HARBIYDA TUSHGAN BIR MISOL YECHIMI

Misol. $\{2-\sqrt{2}\}+\{2+\sqrt{2}\}+\{3-\sqrt{3}\}+\{3+\sqrt{3}\}+...+\{2017-\sqrt{2017}\}+\{2017+\sqrt{2017}\}$ yig'indini hisoblang.

Yechish: Keling avval shu misolning **orginal** yechimini keltirib o'taylik. Shundan so'ng qisqa hulosani chiqarib olamiz.

I. $a\in N,\ \sqrt{a}\notin N\ da\ \{a-\sqrt{a}\}+\{a+\sqrt{a}\}=1$ ekanligini ko'rsatamiz. a = 2 holni qaraymiz.

1) $y=\{x\}$ ning asosiy davri T = 1 ekanligi uchun $f(x)=f(x-n\cdot T)=f(x+n\cdot T)$ qoida asosida $\{2-\sqrt{2}\}+\{2+\sqrt{2}\}=\{2-\sqrt{2}-2\cdot1\}+\{2+\sqrt{2}-2\cdot1\}=\{-\sqrt{2}\}+\{\sqrt{2}\}$ kabi yozib olamiz.

2) $-\sqrt{2}=-1{,}41...$ ekanligidan bu sonning butun qismi $[-\sqrt{2}]=[-1{,}41...]=-2$ bo'ladi. Sonning kasr qismi quyidagi tenglikdan topiladi:

$$\{x\}=x-[x]\Rightarrow\ \{-\sqrt{2}\}=-\sqrt{2}-(-2)=2-\sqrt{2}.$$

Xuddi shunday,

$\sqrt{2}=1{,}41...$ ekanligidan bu sonning butun qismi $[\sqrt{2}]=[1{,}41...]=1$ bo'ladi.

$$\{x\}=x-[x]\text{ dan }\{\sqrt{2}\}=\sqrt{2}-1=\sqrt{2}-1.$$

3) Bu ikki kasr sonlar yig'indisi $\{-\sqrt{2}\}+\{\sqrt{2}\}=2-\sqrt{2}+\sqrt{2}-1=1$ ga teng.

Demak, $a\in N,\ \sqrt{a}\notin N\ da\ \{a-\sqrt{a}\}+\{a+\sqrt{a}\}=1$ tenglik to'g'ri ekan.

II. $a\in N,\ \sqrt{a}\in N\ da\ \{a-\sqrt{a}\}+\{a+\sqrt{a}\}=0$ ekanligini ko'rish qiyin emas. Masalan, a = 4 holni qaraymiz. $\{4-\sqrt{4}\}+\{4+\sqrt{4}\}=\{4-2\}+\{4+2\}=\{2\}+\{6\}=0+0=0$.

Berilgan misolda yig'indisi 0 ga teng bo'ladigan juftliklar $[\sqrt{2017}]-1=43$ tani tashkil etadi. Ya'ni $\{4-\sqrt{4}\}=\{2^2-\sqrt{2^2}\}$ dan $\{1936-\sqrt{1936}\}=\{44^2-\sqrt{44^2}\}$ gacha.

Demak, berilgan yig'indi quyidagiga teng ekan:

$$\underbrace{\{2-\sqrt{2}\}+\{2+\sqrt{2}\}}_{1}+\underbrace{\{3-\sqrt{3}\}+\{3+\sqrt{3}\}}_{1}+...+\underbrace{\{2017-\sqrt{2017}\}+\{2017+\sqrt{2017}\}}_{1}=$$

$$=\underbrace{1+1+0+...+1}_{2016ta}=2016-43=1973.$$

Xulosa: $\{a-\sqrt{a}\}+\{a+\sqrt{a}\}=\begin{cases}1,\ agar\ \ a\in N,\ \sqrt{a}\notin N\\0,\ agar\ \ a\in N,\ \sqrt{a}\in N\end{cases}.$

$$+\begin{cases}\{2-\sqrt{2}\}+\{2+\sqrt{2}\}=1,\\\{3-\sqrt{3}\}+\{3+\sqrt{3}\}=1,\\\{4-\sqrt{4}\}+\{4+\sqrt{4}\}=0,\\..................................\\\{2017-\sqrt{2017}\}+\{2017+\sqrt{2017}\}=1\end{cases}$$

$$\underbrace{1+1+0+...+1}_{2016-43=1973ta}=1973.$$

Javob: 1973.

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