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@matematikaguruh

M (2017). $a^2\frac{(x-b)(x-c)}{(a-b)(a-c)}+b^2\frac{(x-a)(x-c)}{(b-a)(b-c)}+c^2\frac{(x-a)(x-b)}{(c-a)(c-b)}$ **ni soddalashtiring.**
(Bu yerda $(a-b)(a-c)(b-c)\neq 0$.)

Yechish: $f(x)=a^2\frac{(x-b)(x-c)}{(a-b)(a-c)}+b^2\frac{(x-a)(x-c)}{(b-a)(b-c)}+c^2\frac{(x-a)(x-b)}{(c-a)(c-b)}$ **(1)**
belgilash kiritamiz. Bilamizki bu kvadrat funksiya hisoblanib, uni
$f(x)=mx^2+nx+k$ **(2)**
ko'rinishda izlaymiz.
(1) funksiyada $f(a)=a^2$; $f(b)=b^2$ **va** $f(c)=c^2$
tengliklar o'rinli. Bularni **(2)** qo'yib,
$\begin{cases} ma^2+na+k=a^2 \\ mb^2+nb+k=b^2 \\ mc^2+nc+k=c^2 \end{cases} \Rightarrow m=1; n=0; k=0$ bo'lishini ko'rish qiyin emas.
Topilgan m, n va k larni **(2)** ga qo'yib, $f(x)=x^2$ bo'lishi topamiz.
Javob: x^2. **M(Xo). @matematika**

Xususan, $x=\pm 1$ **da** $x^2=(\pm 1)^2=1$.

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M (2017). $x^2 - (k+1)x + k^2 + k - 22 = 0$ **tenglama ildizlaridan biri 2 dan kata, ikkinchisi esa 2 dan kichik bo'lsa, k ning butun qiymatlari yig'indisini toping.**

Yechish: Tenglikning chap qismi kvadrat funksiya bo'lib, parabolaning tarmoqlari yuqoriga yo'nalgan va shartga ko'ra Ox o'qini 2 ta nuqtada ya'ni $x = 2$ nuqtaning o'ng va chap tomonidan kesib o'tishi kerak. Demak, shu funksiya $x = 2$ da manfiy qiymatga ega bo'lishi kerak:

$(-2)^2 - (k+1)(-2) + k^2 + k - 22 < 0 =>$

$k^2 - k - 20 < 0 => -4 < k < 5.$

Bundan k ning butun qiymatlari yi'gindisi

$-3 + (-2) + (-1) + 0 + 1 + 2 + 3 + 4 = 4$ ga teng.

Javob: 4. @matematika

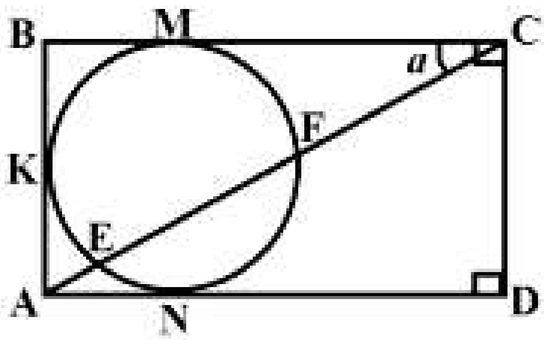
@matematikaguruh

M.2015: $4x \cdot arccos(x^2 - 4x + 5) > x^2$ **tengsizlikni yeching.**

Yechish: Bu nostandart tengsizlikni yechimlarini uning aniqlanish sohasidan izlaymiz: $-1 \leq x^2 - 4x + 5 \leq 1$ Bu qo'sh tengsizlikni faqat $x = 2$ soni qanoatlantiradi. Endi biz $x = 2$ bo'lganida berilgan tengsizlikni tekshirib ko'ramiz.

$4 \cdot 2 \cdot \arccos(2^2 - 4 \cdot 2 + 5) > 2^2$ bundan esa $0 > 4$ sonli tengsizlik hosil bo'ladi. Bu taqqoslash no'to'g'ri bo'lgani sababli $x = 2$ soni tengsizlikni yechimi bo'lmaydi.

Javob: $\emptyset$ @matematika



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Masala: **ABCD – to'g'ri to'rtburchak yuzi 50 ga teng. Aylana AB, BC va AD tomonlarga mos ravishda K, M va N nuqtalarda urinadi. AC diagonal aylanani E va F nuqtalarda kesib o'tadi. Agar ∠BCA=α va EF=7 bo'lsa, sinα ning qiymatini toping.**

Yechish: Zarur belgilashlarni kiritamiz: aylana radiusi $R,\ AE = x,\ FC = y,\ AC = d,$ $AK = KB = BM = R,\ AB = 2R,\ CM = BC - R.$ To'rtburchak yuzi 50 ekanligidan $\boldsymbol{BC} = \frac{50}{AB} = \frac{25}{R} => CM = \frac{25}{R} - R.$ Aylanaga urinma va kesuvchi haqidagi teoremaga ko'ra, $x(x+7) = R^2 => \left(x + \frac{7}{2}\right)^2 = R^2 + \frac{49}{4} => x + 3{,}5 = \sqrt{R^2 + \frac{49}{4}}$ **(1)**

$y(y+7) = \left(\frac{25}{R} - R\right)^2 => y + \frac{7}{2} = \sqrt{\left(\frac{25}{R} - R\right)^2 + \frac{49}{4}}$ **(2)**

(1) va **(2)** tengliklarni qo'shib, $\boldsymbol{d = x + y + 7} = \sqrt{R^2 + \frac{49}{4}} + \sqrt{\left(\frac{25}{R} - R\right)^2 + \frac{49}{4}}$ **(3).**

ABC to'g'ri burchakli uchburchakda **Pifagor** teoremasiga ko'ra, $4R^2 + \frac{625}{R^2} = d^2$ **(4)**

(1) va **(4)** dan $4R^2 + \frac{625}{R^2} = \sqrt{R^2 + \frac{49}{4}} + \sqrt{\left(\frac{25}{R} - R\right)^2 + \frac{49}{4}}$ buni kvadratga ko'tarib,

$4R^2 + \frac{625}{R^2} = R^2 + \frac{49}{4} + \frac{625}{R^2} - 50 + R^2 + \frac{49}{4} + 2 \cdot \sqrt{R^2 + \frac{49}{4}} \cdot \sqrt{\left(\frac{25}{R} - R\right)^2 + \frac{49}{4}}$ buni soddalashtirib, $4R^2 + 51 = \sqrt{4R^2 + 49} \cdot \sqrt{\left(\frac{50}{R} - 2R\right)^2 + 49}$ natijani olamiz va yana kvadratga ko'tarib, $16R^4 + 408R^2 + 2601 = (4R^2 + 49) \cdot \left(\left(\frac{50}{R} - 2R\right)^2 + 49\right)$

$$16R^4 + 408R^2 + 2601 = (100 - 4R^2)^2 + 196R^2 + \left(\frac{350}{R} - 14R\right)^2 + 2401$$

$$16R^4 + 212R^2 + 200 = 10000 - 800R^2 + 16R^4 + \frac{350^2}{R^2} - 9800 + 196R^2$$

$816R^2 = \frac{350^2}{R^2} => \boldsymbol{R^2} = \frac{175}{2\sqrt{51}}$ ekanligini topamiz.

$$\mathbf{tg}\,\boldsymbol{\alpha} = \frac{AB}{BC} = \frac{2R}{\frac{25}{R}} = \frac{2R^2}{25} = \frac{2}{25} \cdot \frac{175}{2\sqrt{51}} = \frac{7}{\sqrt{51}} => \mathbf{sin}\,\boldsymbol{\alpha} = \frac{\mathrm{tg}\,\alpha}{\sqrt{1+\mathrm{tg}^2\,\alpha}} = \frac{\frac{7}{\sqrt{51}}}{\sqrt{1+\frac{49}{51}}} = \frac{7}{10}.$$

Javob: $\frac{7}{10}$

@matematika

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M: Ifodani soddalashtiring:

$$\frac{abc}{bc+ac-ab}-\left(\frac{a-1}{a}+\frac{b-1}{b}-\frac{c-1}{c}\right):\left(\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right)$$

Yechish: $\frac{abc}{bc+ac-ab}-\left(\frac{a-1}{a}+\frac{b-1}{b}-\frac{c-1}{c}\right):\left(\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right)=$

$$\frac{1}{\frac{1}{a}+\frac{1}{b}-\frac{1}{c}}-\left(1-\frac{1}{a}+1-\frac{1}{b}-1+\frac{1}{c}\right):\left(\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right)=$$

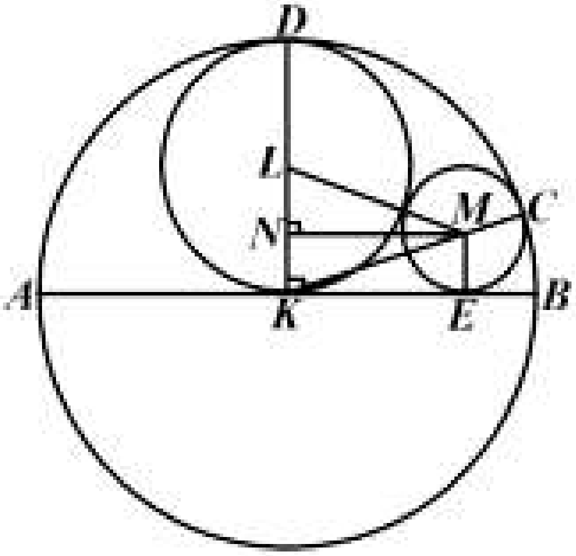
$$\frac{1}{\frac{1}{a}+\frac{1}{b}-\frac{1}{c}}+\left(\frac{1}{a}+\frac{1}{b}-\frac{1}{c}-1\right):\left(\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right)=$$

$$\frac{1}{\frac{1}{a}+\frac{1}{b}-\frac{1}{c}}+\frac{\frac{1}{a}+\frac{1}{b}-\frac{1}{c}-1}{\frac{1}{a}+\frac{1}{b}-\frac{1}{c}}=1$$

Javob: 1 @matematika

@matematikaguruh

M (2016): AB kesma K aylananing diametri bo`lsin. L aylana K aylanaga hamda AB to`g`ri chiziqqa K aylananing markazida urinadi. M aylana K va L aylanaga hamda AB to`g`ri chiziqqa urinadi. Agar M aylana radiusi $R_M$ ga teng bo`lsa, L va K aylanalarning R_L va R_K radiuslarini toping.



Yechish: L aylananing radiusi $KL = R_L$ bo'lsin. U holda K aylananing radiusi $R_K = 2R_L$ bo'ladi. Shartga ko'ra $MC = ME = NK = R_M$. Demak, chizmaga asosan $LN = R_L - R_M$, $LM = R_L + R_M$, $KM = 2R_L - R_M$, $MN = KE$. LMN va KME to'g'ri burchakli uchburchaklarga ko'ra,

$$\begin{cases} KM^2 - ME^2 = KE^2 \\ LM^2 - LN^2 = MN^2 \end{cases} => KM^2 - ME^2 = LM^2 - LN^2 =>$$

$$(2R_L - R_M)^2 - R_M^2 = (R_L + R_M)^2 - (R_L - R_M)^2$$

$$4R_L^2 - 4R_LR_M + \boldsymbol{R_M^2} - \boldsymbol{R_M^2} = R_L^2 + 2R_LR_M + \boldsymbol{R_M^2} - R_L^2 + 2R_LR_M - \boldsymbol{R_M^2}$$

$4R_L^2 = 8R_LR_M$ => $R_L = 2R_M$ va $R_K = 2R_L = 4R_M$ bo'lishini topamiz.

Javob: $R_L = 2R_M$ va $R_K = 4R_M$. @matematika

M: $9 \cdot 16^x - 7 \cdot 12^x - 16 \cdot 9^x = 0$ tenglamani yeching.

Yechish: Tenglikning ikkala tomonini 9^x ga bo'lamiz:

$$9 \cdot \left(\frac{16}{9}\right)^x - 7 \cdot \left(\frac{12}{9}\right)^x - 16 = 0$$

$$9 \cdot \left(\frac{4}{3}\right)^{2x} - 7 \cdot \left(\frac{4}{3}\right)^x - 16 = 0$$

$\left(\frac{4}{3}\right)^x = t$ belgilash kiritamiz.

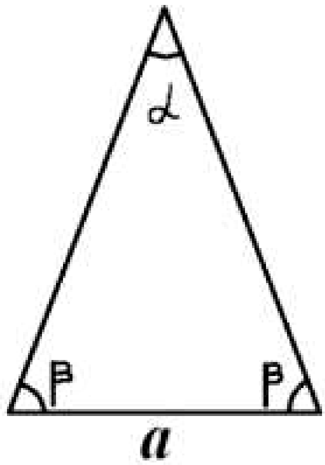
$9 \cdot t^2 - 7 \cdot t - 16 = 0$ bu tenglamani yechib,

$t_1 = -1;\ t_2 = \frac{16}{9}$ yechimlarni hosil qilamiz.

Bularni belgilashga qo'yib, $\left(\frac{4}{3}\right)^x = -1 => x \in \emptyset$ va

$\left(\frac{4}{3}\right)^x = \frac{16}{9} => x = 2$ yechimlarni hosil qilamiz.

Javob: $x = 2$



M: Teng yonli uchburchakning uchidagi burchagi α ga teng. Uchburchakka ichki va tashqi chizilgan doiralar radiuslari nisbatini toping.

Yechish: Uchburchakning asosi uzunligi a ga va asosidagi burchagi esa β ga teng bo'lsin. U holda $\beta = \frac{\pi-\alpha}{2}$, bo'lib, $R = \frac{a}{2sin\alpha}$ va $r = \frac{a}{2}tg\frac{\beta}{2} = \frac{a}{2}tg\frac{\pi-\alpha}{4}$ bo'ladi. Bundan esa

$$\frac{r}{R} = \frac{a}{2sin\alpha} : \frac{a}{2}tg\frac{\pi-\alpha}{4} = tg\frac{\pi-\alpha}{4}sin\alpha.$$

Javob: $tg\frac{\pi-\alpha}{4}sin\alpha$

@matematikaguruh **Test (5.07.2017)**

Agar $x = \frac{\sqrt{11}+1}{2}$ bo'lsa, $\frac{x^3-3x^2+6,5x-2}{x^2-x+1}$ kasrning qiymatini hisoblang.

A) $\sqrt{11}+1$ B) $\sqrt{11}-1$ C) $1-\sqrt{11}$ D) $\sqrt{11}-2$

Yechih: Berilgan $x = \frac{\sqrt{11}+1}{2}$ shartdan

$2x = \sqrt{11}+1$ **(1)** => $2x-1 = \sqrt{11}$ **(2)**

(2) tenglikni ikkala tomonini kvadratga ko'tarib,

$4x^2-4x+1 = 11 => x^2-x = 2,5$ **(3)**

ni hosil qilamiz. Endi so'ralgan ifodada **(3)** ko'rinishidagi ifodalarni hosil qilib, uning qiymatidan va **(1)** dan foydalangan holda kasrning qiymatini hisoblaymiz:

$$\frac{x^3-3x^2+6,5x-2}{x^2-x+1} = \frac{x(x^2-x)-2(x^2-x)+4,5x-2}{(x^2-x)+1} = \frac{2,5x-5+4,5x-2}{2,5+1} =$$

$$\frac{7x-7}{3,5} = 2x-2 = \sqrt{11}+1-2 = \sqrt{11}-1.$$

Javob: $\sqrt{11}-1$ **(B)** **@matematika**

M:2015. **@matematikaguruh** $x^2 - 4|x| - a + 3 = 0$ **tenglama 2 ta musbat ildizga ega bo'ladigan a ning butun qiymatlarining o'rta arifmetigini toping.**

Yechish: $|x| = t$ (1) belgilash kiritamiz. Bu (1) tenglama ikkita musbat ildizga ega bo'lishi uchun t ning 2 ta musbat qiymati mavjud bo'lishi kerak. Shuning uchun $t^2 - 4t - a + 3 = 0$ tenglama 2 ta musbat ildizga ega bo'ladigan a ning qiymatlarini topamiz.

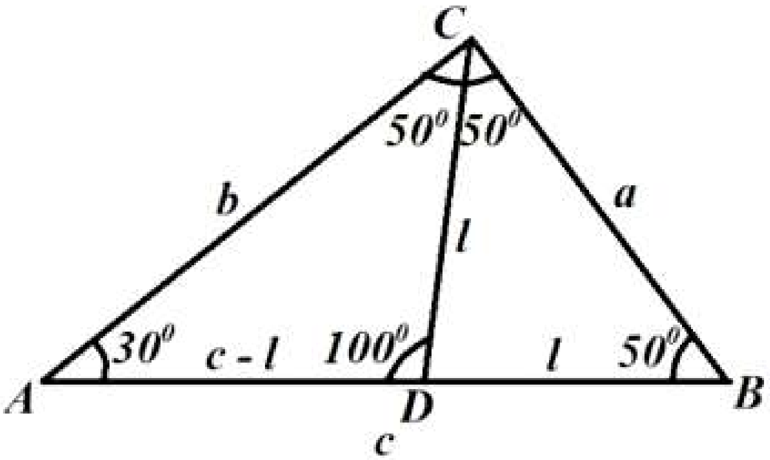
$$\begin{cases} (-4)^2 - 4 \cdot 1 \cdot (-a+3) > 0 \\ -a + 3 > 0 \end{cases} => \begin{cases} a > -1 \\ a < 3 \end{cases} => a \in (-1; 3)$$

a ning butun qiymatlari o'rta arifmetigini topamiz: $\frac{0+1+2}{3} = 1.$

Javob: 1 **@matematika**

@matematikaguruh

M: ABC uchburchakning A va B burchaklari mos ravishda 30° va 50° ga teng bo'lsa, uning *a*, *b* va *c* tomonlari orasidagi o'zaro munosabatni aniqlang.



Yechish: Shartda berilgan va zarur belgilashlarni kiritamiz (yuqoridagi chizmaga qarang).

ACD va ABC uchburchaklar o'xshashligidan

$$\frac{CD}{AC} = \frac{BC}{AB} => \frac{l}{b} = \frac{a}{c} => l = \frac{ab}{c}.$$

Bissektrisa xossasiga ko'ra

$$\frac{BC}{AC} = \frac{BD}{AD} => \frac{a}{b} = \frac{l}{c-l} => l = \frac{ac}{a+b}.$$

Demak $l = \frac{ab}{c}$ va $l = \frac{ac}{a+b}$ tengliklardan

$\frac{ab}{c} = \frac{ac}{a+b} => a = \frac{c^2 - b^2}{b}$ munosabatni hosil qilamiz.

Javob: $a = \frac{c^2 - b^2}{b}$.

@matematika

Savol(5.07.2017): **Agar $\vec{a}$, $\vec{b}$ va $\vec{c}$ birlik vektorlar $\vec{a}+\vec{b}+\vec{c}=0$ shartni qanoatlantirsa, $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ ning qiymatini hisoblang.**

Yechish: **Shartga ko'ra, $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$ va $\vec{a}^2=|\vec{a}|^2$ tenglik o'rinli ekanligini inobatga olib, $\vec{a}+\vec{b}+\vec{c}=0$ ning ikkala tomonini kvadratga ko'taramiz:**

$$\left(\vec{a}+\vec{b}+\vec{c}\right)^2=0^2 => \vec{a}^2+\vec{b}^2+\vec{c}^2+2\cdot\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=0$$

$$1+1+1+2\cdot\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=0$$

$$2\cdot\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=-3$$

$$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-1,5$$

@matematika

Javob: $-1,5$.

@matematikaguruh

Test: 2017 ***@matematika***

Misol. $(a^2 + b^2 + 4)x^2 + 2(a + b + 2)x + 3 = 0$ **tenglama haqiqiy yechimlarga ega bo'lsa,** $3a - b$ **ni toping.**

A) 3 **B) −4** **C) 4** **D) – 3**

Yechish: Berilgan tenglamada qavslarni ochamiz:

$$a^2x^2 + b^2x^2 + 4x^2 + 2ax + 2bx + 4x + 3 = 0$$

Buni quyidagicha ko'rinishda yozamiz:

$$(ax + 1)^2 + (bx + 1)^2 + (2x + 1)^2 = 0$$

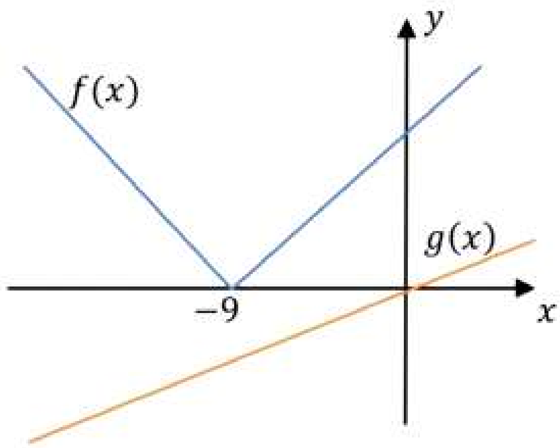
Bundan esa $\begin{cases} ax + 1 = 0 \\ bx + 1 = 0 \\ 2x + 1 = 0 \end{cases} => \left[\begin{array}{l} a = b = 2 \\ x = -0{,}5 \end{array}\right.$ yechimlarni hosil qilamiz. Demak, $3a - b = 2a = 4$

Javob: 4 C. ***@matematikaguruh***

@matematikaguruh

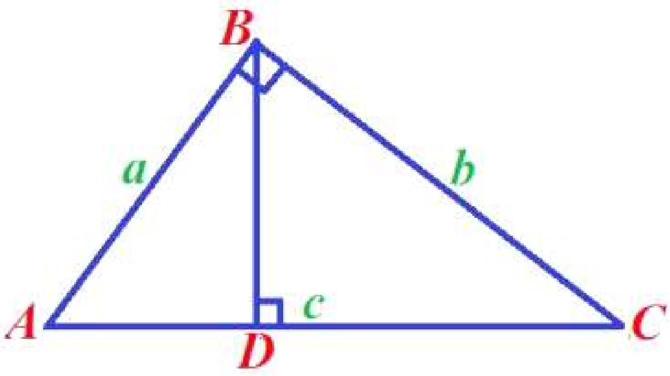
Misol (5.07.2017):

$|x+9| = \frac{x}{2} + a$ tenglama a ning nechta natural qiymatida yechimga ega emas?



Yechish: $f(x) = |x+9|$ funksiya $x = -9$ da eng kichik qiymat **nol**ga erishadi. $g(x) = \frac{x}{2} + a$ funksiyaning $x = -9$ nuqtadagi qiymati $f(x)$ funksiyaning eng kichik qiymati(nol)dan kichik bo'lsa, berilgan tenglama yechimga ega bo'lmaydi. $g(-9) < 0 \;=>\; \frac{-9}{2} + a < 0 \;=>\; a < 4,5.$ Demak, a ning natural qiymatlari to'plami $A = \{1; 2; 3; 4\}$ dan iborat bo'lib, uning soni $n(A) = 4$ ga teng. **Javob: 4.** @matematika

@matematikaguruh



M: **ABC to'g'ri burchakli uchburchakda gipotenuzaga BD balandlik o'tkazilgan. Agar ABD va CBD uchburchaklarning perimetrlari mos ravishda p_1 *va* p_2 bo'lsa, ABC uchburchakning perimetrini toping.**

Yechish: **ABC** uchburchakning perimetri p, **AB**=a, **BC**=b va **AC**=c bo'lsin. Δ**ABC** - Δ**ABD** ***va*** ΔCBD ***lar bilan o'xshash, shuning uchun ularning gipotenuzalari nisbati perimetrlari nisbatiga teng***, ya'ni

$\frac{a}{c} = \frac{p_1}{p}$ va $\frac{b}{c} = \frac{p_2}{p}$. ***Pifagor*** teoremasiga ko'ra, $\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$ tenglikdan foydalanib, $\left(\frac{p_1}{p}\right)^2 + \left(\frac{p_2}{p}\right)^2 = 1$ => $p = \sqrt{p_1^2 + p_2^2}$

Javob: $\sqrt{p_1^2 + p_2^2}$.   @matematika

@matematikaguruh

Misol (5.07.2017). $cos9\alpha = 4cos\alpha$ **bo'lsa,** $(4\cos^2 3\alpha - 3)(4\cos^2 \alpha - 3)$ **ning qiymatini hisoblang.** $\alpha \neq \frac{\pi}{2} + \pi n, n \in Z$

Yechish: $(4\cos^2 3\alpha - 3)(4\cos^2 \alpha - 3) = \frac{cos3\alpha(4\cos^2 3\alpha - 3)}{cos3\alpha} \cdot \frac{cos\alpha(4\cos^2 \alpha - 3)}{cos\alpha} =$

$\frac{4\cos^3 3\alpha - 3cos3\alpha}{cos3\alpha} \cdot \frac{4\cos^3 \alpha - 3cos\alpha}{cos\alpha}$

$4\cos^3 \alpha - 3cos\alpha = cos3\alpha$ **formulaga ko'ra,**

$\frac{cos9\alpha}{cos3\alpha} \cdot \frac{cos3\alpha}{cos\alpha} = \frac{cos9\alpha}{cos\alpha} = \frac{4cos\alpha}{cos\alpha} = 4.$

Javob: 4. @matematika

M: $1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1}$ yig'indini hisoblang.

Yechish: Bu yig'indini qiymati S bo'lsin.

$1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1} = S$ (1)

Agar $x = 0$ bo'lsa, $S = 1$ bo'ladi. $x = 1$ bo'lsa, $S = \frac{1+n}{2} \cdot n$ bo'ladi. Endi biz $x \neq 0, x \neq 1$ bo'lgan holda tenglikning ikkala tomonini x ga ko'paytiramiz.

$1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1} = S / \cdot x$

$x + 2x^2 + 3x^3 + 4x^4 + \cdots + (n-1)x^{n-1} + nx^n = S \cdot x$ (2)

(1) va (2) tenliklarni hadma had ayiramiz: @matematika

$1 + x + x^2 + x^3 + x^4 + \cdots + x^{n-1} - nx^n = S - S \cdot x$ @matematikaguruh

$\frac{1 \cdot (1-x^n)}{1-x} - nx^n = S(1-x) \Rightarrow S = \frac{nx^{n+1} - (n+1)x^n + 1}{(1-x)^2}$.

Javob: $1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1} = \begin{cases} \frac{1+n}{2} \cdot n, & x = 1 \text{ bo'lsa}, \\ \frac{nx^{n+1} - (n+1)x^n + 1}{(1-x)^2} & x \neq 1 \text{ bo'lsa}. \end{cases}$

M: 2017. @matematika Agar $a + b + c = m$ va $\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} = n$ bo'lsa, $a + b + c - \left(\frac{c}{a+b} + \frac{a}{b+c} + \frac{b}{c+a}\right)$ ifodaning qiymatini toping (bunda $m \neq 0$).

Yechish: $a = m - b - c$; $b = m - c - a$; $c = m - a - b$

$$a + b + c - \left(\frac{c}{a+b} + \frac{a}{b+c} + \frac{b}{c+a}\right) =$$

$$m - \left(\frac{m-a-b}{a+b} + \frac{m-b-c}{b+c} + \frac{m-c-a}{c+a}\right) =$$

$$m - \left(\frac{m}{a+b} + \frac{m}{b+c} + \frac{m}{c+a} - 3\right) =$$

$$m + 3 - m \cdot \left(\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a}\right) = m + 3 - mn.$$

Javob: $m + 3 - mn$ *@matematikaguruh*

M: $\begin{cases} \frac{4x^2}{1+4x^2} = y \\ \frac{4y^2}{1+4y^2} = z \\ \frac{4z^2}{1+4z^2} = x \end{cases}$ **tenglamalar sistemasini yeching:**

Yechish: Quyidagi 2 ta holatni ko'rib chiqamiz:

1-hol. Sistemadagi noma'lumlardan birortasi, masalan $x = 0$ bo'lsin. U holda $y = 0$, $z = 0$ bo'lishi kelib chiqadi.

2-hol. $x \neq 0$ bo'lsin. Bunda sistemani quyidagicha yozib yechish mumkin:

$$\begin{cases} \frac{1+4x^2}{4x^2} = \frac{1}{y} \\ \frac{1+4y^2}{4y^2} = \frac{1}{z} \\ \frac{1+4z^2}{4z^2} = \frac{1}{x} \end{cases} => \begin{cases} \frac{1}{4x^2} + 1 = \frac{1}{y} \\ \frac{1}{4y^2} + 1 = \frac{1}{z} \\ \frac{1}{4z^2} + 1 = \frac{1}{x} \end{cases}$$

Bu sistemadagi barcha tenglamalarni qo'shamiz.

$$\frac{1}{4x^2} + 1 + \frac{1}{4y^2} + 1 + \frac{1}{4z^2} + 1 = \frac{1}{y} + \frac{1}{z} + \frac{1}{x}$$

$$\frac{1}{4x^2} - \frac{1}{x} + 1 + \frac{1}{4y^2} - \frac{1}{y} + 1 + \frac{1}{4z^2} - \frac{1}{z} + 1 = 0$$

$$\left(\frac{1}{2x} - 1\right)^2 + \left(\frac{1}{2y} - 1\right)^2 + \left(\frac{1}{2z} - 1\right)^2 = 0.$$

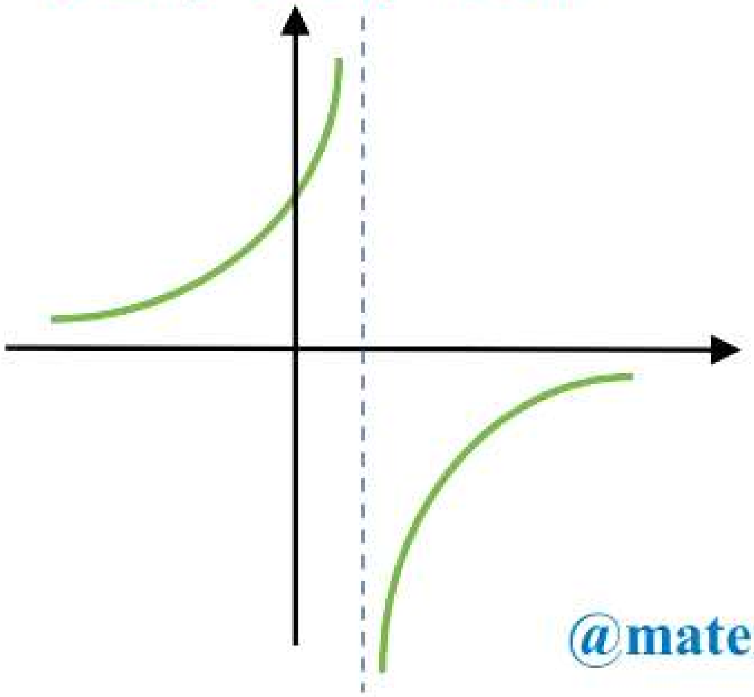
Bundan esa quyidagi sistema hosil bo'ladi.

$$\begin{cases} \frac{1}{2x} - 1 = 0 \\ \frac{1}{2y} - 1 = 0 \\ \frac{1}{2z} - 1 = 0 \end{cases} => \begin{cases} x = \frac{1}{2} \\ y = \frac{1}{2} \\ z = \frac{1}{2} \end{cases}$$

Javob: **(0; 0; 0) yoki** $\left(\frac{1}{2}; \frac{1}{2}; \frac{1}{2}\right)$

$y = a + \frac{2}{bx+c}$ **funksiya uchun quyidagi munosabatlar o'rinli.**

$a = 0; \quad b < 0; \quad c > 0$



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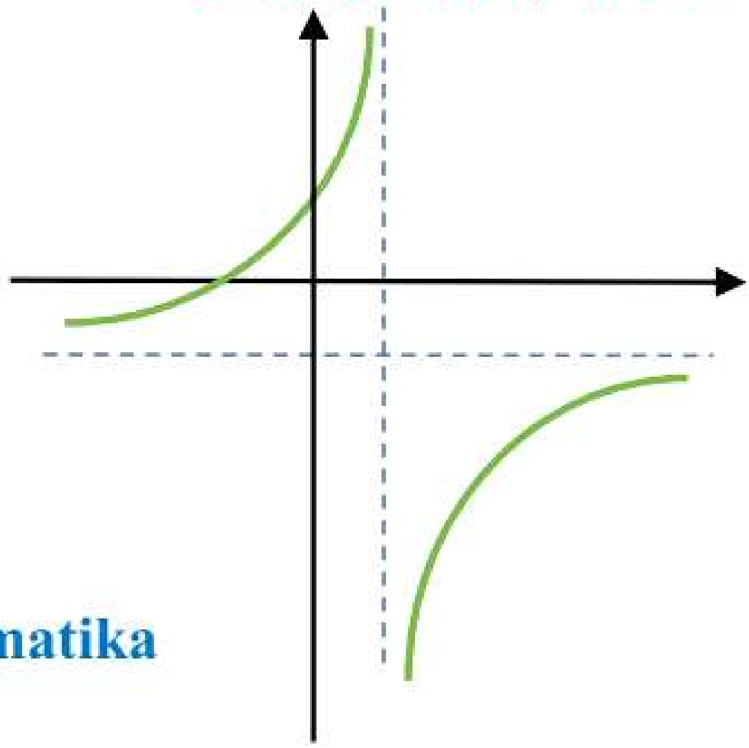

@matematika

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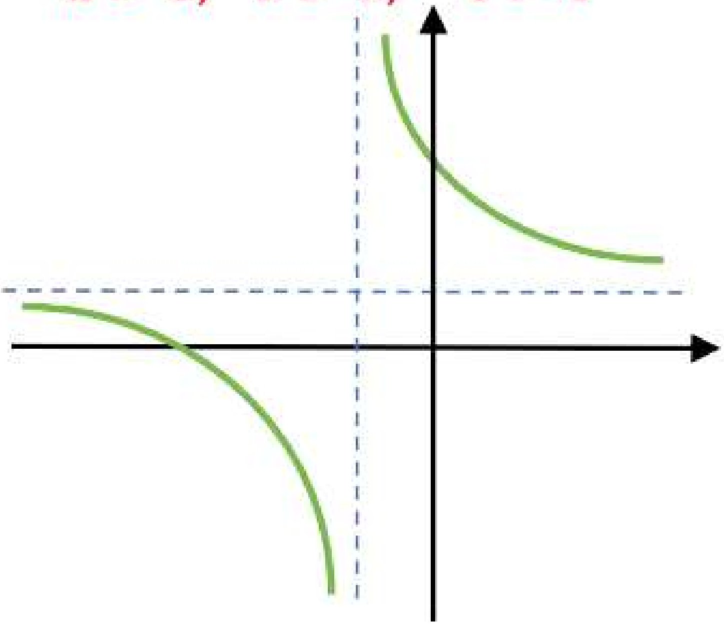

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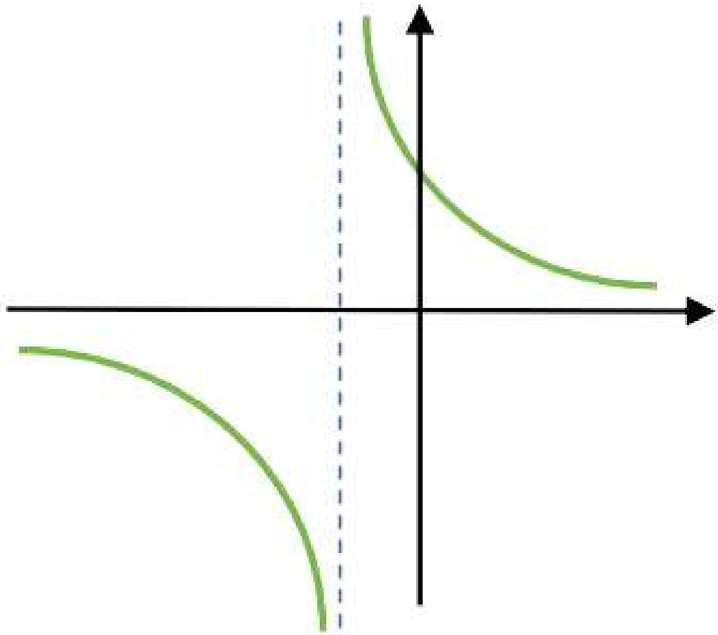
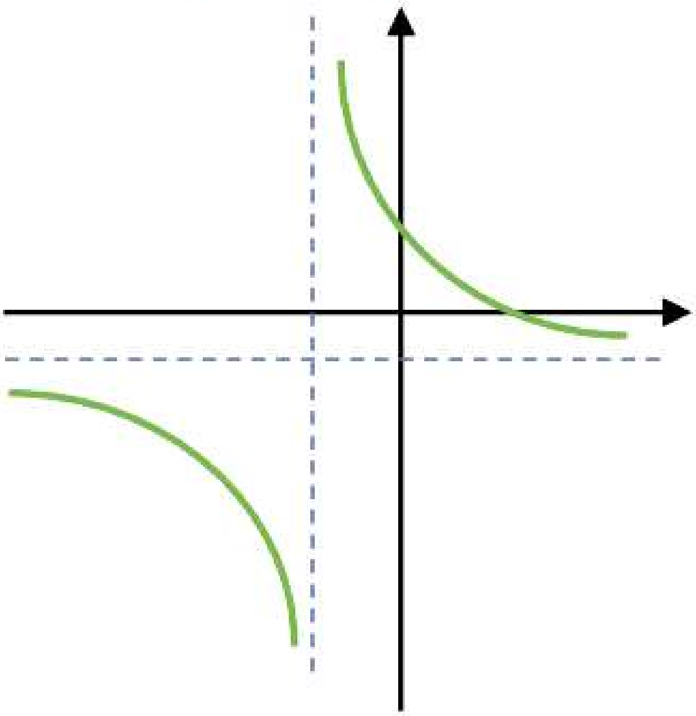
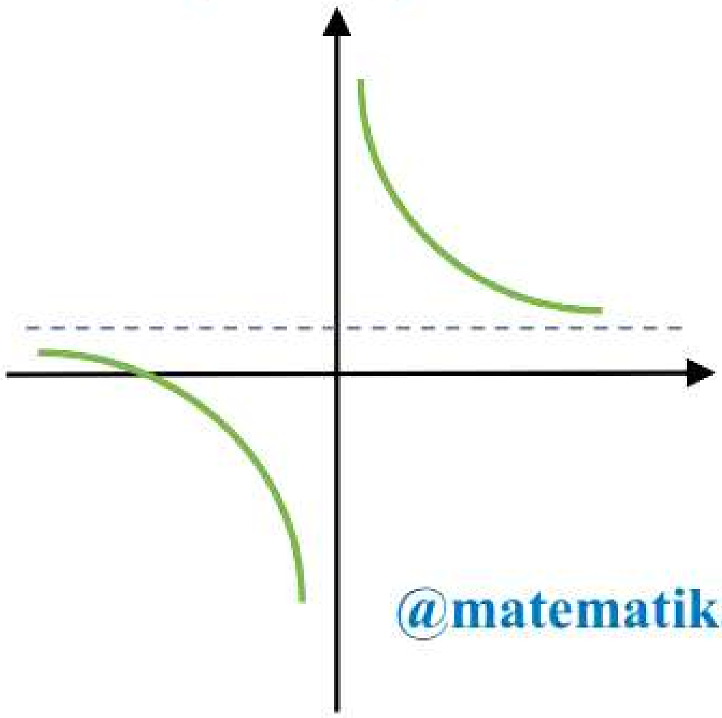


$a < 0; \quad b > 0; \quad c > 0$



$a > 0; \quad b > 0; \quad c = 0$



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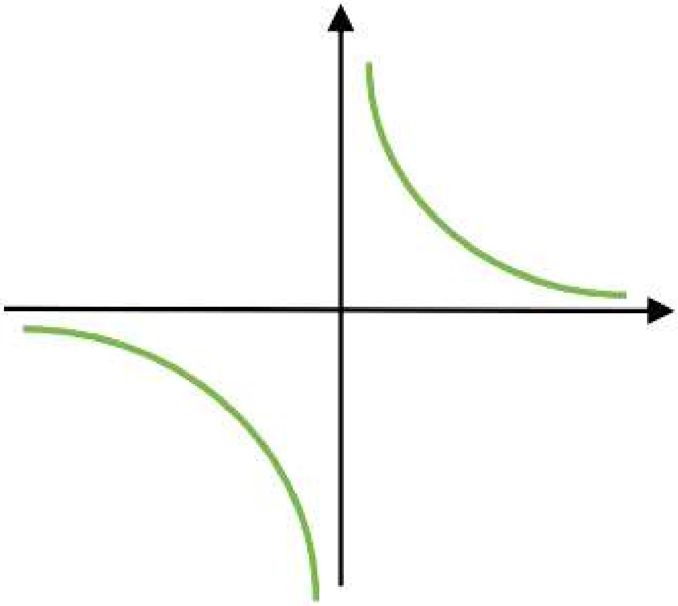
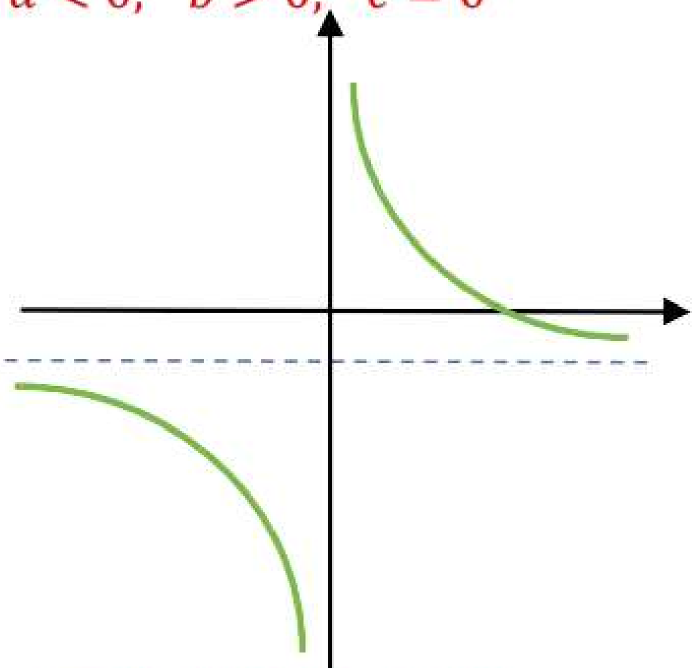


$a < 0; \quad b > 0; \quad c = 0$



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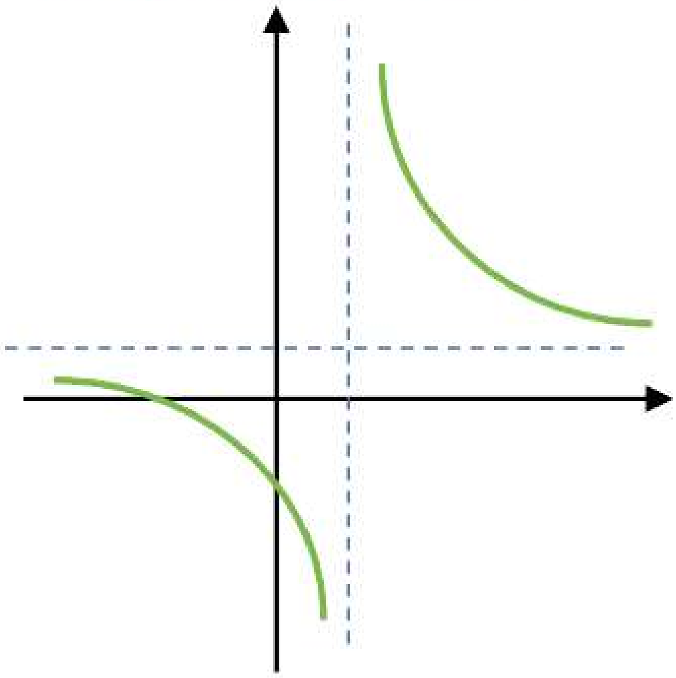

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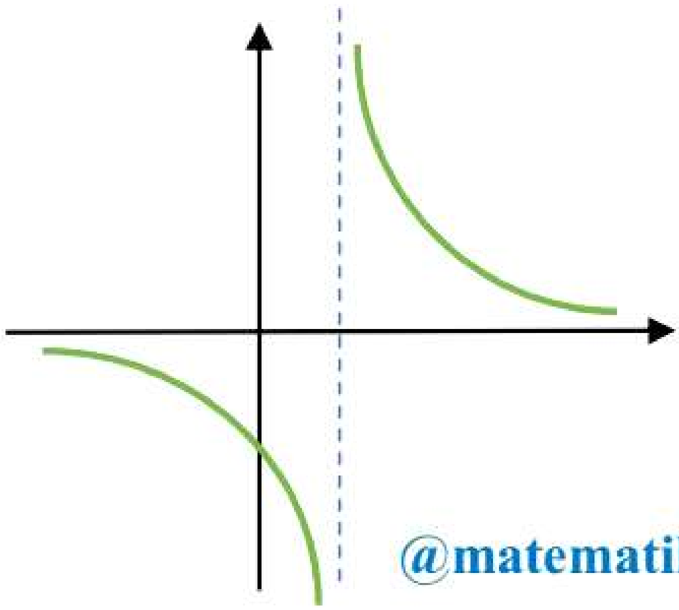

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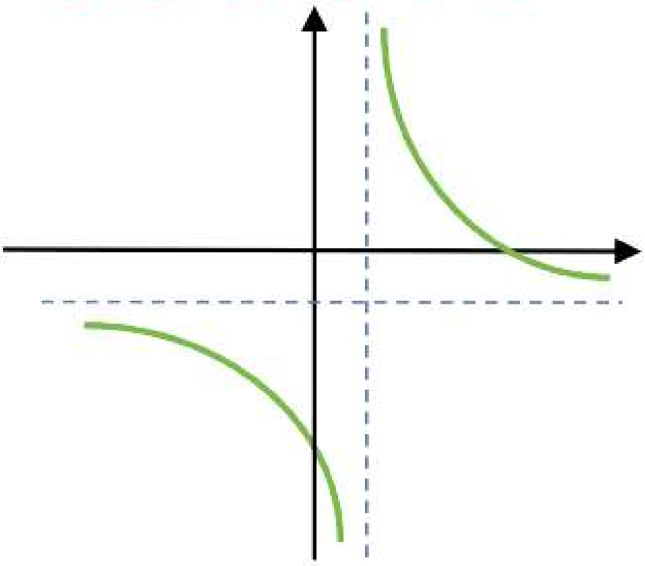
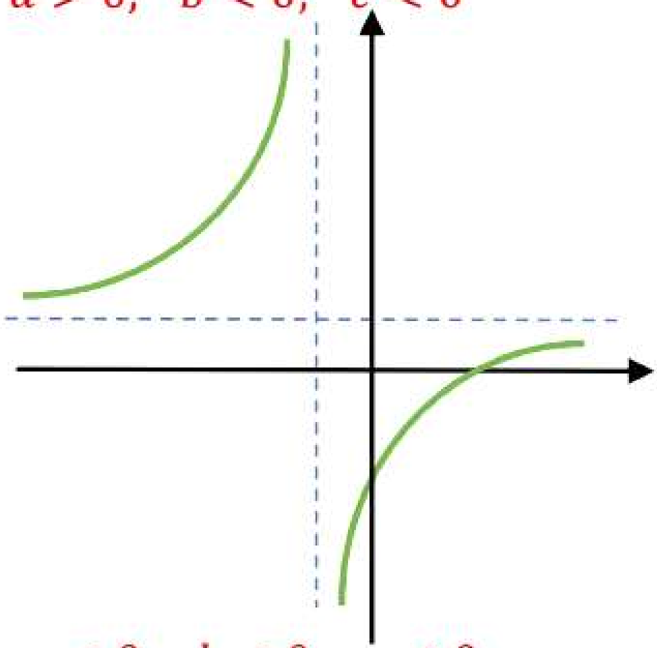
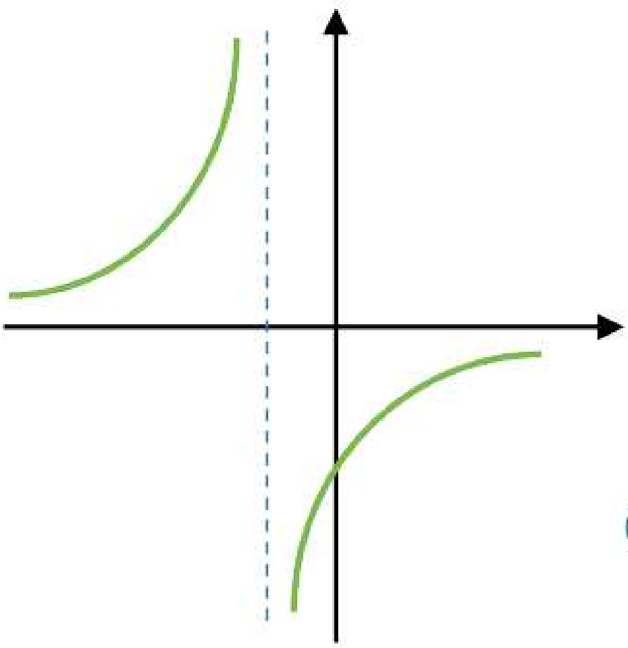


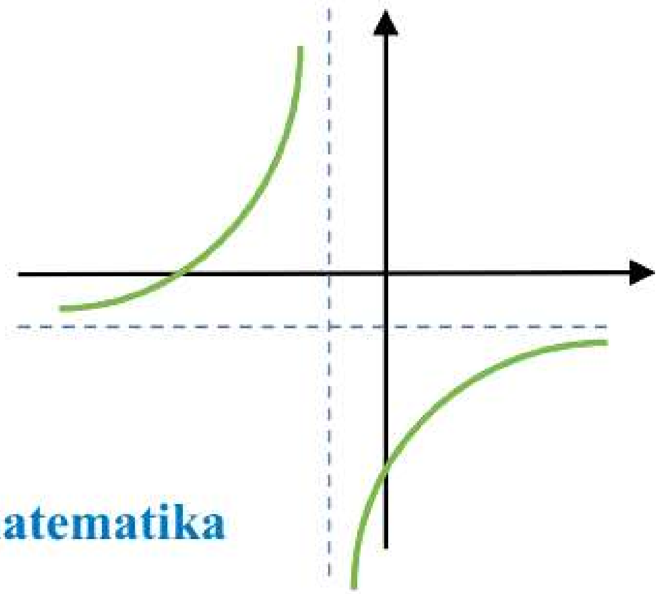
$a > 0; \quad b < 0; \quad c < 0$



$a = 0; \quad b < 0; \quad c < 0$



$a < 0; \quad b < 0; \quad c < 0$



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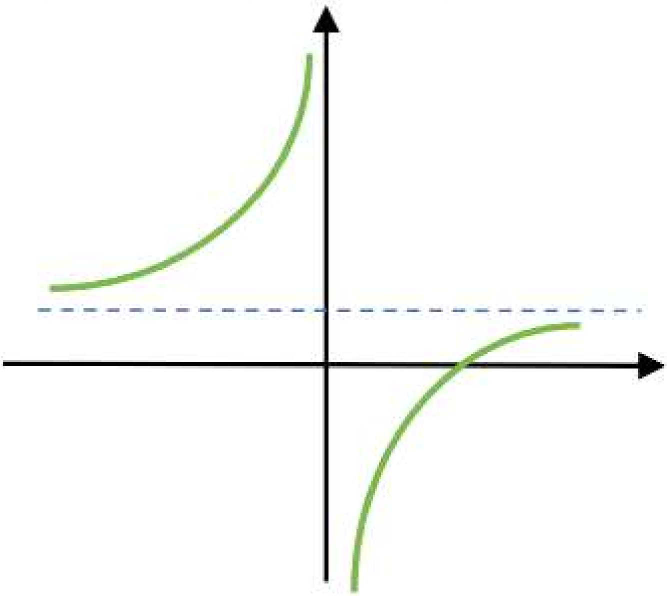
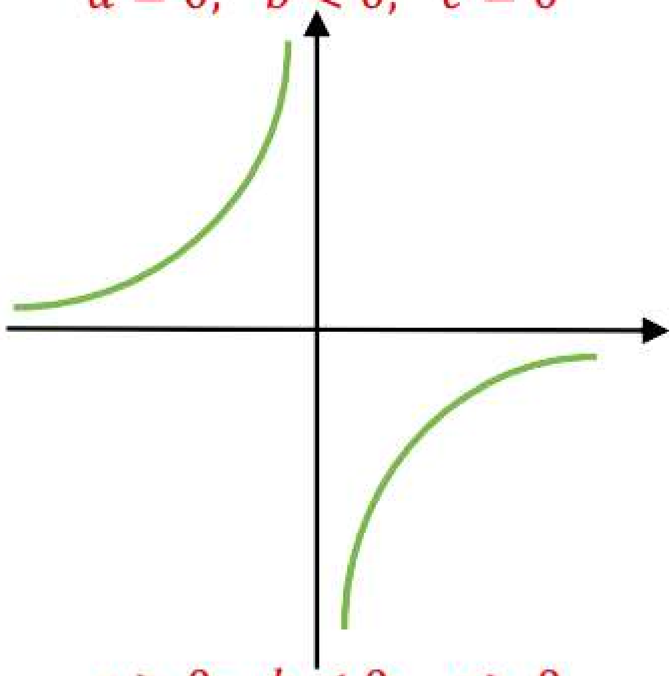
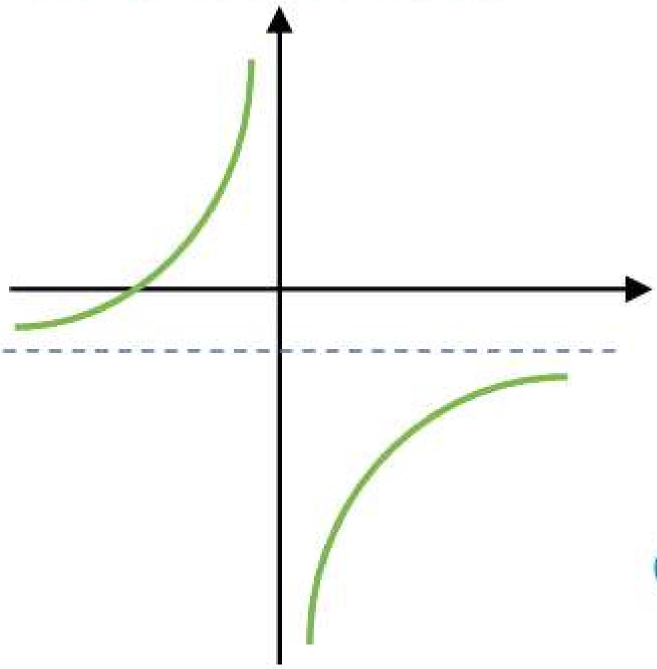


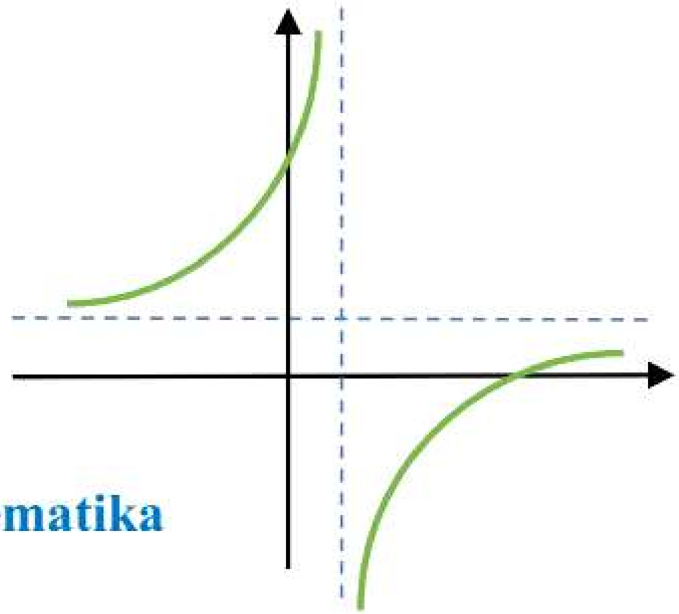
$a = 0; \quad b < 0; \quad c = 0$



$a < 0; \quad b < 0; \quad c = 0$



$a > 0; \quad b < 0; \quad c > 0$



@matematika

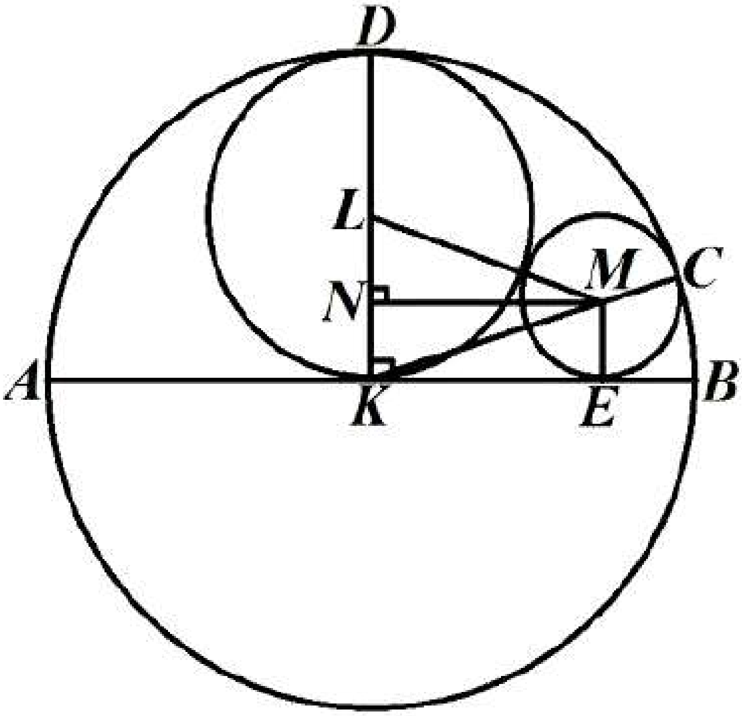
M: $f(x)$ funksiya berilgan (a, b) intervalda noldan farqli va differensiallanuvchi bo'lsin. $(f(x))^{-1}$ funskiyaning (a, b) intervaldagi hosilasini toping.

A) $-(f(x))^{-2} \cdot f'(x)$ B) $(f(x))^{-2} \cdot f'(x)$

C) $(f(x))^{-2}$ D) $2(f(x))^{-2} \cdot f'(x)$

Yechish: (a, b) oraliqda dufferensiallanuvchi degani – shu (a, b) oraliqda hosilasi mavjud degani.

Demak, $(f(x))^{-1} \Rightarrow (-1)f'(x)(f(x))^{-2} = -(f(x))^{-2} \cdot f'(x)$.

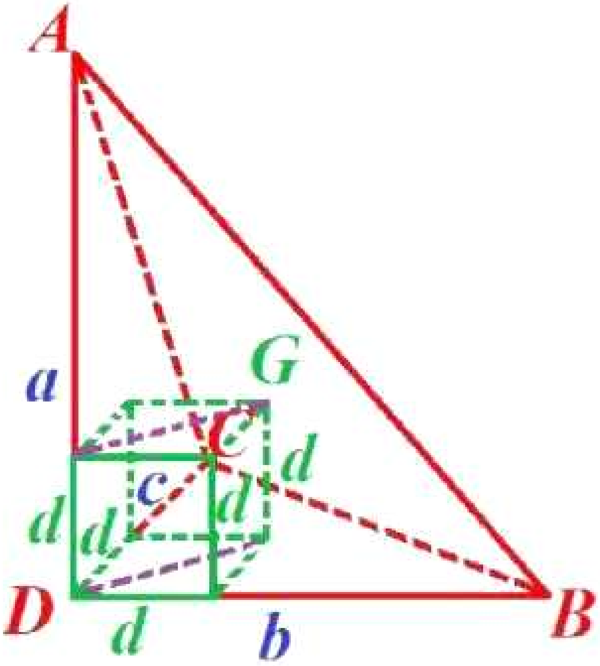


AB kesma K aylananing diametri bo`lsin. L aylana K aylanaga hamda AB to`g`ri chiziqqa K aylananing markazida urinadi. M aylana K va L aylanaga hamda AB to`g`ri chiziqqa urinadi. Agar M aylana radiusi 1 ga teng bo`lsa, L aylana radiusini toping.

Yechish: L aylananing radiusi $KL = R$ bo'lsin. U holda K aylananing radiusi $KB = KD = KC = 2KL = 2R$ bo'ladi. Shartga ko'ra $MC = ME = NK = 1$. Bunga ko'ra chizmaga asosan $LN = R - 1,\ LM = R + 1,\ KM = 2R - 1$, $MN = KE$ bo'lishini aniqlashimiz mumkin. LMN va KME to'g'ri burchakli uchburchaklarga ko'ra $KM^2 - ME^2 = KE^2$ va $LM^2 - LN^2 = MN^2$. Demak $KM^2 - ME^2 = LM^2 - LN^2 \Rightarrow (2R-1)^2 - 1^2 = (R+1)^2 - (R-1)^2$.

Bu tenglamani yechib, $R = 2$ bo'lishini topamiz.

Javob: 2

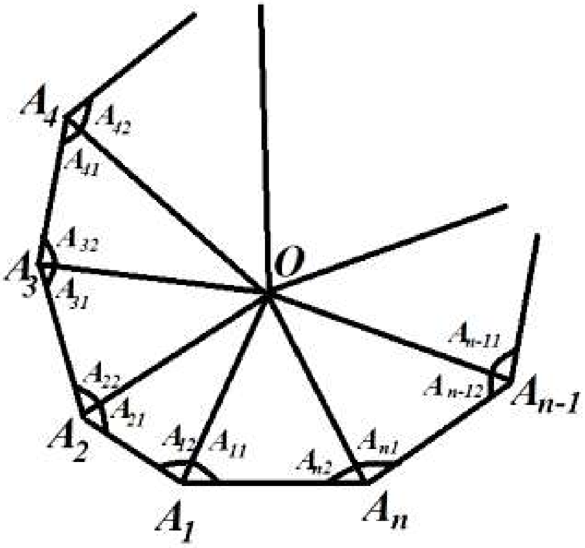


$ABCD$ tetradrni D uchidan chiquvchi uchta qirralari o'zaro perpendikulyar hamda uzunliklari $DA = a$; $DB = b$; $DC = c$ ga teng. Unga ichki chizilgan kubni bir uchi D nuqtada bu nuqtaga qarama-qarshi G uchi esa ABC yoqda yotadi. Kubning qirrasi uzunligi d ning qiymatini quyidagi formula yordamida hisoblanadi.

$$d = \frac{a \cdot b \cdot c}{a \cdot b + b \cdot c + c \cdot a}$$

@matematikaguruh

Teorema: **Qavariq ko'pburchak ichidagi ixtiyoriy nuqtadan uchlariga o'tkazilgan kesmalar uning burchaklarini chizmadagidek bo'laklarga bo'ladi va quyidagi tenglik o'rinli bo'ladi.**



$$\frac{\sin A_{11}}{\sin A_{12}} \cdot \frac{\sin A_{21}}{\sin A_{22}} \cdot \frac{\sin A_{31}}{\sin A_{32}} \cdot \frac{\sin A_{41}}{\sin A_{42}} \cdot \ldots \cdot \frac{\sin A_{n-11}}{\sin A_{n-12}} \cdot \frac{\sin A_{n1}}{\sin A_{n2}} = 1$$

@riyoziyot

Isbot: OA_1A_2; OA_2A_3; OA_3A_4; ... ; $OA_{n-1}A_n$; OA_nA_1 uchburchaklarda sinuslar teoremasiga ko'ra,

$$\frac{\sin A_{21}}{\sin A_{12}} = \frac{OA_1}{OA_2};\ \frac{\sin A_{31}}{\sin A_{22}} = \frac{OA_2}{OA_3};\ \frac{\sin A_{41}}{\sin A_{32}} = \frac{OA_3}{OA_4};\ \ldots;\ \frac{\sin A_{n1}}{\sin A_{n-12}} = \frac{OA_{n-1}}{OA_n};\ \frac{\sin A_{11}}{\sin A_{n2}} = \frac{OA_n}{OA_1}$$

bu tengliklarni ko'paytirib,

$$\frac{\sin A_{21}}{\sin A_{12}} \cdot \frac{\sin A_{31}}{\sin A_{22}} \cdot \frac{\sin A_{41}}{\sin A_{32}} \cdot \ldots \cdot \frac{\sin A_{n1}}{\sin A_{n-12}} \cdot \frac{\sin A_{11}}{\sin A_{n2}} = \frac{OA_1}{OA_2} \cdot \frac{OA_2}{OA_3} \cdot \frac{OA_3}{OA_4} \cdot \ldots \cdot \frac{OA_{n1}}{OA_n} \cdot \frac{OA_n}{OA_1} \Rightarrow$$

$$\frac{\sin A_{11}}{\sin A_{12}} \cdot \frac{\sin A_{21}}{\sin A_{22}} \cdot \frac{\sin A_{31}}{\sin A_{32}} \cdot \frac{\sin A_{41}}{\sin A_{42}} \cdot \ldots \cdot \frac{\sin A_{n-11}}{\sin A_{n-12}} \cdot \frac{\sin A_{n1}}{\sin A_{n2}} = 1.$$

@matematika

@matematikaguruh

Masala: **10 ta xaltada 10 tadan tanga bor. Bitta xaltadagi tangalar soxta. Soxta tanga haqiqiysidan 5 grammga yengil. Tarozida bir marta tortish orqali qaysi xaltada soxta tanga borligini aniqlang.**

Yechish: Faraz qilaylik $n-$ nomerli xaltada $(1 \leq n \leq 10)$ soxta tangalar bo'lsin. $1-$xaltadan 1 ta, $2-$xaltadan 2 ta, $3-$xaltadan 3 ta, va xakazo… $10-$ xaltadan 10 ta tanga olamiz. Bunda biz jami $1+2+3+\cdots+10=55$ tangaga ega bo'lib, bundan n tasi soxta. Endi soxta aralash olingan 55 ta tangalarni shartagi **bir marta o'lchash** imkoniyatimizni ishga solib, tarozida tortamiz va natijani 11 ga (aslida 55 ga bo'linadi lekin soxta tanganing haqiqiydan farqi 5 ham va 55 ham 5 ga bo'lingani uchun $55:5=11$ usuli qulayroq)bo'lamiz. Agar soxta tangalar

$\mathbf{1}-$ chi xaltada bo'lsa, qoldiq $\mathbf{11-1=10}$ $(55-5\cdot n$ ga ko'ra 5 ga bo'lingan ko'rinishi)ga,

$\mathbf{2}-$ chi xaltada bo'lsa, qoldiq $\mathbf{11-2=9}$ ga,

$\mathbf{3}-$ chi xaltada bo'lsa, qoldiq $\mathbf{11-3=8}$ ga,

va hakazo…

$\boldsymbol{n}-$ chi xaltada bo'lsa, qoldiq $\mathbf{11}-\boldsymbol{n}$ ga,

va hakazo…

$\mathbf{10}-$chi xaltada bo'lsa, qoldiq $\mathbf{11-10=1}$ ga teng bo'ladi.

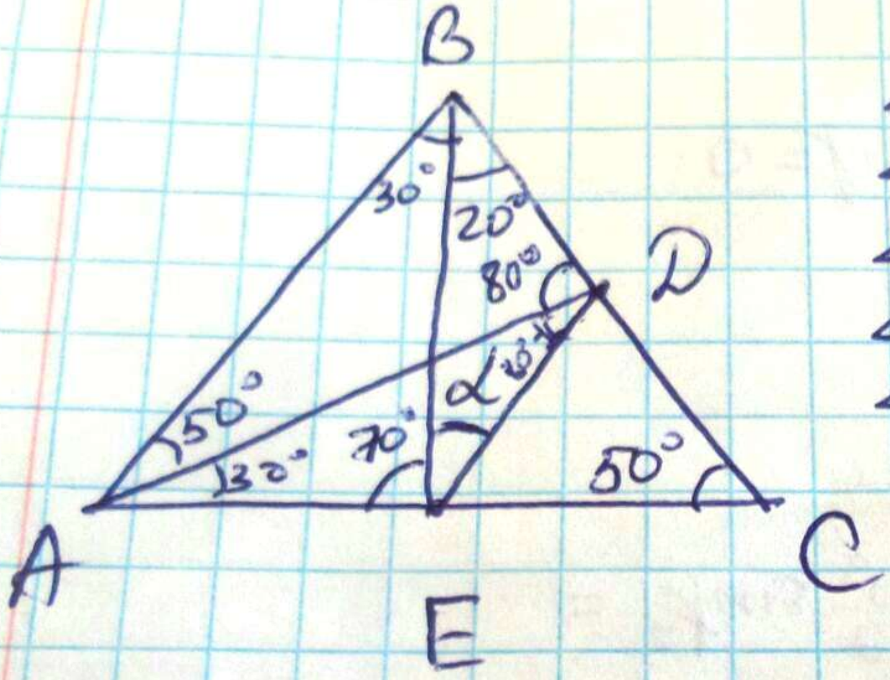
Bu qoldiqlar nomerlar bo'yicha teskari tartibda ekanligini ko'rib turibsiz. Buni to'g'ri tartibga keltirish uchun 11 dan shu qoldiqlarni ayiramiz va bundan quyidagi sonli qator hosil bo'ladi:

$1,2,3,\ldots,n,\ldots,10$ **(1)**

(1) qatordagi son **soxta tangalar joylashgan xaltaning nomer**ini bildiradi.

Hulosa. **"Yuqoridagi tartibda olingan 55 ta tangani tarozida bir marta o'lchashdan chiqqan natijani 11 ga bo'lib, hosil bo'lgan qoldiqni 11 dan ayrilganda kelib chiqadigan son – soxta tangalar joylashgan xalta nomerini ko'rsatadi".** @matematika

7. ABC uchburchakning BC va AC tomonlarida mos ravishda D va E nuqtalar shunday olinganki bunda burchak BAD=50^0, burchak ABE=30^0. Agar burchak ABC=ACB=50^0 bo`lsa, burchak BED ni toping?



$\angle BAD = 50°$
$\angle DAC = 30°$
$\angle AEB = 70°$
$\angle BED = \alpha$
$\angle ADE = 80° - \alpha$
$\angle ADB = 80°$
$\angle DBE = 20°$
$\angle ABE = 30°$

$$\frac{\sin 50°}{\sin 30°} \cdot \frac{\sin 70°}{\sin \alpha} \cdot \frac{\sin(80° - \alpha)}{\sin 80°} \cdot \frac{\sin 20°}{\sin 30°} = 1$$

$$\frac{\sin 20° \cdot \cos 20° \cdot \cos 40° \cdot \sin(80° - \alpha)}{\frac{1}{4} \cdot \sin 80° \cdot \sin \alpha} = 1$$

$$\sin(80° - \alpha) = \sin \alpha$$

$$\alpha = 40°$$

65. Agar $\begin{cases} x^2 + (y+a)^2 - 1 = 0 \\ x^2 + y = b \end{cases}$ tenglamalar sistemasi yagona yechimga ega bo'lsa, $a + b$ ni toping.

A) *1* B) *bir qiymatli aniqlanmaydi* C) *-1* D) *0*

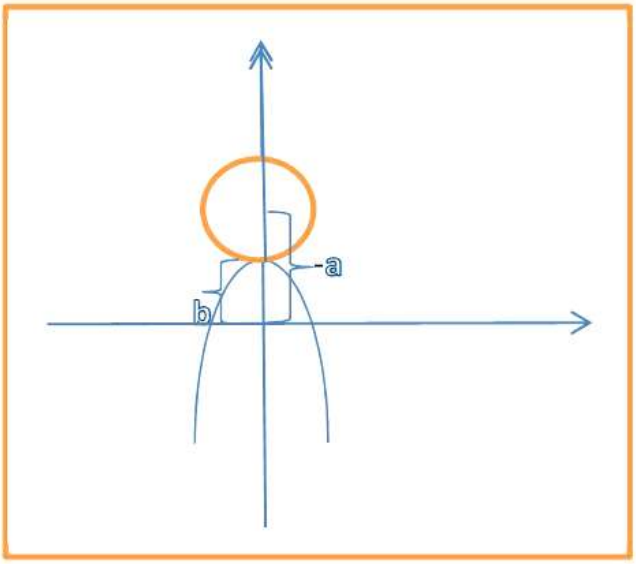
ECHILISHI:

$y = b - x^2$ shoxlari pasga qaragan parabola, uchi Oy o'qda yotadi;

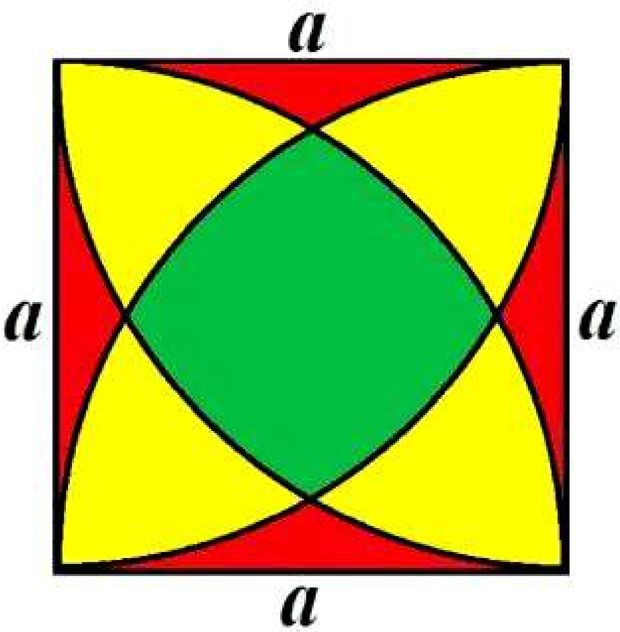
Birinchi tenglik esa aylana tenglamasi bo'lib radiusi 1 ga teng, markazi (0; -a) nuqtada;

Bu sistema bir echimga ega bo'lsa tengliklar ifodalaydigan chiziqlar bitta umumiy nuqtaga ega bo'ladi;

Parabola uchi Oy o'qida, aylana markazi Oy o'qida bo'lgani, uchun chiziqlarning umumiy nuqtasi Oy o'qida bo'ladi, aylana parabola tepasida joylashadi;



-a-b=1 bundan a+b=1 bo'ladi;



S=? S=? S=?

FORMULALAR:

$S_{kv} = a^2;$

$S = \left(1 - \sqrt{3} + \frac{\pi}{3}\right) \cdot a^2;$

$S = \left(2\sqrt{3} - 4 + \frac{\pi}{3}\right) \cdot a^2;$

$S = \left(4 - \sqrt{3} - \frac{2\pi}{3}\right) \cdot a^2$

M: Ketma-ket x, y, z natural sonlar uchun $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+\frac{y}{x}+\frac{x}{z}+\frac{z}{y}$ son butun son bo'lsa, y ni toping. A) 3 B) 2 C) 1 D) 4

Yechish: $x=n-1,\ y=n,\ z=n+1\ (n>1,\ n\in N)$ bo'lsin.

$$\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+\frac{y}{x}+\frac{x}{z}+\frac{z}{y} \Leftrightarrow \frac{n-1}{n}+\frac{n}{n+1}+\frac{n+1}{n-1}+\frac{n}{n-1}+\frac{n-1}{n+1}+\frac{n+1}{n}$$

Guruhlaymiz: $\left(\frac{n+1}{n-1}+\frac{n}{n-1}\right)+\left(\frac{n-1}{n}+\frac{n+1}{n}\right)+\left(\frac{n}{n+1}+\frac{n-1}{n+1}\right)\in Z$

Ifoda butun son bo'lishi uchun har bir guruh butun son bo'lishi kerak bo'ladi.

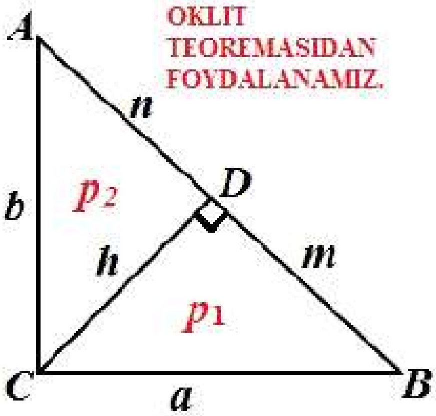
$\left(\frac{n+1}{n-1}+\frac{n}{n-1}\right)=\frac{2n+1}{n-1}\in N;\ \left(\frac{n-1}{n}+\frac{n+1}{n}\right)=2\in N;\ \left(\frac{n}{n+1}+\frac{n-1}{n+1}\right)=\frac{2n-1}{n+1}\in Z$

Uchingi guruhni qaraymiz: surati maxrajiga karrali bo'lsagina bu butun son bo'ladi. Deylik k karrali son bo'lsin. Unda

$$\frac{2n-1}{n+1}\Leftrightarrow 2n-1=(n+1)k \Leftrightarrow n=\frac{k+1}{2-k}>1 \Leftrightarrow \frac{2k-1}{k-2}<0 \Leftrightarrow 0{,}5<k<2 \Leftrightarrow k=1$$

$2n-1=(n+1)k \Leftrightarrow 2n-1=n+1 \Leftrightarrow n=2 \Leftrightarrow y=2$. ***Javob:*** **B**.

M: ABC to'g'ri burchakli uchburchakning gipotenuzasiga tushirilgan CD balandligi uchburchakni ikkita BCD va ACD uchburchaklarga ajratadi. Agar BCD uchburchakning perimetri p_1 ga, ACD uchburchakning yarim perimetri p_2 ga teng bo'lsa, ABC uchburchakning yarim perimetrini toping.



***Yechish*:** $\mathrm{p} = \dfrac{a+b+c}{2}$ – yarim perimetr $\Leftrightarrow a+b+c=2\mathrm{p}$.

$$\begin{cases} a+h+m=2\mathrm{p}_1 \\ b+h+n=2\mathrm{p}_2 \end{cases} \Leftrightarrow \begin{cases} a+\dfrac{ab}{c}+\dfrac{a^2}{c}=2\mathrm{p}_1 \\ b+\dfrac{ab}{c}+\dfrac{b^2}{c}=2\mathrm{p}_2 \end{cases} \Leftrightarrow \begin{cases} a\cdot(a+b+c)=2c\mathrm{p}_1 \\ b\cdot(a+b+c)=2c\mathrm{p}_2 \end{cases}$$

$$\Leftrightarrow \begin{cases} a\cdot 2\mathrm{p}=2c\mathrm{p}_1 \\ b\cdot 2\mathrm{p}=2c\mathrm{p}_2 \end{cases} \Leftrightarrow \begin{cases} \dfrac{a}{c}=\dfrac{\mathrm{p}_1}{\mathrm{p}} \\ \dfrac{b}{c}=\dfrac{\mathrm{p}_2}{\mathrm{p}} \end{cases} \Leftrightarrow \left(\dfrac{a}{c}\right)^2+\left(\dfrac{b}{c}\right)^2=1 \Leftrightarrow \left(\dfrac{\mathrm{p}_1}{\mathrm{p}}\right)^2+\left(\dfrac{\mathrm{p}_2}{\mathrm{p}}\right)^2=1 \Leftrightarrow \mathrm{p}=\sqrt{\mathrm{p}_1^2+\mathrm{p}_2^2}$$

Javob: $\mathrm{p}=\sqrt{\mathrm{p}_1^2+\mathrm{p}_2^2}$

@super_matematika

YIG'INDI FORMULALARI

I. $a_1 + a_2 + a_3 + ... + a_n = \sum_{k=1}^{n} a_k$. *@super_matematika*

Bunda $\sum \rightarrow$ yig'indi belgisi. $\boldsymbol{k} = 1$ – boshlang'ich qiymat. $\boldsymbol{n}$ – oxirgi qiymat, $\boldsymbol{a_k}$ – esa yig'indini har bir hadini hosil qiluvchi umumiy formula.

1 – misol. $1+2+3+...+15$ yig'indini qisqa shaklda yozing.
Yechish: Bu 1 dan 15 gacha ketma-ket yozilgan natural sonlar yig'indisini tashkil etmoqda.

Demak, $a_1 + a_2 + a_3 + ... + a_n = \sum_{k=1}^{n} a_k \Rightarrow$ formula bo'yicha $k = 1$, $n = 15$, $a_k = k$.

Javob: $1+2+3+...+15 = \sum_{k=1}^{15} k$

2 – misol. $4!+5!+6!+...+13!$ yig'indini qisqa shaklda yozing.

Yechish: Bu 4 dan 13 gacha bo'lgan sonlarning faktorialllarining yig'indisini tashkil etmoqda.

Demak, $a_1+a_2+a_3+...+a_n=\sum_{k=1}^{n} a_k \Rightarrow$ formula bo'yicha $k = 4$, $n = 13$, $a_k = k!$.

Javob: $4!+5!+6!+...+13!=\sum_{k=4}^{13} k!$ *@super_matematika*

3 – misol. $\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+...+\frac{60}{61}$ yig'indini qisqa shaklda yozing.

Yechish: Yig'indining har bir hadi $\frac{k}{k+1}$ formula bilan hosil qilinganini bilish qiyin emas. Bunda k ning boshlang'ich qiymati $k = 3$ bo'lmoqda, oxirgi qiymati esa $k = 60$ bo'lmoqda.

Demak, $\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+...+\frac{60}{61}=\sum_{k=3}^{60}\frac{k}{k+1}$ ekan.

Javob: $\sum_{k=3}^{60}\frac{k}{k+1}$

4 – misol. $5+10+17+...+82$ yig'indini qisqa shaklda yozing.

***Yechish*:** Yig'indining har biri hadi qandau hosil qilinmoqda?

Mana bunday: $5+10+17+...+82=(2^2+1)+(3^2+1)+(4^2+1)+...+(9^2+1)$

Demak, Umumiy formula k^2+1 ekan va bunda $k=2$, $n=9$.

***Javob*:** $5+10+17+...+82=\sum_{k=2}^{9}k^2+1$.

@super_matematika

5 – misol. $16+19+22+...+61$ yig'indini qisqa shaklda yozing.

***Yechish*:** Yig'indining har biri hadi qandau hosil qilinmoqda?

Mana bunday: $16+19+22+...+61=(3\cdot5+1)+(3\cdot6+1)+(3\cdot7+1)+...+(3\cdot20+1)$

Demak, Umumiy formula $3k+1$ ekan va bunda $k=5$, $n=20$.

***Javob*:** $16+19+22+...+61=\sum_{k=5}^{20}3k+1$.

KOMBINATORIKA

Matematika va uning tatbiqlarida ko'pincha turli ko'rinishidagi to'plamlar va uning qism to'plamlari bilan ish ko'rishga to'g'ri keladi: ularning har biri elementlari orasidagi bog'lanishni topish, ma'lum xossaga ega bo'lgan to'plamlar yoki ularning qism to'plamlarining sonini aniqlash va h.k.

To'plamlar va ularning qism to'plamlarini tuzish usullarini hamda miqdorlarini o'rganuvchi fan **kombinatorika** deyiladi.

Kombinatorika asosan, XVII – XIX asrlarda mustaqil fan sifatida yuzaga kelgan bo'lib, uning rivojiga B. Paskal, P. Ferma, G.Leybnis, Y. Bernulli, L. Eyler kabi olimlar katta hissa qo'shganlar.

Berilgan chekli to'plamning elementlaridan tuzilgan har bir tayin guruhlar **birlashma** deyiladi. Boshqacha qilib aytganda, har qanday narsalardan tuzilgan va bir-biridan shu narsalarning yo tartibi bilan, yo o'zi bilan farq qiluvchi guruhlar **birlashmalar** deyiladi.

Masalan, 1; 2; 3; 4; 5 raqamlardan 123; 213;125; 124; 2345; 3241 va hokazo to'plamlarni tuzib tekshiramiz: 123 va 213 lar bir-biridan raqamlarining tartibi bilan, 123 va 125 lar raqamlarining o'zi bilan farq qiladi. Birlashmalarni tashkil etuvchi narsalar **elementlar** deyiladi.

Elementlarni a, b, c, ... lotin alifbosining kichik harflarini bilan begilaymiz.

Agar to'plamda tartib munosabati kiritlgan bo'lsa, ya'ni to'plamning qaysi elementi qaysi elementdan keyin kelishi yoki qaysinisidan oldin kelishi aniqlangan bo'lsa, bunday to'plam tartiblangan deyiladi. Agar tartiblangan to'plamda elementlarning joylashish tartibi o'zgartirilsa, dastlabki to'plamdan farqli yangi to'plam hosil bo'ladi.

Biz birlashmalarning uchta asosiy xili bilan tanishamiz:

1. O'rinlashtirish; 2. O'rin almashtirish; 3. Guruhlashtirish.

Bular bilan quyida ba'tafsil tanishib chiqamiz:

1. (Takrorlanadigan) O'rinlashtirishlar

Ta'rif. m elementni n tadan o'rinlashtirish deb shunday birlashmalarga aytiladiki, ularning har birida berilgan m elementdan olingan n ta element bo'lib, ular bir-birlaridan yo elementlari bilan, yoki elementlarining tartibi bilan farq qiladi ($n \le m$ bo'lishi shart). m ta elementdan n tadan tuzilgan o'rinlashtirishlar soni A_m^n belgi bilan begilanadi. (A – fransuzcha "arrangement", ya'ni o'rinlashtirish degan so'zning bosh harfi hisoblanadi.)

O'rinlashitirshlar sonining formulasini chiqarish.

Masalan, uchta *a*, *b*, *c* elementlardan bittadan olib: *a*, *b*, *c*; ikkitadan olib: *ab*, *ac*, *bc*, *ba*, *ca*, *cb*; uchtadan olib: *abc*, *acb*, *bac*, *bca*, *cab*, *cba* birlashmalarni tuzamiz. Bulardan *ab*, *ac*, *bc*, *ca* va *cb* larni olib tekshiramiz.

Bunda *ab*, *ac*, *bc* lar elementlari bilan, *ab* va *ba*, *ac* va *ca*, *bc* va *cb* lar elementlarining tartibi bilan farq qiladi. Lekin har ikkovida ham elementlar soni bir xil. Bunday birlashmalar uch elementdan 2 tadan o'rinlashitirish deyiladi va A_3^2 shaklda yoziladi. Umuman bizga *m* ta: *a*, *b*, *c*, …, *k*, *e* elementlar berilgan bo'lsin.

Bittadan tuzilgani *m* ga teng: $A_m^1 = m$.

Ikkitadan tuzish uchun: *a* ning yoniga qolgan *b*, *c*, …, *k*, *e* (*m*–1) tasini, *b* ning yoniga qolgan *a*, *c*, …, *k*, *e* (*m*–1) tasini va hokazo qo'yib chiqamiz. U holda *m* elementdan 2 tadan o'rinlashtirish:

$$m \text{ ta} \begin{cases} ab, ac, ..., ak, ae & (m-1) \text{ ta o'rinlashtirish;} \\ ba, bc, ..., bk, be & (m-1) \text{ ta o'rinlashtirish;} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ ea, eb, ec, ..., ek & (m-1) \text{ ta o'rinlashtirish;} \end{cases}$$

Demak, *m* elementdan 2 tadan hamma o'rinlashtirishlar soni $A_m^2 = m(m-1)$ bo'ladi.

Endi uchtadan tuzish uchun tuzilgan 2 tadan o'rinlashtirishlardan har birining yoniga qolgan (*m*–2) ta elementni bittadan qo'yib chiqamiz.

$$m(m-1) \text{ ta} \begin{cases} abc, abd, ..., abk, abe & (m-2) \text{ ta o'rinlashtirish;} \\ acb, acd, ..., ack, ace & (m-2) \text{ ta o'rinlashtirish;} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ eka, ekb, ... & (m-2) \text{ ta o'rinlashtirish;} \end{cases}$$

Demak, m elementdan 3 tadan hamma o'rinlashtirishlar soni $A_m^2 = m(m-1)(m-2)$ bo'ladi. Shunga o'xshash: $A_m^4 = m(m-1)(m-2)(m-3)$ bo'ladi va hokazo. Umuman: $A_m^n = m(m-1)(m-2)(m-3)...(m-(n-1))$.

Demak,

$$\boxed{A_m^n = m(m-1)(m-2)(m-3)...(m-n+1).}$$

Bu *o'rinlashtirishlar sonini topish formulasi* deyiladi.

Bu formulani qisqacha ko'rnishda ham yozib olsak bo'ladi. Bundan oldingi jurnal sonida faktorial haqida ma'lumot berib o'tgan edik. Aynan shu mavzuga bog'lab yozib olsak, formula ishlash juda qulay bo'ladi.

Ya'ni

$$A_m^n = \frac{n!}{(n-m)!}$$

kabi ko'rinishda.

***Misol* 1.** $A_8^3 = 8 \cdot 7 \cdot 6 = 336$ yoki $A_8^3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 6 \cdot 7 \cdot 8 = 336.$

***Misol* 2.** Musobaqada 11 ta komanda ishtirok etadi. To'rtta turli medalni necha xil usul bilan ularga taqsimash mumkin?

Yechish: Masala o'rinlashtirishlar sonini aniqlash bilan yechiladi.

$$A_{11}^4 = 11 \cdot 10 \cdot 9 \cdot 8 = 7920 \text{ yoki } A_{11}^4 = \frac{11!}{(11-4)!} = \frac{8!}{7!} = 8 \cdot 9 \cdot 10 \cdot 11 = 7920.$$

***Misol* 3.** Bir sinfda 9 ta fandan dars bo'lib, har kuni 5 xil fandan dars o'tiladi. Bir kunlik dars necha xil usul bilan taqsimlanishi mumkin?

Yechish: Bu masala ham o'rinlashtirishlar sonini aniqlash bilan yechiladi.

$$A_9^5 = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15120 \text{ yoki } A_9^5 = \frac{9!}{(9-5)!} = \frac{9!}{4!} = 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 = 15120.$$

2. O'rin almashtirish

Ta'rif. Faqat elementlarining tartibi bilangina farq qilgan (ya'ni $n = m$) o'rinlashtirishlar o'rin almashtirishlar deyiladi.

m ta elementdan tuzilgan o'rin almashtirishlar soni P_m belgi bilan begilanadi. (P – fransuzcha "Permution", ya'ni o'rin almashtirish so'zining bosh harfidir.)

Formulasini keltirib chiqarish.

Ta'rifga ko'ra:

$$P_m = A_m^m = m(m-1)(m-2)...(m-(m-2))(m-(m-1)) = m \cdot (m-1)(m-2)...2 \cdot 1$$

Ya'ni, $P_m = 1 \cdot 2 \cdot 3...(m-2)(m-1)m$ yoki $P_m = m!$ ko'rinishda yozib olsak bo'ladi.

$$P_m = 1 \cdot 2 \cdot 3...(m-2)(m-1)m \text{ yoki } P_m = m!$$

Bu formula *o'rin almashtirishlar sonini topish formulasi* deyiladi.

Misol 1. $P_3 = 3! = 1\cdot2\cdot3 = 6$; $P_5 = 5! = 1\cdot2\cdot3\cdot4\cdot5 = 120$.

Misol 2. 6 ta stul qo'yilgan. Unga 6 kishini necha xil usul bilan o'tqazish mumkin.

Yechish: Bu masala o'rin almashtirishlar sonini aniqlash bilan yechiladi.

$P_6 = 6! = 1\cdot2\cdot3\cdot4\cdot5\cdot6 = 720$ xil usul bilan.

3. Guruhlashtirish

Ta'rif. m ta elementdan n tadan tuzilgan guruhlashtirish deb, m elementdan n tadan tuzilgan o'rinlashtirishlardan bir-biridan eng kamida bitta elementi bilan farq qiladigan o'rinlashtirishlarga aytiladi.

m elementdan n tadan guruhalshtirish soni C_m^n belgi bilan begilanadi (C – fransuzcha "Combinasion", ya'ni guruhlashtirish degan so'zning bosh harfidir.)

Masalan, to'rt element *a*, *b*, *c*, *d* dan 3 tadan tuzilgan *abc*, *abd*, *acd*, *bcd* guruhlarni olib tekshiramiz.Bu guruhlarning har birida mumkin bo'lgan barcha o'rin almashtirishlarni qilsak, to'rt elementdan 3 tadan mumkin bo'lgan barcha o'rinlashtirishlarni hosil qilamiz:

abc	*abd*	*acd*	*bcd*
acb	*adb*	*adc*	*bdc*
bac	*bad*	*cad*	*cbd*
bca	*bda*	*cda*	*cdb*
abc	*dab*	*dac*	*dbc*
cab	*dba*	*dca*	*dcb*

Bunday o'rinlashtirshlarning soni = 6 · 4 = 24. Bunda: 6 – o'rin almashtirishlar soni, 4 – guruhlar soni. 24 – o'rin alamshitirshlar soni.

Demak, $A_4^3 = C_4^3 P_3$. Shunga o'xshash: $A_8^5 = C_8^5 P_5$; $A_{15}^9 = C_{15}^9 P_9$ va hokazo.

Umuman: $A_m^n = C_m^n P_n$. Bundan:

$$C_m^n = \frac{A_m^n}{P_n} = \frac{m!}{(m-n)!\cdot n!} \text{ yoki } C_m^n = \frac{m(m-1)(m-2)...(m-n+1)}{1\cdot2\cdot3...n}$$

Bu formula *guruhlashtirishlar sonini topish formulasi* deyiladi.

Bunda $C_m^0 = 1$ deb qabul qilingan.

Misol 1. a) $C_8^3=\frac{8!}{(8-3)!\cdot 3!}=\frac{8!}{5!\cdot 3!}=\frac{6\cdot 7\cdot 8}{1\cdot 2\cdot 3}=56$;

b) $C_{20}^5=\frac{20!}{15!\cdot 5!}=\frac{16\cdot 17\cdot 18\cdot 19\cdot 20}{1\cdot 2\cdot 3\cdot 4\cdot 5}=15504$.

Misol 2. $C_{2x}^2=1$ tenglamani yeching.

Yechish: $C_{2x}^2=\frac{(2x)!}{(2x-2)!\cdot 2!}=\frac{(2x-1)2x}{1\cdot 2}=1\Leftrightarrow 2x^2-x-1=0\Leftrightarrow \begin{cases} x_1=1 \\ x_2=-\frac{1}{2} \end{cases}$.

Bundan $x_1=1$ berilgan tenglamaning ildizi bo'la oladi. $x_2=-0,5$ berilgan tenglamaning chet ildizi hisoblanadi (manfiy son ishlatib bo'lmaydi).

Misol 3. Tenglamani yeching. $C_{2x+8}^{2x+3}=13A_{2x+6}^3$

Yechish: $C_{2x+8}^{2x+3}=\frac{(2x+8)!}{(2x+3)!\cdot(2x+8-2x-3)!}=\frac{(2x+8)!}{(2x+3)!\cdot 5!}$.

$A_{2x+6}^3=\frac{(2x+6)!}{(2x+6-3)!}=\frac{(2x+6)!}{(2x+3)!}$

$\frac{(2x+8)!}{(2x+3)!\cdot 5!}=13\frac{(2x+6)!}{(2x+3)!}\Rightarrow\frac{(2x+7)(2x+8)}{120}=13\Rightarrow$

$=(2x+7)(2x+8)=13\cdot 120\Rightarrow x=16.$

Misol 4. Tenglamalar sistemasini yeching. $\begin{cases} A_{m-2}^n : A_{m-2}^{n-1}=3 \\ C_{m-2}^n : C_{m-2}^{n-1}=0,6 \end{cases}$

Yechish: 1 – tenglamalardagi har ifodani topib olamiz.

$A_{m-2}^n=\frac{(m-2)!}{(m-2-n)!}=\frac{(m-2)!}{(m-n-2)!}$, $A_{m-2}^{n-1}=\frac{(m-2)!}{(m-2-(n-1))!}=\frac{(m-2)!}{(m-n-1)!}$

$A_{m-2}^n : A_{m-2}^{n-1}=\frac{(m-2)!}{(m-n-2)!}:\frac{(m-2)!}{(m-n-1)!}=\frac{(m-2)!}{(m-n-2)!}\cdot\frac{(m-n-1)!}{(m-2)!}=$

$=\frac{(m-n-1)!}{(m-n-2)!}=m-n-1=3$. (1)

2 – tenglamalardagi har ifodani topib olamiz.

$$C_{m-2}^{n}=\frac{(m-2)!}{n!(m-2-n)!}=\frac{(m-2)!}{n!(m-n-2)!},$$

$$C_{m-2}^{n-1}=\frac{(m-2)!}{n!(m-2-(n-1))!}=\frac{(m-2)!}{(n-1)!(m-n-1)!}.$$

$$C_{m-2}^{n}:C_{m-2}^{n-1}=\frac{(m-2)!}{n!(m-n-2)!}\cdot\frac{(n-1)!(m-n-1)!}{(m-2)!}=\frac{(n-1)!(m-n-1)!}{n!(m-n-2)!}=$$

$$=\frac{(m-n-1)}{n}=0,6. \ (2)$$

Bu (1) va (2) yechimlarni bitta sistema qilib olamiz.

$$\begin{cases} m-n-1=3, \\ \frac{m-n-1}{n}=\frac{3}{5}. \end{cases} \Rightarrow \frac{m-n-1}{n}=\frac{3}{5} \Rightarrow \frac{3}{n}=\frac{3}{5} \Rightarrow n=5 \Rightarrow m-5-1=3,\ m=9.$$

Javob: $n=5,\ m=9$.

***Misol* 5**. Sinfdagi 24 o'quvchidan ko'rikda ishtirok etish uchun to'rt o'quvchini necha xil usul bilan tanlash mumkin?

Yechish: Ko'rik ishtirkchilarining tartibi ahamiyatga ega bo'lmagi uchun 24 elementli to'plamning 4 elementli qism to'plamlari soni nechtaligini topamiz:

$$C_{24}^{4}=\frac{24!}{(24-4)!4!}=\frac{21\cdot22\cdot23\cdot24}{1\cdot2\cdot3\cdot4}=10626.$$

Demak, to'rt o'quvchini 10626 usul bilan tanlash mumkin ekan.

C_m^k **ko'rinishdagi sonlarning xossalari.**

1. $C_m^n=C_m^{m-n}$. **2.** $C_m^n=C_{m-1}^{n-1}+C_{m-1}^{n}$; **3.** $C_m^0=C_m^m=1$.

Bu xossalarni isbotini o'zizga qoldiramiz.

Tengsizliklarni qo'shish va ayirish, ko'paytirish, bo'lish.

Qo'sh tengsizliklar bo'yicha.

M: $3 \le x \le 7$ va $-4 \le y \le 0$ bo'lsa, $x + y$ ni va $x - y$ ni toping.

Yechish:

$$+\begin{array}{l} -3 \le x \le 7 \\ -4 \le y \le 0 \\ \hline \end{array}$$
$$3+(-4) \le x + y \le 7+0$$
$$-1 \le x + y \le 7$$
$$+\begin{array}{l} -3 \le x \le 7 \\ -4 \le y \le 0 \\ \hline \end{array}$$
$$3-0 \le x - y \le 7-(-4)$$
$$3 \le x - y \le 11$$

M: $2 \le x < 4$ va $5 \le y < 8$ bo'lsa, $x + y$ ni va $x - y$ ni toping.

Yechish:

$$+\begin{array}{l} 2 \le x < 4 \\ 5 \le y < 8 \\ \hline 7 \le x + y < 12 \end{array} \qquad -\begin{array}{l} 2 \le x < 4 \\ 5 \le y < 8 \\ \hline -6 < x - y < -1 \end{array}$$

M: $1 < a < 2$ va $2 < b < 3$ bo'lsa, $2a - 3b$ qaysi oraliqda bo'ladi?

Yechish:

$$\times\begin{array}{l} 1 < a < 2 \\ 2 \quad 2 \quad 2 \\ \hline 2 < 2a < 4 \end{array} \qquad \times\begin{array}{l} 2 < b < 3 \\ 3 \quad 3 \quad 3 \\ \hline 6 < 3b < 9 \end{array}$$
$$-\begin{array}{l} 2 < 2a < 4 \\ 6 < 3b < 9 \\ \hline -7 < 2a - 3b < -2 \end{array}$$

M: $-2 < -x < 3$ bo'lsa, $2x$ qaysi oraliqda?

Yechish:

$$\times\begin{array}{l} -2 < -x < 3 \\ -2 \quad -2 \quad -2 \\ \hline 4 > 2x > -6 \end{array}$$
Javob: $-6 < 2x < 4$.

@super_matematika

M: $1 < \frac{x}{2} < 5$ va $-3 < -\frac{y}{3} < 1$ bo'lsa, $x + y$ ni toping.

Yechish:

$$\times\begin{array}{l} 1 < \frac{x}{2} < 5 \\ 2 \quad 2 \quad 2 \\ \hline 2 < x < 10 \end{array} \qquad \times\begin{array}{l} -3 < -\frac{y}{3} < 1 \\ -3 \quad -3 \quad -3 \\ \hline 9 > y > -3 \end{array}$$
$$+\begin{array}{l} 2 < x < 10 \\ -3 < y < 10 \\ \hline -1 < x + y < 19 \end{array}$$
Javob: $-1 < x + y < 19$.

M: $8 < x \le 10$ va $2 \le y < 4$ bo'lsa, x/y va $x \cdot y$ ni toping.

Yechish:

$$\div\begin{array}{l} 8 < x \le 10 \\ 2 \le y < 4 \\ \hline \frac{8}{4} < \frac{x}{y} \le \frac{10}{2} \end{array} \qquad \times\begin{array}{l} 8 < x \le 10 \\ 2 \le y < 4 \\ \hline 16 < xy < 40 \end{array}$$

Javob: $2 < \frac{x}{y} \le 5$; $16 < xy < 40$.

M: $2 < x < 3$ va $4 < y/x < 8$ bo'lsa, y qaysi oraliqda joylashadi?

Yechish:

$$\times\begin{array}{l} 2 < x < 3 \\ 4 < \frac{y}{x} < 8 \\ \hline 8 < y < 24 \end{array}$$
Javob: $8 < y < 24$.

M: $3 < a < 5$ bo'lsa, $\frac{3}{a}$ qaysi oraliqda?

Yechish:

$$\times\begin{array}{l} \frac{1}{3} > \frac{1}{a} > \frac{1}{5} \\ 3 \quad 3 \quad 3 \\ \hline 1 > \frac{3}{a} > \frac{3}{5} \end{array}$$
Javob: $\frac{3}{5} < \frac{3}{a} < 1$.

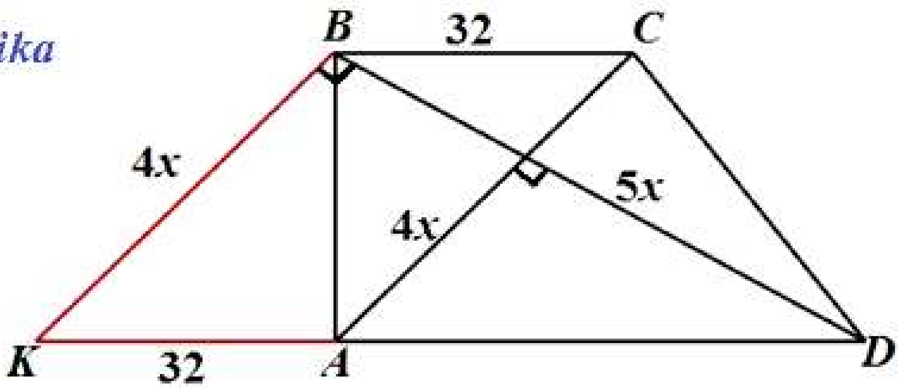
M: $2 < x < 7$ va $4 < y < 5$ bo'lsa, $x + x \cdot y = ?$

Yechish: $x + x \cdot y = x(1 + y)$

$$\times\begin{array}{l} 5 < 1 + y < 6 \\ 2 < x < 7 \\ \hline 10 < x(1 + y) < 42 \end{array}$$
Javob: $10 < x + x \cdot y < 42$

3. $ABCD$ to'g'ri burchak trapetsiyada $BC \parallel AD$, $\angle BAD = 90^o$, $BC = 32$ sm, $AC \perp BD$ va $AC : BD = 4:5$ kabi bo'lsa, AD katta asos uzunligini toping. *@super_matematika*

3. *Yechish*: Masala shartiga mos chizma chizib olamiz va AC diagonalga parallel bo'lgan KB chiziqni o'tkazdiramiz. U holda $BC = KA = 32$ va $\angle KBD = 90^o$ bo'ladi.



ABD to'g'ri burchakli uchburchak uchun quyidagilar o'rinli:

$$\begin{cases} KD = \sqrt{KB^2 + BD^2} \\ KB^2 = KA \cdot KD \end{cases} \Leftrightarrow \begin{cases} KD = \sqrt{(4x)^2 + (5x)^2} = \sqrt{41}x \\ 16x^2 = 32 \cdot KD \end{cases}$$

$$KD = \frac{x^2}{2} = \sqrt{41}x \Leftrightarrow x = 2\sqrt{41} \Rightarrow KD = \frac{x^2}{2} = \frac{4 \cdot 41}{2} = 82.$$

$AD = KD - KA \Leftrightarrow AD = 82 - 32 = 50.$ ***Javob*: 50 sm**

Расчет площади

сторона и высота

$$S = \frac{a \cdot h_a}{2}$$

Данные две стороны и угол между ними

$$S = \frac{a \cdot b \cdot \sin\gamma}{2}$$

известной радиус окружности вписанной или описанной, известны все углы или все стороны

$$S = r \cdot p$$

$$S = \frac{a \cdot b \cdot c}{4 \cdot R}$$

$$S = 2R^2 \sin\alpha \cdot \sin\beta \cdot \sin\gamma$$

формула Герона

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

1. Tenglamalar sistemasini yeching:

$$\begin{cases} \frac{xy}{9}+\frac{xz}{25}-\frac{yz}{49}=1+2\ln\frac{7x}{15} \\ \frac{xy}{9}+\frac{yz}{49}-\frac{xz}{25}=1+2\ln\frac{5y}{21} \\ \frac{yz}{49}+\frac{xz}{25}-\frac{xy}{9}=1+2\ln\frac{3z}{35} \end{cases}$$

1. *Yechish*: Sistema ikkta-ikkita qo'shib olib quyidagi tengliklarni hosil qilamiz: ($x > 0$, $y > 0$, $z > 0$)

$$\begin{cases} \frac{xy}{9}+\frac{xz}{25}-\frac{yz}{49}=1+2\ln\frac{7x}{15}, \\ \frac{xy}{9}+\frac{yz}{49}-\frac{xz}{25}=1+2\ln\frac{5y}{21}, \\ \frac{yz}{49}+\frac{xz}{25}-\frac{xy}{9}=1+2\ln\frac{3z}{35}, \end{cases} \Leftrightarrow \begin{cases} \frac{xy}{9}=1+\ln\frac{xy}{9}, \\ \frac{yz}{49}=1+\ln\frac{yz}{49}, \\ \frac{xz}{25}=1+\ln\frac{xz}{25}. \end{cases}$$

Bu tengliklar bajarilishi uchun $\begin{cases} \frac{xy}{9}=1, \\ \frac{yz}{49}=1, \\ \frac{xz}{25}=1. \end{cases}$ bo'lishi kerak bo'ladi.

Yuqoridagi tenglikladan $\times\begin{cases} xy=9, \\ yz=49, \\ xz=25. \end{cases} \Leftrightarrow (xyz)^2=105^2 \Rightarrow xyz=105.$

Endi noma'lumlarni topamiz:

$$x=\frac{xyz}{yz}=\frac{105}{49}=\frac{15}{7};\ y=\frac{xyz}{xz}=\frac{105}{25}=\frac{21}{5};\ z=\frac{105}{9}=\frac{35}{3}.$$

Javob: $(x, y, z)\in\left(\frac{15}{7};\frac{21}{5};\frac{35}{3}\right).$

3. Tekislikda katetlarining uzunliklari a ga teng bo'lgan teng yonli to'g'ri burchakli uchburchak yotibdi. Bu uchburchakni to'g'ri burchagi uchi atrofida $45°$ ga burish orqali boshqa teng yonli uchburchak hosil bo'ladi. Bu ikkita uchburchakning umumiy qismi bo'lgan to'rtburchakning yuzini toping.

3. *Yechish*: Misol shartiga mos chizma chizib olamiz va uchburchakning katetlarini a deb olamiz.

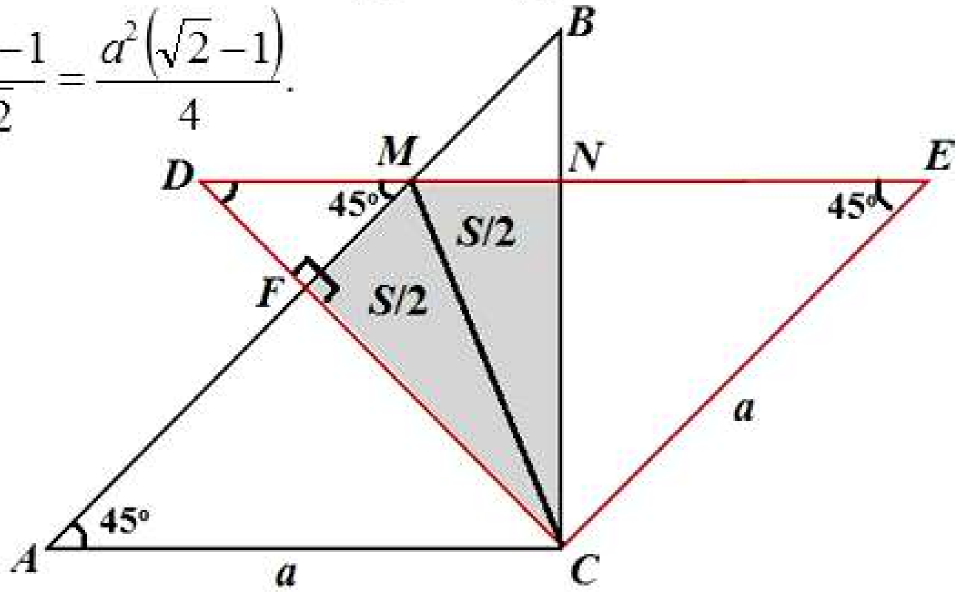
@super_matematika

$\Delta FCM = \Delta MCN$ bo'lganligidan $S_{CFMN} = 2S_{FMC}$,

$S_{FMC} = \frac{1}{2} MF \cdot FC$, $FC = \frac{a}{\sqrt{2}}$, $MF = DF = a - \frac{a}{\sqrt{2}} = a\frac{\sqrt{2}-1}{\sqrt{2}}$.

Demak, $S_{FMC} = \frac{1}{2} \cdot \frac{a}{\sqrt{2}} \cdot a\frac{\sqrt{2}-1}{\sqrt{2}} = \frac{a^2\left(\sqrt{2}-1\right)}{4}$.

$S_{CFMN} = 2S_{FMC} = \frac{a^2\left(\sqrt{2}-1\right)}{2}$.



Javob: $S_{CFMN} = \frac{a^2\left(\sqrt{2}-1\right)}{2}$.

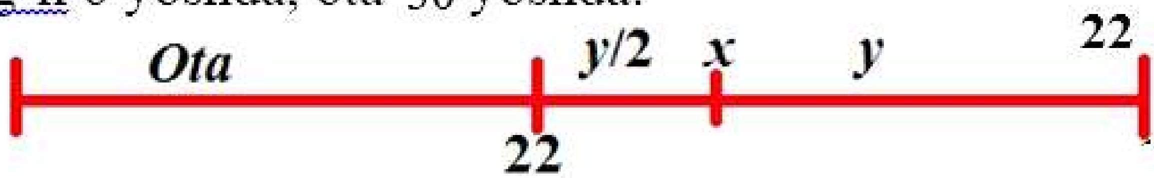
2. Ota o'g'lidan 5 marta katta. Ota institutni 22 yoshida tugatgan. Shundan beri o'g'il ham 22 yoshga yetishi uchun ketgan vaqtining yarmisicha vaqt o'tdi. Hozir o'g'il necha yoshda va ota necha yoshda?

2. *Yechish*: Masala shartidan ushbu tenglamalar sistemasini yozamiz:

$$\begin{cases} 22+\dfrac{y}{2}=5x \\ x+y=22 \end{cases}$$

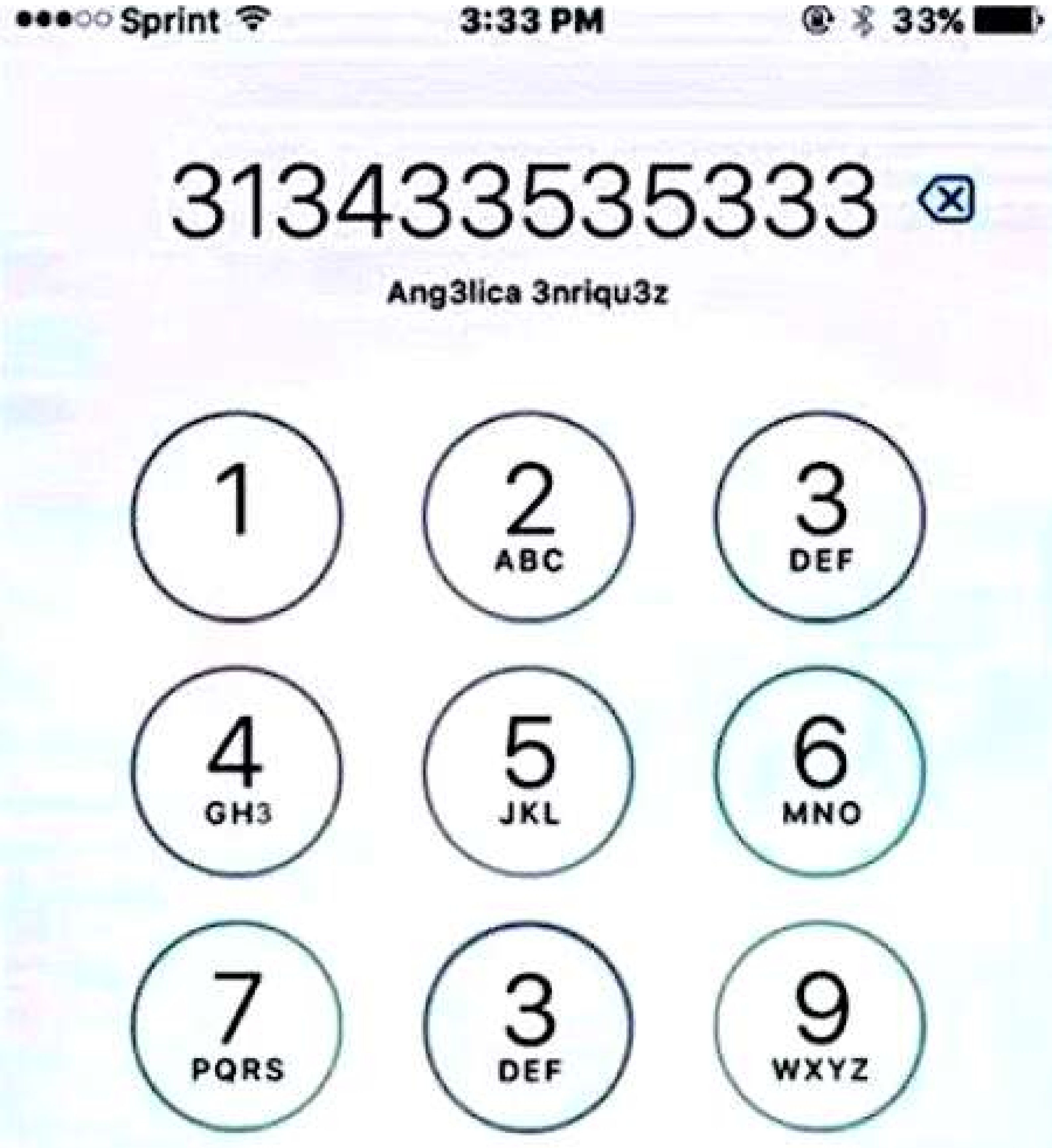
. Bundan $x=6$, $y=16$ ekanligi kelib chiqadi.

Demak, o'g'il 6 yoshda, ota 30 yoshda.



Javob: 6 yosh, 30 yosh.

@super_matematika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Misol. $\{x \mid x \in Z, -3 \le x < 4\}$ to'plamning bo'sh bo'lmagan qism to'plamlar sonini toping.

Yechish: To'plamlarga oid testlarda ko'pincha quyidagi masalalar ko'p uchraydi: to'plamning qism to'plamlari soni, to'plamni ikkita kesishmaydigan qism to'plamlarga ajratish usullari, to'plamning bo'sh bo'lmagan qism to'plamlarini sonini topish kabilar. Bu misol ham shular jumlasidandir. To'plam elementlari butun sonligi uchun $(x \in Z)$ uning sonini topib olamiz: $x \in \{-3; -2; -1; 0; 1; 2; 3\} \Leftrightarrow n = 8$ ta ekan.

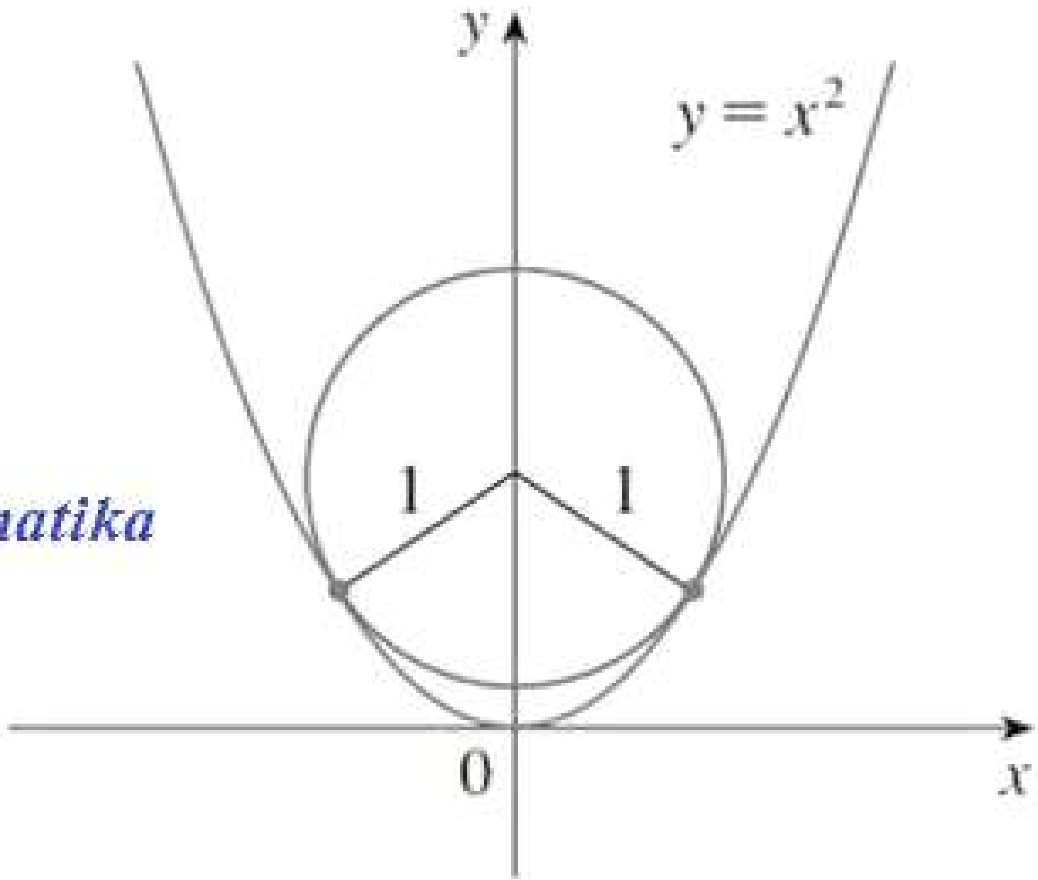
Agar A to'plamning elementlari soni n $(n \ge 1)$ ta bo'lsa, u holda, uning

1) qism – to'plamlari soni $\mathbf{2^n}$ ta bo'ladi.
2) bo'sh bo'lmagan qism to'plamlari soni $\mathbf{2^n - 1}$ ta bo'ladi.
3) ikkita kesishmaydigan qism – to'plamlarga ajratish usullari soni $\mathbf{2^{n-1}}$ ta bo'ladi.

U holda yuqrodagi 2- formulaga ko'ra $\{x \mid x \in Z, -3 \le x < 4\}$ to'plamning bo'sh bo'lmagan qism – to'plamlari soni $2^n - 1 = 2^8 - 1 = 127$ ta ekan.

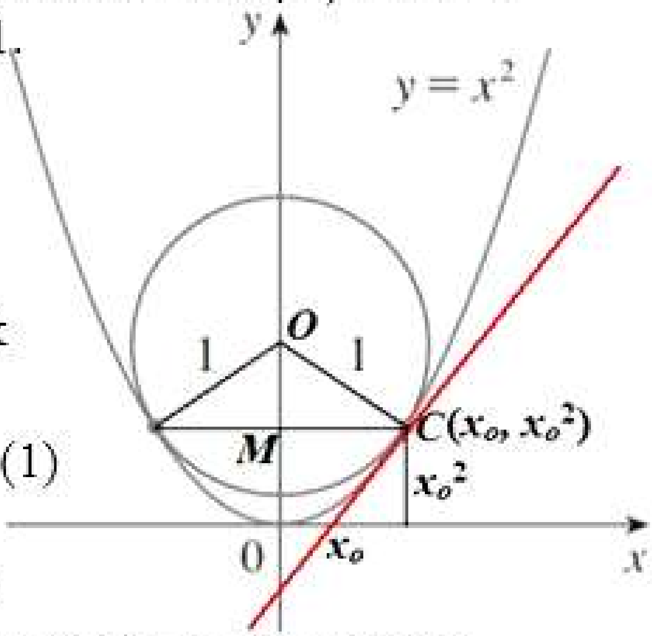
Javob: 127 ta. *@super_matematika*

2. Quyidagi chizmada $y = x^2$ parabola va unga ichki chizilgan, radiusi 1 ga teng bo'lgan aylana tasvirlangan. Aylananing markazining koordinatalarini toping.



2. *Yechish*: aylana markazi Oy o'qida bo'lgani uchun $O(0;\, b)$ bo'ladi.

ΔOMC da $OM = b - x_o^2$, $MC = x_o$, $OC = 1$.



C nuqtaning koordinatalarini topish uchun shu nuqtadan aylana va parabola uchun umumiy urinma o'tkazamiz. Bu urinma umumiy bo'lgani uchun har bir funksiya uchun tenglamani yozib burchak koeffitsientlarini tenglashtiramiz.

1) Parabola uchun: $y = x^2 \Rightarrow y'(x_o) = 2x_o$ (1)

2) Aylana tenglamasi uchun topishdan oldin uning tenglamasini tuzib olamiz:

$x^2 + (y-b)^2 = 1$. Endi ikkala tomondan y bo'yicha hosila olamiz:

$$2x + 2(y-b)y' = 0 \Leftrightarrow y' = \frac{x}{b-y} \Leftrightarrow y'(x_o) = \frac{x_o}{b - x_o^2} \qquad (2)$$

Endi (1) va (2) tengliklarni tenglashtiramiz: $2x_o = \dfrac{x_o}{b - x_o^2}$ (3)

(3) dan $b - x_o^2 = \dfrac{1}{2}$ kelib chiqadi. Buni ΔOMC uchun qo'llasak,

$$(b - x_o^2)^2 + x_o^2 = 1 \Leftrightarrow \left(\frac{1}{2}\right)^2 + x_o^2 = 1 \Leftrightarrow x_o^2 = \frac{3}{4}.$$

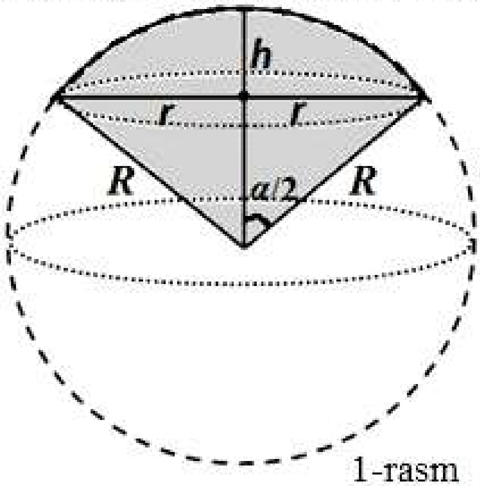
Demak, $b - x_o^2 = \dfrac{1}{2} \Leftrightarrow b - \dfrac{1}{2} = \dfrac{3}{4} \Leftrightarrow b = \dfrac{5}{4}$, $O\left(0;\, \dfrac{5}{4}\right)$ – ekan.

3. Sharning radiusi R bo'lib, shar sektorining markaziy burchagi α ga teng bo'lsa, shar sektori to'la sirtining yuzi hisoblansin.

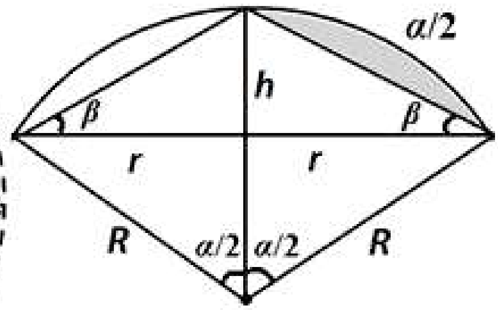
3. *Yechish*: Shar sektorini tasvirlab olamiz (1-rasm). Bunda R – shar radiusi, r – shar sektori (segmenti) radiusi, h – shar sektori (segmenti) balandligi. α – sektorning markaziy burchagi.

Sektorning yon tarafdan ko'rinishini tasvirlab olamiz (2-rasm).

$$\begin{cases} \sin\frac{\alpha}{2} = \frac{r}{R} \Leftrightarrow r = R\sin\frac{\alpha}{2} \\ \beta = \frac{\alpha}{4} \\ tg\beta = \frac{h}{r} \Leftrightarrow h = R\sin\frac{\alpha}{2}tg\frac{\alpha}{4} \end{cases}$$



1-rasm



2-rasm

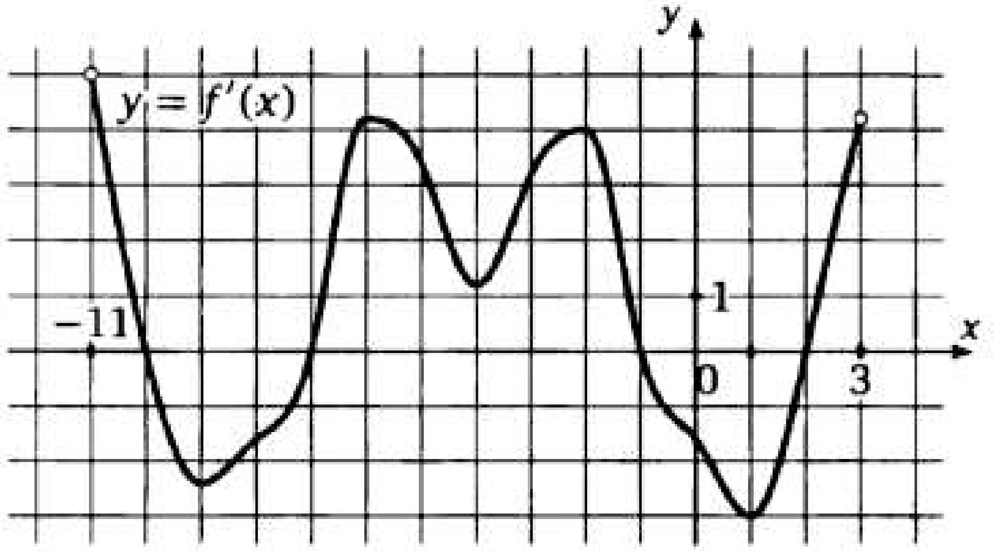
$S_{To'la} = \pi R(2h + r)$ – shar sektori to'la sirti.

$$S_{To'la} = \pi R\left(2R\sin\frac{\alpha}{2}tg\frac{\alpha}{4} + R\sin\frac{\alpha}{2}\right) = \pi R^2\sin\frac{\alpha}{2}\left(2tg\frac{\alpha}{4} + 1\right)$$

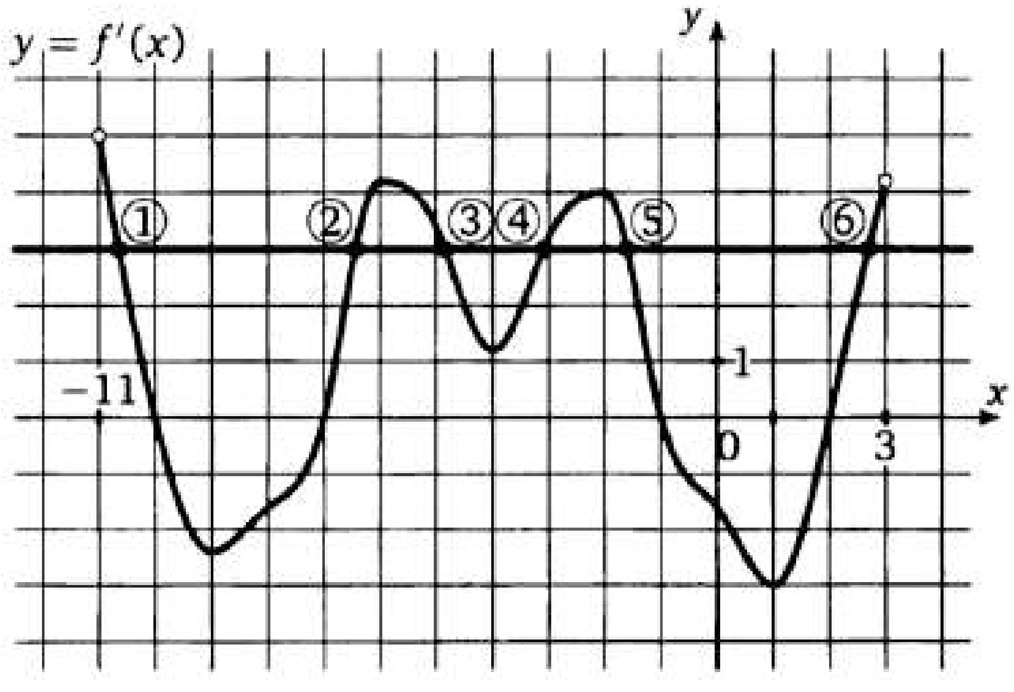
Javob: $S_{To'la} = \pi R^2\sin\frac{\alpha}{2}\left(2tg\frac{\alpha}{4} + 1\right)$.

Решение задачи 15 диагностической работы

15. На рисунке изображен график производной функции $f(x)$, определенной на интервале $(-11; 3)$. Найдите количество таких чисел x_i, что касательная к графику функции $f(x)$ в точке с абсциссой x_i параллельна прямой $y = 3x - 11$ или совпадает с ней.



Решение. Если касательная к графику функции $f(x)$ параллельна прямой $y = 3x - 11$ или совпадает с ней, то ее угловой коэффициент равен 3, а значит, нам нужно найти количество точек, в которых производная функции $f(x)$ равна 3. Для этого на графике производной проведем горизонтальную черту, соответствующую значению $y = 3$, и посчитаем количество точек графика производной, лежащих на этой линии. В нашем случае таких точек 6.

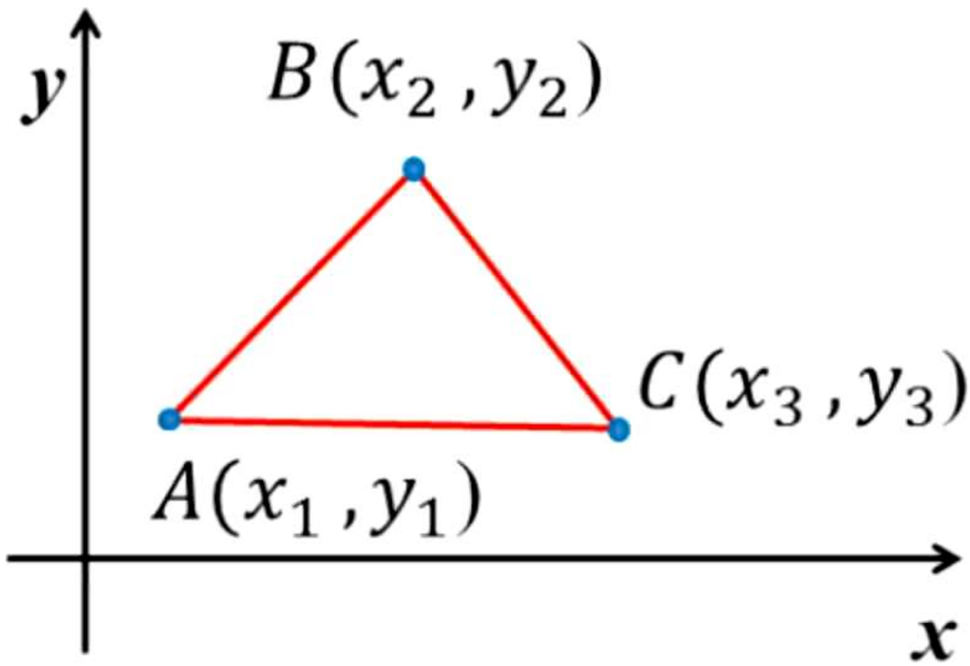


Ответ: 6.

$$A = \pm\frac{1}{2}\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}$$

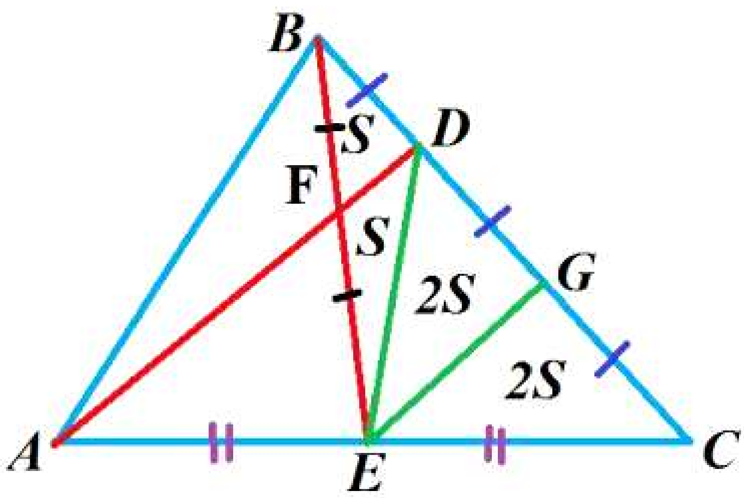
$$= \pm\frac{1}{2}[(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)]$$

$$= \pm\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$



@matematikaguruh va @riyoziyot

M: Berilgan ABC uchburchakda E nuqta - AC tomonning o'rtasi. BC tomonda D nuqta shunday olinganki, $2BD = DC$ tenglik o'rinli. AD va BE to'g'ri chiziqlar F nuqtada kesishgan. Agar $FDCE$ to'rtburchakning yuzi 20 ga teng bo'lsa, BDF uchburchak yuzasini toping.



Yechish: AD ga parallel EG to'g'ri chiziq o'tkazamiz. E nuqta AC ning o'rtasi bo'lgani uchun $DG = GC$ bo'ladi. Shartga ko'ra, $2BD = DC$ => $BD = DG = GC$ va $BF = FE$. $S_{BDF} = \boldsymbol{S}$ bo'lsin. U holda $S_{DFE} = \boldsymbol{S}$; $S_{CEG} = S_{DEG} = S_{BDE} = \boldsymbol{2S}$; $S_{FDCE} = \boldsymbol{5S} = 20 => \boldsymbol{S} = \boldsymbol{4}$.

Javob: 4. @matematika

@matematikaguruh va @riyoziyot

M:EGE $\frac{25^x-5\cdot5^{x+1}+26}{5^x-1}+\frac{25^x-7\cdot5^x+1}{5^x-7}\le 2\cdot5^x-24$ **tengsizlikni yeching.**

Yechish: $5^x=t$

$$\frac{t^2-25\cdot t+26}{t-1}+\frac{t^2-7\cdot t+1}{t-7}\le 2\cdot t-24$$

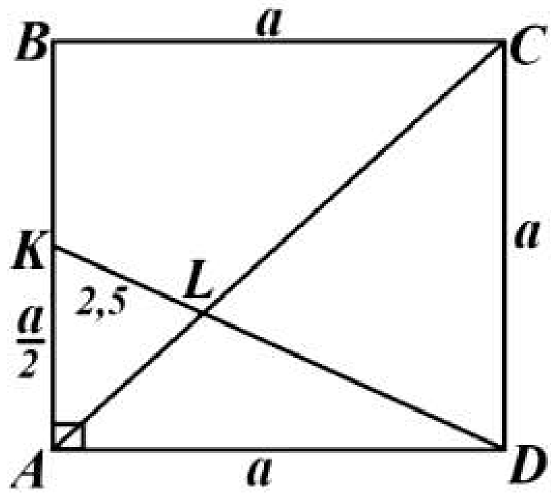
$$t-24+\frac{2}{t-1}+t+\frac{1}{t-7}\le 2\cdot t-24$$

$$\frac{2}{t-1}+\frac{1}{t-7}\le 0 \Rightarrow \frac{t-5}{(t-1)(t-7)}\le 0 \Rightarrow \left[\begin{array}{l} t<1 \\ 5\le t<7 \end{array}\right.$$

$$\Rightarrow \left[\begin{array}{l} 5^x<1 \\ 5\le 5^x<7 \end{array}\right. \Rightarrow \left[\begin{array}{l} x<0 \\ 1\le x<\log_5 7 \end{array}\right.$$

Javob: $(-\infty;0)\cup[1;\log_5 7)$ **@matematika**

@matematikaguruh va @riyoziyot



M:2015 ABCD kvadaratning D uchidan AB tomonning o'rtasiga DK kesma o'tkazilgan. AC diagonal uni L nuqtada kesib o'tib undan KL=2,5 sm uzunlikdagi kesma ajratdi. Kvadratning tomoni toping.

Yechish: AC bissektrisa bo'lganligi sababli ADK uchburchakda AL kesma DK ga o'tkazilgan bissektrisa bo'ladi. Demak,

$$\frac{AD}{AK} = \frac{DL}{KL} \Rightarrow \frac{a}{\frac{a}{2}} = \frac{DL}{2,5} \Rightarrow DL = 5 \Rightarrow DK = 7,5$$

$AK^2 + AD^2 = DK^2$ ekanligiga ko'ra,

$$\frac{a^2}{4} + a^2 = 7,5^2 \Rightarrow 5a^2 = 225 \Rightarrow a = 3\sqrt{5}$$

Javob: $3\sqrt{5}$.

@matematika

M:EGE. Tengsizlikni yeching: (@matematikaguruh va @riyoziyot)

$$\frac{1}{2}\log_{x-1}(x^2-8x+16)+\log_{4-x}(-x^2+5x-4)>3.$$

Yechish: Tengsizlikni quyidagi ko'rinishga keltiramiz:

$$\frac{1}{2}\log_{x-1}(x-4)^2+\log_{4-x}[(x-1)(4-x)]>3.$$

$4-x>0$ va $x-1>0$ ekanligini e'tiborga olgan holda tengsizlik quyidagi ko'rinishga o'tadi:

$\log_{x-1}(4-x)+\log_{4-x}(x-1)+\log_{4-x}(4-x)>3$ bundan esa $\log_{x-1}(4-x)+\log_{4-x}(x-1)>2$ hosil bo'ladi. $\log_{x-1}(4-x)=y$ belgilash kiritamiz: $y+\frac{1}{y}>2 \Rightarrow \frac{(y-1)^2}{y}>0 \Rightarrow \left[\begin{array}{l}0<y<1\\ y>1\end{array}\right. \Rightarrow$

$\left[\begin{array}{l}0<\log_{x-1}(4-\mathrm{x})<1\\ \log_{x-1}(4-\mathrm{x})>1.\end{array}\right.$ Bu tengsizliklarni quyidagi tengsizliklar sistemalariga ajratamiz:

$$\left[\begin{array}{l}\begin{cases}0<x-1<1\\ 1>4-x>x-1\\ 4-x>0\end{cases}\\ \begin{cases}0<x-1<1\\ 1>4-x>x-1\\ 4-x>0\end{cases}\\ \begin{cases}0<x-1<1\\ 4-x<x-1\\ 4-x>0\end{cases}\\ \begin{cases}x-1>1\\ 4-x>x-1\\ 4-x>0\end{cases}\end{array}\right. \Rightarrow \left[\begin{array}{l}\begin{cases}1<x<2\\ x>3\\ x<2,5\\ x<4\end{cases}\\ \begin{cases}x>2\\ x<3\\ x>2,5\\ x<4\end{cases}\\ \begin{cases}1<x<2\\ x>2,5\\ x<4\end{cases}\\ \begin{cases}x>2\\ x<2,5\\ x>4\end{cases}\end{array}\right. \Rightarrow \left[\begin{array}{l}\emptyset\\ 2,5<x<3\\ \emptyset\\ 2<x<2,5\end{array}\right. \Rightarrow$$

$$\left[\begin{array}{l}2,5<x<3\\ 2<x<2,5\end{array}\right. \Rightarrow (2;2,5)\cup(2,5;3).$$

Javob: $(2;2,5)\cup(2,5;3)$. **@matematika**

@matematikaguruh va @riyoziyot

M: $\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+\cdots+\frac{1}{1+2+3+\cdots+10}$ **yig'indining qiymatini hisoblang.**

Yechish: Kasrlarning mahrajini arifmetik progressiya yordamida hisoblab, yig'indining qiymatini aniqlaymiz:

$$\frac{1}{\frac{1+2}{2}\cdot 2}+\frac{1}{\frac{1+3}{2}\cdot 3}+\frac{1}{\frac{1+4}{2}\cdot 4}+\cdots+\frac{1}{\frac{1+10}{2}\cdot 10}=$$

$$\frac{2}{2\cdot 3}+\frac{2}{3\cdot 4}+\frac{2}{4\cdot 5}+\cdots+\frac{2}{10\cdot 11}=$$

$$\frac{2}{1}\cdot\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\cdots+\frac{1}{10}-\frac{1}{11}\right)=$$

$$2\cdot\left(\frac{1}{2}-\frac{1}{11}\right)=\frac{9}{11}.$$

Javob: $\frac{9}{11}$. @matematika

@matematikaguruh va @riyoziyot

M: 5 ta ruchka, 3 ta qalam va 4 ta flomaster bor. Ikkita xildagi predmetlardan tashkil topgan nechta to'plamni tuzish mumkin? A) 47 B) 42 C) 60 D) 24

Yechish: Barcha predmetlar (5+3+4=12 ta)uchun 2 tadan guruhlashlar soni $C_{12}^2 = \frac{12!}{2!\cdot 10!} = \mathbf{66}$ ta. Tarkibida bir xil 2 ta predmetlar qatnashgan to'plamlar sonini aniqlaymiz:

ruchka uchun $C_5^2 = \frac{5!}{2!\cdot 3!} = 10$ **ta, qalam uchun** $C_3^2 = \frac{3!}{2!\cdot 1!} = 3$ **ta** va **flomaster uchun** $C_4^2 = \frac{4!}{2!\cdot 2!} = 6$ **ta.**

Demak, ikkita xildagi predmetlardan tashkil topgan to'plamlar soni $\mathbf{66 - 10 - 3 - 6 = 47}$ ta ekan.

Javob: 47 A. **@matematika**

M: Agar $xy + yz + zx = 16$ bo'lsa, $x^2 + y^2 + z^2$ va $(x + y + z)^2$ ifodalarning eng kichik qiymatini toping.

Yechish: $x^2 + y^2 + z^2$ ifodaning eng kichik qiymatini topishning umumiy qoidasini keltirib chiqaramiz: Istalgan a va b sonlari uchun $a^2 + b^2 \geq 2ab$ tengsizlik har doim o'rinli ekanligidan foydalanib,

$x^2 + y^2 \geq 2xy;\ y^2 + z^2 \geq 2yz$ va $z^2 + x^2 \geq 2zx$ (1)

tengsizliklarni yozishimiz mumkin. Bu (1) tengsizliklarni qo'shib, $x^2 + y^2 + z^2 \geq xy + yz + zx$ natijani olamiz. Demak, $x^2 + y^2 + z^2$ ifodaning eng kichik qiymati $x^2 + y^2 + z^2 = 16$ ga teng bo'ladi.

Endi $(x + y + z)^2$ ifodani eng kichik qiymatini topishning umumiy qoidasini keltirib chiqaramiz:

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx) \geq$
$xy + yz + zx + 2(xy + yz + zx) = 3(xy + yz + zx)$.

Demak, $(x + y + z)^2 \geq 3(xy + yz + zx)$ ekan. Bundan esa $(x + y + z)^2$ ifodaning eng kichik qiymati $(x + y + z)^2 = 3(xy + yz + zx) = 3 \cdot 16 = 48$ ga teng bo'ladi.

Javob: 16 va 48. @matematika

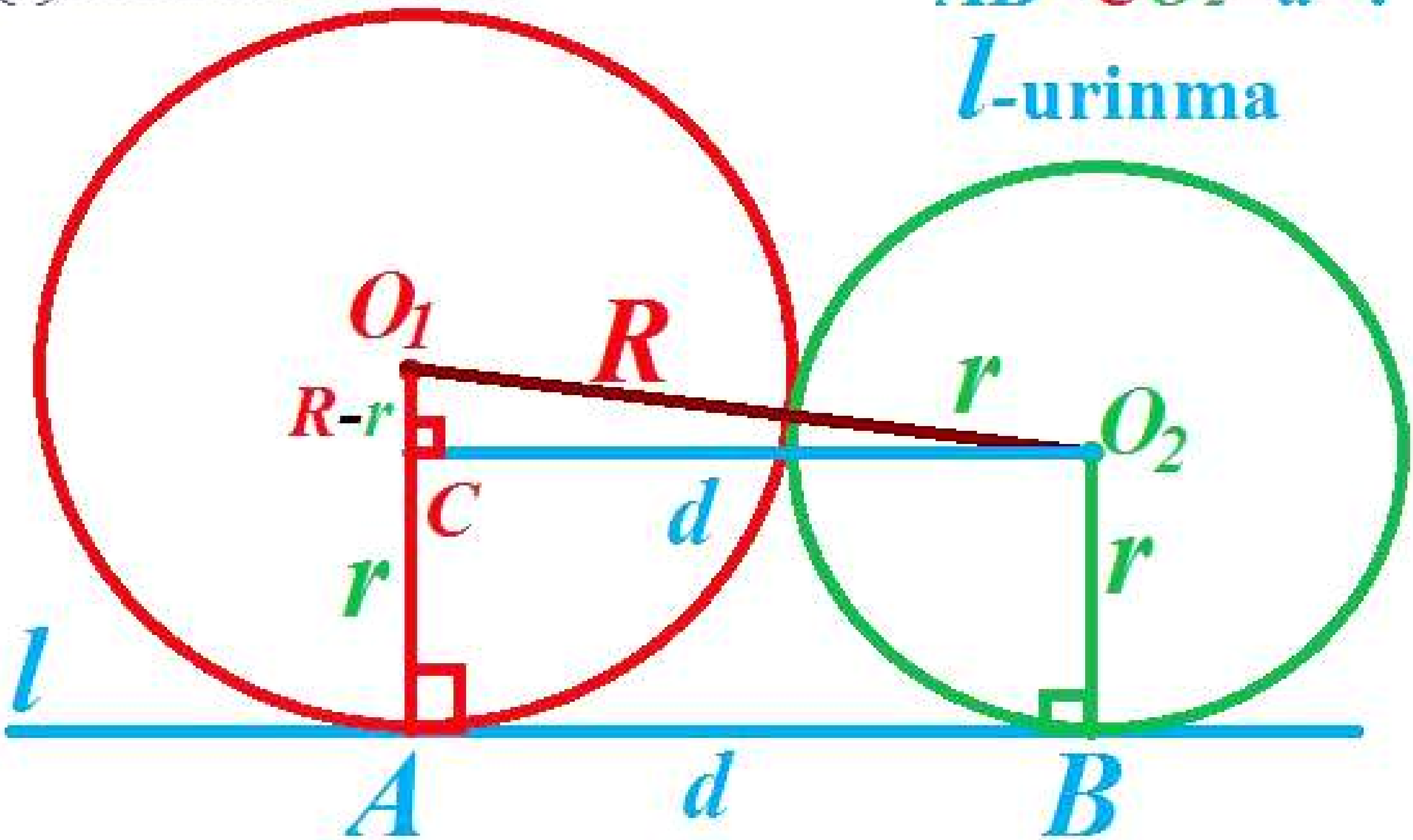
Astronomiyaning ba'zi kattaliklari

Yerning radiusi	$6{,}37 \cdot 10^6$ m
Yerning massasi	$5{,}98 \cdot 10^{24}$ kg
Quyoshning radiusi	$6{,}95 \cdot 10^8$ m
Quyoshning massasi	$1{,}98 \cdot 10^{30}$ kg
Oyning radiusi	$1{,}74 \cdot 10^6$ m
Oyning massasi	$7{,}33 \cdot 10^{22}$ kg
Yerning markazidan Quyoshning markazigacha bo'lgan masofa	$1{,}49 \cdot 10^{11}$ m
Yerning markazidan Oyning markazigacha bo'lgan masofa	$3{,}84 \cdot 10^8$ m
Oyning Yer atrofida aylanish davri	27,3 kech.kun.= $2{,}36 : 10^6$ s

$O_1A=R$; $O_2B=CA=r$; $O_1O_2=R+r$; $O_1C=R-r$;

$AB=CO_2=d=?$

l-urinma



$$d^2=(R+r)^2-(R-r)^2 \Rightarrow d=2\cdot\sqrt{Rr}$$

@matematikaguruh va @riyoziyot

M: $6x + 11y$ va $a(x + 7y)$ sonlari 31 ga bo'linsa, $(x, y \in Z)$, a ning eng kichik natural qiymatini toping.

Yechish: $\frac{6x+11y}{31} = n, (n \in Z) => x = \frac{31n-11y}{6}$.

$\frac{a(x+7y)}{31} = a \cdot \left(\frac{31n-11y}{6} + 7y\right) \cdot \frac{1}{31} = \frac{a}{6} \cdot (31n + 31) \cdot \frac{1}{31} = \frac{a}{6} \cdot (n + 1) => a = 6.$

Javob: $a = 6$. @matematika

@matematikaguruh va @riyoziyot

M: $\frac{3^2+1}{3^2-1}+\frac{5^2+1}{7^2-1}+\frac{7^2+1}{7^2-1}+\cdots+\frac{(2n+1)^2+1}{(2n-1)^2-1}$ **yig'indini hisoblang.** $(n \in N)$

Yechish: $\frac{3^2+1}{3^2-1}+\frac{5^2+1}{5^2-1}+\frac{7^2+1}{7^2-1}+\cdots+\frac{(2n+1)^2+1}{(2n+1)^2-1}=$

$$\left(1+\frac{2}{3^2-1}\right)+\left(1+\frac{2}{5^2-1}\right)+\left(1+\frac{2}{7^2-1}\right)+\cdots+\left(1+\frac{2}{(2n+1)^2-1}\right)=$$

$$n+\frac{2}{2\cdot 4}+\frac{2}{4\cdot 6}+\frac{2}{6\cdot 8}+\cdots+\frac{2}{(2n)\cdot(2n+2)}=$$

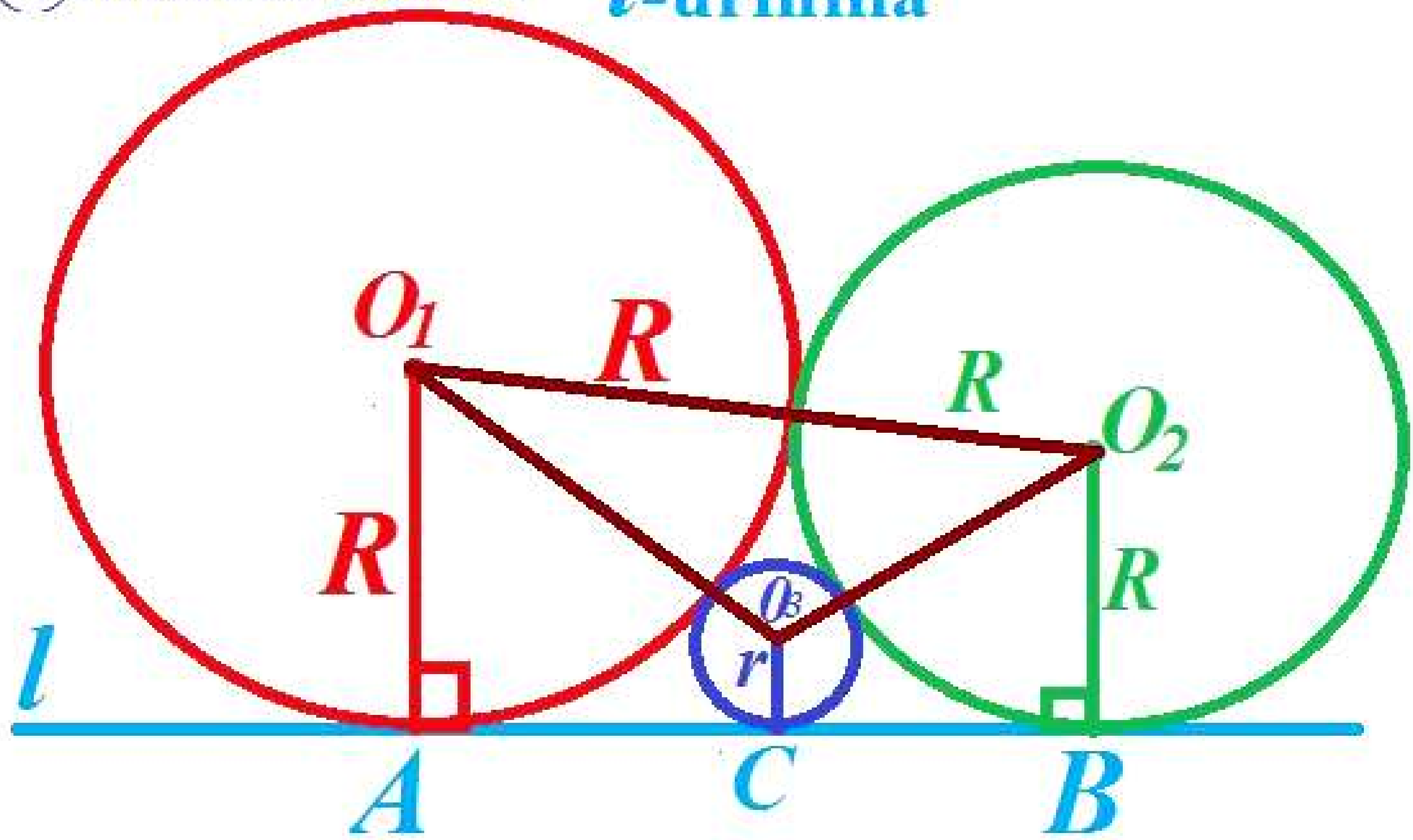
$$n+\frac{1}{2}-\frac{1}{4}+\frac{1}{4}-\frac{1}{6}+\frac{1}{6}-\frac{1}{8}+\cdots+\frac{1}{2n}-\frac{1}{2n+2}=$$

$$n+\frac{1}{2}-\frac{1}{2n+2}=n+\frac{n}{2n+2}.$$

Javob: $n+\frac{n}{2n+2}$. **@matematika**

$O_1A=R;\ O_2B=R;\ O_1O_2=R+r;\ O_3C=r;\ r=?$

@matematika l-urinma



$AC=2\cdot\sqrt{Rr};\ CB=2\cdot\sqrt{Rr};\ AB=2\cdot\sqrt{RR}$

$AC+CB=AB \Rightarrow 2\cdot\sqrt{Rr}+2\cdot\sqrt{Rr}=2\cdot\sqrt{RR} \Rightarrow$

$\sqrt{r}\cdot(\sqrt{R}+\sqrt{R})=\sqrt{RR}$

$$\frac{\sqrt{R}+\sqrt{R}}{\sqrt{RR}}=\frac{1}{\sqrt{r}} \Rightarrow \frac{1}{\sqrt{r}}=\frac{1}{\sqrt{R}}+\frac{1}{\sqrt{R}}$$

M. Hisoblang: $\lim\limits_{x\to\pi}\frac{1}{x-\pi}\int_{\pi}^{x}\frac{cost}{1-cost}dt.$

Y: $\frac{cosx}{1-cosx}$ funksiyaning boshlang'ich funksiyasi $F(x)$ bo'lsin. $\int_{\pi}^{x}\frac{cost}{1-cost}dt = F(t)\Big|_{\pi}^{x} = F(x)-F(\pi);$

$\left(\int_{\pi}^{x}\frac{cost}{1-cost}dt\right)' = \left(F(x)-F(\pi)\right)' = F'(x)-F'(\pi) = \frac{cosx}{1-cosx}.$

$\lim\limits_{x\to\pi}\int_{\pi}^{x}\frac{cost}{1-cost}dt = \lim\limits_{x\to\pi}\left(F(x)-F(\pi)\right) = F(\pi)-F(\pi) = 0.$

$\lim\limits_{x\to\pi}\frac{1}{x-\pi}\int_{\pi}^{x}\frac{cost}{1-cost}dt = \lim\limits_{x\to\pi}\frac{\int_{\pi}^{x}\frac{cost}{1-cost}dt}{x-\pi}$ bundan ko'rinib turibdiki, kasr $\frac{0}{0}$ aniqmaslikni ifodalaydi va **Lopital** qoidasiga ko'ra,

$\lim\limits_{x\to\pi}\frac{\left(\int_{\pi}^{x}\frac{cost}{1-cost}dt\right)'}{(x-\pi)'} = \lim\limits_{x\to\pi}\frac{\frac{cosx}{1-cosx}}{1} = \lim\limits_{x\to\pi}\frac{cosx}{1-cosx} = \frac{cos\pi}{1-cos\pi} = -\frac{1}{2}.$

Javob: $-\frac{1}{2}.$

@matematikaguruh va @riyoziyot

ESLAB QOLING!!!

Hulosa. $y = f(x)$ **funksiya grafigini** Ox **o'qidan** n **marta,** Oy **o'qidan** k **marta cho'zish(qisish) uchun shu funksiyada y ni o'rniga** $\frac{y}{n}$ (ny) **va** x **ning o'rniga esa** $\frac{x}{k}$ (kx) **qo'yish kerak!**

$$\frac{y}{n} = f\left(\frac{x}{k}\right) \;=>\; y = n \cdot f\left(\frac{x}{k}\right)$$

$$\left(ny = f(kx) \;=>\; y = \frac{1}{n} \cdot f(kx)\right).$$

@matematika

@matematikaguruh va @riyoziyot

ESLAB QOLING!!!

! $y = f(x)$ funksiya grafigini Ox **o'qidan** n **marta,** Oy **o'qidan** k **marta cho'zilsa(qisilsa),** $y = n \cdot f\left(\frac{x}{k}\right)$ $\left(y = \frac{1}{n} \cdot f(kx)\right)$ funksiya grafigi hosil bo'ladi.

! $y = f(x)$ funksiya grafigini Ox **o'qidan** n **marta cho'zilsa(qisilsa),** Oy **o'qidan esa** k **marta qisilsa(cho'zilsa),** $y = n \cdot f(kx)$ $\left(y = \frac{1}{n} \cdot f\left(\frac{x}{k}\right)\right)$ funksiya grafigi hosil bo'ladi.

Bu yerda $n \neq 0$; $k \neq 0$. *@matematika*

@matematikaguruh va @riyoziyot

M. **Agar a, b, c, d sonlar uchun $a^2 + b^2 + c^2 + d^2 = 4$ tenglik o'rinli bo'lsa, u holda $(2 + a)(2 + b) \geq cd$ bo'lishini isbotlang.** **@matematika**

Yechish: $(2 + a)(2 + b) = 4 + 2a + 2b + ab =$

$\frac{1}{2}(4 + 4 + 4a + 4b + 2ab) =$

$\frac{1}{2}(a^2 + b^2 + c^2 + d^2 + 4 + 4a + 4b + 2ab) =$

$\frac{1}{2}((a + b + 2)^2 + c^2 + d^2) \geq \frac{c^2+d^2}{2} \geq cd.$

@matematikaguruh va @riyoziyot

M. $\begin{cases} x+\frac{3x-y}{x^2+y^2}=3 \\ y-\frac{x+3y}{x^2+y^2}=0 \end{cases}$ **(1) tenglamalar sistemasini yeching.**

Y: $x = ay$ **(2)** bo'lsin. U holda $\begin{cases} ay+\frac{3a-1}{y(a^2+1)}=3 \\ y-\frac{a+3}{y(a^2+1)}=0 \end{cases}$ **(3)** bo'ladi.

(2) tenglamadan $y^2=\frac{a+3}{a^2+1}$ **(4)**. (1) tenglamadan

$y^2a(a^2+1)+3a-1=3y(a^2+1) \Rightarrow$

$a^2+3a+3a-1=3y\cdot\frac{a+3}{y^2} \Rightarrow y=\frac{3(a+3)}{a^2+6a-1}$ **(5).**

(5) ni (4) ga qo'yamiz: $\frac{a+3}{a^2+1}=\left(\frac{3(a+3)}{a^2+6a-1}\right)^2$. Bundan

$(a+3)\left((a^2+6a-1)^2-9(a+3)(a^2+1)\right)=0$

$(a+3)(a+1)(a-2)(a^2+4a+13)=0 \Rightarrow$

$a_1=-3; a_2=-1; a_3=2$ **(6)**.

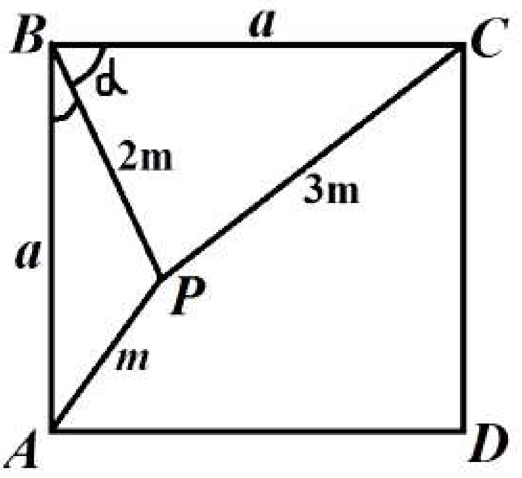
(6) ni (5) ga qo'yamiz: $y_1=0;\ y_2=-1; y_3=1$ **(7)**

(6) va (7) ni (2) ga qo'yamiz: $x_1=0;\ x_2=1;\ x_3=2$ **(8)**

(7) va (8) yechimlar juftligidan faqat $(1;\ -1)$ va $(2;1)$ yechimlar (1) ni qanoatlantiradi.

Javob: $(1;\ -1)$ **va** $(2;1)$ ***@matematika***

Masala: **@matematika** **P nuqta $ABCD$ kvadratning ichida joylashgan $AP:BP:CP = 1:2:3$ munosabat o'rinli bo'lsa, $\sin(\angle CBP)$ ni toping.**



$A)\ \frac{\sqrt{2}+1}{\sqrt{5+2\sqrt{2}}}$ $B)\ \frac{2\sqrt{2}+1}{\sqrt{10+4\sqrt{2}}}$

$C)\ \frac{\sqrt{2}}{\sqrt{5+2\sqrt{2}}}$ $D)\ \frac{1}{\sqrt{10+4\sqrt{2}}}$

Yechish: Kvadratning tomoni uzunligi a ga va $\angle CBP = \alpha$ ga teng bo'lsin. U holda shartga ko'ra $AP = m;\ BP = 2m$ va $PC = 3m$ va $\angle ABP = 90° - \alpha$ bo'ladi. ABP va CBP uchburchaklarda kosinuslar teoremasiga ko'ra,

$$\begin{cases} m^2 = a^2 + 4m^2 - 4am\cos(90° - \alpha) \\ 9m^2 = a^2 + 4m^2 - 4am\cos\alpha \end{cases} \Rightarrow \begin{cases} 4am\sin\alpha = a^2 + 3m^2 \\ 4am\cos\alpha = a^2 - 5m^2 \end{cases}$$

bu tenglamalarni ayirib $m = \frac{a}{2} \cdot (\sin\alpha - \cos\alpha)$ ni topamiz va buni sistemaning birinchi tenglamasiga qo'yib,

$$2a^2 \cdot \sin\alpha \cdot (\sin\alpha - \cos\alpha) = a^2 + \frac{3a^2}{4} \cdot (\sin\alpha - \cos\alpha)^2.$$

Buni soddalashtirsak, quyidagi ko'rinishga keladi:

$\sin^2\alpha - 2\sin\alpha\cos\alpha - 7\cos^2\alpha = 0$ va buni $\cos^2\alpha$ bo'lib,

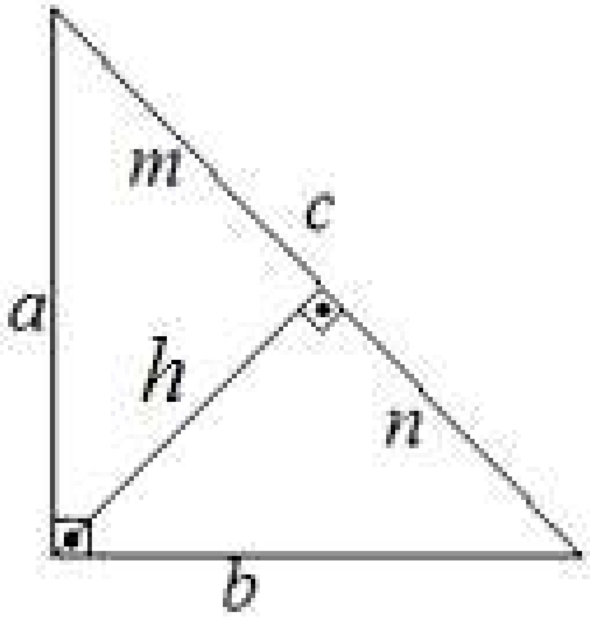
$\mathrm{tg}^2\,\alpha - 2\,\mathrm{tg}\,\alpha - 7 = 0 \Rightarrow \mathrm{tg}\,\alpha = 2\sqrt{2} + 1.$

$0° < \alpha < 90°$ ekanidan va $\sin\alpha = \frac{\mathrm{tg}\,\alpha}{\sqrt{1+\mathrm{tg}^2\,\alpha}}$ formuladan foydalanib,

$$\sin\alpha = \frac{2\sqrt{2}+1}{\sqrt{1+(2\sqrt{2}+1)^2}} = \frac{2\sqrt{2}+1}{\sqrt{10+4\sqrt{2}}}.$$

Javob: $\frac{2\sqrt{2}+1}{\sqrt{10+4\sqrt{2}}}$ (B) **@matematikaguruh**

Katetlarning gipotenuzadagi proyeksiyalari



$m + n = c$

$m - a$ ning proyeksiyasi.

$n - b$ ning proyeksiyasi.

@super_matematika

Oklit teoremalari

1) $m^2 + h^2 = a^2$ va $n^2 + h^2 = b^2$

2) $a \cdot b = c \cdot h$

3) $a^2 = m \cdot c$ va $b^2 = n \cdot c$

4) $\frac{1}{h^2} = \frac{1}{a^2} + \frac{1}{b^2}$ **5)** $\frac{m}{n} = \frac{a^2}{b^2}$

@matematikaguruh va @riyoziyot

M. Hisoblang: $\lim_{x\to 1}\frac{\sqrt{2x+1}-\sqrt{3x}}{\sqrt{x+1}-\sqrt{2x}}$

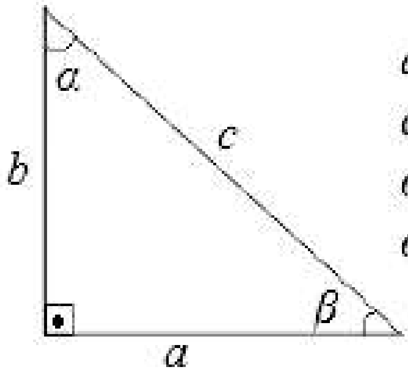
Y: $\lim_{x\to 1}\frac{\sqrt{2x+1}-\sqrt{3x}}{\sqrt{x+1}-\sqrt{2x}}=\lim_{x\to 1}\left(\frac{\sqrt{2x+1}-\sqrt{3x}}{\sqrt{x+1}-\sqrt{2x}}\cdot\frac{\sqrt{2x+1}+\sqrt{3x}}{\sqrt{x+1}+\sqrt{2x}}\cdot\frac{\sqrt{x+1}+\sqrt{2x}}{\sqrt{2x+1}+\sqrt{3x}}\right)=$

$\lim_{x\to 1}\left(\frac{2x+1-3x}{x+1-2x}\cdot\frac{\sqrt{x+1}+\sqrt{2x}}{\sqrt{2x+1}+\sqrt{3x}}\right)=\lim_{x\to 1}\frac{\sqrt{x+1}+\sqrt{2x}}{\sqrt{2x+1}+\sqrt{3x}}=\frac{\sqrt{1+1}+\sqrt{2}}{\sqrt{2+1}+\sqrt{3}}=\sqrt{\frac{2}{3}}$

Javob: $\sqrt{\frac{2}{3}}$.

@matematika

To'g'ri burchakli uchburchak



a, b – katetlar.
c – gipotenuza.
α, β – o'tkir burchaklar.
$\alpha + \beta = 90^\circ$.

Pifagor teoremasi:

$$a^2 + b^2 = c^2$$

Masalan, $a = 3$, $b = 4$, $c = 5$.

@super_matematika

Pifagor uchlik sonlarni hosil qilish.

1) $a,\ b = \dfrac{a^2 - 1}{2},\ c = \dfrac{a^2 + 1}{2};$ a – toq son.

2) $a,\ b = \left(\dfrac{a}{2}\right)^2 - 1,\ c = \left(\dfrac{a}{2}\right)^2 + 1;$ a – juft son.

3) $a = 2mn,\ b = m^2 - n^2,\ c = m^2 + n^2$

$m > n$ musbat son bo'lgan istalgan sonlar.

a katet	b katet	c gipotenuza
3	4	5
5	12	13
7	24	25
9	40	41
11	60	61
12	35	37
13	84	85
16	63	65
17	144	145
19	180	181

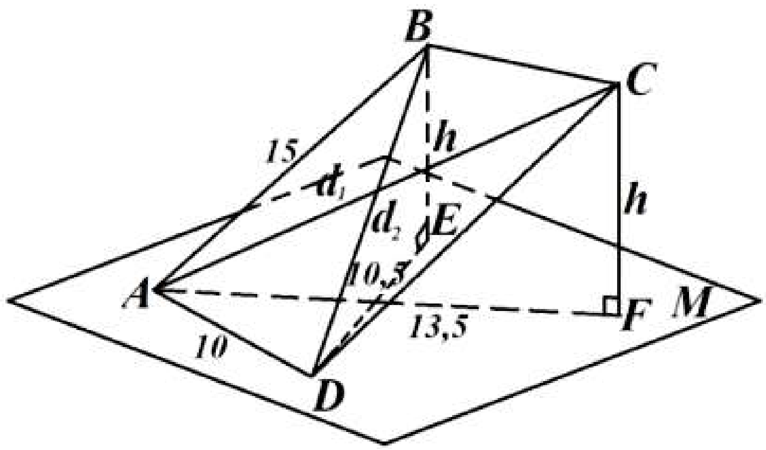
*Agar **a**, **b**, va **c** sonlar Pifagor uchlik sonlar bo'lsa, **ma**, **mb** va **mc** sonlar ham Pifagor sonlari bo'ladi.*

M: $2 \cdot 3$, $2 \cdot 4$ *va* $2 \cdot 5$, ya'ni 6, 8 va 10.

@matematikaguruh va @riyoziyot

M.(Variant 2015)

$ABCD$ parallelogrammning A va D uchlari M tekislikda, B va C uchlari uning tashqarisida, $AD = 10$ sm, $AB = 15$ sm, AC va BD diagonallarining M tekislikdagi proyeksiyalari mos ravishda 13,5 sm va 10,5 sm ga teng. Parallelogrammning diagonallarini toping.



Yechish: **AD//BC** ekanligidan **BC** tomon **M** tekislikka parallel bo'ladi. $AC = d_1, BD = d_2, BE = CF = h$ bo'lsin. Shartga ko'ra $AF = 13,5$ sm, $DE = 10,5$ sm. U holda

$$\begin{cases} AC^2 + BD^2 = 2 \cdot (AB^2 + AD^2) \\ AC^2 = AF^2 + CF^2 \\ BD^2 = DE^2 + BE^2 \end{cases} => \begin{cases} d_1^2 + d_2^2 = 2 \cdot (15^2 + 10^2) \\ d_1^2 = 13,5^2 + h^2 \\ d_2^2 = 10,5^2 + h^2 \end{cases} =>$$

$$\begin{cases} d_1^2 + d_2^2 = 650 \\ d_1^2 - d_2^2 = 72 \end{cases} => \begin{cases} d_1 = 19 \\ d_2 = 17 \end{cases}$$

Javob: 17 sm va 19 sm. @matematika

@matematikaguruh va @riyoziyot

M. $\alpha+\beta+\gamma=\pi$ va $\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}=a$ bo'lsa, $\sin\alpha+\sin\beta+\sin\gamma$ ni toping.

Y: $\alpha+\beta=\pi-\gamma$. $\sin\alpha+\sin\beta+\sin\gamma=$
$2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+\sin\gamma=2\sin\frac{\pi-\gamma}{2}\cos\frac{\alpha-\beta}{2}+\sin\gamma=$
$2\cos\frac{\gamma}{2}\cos\frac{\alpha-\beta}{2}+2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}=2\cos\frac{\gamma}{2}\left(\cos\frac{\alpha-\beta}{2}+\sin\frac{\gamma}{2}\right)=$
$2\cos\frac{\gamma}{2}\left(\cos\frac{\alpha-\beta}{2}+\sin\frac{\pi-\alpha-\beta}{2}\right)=2\cos\frac{\gamma}{2}\left(\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta}{2}\right)=$
$2\cos\frac{\gamma}{2}\left(2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\right)=4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}=4a.$

Javob: $4a$. @matematika

@matematikaguruh *va* @riyoziyot

M. $\alpha+\beta+\gamma=\pi$ va $\sin\frac{\alpha}{2}\cdot\sin\frac{\beta}{2}\cdot\sin\frac{\gamma}{2}=a$ bo'lsa, $\cos\alpha+\cos\beta+\cos\gamma$ ni toping.

Y: $\alpha+\beta=\pi-\gamma$. $\cos\alpha+\cos\beta+\cos\gamma=$

$2\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}+\cos\gamma=2\cos\frac{\pi-\gamma}{2}\cdot\cos\frac{\alpha-\beta}{2}+\cos\gamma=$

$2\sin\frac{\gamma}{2}\cdot\cos\frac{\alpha-\beta}{2}+1-2\sin^2\frac{\gamma}{2}=$

$2\sin\frac{\gamma}{2}\cdot\left(\cos\frac{\alpha-\beta}{2}-\sin\frac{\gamma}{2}\right)+1=$

$2\sin\frac{\gamma}{2}\cdot\left(\cos\frac{\alpha-\beta}{2}-\sin\frac{\pi-\alpha-\beta}{2}\right)+1=$

$2\sin\frac{\gamma}{2}\cdot\left(\cos\frac{\alpha-\beta}{2}-\cos\frac{\alpha+\beta}{2}\right)+1=$

$2\sin\frac{\gamma}{2}\cdot\left(2\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right)+1=4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}+1=4a+1.$

Javob: $4a+1$. @matematika

@matematikaguruh ***va*** *@riyoziyot*

M: $\frac{11n+3}{13n+4}$ **kasr qisqaradigan $[1;25]$ kesmaga tegishli natural n sonlar nechta ?**

Y: $\frac{11n+3}{13n+4}$ **bu kasr qisqaradigan sonni topish uchun uning surat va mahraji EKUBini Evklid algaritmi yordamida aniqlaymiz:**

1) $13n+4 = 1\cdot(11n+3) + (2n+1)$: $\begin{array}{r|l} 13n+4 & 11n+3 \\ -\,11n+3 & 1 \\ \hline 2n+1 & \end{array}$

2) $\begin{array}{r|l} 11n+3 & 2n+1 \\ -\,10n+5 & 5 \\ \hline n-2 & \end{array}$

3) $\begin{array}{r|l} 2n+1 & n-2 \\ -\,2n-4 & 2 \\ \hline 5 & \end{array}$

4) $\begin{array}{r|l} 5n-10 & 5 \\ -\,5n & n-2 \\ \hline -10 & \\ -\,-10 & \\ \hline 0 & \end{array}$

$$EKUB\{(11n+3);(13n+4)\} = 5 \Rightarrow \frac{11n+3}{5} = 2n + \frac{n+3}{5}$$

$$\frac{n+3}{5} = m, (m \in N) \quad \Rightarrow \quad n = 5m-3 \qquad (1)$$

Shartga ko'ra, $1 \le n \le 25 \quad \Rightarrow \quad 1 \le 5m-3 \le 25 \Rightarrow$

$0,8 \le m \le 5,6 \quad \Rightarrow \quad m \in \{1;2;3;4;5\} \qquad (2)$

(2) ni (1) ga qo'yib, $n \in \{2;7;12;17;22\}$ **bo'lib, uning 5 ta natural qiymati mavjudligini ko'rishimiz mumkin.**

Javob: 5 ta. **@matematika**

M. $\alpha+\beta+\gamma=\pi$ va $sin2\alpha\cdot sin2\beta\cdot sin2\gamma=a$ bo'lsa, $sin4\alpha+sin4\beta+sin4\gamma$ ni toping.

Y: $\alpha+\beta=\pi-\gamma$. $sin4\alpha+sin4\beta+sin4\gamma=$
$2sin(2\alpha+2\beta)\cdot cos(2\alpha-2\beta)+sin4\gamma=$
$2sin(2\pi-2\gamma)\cdot cos(2\alpha-2\beta)+sin4\gamma=$
$-2sin2\gamma\cdot cos(2\alpha-2\beta)+2sin2\gamma cos2\gamma=$
$-2sin2\gamma\cdot(cos(2\alpha-2\beta)-cos2\gamma)=$
$-2sin2\gamma\cdot\left(cos(2\alpha-2\beta)-cos(2\pi-2\alpha-2\beta)\right)=$
$-2sin2\gamma\cdot\left(cos(2\alpha-2\beta)-cos(2\alpha+2\beta)\right)=$
$-2sin2\gamma\cdot(2sin2\beta sin2\alpha)=-4sin2\alpha sin2\beta sin2\gamma=-4a.$

Javob: $-4a$.

Nyuton binomi @matematikaguruh va @riyoziyot

$$(a+b)^n = C_n^0 a^n + C_n^1 a^{n-1} b + C_n^2 a^{n-2} b^2 + \dots + C_n^{n-1} a b^{n-1} + C_n^n b^n$$

Bu yerdan ko'rinib turibdiki, a va b oldidagi koeffitsiyent 1 ga teng va uning $n-$ darajasi ham 1 ga teng: $C_n^0 = C_n^n = 1^n = 1$.

$C_n^0;\ C_n^1;\ C_n^2; \dots; C_n^n$ lar esa binomial koeffitsiyentlar deyiladi.

$C_n^m = \frac{n!}{m!\cdot(n-m)!}$; $\quad n! = 1\cdot 2\cdot 3\cdot 4\cdot \dots \cdot n$

$0! = 1$ deb qabul qilingan.

Sababi: $1 = C_n^0 = \frac{n!}{0!\cdot(n-0)!} = \frac{n!}{0!\cdot n!} = \frac{1}{0!} \Leftrightarrow 1 = \frac{1}{0!} \Rightarrow 0! = 1$.

Xossalari.

1. Yoyilma $n+1$ ta haddan iborat.

2. Barcha binomial koeffitsiyentlar yig'indisi $C_n^0 + C_n^1 + C_n^2 + \dots + C_n^n = 2^n$ ga teng.

3. Boshi va oxiridan teng uzoqlikda turuvchi binomial koeffitsiyentlar o'zaro teng ya'ni: $C_n^m = C_n^{n-m}$.

4. Toq o'rinda turgan binomial koeffitsiyentlar yig'indisi juft o'rinda turgan binomial koeffitsiyentlar yig'indisiga teng.

5. Binom yoyilmasi a o'zgaruvchiga nisbatan $n-$ darajani ko'phab bo'ladi.

6. Binom ko'rsatkichi toq bo'lganda yoyilmada ikkita o'rta had, juft son bo'lganda esa bitta o'rta had bo'ladi.

7. Yoyilmaning istalgan hadi: $T_{m+1} = C_n^m \cdot a^{n-m} \cdot b^m$ ga teng.

Masala

Ikkita zaryadsiz sfera bir-biridan uzoq masofada joylashgan. Zaryad miqdori q bo'lgan zaryadlangan sharcha avval birinchi sferaga, so'ngra esa ikkinchisiga tekkizildi. Natijada ikkinchi sferadagi zaryad miqdori $0{,}09q$ bo'ldi. Sharchada qanday zaryad miqdori qolgan?

A) $0{,}01q$ B) $0{,}9q$ C) $0{,}09q$ D) $0{,}81q$

Yechimi:

R—sfera radiuslari; r—zaryadlangan sharcha radiusi bo'lsin. 1—sferaga tekkizilganda u q_1 ga teng zaryadla—nib qolsin. Bu hol uchun:

$$\frac{R}{r}=\frac{q_1}{q-q_1}$$

2—sferaga tekkizilgandan keyingi hol uchun:

$$\frac{R}{r}=\frac{0{,}09q}{(q-q_1)-0{,}09q}\Rightarrow\frac{R}{r}=\frac{0{,}09q}{0{,}91q-q_1}$$

Tenglashtirsak:

$$\frac{q_1}{q-q_1}=\frac{0{,}09q}{0{,}91q-q_1}\Rightarrow q_1^2-qq_1+0{,}09q^2=0$$

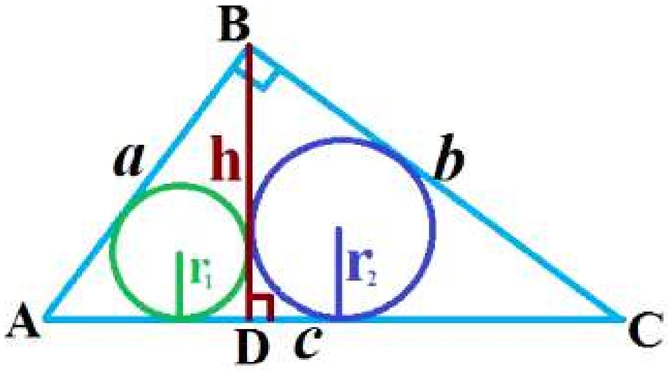
Bundan $q_1=0{,}9q$

Sharchada qolgan zaryad:

$0{,}91q-q_1=0{,}91q-0{,}9q=0{,}01q$

Javob: A

4-M: ABC to'g'ri burchakli uchburchakda gipotenuzaga BD balandlik o'tkazilgan. Agar ABD va CBD uchburchaklarga ichki chizilgan aylana radiuslari mos ravishda r_1 va r_2 bo'lsa, ABC uchburchakning BD balandligini toping.



Yechish: ABC uchburchakda $AB = a, BC = b, AC = c$, ichki chizilgan aylana radiusi r va $BD = h$ bo'lsin. $r = \frac{a+b-c}{2} = \sqrt{r_1^2 + r_2^2}$.

$$\begin{cases} \frac{|AD|+h-a}{2} = r_1 \\ \frac{|DC|+h-b}{2} = r_2 \end{cases} + \Rightarrow |AD| + |DC| + 2h - a - b = 2r_1 + 2r_2 \Rightarrow$$

$$2h = 2r_1 + 2r_2 + a + b - c \Rightarrow h = r_1 + r_2 + \frac{a+b-c}{2} = r_1 + r_2 + r.$$

$$h = r_1 + r_2 + \sqrt{r_1^2 + r_2^2}.$$

Javob: $r_1 + r_2 + \sqrt{r_1^2 + r_2^2}$.

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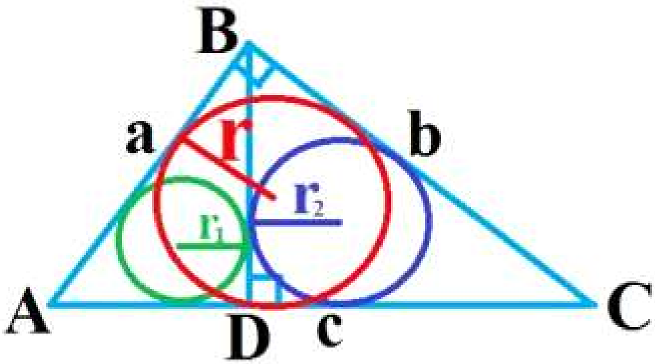
3-M: **ABC to'g'ri burchakli uchburchakda gipotenuzaga BD balandlik o'tkazilgan. Agar ABD va CBD uchburchaklarga ichki chizilgan aylana radiuslari mos ravishda r_1 va r_2 bo'lsa, ABC uchburchakka ichki chizilgan aylana radiusini toping.**



Yechish: **ABC** uchburchakda **AB**=***a***, **BC**=***b*** va **AC**=***c*** va ichki chizilgan aylana radiusi r bo'lsin. ΔABC - ΔABD *va* Δ*CBD lar bilan o'xshash, shuning uchun ularning gipotenuzalari nisbati, ichki chizilgan aylanalar radiuslari nisbatiga teng*, ya'ni $\frac{a}{c} = \frac{r_1}{r}$ va $\frac{b}{c} = \frac{r_2}{r}$.

Pifagor teoremasiga ko'ra, $\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$ tenglikdan foydalanib,

$$\left(\frac{r_1}{r}\right)^2 + \left(\frac{r_2}{r}\right)^2 = 1 \quad => \quad r = \sqrt{r_1^2 + r_2^2}$$

Javob: $\sqrt{r_1^2 + r_2^2}$.

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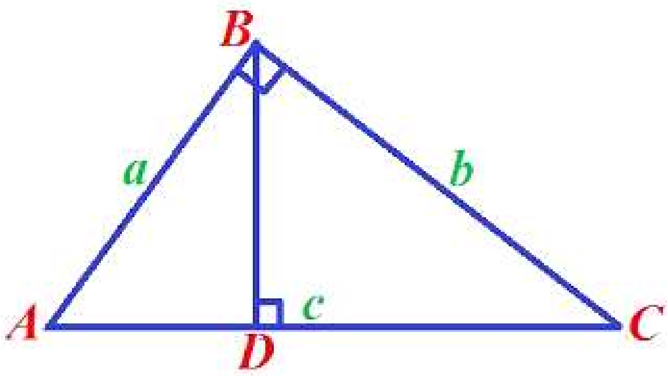

2-M: **ABC to'g'ri burchakli uchburchakda gipotenuzaga BD balandlik o'tkazilgan. Agar ABD va CBD uchburchaklarning yarim perimetrlari mos ravishda p_1 *va* p_2 bo'lsa, ABC uchburchakning yarim perimetrini toping.**

Yechish: **ABC** uchburchakning yarim perimetri p, **AB**=a, **BC**=b va **AC**=c bo'lsin. ΔABC - *ΔABD va ΔCBD lar bilan o'xshash, shuning uchun ularning gipotenuzalari nisbati yarim perimetrlari nisbatiga teng*, ya'ni

$\frac{a}{c} = \frac{p_1}{p}$ va $\frac{b}{c} = \frac{p_2}{p}$. ***Pifagor*** teoremasiga ko'ra, $\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$ tenglikdan foydalanib, $\left(\frac{p_1}{p}\right)^2 + \left(\frac{p_2}{p}\right)^2 = 1$ => $p = \sqrt{p_1^2 + p_2^2}$

Javob: $\sqrt{p_1^2 + p_2^2}$.

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M(2016). $\underbrace{\mathrm{tg}(\mathrm{tg}(\mathrm{tg}\ldots(\mathrm{tg}\,x)\ldots))}_{n\ ta} = 2012$ **tenglama** $\left[0;\frac{\pi}{3}\right]$ **kesmada cheksiz ko'p yechimga ega bo'lsa,** n **ning eng kichik qiymatini toping.**

Y: 1) $n = 1$ da $\mathrm{tg}\,x = 2012 \Rightarrow x = \mathrm{arctg}\,2012 + \pi n_1, n_1 \in Z$.
$n_1 = 0$ da $x = \mathrm{arctg}\,2012$, $\frac{\pi}{3} < \mathbf{arctg\,2012} < \frac{\pi}{2} < \sqrt{3}$.

2) $n = 2$ da $\mathrm{tg}(\mathrm{tg}\,x) = 2012 \Rightarrow \mathrm{tg}\,x = \mathrm{arctg}\,2012 + \pi n_1$
$x = \mathrm{arctg}(\mathrm{arctg}\,2012 + \pi n_1) + \pi n_2, n_1, n_2 \in Z$. Bunda $n_2 = 0$ bo'lganda $x = \mathrm{arctg}(\mathrm{arctg}\,2012 + \pi n_1)$ yechim n_1 ning istalgan natural qiymatlarida $\mathbf{arctg(arctg\,2012 + \pi n_1) < arctg(+\infty)} < \frac{\pi}{2}$ tengsizlik o'rinli.

3) $n = 3$ da $\mathrm{tg}(\mathrm{tg}(\mathrm{tg}\,x)) = 2012 \Rightarrow \mathrm{tg}(\mathrm{tg}\,x) = \mathrm{arctg}\,2012 + \pi n_1 \Rightarrow$
$\mathrm{tg}\,x = \mathrm{arctg}(\mathrm{arctg}\,2012 + \pi n_1) + \pi n_2 \Rightarrow$
$x = \mathrm{arctg}(\mathrm{arctg}(\mathrm{arctg}\,2012 + \pi n_1) + \pi n_2) + \pi n_3$, $n_1, n_2, n_3 \in Z$.
Bunda $n_2 = 0$ va $n_3 = 0$ da ***n_1 ning istalgan natural qiymatida*** $\mathbf{arctg(arctg((arctg\,2012 + \pi n_1))) < arctg(arctg(+\infty)) <}$ $\mathbf{arctg}\left(\frac{\pi}{2}\right) < \frac{\pi}{3}$ ***tengsizlik o'rinli.*** Demak, $n \geq 3$ ***ning barcha natural qiymatlarida berilgan tenglama*** $\left[0;\frac{\pi}{3}\right]$ ***kesmada cheksiz ko'p yechimga ega*** bo'ladi deb hulosa chiqarish mumkin.

Javob: 3. **@matematika**

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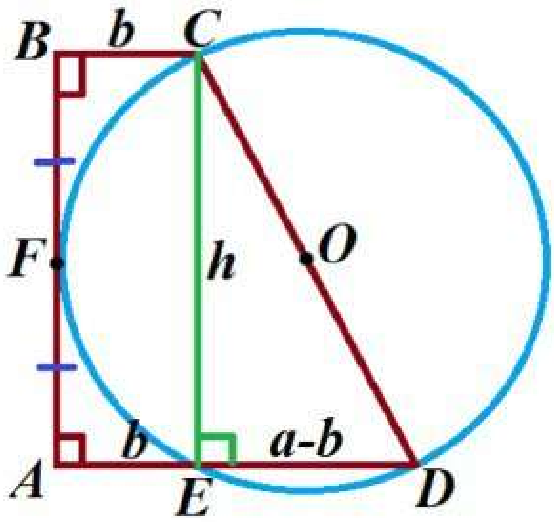

M(2016). To'g'ri burchakli trapetsiyaning balandligi *h*, uning asosiga perpendikulyar bo'lmagan tomonini diametr qilib chizilgan aylana trapetsiyaning qarama-qarshi tomoniga urinadi. Katetlari trapetsiyaning asoslari bo'lgan to'g'ri burchakli uchburchakning yuzini toping.

Y: Trapetsiyaning asoslari uzunliklari $AD = a$ va $BC = b$ bo'lsin. Biz esa katetlari a va b bo'lgan to'g'ri burchakli uchburchakning yuzi $\frac{ab}{2}$ ni topishimiz kerak. Shartga ko'ra $AB = h$. **AD** ning aylana bilan kesishish nuqtasi **E** ga **CE** vatar o'tkazamiz va bunda **CED** uchburchak to'g'ri burchakli bo'lib, **ABCE** to'g'ri to'rtburchak bo'lishi kelib chiqadi. Bundan $AE = BC = b, AF = \frac{AB}{2} = \frac{h}{2}$. AF –urinma AD –kesuvchi bo'lgani uchun $AF^2 = AE \cdot AD \Rightarrow \left(\frac{h}{2}\right)^2 = b \cdot a \Rightarrow \frac{ab}{2} = \frac{h^2}{8}$.

Javob: $\frac{h^2}{8}$.

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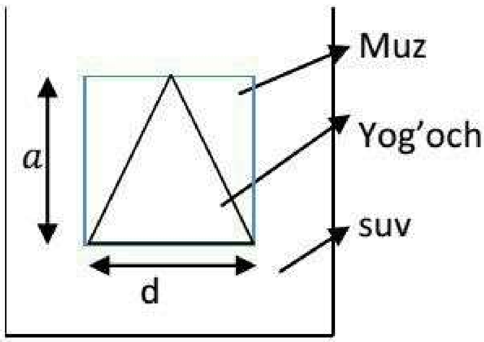
Kub shaklidagi muzning ichiga joylashtirilgan yog‘ochdan yasalgan konus asosining diametri va uning balandligi kub qirrasi uzunligiga teng. Agar ushbu muz suvda suzib yurgan bo‘lsa, uning qanday qismi (%) suvga botmagangan bo‘ladi? Yog‘och, muz va suvning zichliklari mos ravishda 0,8; 0,9; 1,0 (g/sm^3) ga teng. $\pi \approx 3,0$ deb oling.

A) 14,6   B) 20,7   C) 27,8   D) 12,5



$a = d = h$; $F_a = mg$; $\rho_s g V_b = g(\rho_m V_m + \rho_y V_y)$;

$V_y = \frac{1}{3}\pi R^2 h = \frac{\pi a^3}{12}$; $V_m = a^3 - V_y = a^3 - \frac{\pi a^3}{12}$;

$\rho_s V_b = \rho_m a^3 \left(1 - \frac{\pi}{12}\right) + \rho_y \frac{\pi a^3}{12}$

$\rho_s V_b = a^3 \left(\rho_m \left(1 - \frac{\pi}{12}\right) + \rho_y \frac{\pi}{12}\right)$

$\rho_s V_b = \frac{V}{12}\left(12\rho_m - \pi(\rho_m - \rho_y)\right)$

$\frac{V_b}{V} = \frac{12\rho_m - \pi(\rho_m - \rho_y)}{12\rho_s}$

$1 - \frac{V_b}{V} = 1 - \frac{12\rho_m - \pi(\rho_m - \rho_y)}{12\rho_s} = \frac{12(\rho_s - \rho_m) - \pi(\rho_y - \rho_m)}{12\rho_s} = \frac{12(1-0,9) - \pi(0,8-0,9)}{12} = 0,125 = 12,5\%$