2}{2a}$; 6) $\frac{5x}{x^3y}$.

463. 1) $\frac{a^2}{a^3}$; 2) $\frac{b^3}{b^7}$; 3) $\frac{a^5}{a^4}$; 4) $\frac{b^6}{b^4}$.

Kasrni qisqartiring **(464 — 474):**

464. 1) $\frac{6ab}{4a}$; 3) $\frac{a^4b}{ab^3}$; 5) $\frac{12a^4b^2}{18a^3b^3}$;

2) $\frac{14c}{49c}$; 4) $\frac{3a^2b}{9a^3}$; 6) $\frac{25a^3bc^2}{125ac^3}$.

465. 1) $\frac{4(m+n)}{5(m+n)}$; 3) $\frac{2b(m-n)}{8b(m-n)(m-n)}$; 5) $\frac{2(a-b)}{b-a}$;

2) $\frac{7a(a-b)}{5(a-b)}$; 4) $\frac{3a(a+b)}{9a(a+b)(a-b)}$; 6) $\frac{5(x-y)}{15(y-x)}$.

466. 1) $\frac{(a-b)^2}{a-b}$; 3) $\frac{m-n}{(n-m)^2}$; 5) $\frac{3m(1-x)^2}{9m^2(x-1)^2}$;

2) $\frac{m+n}{(m+n)^4}$; 4) $\frac{(2x-3y)^2}{3y-2x}$; 6) $\frac{8a^2b(a-b)}{4a^3b(b-a)^2}$.

467. 1) $\frac{3x+3y}{6c}$; 3) $\frac{2a+2b}{4a-4b}$; 5) $\frac{ac-bc}{ac+bc}$;

2) $\frac{8a}{4m-4n}$; 4) $\frac{12a-3}{6a+9}$; 6) $\frac{a+ab}{a-ab}$.

468. 1) $\frac{a^2}{a^2+ab}$; 3) $\frac{7a+14b}{3a+6b}$; 5) $\frac{3a-6b}{12b-6a}$;

2) $\frac{pq^3}{p^2q-pq^2}$; 4) $\frac{2m^2-mn}{2mn-n^2}$; 6) $\frac{x^2-2xy}{2y^2-xy}$.

469. 1) $\frac{12x^2-30xy}{30x^2-12xy}$; 2) $\frac{36a^2+24ab}{24a^2+36ab}$; 3) $\frac{m^3-3m^2n}{3m^2n-3m^3}$; 4) $\frac{a^3-2a^2b}{2a^3b^2-a^4b}$.

470. 1) $\frac{a^2-b^2}{a+b}$; 3) $\frac{4c^2-9x^2}{2c-3x}$; 5) $\frac{3a(a-b)}{6a^2(b-a)}$;

2) $\frac{a-b}{a^2-b^2}$; 4) $\frac{25-x^2}{5-x}$; 6) $\frac{5a(c^2-4)}{10a^2(2-c)}$.

471. 1) $\frac{8-3c}{9c^2-64}$; 3) $\frac{2y-10}{25-y^2}$; 5) $\frac{b^2-c^2}{b^4n-c^4n}$;

2) $\frac{100-49b^2}{7b+10}$; 4) $\frac{5y-y^2}{25-y^2}$; 6) $\frac{5a^3b+5ab^3}{a^4-b^4}$.

472. 1) $\frac{d^2-6d+9}{d-3}$; 2) $\frac{b+7}{b^2+14b+49}$; 3) $\frac{9-6a+a^2}{3-a}$; 4) $\frac{1-2p}{1-4p+4p^2}$.

473. 1) $\frac{4y^2-4y+1}{4y^2-1}$; 3) $\frac{3a^2-6ab+3b^2}{6a^2-6b^2}$;

2) $\frac{16a^2-1}{16a^2-8a+1}$; 4) $\frac{50m^2+100mn+50n^2}{15m^2-15n^2}$.

474. 1) $\frac{1-a^2}{(a-1)^2}$; 3) $\frac{4y^2-4y+1}{2-4y}$;

2) $\frac{(m-n)^2}{n-m}$; 4) $\frac{5-2x}{4x^2-20x+25}$.

475. Kasrni qisqartiring:

1) $\frac{9c^2-16}{16-24c+9c^2}$; 4) $\frac{36c-c^3}{c^3+12c^2+36c}$;

2) $\frac{16x^2-24xy+9y^2}{9y^2-16x^2}$; 5) $\frac{25b-49b^3}{49b^3-70b^2+25b}$;

3) $\frac{4x^2-4xy+y^2}{y^2-4x^2}$; 6) $\frac{4b^2-12bc+9c^2}{-2ab+3ac}$.

476. Kasrni qisqartiring:

1) $\frac{2a^5-128a^2}{(2a^2+8a+32)(a^4-4a^3)}$; 3) $\frac{3a^3+ab-6a^2b-2b^3}{9a^5-ab^4-18a^4b+2b^5}$;

2) $\frac{2a^4+3a^3+2a+3}{(a^2-a+1)(2a+3)}$; 4) $\frac{3ac^2+3bc^2-3ab^2-3b^3}{6ac^2+6bc^2-6ab^2-6b^3}$.

25- §. Kasrlarni umumiy maxrajga keltirish

Oddiy kasrlarni qo‘shishda avval kasrlarni umumiy maxrajga keltirib olinadi. Masalan, $\frac{1}{4}, \frac{3}{25}, \frac{7}{10}$ kasrlar uchun umumiy maxraj 100 soni bo‘ladi, bu son 4, 25, 10 sonlarining eng kichik umumiy karralisidir.

Algebraik kasrlarning umumiy maxraji shu kasrlar maxrajlarining eng kichik umumiy karralisidir. Kasrlarni umumiy maxrajga keltirishda kasrning asosiy xossasidan foydalaniladi.

1- masala. $\frac{m}{3a^2b}$, $\frac{n}{6ab^2}$ va $\frac{p}{4ac}$ algebraik kasrlarni umumiy maxrajga keltiring.

△ Berilgan kasrlarning umumiy maxraji har bir kasrning maxrajiga bo‘linishi kerak. Demak, u 3 ga, 6 ga, 4 ga, ya’ni 12 ga; a^2 ga, a ga va a ga, ya’ni a^2 ga; b ga va b^2 ga, ya’ni b^2 ga; c ga bo‘linishi kerak.

Shunday qilib, kasrlarning umumiy maxraji 12, a^2, b^2 va c ko‘paytuvchilarni o‘z ichiga olishi kerak. Umumiy maxraj sifatida $12a^2b^2c$ ko‘paytmani olish lozim bo‘ladi. Bu umumiy maxrajni birinchi kasrning maxrajiga bo‘lib, uning surat va maxrajini ko‘paytirish kerak bo‘lgan birhadni topamiz. Bu birhad berilgan *kasrning qo‘shimcha ko‘paytuvchisi* deyiladi. Birinchi kasr uchun bunday birhad $4bc$ ga teng. Xuddi shunday yo‘l bilan ikkinchi va uchinchi kasrlar uchun qo‘shimcha ko‘paytuvchilarni topamiz: $2a$ va $3ab^2$.

Birinchi, ikkinchi va uchinchi kasrlarning surati va maxrajini mos ravishda $4bc$, $2ac$ va $3ab^2$ ga ko‘paytirib, ularni $12a^2b^2c$ umumiy maxrajga keltiramiz:

$$\frac{m}{3a^2b}=\frac{4mbc}{12a^2b^2c},\quad \frac{n}{6ab^2}=\frac{2nac}{12a^2b^2c},\quad \frac{p}{4ac}=\frac{3pab^2}{12a^2b^2c}.\ \blacktriangle$$

2- masala. Kasrlarni umumiy maxrajga keltiring:

$$\frac{a}{x^2-y^2};\quad \frac{b}{2x^2-4xy+2y^2};\quad \frac{c}{3x^2+6xy+3y^2}.$$

△ Kasrlarning maxrajini ko‘paytuvchilarga ajratamiz:

$$x^2 - y^2 = (x - y)\,(x + y);$$

$$2x^2 - 4xy + 2y^2 = 2\left(x^2 - 2xy + y^2\right) = 2\,(x - y)^2;$$

$$3x^2 + 6xy + 3y^2 = 3\left(x^2 + 2xy + y^2\right) = 3\,(x + y)^2.$$

Umumiy maxraj berilgan kasrlarning har birining maxrajiga bo‘linishi kerak.

Umumiy maxraj birinchi kasrning maxrajiga bo‘linishi uchun uning tarkibida $(x - y)\,(x + y)$ ko‘paytma bo‘lishi kerak.

So‘ngra, umumiy maxraj ikkinchi kasrning maxrajiga bo‘linishi kerak va shuning uchun unda $2(x - y)^2$ ko‘paytuvchi bo‘lishi kerak. Demak, birinchi kasr maxrajiga $2(x - y)$ ko‘paytuvchini yozib qo‘yish kerak, ya’ni umumiy maxraj tarkibida

$$2(x - y)^2(x + y)$$

ko‘paytma bo‘lishi lozim.

Umumiy maxraj uchinchi kasrning $3(x + y)^2$ maxrajiga bo‘linishi uchun hosil qilingan ko‘paytmaga $3(x + y)$ ko‘paytuvchini yozib qo‘yish kerak. Demak, uchala kasrning umumiy maxraji

$$6(x - y)^2(x + y)^2$$

ga teng bo‘ladi.

Kasrlarni umumiy maxrajga keltirish uchun ularning surat va maxrajini qo‘shimcha ko‘paytuvchilarga ko‘paytirish kerak, ular esa umumiy maxrajni har bir kasrning maxrajiga bo‘lish yo‘li bilan topiladi; berilgan kasrlar uchun ular mos ravishda quyidagilarga teng:

$$6(x - y)(x + y),\ 3(x + y)^2,\ 2(x - y)^2.$$

Demak, berilgan kasrlarni bunday yozib olish mumkin:

$$\frac{a}{x^2 - y^2} = \frac{6a\,(x - y)\,(x + y)}{6\,(x - y)^2\,(x + y)^2};\quad \frac{b}{2x^2 - 4xy + 2y^2} = \frac{3b\,(x + y)^2}{6\,(x - y)^2\,(x + y)^2};$$

$$\frac{c}{3x^2 + 6xy + 3y^2} = \frac{2c\,(x - y)^2}{6\,(x - y)^2\,(x + y)^2}. \quad \blacktriangle$$

Shunday qilib, algebraik kasrlarni umumiy maxrajga keltirish uchun:

1) berilgan kasrlarning umumiy maxrajini topish;
2) har bir kasr uchun qo‘shimcha ko‘paytuvchini topish;
3) har bir kasrning suratini uning qo‘shimcha ko‘paytuvchisiga ko‘paytirish;
4) har bir kasrni topilgan surat va umumiy maxraj bilan yozish kerak.

Mashqlar

Quyidagi mashqlarda kasrlarni umumiy maxrajga keltiring **(477—484):**

477. 1) $\frac{1}{2}$ va $\frac{2}{3}$; 3) $\frac{5}{7}$ va $\frac{3}{14}$; 5) $\frac{x}{2y}$ va $\frac{x}{3y}$;

2) $\frac{1}{a}$ va $\frac{2}{b}$; 4) $\frac{a}{b}$ va $\frac{a}{2b}$; 6) $\frac{8}{15}$ va $\frac{5}{12}$.

478. 1) $\frac{3}{4a}, \frac{1}{5b}$ va $\frac{7}{20ab}$; 3) $\frac{7}{a^2}$ va $\frac{8}{a^3}$;

2) $\frac{3x}{4y}, \frac{6}{xy}$ va $\frac{4y}{3x}$; 4) $\frac{a}{2x}$ va $\frac{b}{4x^3}$.

479. 1) a va $\frac{b^2}{a}$; 2) $3b$ va $\frac{a^2}{2b}$;

3) a^2 va $\frac{c}{2ab}$; 4) $\frac{b}{3a}, \frac{3c}{2b}$ va ab.

480. 1) $\frac{1}{2p^2}, \frac{1}{6pk}$ va $\frac{1}{3k^2}$; 3) $\frac{2a}{b^2}, \frac{4}{15a^2b}$ va $\frac{3}{20a^3b^4}$;

2) $\frac{1}{6b^2}, \frac{a^2+b^2}{9a^2b^2}$ va $\frac{3-a^2}{18ab^2}$; 4) $\frac{7}{20x^4y}, \frac{31}{6xy^3}$ va $\frac{4}{3x^2y^4}$.

481. 1) $\frac{3}{x+y}$ va $\frac{5}{x}$; 3) $\frac{7x}{2(x-1)}$ va $\frac{5x}{x-1}$;

2) $\frac{6}{a-1}$ va $\frac{2}{a}$; 4) $\frac{2a^2}{3(a+1)}$ va $\frac{5a^2}{4(a+1)}$.

482. 1) $\frac{1}{x-y}$ va $\frac{1}{x+y}$;

2) $\frac{7a}{3x-y}$ va $\frac{6b}{3x+y}$;

3) $\frac{5x}{2x-2}$ va $\frac{3}{4x-4}$.

4) $\frac{3x}{4x+4y}$ va $\frac{x}{8x+8y}$;

483. 1) $\frac{3b}{b-2}$ va $\frac{4}{b^2-4}$;

2) $\frac{7a}{x^2-9}$ va $\frac{a}{x+3}$;

3) $\frac{1}{1-a}$, $\frac{2a}{1+a}$ va $\frac{a^2}{1-a^2}$;

4) $\frac{6x}{x-y}$, $\frac{7xy}{x+y}$ va $\frac{3}{x^2-y^2}$.

484. 1) $\frac{m}{2m+2n}$, $\frac{n}{8m-8n}$ va $\frac{mn}{6m^2-6n^2}$;

2) $\frac{2c}{5b-5c}$, $\frac{3a^2}{35b^2-35c^2}$ va $\frac{7b}{14b+14c}$;

3) $\frac{1}{a^2-4b^2}$, $\frac{1}{3a^2+6ab}$ va $\frac{1}{2ab-a^2}$;

4) $\frac{5}{4x-4}$, $\frac{4x}{1-x^2}$ va $\frac{1}{3x^2+3x}$.

№ 9 | Bir qurt yerdan daraxtning uchiga chiqmoqchi bo‘libdi. Daraxt bo‘ylab kechasi u 2 m balandlikka chiqqach, kunduzi esa 1m pastga tushar ekan. 9- kechada u daraxtning uchiga chiqib olibdi. Daraxtning balandligi necha metr ekan?

26-§ *Algebraik kasrlarni qo‘shish va ayirish*

Bir xil maxrajli kasrlarni qo‘shish va ayirish qoidalarini bunday yozish mumkin:

$$\frac{a}{m}+\frac{b}{m}=\frac{a+b}{m};$$

$$\frac{a}{m}-\frac{b}{m}=\frac{a-b}{m}.$$

1- masala. $\frac{a-b}{a+b}$, $\frac{2a-b}{a+b}$ va $\frac{a-2b}{a+b}$ kasrlarni qo‘shing.

$\triangle\ \dfrac{a-b}{a+b}+\dfrac{2a-b}{a+b}+\dfrac{a-2b}{a+b}=\dfrac{a-b+2a-b+a-2b}{a+b}=\dfrac{4a-4b}{a+b}=\dfrac{4\left(a-b\right)}{a+b}.$ ▲

2- masala. $\dfrac{a^2}{a+b}$ va $\dfrac{b^2}{a+b}$ kasrlarning ayirmasini toping.

$\triangle\ \dfrac{a^2}{a+b}-\dfrac{b^2}{a+b}=\dfrac{a^2-b^2}{a+b}=\dfrac{\left(a+b\right)\left(a-b\right)}{a+b}=a-b.$ ▲

Har xil maxrajli kasrlarni qo‘shish va ayirish uchun bu kasrlarni umumiy maxrajga keltirish va bir xil maxrajli kasrlarni qo‘shish yoki ayirish qoidasidan foydalanish kerak.

3- masala. $\dfrac{1}{a^3}$, $\dfrac{1}{2a^2b}$ va $\dfrac{1}{3ab^2}$ kasrlarni qo‘shing.

△ Berilgan kasrlarning umumiy maxraji $6a^3b^2$ ko‘paytma bo‘ladi. Demak,

$$\frac{1}{a^3}+\frac{1}{2a^2b}+\frac{1}{3ab^2}=\frac{6b^2}{6a^3b^2}+\frac{3ab}{6a^3b^2}+\frac{2a^2}{6a^3b^2}=\frac{2a^2+3ab+6b^2}{6a^3b^2}.$$ ▲

4- masala. $\dfrac{a}{3b^2c}$ va $\dfrac{c}{15ab^2}$ kasrlarning ayirmasini toping.

$\triangle\ \dfrac{a}{3b^2c}-\dfrac{c}{15ab^2}=\dfrac{5a^2}{15ab^2c}-\dfrac{c^2}{15ab^2c}=\dfrac{5a^2-c^2}{15ab^2c}.$ ▲

5- masala. $\dfrac{1}{x^2-x}$ va $\dfrac{3}{x^2-1}$ kasrlarni qo‘shing.

△ Kasrlarning maxrajlarida turgan ko‘phadlarni ko‘paytuvchilarga ajratamiz:

$$x^2-x=x\left(x-1\right),\ x^2-1=\left(x-1\right)\left(x+1\right).$$

Kasrlarning umumiy maxraji $x\left(x-1\right)\left(x+1\right)$ ko‘paytma bo‘ladi. Kasrlarni umumiy maxrajga keltirib, topamiz:

$$\frac{1}{x^2-x}+\frac{3}{x^2-1}=\frac{1}{x(x-1)}+\frac{3}{(x-1)(x+1)}=\frac{x+1}{x(x^2-1)}+\frac{3x}{x(x^2-1)}=$$

$$=\frac{x+1+3x}{x(x^2-1)}=\frac{4x+1}{x(x^2-1)}. \blacktriangle$$

Shunday qilib, turli maxrajli kasrlarni qo‘shish va ayirishni ushbu tartibda bajarish mumkin:

1) kasrlarning umumiy maxraji topiladi;
2) kasrlarni umumiy maxrajga keltiriladi;
3) hosil bo‘lgan kasrlarni qo‘shiladi;
4) mumkin bo‘lsa, natijani soddalashtiriladi.

6- masala. $\frac{1}{a^2+4a+4}-\frac{4}{a^4+4a^3+4a^2}+\frac{4}{a^3+2a^2}$ ifodaning son qiymatini $a=0{,}5$ bo‘lganda hisoblang.

△ Berilgan ifodani quyidagicha almashtirish mumkin:

$$\frac{1}{(a+2)^2}-\frac{4}{a^2(a^2+4a+4)}+\frac{4}{a^2(a+2)}=\frac{1}{(a+2)^2}-\frac{4}{a^2(a+2)^2}+\frac{4}{a^2(a+2)}=$$

$$=\frac{a^2-4+4(a+2)}{a^2(a+2)^2}=\frac{a^2+4a+4}{a^2(a+2)^2}=\frac{1}{a^2}.$$

Demak, izlanayotgan son qiymat: $\frac{1}{0{,}5^2}=\frac{1}{0{,}25}=\frac{100}{25}=4.$ ▲

Mashqlar

Kasrlarning yig‘indisini (ayirmasini) toping **(485—491):**

485. 1) $\frac{p}{q^2}+\frac{3p}{q^2}$;

2) $\frac{8a}{b^3}-\frac{3a}{b^3}$;

3) $\frac{a}{a+b}+\frac{c}{a+b}$;

4) $\frac{x}{n+a}-\frac{y}{n+a}$.

486. 1) $\frac{c+d}{2a}+\frac{2c-d}{2a}$; 2) $\frac{a+2b}{3c^2}+\frac{5a-2b}{3c^2}$; 3) $\frac{a+b}{2c}-\frac{a-b}{2c}$;

4) $\frac{10a-b}{a^3}-\frac{3a-b}{a^3}$; 5) $\frac{(1+b)^2}{5d}+\frac{(1-b)^2}{5d}$; 6) $\frac{(2+a)^2}{a^2b}-\frac{(2-a)^2}{a^2b}$.

487. 1) $\frac{2}{5}+\frac{3}{7}$; 3) $\frac{2}{3a}+\frac{1}{a}$; 5) $\frac{c}{15a}+\frac{d}{3}$;

2) $\frac{4}{7}-\frac{5}{28}$; 4) $\frac{1}{b}-\frac{2}{5b}$; 6) $\frac{a}{4}-\frac{b}{12d}$.

488. 1) $\frac{m}{2}-\frac{1}{n}$; 2) $\frac{3}{a}+\frac{b}{5}$; 3) $5-\frac{1}{a}$; 4) $\frac{2}{b}+7$.

489. 1) $5-\frac{2}{b}+\frac{3}{b^2}$; 2) $\frac{2}{c}+4-\frac{3}{c^2}$; 3) $d-\frac{c}{d}+\frac{c^2}{d^2}$; 4) $\frac{m}{n}-k+\frac{m^2}{n^2}$.

490. 1) $\frac{1}{ab}+\frac{1}{bc}$; 3) $\frac{a}{bc}-\frac{a}{bd}$; 5) $\frac{3}{m^2}+\frac{4}{mn}$;

2) $\frac{1}{mn}-\frac{1}{mk}$; 4) $\frac{b}{ac}+\frac{b}{cd}$; 6) $\frac{2}{mn}-\frac{3}{n^3}$.

491. 1) $\frac{3c}{4a^3b}+\frac{5d}{6ab^3}$; 3) $\frac{2}{3y^3}-\frac{1}{6x^2y}+\frac{5}{12xy^2}$; 5) $\frac{a}{b^2}+\frac{b}{c^2}+\frac{c}{a^2}$;

2) $\frac{2a}{9b^4}-\frac{7c}{6a^3b}$; 4) $\frac{5}{7x^2y}-\frac{3}{4xy^2}+\frac{11}{14x^2y^2}$; 6) $\frac{b}{c}+\frac{b}{c^2d}+\frac{b}{cd^2}$.

Algebraik kasrlarni qo‘shing va ayiring **(492—503):**

492. 1) $\frac{2x}{3(a-b)}+\frac{x}{a-b}$; 3) $\frac{2a^2}{3(a+1)}+\frac{5a^2}{4(a+1)}$;

2) $\frac{7x}{2(x-1)}-\frac{5x}{x-1}$; 4) $\frac{4y}{5(y-3)}-\frac{5x}{2(y-3)}$.

493. 1) $\frac{5}{2x-2}+\frac{3}{4x-4}$; 3) $\frac{a}{3a+3b}-\frac{2a}{6a+6b}$;

2) $\frac{7}{5b+5}-\frac{3}{10b+10}$; 4) $\frac{3x}{4x+4y}-\frac{x}{8x+8y}$.

494. 1) $\frac{3}{a^2+a}+\frac{5a}{ab+b}$; 3) $\frac{y+a}{b^2+ba}+\frac{y-b}{ab+a^2}$;

2) $\frac{5b}{ax+ay}-\frac{2a}{bx+by}$; 4) $\frac{y-b}{a^2-ab}-\frac{y-a}{ab-b^2}$.

495. 1) $\frac{3}{x+y}-\frac{5}{x}$; 3) $\frac{1}{x(x-3)}+\frac{1}{x(x+3)}$;

2) $\frac{6}{a}-\frac{10}{a-1}$; 4) $\frac{4}{5(a-b)}-\frac{7}{8(a+b)}$.

496. 1) $\frac{a}{1-b^2}+\frac{1}{1+b}$; 3) $\frac{5+p^2}{p^2-36}-\frac{p}{6+p}$;

2) $\frac{2}{x^2-9}+\frac{1}{x+3}$; 4) $\frac{2x}{x-4}-\frac{5x-2}{x^2-16}$.

497. 1) $\frac{2x}{x-4}-\frac{5x-2}{16-x^2}$; 3) $\frac{c^2-8}{2c+3}-\frac{16c-2c^3}{9-4c^2}$;

2) $\frac{12n-5}{n^2-49}+\frac{6}{7-n}$; 4) $\frac{21y^2+1}{1-9y^2}-\frac{y}{3y-1}$.

498. 1) $\frac{3}{a+2}+\frac{2a}{(a+2)^2}$; 2) $\frac{a}{(3a+1)^2}+\frac{4}{3a+1}$.

499. 1) $\frac{2y+8}{y^2-4y+4}-\frac{7}{y-2}$; 4) $\frac{4}{(m-n)^2}-\frac{7}{n-m}$;

2) $\frac{4-5x}{1+6x+9x^2}-\frac{2}{3x+1}$; 5) $\frac{2a}{25-10a+a^2}+\frac{10}{a^2-25}$;

3) $\frac{7}{(a-b)^2}-\frac{5}{b-a}$; 6) $\frac{1}{x^2-6x+9}+\frac{1}{(x+3)^2}$.

500. 1) $a+\frac{a}{a-1}$; 2) $b-\frac{b}{b-2}$; 3) $c+1-\frac{c^2}{c-1}$; 4) $\frac{a^2}{a+1}-a+1$.

501. 1) $\frac{7}{a+b}+\frac{8}{a-b}-\frac{16b}{a^2-b^2}$;

2) $\frac{6x}{x^2-y^2}-\frac{3}{x-y}-\frac{4}{x+y}$;

3) $\frac{3}{a+3}+\frac{2}{3-a}-\frac{6}{a^2-9}$;

4) $\frac{3}{4a^2-9}-\frac{8}{2a+3}-\frac{7}{3-2a}$.

502. 1) $\frac{a+b}{a}-\frac{a}{a-b}-\frac{b}{a^2-ab}$;

2) $\frac{5b-1}{3b^2-3}+\frac{b+2}{2b+2}-\frac{b+1}{b-1}$;

3) $\frac{6a}{9a^2-1}+\frac{3a+1}{3-9a}+\frac{3a-1}{6a+2}$;

4) $\frac{7}{m}-\frac{4}{m-2n}-\frac{m-n}{4n^2-m^2}$;

5) $x-\frac{xy}{x+y}-\frac{x^3}{x^2-y^2}$;

6) $a-2+\frac{4a}{2+a}-\frac{a^3+b}{a^2+2a}$.

503. 1) $\frac{a+1}{a^3-1}-\frac{1}{a^2+a+1}$;

2) $\frac{a^2+4}{a^3+8}-\frac{1}{a+2}$;

3) $\frac{a+b}{a^2-ab+b^2}-\frac{1}{a+b}$;

4) $\frac{m^2-3m+9}{m^3-27}-\frac{1}{m-3}$.

504. Ifodani soddalashtirib, so‘ngra son qiymatini toping:

1) $\frac{8a^2}{a^3-1}+\frac{a+1}{a^2+a+1}$, bunda $a=2$;

2) $\frac{3c^2-c+3}{c^3-1}-\frac{c-1}{c^2+c+1}+\frac{2}{1-c}$, bunda $c=1\frac{1}{2}$.

27- §. *Algebraik kasrlarni ko‘paytirish va bo‘lish*

Algebraik kasrlarni ko‘paytirish va bo‘lish ham oddiy kasrlarni ko‘paytirish va bo‘lish qoidalari bo‘yicha bajariladi:

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd};$$

$$\frac{a}{b}:\frac{c}{d}=\frac{ad}{bc}.$$

1- masala. Kasrlarni ko‘paytiring:

$$\frac{1}{2xy},\ \frac{4x^2y^3}{5z} \text{ va } \frac{10z^2}{3x^3}.$$

$$\triangle\ \frac{1}{2xy}\cdot\frac{4x^2y^3}{5z}\cdot\frac{10z^2}{3x^3}=\frac{1\cdot 4x^2y^3\cdot 10z^2}{2xy\cdot 5z\cdot 3x^3}=\frac{4y^2z}{3x^2}.\ \blacktriangle$$

2- masala. $\frac{a-b}{a^2+ab}$ va $\frac{b^2+ab}{(a-b)^2}$ kasrlarni ko‘paytiring.

$\triangle$ Ko‘paytuvchilarga ajratib, topamiz:

$$\frac{a-b}{a^2+ab}\cdot\frac{b^2+ab}{(a-b)^2}=\frac{(a-b)b(a+b)}{a(a+b)(a-b)^2}=\frac{b}{a(a-b)}.\ \blacktriangle$$

3- masala. $\frac{m+n}{9m^2n^3}$ va $\frac{m^2-n^2}{27mn^2}$ kasrlarni bo‘ling.

$$\triangle\ \frac{m+n}{9m^2n^3}:\frac{m^2-n^2}{27mn^2}=\frac{(m+n)\cdot 27mn^2}{9m^2n^3(m^2-n^2)}=\frac{(m+n)3}{mn(m-n)(m+n)}=\frac{3}{mn(m-n)}.\ \blacktriangle$$

Algebraik kasrni darajaga ko‘tarishda ushbu formuladan foydalaniladi:

$$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}.$$

Masalan,

$$\left(\frac{4a^2}{b}\right)^2=\frac{16a^4}{b^2};\quad \left(\frac{a+b}{3c}\right)^3=\frac{(a+b)^3}{27c^3}.$$

Mashqlar

Kasrlarni ko‘paytiring **(505—506):**

505. 1) $\frac{85}{24}\cdot\frac{72}{17}$; 2) $\frac{256}{169}\cdot\frac{13}{64}$; 3) $50\cdot\frac{7}{625}$; 4) $\frac{5}{26}\cdot 39$.

506. 1) $\frac{a^3b}{c}\cdot\frac{c^2}{a^4}$; 3) $\frac{6a}{5b}\cdot\frac{15c}{2d}$; 5) $\frac{2a}{3b}\cdot 3c$;

2) $\frac{m^2n^2}{k}\cdot\frac{k^3}{m^3n^3}$; 4) $\frac{4m}{9n}\cdot\frac{27k}{16d}$; 6) $14a^2\cdot\frac{b^2}{7c^3}$.

507. Kasrlarni bo‘ling:

1) $\frac{3}{5}:\frac{3}{7}$; 3) $\frac{a}{8}:\frac{1}{3}$; 5) $\frac{2}{a}:\frac{6}{7}$;

2) $\frac{11}{12}:\frac{2}{5}$; 4) $\frac{6}{c}:\frac{m}{13}$; 6) $\frac{9}{35}:\frac{b}{5}$.

508. Kasrlarni bo‘ling:

1) $\frac{8}{17}:\frac{8}{17}$; 3) $\frac{3a}{7b}:\frac{a}{b}$; 5) $\frac{2a}{3b}:\frac{a^2}{bc}$;

2) $\frac{a}{b}:\frac{a}{b}$; 4) $\frac{c}{2d}:\frac{4c^2}{5d}$; 6) $\frac{5m}{n^2}:\frac{10m^3}{n}$.

509. Kasrlarni bo‘ling:

1) $\frac{17}{12}:\frac{34}{39}$; 3) $\frac{4}{13}:5$; 5) $12:\frac{8}{9}$;

2) $\frac{54}{25}:\frac{81}{75}$; 4) $\frac{a}{b}:c$; 6) $a:\frac{b}{c}$.

510. Kasrlarni bo‘ling:

1) $\frac{a^2b}{c}:\frac{a^4}{c^2}$; 3) $\frac{4a}{5b}:\frac{12c}{25d}$; 5) $\frac{6a}{5b}:(5c)$;

2) $\frac{mn}{k}:\frac{m^2n^2}{k^3}$; 4) $\frac{8m}{9n}:\frac{16k}{27d}$; 6) $12a^2:\frac{4d}{5c^2}$.

Ko‘rsatilgan amallarni bajaring **(511—517):**

511. 1) $\left(\frac{5a}{7b}\right)^2\cdot\frac{14b^2}{25a^3}$; 3) $\frac{2a^2}{5b^2}:\frac{12a^2}{15b^2}$; 5) $\left(\frac{ab}{cd}\right)^2\cdot acd$;

2) $\left(\frac{3a^2}{2b}\right)^3\cdot\frac{16b^3}{21a^4}$; 4) $\frac{3a^3}{7b}:\frac{9a^4}{21b}$; 6) $abc^2\cdot\left(\frac{ab}{cd}\right)^2$.

512. 1) $\frac{8a^2b}{9c}\cdot\frac{36c^3}{5a^3b}$; 3) $\frac{16x^2y}{7z}:\frac{20xy^3}{21z^2}$; 5) $\frac{18m^3n^5}{7k}:\left(9n^2\right)$;

2) $\frac{7b^4}{9c^5y}:\frac{35b^4c^2}{18c^4y^2}$; 4) $\frac{46d^3c}{15a}:\frac{23dc^2}{5a^3}$; 6) $24k^2:\frac{12m^4k^2}{11p^3n}$.

513. 1) $\frac{3x^2y}{4a^2b}\cdot 4a^2b$; 3) $15xy:\frac{30xy}{7a^2b}$;

2) $\frac{5a^2b}{7xy^2}\cdot 14xy^2$; 4) $\frac{7x^2y}{2a^2b}:\left(14x^2y\right)$.

514. 1) $\frac{7-x}{a+b}\cdot\frac{a-b}{7-x}$; 3) $\frac{c+d}{c-d}:\frac{c}{c-d}$; 5) $\frac{a^2-ab}{b}\cdot\frac{b}{a}$;

2) $\frac{x-y}{2a}\cdot\frac{4b}{x-y}$; 4) $\frac{a-b}{2b}:\frac{a-b}{6b^2}$; 6) $\frac{ab+b^2}{9}:\frac{b^2}{3a}$.

515. 1) $\frac{a+1}{b}\cdot\frac{4b^2}{a^2-1}$; 4) $\frac{5m}{m^2-n^2}:\frac{15m^3}{m-n}$;

2) $\frac{1-a}{3b^2}\cdot\frac{b^3}{1-a^2}$; 5) $\frac{3\left(x+y\right)}{4y^2\left(x^2+y^2\right)}\cdot\frac{x^2+y^2}{x^2-y^2}$;

3) $\frac{a^2-b^2}{9b^2}:\frac{a+b}{3b}$; 6) $\frac{5\left(a-b\right)}{3\left(a^2+b^2\right)}:\frac{\left(a-b\right)^2}{a^2+b^2}$.

516. 1) $\frac{a^2-b^2}{3a+3b}\cdot\frac{3a^2}{5b-5a}$; 4) $\frac{3n^2-3m^2}{n^2+np}\cdot\frac{6m-6n}{n+p}$;

2) $\frac{5x^2-5y^2}{x^2+y^2}\cdot\frac{3x^2}{10y-10x}$; 5) $\frac{a^2+b^2}{x^3+x^2y}\cdot\frac{x^2-y^2}{a^4-b^4}$;

3) $\frac{a^2-25}{a^2-3a}:\frac{a+5}{9-a^2}$; 6) $\frac{a^2+b^2}{a^2-ab}:\frac{a^4b-b^5}{a^2b-ab^2}$.

517. 1) $\dfrac{a-5}{a^2+6a+9}\cdot\dfrac{(a+3)^2}{a^2-25}$; 3) $\dfrac{a^2-49}{a^2+2ab+b^2}\cdot\dfrac{a+b}{a-7}$;

2) $\dfrac{b^2-8b+16}{b+3}:\dfrac{(b-4)^2}{b^2-9}$; 4) $\dfrac{a^2-2a+1}{2a+1}:\dfrac{a-1}{4a^2-1}$.

28-§ *Algebraik kasrlar ustida birgalikda bajariladigan amallar*

Algebraik kasrlar ustida birgalikda bajariladigan amallarga doir misollar ko‘ramiz.

1- masala. Ifodani soddalashtiring: $\left(\dfrac{a+1}{2a-2}-\dfrac{1}{2a^2-2}\right)\cdot\dfrac{2a+2}{a+2}$.

△ Qavs ichidagi ifodalarni soddalashtiraylik:

$$\frac{a+1}{2a-2}-\frac{1}{2a^2-2}=\frac{a+1}{2(a-1)}-\frac{1}{2(a^2-1)}=\frac{(a+1)^2-1}{2(a^2-1)}=$$

$$=\frac{(a+1-1)(a+1+1)}{2(a^2-1)}=\frac{a(a+2)}{2(a+1)(a-1)}.$$

Ko‘paytmani topamiz:

$$\frac{a(a+2)}{2(a+1)(a-1)}\cdot\frac{2a+2}{a+2}=\frac{a(a+2)2(a+1)}{2(a+1)(a-1)(a+2)}=\frac{a}{a-1}.$$ ▲

2- masala. Ko‘rsatilgan amallarni bajaring:

$$\left(\frac{a+b}{a-b}-\frac{a-b}{a+b}\right):\left(\frac{a+b}{a-b}-1\right).$$

Birinchi qavs ichidagi amalni bajaramiz:

$$\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{(a+b)^2-(a-b)^2}{(a-b)(a+b)}=\frac{(a+b+a-b)(a+b-a+b)}{a^2-b^2}=$$

$$=\frac{2a\cdot 2b}{a^2-b^2}=\frac{4ab}{a^2-b^2}.$$

Ikkinchi qavs ichidagi amalni bajaramiz:

$$\frac{a+b}{a-b}-1=\frac{a+b-a+b}{a-b}=\frac{2b}{a-b}.$$

Bo‘lamiz:

$$\frac{4ab}{a^2-b^2}:\frac{2b}{a-b}=\frac{4ab(a-b)}{(a^2-b^2)2b}=\frac{2a}{a+b}.\blacktriangle$$

3- masala. Hovuz birinchi quvur orqali a soatda, ikkinchisi orqali b soatda to‘ladi. Agar bir vaqtda ikkala quvurni ochib qo‘yilsa, hovuz necha soatda to‘ladi?

$\triangle$ Hovuzning hajmi V bo‘lsin, deylik. Bir soatda birinchi quvur $\frac{V}{a}$ ga teng hajmni, ikkinchisi $\frac{V}{b}$ ga teng hajmni to‘ldiradi, ikkala quvur esa bir soatda $\frac{V}{a}+\frac{V}{b}$ ga teng hajmni to‘ldiradi. Qidirilayotgan vaqt t bo‘lsin. t soatda ikkala quvur hovuzni butunlay to‘ldirishi kerak, ya’ni

$$\left(\frac{V}{a}+\frac{V}{b}\right)\cdot t=V.$$

Tenglikning ikkala qismini V ga bo‘lib,

$$\left(\frac{1}{a}+\frac{1}{b}\right)t=1$$

ni hosil qilamiz. Qavs ichida turgan kasrlarning yig‘indisi $\frac{a+b}{ab}$ ga teng. Shuning uchun $\frac{a+b}{ab}\cdot t=1$, bundan $t=\frac{ab}{a+b}.\blacktriangle$

Mashqlar

Ko‘rsatilgan amallarni bajaring **(518—523):**

518. 1) $\left(\frac{a}{2}-\frac{a}{3}\right)\cdot\frac{1}{a^2}$; 3) $\frac{a-b}{a+b}\left(\frac{a}{5}+\frac{b}{5}\right)$; 5) $1:\left(1+\frac{1}{a}\right)$;

2) $\frac{a^2}{3}\cdot\left(\frac{2}{a}+\frac{2}{a^2}\right)$; 4) $\frac{ab}{a-b}\left(\frac{1}{b}-\frac{1}{a}\right)$; 6) $b:\left(b+\frac{1}{b}\right)$.

519. 1) $\left(1+\frac{1}{a}\right):\left(1-\frac{1}{a}\right)$; 3) $\left(\frac{b}{a}+\frac{a}{b}-2\right):\left(\frac{1}{b}-\frac{1}{a}\right)$;

2) $\left(a+\frac{a}{b}\right)\left(a-\frac{a}{b}\right)$; 4) $\left(\frac{m}{n}+\frac{n}{m}+2\right)\left(1+\frac{m-n}{m+n}\right)$.

520. 1) $\left(1-\frac{a-b}{a+b}\right)\left(2+\frac{2b}{a-b}\right)$;

2) $\left(1+\frac{a+b}{a-b}\right)\left(2-\frac{2a}{a+b}\right)$;

3) $\left(\frac{6}{a-b}-\frac{5}{a+b}\right)\cdot\frac{a-b}{a+11b}$;

4) $\left(\frac{3}{c}+\frac{3}{c+d}\right)\cdot\frac{c}{18(2c+d)}$.

521. 1) $\left(\frac{2m+1}{2m-1}-\frac{2m-1}{2m+1}\right):\frac{4m}{10m-5}$;

2) $\left(\frac{z+6}{3z+9}-\frac{1}{z+3}\right):\frac{z+2}{27z}$;

3) $\frac{y-1}{y}:\left(\frac{y^2+1}{y^2+2y}-\frac{2}{y+2}\right)$;

4) $\frac{m-2}{m-5}:\left(\frac{m^2+24}{m^2-25}-\frac{4}{m-5}\right)$.

522. 1) $\frac{a^2+ab}{a^2+b^2}\left(\frac{a}{a-b}-\frac{b}{a+b}\right)$;

2) $\frac{ab-b^2}{a^2+b^2}\left(\frac{a}{a+b}+\frac{b}{a-b}\right)$;

3) $\left(\frac{c+d}{c}-\frac{2c}{c-d}\right)\cdot\frac{d-c}{c^2+d^2}$;

4) $\left(\frac{2c}{c+d}+\frac{d-c}{c}\right)\cdot\frac{c+d}{c^2+d^2}$.

523. 1) $\left(\frac{a+1}{2a-2}+\frac{6}{2a^2-2}-\frac{a+3}{2a+2}\right)\cdot\frac{4a^2-4}{3}$;

2) $\left(\frac{b}{a^2+ab}+\frac{2}{a+b}+\frac{a}{b^2+ab}\right):\frac{a^2-b^2}{4ab}$;

3) $\frac{a^2-c^2}{a+b}\cdot\frac{a^2-b^2}{ac+c^2}\cdot\left(a+\frac{ac}{a-c}\right)$;

4) $\frac{c^2-ac}{a^2-b^2}\cdot\frac{a-b}{c^2-a^2}:\left(c-\frac{ac}{a+c}\right)$.

524. Hajmi V bo‘lgan muz bo‘lagining massasi p kilogrammga teng. Hajmi V_1 bo‘lgan bo‘lakning massasi nimaga teng?

525. Avtomobil soatiga v km tezlik bilan harakat qilib, s kilometr yo‘l bosib o‘tdi. Agar mototsiklchining tezligi soatiga u kilometr bo‘lsa, shu vaqt ichida u qancha yo‘l bosib o‘tadi?

526. Motorli qayiqning turg‘un suvdagi tezligi soatiga v kilometr, daryo oqimining tezligi esa v_1 kilometr. Qayiq oqim bo‘yicha harakat qilib, s kilometr o‘tdi. Motorli qayiq oqimga qarshi shu vaqt ichida qancha masofani bosib o‘tadi?

527. *(Qadimiy masala.)* Ikki buyumdan birining 10 tasi bir dinor va ikkinchisining 15 tasi bir dinor. Bir dinorga ikkala buyumdan bir xil miqdorda necha donadan sotib olish mumkin?

O‘zingizni tekshirib ko‘ring!

1. Harflarning joiz qiymatlarini toping:

$$\frac{a}{b};\ \frac{3}{a-1};\ \frac{a}{b+2}.$$

2. Amallarni bajaring:

1) $4a+\frac{1-4a^2}{a}$;

2) $\frac{a+b}{a-b}-\frac{a-b}{a+b}$;

3) $\frac{2a-4}{3b}\cdot\frac{6b}{a-2}$;

4) $\frac{a^2-b^2}{b^2}:\frac{a+b}{b}$.

3. Ifodani soddalashtiring va uning $x=2\frac{2}{3}$ bo‘lgandagi son qiymatini toping:

$$\frac{1+2x}{x-3}-\frac{x^2+3x}{5}\cdot\frac{10}{x^2-9}.$$

V bobga doir mashqlar

Kasrlarni umumiy maxrajga keltiring:

528. 1) $\frac{5a}{a^3-27}$, $\frac{a-3}{a^2+3a+9}$ va $\frac{1}{a-3}$;

2) $\frac{3}{x+2}$, $\frac{x+1}{x^3+8}$ va $\frac{x+2}{x^2-2x+4}$.

Amallarni bajaring **(529—530):**

529. 1) $\frac{a+3}{5}+\frac{7+a}{10}+\frac{a-3}{2}$;

2) $\frac{b-7}{4}+\frac{5b-2}{3}+\frac{3b-1}{8}$;

3) $\frac{a-2}{45}-\frac{a+5}{15}-\frac{a-9}{9}$;

4) $\frac{b}{12}-\frac{3b+1}{9}-\frac{2b-1}{4}$.

530. 1) $\frac{y}{n-2}+\frac{z}{2-n}$;

3) $\frac{2m}{3-5n}-1+\frac{7n-4}{5n-3}$;

2) $\frac{p+2q}{3p-q}-\frac{5q-2p}{q-3p}$; 4) $4-\frac{3a}{5-2b}+\frac{5(a-10)}{2b-5}$.

Ko‘rsatilgan amallarni bajaring **(531—533):**

531. 1) $\frac{a^2-2ab+b^2}{a^2-ab+b^2}:\frac{8a-8b}{a^3+b^3}$; 3) $\frac{n^3-m^3}{n^2-m^2}:\frac{n^2+nm+m^2}{n^2+2nm+m^2}$;

2) $\frac{a^2+2ab+b^2}{a^2+ab+b^2}\cdot\frac{a^3-b^3}{7a+7b}$; 4) $\frac{m^2+2mn+n^2}{p^3+c^3}\cdot\frac{p+c}{2m+2n}$.

532. 1) $\frac{64x^2-1}{x^2-4}\cdot\frac{(x+2)^2}{x^2-4}\cdot\frac{(x-2)^2}{8x+1}$;

2) $\frac{x-6}{x^2+6x+9}\cdot\frac{x^2+4x+4}{(x^2+2)(x-2)}\cdot\frac{x^3-9x}{(x-6)(x+2)}$;

3) $\frac{am^2-an^2}{m^2+2mn+n^2}:\frac{am^2+2amn+an^2}{3m+3n}$;

4) $\frac{ab-4b-2a+8}{2a+8-ab-4b}:\frac{2a-8-ab+4b}{ab+4b-2a-8}$.

533. 1) $(x^2-1)\left(\frac{1}{x-1}-\frac{1}{1+x}+1\right)$; 3) $\left(\frac{x+y}{x-y}-\frac{x-y}{x+y}\right):\left(\frac{x-y}{x+y}+\frac{x+y}{x-y}\right)$;

2) $\left(1+a-\frac{a^2+3}{a+1}\right)(1-a^2)$; 4) $\left(\frac{2-a}{2+a}-\frac{a+2}{a-2}\right):\left(\frac{2+a}{2-a}+\frac{a-2}{a+2}\right)$.

№ 10 | *n* sonning raqamlari yig‘indisi 2006 ga teng. *n* sonni ikkita o‘zaro teng sonlar ko‘paytmasi ko‘rinishida tasvirlash mumkinmi?

V bobga doir sinov mashqlari — testlar

1. Kasrni qisqartiring: $\frac{27a^2-36ab+12b^2}{9a^2-4b^2}$.

A) $\frac{3(3a-2b)}{3a+2b}$; B) $\frac{3a-2b}{3a+2b}$; C) $\frac{39-36ab}{5}$;

D) $\frac{3a^2-36ab+3b^2}{a^2-b^2}$; E) $\frac{3a+2b}{3a-2b}$.

2. Kasrni qisqartiring: $\frac{7a^2(ab^2-9a)}{3a(21a-7ab)}$.

A) $\frac{7a(ab^2-9a)}{3(21a-7ab)}$; B) $\frac{-a(b+3)}{3}$; C) $\frac{7(ab^2-9a)}{3(21-7b)}$;

D) $\frac{a(b-3)}{3}$; E) $-a(b+1)$.

3. Kasrni qisqartiring: $\frac{8a^3+12a^2+6a+1}{4a^2+4a+1}$.

A) $2a-1$; B) $\frac{2a-1}{2a+1}$; C) $\frac{24a^4-15}{17-14a^2}$;

D) $\frac{2a-16a^3+1}{-14a^2+5}$; E) $2a+1$.

4. Amallarni bajaring: $\frac{4}{a+b}+\frac{5}{a-b}-\frac{10b}{a^2-b^2}$.

A) $\frac{9}{a-b}$; B) $\frac{9}{a+b}$; C) $\frac{-9}{a+b}$;

D) $\frac{9(a+b)}{a-b}$; E) $\frac{9(a-b)}{a+b}$.

5. Kasrlarni ayiring: $\frac{a^2+9}{a^3+27}-\frac{1}{a+3}$.

A) $\frac{1}{a^2+9}$; B) $\frac{3}{a^2+9}$; C) $\frac{a}{a^3+9}$;

D) $\frac{3a}{a^3+27}$; E) $\frac{3a+18}{a^3+27}$.

6. Kasrlarni ko‘paytiring: $\frac{9a^2-16b^2}{6a+8b}\cdot\frac{6a^2}{12b-9a}$.

A) a^2; B) $-a^2$; C) $\frac{a^2}{3a-4b}$;

D) $\frac{6}{3a+4b}$; E) to‘g‘ri javob berilmagan.

7. Kasrlarni bo‘ling: $\frac{4a^2-20ab+25b^2}{5b+4}:\frac{(2a-5b)^2}{25b^2-16}$.

A) $\frac{5b+4}{2a-5b}$; B) $\frac{2a-5b}{5b-4}$; C) $5b-4$;

D) $5b+4$. E) $\frac{2a+5b}{5b-4}$.

8. Amallarni bajaring: $\left(\frac{4a+5b}{4a-5b}-\frac{4a-5b}{4a+5b}\right):\left(\frac{4a+5b}{4a-5b}-1\right)$.

A) $\frac{8a}{16a^2-25b^2}$; B) $\frac{4a-5b}{4a+5b}$; C) $\frac{8a}{4a-5b}$;

D) $\frac{8}{4a+5b}$; E) $\frac{8a}{4a+5b}$.

9. Amallarni bajaring: $\left(\frac{3x+1}{3x-1}-\frac{1}{9x^2-1}\right)\cdot\frac{3x+1}{3x+2}$.

A) $\frac{3x}{3x-1}$; B) $\frac{3x-1}{3x}$; C) $\frac{3x}{3x+1}$;

D) $\frac{3x+1}{3x-1}$; E) to‘g‘ri javob berilmagan.

10. Amallarni bajaring: $\left(\frac{2-3a}{2+3a}-\frac{3a+2}{3a-2}\right):\left(\frac{2+3a}{2-3a}+\frac{3a-2}{3a+2}\right)$.

A) $\frac{9a^2-4}{12a}$; B) $\frac{9a^2+4}{12a}$; C) $\frac{a^2+4}{12a}$;

D) $\frac{a^2+4}{a}$; E) $\frac{9a^2+4}{3a+2}$.

Tarixiy masalalar

① *Evklid (eramizdan avvalgi III asr) masalasi.*
a, b, c, d — musbat sonlar va a ularning eng kattasi bo‘lsin. Agar $\frac{a}{b}=\frac{c}{d}$ bo‘lsa, u holda $a+d>b+c$ bo‘lishini isbotlang.

② *Eyler masalasi.*
Tenglikning to‘g‘riligini tekshiring:

$$a^3+b^3+\left[\frac{b\cdot(2a^3+b^3)}{a^3-b^3}\right]^3=\left[\frac{a(a^3+2b^3)}{a^3-b^3}\right]^3.$$

③ *Eyler masalasi.*
$4a^4+1$ ifodani ko‘paytuvchilarga ajrating.

④ *Al-Karaji masalasi.*
Quyidagi tengliklarning to‘g‘riligini ko‘rsating:

$$\frac{1}{2}\left[\frac{a^2-b^2}{a-b}+(a-b)\right]=a; \qquad \frac{1}{2}\left[\frac{a^2-b^2}{a-b}-(a-b)\right]=b.$$

Tarixiy ma’lumotlar

Qisqa ko‘paytirish formulalari, algebraik kasrlarga oid ma’lumot qadimgi risolalarda uchraydi. Masalan, al-Karajining „Al-Fahri“, Misr olimi Abu Komil (850—930) ning „Kitab al-jabr val-muqobala“ asarlarida ham algebraik kasrlar o‘rganilgan. Abu Komil al-Xorazmiydan keyin algebraga doir kitob yozgan birinchi olimdir. Abu Komil o‘z asarida

$\left(\frac{a}{b}\right)\cdot b=a$, $\frac{a}{b}=\frac{a^2}{ab}$, $\frac{a}{b}\cdot\frac{b}{a}=1$, $\frac{a}{b}+\frac{b}{a}=\frac{a^2+b^2}{ab}$ kabi sodda munosabatlarga ham e’tibor qaratadi.

Algebraik kasrlarga I. Nyutonning „Umumiy arifmetika“ kitobida ham yetarlicha o‘rin berilgan. „$\frac{a}{b}$ kasr a ni b ga bo‘lish natijasida hosil bo‘lgan kattalikdir. Xuddi shuningdek, $\frac{ab-bb}{a+x}$ kattalik $ab-bb$ ni $a+x$ ga bo‘lish natijasida hosil bo‘ladi,“ — deydi Nyuton.

534. Sonli ifodaning qiymatini toping:

1) $2\frac{7}{8}+5\frac{5}{6}+7\frac{1}{8}+\frac{5}{6}$; 2) $13\frac{5}{6}\cdot\frac{1}{7}+\frac{1}{6}\cdot\frac{1}{7}$.

535. Tenglik to‘g‘rimi:

1) $\frac{2-\frac{3}{5}+0{,}7}{1\frac{4}{5}-1+0{,}4}=\frac{7}{4}$; 2) $\frac{\left(\frac{4}{7}-7-0{,}2\right)\cdot 3{,}5}{2{,}26}=-10$;

3) $\left(\frac{4{,}752}{3{,}2}+\frac{0{,}608}{3{,}8}\right):\left(7{,}5-\frac{3{,}55}{1{,}42}\right)=0{,}0617$.

536. Ikki sondan biri a ga teng, ikkinchisi undan 7 ta ortiq. Shu sonlar ko‘paytmasining ikkilanganini toping. Shu ko‘paytmaning qiymatini $a=\frac{1}{2}$ bo‘lganda hisoblang.

537. Ikki sonning yig‘indisi 30 ga teng. Sonlardan biri a. Shu sonlarning ikkilangan ko‘paytmasini yozing. Shu ko‘paytmaning qiymatini $a=-2$ bo‘lganda hisoblang.

538. a ta yuzlik, b ta o‘nlik va c ta birlikdan tuzilgan natural sonda nechta birlik borligini ko‘rsatuvchi formula tuzing. Xuddi shu raqamlar yordamida, lekin teskari tartibda yozilgan sonda nechta birlik bor?

539. a kilogramm va c gramm necha grammni tashkil qiladi? Grammlar sonini x harfi bilan belgilab, javobni formula bilan yozing.

540. Algebraik ifodaning son qiymatini toping:

1) $\frac{2a+b}{b-2a}$, bunda $a=-\frac{1}{2}$, $b=-3$;

2) $\frac{1}{8}a^3-27ab^2$, bunda $a=2$, $b=-\frac{1}{3}$;

3) $\frac{2}{3}a^2b-\frac{3}{2}ab^2$, bunda $a=4,\ b=1$;

4) $\frac{abc-4a+3b}{a+b-c}$, bunda $a=-\frac{1}{3},\ b=-\frac{3}{4},\ c=-\frac{4}{3}$.

541. Birhadlarning ko‘paytmasini toping:

1) $-12a^2bc^2d\cdot 5ac^3d^4\cdot\left(-3b^3cd^2\right)$;

2) $49a^2bc^2\cdot\left(-\frac{2}{7}ab\right)\cdot\left(\frac{1}{14}ac\right)$;

3) $8a^2b\cdot\left(-4ab^3\right)\cdot\left(-7a^3b^2\right)$;

4) $\left(-\frac{2}{3}a^4b^2c\right)\cdot\frac{15}{2}abc^3$.

542. Birhadni darajaga ko‘taring:

1) $\left(-2ab^2\right)^3$; 2) $\left(-0{,}8ac^2\right)^2$; 3) $\left(-\frac{3}{5}abc^3\right)^3$.

543. Ifodani soddalashtiring:

1) $2a^2+2ab+3b^2-a^2-2b^2$;

2) $7a^2+2b^2-\left(6a^2+b^2\right)$;

3) $\frac{2}{3}a^2-b^2+\frac{4}{3}a^2-\frac{5}{7}b^2$;

4) $\frac{1}{7}a^2b\cdot 23m-\frac{2}{7}a^2bm$.

544. Ifodaning son qiymatini toping:

1) $5a^2-2ab+6a-7ab-6a^2-6a$, bunda $a=5,\ b=-\frac{1}{9}$.

545. Ko‘phadni birhadga ko‘paytiring:

1) $\left(a^2-ab+b^2\right)3ab^3$; 2) $\left(6a^2-4ab^2+1\right)\cdot\frac{1}{2}ab$.

546. Ko‘phadlarni ko‘paytiring:

1) $\left(a^2+3ab+b^2\right)(7a-5b)$;

2) $(a+3b-4c)(a-3b-4c)$;

3) $\left(\frac{1}{3}a^2b-\frac{2}{5}ab^2\right)(15a-30b)$;

4) $\left(\frac{1}{2}a^2+4a+1\right)(3a-1)$.

Tenglamani yeching **(547—551):**

547. 1) $4(2x-1)+3(1-2x)=7$;

2) $4(x+2)-2(3x-2)=14x-5(x+3)$.

548. 1) $\frac{x-2}{4}-\frac{1}{2}=\frac{x+7}{6}$; 2) $\frac{2(3x-1)}{5}=4-\frac{x+2}{2}$.

549. 1) $7-\frac{x}{2}=3+\frac{7x}{2}$; 2) $\frac{x+3}{2}=x-4$.

550. 1) $\frac{x}{2}+\frac{x}{3}+\frac{x}{6}=12$; 2) $\frac{2x-1}{5}-\frac{x+1}{5}=\frac{3(1-x)}{10}$.

551. 1) $\frac{6x+7}{7}+\frac{3+5x}{8}=3$; 3) $1+x=\frac{5x-2}{2}$;

2) $5-\frac{2x-5}{3}=\frac{4x+2}{3}$; 4) $\frac{1-x}{9}-1=7x$.

552. Uchta qutida 119 ta qalam bor. Birinchi qutida ikkinchidagiga qaraganda 4 ta ortiq va uchinchidagiga qaraganda 3ta kam qalam bor. Har bir qutida nechtadan qalam bor?

553. Otasi 30 yoshda, o‘g‘li esa 4 yoshda. Necha yildan keyin otasi o‘g‘lidan uch marta katta bo‘ladi?

554. O‘g‘li 6 yoshda, otasi esa undan 6 marta katta. Necha yildan keyin o‘g‘li otasidan 4 marta yosh bo‘ladi?

555. Ikki velosipedchi bir vaqtda bitta yo‘l ustidagi qishloqdan bir-biriga qarab yo‘lga chiqdilar. Birinchisi 15 km/soat, ikkinchisi esa 12 km/soat tezlik bilan harakat qilmoqda. Agar qishloqlar orasidagi masofa 40,5km bo‘lsa, qancha vaqtdan keyin uchrashuv sodir bo‘ladi?

556. Ikki velosipedchi bir yo‘ldagi ikkita qishloqdan bir vaqtda bir xil yo‘nalishda yo‘lga chiqdi. Ikkinchi velosipedchi oldinda, birinchisi orqada bormoqda. Birinchi velosipedchining tezligi 15 km/soat, ikkinchisiniki esa 12 km/soat. Agar qishloqlar orasidagi masofa 4,5 km bo‘lsa, birinchi velosipedchi ikkinchisini qancha vaqtda quvib yetadi?

Soddalashtiring: **(557—559).**

557. 1) $(a+1)(a-1)(a^2+1)$; 2) $\left(\frac{a}{2}-5\right)\left(5+\frac{a}{2}\right)+25$.

558. 1) $(a+3)^2+(a-3)^2$; 2) $(4a+b)^2-(4a-b)^2$.

559. 1) $(1-a)(1+a+a^2)+a^3$; 2) $\left(\frac{1}{2}-c^2\right)\left(\frac{1}{4}+\frac{1}{2}c^2+c^4\right)+c^6$.

Ko‘paytuvchilarga ajrating **(560—561).**

560. 1) $a^4+6a^3+9a^2$; 2) $25-(2-3a)^2$.

561. 1) $(a+1)^2-(4-3a)^2$; 3) $(2a+b)^2-9(a+b)^2$;

2) $(8b-1)^2-(2b+3)^2$; 4) $4(a-2b)^2-25(3a-b)^2$.

562. Kasrni qisqartiring:

1) $\frac{a^2-16}{a^2-8a+16}$; 2) $\frac{4x^2-9}{2x+3}$.

Amallarni bajaring **(563—566):**

563. 1) $\frac{b+3}{5}+\frac{7+b}{10}+\frac{b-3}{2}$; 2) $\frac{a^2+5a-4}{16-a^2}+\frac{2a}{8a+2a^2}$.

564. 1) $\frac{a}{a^2-1}-\frac{1}{1-a^2}$; 2) $\frac{4x^2}{2x-3y}+\frac{12xy}{3y-2x}+\frac{9y^2}{2x-3y}$.

565. 1) $\frac{a-b}{ab}-\frac{a-c}{ac}$; 2) $\frac{1}{14x^3}-\frac{1}{21x^2y}+\frac{1}{4xy^2}$.

566. 1) $\frac{x^2-y^2}{6xy}\cdot\frac{12x^2y}{x+y}$; 2) $\frac{a^2+4a}{a^2-16}:\frac{4a+16}{a^2-4a}$.

Amallarni bajaring **(567—570)**:

567. 1) $\left(\frac{a}{a+1}+1\right):\left(1-\frac{a}{a+1}\right)$; 2) $\frac{1-a^2}{1+b}\cdot\frac{1-b^2}{a+a^2}\cdot\left(1+\frac{a}{1-a}\right)$.

568. 1) $1+3a+\dfrac{9a^2}{1+3a}+\dfrac{1}{3a-1}+\dfrac{6a}{1-9a^2}$;

2) $\left(\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}\right):\left(\dfrac{a^2+b^2}{a^2-b^2}+\dfrac{a^2-b^2}{a^2+b^2}\right)$.

569. 1) $\left(\dfrac{9m^2-3n^2}{4mn}-\dfrac{m-4n}{5n}\right):\left(\dfrac{2m+n}{3m}-\dfrac{5n^2-3m^2}{16m^2}\right)$;

2) $\left(\dfrac{a+4b}{2b}+\dfrac{6b}{4b-a}\right)\left(1-\dfrac{a^2-2ab+4b^2}{a^2-4b^2}\right)$.

570. 1) $\dfrac{3}{2}\left(\dfrac{2a}{3}-\dfrac{a}{7}\right)-\dfrac{12(a-5)}{7}+a+\dfrac{1}{3}\left(\dfrac{a}{2}-5a\right)$;

2) $2-\dfrac{x-a}{x+a}-\dfrac{x}{x-a}+\left(\dfrac{1}{a^2}-\dfrac{1}{ax}+\dfrac{2}{x^2}\right):\left(\dfrac{1}{a^2}-\dfrac{1}{x^2}\right)$.

571. Sayyoh Ko‘ksuv daryosi bo‘yida joylashgan bir oromgohdan velosipedda yo‘lga chiqib, boshqa bir oromgohga tayinlangan vaqtda yetib bormoqchi bo‘ldi. Dastlabki 1 soatda u 10,5 km yo‘l bosdi. Agar qolgan masofani ham shunday tezlik bilan o‘tsa, manzilga mo‘ljallagan vaqtdan 1 soat kechikishini hisoblab bildi. Sayyoh qolgan yo‘lni soatiga 15 km tezlik bilan o‘tdi va manzilga belgilangan vaqtdan yarim soat oldin yetib keldi. Oromgohlar orasidagi masofani toping.

572. Hozir soat 5. Qancha vaqtdan so‘ng soatning minut mili soat milini „quvib yetadi“?

573. Ikki xonali sonning o‘nliklar xonasidagi raqam birliklar xonasidagi qaramdan 4 marta katta. O‘quvchi 507 ni shu ikki xonali songa ko‘paytirmoqchi edi. Ammo u ikki xonali sonning raqamlari o‘rnini almashtirib yozib qo‘ydi. Natijada u topgan ko‘paytma masalaning javobidan 27378 ga kichik chiqdi? To‘g‘ri javob nechaga teng ekan?

574. Mis va ruxdan iborat qotishmaning og‘irligi 36 N ga teng. Qotishmani suvga botirilganda u o‘z og‘irligining $4\frac{1}{3}$ N ini yo‘qotdi. Mis suvga botirilganda o‘z og‘irligining $11\frac{1}{9}$ % ini, rux esa $14\frac{2}{7}$ % ini yo‘qotishi ma’lum. Qotishmadagi mis va rux og‘irligini aniqlang.

575. Tarkibi kumush va misdan iborat qotishmaning massasi 3,5 kg. Undagi kumush tarkibi mis tarkibining $16\frac{2}{3}\ \%$ ini tashkil qiladi. Qotishmadagi kumush massasini toping.

576. 3 ta qopda 120 kg un bor. 1- qopdagi un 2- qopdagi unning $\frac{3}{5}$ qismiga, 3- qopdagi un esa 2- qopdagi unning 80 % iga teng. Har bir qopda necha kilogramm un bor?

577. Ahmad A qishloqdan B qishloqqacha velosipedda 14 km/soat tezlik bilan, qaytishda esa 10 km/soat tezlik bilan yurdi. Agar Ahmad qaytishga 1 soat ortiq vaqt sarflagan bo‘lsa, qishloqlar orasidagi masofani toping.

578. Vertolyot ikki qishloq orasidagi masofani shamol yo‘nalishida 1,5 soatda, shamol yo‘nalishiga qarshi esa 2 soatda uchib o‘tadi. Agar shamolning tezligi 10 km/soat bo‘lsa, shu qishloqlar orasidagi masofa qancha?

579. Firma reja bo‘yicha bir neshta mahsulotni 10 kun muddat ichida tayyorlashi kerak edi. Lekin u har kuni rejaga qo‘shimcha 2 tadan mahsulot tayyorlab, muddatiga bir kun qolganda faqat topshiriqni bajaribgina qolmasdan, balki rejadan yana 3 ta mahsulot ortiq tayyorladi. Firma reja bo‘yicha nechta mahsulot tayyorlashi kerak edi?

580. Kasrlarni umumiy maxrajga keltiring:

1) $\frac{5x}{x^2-4}$, $\frac{3x+y}{x^2+4x+4}$ va $\frac{y-x}{x^2-4x+4}$;

2) $\frac{3a}{2a-3}$, $\frac{4a}{2a+3}$ va $\frac{5b}{4a^2c-9c}$;

3) $\frac{4b}{b^2-2bc+c^2}$, $\frac{2a}{c-b}$ va $\frac{1}{4ac+4ab}$;

4) $\frac{1}{4x^2-9y^2}$, $\frac{1}{4x^2y+12xy^2+9y^3}$ va $\frac{1}{3y-2x}$;

5) $\frac{c-b}{8bc+16c^2}$, $\frac{c+b}{2bc}$ va $\frac{1}{b^2c+4bc^2+4c^3}$;

6) $\frac{2x}{y^3-x^3}$, $\frac{3}{x^2y-xy^2}$ va $\frac{1}{x^2y+xy^2+y^2}$.

Amallarni bajaring **(581—585):**

581. 1) $\dfrac{a^2+2a+1}{b^2-4}\cdot\dfrac{b+2}{a+b}-\dfrac{a}{b+2}$; 2) $\dfrac{a^2-2a+1}{b-2}:\dfrac{a^2-1}{b^2-4}-\dfrac{2a-b}{a+1}$;

3) $\left(\dfrac{(a+1)^2}{a^2-1}-1\right)\left(1-\dfrac{a}{a+1}\right)$;

4) $\left(\dfrac{x^2}{x+y}-\dfrac{x^3}{x^2+2xy+y^2}\right):\left(\dfrac{x}{x+y}-\dfrac{x^2}{x^2-y^2}\right)$.

582. 1) $\left(\dfrac{c-d}{c^2+dc}-\dfrac{c}{d^2+cd}\right):\left(\dfrac{d^2}{c^3-cd^2}+\dfrac{1}{c+d}\right)$;

2) $\left(\dfrac{2n}{k+2n}-\dfrac{4n^2}{k^2+4nk+4n^2}\right):\left(\dfrac{2n}{k^2-4n^2}+\dfrac{1}{2n-k}\right)$;

3) $\left(\dfrac{b^2}{b+x}-\dfrac{b^3}{b^2+x^2+2bx}\right):\left(\dfrac{b}{b+x}-\dfrac{b^2}{b^2-x^2}\right)$;

4) $\left(\dfrac{2q}{2q+m}-\dfrac{4q^2}{4q^2+4mq+m^2}\right):\left(\dfrac{2q}{4q^2-m^2}+\dfrac{1}{m-2q}\right)$.

583. 1) $1+a-\dfrac{a-1}{a}+\dfrac{a^2-1}{2a}-\dfrac{3a}{2}$;

2) $\dfrac{m+1}{m^2+m+1}-\dfrac{2}{1-m}+\dfrac{3m^2+2m+4}{1-m^3}$;

3) $\dfrac{m+n}{3}-m+2n$; 4) $m+n-\dfrac{2m-n}{5}-\dfrac{m+n}{2}$.

584. 1) $\dfrac{a^3+2a^2}{a^2-1}\cdot\dfrac{(a+1)^3(a-1)}{a^2(a+2)}$; 2) $\dfrac{(a^2+ab)^2}{a^2-b^2}:\dfrac{(a+b)^2}{(ab-b^2)^2}$.

585. 1) $\dfrac{3}{2}\left(\dfrac{2a}{3}-\dfrac{a}{7}\right)-\dfrac{12(a-5)}{7}+a+\dfrac{1}{3}\left(\dfrac{a}{2}-5a\right)$;

2) $2-\dfrac{x-a}{x+a}-\dfrac{x}{x-a}+\left(\dfrac{1}{a^2}-\dfrac{1}{ax}+\dfrac{2}{x^2}\right):\left(\dfrac{1}{a^2}-\dfrac{1}{x^2}\right)$.

Tenglamani yeching **(586—587):**

586. 1) $\frac{4x-3}{2}-\frac{5-2x}{3}-\frac{3x-7}{6}=0$; 2) $\frac{x+4}{5}-\frac{x+3}{3}=x-5-\frac{x-2}{2}$.

587. 1) $(2x-3)(x+5)-(3-x)(5-2x)=-30$;

2) $5(x-1)^2-2(x+3)^2=3(x+2)^2$;

3) $(x-3)(3+x)+(12x-6):3=(x-2)(x+2)-7$;

4) $(x+1)^2-(x+2)^2=(x+3)^2-(x+5)^2$.

588. Avtomobil shahardan qishloqqacha bo‘lgan masofani 80 km/soat tezlik bilan bosib o‘tdi. Orqaga qaytishda u masofaning 75 % ini avvalgi tezlik bilan, qolgan yo‘lni esa 60 km/soat tezlik bilan bosib o‘tdi va shuning uchun ham qaytishda yo‘lga shahardan qishloqqa borishdagiga qaraganda 10 minut ortiq vaqt sarf qildi. Shahardan qishloqqacha bo‘lgan masofani toping.

589. Qayiq daryo oqimiga qarshi 4,5 soat va oqim bo‘yicha 2,1 soat suzdi. Qayiq hammasi bo‘lib 52,2 km suzdi. Agar daryo oqimining tezligi 3 km/soat bo‘lsa, qayiqning turg‘un suvdagi tezligini toping.

590. Oralaridagi masofa 340 km bo‘lgan ikki bekatdan bir vaqtda bir-birlariga qarab ikki poyezd yo‘lga chiqdi. Ulardan birining tezligi ikkinchisinikidan 5 km/soat ortiq. Agar harakat boshlanganidan 2 soat o‘tgandan keyin poyezdlar orasidagi masofa 30 km ekanligi ma’lum bo‘lsa, ularning tezligini toping.

Ifodaning son qiymatini toping:

591. 1) $(0{,}13x^2-2x)^2-6x^2\left(\frac{2}{3}x^2-0{,}3x\right)$, bunda $x=-\frac{1}{3}$;

2) $-\frac{2}{3}(x-1)^2-2\frac{1}{3}(x-3)(x+3)$, bunda $x=3$;

3) $\left(\frac{1}{3}x+2a\right)\left(2a-\frac{1}{3}x\right)-\left(\frac{2}{9}x+\frac{4}{7}a\right)\left(7a-\frac{1}{2}x\right)+1\frac{5}{9}ax$, bunda $a=\frac{5}{16}$, $x=28$;

4) $(3x+2y)(9x^2-6xy+4y^2)-8(x^3+y^3)$, bunda $x=0{,}1$, $y=4$.

1. 2) 7; 4) 5,86. **2.** 2) $\frac{9}{56}$; 4) 0,5. **4.** 2) Noto‘g‘ri; 4) To‘g‘ri. **5.** $40\cdot0{,}03 = 6 : 5$. **6.** 2) $3\cdot(2 + 6)=2\cdot(2\cdot 6)$. **8.** 2) $\frac{9}{56}$; 4) $4\frac{6}{7}$; **9.** 2) $-0{,}02$; 4) 3. **10.** 2) 0; 4) 5. **11.** 2) -2; 4) 0. **12.** $(7m)$t; 168 t. **13.** 1) $(60m)$ min.; 2) $\frac{p}{60}$ min; 3) $(60m + l + \frac{p}{60})$ min. **14.** $3(x - y)$; 2) 4,5; 4) 2,5. **15.** $(x + y)(x - y)$; 2) $-\frac{11}{64}$; 4) 0,104. **16.** 2) $-1\frac{2}{3}$. **17.** 2) 4. **18.** 1, 3, 15, 21. **19.** 2) $(m - 1)m$; 4) $(2p + 1)(2p + 3)(2p + 5)$. **21.** $(p - q)$ t; 1) 5t; 2) q p dan katta bo‘lmaydi; q p ga teng bo‘lishi mumkin. **22.** $400n + 500m$; 155000; 155000. **24.** 187200 m^3, $(37440m)$ m^3. **25.** $s = 3\frac{1}{6}c + 1\frac{2}{3}a + 2\frac{1}{2}b$, 53 km. **26.** 2) $a - b$; 4) $2mn$; 6) $(a+b)(a-b)$. **28.** 5000; 150000. **29.** $3a$; $8a$; $10a$; 500; 400; $\frac{sa}{100}$. **30.** 2) 30 kg. **31.** 2) $(5k)$ km. **32.** $(50a)$ kg. **33.** $(15a)$ ga. **34.** $(x\cdot 6 + y\cdot 3)$ so‘m. **35.** $(a\cdot 15 + b\cdot 20)$ kg. **36.** $(km + cn)$ kg. **37.** $S = a(a - b)$. **38.** $mn + k$; 810 o‘rin. **39.** 4 soat 35 min. **40.** b) $p = (m + n)\cdot 2$; $S = mn - xy$; e) $p = 2(a + m + n + x)$, $S = mn - ab - xy$. **41.** 2) $2(2a + 4)m$; 3) $(a + 8)(a - 4)m^2$. **42.** $\frac{s}{t-1}$ km/soat. **44.** $\frac{a-1500}{20}$ m^2. **45.** $500(100 + p)$ so‘m. **47.** $t = \frac{s-3}{v}$, ulgurmaydi. **49.** 2) 40; 4) -41. **50.** 2) $3y - 2x$; 4) $8{,}7 - 2\frac{1}{3}m + 1\frac{2}{3}n$. **51.** 2) $3 - 2{,}7b$; 4) $\frac{2}{3}y + \frac{1}{3}b - 3$; 6) $5p$. **52.** 2) $x + 5$; 4) $58c + 14d$. **53.** 2) 67,048; 4) $-11{,}221$. **54.** 2) 0,28; 4) $7\frac{37}{112}$. **55.** 2) $-4 - 9 + 1\,1$; 4) $2a - 3b - 4c$. **57.** 2) $2 + b + (-c)$; 4) $3 + a + (-b) + (-c)$. **58.** 2) $a - 2b + 3c$; 4) $-a + 2b - 3c$. **59.** 2) $a - b + c - d$; 4) $a - b - c + d - k$. **60.** 2) $8x - 2y$; 4) $3a - 3$. **61.** 2) $a - 2b + (m + c)$; 4) $a + (-m + 3b^2 - 2a^3)$. **62.** 2) $2a + b - (-m - 3c)$; 4) $a - (m - 3b^2 + 2a^3)$. **63.** 2) $a - (b - 1)$; 4) $(a - 2b) + 8$. **64.** 2) $2x^2 + 5x^2y + (-4xy^2 - y^3)$; 4) $-(-2x^2 - 5x^2y) - 4xy^2 - y^3$. **65.** 2) $c + (-a + b)$; 4) $n + (-d + l)$. **66.** 2) $4a - 4b$; 4) $5x - 3y$. **67.** 2) $x = 1$; 4) $x = 5$. **68.** 2) $-1{,}16$; 4) -3. **69.** 2) -1; 4) 9; 6) 9; 8) 3,9. **70.** 2) 147; 4) 144. **71.** 2) -132; 4) 7. **72.** 2) 1,08; 4) 6,12. **73.** 2) 42; 4) -1. **74.** 2) $-\frac{6}{5}$; 4) $\frac{3}{16}$. **75.** 2) $7\frac{1}{4}$. **78.** 6 dirham. **80.** 2) 3. **85.** 2) $x = -27$; 4) $x = 1{,}009$. **86.** 2) $x = \frac{5}{7}$; 4) $x = \frac{2}{3}$. **87.** 2) $x = -1{,}3$; 4) $x = 0{,}05$. **88.** 2) $x = 64$; 4) $x = 1$. **89.** 2) $x = -\frac{4}{25}$; 4) $x = -\frac{1000}{3}$. **90.** 2) $x = \frac{3}{7}$; 4) $x = \frac{1}{3}$. **91.** 2) $x = 17$; 4) $y = -1$. **92.** 2) $x = 7\frac{1}{2}$; 4) $y = 24$. **93.** 2) $z = 6$; 4) $x = 0{,}6$. **94.** 2) $y = 13$; 4) $x = 1$. **95.** 2) $y = 319$; 4) $x = 5$. **96.** 2) $x = 37$;

4) $x = 1{,}1$. **99.** 2) $x = 1$; 4) $x = 1$. **100.** 2) $x = 0{,}2$; 4) $x = 4$. **101.** 2) 10. **102.** 2) 16, 20, 24. **103.** 2) 144, 432, 293. **104.** 2) 8, 8, 6. **105.** 2) 20, 40. **106.** 25, 27, 29. **107.** 4, 6, 8 va 10. **108.** 2) 12. **109.** 2) 8 yildan keyin. **110.** 2) 2200 t va 1100 t. **111.** 2) 108 km. **112.** 2) 40 kg. **113.** 2) 150 ta mashina. **114.** 2) $y = 0$; 4) $x = 0{,}8$. **115.** 2) $x = 13$; 4) $x = -153$. **116.** 83,6 kg, 508, 8 kg, 1327 kg. **117.** 8 km/soat. **119.** 32 kundan so‘ng. **120.** 30 km/soat, 40 km/soat yoki $36\frac{2}{3}$ km/soat, $46\frac{2}{3}$ km/soat. **123.** 2) $\left(\frac{1}{3}\right)^5$; 4) $(-2{,}7)^4$. **124.** 2) m^5; 4) $(-3b)^4$. **125.** 2) $(a+b)^2$; 4) $\left(\frac{m}{n}\right)^5$. **126.** 2) $4^4 \cdot 21$; 4) $6^2 \cdot 7^2 \cdot 3^3$. **127.** 2) $(0{,}5)^3 \cdot 2^2 \cdot 4^2$; 4) $\left(\frac{2}{3}\right)^3 \cdot (2,3)^2$. **128.** 2) $x^4 \cdot 3^2$; 4) $\left(\frac{a}{b}\right)^2 (8a-b)^3$. **129.** 2) $a^2 + b^4$; 4) $2x^3$. **130.** 2) na^3; 4) $5^k + a^{17}$. **132.** 2) 9; 4) 125. **133.** 2) -1; 4) 0. **134.** 2) $\frac{9}{25}$; 4) $12\frac{19}{27}$. **135.** 2) 2,89; 4) $\frac{1}{625}$. **136.** 2) -125; 4) $-5\frac{1}{16}$. **137.** 2) 270; 4) 4. **138.** 2) 40; 4) -6. **139.** 2) 18; 4) 72. **140.** $-2\frac{1}{4}$, $2\frac{1}{4}$, $-3\frac{3}{8}$; -25, 25, 125. **143.** $a \cdot 10^3 + b \cdot 10^2 + c \cdot 10 + d$. **145.** 2) 3532037; 4) 101001. **146.** 2) 7^6; 4) 5^6. **147.** 2) a^7; 4) $(3b)^7$. **148.** 2) $(-3)^4$; 4) $(-1{,}2)^7$. **149.** 2) 3^{10}; 4) $(-6)^{12}$. **150.** 2) $\left(\frac{2}{3}\right)^8$; 4) b^{15}. **151.** 2) $\left(\frac{-5x}{6}\right)^{12}$; 4) $(n+m)^{20}$. **152.** 2) 3^{8+n}; 4) a^{n+13}. **154.** 2) 2^2; 4) 2^7. **155.** 2) 2^6; 4) 2^{10}. **156.** 2) 2^{14}; 4) 2^9. **157.** 2) 2^{23}; 4) 2^{4+n}. **158.** 2) 3^1; 4) 3^4. **159.** 2) 3^5; 4) 3^7. **160.** 2) 3^{18}; 4) 3^6. **161.** 2) 3^{n+1}; 4) 3^{3+n}. **162.** 2) 4^2; 4) 10^8. **163.** 2) $\frac{1}{17}$; 4) d^{12}. **164.** 2) $(2a)^2$; 4) $(m+n)^5$. **165.** 2) 2^2; 4) 2^2. **166.** 2) 2^3; 4) 2^9. **167.** 2) 3^3; 4) 3. **168.** 2) 3^2; 4) 3^4. **169.** 2) 6; 4) 25. **170.** 2) 44; 4) 9. **171.** 2) -6; 4) 12. **172.** 2) $x = 64$; 4) $x = 27$. **173.** 2) $x = 16$; 4) $x = 4$. **174.** 2) $x = 243$; 4) $x = 9$. **175.** 1) a^{56}; 2) a^{21}. **176.** 2) a^{15}; 4) a^{23}. **177.** 2) a^9; 4) a^{12}. **178.** 2) $n = 7$; 4) $n = 2$. **179.** 2) $\left(\frac{5}{6}\right)^2$; 4) $(0{,}02)^2$. **180.** 2) $(7^3)^2$; 4) $\left(\left(-\frac{2}{3}\right)^{12}\right)^2$. **181.** 2) $(b^3)^2$; 4) $(x^{10})^2$. **182.** 2) $7^5 \cdot 6^5$; 4) $4^3 \cdot \left(\frac{1}{7}\right)^3$. **183.** 2) $81x^4$; 4) $64b^2$. **184.** 2) 6^6y^6; 4) $27n^3m^3$. **185.** 2) $x^7y^7z^7$; 4) $2^9 \cdot 4^9 \cdot 9^9$. **186.** 2) a^6b^3; 4) $0{,}01c^6$. **187.** 2) $512a^{12}b^{21}$; 4) $16n^4m^{12}$. **189.** 2) $(3{,}4 \cdot b)^4$; 4) $\left(-\frac{2}{3}a\right)^2$. **190.** 2) $(9 \cdot r)^2$; 4) $(15 \cdot a \cdot b)^3$. **191.** 2) $(a^2b^3)^2$; 4)

$(9m)^2$. **192.** 2) $(xy^2z^4)^2$; 4) $(10c^4x^3)^2$. **193.** 2) $(0{,}7nm^5)^2$; 4) $\left(\frac{4}{25}a^5b^8\right)^2$. **194.** 2) $(b^3)^3$; 4) $(4^2)^3$. **195.** 2) $\left(\left(-\frac{2}{3}\right)^5\right)^3$; 4) $(-0{,}1)^3$. **196.** 2) $(a^2b)^3$; 4) $(x^4y^3z^2)^3$. **197.** 2) $(-10b^2)^3$; 4)$(-0{,}2xy^3)^3$. **198.** 2) 1; 4) -1. **199.** 2) 1; 4) $\frac{1}{32}$. **200.** 2) 144; 4) 14. **201.** 2) 1; 4) 4. **202.** 2) 14; 4) 16. **203.** 2) $\frac{25}{49}$; 4) $\frac{b^3}{8^3}$. **204.** 2) $\frac{169}{n^2}$; 4) $-\frac{64}{c^3}$. **205.** 2) $\frac{81b^4}{625c^4}$; 4) $\frac{5^6}{7^{12}}$;. **206.** 2) $\frac{49}{(2+c)^2}$; 4) $\frac{(a+b)^7}{(a-b)^7}$. **207.** 2) $(\frac{2}{5})^5$; 4) $\left(\frac{5}{a}\right)^7$. **208.** 2) $\left(\frac{a}{b}\right)^3$; 4) $\left(\frac{7}{10}\right)^2$. **209.** 2) $\left(\frac{4x}{3y}\right)^4$; 4)$\left(-\frac{1}{3}\right)^3$. **210.** 2) $\frac{1}{35}$; 4) $\frac{16}{9}$. **211.** 2) $\frac{2}{5}$; 4) $\frac{7}{3}$. **212.** 1) $\cong 3{,}3\cdot10^5$ marta; 2) $\cong$9 yil. **213.** 2) $\frac{3}{10}$. **214.** 2) 3^{5n+2}; 4) b^{4n}. **215.** 2) 7; 4) 5. **216.** 2) $81x^8y^6z^{14}$; 4) $-2{,}48832a^{15}b^{10}c^{20}$. **217.** 2) a^2; 4) a^4. **218.** 2) $10^{20} > 20^{10}$; 4) $3^{40} > 6^{20}$. **219.** 2) 4; 4) $\frac{1}{7}$. **220.** 2) $\frac{1}{3}$; 4) 13,2. **221.** 2) $8{,}647\cdot10^6$. **222.** 2) $3bc$; 4) ab^2. **223.** 2) $3a^2b$. **224.** 2) $100n$ (sm). **225.** 2) $4a^2$, $4a^2$, $2a^2$, $6a^2$. **226.** 2) 8; 4) 1; 6) 18. **227.** 2) z^{11}; 4) m^4; 6) $72p^3q^2$; **228.** 2) 2. **229.** $\frac{12}{25}$ kun. **230.** 2) $6ab$; 4) $-2a^3$. **231.** 2) $35m^2n$; 4) $-4b^5$. **232.** 2) $-2m^3n$; 4) $\frac{5}{14}b^3c^2$. **233.** 2) $28x^3y^3$; 4) $2a^2b^2c^2$. **234.** 2) $-21a^6b^6c^2$; 4) $-\frac{9}{8}a^4x^3y^4$. **235.** 2) $-7{,}5m^7r^7n^5$; 4) $-7{,}5a^5b^7c^7$. **236.** 2) $-15m^3n^2$; 4) $-26a^4b^4c^5$. **237.** 2) $30a^4b^3$; 4) $4a^3b^2c^3$. **238.** 2) $25b^2$; 4) $4a^6$. **239.** 2) $16a^2b^2$; 4) $-8x^3y^3z^3$. **240.** 2) $-a^{10}b^5c^5$; 4) $16x^8y^{12}$. **241.** 2) $\frac{1}{81}m^8n^8$. **242.** 2) $-2a^4$; 4) $a^2b^5c^2y^2$. **243.** 2) x^5y^5; 4) $-4a^{10}b^{11}$. **244.** 2) $(4x^2)^2$; 4) $(9x^3y)^2$. **245.** 2) 204,8; 4) 1,008. **246.** $7\frac{1}{5}$ qarich. **250.** 2) $6a^2b^3-24a^4b$; 4) $-bc^5+5x^2y^4$. **251.** 2) $-6xy^4z-20m^3n^2k^3$; 4) $\frac{1}{3}a^2b^2-2a^2b^3$. **252.** 2) 2; 4) 0. **253.** 2) $-7{,}6$; 4) -252. **254.** 2) $\frac{1}{3}y$; 4) $\frac{13}{16}a^2b$. **255.** 2) $2a+b$; 4) $2a^2-3b^2$. **256.** 2) $-y$; 4) $3{,}8a^2$. **257.** 2) a^2; 4) $2xy-2{,}2y^2$. **258.** 2) $-\frac{7}{8}ab^2+\frac{3}{8}a^2b$; 4) $4x-2{,}46y$. **259.** 2) $x^3-x^2y-xy^2$. 4) ab^2+2ab. **260.** 2) $8b^2-19bc-15c^2$; 4) $2x^2y$. **261.** 2) $-\frac{1}{3}a^2bc-4a^2c$. **262.** 2) $3x+3y$; 4) $3x+1$. **263.** 2) $5a^2-b^2$; 4) $-\frac{1}{2}b^2+1\frac{1}{4}$. **264.** 2) $0{,}1c^2$; 4) $6a+22b$. **265.** 2) $-2a^2-6ab+6b^2$; 4)$25z+30az^2$. **266.** 2) $-2b$; 4) $9x^3$. **267.** 2) $3x^2$; 4) $8a^2-b^2-ab$. **268.** 2) $-0{,}07x^2+0{,}06y^2$; $0{,}27x^2-0{,}1y^2$; 4) $0{,}61a^3+1{,}12b^3$; $1{,}39a^3-0{,}88b^3$. **269.** 2) $3x^2+3x^2y^2-x^3$. **270.** 2) $-5b^2+3b$. **271.** 2) q^3; 4) $-5ab+8b^2$. **273.** $k+2m-n$. **274.** 2) $1-\frac{1}{2}x$; 4)$20m-30n$; 6) $-\frac{1}{3}m+\frac{1}{3}n-\frac{1}{3}p$; 8) $-15x^3-35x^2+5x$. **275.** 2)$-10xz+8yz$; 4) x^3-x^2+x. **276.** 2) $75a^2b^2+15a^2b$; 4) $3x^2y^3-6x^4y^2$. **277.** 2) $16ab^2-24a^2bc+8abc^2$; 4)

$x^3yz + 2xy^3z + 3xyz^3$. **278.** 2) $a^3b^7 + \frac{3}{4}a^4b^4$. **279.** 2) $-3a + 7b$; 4) $-14p - 9$. **280.** 2) $-a^2b + 6b^2$; 4) $19x - 12$. **281.** 2) $2x - 3{,}5$; 4) $0{,}5y - 1{,}7$. **282.** 2) 5; 4) 204. **283.** 2) $z^2 + 3z - 4$; 4) $bc + 4c + 5b + 20$. **284.** 2) $-a^2 + 8a + 20$; 4) $p - q + pq - q^2$. **285.** 2) $10a^2 + 7a - 12$; 4) $20p^2 - 17pq + 3q^2$. **286.** 2) $0{,}09 - m^2$; 4) $0{,}04a^2 - 0{,}25x^2$. **287.** 2) $30x^4 + 30y^4 - 61x^2y^2$; 4) $x^3 + 5x^2 + 7x + 3$. **288.** 2) $27a^3 - 8b^3$; 4) $27a^3 + 8b^3$. **289.** 2) $-20b^2 + 17bc - 3c^2 - 16by + 4cy$; 4) $9a^2 - 24ab + 12ac + 15b^2 - 20bc$. **290.** 2) $0{,}3x^2 + xz - 0{,}3y^2 + yz$; 4) $0{,}3a^4 - 0{,}9a^3 + 2a^2 + 3a - 10$. **291.** 2) $a^3 - ab^2 + 3a^2b - 3b^3$; 4) $12x^3 - 29x^2 + 7x + 6$. **293.** 2) $12\frac{2}{3}$. **295.** 2) y^4; 4) 1. **296.** 2) $-3a$; 4) $-5c$. **297.** 2) $\frac{2}{15}a$; 4) $-9c$. **298.** 2) $9m$; 4) $\frac{4}{5}b$. **299.** 2) 8; 4) 7. **300.** 2) 3; 4) -3. **301.** 2) $-\frac{5}{3}$; 4) $-1{,}3$. **302.** 2) $-\frac{5}{3}p$; 4) $0{,}4c$. **303.** 2) $7m^6$; 4) $\frac{7}{6}$. **304.** 2) $\frac{9}{4}ab^2$; 4) $3ab$. **305.** 2) $-\frac{1}{13}axy^2$; 2) $\frac{1}{2}a^3b$. **306.** 2) $81x^4y$; 4) $x^7y^{11}z^3$. **307.** 2) $2b - 1$; 4) $2 - x$. **308.** 2) $4a - 3b$; 4) $-c + 1$. **309.** 2) $-\frac{2}{3}cb - 1$; 4) $-\frac{1}{4}ab + \frac{3}{4}a^2$. **310.** 2) $-2x - 3y + 4$; 4) $a + 3a^2b - 2$. **311.** 2) 1; 4) $-3a$. **312.** 2) 24; 4) 0. **313.** 2) a^3; 4) $c^2 + 3^2$. **314.** 2) $n^2 - m^2$; 4) $(\frac{1}{2})^3 - b^3$. **315.** $4c$ sm, c^2 м^2. **316.** $6k^2$ см^2, k^3 см^3. **317.** $3x^2$ yoki $\frac{1}{3}x^2$. **318.** 10 km. **319.** 108000. **320.** Yo‘q. **321.** 2) $3{,}08 \cdot 10^{13}$. **322.** $5{,}1 \cdot 10^8$; 10^{12}. **323.** 10 kg. **324.** 2) xy; 4) $10mn^2k$. **325.** 2) $13\frac{3}{4}$. **326.** 2) $3x^2$; 4) $8a^2 + b^2 - ab$. **327.** 2) $0{,}5x^2 + xz - 0{,}5y^2 + yz$; 4) $a^4 - 2a^3 + 3a^2 + 4a - 10$. **328.** 2) $2a^3 - 2ab^2 + 3a^2b - 3b^3$; 4) $6x^3 - 17x^2 - 4x + 3$. **329.** 2) $5x^3 + 8x^2 + 9x - 1$; 4) $1\frac{1}{4}a^5 + 2a^2x - 1\frac{1}{2}x^2$. **332.** 2) 180,7; 4) 12,5. **333.** 2) $2x^2 - 2x$; 4) $a^3 + ab - a^2b^2 - b^3$. **334.** 240 km. **335.** 2) 150000; 4) 4. **336.** 2) $3(a - x)$; 4) $6(a + 2)$. **337.** 2) $2(4a - 2b - 1)$; 4) $3(3x - y + 4z)$. **338.** 2) $c(d + b)$; 4) $x(3 - y)$. **339.** 2) $3b(d - a)$; 4) $3p(2k - 1)$. **340.** 2) $x(y - x + z)$; 4) $4b(b + 2a - 3a^2)$. **341.** 2) $a^3(a - 3)$; 4) $x^2y^2(y - x)$. **342.** 2) $6x^2(x^2 - 4)$; 4) $3a^2(2a^3 + 1)$. **343.** 2) $4x^2y(5xy + 1)$; 4) $3xyz(3z - 4y)$. **344.** 2) $5a^3(4a - 1 + 3a^2)$; 4) $2x^2y^2(y^2 - x^2 + 3xy)$. **345.** 2) 18700; 4) $-1{,}62$. **346.** 2) $(a+5)(b-c)$; 4) $(y - 3)(1 + b)$. **347.** 2) $(m - 3)(3n + 5m)$; 4) $(c - d)(7a - 2b)$. **348.** 2) $(x + y)(a^2 - b^2)$; 4) $(a^2 - 2b^2)(x + y)$. **349.** 2) $(p - q)(c - a + d)$; 4) $(x^2 + 1)(m - n - l)$. **350.** 2) $(b - c)(a + c)$; 4) $(x - y)(2b + 1)$. **351.** 2) $(a - 2)(6 - a)$; 4) $(m - 2)(a^2 - b)$. **352.** 2) $(x - y)(x - y - 3)$; 4) $(3 - b)(-a + 1 - b)$. **353.** 2) $x = 1$; 4) $x = 0{,}49$. **354.** Ulguradi. **355.** 2) $(m - n)(1 + p)$; 4) $(x - y)(1 + 2a)$. **356.** 2) $(a - b)(a - b + 1)$; 4) $(p - 1)(4q + p - 1)$. **357.** 2) $(p - 1)(4q + 1)$; 4) $(p - 1)(4q - 1)$. **358.** 2) $(b + c)(a + d)$; 4) $2(x - 1)(3x - 4y)$. **359.** 2) $(c + d)(a - 3b)$; 4) $(a - 3b)(x + 5y)$.

360. 2) $(b + c - a)(y - x^2)$; **361.** 2) 12500; 4) 28. **362.** 2) $-0{,}625$; 4) $-0{,}33$. **363.** 2) 906. **364.** 2) $t = -7$, $t = 4$. **365.** 2) $x^2 - 2xy + y^2$; 4) $x^2 + 2x + 1$; 6) $49 + 14m + m^2$. **366.** 2) $x^2 - 6x + 9$; 4) $y^2 - 12y + 36$; 6) $b^2 + b + \frac{1}{4}$. **367.** 2) $9x^2 + 12xy + 4y^2$; 4) $25z^2 - 10zt + t^2$. **368.** 2) $a^4 + 2a^2 + 1$; 4) $x^4 + 2x^2y^2 + y^4$. **369.** 2) $a^2 - \frac{2}{3}a + \frac{1}{9}$; 4) $\frac{x^2}{9} + \frac{xy}{6} + \frac{y^2}{16}$. **370.** 2) $0{,}16b^2 - 0{,}4bc + 0{,}25c^2$; 4) $\frac{1}{16}a^6 - \frac{2}{5}a^3 + \frac{16}{25}$. **371.** 2) $\frac{1}{9}x^4 + \frac{1}{3}x^3 + \frac{1}{4}x^2$; 4) $100x^4 -$ $- 60x^3y^3 + 9x^2y^6$. **372.** 2) $9b^4 + 12ab^3 + 4a^2b^2$; 4) $16x^2y^2 + 4xy^3 + 0{,}25y^4$. **373.** 2) 1681; 4) 9604. **374.** 2) 1006009; 4) 1521. **375.** 2) 3249; 4) 1002001. **376.** 2) $4xy$; 4) $8a^2 + 2b^2$. **377.** 2) $7a^2 - 52a + 112$; 4) $4x^2 - 16x - 4$. **378.** 2) $x = 2$; 4) $x = -0{,}5$. **379.** 2) $y = 3$; 4) $y = \frac{2}{3}$. **380.** 2) -11; 4) -17. **382.** 2) $(5 + x)^2$; 4) $(p - 0{,}8)^2$. **386.** 2) $p^2 - q^2$; 4) $m^2 - n^2$; 6) $m^2 - 4$. **387.** 2) $a^2 - 9$; 4) $x^2 - 49$; 6) $b^2 - 1$. **388.** 2) $c^2 - 9d^2$; 4) $9m^2 - 4n^2$. **389.** 2) $\frac{25}{36}a^2 - b^2$; 4) $\frac{4}{9}m^2 - \frac{9}{16}n^2$. **390.** 2) $a^4 - b^6$; 4) $m^6 - n^6$. **393.** 2) $25a^2b^4 - 4a^4b^2$; 4) $a^2b^6 - 16x^2y^2$. **394.** 2) $x^4 - 1$; 4) $81a^4 - 16b^4$. **395.** 2) 4896; 4) 2491. **396.** 2) 1584; 4) 39999. **397.** 2) $2a^2 + 4a$; 4) $24ab - 32b^2$. **399.** 2) $x = \frac{4}{3}$; 4) $x = 2$. **400.** 64 sm^2 ga kamaydi. **401.** -10. **402.** 2) 980; 4) 5,87. **405.** 2) $(2a - 3)(2a + 3)$; 4) $(9a - 4b)(9a + 4b)$. **406.** 2) $(ab - 4)(ab + 4)$; 4) $(4x - 5y)(4x + 5y)$. **407.** 2) $(\frac{2}{3}a -$ $- \frac{1}{4}b)(\frac{2}{3}a + \frac{1}{4}b)$; 4) $(0{,}3x - 0{,}4y)(0{,}3x + 0{,}4y)$. **408.** 2) $(xy^2 - 4)(xy^2 + 4)$; 4) $(5a -$ $- 3b^3)(5a + 3b^3)$. **409.** 2) $(a^2 - b^4)(a^2 + b^4)$; 4) $(b^2 - 9)(b^2 + 9)$. **410.** 2) $(m - n - k)(m - n + k)$; 4) $3(x - y)(3x + y)$. **411.** 2) $(a + 2b + c)(a - c)$; 4) $4(2a - b)(-a - 2b)$. **412.** 2) $(1 + c)^2$; 4) $(9 - x)^2$. **413.** 2) $(10 - 3a)^2$; 4) $(a + 5b)^2$. **414.** 2) $(p^2 - q)^2$; 4) $(5a^3 + 3b)^2$. **415.** 2) $(b^2 - 9)^2$; 4) $(4 - a^2b^2)^2$. **416.** 2) $-(3 - b)^2$; 4) $3(a + 2b)^2$. **417.** 2) 60 000; 4) 216. **418.** 2) $x = \frac{1}{2}$, $x = -\frac{1}{2}$; 4) $x = 5$. **419.** 2) 10000; 4) $\frac{2}{3}$. **420.** 2) $x^2 + 2xy + y^2$; 4) $x^2 -$ $- 2xy + y^2$. **421.** $(c + d)(c^2 - cd + d^2)$; 4) $(a - 3)(a^2 + 3a + 9)$; 6) $(a + 1)(a^2 - a + 1)$; 8) $(5 - b)(25 + 5b + b^2)$. **422.** 2) $(4 - 5y)(16 + 20y + 25y^2)$; 4) $(4y + \frac{1}{3})(16y^2 - \frac{4}{3}y + \frac{1}{9})$. **423.** 2) $(1 + 3b)(1 - 3b + 9b^2)$; 4) $(\frac{1}{2}a^2 + 5b)(\frac{1}{4}a^4 - \frac{5}{2}a^2b + 25b^2)$. **424.** 2) $(a + b)(a - b) \times$ $\times (a^4 + a^2b^2 + b^4)$; 4) $(2 + y)(2 - y)(16 + 4y^2 + y^4)$. **425.** 2) $y^3 + 8$; 4) $64c^3 - 125d^3$. **426.**

2) $a^6b^6-125a^3$; 4) $\frac{1}{8}x^3-\frac{1}{27}y^3$. **427.** 2) $16a^2(4a+5b)$; 4) $(a-b)(a^2+ab+b^2+a-b)$. **428.** 2) 0,02. **429.** 2) 5; 4) 26. **430.** 2) $x=3$; 4) $x=0{,}2$. **441.** 2) $x=2$. **442.** 2 km/soat, 16 km/soat. **443.** 2) $(x-y)(4+3x-3y)$; 4) $(b-a)(b-a-1)$. **444.** 2) $y(x+y)^2$; 4) $(b-a)^2(a-1)$. **445.** 2) $24x^2(y-z)$; 4) $4(2x-y)(2x-3y-1)$. **446.** 2) $5(x+y)(2x+1)$; 4) $(3z^2+2y^2)(16x-5y)$. **447.** 2) $(2nk+5m)(3mk-7n^2)$; 4) $(5c-3x)(8b-3c)$. **448.** 2) $16x+2$; 4) $-19y+6$. **450.** 2) $\frac{5}{8}$; 4) $\frac{11}{8}$. **454.** $\frac{a^2-b^2}{(a-b)^2}$. **456.** 2) 5; 4) 1,9; 6) 4. **457.** 2) $v=\frac{m}{p}$; 4) $a=\frac{p}{2}-b$. **458.** $x=\frac{np}{1000t}$, $x=3$. **459.** $t=\frac{a}{cn}$, $t=15$. **461.** 2) $\frac{4}{5}$, 4) -2. **462.** 2) $\frac{2}{3}$; 4) $\frac{b}{2c}$. **463.** 2) $\frac{1}{b^4}$; 4) b^2. **464.** 2) $\frac{2}{7}$; 4) $\frac{b}{3a}$; 6) $\frac{a^2b}{5c}$. **465.** 2) $\frac{7a}{5}$; 4) $\frac{1}{3(a-b)}$; 6) $-\frac{1}{3}$. **466.** 2) $\frac{1}{(m+n)^3}$; 4) $3y-2x$; 6) $\frac{2}{a(a-b)}$. **467.** 2) $\frac{2a}{m-n}$; 4) $\frac{4a-1}{2a+3}$; 6) $\frac{1+b}{1-b}$. **468.** 2) $\frac{q^2}{p-q}$; 4) $\frac{m}{n}$; 6) $-\frac{x}{y}$. **469.** 2) $\frac{3a+2b}{2a+3b}$; 4) $-\frac{1}{ab}$. **470.** 2) $\frac{1}{a+b}$; 4) $5+x$; 6) $-\frac{c+2}{2a}$. **471.** 2) $10-7b$; 4) $\frac{y}{5+y}$; 6) $\frac{5ab}{a^2-b^2}$. **472.** 2) $\frac{1}{b+7}$; 4) $\frac{1}{1-2p}$. **473.** 2) $\frac{4a+1}{4a-1}$; 4) $\frac{10(m+n)}{3(m-n)}$. **474.** 2) $n-m$; 4) $\frac{1}{5-2x}$. **475.** 2) $\frac{3y-4x}{3y+4x}$; 4) $\frac{6-c}{6+c}$; 6) $\frac{3c-2b}{a}$. **476.** 2) $a+1$; 4) $\frac{1}{2}$. **477.** 2) $\frac{b}{ab}$ va $\frac{2a}{ab}$; 4) $\frac{2a}{2b}$ va $\frac{a}{2b}$; 6) $\frac{32}{60}$ va $\frac{25}{60}$. **478.** 2) $\frac{9x^2}{12xy}$, $\frac{72}{12xy}$ va $\frac{16y^2}{12xy}$; 4) $\frac{2ax^2}{4x^3}$ va $\frac{b}{4x^3}$. **479.** 2) $\frac{6b^2}{2b}$ va $\frac{a^2}{2b}$; 4) $\frac{2b^2}{6ab}$, $\frac{9ac}{6ab}$, $\frac{6a^2b^2}{6ab}$. **480.** 2) $\frac{3a^2}{18a^2b^2}$, $\frac{2(a^2+b^2)}{18a^2b^2}$ va $\frac{a(3-a^2)}{18a^2b^2}$; 4) $\frac{21y^3}{60x^4y^4}$, $\frac{310x^3y}{60x^4y^4}$ va $\frac{80x^2}{60x^4y^4}$. **481.** 2) $\frac{6a}{(a-1)a}$ va $\frac{2(a-1)}{(a-1)a}$; 4) $\frac{8a^2}{12(a+1)}$ va $\frac{15a^2}{12(a+1)}$. **482.** 2) $\frac{7a(3x+y)}{9x^2-y^2}$ va $\frac{6b(3-y)}{9x^2-y^2}$; 4) $\frac{6x}{8x+8y}$ va $\frac{x}{8x+8y}$. **483.** 2) $\frac{7a}{x^2-9}$ va $\frac{a(x-3)}{x^2-9}$; 4) $\frac{6x(x+y)}{x^2-y^2}$, $\frac{7xy(x-y)}{x^2-y^2}$ va $\frac{3}{x^2-y^2}$. **484.** 2) $\frac{28c(b+c)}{70(b^2-c^2)}$, $\frac{6a^2}{70(b^2-c^2)}$ va $\frac{35b(b-c)}{70(b^2-c^2)}$; 4) $\frac{15x(x+1)}{12x(x^2-1)}$; $\frac{-48x^2}{12x(x^2-1)}$ va $\frac{4(x-1)}{12x(x^2-1)}$. **485.** 2) $\frac{5a}{b^3}$; 4) $\frac{x-y}{n+a}$. **486.** 2) $\frac{2a}{c^3}$; 4) $\frac{7}{a^2}$; 6) $\frac{8}{ab}$. **487.** 2) $\frac{11}{28}$; 4) $\frac{3}{5b}$; 6) $\frac{3ad-b}{12d}$. **488.** 2) $\frac{15+ab}{5a}$;

4) $\frac{2+7b}{b}$. **489.** 2) $\frac{2c+4c^2-3}{c^2}$; 4) $\frac{mn-kn^2+m^2}{n^2}$. **490.** 2) $\frac{k-n}{mnk}$; 4) $\frac{bd+ba}{acd}$;

6) $\frac{2n^2-3m}{mn^3}$. **491.** 2) $\frac{4a^4-21cb^3}{18a^3b^4}$; 4) $\frac{20y-21x+22}{28x^2y^2}$; 6) $\frac{b(cd^2+d+c)}{(cd)^2}$. **492.** 2) $\frac{3x}{2(1-x)}$;

4) $\frac{8y-25x}{10(y-3)}$. **493.** 2) $\frac{11}{10(b+1)}$; 4) $\frac{5x}{8(x+y)}$. **494.** 2) $\frac{5b^2-2a^2}{ab(x+y)}$; 4) $\frac{a+b-y}{ab}$. **495.** 2) $\frac{2(2a+3)}{a(1-a)}$;

4) $\frac{67b-3a}{40(a^2-b^2)}$. **496.** 2) $\frac{x-1}{x^2-9}$; 4) $\frac{2x^2+3x+2}{x^2-16}$. **497.** 2) $\frac{6n-47}{n^2-49}$; 4) $\frac{24y^2+y+1}{1-9y^2}$.

498. 2) $\frac{13a+4}{(3a+1)^2}$. **499.** 2) $\frac{2-11x}{(3x+1)^2}$; 4) $\frac{4-7n+7m}{(n-m)^2}$; 6) $\frac{2x^2+18}{(x^2-9)^2}$. **500.** 2) $\frac{b^2-3b}{b-2}$;

4) $\frac{1}{a+1}$. **501.** 2) $-\frac{1}{x+y}$; 4) $\frac{2(24-a)}{4a^2-9}$. **502.** 2) $\frac{b-3b^2-14}{6(b^2-1)}$; 4) $\frac{28n^2-4m^2+9mn}{m(4n^2-m^2)}$;

6) $\frac{4a^2-4a-b}{a^2+2a}$. **503.** 2) $\frac{2a}{a^3+8}$; 4) $-\frac{6m}{m^3-27}$. **504.** 2) $-\frac{12}{19}$. **505.** 2) $\frac{4}{13}$; 4) $\frac{15}{2}$.

506. 2) $\frac{k^2}{mn}$; 4) $\frac{3mk}{4nd}$; 6) $\frac{2a^2b^2}{c^3}$. **509.** 2) 2; 4) $\frac{a}{bc}$; 6) $\frac{ac}{b}$. **510.** 2) $\frac{k^2}{mn}$; 4) $\frac{3md}{2nk}$;

6) $\frac{15a^2c^2}{d}$. **511.** 2) $\frac{18a^2}{7}$; 4) $\frac{1}{a}$; 6) $\frac{a^3b^3}{d^2}$. **512.** 2) $\frac{2y}{5c^3}$; 4) $\frac{2d^2a^2}{3c}$; 6) $\frac{22p^3n}{m^4}$. **513.** 2) $10a^2b$;

4) $\frac{1}{4a^2b}$. **514.** 2) $\frac{2b}{a}$; 4) $3b$; 6) $\frac{(a+b)a}{3b}$. **515.** 2) $\frac{b}{3(1+a)}$; 4) $\frac{1}{3m^2(m+n)}$; 6) $\frac{5}{3(a-b)}$.

516. 2) $\frac{3x^2(x+y)}{2(x^2+y^2)}$; 4) $\frac{-18(n-m)^2(n+m)}{n(n+p)^2}$; 6) $\frac{1}{a^2-b^2}$. **517.** 2) $b-3$; 4) $(a-1)(2a-1)$.

518. 2) $\frac{2(a+1)}{3}$; 4) 1; 6) $\frac{b^2}{b^2+1}$. **519.** 2) $\frac{a^2(b^2-1)}{b^2}$; 4) $\frac{2(m+n)}{n}$. **520.** 2) $\frac{4ab}{a^2-b^2}$;

4) $\frac{1}{6(c+d)}$. **521.** 2) $\frac{9z}{z+2}$; 4) $\frac{m+5}{m-2}$. **522.** 2) $\frac{b}{a+b}$; 4) $\frac{1}{c}$. **523.** 2) $\frac{4}{a-b}$; 4) $\frac{1}{c(a+b)}$.

526. $\frac{v-v_1}{v+v_1}\cdot s$ km. **527.** 6 donadan. **528.** 2) $\frac{3(x^2-2x+4)}{x^3+8}$, $\frac{x+1}{x^3+8}$ va $\frac{(x+2)^2}{x^3+8}$.

529. 2) $\frac{55b-61}{24}$; 4) $\frac{5-27b}{36}$. **530.** 2) $\frac{7q-p}{3p-q}$; 4) $\frac{8a+8b-70}{2b-5}$. **531.** 2) $\frac{a^2-b^2}{7}$;

4) $\frac{m+n}{2(p^2-pc+c^2)}$. **532.** 2) $\frac{x(x+2)(x-3)}{(x-2)(x+3)(x^2+2)}$; 4) 1. **533.** 2) $-2(a-1)^2$; 4) $\frac{a^2+4}{4a}$. **534.** 2) 2. **535.** 2) Noto‘g‘ri. **536.** $7\frac{1}{2}$. **537.** $2a(30-a)$; -128. **538.** $a\cdot 100+b\cdot 10+c$; $c\cdot 100+b\cdot 10+a$; *a ta*. **539.** $x=1000a+c$. **540.** 3) $4\frac{2}{3}$. **541.** 2) $-a^4b^2c^3$. **542.** 2) $0{,}64a^2c^4$. **543.** 4) $3a^2bm$. **546.** 4) $1{,}5a^3+11{,}5a^2-a-1$. **547.** 2) $x=2\frac{5}{11}$. **551.** 4) $x=-\frac{1}{8}$. **552.** 40, 36, 43. **553.** 9 yildan so‘ng. **554.** 4 yildan so‘ng. **555.** 1,5 soatda. **556.** 1,5 soatda . **557.** 2) $\frac{a}{4}$. **558.** 2) $16ab$. **560.** 2) $3(1+a)(7-3a)$. **561.** 2) $4(3b-2)(5b+1)$; 4) $(17a-9b)(b-13a)$. **562.** 2) $(m-1)(m^2+1)$; 4) $(1+a-b)(1-a+b)$. **570.** 2) 1. **571.** 63 km. **572.** $27\frac{3}{11}$ minutdan so‘ng. **573.** 41574. **574.** Mis — 25,5N; rux — 10,5 N. **575.** $\frac{1}{2}$ kg. **577.** 35 km. **578.** 120 km. **579.** 150 ta. **580.** 2) $\frac{3ac(2a+3)}{c(4a^2-9)}$, $\frac{4ac(2a-3)}{c(4a^2-9)}$ va $\frac{5b}{c(4a^2-9)}$. **581.** 2) $\frac{ab-2}{a+1}$; 4) $\frac{x(y-x)}{y+x}$. **582.** 2) $\frac{2n(2n-k)}{2n+k}$; 4) $\frac{2q(m-2q)}{m+2q}$. **583.** 4) $\frac{m+7n}{10}$. **585.** 2) 1. **586.** $x=6$. **587.** 2) $x=-\frac{25}{34}$; 4) $x=-6{,}5$. **588.** 160 km. **589.** 9 km/soat. **590.** 80 km/soat; 75 km/soat. **591.** 2) $-\frac{2}{3}$; 4) 0,019.

„O‘zingizni tekshirib ko‘ring“ topshiriqlariga javoblar

I bob. 1. 1) 120,3; 2) $-3\frac{1}{6}$; **2.** $3x+4y$; $\frac{1}{3}$. **3.** $10a+15b$.

II bob. 1. Ha, $x=-4$; **2.** 1) $x=\frac{1}{3}$; 2) $x=3$. **3.** 5 kg; 3 kg.

III bob. 1. 5^5; 3^2; 2^{12}; 6^5. **2.** $3b+d$. **3.** $-1{,}25\,a^4b^3c^2$; $0{,}7m-2n-1$. **4.** $3m^2-4$; $-3{,}8125$.

IV bob. 1. $2a^2+12a$. **2.** 1) $y(x-2)$; 2) $(4a-9)(4a+9)$; 3) $3x^2\cdot(1-2x)$; 4) $(x-5)^2$; 5) $(x-1)(3+y)$; 6) $2(a-b)^2$. **3.** $(a-3b)(a+3)$; 8.

V bob. 1. $b\neq 0$, $a\neq 1$, $a\neq -2$. **2.** 1) $\frac{1}{a}$; 2) $\frac{4ab}{a^2-b^2}$; 3) 4; 4) $\frac{a-b}{b}$. **3.** $\frac{1}{x-3}$; -3.

Qiziqarli masalalarga javoblar

1. 99 + 9 : 9. **2.**44 ta uchburchak, 10 kvadrat, 8 ta to‘g‘ri to‘rtburchak. **3.** 5 yoshda. **4.** 18 minut. **5.** 1) 6; 2) 3; 3) 4; 4) 9. **6.** 24 000 km. **7.** 6 ta **8.** 1) 7; 2) 4 o‘g‘il, 3 qiz. **9.** 10 metr. **10.** Mumkin emas.

I bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
E	A	B	C	D	B	C	E	B	D	A	B	C	D	E	A	D	B

II bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
A	B	C	D	E	A	B	C	D	E	A	B	C	D	E	A	B	C	D

III bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C	E	B	D	B	D	A	B	D	E	B	C	E	A	B	C	D	E	D	A

IV bob

1	2	3	4	5	6	7	8	9	10	11	12	13
A	D	E	A	B	D	E	B	A	B	D	E	B

V bob

1	2	3	4	5	6	7	8	9	10
A	B	E	B	D	B	C	E	A	B

TUSHUNCHALAR KO‘RCATKICHI

ISMLAR KO‘RSATKICHI

MUNDARIJA

IV bob. KO'PHADNI KO'PAYTUVCHILARGA AJRATISH

V bob. ALGEBRAIK KASRLAR

SHAVKAT ARIFJANOVICH ALIMOV,
ALIMDJAN RAXIMOVICH XALMUXAMEDOV,
MIRFAZIL ABDILXAKOVICH MIRZAXMEDOV

ALGEBRA

Umumiy ta'lim maktablarining
7- sinfi uchun darslik

3- n a s h r i

„O'qituvchi" nashriyot-matbaa ijodiy uyi
Toshkent — 2009

Muharrirlar: *O' Husanov, N.G'oipov, X. Alimov*
Rasmlar muharriri *Sh. Xo'jayev*
Tex. muharrir *T. Greshnikova*
Komputerda sahifalovchi *N. Ahmedova*
Musahhih *Z. Sodiqova*

Original-meketdan bosishga ruxsat etildi 26.01.2009. Bichimi $70\times90^{1}/_{16}$. Tayms garn. Kegli 12 shponli. Ofset bosma usulida bosildi. Shartli b.t. 12,87. Nashr t. 10,0. nusxada bosildi. Buyurtma № ... . Bahosi ... .

O'zbekiston Matbuot va axborot agentligining „O'qituvchi" nashriyot-matbaa ijodiy uyi. Toshkent, 129. Navoiy ko'chasi, 30- uy.//Toshkent, Yunusobod dahasi, Murodov ko'chasi, 1- uy. Shartnoma №

A 50 **Alimov Sh.A.**
Algebra: Umumiy ta'lim maktablarining 7- sinfi uchun darslik/Sh.A. Alimov, O.R. Xolmuhamedov, M.A. Mirzaahmedov. 3- nashri. — T.: „O'qituvchi" NMIU, 2009. — 176 b.

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