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QO`RG`ONTEPA TUMAN XALQ TA`LIMI BO`LIMI

35 – UMUMIY O`RTA TA`LIM MAKTABI
MATEMATIKA FANI O`QITUVCHISI

TO`YCHIYEV
MAMASODIQ

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KIRUVCHILAR UCHUN

MATEMATIKADAN FOYDALI FORMULALAR

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SIZGA OMAD YOR BO`LSIN

Bilim shunday boylikki,
uni boshqalarga qancha bersang
o`zingda shuncha ko`payadi.

So'rab o`rgangan olim
Orlanib so`ramagan o`ziga zolim.

Иллат излаганга – иллатдир дунё,
Ғурбат излаганга ғурбатдир дунё.
Ким нени изласа топур бегумон.
Ҳикмат излаганга ҳикматдир дунё.

Natural sonlarga doir formulalar

1. ***1*** dan ***N*** gacha barcha natural sonlarni yozib chiqish uchun nechta raqam ketadi?

$$K = n(N+1) - \overbrace{111\ldots1}^{n}$$

Bunda, n N ning raqamlari soni

2. $\overline{1234\ldots n0}$ sonining raqamlari **y**ig`indisi quyidagi formula bilan topiladi:

$$\mathrm{Ry} = 5\mathrm{n}^2 + 41\mathrm{n}$$

3. $\overline{m1m2m3\ldots n0}$ sonining raqamlari **y**ig`indisi quyidagi formula bilan topiladi:

$$\mathrm{Ry} = 5(\mathrm{n}^2 - \mathrm{m}^2) + 41(\mathrm{n}-\mathrm{m})$$

4. $\overline{1234\ldots nt}$ sonining raqamlari **y**ig`indisi quyidagi formula bilan topiladi:

$$Ry = 5n^2 + (41+n)t + \frac{t(t+1)}{2}$$

5. $\overline{mk\ldots\ldots\ldots.nt}$ sonning raqmlari **y**ig`indisi quyidagi formula bilan topiladi:

$$Ry = 5(n^2 - m^2) + (41+t)n - (40+k)m + \frac{(t+k)(t-k+1)}{2}$$

6. ***1*** dan ***N*** gacha barcha natural sonlar ichida ***m*** ga ham, ***n*** ga ham bo`linmaydiganlari nechta?

$$K = N + \left[\frac{N}{mn}\right] - \left[\frac{N}{m}\right] - \left[\frac{N}{n}\right]$$

Bunda, [x] – x ning butun qismi.

7. $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}$ tenglamaning natural ildizlari soni n^2 ning bo`luvchilari soniga teng.

Butun ildizlari soni esa n^2 ning bo`luvchilari sonining ikki baravaridan bitta kam.
Bu tenglamaning barcha natural ildizlarini topish uchun
$\mathbf{(x - n)(y - n) = n^2}$ tenglamani yechish kifoya.

8. $|x| + |y| \le n$ sohaga tegishli butun nuqtalar soni $n^2 + (n+1)^2$ ga teng.

9. $|x| + |y| < n$ sohaga tegishli butun nuqtalar soni $(n - 1)^2 + n^2$ ga teng.

10. Soat ***a*** dan ***b*** minut o`tganda soat va minut millari orasidagi burchak quyidagi fo`rmula bilan topiladi.

$$x=\left|30a-\frac{11b}{2}\right| \text{ gradus}$$

Agar, $N=p_1^{\alpha_1}\cdot p_2^{\alpha_2}\cdot\ldots\cdot p_n^{\alpha_n}$ bo`lsa, u xolda:

11. Natural bo`luvchilari soni → $NBS=(\alpha_1+1)\cdot(\alpha_2+1)\ldots(\alpha_n+1)$

12. Natural bo`luvchilari yig`indisi → $NBY=\frac{p_1^{\alpha_1+1}-1}{p_1-1}\cdot\frac{p_2^{\alpha_1+1}-1}{p_2-1}\ldots\frac{p_n^{\alpha_n+1}-1}{p_n-1}$

13. *Barcha natural sonlar ketma-ket yozilsa, K – o`rinda qaysi raqam turadi?*
Bumasalani yechishda quyidagi formuladan fodalanamiz.

Bunda N- biz raqamini izlayotgan son

$$N=\frac{K-n+\overbrace{111...1}^{n}}{n}$$

Bunda, n N ning raqamlari soni

Qoldiqdan foydalanamiz….

10. ***m*** dan ***n*** gacha barcha natural sonlar ichida ***k*** ga qoldiqsiz bo`linadigan sonlar nechta?

$$x=\left[\frac{n}{k}\right]-\left[\frac{m}{k}\right]$$

Foizga doir formulalar

1. ***a*** kg mevaning tarkibida ***p*** **%** suv bor. Ma`lum muddat quritilgandan keyin suv ***q*** **%** , og`irligi esa ***b*** kg bo`lib qoldi. Uholda: $a(100-p)=b(100-q)$ tenglik o`rinli.

2. ***p*** **%** li ***a*** kg suyuqlik bilan ***q*** **%** li ***b*** kg suyuqlikni aralashtirsak, xosil bo`lgan ***a+b*** kg aralashmaning konsentratsiyasi quyidagi formula bilan topiladi: $x=\frac{ap+bq}{a+b}$

3. Biror buyumning narhi avval p % ga, keyin yana q % ga oshirilsa, jami necha foiz oshgan bo`ladi?

$$x=p+q+\frac{pq}{100}$$

4. Biror buyumning narhi avval p % ga, keyin yana q % ga kamaytirilsa, jami necha foiz kamaygan bo`ladi?

$$x=p+q-\frac{pq}{100}$$

5. Biror buyumning narhi avval p % ga oshirilib, keyin q % ga kamaytirilsa, jami necha foiz o`zgargan bo`ladi?

$$x=p+q-\frac{pq}{100}$$

Bunda: x>0 bo`lsa, ortgan,
x<0 bo`lsa, kamaygan bo`ladi

6. Biror buyumning narhi p % ga oshirildi. Uni o`z harhiga qaytarish uchun yangi narhni necha foiz kamaytirish kerak?

$$x=\frac{100p}{100+p}$$

7. Biror buyumning narhi p % ga kamaytirildi. Uni o`z harhiga qaytarish uchun yangi narhni necha foiz oshirish kerak?

$$x=\frac{100p}{100-p}$$

8. Bankka oddiy foiz hisobida yillik p % daromad bilan a so`m qo`yilsa, t yildan keyin omonat miqdori qancha bo`ladi?

$$x=a\left(1+\frac{pt}{100}\right)$$

9. Bankka murakkab foiz hisobida yillik p % daromad bilan a so`m qo`yilsa, t yildan keyin omonat miqdori qancha bo`ladi?

$$x=a\left(1+\frac{p}{100}\right)^t$$

10. $\boldsymbol{a}$ va $\boldsymbol{b}$ sonlarining protsent nisbatini: $x=\frac{a}{b}\cdot 100$

11. Agar $\boldsymbol{a}$ soni $\boldsymbol{b}$ sonining $\boldsymbol{p}$ **%** ini tashkil qilsa, u xolda $\boldsymbol{b}$ soni $\boldsymbol{a}$ sonining

$\frac{100^2}{p}$ % ini tashkil qiladi.

Modulli tenglamalar.

1. $|x-a|+|x-b|=c$ tenglamaning ildizlari :

1. $|a-b|>c$ bo`lsa, ildizlari mavjud emas.
2. $|a-b|=c$ bo`lsa, cheksiz ko`p: $x\in[a;b]$
3. $|a-b|<c$ bo`lsa, ikkita ildizga ega: $x_1=\frac{a+b-c}{2}$

$x_2=\frac{a+b+c}{2}$

2. $|x-a|-|x-b|=c$ tenglamaning ildizlari :

1. $|a-b|>|c|$ bo`lsa, bitta, $x=\frac{a+b+c}{2}$.
2. $|a-b|=|c|$ bo`lsa, cheksiz ko`p: *a<b da x€[b; ∞)*

a>b da x€(-∞;b]

3. $|a-b|<|c|$ bo`lsa, mavjud emas, Ǿ (*bo`sh to`plam*)

3. $|x-a|+|x-b|+|x-c|=d$ (a<b<c) tenglamaning ildizlari :

1. $|a-c|>d$ bo`lsa, u holda ildizga ega emas. Ø
2. $|a-c|=d$ bo`lsa, 1 ta ildizga ega: $\boxed{x=b}$
3. $|a-c|<d$ bo`lsa, 2 ta ildizga ega:

$$\boxed{x_1=\frac{a+b+c-d}{3}} \quad \text{va} \quad \boxed{x_2=\frac{a+b+c+d}{3}}$$

Funksiyalarga oid formulalar

1. ***f(x)*** ko`phadning barcha koeffitsiyentlari yig`indisi ***f(1)*** ga teng.

2. ***f(x)*** toq darajali hadlarining koeffitsiyentlari yig`indisi $\frac{f(1)-f(-1)}{2}$ ga teng.

3. ***f(x)*** juft darajali hadlarining koeffitsiyentlari yig`indisi $\frac{f(1)+f(-1)}{2}$ ga teng.

4. $M(x_0\,;\,y_0)$ nuqtadan $ax+by+c=0$ to`g`ri chiziqqacha bo`lgan masofa:

$$\boxed{d=\left|\frac{ax_0+by_0+c}{\sqrt{a^2+b^2}}\right|}$$

**5`**. $M(x_0\,;\,y_0)$ nuqtadan $y=kx+b$ to`g`ri chiziqqacha bo`lgan masofa:

$$\boxed{d=\left|\frac{kx_0-y_0+b}{\sqrt{k^2+1}}\right|}$$

6. $y=a(x-m)^2+n$ parabolani $\vec{a}(x_0\,;y_0)$ vektor bo`yicha siljitilsa,

$y=a(x-m-x_0)^2+n+y_0$ parabola hosil bo`ladi.

7. Kvadrat ildizlar yig`indisi va ayirmasiga oid ajoyb formula:

$$\boxed{\sqrt{a+\sqrt{b}}\pm\sqrt{a-\sqrt{b}}=\sqrt{2a\pm2\sqrt{a^2-b}}}$$

8. Radikallar formulasi:

$$\sqrt{a\pm\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}\pm\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

9. $y=kx+a$ *va* $y=kx+b$ *parallel to`g`ri chiziqlar orasidagi masofa:*

$$d=\frac{|a-b|}{\sqrt{k^2+1}}$$

10. $y=a$ *ga nisbatan* $y=kx+b$ *to`g`ri chiziqqa simmetrik bo`lgan to`g`ri chiziqning tenglamasi:*

$$y=2a-kx-b$$

11. $y=a$ *ga nisbatan* $y=f(x)$ *funksiya grafigiga simmetrik bo`lgan funksiya tenglamasi:*

$$y=2a-f(x)$$

12. Ox o`qiga nisbatan* $y=f(x)$ *funksiya grafigiga simmetrik bo`lgan funksiya tenglamasi:

$$y=-f(x)$$

13. Oy o`qiga nisbatan* $y=f(x)$ *funksiya grafigiga simmetrik bo`lgan funksiya tenglamasi:

$$y=f(-x)$$

14. $y=x$ *ga nisbatan* $y=f(x)$ *funksiya grafigiga simmetrik bo`lgan funksiya tenglamasi shu funksiyaga teskari bo`lgan funksiyadir, ya`ni:*

$$x=g(y)\quad\Rightarrow\quad y=g(x)$$

15. $y=kx+b$ *to`g`ri chiziqqa nisbatan* $A(m;n)$ *nuqtaga simmerik bo`lgan* $A`(x;y)$ *nuqtaning koordinatalari quyidagi formula bilan topiladi:*

$$x=-\frac{k^2m-2kn+2b-m}{k^2+1}$$

$$y=\frac{k^2n+2km+2b-n}{k^2+1}$$

16. $y=mx+a$ *va* $y=nx+b$ *kesishuvchi to`g`ri chiziqlar orasidagi burchak quyidagi formula bilan topiladi:*

$$tg\varphi=\left|\frac{m-n}{1+mn}\right|$$

17. $\log_x y>0$ *tengsizlikni yechish uchun* ***(x-1)(y-1)>0*** *tengsizlikni yechish kifoya.*

18. $\log_x y<0$ *tengsizlikni yechish uchun* ***(x-1)(y-1)<0*** *tengsizlikni yechish kifoya.*

*19. Agar **a** ta qush **b** ta qurtni **c** minutda yeb tugatsa, **p** ta qush **q** ta qurtni necha minutda yeb tugatadi?*

$$x=\frac{aqc}{bp}$$

20. ***a[x] = b{x}*** *tenlamaning ildizlari quyidagi fo`rmula bilan topiladi:*

$$x=\frac{a+b}{b}\cdot n$$

bunda, $n = \overline{0;\left[\frac{b}{a}\right]}$ *Demak, bu tenglama* $\left[\frac{b}{a}\right]+1$ *ta ildizga ega bo`ladi.*

21. *Kemaning oqim bo`ylab tezligi* ***a****, oqimga qarshi tezligi* ***b*** *bo`lsa, u holda:*

O`z tezligi $v = \frac{a+b}{2}$ *ga teng., Suvning tezligi esa* $v_s = \frac{a-b}{2}$ *ga teng.*

22. *Kema ma`lum masofani oqim bo`ylab* ***a*** *soatda, oqimga qarshi* ***b*** *soatda bosib o`tsa,*

sol shu masofani $\frac{2ab}{b-a}$ *soatda bosib o`tadi.*

23. *Eskalator tinch turgan odamni yuqoriga* ***a*** *minutda olib chiqadi. Odam ham eskalator bilan birga xarakatlansa* ***b*** *minutda yuqoriga chiqadi. To`xtab turgan eskalatorda odam o`zi yursa*

$t = \frac{ab}{a-b}$ *minutda yuqoriga chiqadi?*

24. ***a*** *va* ***b*** *sonlari o`rtasiga uchta musbat son shunday qo`yilganki, ular berilgan sonlar bilan birgalikda geometrik progressiya hosil qiladi. Qo`yilgan sonlar yig`indisini toping.*

Avvalo ***q*** *ni topamiz:* $q = \sqrt[4]{\frac{b}{a}}$ $\quad S = aq(q^2 + q + 1)$

Yoki, ***q*** *ni topib o`tirmasdan:* $S = \sqrt{ab} + \sqrt[4]{a^3 b} + \sqrt[4]{ab^3}$

25. $M(x_0 ; y_0)$ nuqtadan $(x-a)^2 + (y-b)^2 = R^2$ aylanagacha bo`lgan masofa:

$$d = \sqrt{(a-x_o)^2 + (b-y_o)^2} - R$$

26. *Agar* ***n*** *juft natural son bo`lsa, quyidagi formulalar o`rinli:*

$$\sqrt[n]{(a-b)^n} = a-b \quad agar, \quad a > b \quad bo`lsa.$$
$$\sqrt[n]{(a-b)^n} = b-a \quad agar, \quad a < b \quad bo`lsa.$$

27. *Ko`paytuvchilarga ajratish:*

1) $a^2(c-b) + b^2(a-c) + c^2(b-a) = (a-b)(b-c)(c-a)$

2) $a(c+b)^2+b(a+c)^2+c(b+a)^2-4abc = (a+b)(b+c)(c+a)$

3) $a^3(c-b) + b^3(a-c) + c^3(b-a) = (a-b)(b-c)(c-a)(a+b+c)$

4) $a(c^2-b^2) + b(a^2-c^2) + c(b^2-a^2) = (a-b)(b-c)(c-a)$

Aralshmalarga doir masalalar.

*1. **1** – aralashmaning 1 kilogrammi **a** so`m, **2** – aralashmaning 1 kilogrammi esa **b** so`m turadi. Ulardan **p : q** nisbatda olib tayyorlangan yangi aralashmaning 1 kilogrammi qancha turadi?*

$$X = \frac{ap + bq}{p + q}$$

*2. **1** – idishda **p %** li, **2** –idishda **q %** li aralashma bor edi. 1- idishdan **a** litr 2 - idishdan esa **b** litr olib tayyorlangan yangi aralashmaning konsentratsiyasini aniqlang.*

$$X = \frac{ap + bq}{a + b}$$

*3. Ikki buyumning birgalikdagi narxi **a** so`m, ulardan birinchisi **p** % kamaytirilib, ikkinchisini **q** % oshirildi , endi ularning birgalikdagi narxi **b** so`m bo`lib qoldi. Dastlabki narxlarini toping.*

$$BBDN = \frac{aq + 100(a - b)}{p + q} \qquad IBDN = \frac{ap - 100(a - b)}{p + q}$$

(BBDN – Birinchi Buyumning Dastlabki Narxi. IBDN - Ikkinchi Buyumning Dastlabki Narxi)

*4. Imtihonda **p**% o`quvchi birorta ham masalani yecha olmagan, **a** ta o`quvchi xatoga yo`l qo`ygan. Agar barcha masalalarni to`liq yechgan o`quvchilarning masalalarni umuman yecha olmagan o`quvchilarga nisbati **m : n** kabi bo`lsa, qancha o`quvchi imtihon topshirgan?*

$$X = \frac{100an}{(100 - p)n - pm}$$

5. Birinchi kuni ish normasining $\frac{m}{n}$ qismi bajarildi. Ikkinchi kuni birinchi kuni bajarilgan ishning $\frac{p}{q}$ qismicha ko`p ish bajarildi. Shu ikki kunda ish no`rmasining qancha qismi bajarilgan?

$$X = \frac{m(p + 2q)}{nq}$$

Qoldiqli bo`lishga doir masalalar.

*1. Ketma-ket kelgan uchta natural sondan kattasining kvadrati qolgan ikkitasining ko`paytmasidan **a** taga katta. Berilgan sonlardan kattasini toping.*

$$X = \frac{a + 2}{3}$$

*2. Ketma-ket kelgan uchta natural sondan kichigining kvadrati qolgan ikkitasining ko`paytmasidan **a** taga kam. Berilgan sonlardan kichigini toping.*

$$X = \frac{a - 2}{3}$$

*3. Ikkita sonning ayirmasi **a** ga teng, ulardan biri ikkinchisidan **n** marta katta.*

Shu sonlardan kattasini toping.

$$X = \frac{an}{n-1}$$

4. *Ikkita sonning ayirmasi* ***a*** *ga teng, ulardan biri ikkinchisidan* ***n*** *marta kichik. Shu sonlardan kichigini toping.*

$$X = \frac{a}{n-1}$$

5. *Qoldiqli bo`lish fo`rmulasi:* $N = bk+q$ ***N*** *sonini* ***b*** *ga bo`lganda* ***k*** *tadan tegib* ***q*** *qoldiq qoladi.*

Bunda: $0 \leq q < b$

6. *Agar,* ***N*** *sonini* ***a*** *ga,* ***b*** *ga va* ***c*** *ga bo`lganda bir xil* ***q*** *qoldiq qolsa, u holda:*

$$N = EKUK(a,b,c) \cdot k + q$$

7. ***M*** *va* ***N*** *sonlarini* ***a*** *ga bo`lganda bir xil qoldiq qolsa, u holda* $M - N$ *ayirma* ***a*** *ga qoldiqsiz bo`linadi.*

Geometrik masalalar.

1. *Balandligi* ***H****, asosining radiusi* ***R*** *bo`lgan silindr ichida uzunligi* ***a*** *ga teng kesma joylashgan. Shu kesmadan o`qqacha bo`lgan masofani toping.*

$$d = \sqrt{R^2 - \frac{a^2 - H^2}{4}}$$

2. *Aylananing ikkita kesishuvchi vatarlaridan biri kesishish nuqtasida uzunliklari* ***a*** *va* ***b*** *bo`lgan kesmalarga, ikkinchisi uzunliklari* ***p:q*** *nisbatda bo`lgan kesmalarga ajraladi. Ikkinchi vatarning uzunligini toping.*

$$CD = (p+q) \cdot \sqrt{\frac{ab}{pq}}$$

3. *k va l ning qanday qiymatlarida* $y = \frac{k}{x}$ *va* $y = kx + l$ *chiziqlar* $M(x_o\, ; y_o)$ *nuqtadan o`tadi?*

$$k = x_o y_o \qquad va \qquad l = y_o - kx_o$$

4. $M(x_o\, ; y_o)$ *nuqtadan o`tib,* $\vec{a}\,(m\, ;\, n)$ *vektorga perpendikulyar bo`lgan to`g`ri chiziq tenglamasi quyidagi formula bilan topiladi:*

$$mx + ny = mx_o + ny_o$$

5. $ax + by = c$ *to`g`ri chiziqqa perpendikulyar bo`lgan* $\vec{a}\,(m\, ;\, n)$ *vektorning koordinatalari quyidagi formula bilan topiladi:* $m = \frac{kc}{b};\quad n = \frac{kc}{a}$ *(bunda* ***k*** *ixtiyoriy son.)*

6. *Muntazam n burchakka ichki chizilgan aylana va tashqi chizilgan aylanalarning*

Radiuslari nisbati: $\frac{r}{R} = \cos\frac{\pi}{n}$ *Aylanalar uzunliklari nisbati:* $\frac{c}{C} = \cos\frac{\pi}{n}$

Doiralar yuzlarining nisbati:

$$\frac{s}{S} = \cos^2\frac{\pi}{n}$$

7. Tomoni diagonallarining o`rtaproporsionaliga teng bo`lgan rombning o`tmas burchagi 150$^o$ ga, o`tkir burchagi esa 30^o ga teng bo`ladi va aksincha o`tkir burchagi 30^o ga teng bo`lgan rombning tomoni diagonallarining o`rta proporsionaliga teng bo`ladi.

$$a = \sqrt{d_1 d_2}$$

8. *ABC teng yonli uchburchakda AB = BC = 2a , AC = a. AE – mediana, CD – bissektrisa bo`lsa, DE kesma uzunligini quyidagi formula bilan topiladi:*

$$DE = \frac{2a}{3}$$

9. "O`rtalar"

1. *O`rta garmonik:* $\frac{2ab}{a+b}$ *(Trapetsiyaning diagonallari kesishgan nuqtadan o`tuvchi kesma)*
2. *O`rta geometrik:* $\sqrt{ab}$ *(Trapetsiyani 2 ta o`xshash trapetsiyaga ajratuvchi kesma)*
3. *O`rta arifmetik:* $\frac{a+b}{2}$ *(Trapetsiyaning o`rta chizig`i)*
4. *O`rta kvadratik:* $\sqrt{\frac{a^2+b^2}{2}}$ *(Trapetsiyaning yuzini teng ikkiga bo`luvchi kesma)*

Bunda:

$$\frac{2ab}{a+b} \le \sqrt{ab} \le \frac{a+b}{2} \le \sqrt{\frac{a^2+b^2}{2}}$$

I. Trigonometriya.

1. Uchlangan burchak formulalari:

1) $\sin 3x = 3\sin x - 4\sin^3 x$ **2)** $\cos 3x = 4\cos^3 x - 3\cos x$

3) $$tg3\alpha = \frac{3tg\alpha - tg^3\alpha}{1 - 3tg^2\alpha}$$

4) $$ctg3\alpha = \frac{ctg^3\alpha - 3ctg\alpha}{3ctg^2\alpha - 1}$$

2. Yig`indiga oid formulalar.

1) $$\cos\frac{\pi}{2n+1} + \cos\frac{3\pi}{2n+1} + \cdots + \cos\frac{(2n-1)\pi}{2n+1} = \frac{1}{2}$$

2) $$\cos\frac{2\pi}{2n+1} + \cos\frac{4\pi}{2n+1} + \cdots + \cos\frac{2n\pi}{2n+1} = -\frac{1}{2}$$

3) $$\cos\frac{\pi}{2n+1} + \cos\frac{2\pi}{2n+1} + \cos\frac{3\pi}{2n+1} + \cdots + \cos\frac{2n\pi}{2n+1} = 0$$

4) *Agar,* $\alpha + \beta = 45^o$ *bo`lsa, u holda* $(1 + tg\alpha)\cdot(1 + tg\beta) = 2$ *bo`ladi.*

3. Ko`paytmaga oid formulalar.

1) $$\cos\frac{\pi}{2n+1}\cdot\cos\frac{3\pi}{2n+1}\cdot\ldots\cdot\cos\frac{(2n-1)\pi}{2n+1} = \frac{1}{2^n}(\sin\frac{n\pi}{2} - \cos\frac{n\pi}{2})$$

2) $$\cos\frac{2\pi}{2n+1}\cdot\cos\frac{4\pi}{2n+1}\cdot\ldots\cdot\cos\frac{2n\pi}{2n+1} = \frac{1}{2^n}(\cos\frac{n\pi}{2} - \sin\frac{n\pi}{2})$$

3) $$\cos\frac{\pi}{2n+1}\cdot\cos\frac{2\pi}{2n+1}\cdot\cos\frac{3\pi}{2n+1}\cdot\ldots\cdot\cos\frac{2n\pi}{2n+1} = -\frac{1}{4^n}$$

4) $$\cos\frac{\pi}{2^n-1}\cdot\cos\frac{2\pi}{2^n-1}\cdot\cos\frac{4\pi}{2^n-1}\cdot\ldots\cdot\cos\frac{2^{n-1}\pi}{2^n-1} = -\frac{1}{2^n}$$

4. Trigonometrik tenglamalarga oid formulalar.

1. **sinx + cosx = a** *tenlama,* $-\sqrt{2} \le a \le \sqrt{2}$ *da yechimga ega.*

2. **$\sin^4 x + \cos^4 x = a$** *tenglama,* $\frac{1}{2} \le a \le 1$ *da yechimga ega.*

3. **$\sin^6 x + \cos^6 x = a$** *tenglama,* $\frac{1}{4} \le a \le 1$ *da yechimga ega.*

4. **sinx = siny** *tenglamada* **x - y = 2kπ** *yoki* **x + y = π + 2nπ**

5. **cosx = cosy** *tenglamada* **x - y = 2kπ** *yoki* **x + y = 2nπ**

6. **tgx = tgy** *tenglamada* **x - y = kπ** *(Aniqlanish soxasiga kirmaganlar chiqarib tashlanadi)*

7. **ctgx = ctgy** *tenglamada* **x - y = kπ** *(Aniqlanish soxasiga kirmaganlar chiqarib tashlanadi)*

8. **sinx = cosy** *tenglamada* **$x - y = 90^o + 2k\pi$** *yoki* **$x + y = 90^o + 2n\pi$**

9. **tgx = ctgy** *tenglamada* **$x + y = 90^o + k\pi$** *(Aniqlanish soxasiga etibor bering)*

10. **asinx + bcosx = c** *tenglama quyidagi shart bajarilganda yechimga ega:*

$$-\sqrt{a^2+b^2} \le c \le \sqrt{a^2+b^2}$$

5. Teskari trigonometrik funksiyalarga oid formulalar.

1) $\arcsin x = -\arcsin(-x) = \frac{\pi}{2} - \arccos x = \operatorname{arctg}\frac{x}{\sqrt{1-x^2}}$

2) $\arccos x = \pi - \arccos(-x) = \frac{\pi}{2} - \arcsin x = \operatorname{arcctg}\frac{x}{\sqrt{1-x^2}}$

3) $\operatorname{arctg} x = -\operatorname{arctg}(-x) = \frac{\pi}{2} - \operatorname{arcctg} x = \arcsin\frac{x}{\sqrt{1+x^2}}$

4) $\operatorname{arcctg} x = \pi - \operatorname{arcctg}(-x) = \frac{\pi}{2} - \operatorname{arctg} x = \arccos\frac{x}{\sqrt{1+x^2}}$

5) $\operatorname{arctg}1 + \operatorname{arctg}2 + \operatorname{arctg}3 = \pi$

6) $Agar,\ x+y+z=0\ va\ \sin\frac{x}{2}\cdot\sin\frac{y}{2}\cdot\sin\frac{z}{2} = a\ bo`lsa,$

$\sin x + \sin y + \sin z = -4a\ \ bo`ladi.$

6. Trigonometrik funksiyalarning darajasini pasaytirish.

(Bu formulalar boshlang`ich funksiyani topishda juda foydali)

1. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

2. $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

3. $\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$

4. $\cos^3 x = \frac{1}{4}(3\cos x + \cos 3x)$

5. $\sin^4 x = \frac{1}{8}(3 - 4\cos 2x + \cos 4x)$

6. $\cos^4 x = \frac{1}{8}(3 + 4\cos 2x + \cos 4x)$

7. Soddalashtirishda va hisoblashda ishlatiladigan ajoyib fo`rmulalar.

1. $\sin\alpha \cdot \sin(60^o - \alpha) \cdot \sin(60^o + \alpha) = \frac{1}{4}\sin 3\alpha$

2. $\cos\alpha \cdot \cos(60^o - \alpha) \cdot \cos(60^o + \alpha) = \frac{1}{4}\cos 3\alpha$

3. $tg\alpha \cdot tg(60^o - \alpha) \cdot tg(60^o + \alpha) = tg 3\alpha$

4. $ctg\alpha \cdot ctg(60^o - \alpha) \cdot ctg(60^o + \alpha) = ctg 3\alpha$

Masalan:

1. $\sin 10^o \cdot \sin 50^o \cdot \sin 70^o = \frac{1}{4}\sin 30^o = \frac{1}{4}\cdot\frac{1}{2} = \frac{1}{8}$

2. $\cos 20^o \cdot \cos 40^o \cdot \cos 80^o = \frac{1}{4}\cos 60^o = \frac{1}{4}\cdot\frac{1}{2} = \frac{1}{8}$

II. Uchburchaklar.

1. *To`g`ri burchakli uchburchakka ichkii chizilgan aylana urunish nuqtasida gipotenuzani uzunligi* ***m*** *va* ***n*** *ga teng bo`lgan qismlarga ajratsa, uning yuzi* $\boldsymbol{S = mn}$ *fo`rmula bilan topiladi.*

2. *To`g`ri burchakli uchburchakning to`g`ri burchagi uchidan gipotenuzaga tushirilgan mediana va balandlik* ***m*** *va* ***h*** *ga teng bo`lsa, uning yuzi* $\boldsymbol{S = mh}$ *fo`rmula bilan topiladi.*

3. *To`g`ri burchakli uchburchakning to`g`ri burchagi uchidan gipotenuzaga tushirilgan bissektrisa va balandlik* ***l*** *va* ***h*** *ga teng bo`lsa, uning yuzi*

$$S = \frac{h^2 l^2}{2h^2 - l^2}$$

fo`rmula bilan topiladi.

4. *To`g`ri burchakli uchburchakning to`g`ri burchagi uchidan gipotenuzaga tushirilgan bissektrisa va mediana* ***l*** *va* ***m*** *ga teng bo`lsa, uning yuzi*

$$S = \frac{l}{4}(l + \sqrt{l^2 + 8m^2})$$

fo`rmula bilan topiladi.

5. *Yon tomoni* ***a****, uchidagi burchagi* ***α*** *bo`lgan teng yonli uchburchakka tashqi chizilgan aylana radiusini toping.*

$$R = \frac{a}{2\cos\alpha}$$

6. *Yon tomoni* ***a,*** *uchudagi burchagi* ***α*** *ga teng bo`lgan teng yonli uchburchakka ichki chizilgan aylananing radiusi:*

$$r = a \cdot tg\frac{\alpha}{2}(1 - \sin\frac{\alpha}{2})$$

7. *To`g`ri burchakli uchburchakka tashqi chizilgan aylana radiusi* ***R*** *va ichki chizilgan aylana radiusi* ***r*** *bo`lsa, uning yuzi quyidagi formula bilan topiladi:*

$$\boldsymbol{S = 2Rr + r^2}$$

8. *Tomonlari a, b, c ga teng bo`lgan ABC uchburchakning AD, BE, CF balandliklari o`tkazilgan.* ***DEF*** *uchburchakning yuzi quyidagi formula bilan topiladi:*

$$S_{DEF} = 2\cos\alpha \cdot \cos\beta \cdot \cos\gamma \cdot S_{ABC}$$

9. *Tomonlari a, b, c ga teng bo`lgan ABC uchburchakning AD, BE, CF medianalari o`tkazilgan.* ***DEF*** *uchburchakning yuzi quyidagi formula bilan topiladi:*

$$S_{DEF} = \frac{1}{4} S_{ABC}$$

10. *Tomonlari a, b, c ga teng bo`lgan ABC uchburchakning AD, BE, CF bissektrisalari o`tkazilgan.* ***DEF*** *uchburchakning yuzi quyidagi formula bilan topiladi:*

$$S_{DEF} = \frac{2abc}{(a+b)(b+c)(c+a)} \cdot S_{ABC}$$

11. Tomonlari a, b, c ga teng bo`lgan ABC uchburchakka ichki chizilgan aylana uning tomonlariga D, E, F nuqtalarda urunadi. ***DEF*** *uchburchakning yuzi quyidagi formula bilan topiladi:*

$$S_{DEF} = \frac{2S^3}{abcp}$$

12. Uchburchakning ***a*** *asosiga parallel va uning yuzini teng ikkiga bo`luvchi kesmaning uzunligi quyidagi formula bilan topiladi:*

$$X = \frac{a}{\sqrt{2}}$$

13. Tomoni ***a*** *ga teng bo`lgan muntazam uchburchakka ichki chizilgan kvadratning yuzi:*

$$S = 3a^2(7 - 4\sqrt{3})$$

14. Teng yonli uchburchakning asosi ***a*** *ga, uning asosiga tushirilgan balandligi esa asosi va yon tomonlarining o`rtalarini tutashtiruvchi kesmaning uzunligiga teng. Uchburchakning yuzini toping.*

$$S = \frac{a^2\sqrt{3}}{12}$$

15. To`g`ri burchakli uchburchakka aylana ichki chizilgan. Shu aylana urunish nuqtasida uning katetlaridan birini uzunliklari ***m*** *va* ***n*** *bo`lgan kesmalarga ajratadi. Uchburchak yuzini toping.*

$$S = \frac{n+m}{n-m}mn$$

16. To`g`ri burchakli uchburchakning to`g`ri burchagi uchidan tushirilgan balandlik va mediananing nisbati ***p:q*** *kabi. Shu uchburchak kichik katetini katta katetiga nisbatini toping.*

$$\frac{a}{b} = \frac{q - \sqrt{q^2 - p^2}}{p}$$

17. Uchburchakning ***h*** *ga teng balandligi uning asosi uzunligini* ***m : n*** *nisbattda bo`ladi. Shu balandlikka parallel va uchburchakning yuzini teng ikkiga bo`ladigan to`g`ri chiziq kesmasining uzunligini toping.*

$$x = h\sqrt{\frac{m+n}{2n}}$$

18. Katetlari ***a*** *va* ***b*** *bo`lgan to`g`ri burchakli uchburchak to`g`ri burchagining* ***bissektrissasi****:*

$$l_c = \frac{ab\sqrt{2}}{a+b}$$

19. Ixtiyoriy uchburchakning ikki tomoni va ular orasidagi burchagi berilgan bo`lsa, shu burchak ***bissektrisasi*** *quyidagi formula bilan topiladi:*

$$l = \frac{2bc}{b+c}\cos\frac{\alpha}{2}$$

20. Uchburchakning ikki tomoni ***a*** *va* ***b*** *ga teng. Shu tomonlarga tushirilgan medianalar o`zaro perpendikulyar. Uchburchakning* ***yuzini*** *toping.*

$$S = \frac{1}{10}\sqrt{(4a^2 - b^2)(4b^2 - a^2)}$$

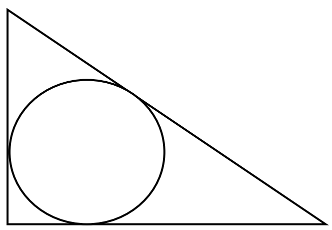
21. Teng yonli uchburchakning asosi ***a*** *ga, uchidan tushirilgan balandlik* ***h*** *ga teng. Yon tomonini diametr qilib aylana chizilgan. Uchburchakning aylana tashqarisidagi qismining yuzini toping.*

$$S = \frac{a^3 h}{2(a^2 + 4h^2)}$$

22. Teng yonli uchburchakning asosi ***a*** *ga, uchidan tushirilgan balandlik* ***h*** *ga teng. Yon tomonini diametr qilib aylana chizilgan. Uchburchakning aylana ichidagi qismining yuzini toping.*

$$S = \frac{2ah^3}{a^2 + 4h^2}$$

23. To`g`ri burchakli uchburchakka aylana ichki chizilgan. Shu aylana urunish nuqtasida uning katetlaridan birini to`g`ri burchak uchidan bo`shlab hisoblaganda uzunliklari* ***m*** *va* ***n*** *bo`lgan kesmalarga ajratadi. U holda



Uchburchakning yuzi:

$$S = \frac{nm(n+m)}{n-m}$$

Uchburchakning perimetri:

$$P = \frac{2n(n+m)}{n-m}$$

24. ABC uchburchakka ichki chizilgan aylana uchburchak tomonlariga A_1, B_1, C_1 nuqtalarda urinadi. U holda:

$$AB_1 = AC_1 = \frac{AB + AC - BC}{2}$$

$$BA_1 = BC_1 = \frac{AB + BC - AC}{2}$$

$$CA_1 = CB_1 = \frac{AC + BC - AB}{2}$$

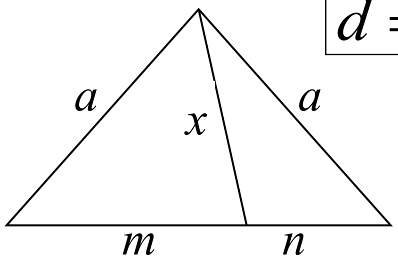
25. Burchaklaridan biri ***60º*** *bo`lgan uchburchakka ichki va tashqi chizilgan aylanalar radiuslari mos ravishda* ***r*** *va* ***R*** *ga teng bo`lsa, shu uchburchakning yuzini toping.*

$$S = r(r+R)\sqrt{3}$$

*26. Ixtiyoriy uchburchakka tashqi chizilgan aylana radiusi **R** va ichki chizilgan aylana radiusi **r** berilgan bo`lsa, u holda shu aylanalar markazlari orasidagi masofa quyidagi formula bilan topiladi:*

$$d=\sqrt{R^2-2Rr}$$

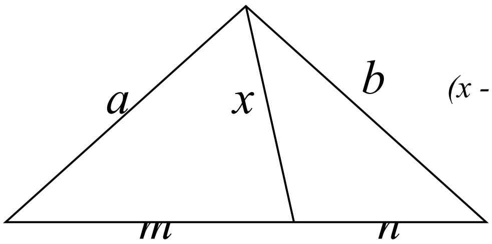
27.



$$x=\sqrt{a^2-mn}$$

(x - ixtiyoriy kesuvchi)

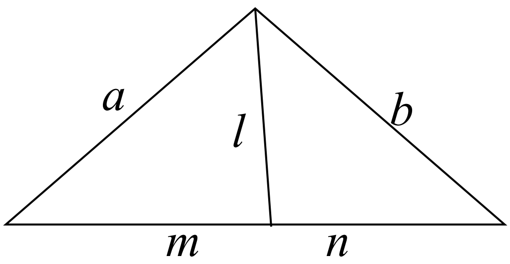
28.



(x - ixtiyoriy kesuvchi)

$$x=\sqrt{\frac{a^2n+b^2m}{m+n}-mn}$$

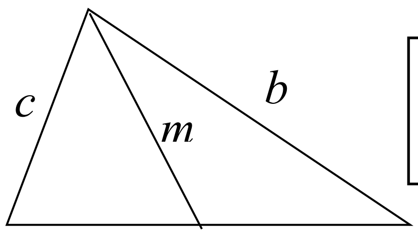
29.



$$l=\sqrt{ab-mn}$$

(l - bissektrisa)

30.



$$S=\frac{1}{4}\sqrt{((b+c)^2-4m^2)(4m^2-(b-c)^2)}$$

(m – mediana)

*Agar, **b+c=p** ba **b-c=q** desak,* $S=\frac{1}{4}\sqrt{(p^2-4m^2)(4m^2-q^2)}$

31 . Agar teng yonli uchburchakning asosi ***a*** ga, yuzi ***S*** ga teng bo`lsa, perimetri quyidagi

formula bilan topiladi: $P=a+\frac{\sqrt{a^4+16S^2}}{a}$

32. Agar to`g`ri burchakli uchburchakning perimetri ***P*** ga, yuzi ***S*** ga teng bo`lsa, u xolda uning

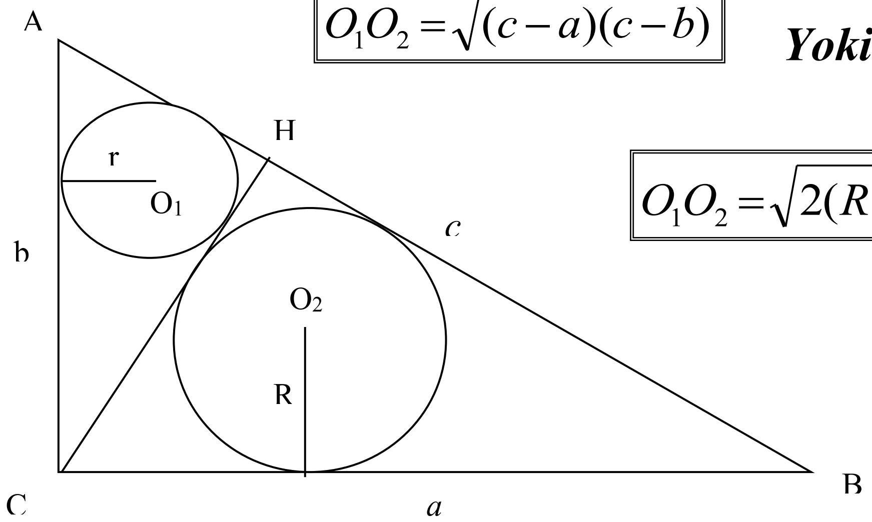
katetlari quyidagi formulalar bilan topiladi: $a=\frac{m+n}{2}$ $b=\frac{m-n}{2}$

Bunda: $m=\frac{P^2+4S}{2P}$ va $n=\sqrt{m^2-8S}$

33. ABC to`g`ri burchakli uchburchakning C to`g`ri burchagi uchidan gipotenuzaga CD balandlik tushirilgan. ADC va BDC uchburchaklarga ichki chizilgan aylanalar markazlari orasidagi masofa quyidagi formula bilan topiladi:

$$O_1O_2=\sqrt{(c-a)(c-b)}$$

Yoki



$$O_1O_2=\sqrt{2(R^2+r^2)}$$

$$S=\frac{R^2+r^2}{2Rr}\left(R+r+\sqrt{R^2+r^2}\right)^2$$

$$CH=R+r+\sqrt{R^2+r^2}$$

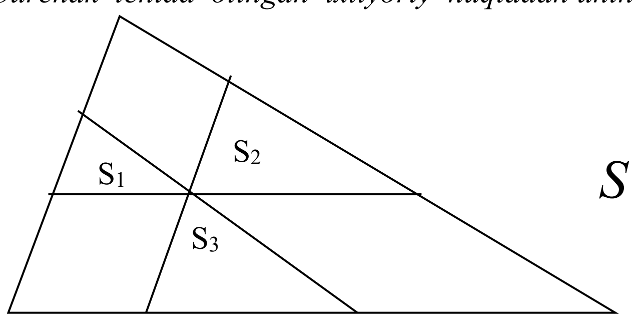
34. To`g`ri burchakli uchburchak medianalari kesishgan nuqtadan gipotenuzagacha bo`lgan masofa:

$$X=\frac{ab}{3c}$$

35. Ihtiyoriy ABC uchburchakda BB_1 va CC_1 bissektrisalar o`tkazilgan. Agar $\angle B_1OC=\alpha$ bo`lsa, u holda $\angle A=180^o-2\alpha$

36. Ihtiyoriy ABC uchburchakka ichki chizilgan aylana markazi O. Agar, OA=m va OB=n bo`lsa, u holda $AB=\sqrt{m^2+n^2+mn\sqrt{2}}$

37. Uchburchak ichida olingan ixtiyoriy nuqtadan uning tomonlariga parallel chiziqlar o`tkazilgan.



$$S=\left(\sqrt{S_1}+\sqrt{S_2}+\sqrt{S_3}\right)^2$$

38. Uchburchakning h_a, h_b va h_c balandliklari yordamida turini aniqlash:

$$\frac{1}{h_a^2}+\frac{1}{h_b^2}>\frac{1}{h_c^2}\quad \textit{bo`lsa, o`tkir burchakli.}$$

$$\frac{1}{h_a^2}+\frac{1}{h_b^2}<\frac{1}{h_c^2}\quad \textit{bo`lsa, o`tmas burchakli.}$$

$$\frac{1}{h_a^2}+\frac{1}{h_b^2}=\frac{1}{h_c^2}\quad \textit{bo`lsa, to`g`ri burchakli.}$$

39. Teng yonli uchdurchakning yon tomoni ***b*** *ga, asosi* ***a*** *ga teng. Shu uchburchak medianalari kesishgan nuqta* ***M****, bissektrisalari kesishgan nuqtq* ***K*** *bo`lsa,* ***MK*** *masofani toping.*

Agar, ***a>b*** *bo`lsa,*

$$MK = \frac{a-b}{3}\sqrt{\frac{2b-a}{2b+a}}$$

Agar, ***a<b*** *bo`lsa,*

$$MK = \frac{a+b}{3}\sqrt{\frac{2b-a}{2b+a}}$$

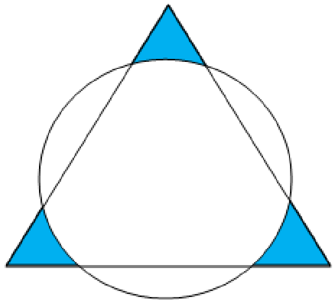
40*. Tomoni* ***a*** *ga teng bo`lgan muntazam uchburchakning markazida radiusi* $\frac{a}{3}$ *ga teng bo`lgan doira chizilgan. Uchburchakning:*

doira ichkarisidagi qismining yuzi:

$$S = \frac{a^2}{36}(3\sqrt{3} + 2\pi)$$

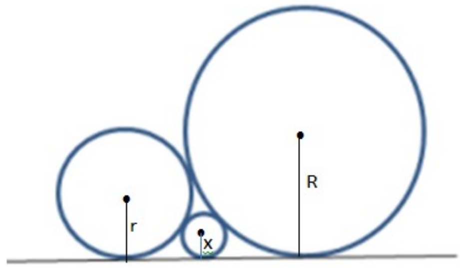
doira tashqarisidagi qismlari yuzi:

$$S = \frac{a^2}{18}(3\sqrt{3} - \pi)$$

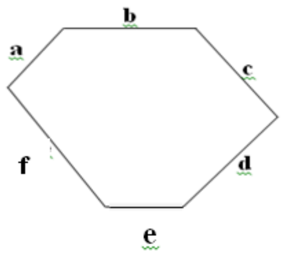


41. *Usbu shaklda ikkita katta aylanalarning radiuslari* ***r*** *va* ***R*** *, eng kichik aylananing radiusi* ***x*** *bo`lsin, u xolda quyidagi formula o`rinli:*

$$x = \frac{Rr}{\left(\sqrt{R} + \sqrt{r}\right)^2}$$



42. *Qavariq oltiburchak tomonlarining uzunliklari* ***a, b, c, d, e, f*** *ga teng bo`lib, ichki burchaklari o`zaro teng bo`lsa, u xolda quyidagi tengliklar o`rinli bo`ladi:*



a+b=d+e

b+c=e+f

c+d=a+f

(buni isbot qilish uchun, tomonlarni davom ettiramiz, muntazam uchburchak xosil bo`ladi…)

43. *Aylanaga tashqi chizilgan xar qanday oltiburchak uchun:* ***a+c+e = b+d+f*** *tenglik o`rinli.*

44. ***n*** *ta to`g`ri chiziq tekislikni eng ko`pi bilan* $\frac{n^2 + n + 2}{2}$ *ta soxaga ajratadi.*

45. ***n*** *ta aylana tekislikni eng ko`pi bilan* $n^2 - n + 2$ *ta soxaga ajratadi.*

46. *Uchta* ***A(x₁;y₁) , B(x₂;y₂) va C(x₃;y₃)*** *nuqtaning bir to`g`ri chiziqda yotish sharti:*

$$\frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1}$$

III. Trapetsiyalar.

1. *Teng yonli trapetsiyaning kichik asosi* ***b*** *ga, perimetri* ***P*** *ga teng. Agar, diagonali o`tmas burchagini teng ikkiga bo`lsa, trapetsiyaning o`rta chizig`i quyidagi formula bilan topiladi.*

$$X = \frac{P + 2b}{6}$$

2. *Teng yonli trapetsiyaning katta asosi* ***a*** *ga, perimetri* ***P*** *ga teng. Agar, diagonali o`tkir burchagini teng ikkiga bo`lsa, trapetsiyaning o`rta chizig`i quyidagi formula bilan topiladi.*

$$X = \frac{P + 2a}{6}$$

3. *Asoslari* ***a*** *va* ***b*** *bo`lgan trapetsiyaning asoslariga parallel va uning yuzini teng ikkiga bo`luvchi kesmaning uzunligi quyidagi formula bilan topiladi:*

$$X = \sqrt{\frac{a^2 + b^2}{2}}$$

4. *Asoslari* ***a*** *va* ***b*** *bo`lgan to`g`ri burchakli trapetsiyaga aylana ichki chizilgan. U holda quyidagi formulalar orinli:*

$$r = \frac{ab}{a + b} \qquad h = \frac{2ab}{a + b} \qquad S = ab$$

5. *Asoslari* ***a*** *va* ***b*** *bo`lgan teng yonli trapetsiyaga aylana ichki chizilgan. U holda quyidagi formulalar orinli:*

$$r = \frac{\sqrt{ab}}{2} \qquad h = \sqrt{ab} \qquad S = \frac{a + b}{2}\sqrt{ab}$$

6. *Diagonali* ***d*** *va o`tkir burchagi* ***φ*** *bo`lgan teng yonli trapetsiyaga tashqi chizilgan aylana radiusi quyidagi formula bilan topiladi:*

$$R = \frac{d}{2\sin\varphi}$$

7. *Ixtiyoriy to`rtburchakning yuzi uning diagonallari va ular orasidagi burchak sinusi ko`paytmasining yarmiga teng.*

$$S = \frac{1}{2} d_1 \cdot d_2 \sin\varphi$$

8. *Trapetsiyaning diagonallari* d_1 *va* d_2, *balandligi* ***h*** *ga teng bo`lsa, uning yuzi:*

$$S = \frac{h}{2}\left(\sqrt{d_1^{\,2} - h^2} + \sqrt{d_1^{\,2} - h^2}\right)$$

*9. Teng yonli trapetsiyaning diagonali **d** ga teng va u asosi bilan **α** burchak tashkil etadi. U xolda:*

O`rta chizig`i: $EF = d\cdot\cos\alpha$

Balandligi $h = d\cdot\sin\alpha$

Yuzi: $S = d^2 \sin\alpha \cos\alpha$

*10. Trapetsiyaning o`rta chizig`i **d** ga, katta asosidagi burchaklari 30° va 60° ga teng. Asoslari o`rtalarini tutashtiruvchi kesmaning uzunligi **c** ga teng bo`lsa, u xolda:*

Katta asosi: $a = d + c$ *va kichik asosi:* $b = d - c$ *bo`ladi.*

*11. Trapetsiyaning yon tomonlari **a** va **b** ga teng. Shu trapetsiyaga aylana ichki chizilgan. Trapetsiyaning o`rta chizig`i uni yuzlarining nisbati **m:n** bo`lgan ikki qismga ajratadi.*

Agar, $\frac{m}{n} = \frac{a+3b}{3a+b}$ *tenglik bajarilsa, u xolda katta asosi **a** ga, kichik asosi **b** ga teng.*

Aks xolda, katta asosi: $\frac{a+b}{n+m}\cdot\frac{3n-m}{2}$ *ga teng. Kichik asosi:* $\frac{a+b}{n+m}\cdot\frac{3m-n}{2}$ *ga teng.*

12. Ham ichki aylana ham tashqi aylana chizish mumkin bo`lgan to`rtburchakning yuzi:

$$S = \sqrt{abcd}$$

*13. Rombning diagonallari kesishgan nuqtadan tomoniga tushirilgan perpendikulyar tomonini uzunliklari **m** va **n** ga teng bo`lgan qismlarga ajratadi. Rombning yuzini toping.*

$$S = 2(m+n)\sqrt{mn}$$

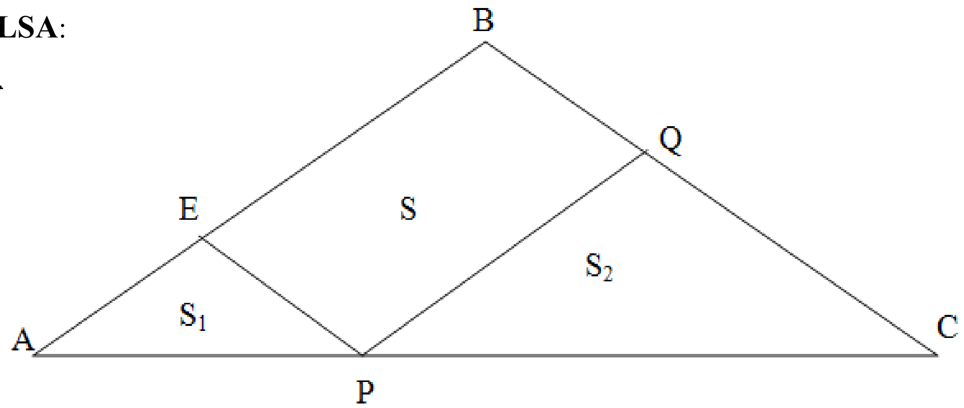
*14. Parallelogrammning bir tomoni **a** ga teng, diagonallari kesishgan nuqtadan ikkinchi tomoniga tushirilgan perpendikulyar bu tomonni uzunliklari **m** va **n** ga teng bo`lgan qismlarga ajratadi. Parallelogrammning yuzini toping.*

$$S = (m+n)\sqrt{a^2-(m-n)^2}$$

15. Uchburchakning bir tomoniga tegishli ixtiyoriy P nuqtadan qolgan tomonlariga harallel PQ va PE chiziqlar o`tkazildi. U holda quyidagi formula o`rinli bo`ladi:

PQ//AB va PE//BC BO`LSA:

$$S = 2\sqrt{S_1 \cdot S_2}$$



IV. Piramidalar.

1. *Uchburchakli muntazam piramidaga tashqi chizilgan sharning markazi uning balandligini* ***a*** *va* ***b*** *ga teng bo`lgan qismlarga ajratadi. Piramidaning hajmini toping.*

$$V=\frac{(a+b)^2(a-b)\sqrt{3}}{4}$$

2. *Piramidaning barcha yon* ***yoqlari*** *asos tekisligi bilan* **α** *burchak tashkil qiladi. Uning asosi tomonlari* ***a, b va c*** *ga teng uchburchakdan iborat.*

Piramidaning balandligi: $$H=\sqrt{\frac{(p-a)(p-b)(p-c)}{p}}\cdot tg\alpha$$

Piramidaning hajmi : $$V=\frac{1}{3}(p-a)(p-b)(p-c)\cdot tg\alpha$$

Agar, piramida muntazam bo`lib asosining tomoni* ***a*** *bo`lsa, $H=\frac{a\sqrt{3}}{6}tg\alpha$ $V=\frac{a^3}{24}tg\alpha$

3. Piramidaning barcha yon **qirralari** asos tekisligi bilan **α** burchak tashkil qiladi. Uning asosi tomonlari **a, b va c** ga teng uchburchakdan iborat.

Piramidaning balandligi: $$H=\frac{abc\cdot tg\alpha}{4\sqrt{p(p-a)(p-b)(p-c)}}$$

Piramidaning hajmi : $$V=\frac{abc}{12}\cdot tg\alpha$$

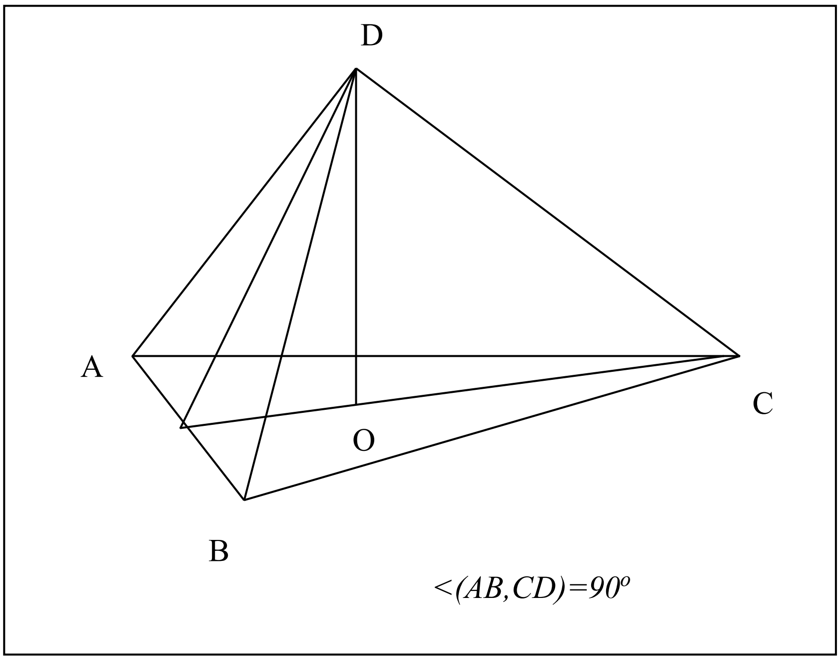
Agar, piramida muntazam bo`lib, asosining tomoni* ***a*** *bo`lsa, $H=\frac{a\sqrt{3}}{3}tg\alpha$ $V=\frac{a^3}{12}tg\alpha$

4. *Muntazam to`rtburchakli piramidaning yon sirti* ***S*** *ga, asosidagi ikki yoqli burchaklari* **α** ga teng bo`lsa, uning balandligi va hajmi quyi**dagi** formulalar bilan topiladi:

1. $$H=\frac{1}{2}\sqrt{S\cdot\cos\alpha}$$

2. $$V=\frac{1}{6}(S\cdot\cos\alpha)^{\frac{3}{2}}$$

5. *Qirrasi a ga teng bo`lgan muntazam tetraedrga oid formulalar.*



1. $$H = \frac{a\sqrt{6}}{3}$$

2. $$S_{ts} = a^2\sqrt{3}$$

3. $$V = \frac{a^3\sqrt{2}}{12}$$

4. $$R = \frac{a\sqrt{6}}{4}$$

5. $$r = \frac{a\sqrt{6}}{12}$$

(x – qo`shni yoqlari orasidagi burchak)
(y – yon qirra bilan asos tekisligi orasidagi burchak)

6. $$x = \arccos\frac{1}{3}$$ *7.* $$y = \arccos\frac{\sqrt{3}}{3}$$

6. *Sharga tashqi chizilgan harqanday piramida uchun quyidagi formula o`rinli:*

$$V = \frac{1}{3}Sr$$

(r - piramidaga ichki chizilgan sharning radiusi, S – to`liq sirt, V – hajm)

7. Sharga balandligi asosining diametriga teng bo`lgan konus ichki chizilgan va konnus asosining radiusi r, sharning radiusi R bo`lsin, u holda quyidagi formulalar o`rinli:

$$R = \frac{5r}{4}$$ *Shar sirtining yuzi:* $$S = \frac{25\pi}{16}r^2$$ *Sharning hajmi:* $$V = \frac{125\pi}{48}r^3$$

8. Muntazam piramidalar uchun quyidagi formula o`rinli:
(bunda, α – piramidaning yon yog`i bilan asos tekisligi orasidagi burchak)

$$\frac{S_{asos}}{S_{yon}} = \cos\alpha$$

9. Muntazam to`rtburchakli piramida asosining markazidan uning yon yog`igacha bo`lgan masofa ***a*** *ga teng. Uning yon yoqlari asos tekisligi bilan* ***α*** *burchak hosil qiladi. Piramidaning hajmini toping.*

$$V = \frac{8a^3}{3\sin\alpha\cdot\sin2\alpha}$$

Agar ***α = 60°*** *bo`lsa,* $$V = \frac{32a^3}{9}$$

Agar ***α = 30°*** *bo`lsa,* $$V = \frac{32a^3}{9}\sqrt{3}$$

Agar ***α = 45°*** *bo`lsa,* $$V = \frac{8a^3}{3}\sqrt{2}$$

10. Muntazam tetraedrning qirrasi ***a*** *ga teng. Asosining markazi orqali yon yog`iga parallel qilib o`tkazilgan kesimning yuzi quyidagi fo`rmula bilan topiladi:*

$$S = \frac{a^2}{9}\sqrt{3}$$

11. Muntazam piramidaning balandligi ***H*** ga, asosidagi barcha ikkiyoqli burchaklari ***α*** ga teng. Shu piramida asosiga ichki chizilgan doiraning radiusini toping:

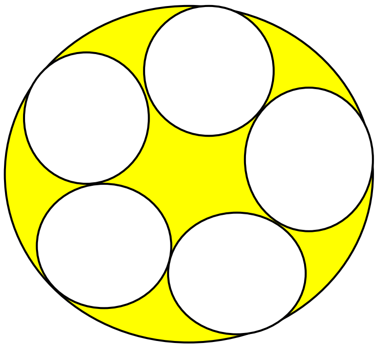
$$r = H \cdot ctg\alpha$$

12. Muntazam to`rtburchakli piramidaning balandligi ***H*** ga, asosining tomoni ***a*** ga teng. Piramida yon yog`iga parallel bo`lib, asosining markazi orqali o`tgan kesim yuzini toping.

$$S = \frac{3a}{16}\sqrt{4H^2 + a^2}$$

13. Radiuslari r ga teng bo`lgan n ta kichik aylana bir – biriga tashqi urinadi va ularning harbiri radiusi R bo`lgan katta aylanaga ichki urinadi. U xolda:

$$\frac{r}{R-r} = \sin\frac{\pi}{n}$$



Yig`indiga oid formulalar.

1. $$1+2+3+...+n=\frac{n(n+1)}{2}$$

3. $$2+4+6+...+2n=n(n+1)$$

2. $$1+3+5+...+(2n-1)=n^2$$

4. $$1^2+2^2+3^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$$

5. $$1^3+2^3+3^3+...+n^3=(1+2+3+...+n)^2=\left(\frac{n(n+1)}{2}\right)^2=\frac{n^2(n+1)^2}{4}$$

6. $$1\cdot2+2\cdot3+3\cdot4+...+n\cdot(n+1)=\frac{n(n+1)(n+2)}{3}$$

7. $$1\cdot2\cdot3+2\cdot3\cdot4+3\cdot4\cdot5+...+n\cdot(n+1)\cdot(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$$

8. $$\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+...+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

9. $$\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+...+\frac{1}{n(n+1)\cdot(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

10. $$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+...+\frac{1}{\sqrt{n}+\sqrt{n+1}}=\sqrt{n+1}-1$$

11. Agar, a_1, a_2, a_3 **....** *ayirmasi* **d** *ga teng arifmetik progressiya bo`lsa, u holda:*

$$\frac{1}{a_1\cdot a_2}+\frac{1}{a_2\cdot a_3}+...+\frac{1}{a_n\cdot a_{n+1}}=\frac{a_{n+1}-a_1}{d\cdot a_1\cdot a_{n+1}}$$

12. *Agar, maxrajdagi sonlar ketma-ket natural sonlarning kvadratlari bo`lsa,*

$$\left(1-\frac{1}{m^2}\right)\cdot.....\cdot\left(1-\frac{1}{n^2}\right)=\frac{(m-1)\cdot(n+1)}{m\cdot n}$$

13. Agar, a_1, a_2, a_3 **....** *lar ketma-ket natural sonlar bo`lsa, u holda:*

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\frac{1}{\sqrt{a_3}+\sqrt{a_4}}+...+\frac{1}{\sqrt{a_n}+\sqrt{a_{n+1}}}=\sqrt{a_{n+1}}-\sqrt{a_1}$$

14. Agar, $a_1, a_2, a_3 ....$ *ayirmasi* d *ga teng arifmetik progressiya bo`lsa, u holda:*

To`ychiyev formulasi

$$a_1^2 + a_2^2 + ... + a_n^2 = \frac{n(n-1)(2n-1)d^2}{6} + na_1a_n$$

15. Toq sonlar kvadratlarining yig`indisi:

$$1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{n(4n^2-1)}{3}$$

16. Juft sonalr kvadratlarining yig`indisi:

$$2^2 + 4^2 + 6^2 + ... + (2n)^2 = \frac{2n(n+1)(2n+1)}{3}$$

17. Toq sonlar kublarining yig`indisi:

$$1^3 + 3^3 + 5^3 + ... + (2n-1)^3 = n^2(2n^2-1)$$

18. Juft sonalr kublarining yig`indisi:

$$2^3 + 4^3 + 6^3 + ... + (2n)^3 = 2n^2(n+1)^2$$

19.

$$1\cdot 3 + 2\cdot 5 + 3\cdot 7 + ... + n(2n+1) = \frac{n(n+1)(4n+5)}{6}$$

20.

$$1\cdot 2 + 2\cdot 5 + 3\cdot 8 + ... + n(3n-1) = n^2(n+1)$$

21. *Agar,* a_1, a_2, ... *lar arifmetik progressiya hadlari bo`lsa, quyidagi formula o`rinli:*

$$1\cdot a_1 + 2\cdot a_2 + 3\cdot a_3 + ... + n\cdot a_n = \frac{n(n+1)(a_1+2a_n)}{6}$$

KVADRAT TENGLAMA ILDIZLARINING BIROR SONGA NISBATAN JOYLASHISHI

№	***f(x) = ax² +bx + c (a≠0), ildizlari x₁ va x₂ (x₁ < x₂)*** ***D=b² – 4ac,*** $x_0 = -\frac{b}{2a}$, ***m va n berilgan sonlar (m<n)***	**Zaruriy va yetarli shartlar**
1	Ikkala ildizlari ham 0 dan katta: $0<x_1<x_2$	$D>0$ $ac>0$ $x_0>0$
2	Ikkala ildizlari ham 0 dan kichik: $x_1<x_2<0$	$D>0$ $ac>0$ $x_0<0$
3	Bitta ildizi 0 dan kichik, ikkinchi ildizi 0 dan katta: $x_1<0<x_2$	$ac<0$
4	Bitta ildizi m dan kichik, ikkinchi ildizi m dan katta: $x_1<m<x_2$	$af(m)<0$
5	Ikkala ildizlari ham m dan katta: $m<x_1<x_2$	$D>0$ $af(m)>0$ $x_0>0$
6	Ikkala ildizlari ham n dan kichik: $x_1<x_2<n$	$D>0$ $af(n)>0$ $x_0<0$
7	Ikkala ildizlari (m;n) intervalda yotadi: $m<x_1<x_2<n$	$D>0$ $af(m)>0$ $af(n)>0$ $m<x_0<n$
8	Faqat kichik ildizi (m;n) intervalda yotadi: $m<x_1<n<x_2$	$af(m)>0$ $af(n)<0$
9	Faqat katta ildiz (m;n) intervalda yotadi: $x_1<m<x_2<n$	$af(m)<0$ $af(n)>0$
10	Bitta ildizi m dan kichik, ikkinchi ildizi n dan katta: $x_1<m<n<x_2$	$af(m)<0$ $af(n)<0$

Funksional tenglamalar.

1. Agar, y = f(x) funksiya monoton o`suvchi bo`lsa, u holda

$$\boxed{\mathbf{f(x) = x \quad va \quad \underbrace{f(f(\ldots f(x))\ldots)}_{\text{(n marta)}} = x}}$$

tenglamalar bir xil yechimlarga ega.

Misol. $\sqrt{6+\sqrt{6+\sqrt{6+x}}} = x$ tenglamani yechish uchun, $\sqrt{6+x} = x$ tenglamani yechish kifoya.

Yuqoridagi qoida yordamida quyidagi tenglamalarni yechish mumkin:

1) $x^2 + 1 = 2\sqrt{2x-1}$

2) $x^2 + 2 = 3\sqrt{3x-2}$

3) $x^2 + 3 = 4\sqrt{4x-3}$

4) $x^2 + 4 = 5\sqrt{5x-4}$

1) $x^3 + 1 = 2\sqrt[3]{2x-1}$

2) $x^3 + 2 = 5\sqrt[3]{5x-2}$

3) $x^3 + 3 = 10\sqrt[3]{10x-3}$

4) $x^3 + 4 = 17\sqrt[3]{17x-4}$

2. Agar, y = f(x) juft funksiya bo`lsa, u holda **f(x) = 0** tenglama x va -x ga nisbatan invariant deyiladi, chunki, f(-x) = f(x)

Masalan, *f(x) + f(c-x)* ifoda *x* va *c-x* ga nisbatan invariant.

Shuningdek, $f(x) + f(\frac{c}{x})$ ifoda x va $\frac{c}{x}$ ga nisbatan invariant.

Bunday holda, x ni c-x bilan yoki x ni $\frac{c}{x}$ bilan almashtirish mumkin.

Misol. 1) Agar, $4f(2-x) - f(x) = 6x^2 - 47x + 56$ *bo`lsa, f(x) ni toping.*

2) Agar, $3f(\frac{3}{x}) - f(x) = 8x$ *bolsa, f(x) ni toping.*

3. $$\boxed{\sqrt[6]{x^2-1} = \sqrt[3]{x-1} + \sqrt[3]{x+1}}$$ Tenglama haqiqiy ildizlarga ega emas!

Vektor.

I. $$y=\sqrt{(x-a)^2+b^2}+\sqrt{(x-c)^2+d^2}$$

funksiyaning **eng kichik** qiymati quyidagi formula bilan topiladi:

$$y_{\min}=\sqrt{(a-c)^2+(b+d)^2}$$

1- 5 funksiyalarning **eng kichik** qiymatini toping.

1. $y=\sqrt{x^2-6x+10}+\sqrt{x^2-4x+8}$

2. $y=\sqrt{x^2-10x+34}+\sqrt{x^2-4x+5}$

3. $y=\sqrt{2x^2-14x+37}+\sqrt{2x^2-12x+20}$

4. $y=\sqrt{4x^2+20x+29}+\sqrt{4x^2-28x+58}$

5. $y=\sqrt{9x^2-6x+17}+\sqrt{9x^2-54x+85}$

II. $$\sqrt{(x-a)^2+(x-b)^2}+\sqrt{(x-c)^2+(x-d)^2}=k$$

Tenglama aqalli bitta ildizga ega bo`ladigan k ning eng kichik qiymati quyidagi formula bilan topiladi:

$$k=\sqrt{(a-c)^2+(b-d)^2}$$

Tenglama aqalli bitta ildizga ega bo`ladigan ***a*** ning eng kichik qiymatini toping:

1. $$\sqrt{(x-7)^2+(x-5)^2}+\sqrt{(x-3)^2+(x-2)^2}=a$$

2. $$\sqrt{(x-5)^2+(x-7)^2}+\sqrt{(x+3)^2+(x-1)^2}=a$$

3. $$\sqrt{(x-8)^2+(x-7)^2}+\sqrt{(x+4)^2+(x+2)^2}=a$$

4. $$\sqrt{(5x+1)^2+(5x+2)^2}+\sqrt{(5x+7)^2+(5x-6)^2}=a$$

5. $$\sqrt{(11x+3)^2+(11x+7)^2}+\sqrt{(11x+15)^2+(11x+2)^2}=a$$

III. $$y = x\left(a\sqrt{b^2 - c^2x^2} + c\sqrt{d^2 - a^2x^2}\right)$$

funksiyaning **eng katta** qiymati quyidagi formula bilan topiladi:

$$\boxed{y_{\max} = b \cdot d}$$

1 - 3 funksiyalarning **eng katta** qiymatini toping.

1. $y = x(\sqrt{1 - 9x^2} + 3\sqrt{4 - x^2})$
2. $y = x(6\sqrt{64 - 49x^2} + 7\sqrt{25 - 36x^2})$
3. $y = x(4\sqrt{81 - 25x^2} + 5\sqrt{49 - 16x^2})$

IV. Quyidagi tenglamalarni yechishda **trigonometriyadan** foydalanish mumkin:

1. $\sqrt{1 - x^2} = 4x^3 - 3x$

2. $\sqrt{8x^3 - 6x} = 1$

3. $x + \dfrac{x}{\sqrt{x^2 - 1}} = \dfrac{35}{12}$

Kardano formulasi.

$$x^3 + ax^2 + bx + c = 0 \qquad (1)$$

Tenglamani yechish uchun belgilash kiritamiz:

$$x = y - \frac{a}{3} \qquad (2)$$

U xolda tenglama quyidagi ko\`rinishga keladi:

$$y^3 + py + q = 0 \qquad (3)$$

Bu tenglamada: $y = u + v$ ***(4)*** ***deb belgilash kiritamiz.***

U xolda (3) tenglama

$$u^3 + v^3 + (u+v)(3uv+p) + q = 0$$ ***(5) ko\`rinishga keladi.***

Bunda $3uv + p = 0$ ***deb tanlab olamiz***

Natijada:

$$\begin{cases} uv = -\dfrac{p}{3} \\ u^3 + v^3 = -q \end{cases}$$

(6) sistemaga ega bo\`lamiz

Bu sistemani yechib, $u = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$ **va** $v = \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$

larni topamiz. Bularni (4) ga qo\`ysak,

$$y = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \qquad (7)$$

(7) ni (2) ga qoyib (1) ning ildizlarini topamiz.

Bu yerda (7) ni Kardano formulasi deyiladi.

HOSILANI HISOBLASH QOIDALARI

$$c' = 0$$
$$x' = 1$$
$$(cx)' = c$$
$$(x^2)' = 2x$$
$$(x^3)' = 3x^2$$
$$(x^n)' = nx^{n-1}$$
$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$
$$\left(\frac{1}{x^2}\right)' = -\frac{2}{x^3}$$
$$\left(\frac{1}{x^n}\right)' = -\frac{n}{x^{n+1}}$$
$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$
$$\left(\sqrt[3]{x}\right)' = \frac{1}{3\sqrt[3]{x^2}}$$
$$\left(\sqrt[n]{x}\right)' = \frac{1}{n\sqrt[n]{x^{n-1}}}$$

$$\left(\sqrt[3]{x^2}\right)' = \frac{2}{3\sqrt[3]{x}}$$
$$\left(\sqrt[5]{x^3}\right)' = \frac{3}{5\sqrt[5]{x^2}}$$
$$\left(\sqrt[n]{x^m}\right)' = \frac{m}{n\sqrt[n]{x^{n-m}}}$$
(bunda $n>m$)

$(kx+b)' = k$

$(ax^2 + bx + c)' = 2ax + b$

$(a^x)' = a^x \ln a$

$(e^x)' = e^x$

$$(\log_a x)' = \frac{1}{x\ln a}$$
$$(\lg x)' = \frac{1}{x\ln 10}$$
$$(\ln x)' = \frac{1}{x}$$
$$(\sin x)' = \cos x$$
$$(\cos x)' = -\sin x$$
$$(tgx)' = \frac{1}{\cos^2 x}$$
$$(ctgx)' = -\frac{1}{\sin^2 x}$$
$$(\sin u)' = u'\cos u$$
$$(\cos u)' = -u'\sin u$$
$$(tgu)' = \frac{u'}{\cos^2 u}$$
$$(ctgu)' = -\frac{u'}{\sin^2 u}$$

$$(\sec u)' = \frac{\sin u}{\cos^2 u}\cdot u'$$
$$(\mathrm{cosec}\, u)' = -\frac{\cos u}{\sin^2 u}\cdot u'$$
$$(\arcsin u)' = \frac{1}{\sqrt{1-u^2}}\cdot u'$$
$$(\arccos u)' = -\frac{1}{\sqrt{1-u^2}}\cdot u'$$
$$(arctgu)' = \frac{1}{1+u^2}\cdot u'$$
$$(arcctgu)' = -\frac{1}{1+u^2}\cdot u'$$
$$(arc\sec u)' = \frac{1}{u\sqrt{u^2-1}}\cdot u'$$
$$(\mathrm{arccosec}\, u)' = -\frac{1}{u\sqrt{u^2-1}}\cdot u'$$
$$(shu)' = \left(\frac{e^u - e^{-u}}{2}\right)' = chu\cdot u'$$
$$(chu)' = \left(\frac{e^u + e^{-u}}{2}\right)' = shu\cdot u'$$
$$(thu)' = \frac{1}{chu^2 u}\cdot u'$$
$$(cthu)' = -\frac{1}{shu^2 u}\cdot u'$$

$$(x^x)' = x^x(1+\ln x)$$
$$(u^v)' = u^v\left(v'\ln u + \frac{u'v}{u}\right)$$

BOSHLANG`ICH FUNKSIYALARNI TOPISH QOIDALARI.

№	Berilgan funrsiya f(x)	Boshlang`ich funksiyasi F(x)	№	Berilgan funrsiya f(x)	Boshlang`ich funksiyasi F(x)
1	f(x)	F(x) + C	21	$\frac{1}{\cos^2 x}$	tgx + C
2	g(x)	G(x) + C	22	$\frac{1}{\sin^2 x}$	-ctgx + C
3	kf(x)	kF(x) + C	23	e^x	$e^x + C$
4	f(x) + g(x)	F(x) + G(x) + C	24	a^x	$\frac{a^x}{\ln a} + C$
5	f(kx+b)	$\frac{1}{k} F(kx+b) + C$	25	$\frac{1}{\sqrt{x}}$	$2\sqrt{x} + C$
6	k (son)	kx + C	26	$\frac{1}{\sqrt[3]{x}}$	$\frac{3\sqrt[3]{x^2}}{2} + C$
7	x	$\frac{x^2}{2} + C$	27	$\frac{1}{\sqrt[n]{x}}$	$\frac{n\sqrt[n]{x^{n-1}}}{n-1} + C$
8	x^2	$\frac{x^3}{3} + C$	28	$\frac{1}{1+x}$	$\ln\|1+x\| + C$
9	x^n	$\frac{x^{n+1}}{n+1} + C$	29	$\frac{1}{x^2+1}$	$arctgx + C$
10	$\frac{1}{x}$ (x>0)	lnx + C	30	$\frac{1}{x^2+n}$	$arctg\frac{x}{\sqrt{n}} + C$
11	$\frac{1}{x^2}$	$-\frac{1}{x} + C$	31	$\frac{1}{\sin x}$	$\ln\left\|tg\frac{x}{2}\right\| + C$
12	$\frac{1}{x^3}$	$-\frac{1}{2x^2} + C$	32	$\frac{1}{\cos x}$	$\ln\left\|tg\frac{2x+\pi}{4}\right\| + C$
13	$\frac{1}{x^n}$	$-\frac{1}{(n-1)x^{n-1}} + C$	33	$\sin^2 x$	$\frac{1}{4}(2x - \sin 2x) + C$
14	$\sqrt{x}$	$\frac{2x\sqrt{x}}{3} + C$	34	$\cos^2 x$	$\frac{1}{4}(2x + \sin 2x) + C$
15	$\sqrt[3]{x}$	$\frac{3x\sqrt[3]{x}}{4} + C$	35	lnx	$xlnx - x + C$
16	$\sqrt[n]{x}$	$\frac{nx\sqrt[n]{x}}{n+1} + C$	36	$\log_a x$	$\frac{x\ln x - x}{\ln a} + C$
17	sinx	-cosx + C	37	$\ln^2 x$	$x(\ln x - 1)^2 + C$
18	cosx	sinx + C	38	$\log_a^2 x$	$\frac{x(\ln x - 1)^2}{\ln^2 a} + C$
19	tgx	-ln(cosx) + C	39	$\sin^4 x$	$\frac{1}{32}(12x - 8\sin 2x + \sin 4x) + C$
20	ctgx	ln(sinx) + C	40	$\cos^4 x$	$\frac{1}{32}(12x + 8\sin 2x + \sin 4x) + C$

INTEGRAL HOSSALARI

1. $(\int f(x)dx)' = f(x)$ **2**. $d(\int f(x)dx) = f(x)dx$ **3**. $\int dF(x) = F(x) + C$

4. $\int kf(x)dx = k\int f(x)dx$ **5**. $\int (f(x) \pm g(x))d(x) = \int f(x)dx \pm \int g(x)dx$

6. $$\int f(kx+b)dx = \frac{1}{k}F(kx+b) + C$$

INTEGRALLAR JADVALI

7. $\int dx = x + c$

8. $\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$

9. $\int \frac{dx}{x} = \ln|x| + c$

10. $\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$

11. $\int a^x dx = \frac{a^x}{\ln a} + C$

12. $\int e^x du = e^x + C$

13. $\int \sin x dx = -\cos x + C$

14. $\int \cos x dx = \sin x + C$

15. $\int \frac{dx}{\cos^2 x} = tgx + C$

16. $\int \frac{dx}{\sin^2 x} = -ctgx + C$

17. $\int tgx dx = -\ln|\cos x| + C$

18. $\int ctgx dx = \ln|\sin x| + C$

19. $\int \frac{dx}{\sin x} = \ln\left|tg\frac{x}{2}\right| + C = \ln\left|\frac{1}{\sin x} - ctgx\right| + C$

20. $\int \frac{dx}{\cos x} = \ln\left|tg(\frac{\pi}{4} + \frac{x}{2})\right| + C = \ln\left|\frac{1}{\cos x} + tgx\right| + C$

21. $\int \frac{du}{a^2 + x^2} = \frac{1}{a}arctg\frac{x}{a} + C = -\frac{1}{a}arcctg\frac{x}{a} + C$

22. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$

23. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C = -\arccos\frac{x}{a} + C$

24. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

25. $\int \sin^n x dx = -\frac{1}{n}\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x dx$

26. $\int \cos^n x dx = -\frac{1}{n}\sin x \cos^{n-1} x + \frac{n-1}{n}\int \cos^{n-2} x dx$

27. $\int \sqrt[n]{x} dx = \frac{nx\sqrt[n]{x}}{n+1} + C$

28. $\int \ln x dx = x\ln x - x + C$

29. $\int \log_a x dx = \frac{x\ln x - x}{\ln a} + C$

30. $\int \ln^2 x dx = x(\ln x - 1)^2 + C$

31. $\int \log^2{}_a x dx = \frac{x(\ln x - 1)^2}{\ln^2 a} + C$

Differentsial belgisi ostiga kiritish:

32. $\int dx = \frac{1}{k}\int d(kx + a)$

33. $\int x dx = \frac{1}{2}\int d(x^2)$

34. $\int \cos x dx = \int d(\sin x)$

35. $\int \frac{dx}{\cos^2 x} = \int d(tgx)$

36. $\int \frac{dx}{1 + x^2} = \int d(arctgx)$

37. $\int \frac{dx}{x} = \int d(\ln x)$

Integralda o`zgaruvchini almashtirish:

38. $\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt$

39. $\int f(\phi(x))\phi'(x)dx = \int f(t)dt$

Bo`laklab integrallash: **40**. $\int udv = uv - \int vdu$

Ba`zi nostandart tenglamalarni yechish qoidalari.

1. Tenglamani yeching: $(x^2 - a^2)^2 = 4ax + 1$

Yechish: Tenglikning har ikki tomoniga $4a^2x^2$ ni qo`shib,

$$(x^2 + a^2)^2 = (2ax + 1)^2$$

ko`rinishga keltiramiz. Bundan $x=a-1$ va $x=a+1$ larni topamiz.

Misollar:

1. $(x^2 - 9)^2 = 12x + 1$
2. $(x^2 - 25)^2 = 20x + 1$
3. $(x^2 - 625)^2 = 100x + 1$
5. $(x^2 - 2017^2)^2 = 8068x + 1$
4. $(x^2 + 9)^2 = 12x - 1$

2. Tenglamani yeching: $$x^2 + \frac{a^2x^2}{(x+a)^2} = b$$

Yechish: Tenglikning har ikki tomoniga a^2 ni qo`shib,

$$\left(x + a - \frac{ax}{x+a}\right)^2 = a^2 + b$$

ko`rinishga keltiramiz. Bundan ikkita kvadrat tenglama kelib chiqadi.

Misollar:

1. $$x^2 + \frac{4x^2}{(x+2)^2} = 5$$

2. $$x^2 + \frac{36x^2}{(x+6)^2} = 13$$

3. $$x^2 + \frac{x^2}{(2x+1)^2} = \frac{5}{16}$$

3. Tenglamaning ildizlari yig`indisini toping: $\boldsymbol{(x^2 - ax + a)(x^2 - x + a) = ax^2}$

$$x_1+x_2+x_3+x_4=\mathbf{a+1}$$

4. Tenglamaning ildizlari yig`indisini toping: $\boldsymbol{(x^2 + ax + a)(x^2 + x + a) = ax^2}$

$$x_1+x_2+x_3+x_4=-\mathbf{(a+1)}$$

5. Tenglamani yeching:

$$x+\frac{ax}{\sqrt{x^2+a}}=\sqrt{a}$$

$$x=\sqrt{\frac{a}{2}\left(a+2\sqrt{a+1}-\sqrt{\left(a+2\sqrt{a+1}\right)^2-4}\right)}$$

50 dan kichik tub sonlar **15** ta:

2	3	5	7	11	13	17	19	23	29	31	37	41	43	47

100 dan kichik tub sonlar **25** ta.

2	3	5	7	11	13	17	19	23	29	31	37	41	43
47	53	59	61	67	71	73	79	83	89	97			

1000 dan kichik tub sonlar **168** ta.

2	3	5	7	11	13	17	19	23	29	31	37	41	43
47	53	59	61	67	71	73	79	83	89	97	101	103	107
109	113	127	131	137	139	149	151	157	163	167	173	179	181
191	193	197	199	211	223	227	229	233	239	241	251	257	263
269	271	277	281	283	293	307	311	313	317	331	337	347	349
353	359	367	373	379	383	389	397	401	409	419	421	431	433
439	443	449	457	461	463	467	479	487	491	499	503	509	521
523	541	547	557	563	569	571	577	587	593	599	601	607	613
617	619	631	641	643	647	653	659	661	673	677	683	691	701
709	719	727	733	739	743	751	757	761	769	773	787	797	809
811	821	823	827	829	839	853	857	859	863	877	881	883	887
907	911	919	929	937	941	947	953	967	971	977	983	991	997

IKKILIK SANOQ SISTEMADAGI SONNI **SAKKIZLIK** SANOQ SISTEMASIGA O`TKAZISH VA AKSINCHA

TRIADALAR

Ikkilik s/s	Sakkizlik s/s	Ikkilik s/s
000 ⇨	0 ⇨	000
001 ⇨	1 ⇨	001
010 ⇨	2 ⇨	010
011 ⇨	3 ⇨	011
100 ⇨	4 ⇨	100
101 ⇨	5 ⇨	101
110 ⇨	6 ⇨	110
111 ⇨	7 ⇨	111

IKKILIK SANOQ SISTEMADAGI SONNI **O`N OLTILIK** SANOQ SISTEMASIGA O`TKAZISH VA AKSINCHA

TETRADALAR

Ikkilik s/s	O`n oltilik s/s	Ikkilik s/s
0000 ⇨	0 ⇨	0000
0001 ⇨	1 ⇨	0001
0010 ⇨	2 ⇨	0010
0011 ⇨	3 ⇨	0011
0100 ⇨	4 ⇨	0100
0101 ⇨	5 ⇨	0101
0110 ⇨	6 ⇨	0110
0111 ⇨	7 ⇨	0111
1000 ⇨	8 ⇨	1000
1001 ⇨	9 ⇨	1001
1010 ⇨	A ⇨	1010
1011 ⇨	B ⇨	1011
1100 ⇨	C ⇨	1100
1101 ⇨	D ⇨	1101
1110 ⇨	E ⇨	1110
1111 ⇨	F ⇨	1111

Ba`zi burchaklar trigonometrik funksiyalarining qiymatlari.

Argument Gradus, radian	F u n k s i y a			
	$\sin\alpha$	$\cos\alpha$	$tg\alpha$	$ctg\alpha$
0° (0)	0	1	0	Mavjud emas
$15^\circ\left(\frac{\pi}{12}\right)$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$2-\sqrt{3}$	$2+\sqrt{3}$
$18^\circ\left(\frac{\pi}{10}\right)$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{5+\sqrt{5}}}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$	$\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$
$30^\circ\left(\frac{\pi}{6}\right)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
$36^\circ\left(\frac{\pi}{5}\right)$	$\frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$
$45^\circ\left(\frac{\pi}{4}\right)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1
$54^\circ\left(\frac{3\pi}{10}\right)$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$
$60^\circ\left(\frac{\pi}{3}\right)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$
$75^\circ\left(\frac{5\pi}{12}\right)$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$2+\sqrt{3}$	$2-\sqrt{3}$
$90^\circ\left(\frac{\pi}{2}\right)$	1	0	Mavjud emas	0

Bular I chorakdagi burchaklar, qolgan burchaklarni shularga keltirish mumkin.

Masalan, $\mathbf{\sin 126^{o} = \sin(90^o + 36^o) = \cos 36^o} = \dfrac{\sqrt{5}+1}{4}$

Axborotning o`lchov birliklari.

1) Axborotning eng kichik o`lchov birligi ***1 bit***
(bit - bu ikkilik raqam degani.)

2) Matndagi har bir harf, belgi, xatto so`zlar orasidagi bo`sh joy ham **1 bayt** axborot hajmiga ega.

3) 8 bit = 1 bayt
4) 1024 bayt = 1 kilo bayt (kbt)
5) 1024 kbt = 1 mega bayt (mbt)
6) 1024 mbt = 1 gigabayt (gbt)
7) 1024 gbt = 1 terabayt (tbt)

$$1024 = 2^{10}$$

BIRLASHMALAR

$$A_n^m = \frac{n!}{(n-m)!} = n(n-1)(n-2)\dots(n-m+1)$$

$$P_n = n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$$

$$P_n = A_n^n$$

$$C_n^m = \frac{n!}{m!(n-m)!} = \frac{A_n^m}{P_m} = \frac{n(n-1)(n-2)\dots(n-m+1)}{1 \cdot 2 \cdot 3 \cdot \dots \cdot m}$$

$$C_n^m = C_n^{n-m}$$