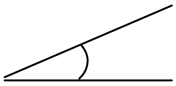
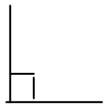
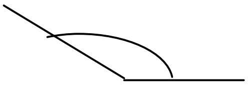
G E O M E T R I Y A

PLANIMETRIYA

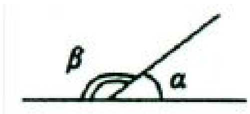
Burchaklar

1. O’lchovi: $1\ rad = \frac{180^0}{\pi} \approx 57^0 17' 45''$; $1^0 = \frac{\pi}{180} rad \approx 0,017453$ rad.

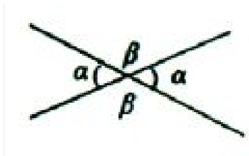
2. Turi: O’tkir: $0 < \alpha < 90^0$,  To’g’ri: $\alpha = 90^0$  .

O’tmas: $90^0 < \alpha < 180^0$  Yoyiq: $\alpha = 180^0$  .

3. Qo'shni burchaklar yig'indisi 180^0 teng, ya`ni
$\alpha + \beta = 180^0$ **α** va **β** - qo'shni burchaklar.

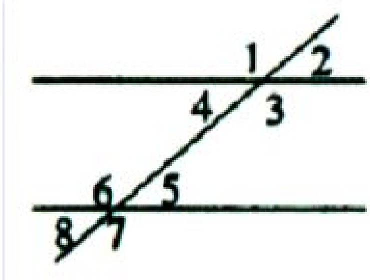


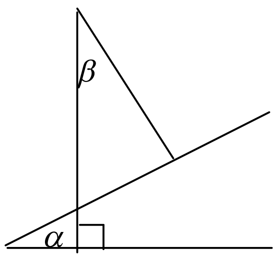
4. Vertikal burchaklar teng: **α = α.**



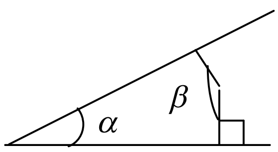
5. To’g’ri chiziqlarning parallelligi
- Mos burchaklar: **2,5; 1,6; 3,7; 4,8;**
- Ichki almashinuvchi burchaklar: **4,5; 3,6;**
- Tashqi almashinuvchi burchaklar: **2,8; 1,7;**
- Ichki bir tomonli burchaklar: **4,6; 3,5;**
- Tashqi bir tomonli burchaklar: **2,7; 1,8;**

$\angle 7 = \angle 3$, $\angle 5 + \angle 3 = 180°$;
$\angle 2 = \angle 5$, $\angle 1 + \angle 4 = 180°$.





$\alpha = \beta$



$\alpha + \beta = 180^0$

Uchburchakda asosiy teoremalar

1. Uchburchak ichki burchaklarining yig'indisi:

$$\alpha + \beta + \gamma = 180^0$$

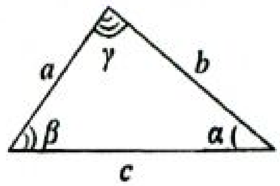
2. Uchburchakning tashqi va ichki burchaklari orasidagi munosabatlar:

$$\alpha + \alpha_1 = 180^0, \quad \beta + \beta_1 = 180^0, \quad \gamma + \gamma_1 = 180^0,$$

$$\alpha_1 = \beta + \gamma, \quad \beta_1 = \alpha + \gamma, \quad \gamma_1 = \alpha + \beta, \quad \alpha_1 + \beta_1 + \gamma_1 = 360^0.$$

3. Uchburchak tengsizligi:

$$\begin{cases} a+b>c, \\ a+c>b, \\ b+c>a; \end{cases} \qquad \begin{cases} |a-b|<c, \\ |a-c|<b, \\ |b-c|<a. \end{cases}$$



4. Sinuslar teoremasi: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma} = 2R$.

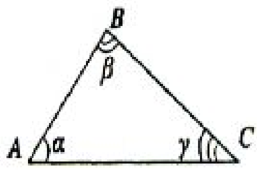
5. Kosinuslar teoremasi:

$$a^2 = b^2 + c^2 - 2bc\cdot cos\,\alpha, \quad b^2 = a^2 + c^2 - 2ac\cdot cos\,\beta,$$

$$c^2 = a^2 + b^2 - 2ab\cdot cos\,\gamma, \quad a = b\cdot cos\,\gamma + c\cdot cos\,\beta,$$

$$b = aCos\,\gamma + cCos\,\alpha, \quad c = a\cdot cos\,\beta + a\cdot cos\,\alpha,$$

$$cos\,\alpha + cos\,\beta + cos\,\gamma \le \frac{3}{2}.$$



6. Tangenslar teoremasi:

$$\frac{a+b}{a-b} = \frac{tg\dfrac{\alpha+b}{2}}{tg\dfrac{\alpha-\beta}{2}} = \frac{ctg\dfrac{\gamma}{2}}{tg\dfrac{\alpha-\beta}{2}}; \quad \frac{a+c}{a-c} = \frac{tg\dfrac{\alpha+\gamma}{2}}{tg\dfrac{\alpha-\gamma}{2}} = \frac{ctg\dfrac{\beta}{2}}{tg\dfrac{\alpha-\gamma}{2}};$$

$$\frac{b+c}{b-c} = \frac{tg\dfrac{\beta+\gamma}{2}}{tg\dfrac{\beta-\gamma}{2}} = \frac{ctg\dfrac{\alpha}{2}}{tg\dfrac{\beta-\gamma}{2}}.$$

7. Mol'veyde formulasi:

$$\frac{a+b}{c} = \frac{cos\dfrac{\alpha-\beta}{2}}{sin\dfrac{\gamma}{2}}; \qquad \frac{a-b}{c} = \frac{sin\dfrac{\alpha-\beta}{2}}{cos\dfrac{\gamma}{2}}.$$

8. $sin\dfrac{\alpha}{2} = \sqrt{\dfrac{(p-b)(p-c)}{bc}}; \quad cos\dfrac{\alpha}{2} = \sqrt{\dfrac{p(p-a)}{bc}}.$

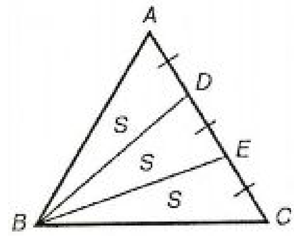
9. c - o'tkir burchakli uchburchakning eng katta tomoni bo'lsa, u holda

$$a^2 + b^2 > c^2.$$

10. c - o'tmas burchakli uchburchakning eng katta tomoni bo'lsa, u holda $a^2 + b^2 < c^2$.

11. $AD = DE = EC, \quad S_{\triangle ABC} = 3S;$

12. $P_{\triangle ABC} = a + b + c, \;\; a, b, c -$ $\triangle ABC$ tomonlari.

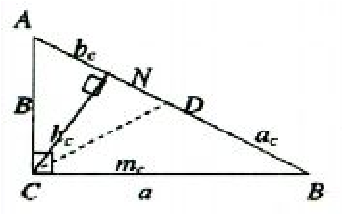


To'g'ri burchakli uchburchak

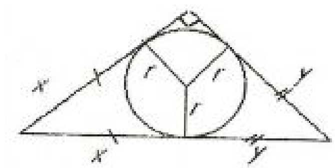
a_c va b_c – a va b katetlarning gipotenuzadagi proyeksiyasi, m_a - $\boldsymbol{a}$ katetga, m_b - $\boldsymbol{b}$ katetga, m_c - $\boldsymbol{c}$ gipotenuzaga tushirilgan mediana. $AN = b_c$, $NB = a_c$, h_c - gipotenuzaga tushirilgan balandlik.

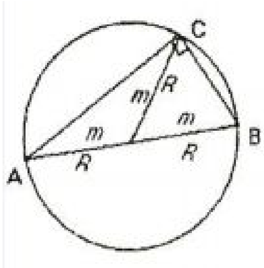
$a^2 + b^2 = c^2$ — Pifagor teoremasi, $c = a_c + b_c$,

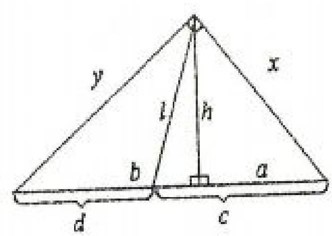
$AD = BD = CD = m_c = R;$



- $a^2 = c \cdot a_c; \;\; b^2 = c \cdot b_c;$
- $h_c = \sqrt{a_c \cdot b_c}; \;\; h_c = \dfrac{a \cdot b}{c};$
- $R = \dfrac{c}{2}; \;\; r = \dfrac{a + b - c}{2};$
- $r + R = \dfrac{a+b}{2}; \quad \dfrac{R}{r} = \dfrac{5}{2} \Rightarrow a : b : c = 3 : 4 : 5;$
- $S = \dfrac{1}{2}ab; \;\; S = \dfrac{1}{2}c \cdot h_c; \;\; S = \dfrac{a^2 ctg\alpha}{2} = \dfrac{c^2 \sin 2\alpha}{4};$
- $S = r^2 + 2Rr; \qquad S = xy;$
- $m_a = \dfrac{1}{2}\sqrt{4b^2 + a^2}; \;\; m_b = \dfrac{1}{2}\sqrt{4a^2 + b^2}; \;\; m_c = \dfrac{c}{2};$
- $\left(\dfrac{d}{c}\right)^2 = \dfrac{b}{a};$
- $\left(\dfrac{y}{x}\right)^2 = \dfrac{b}{a}, \qquad \boldsymbol{l}$ – bissektrisa;
- agar $\dfrac{h_c}{m_c} = \dfrac{p}{q}$ bo\`lsa, $\dfrac{a}{b} = \sqrt{\dfrac{q - \sqrt{q^2 - p^2}}{q + \sqrt{q^2 - p^2}}}$ bo\`ladi.

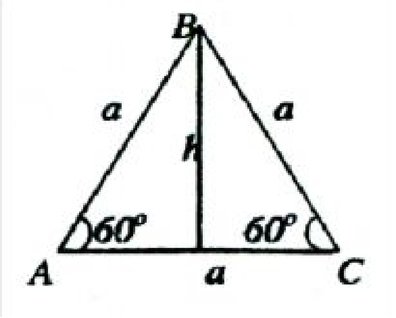






Teng tomonli (muntazam) uchburchak

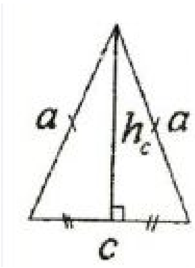
- $AB = AC = BC = a;\ \alpha = \beta = \gamma = 60^\circ$;
- $R = \dfrac{a}{\sqrt{3}}$; $r = \dfrac{a}{2\sqrt{3}}$; $R = 2r$;
- $h = r + R = 1{,}5R = 3r$; $r = \dfrac{1}{3}h$; $R = \dfrac{2}{3}h$;
- $h = l = m = \dfrac{\sqrt{3}}{2}a$; $S = \dfrac{a^2\sqrt{3}}{4}$.

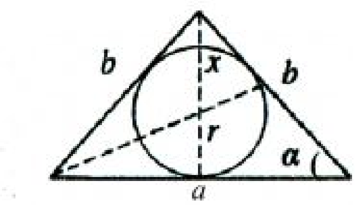


Teng yonli uchburchak

a - asosi, b - yon tomoni, h - balandligi, α -asosidagi burchaklari.

- $r = \dfrac{a}{2} tg \dfrac{\alpha}{2}$; $r = \dfrac{a \cdot h}{a + 2b}$; $R = \dfrac{b^2}{2h}$; $R = \dfrac{a^2}{2h}$;
- $\dfrac{a}{2b} = \dfrac{r}{x}$; $h = x + r$; $\left(\dfrac{a}{2}\right)^2 + (h - R)^2 = R^2$
- $S = \dfrac{a\sqrt{4b^2 - a^2}}{4}$; $S = \dfrac{c\sqrt{4a^2 - c^2}}{4}$
- $r = \dfrac{c(2a - c)}{4h}$.





Ixtiyoriy uchburcak

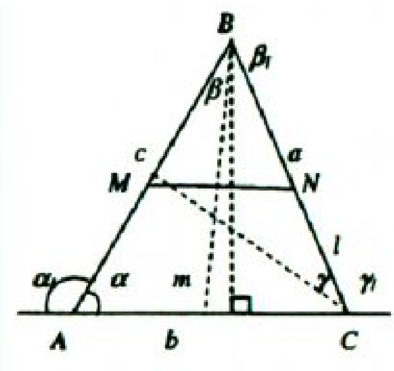
a*, *b*, *c — ΔABC ning tomonlari;

α*, *β*, *γ- uchburchakning ichki burchaklari;

$P = a + b + c$ - uchburchakning perimetri;

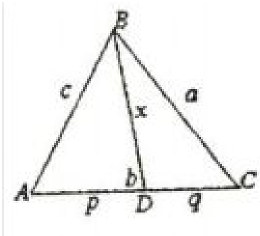
$p = \dfrac{a + b + c}{2}$ -uchburchak yarim perimetri;

α_1, β_1, γ_1 - ΔABC tashqi burchaklari;

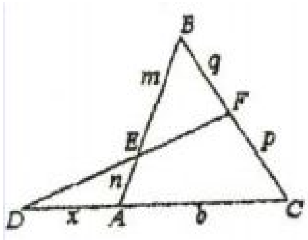


h_a, h_b, h_c - mos ravishda uchburchakning ***a*, *b*, *c*** tomonlariga tusbirilgan balandliklar uzunliklari; MN - uchburchakning **o'rta** chizig'i; R va r - uchburchakka **tashqi** va **ichki** chizilgan aylana radiusi; S - geometrik figuralarning yuzalari; m_a, m_b, m_c—**a, b, c** tomonlarga o'tkazilgan **medianalar** uzunliklari; l_a, l_b, l_c—**a, b, c** tomonlarga o'tkazilgan **bissektrisalar** uzunliklari.

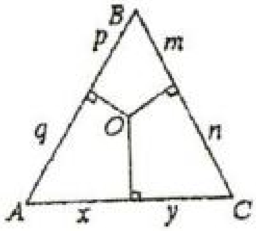
$$x^2=\frac{a^2p+c^2q}{q+p}-pq$$



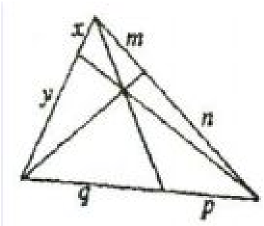
$$\frac{x}{b+x}\cdot\frac{p}{q}\cdot\frac{m}{n}=1$$



$$\frac{q}{p}\cdot\frac{n}{m}\cdot\frac{x}{y}=1$$



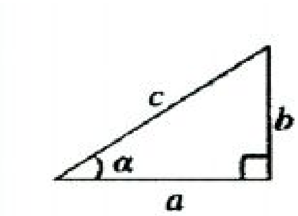
$$x^2+n^2+p^2=y^2+q^2+m^2$$



Burchak sinusi, kosinusi, tangensi va kotangensi

$$\sin\alpha=\frac{b}{c};\quad \cos\alpha=\frac{a}{c};$$

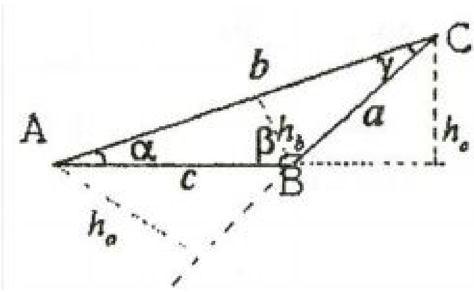
$$tg\alpha=\frac{b}{a};\quad ctg\alpha=\frac{a}{b}.$$



Uchburchak balandligi

1. Uchburchak uchidan chiquvchi va qarshisidagi tomonga perpendikulyar bo’lgan kesma balandlik deyiladi.

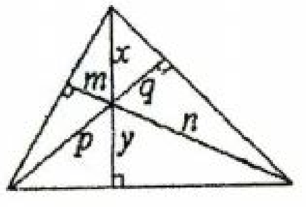
- $h_a=\dfrac{2S}{a}=b\sin\gamma=c\sin\beta;$
- $h_b=\dfrac{2S}{b}=a\sin\gamma=c\sin\alpha;$
- $h_c=\dfrac{2S}{c}=a\sin\beta=b\sin\alpha;$



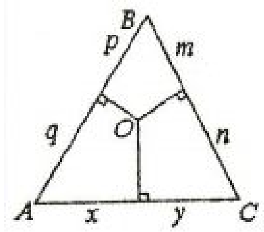
2. Uchburchak tomonlarining o'rtalaridan o'tkazilgan perpendikulyarlaming kesishish nuqtasi unga **tashqi chizilgan**

aylana markazi bo'ladi.

- $R = \dfrac{a \cdot b \cdot c}{4S}$; $\dfrac{1}{r} = \dfrac{1}{h_a} + \dfrac{1}{h_b} + \dfrac{1}{h_c}$, r – ichki chizilgan aylana radiusi;
- $S = \dfrac{1}{2}\sqrt{2h_a \cdot h_b \cdot h_c \cdot R}$; $h_a : h_b : h_c = \dfrac{1}{a} : \dfrac{1}{b} : \dfrac{1}{c} = bc : ac : ab$;
- $x \cdot y = p \cdot q = m \cdot n$;

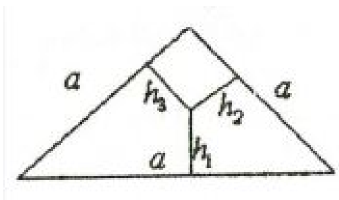


- $x^2 + n^2 + p^2 = y^2 + q^2 + m^2$.



3. Teng tomonli uchburchakning ichidagi ixtiyoriy nuqtadan uning tomonlariga tushirilgan perpendikulyar yig'indisi shu uchburchakning balandligiga teng:

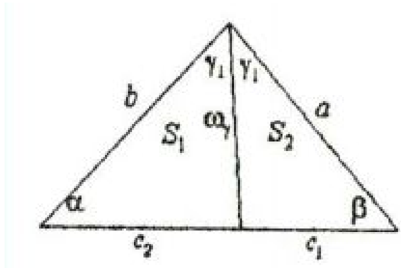
$$h_1 + h_2 + h_3 = h = \frac{\sqrt{3}}{2}a.$$



4. Ixtiyoriy uchburchak uchun: $h_a \le l_a \le m_a$.

Uchburchak bissektrisasi

1. Uchburchakning burchagidan chiqib, shu burchakni teng ikkiga bo'luvchi kesma **bissektrissadir**.

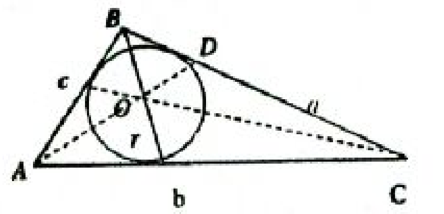


- $\dfrac{a}{b} = \dfrac{c_1}{c_2}$; $\omega_\gamma = \sqrt{b \cdot a - c_1 \cdot c_2}$;
- $\dfrac{S_1}{S_2} = \dfrac{a}{b}$; $2\gamma_1 = \gamma$;
- $\omega_\alpha = \dfrac{2}{b+c}\sqrt{bc(a+b+c)(-a+b+c)} = \dfrac{2bc \cdot \cos\dfrac{\alpha}{2}}{b+c}$;

- $\omega_\beta = \dfrac{1}{a+c}\sqrt{ac(a+b+c)(a-b+c)} = \dfrac{2ac\cdot\cos\dfrac{\beta}{2}}{a+c}$;
- $\omega_\gamma = \dfrac{1}{a+b}\sqrt{ab(a+b+c)(a+b-c)} = \dfrac{2ab\cdot\cos\dfrac{\gamma}{2}}{a+b}$.

2. Uchburchak bissektrisalarining kesishish nuqtasi unga ichki chizilgan **aylana markazi** bo'ladi.

- $r = \dfrac{2S}{a+b+c} = \dfrac{S}{p}$; $\dfrac{OA}{OD} = \dfrac{c+b}{a}$;

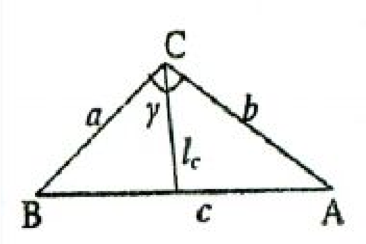


O - uchburchak bissektrisalari kesishgan nuqta.

3. Uchburchakning C uchidan l_c bissektrisa tushirilgan

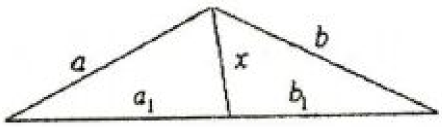
$\angle C = \gamma$ u holda

$l_c \cdot (a+b)\cdot \sin\dfrac{\gamma}{2} = ab\sin\gamma$.

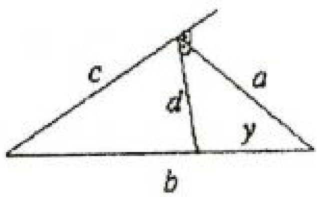


4. Qo’shni burchaklar bissektrisasi orasidagi burchak 90^0 ga teng;

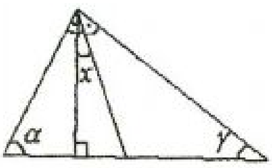
- $x^2 = a\cdot b - a_1\cdot b_1$ x - bissektrisa;



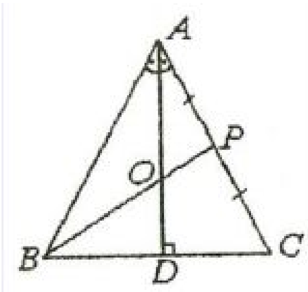
- $\dfrac{y}{b} = \dfrac{d}{c}$, $a = \sqrt{y\cdot b - d\cdot c}$;



- $x = \dfrac{|\alpha - \gamma|}{2}$;



- $AC = BC$, $AP = PC$, $\dfrac{OA}{OD} = \dfrac{OB}{OP}$;

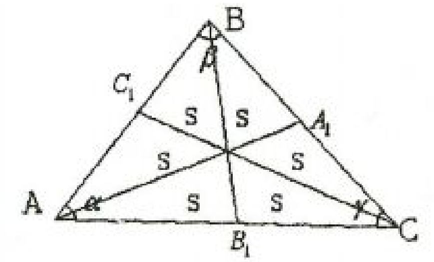


Uchburchak medianasi

Uchburchak uchidan chiqib, qarshisidagi tomonni teng ikkiga bo'luvchi kesma **mediana** deyiladi.

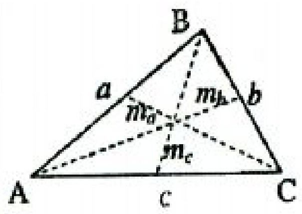
1. Uchburchak medianalari bir nuqrada kesishadi va bu nuqtada uchburchak uchidan boshlab hisoblaganda **2 : 1** nisbatda bo'linadi.

$$BA_1 = CA_1,$$
$$BC_1 = AC_1,$$
$$AB_1 = CB_1.$$



2. m_a - ***a*** tomonga, m_b - ***b*** tomonga, m_c - ***c*** tomonga tushirilgan mediana.

- $AA_1 = m_a = \frac{1}{2}\sqrt{2\left(b^2+c^2\right)-a^2} = \frac{1}{2}\sqrt{b^2+c^2+2bcCos\alpha}$;
- $BB_1 = m_b = \frac{1}{2}\sqrt{2\left(a^2+c^2\right)-b^2} = \frac{1}{2}\sqrt{a^2+c^2+2acCos\beta}$;
- $CC_1 = m_c = \frac{1}{2}\sqrt{2\left(a^2+b^2\right)-c^2} = \frac{1}{2}\sqrt{a^2+b^2+2abCos\gamma}$;



3. $m_a^2 + m_b^2 + m_c^2 = \frac{3}{4}(a^2+b^2+c^2)$.

4. $a = \frac{2}{3}\sqrt{2m_b^2 + 2c_c^2 - m_a^2}$; $b = \frac{2}{3}\sqrt{2m_a^2 + 2m_c^2 - m_b^2}$; $c = \frac{2}{3}\sqrt{2m_a^2 + 2m_b^2 - m_c^2}$;

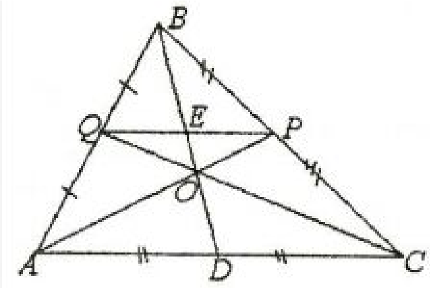
$m_a = \frac{1}{2}(\overline{BC} + \overline{AC})$, m_a - AB tomonga tushirilgan mediana.

5. Medianalar kesishgan nuqtaning koordinatasi:

- **Tekislikda:** $A(x_1;y_1)$, $B(x_2;y_2)$, $C(x_3;y_3)$, $O(x;y)$

$x = \frac{x_1+x_2+x_3}{3}$; $y = \frac{y_1+y_2+y_3}{3}$;

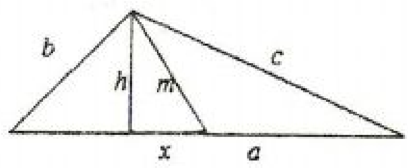
- $BD = m_b$, $CQ = m_c$, $AP = m_a$, $OE = \frac{1}{6}BD$;
- $S_{\Delta\ EOP} = S_{\Delta\ EOQ} = \frac{1}{24}S_{\Delta\ ABC}$;
- $S_{\Delta BQE} = S_{\Delta BEP} = \frac{1}{8}S_{\Delta ABC}$;



- **Fazoda:** $A(x_1;y_1;z_1)$, $B(x_2;y_2;z_2)$, $C(x_3;y_3;z_3)$, $O(x;y;z)$
  $x=\dfrac{x_1+x_2+x_3}{3}$; $y=\dfrac{y_1+y_2+y_3}{3}$; $z=\dfrac{z_1+z_2+z_3}{3}$.

6. Balandlik va mediana ajratgan kesma:

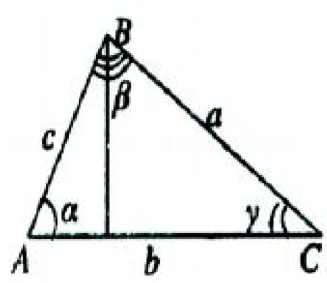
- $x=\dfrac{\left|b^2-c^2\right|}{2a}$



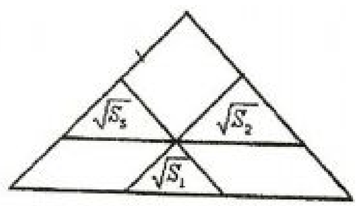
- Uchburchakning medianasi uning yuzini teng ikkiga bo'ladi.

Uchburchakning yuzi

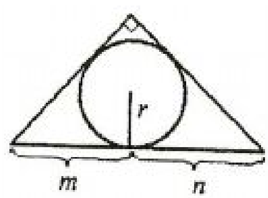
- $S=\dfrac{1}{2}a\,h_a$, $S=\dfrac{1}{2}b\,h_b$, $S=\dfrac{1}{2}c\,h_c$ - tomon va balandlik orqali;
- $S=\sqrt{p(p-a)(p-b)(p-c)}$, $p=\dfrac{a+b+c}{2}$ - Geron formulasi;
- $S=\dfrac{abc}{4R}$, $S=pr$ - ichki va tashqi chizilgan aylana radiuslari orqali;
- $S=\dfrac{4}{3}\sqrt{m(m-m_a)(m-m_b)(m-m_c)}$;
- $m=\dfrac{m_a+m_b+m_c}{2}$ -medianalar orqali;
- $S=\dfrac{a^2\sin\beta\cdot\sin\gamma}{2\sin\alpha}$; $S=\dfrac{b^2\sin\alpha\cdot\sin\gamma}{2\sin\beta}$;
  $S=\dfrac{c^2\sin\alpha\cdot\sin\beta}{2\sin\gamma}$; $S=\dfrac{1}{2}b\,c\cdot\sin\alpha$;
  $S=\dfrac{1}{2}a\,c\cdot\sin\beta$; $S=\dfrac{1}{2}a\,b\cdot\sin\gamma$.



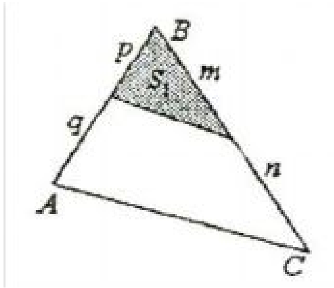
- $S=\left(\sqrt{S_1}+\sqrt{S_2}+\sqrt{S_3}\right)^2$;



- $S=m\cdot n$;

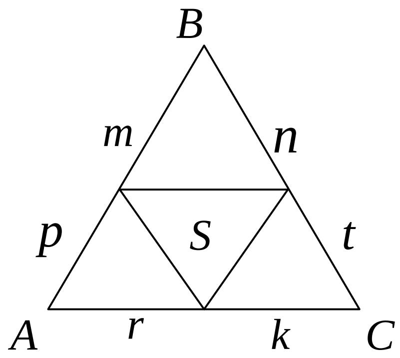


- $S_1 = \dfrac{mp}{(p+q)(m+n)} S_{ABC}$



- $\dfrac{S}{S_{\triangle ABC}} = \dfrac{rtm + knp}{abc}$,

  $a = p+m,\ b = n+t,\ \ c = r+k.$

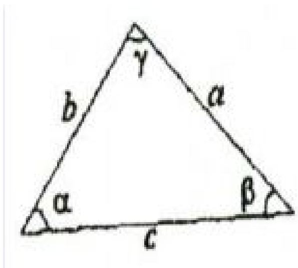


- Uchburchak uchlarining koordinatalari $A(x_1;y_1)$, $B(x_2;y_2)$, va $C(x_3;y_3)$ bo`lsa, uning **yuzi:**

  $$S_{\triangle ABC} = \frac{1}{2}\left|(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)\right|.$$

Uchburchakka tashqi chizilgan aylana radiusi

1. Ichki chizilgan aylana markazi **bissektrisalar kesishgan nuqtada** bo’ladi.
2. Tashqi chizilgan aylana markazi **o’rta perpendikulyar kesishgan nuqtada** bo’ladi.
3. Uchburchakka tashqi va ichki chizilgan aylanalar radiusi R va r, **aylana markazlari orasidagi masofa** d ga teng bo'lsa, u holda $d^2 = R^2 - 2R\cdot r$ bo'ladi.
4. $r = \dfrac{S}{p} = \dfrac{\sqrt{p(p-a)(p-b)(p-c)}}{p}$, $p = \dfrac{a+b+c}{2}$.
5. $R = \dfrac{abc}{4S} = \dfrac{abc}{4\sqrt{p(p-a)(p-b)(p-c)}}$.
6. $R = \dfrac{p}{4\cos\dfrac{\alpha}{2}\cdot\cos\dfrac{\beta}{2}\cdot\cos\dfrac{\gamma}{2}}$.



7. $r=(p-a)tg\frac{\alpha}{2}=(p-b)tg\frac{\beta}{2}=(p-c)tg\frac{\gamma}{2}=$

$$=p\cdot tg\frac{\alpha}{2}tg\frac{\beta}{2}tg\frac{\gamma}{2}=4R\cdot Sin\frac{\alpha}{2}Sin\frac{\beta}{2}Sin\frac{\gamma}{2}.$$

Uchburchaklarnlng o'xshashligi

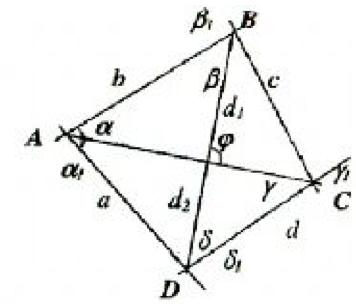
a_1, b_1, c_1 va a_2, b_2, c_2 - o'xshash uchburchaklar **tomoni**, P_1 va P_2 - **perimetri**, S_1 va S_2 - **yuzlari**.

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{P_1}{P_2}, \quad \frac{S_1}{S_2}=\left(\frac{a_1}{a_2}\right)^2=\cdots=\left(\frac{P_1}{P_2}\right)^2.$$

Ixtiyoriy qavariq to'rtburchak

1. d_1 va d_2 - **diagonallar** uzuniigi. φ - diagonallar orasidagi **burchak**.

2. Qavariq to'rtburchakni **yuzi:** $S=\frac{1}{2}d_1d_2\sin\varphi$.



3. $\alpha, \beta, \gamma, \delta$ - **ichki** burchaklari:

$$\alpha+\beta+\gamma+\delta=360^\circ.$$

4. $\alpha_1, \beta_1, \gamma_1, \delta_1$ - **tashqi** burchaklari:

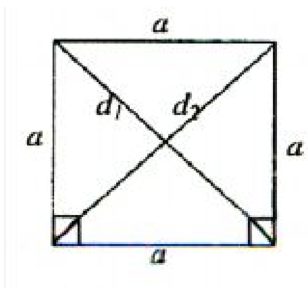
$$\alpha_1+\beta_1+\gamma_1+\delta_1=360^\circ.$$

5. P - perimetri, $P=a+b+c+d$,

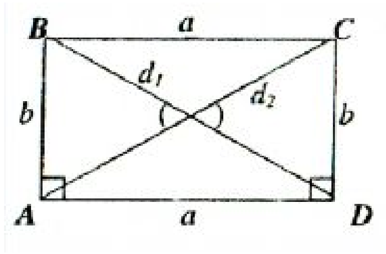
bunda ***a, b, c, d*** – to’rtburchak tomonlarining uzunligi.

Kvadrat

- $d_1=d_2=d$, $d_1\perp d_2$, $d=\sqrt{2}a$;
- $S=a^2$, $S=\frac{1}{2}d^2$, $R=\frac{d}{2}$, $r=\frac{a}{2}$,
- $P=4a$.



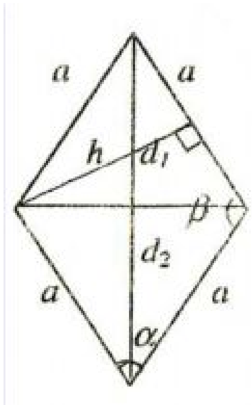
To'g'ri to'rtburchak



- $\angle A = \angle B = \angle C = \angle D = 90^\circ$, $R = \dfrac{d}{2}$;
- $d_1 = d_2 = d$, $d = \sqrt{a^2 + b^2}$;
- $S = \dfrac{1}{2} d^2 \sin\varphi$, $S = ab$;
- $P = 2(a+b)$ ***a*, *b*** - to'g'ri to'rtburchak tomonlari, ***d*** – diagonali.

Romb

a - romb tomoni, d_1, d_2 - diagonallari, h - balandligi, α - o'tkir burchagi, β - o'tmas burchagi.



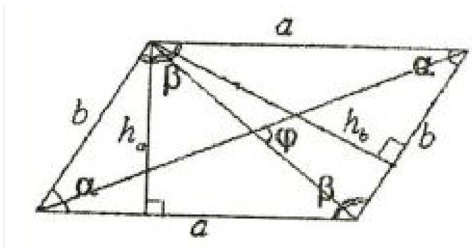
- $d_1 \perp d_2$, $\alpha + \beta = 180^\circ$, $P = 4a$;
- $S = ah = 2ar$, $S = \dfrac{d_1 \cdot d_2}{2}$, $S = a^2 \sin\alpha$;
- $d_1^2 + d_2^2 = 4a^2$, $d_1 = 2a\cos\dfrac{\beta}{2}$, $d_2 = 2a \cdot \sin\dfrac{\beta}{2}$;
- $r = \dfrac{h}{2}$, $r = \dfrac{1}{2}\sin\alpha$, $r = \dfrac{S}{2a}$.

Parallelogramm

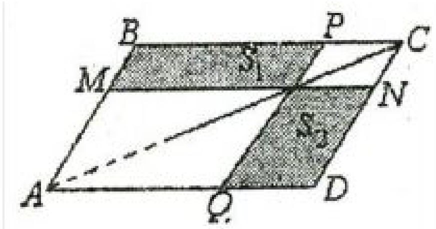
$a = BC = AD$, $b = AB = CD$-parallelogramm tomonlari, φ - diagonallar orasidagi burchak, α - o'tkir burchagi, β - o'tmas burchagi, d_1, d_2 - parallelogrammning diagonallari, O–diagonallar kesish nuqtasi.

- $AO = OC = \dfrac{d_1}{2}$, $BO = OD = \dfrac{d_2}{2}$, $\alpha + \beta = 180^\circ$; $P = 2(a+b)$;
- $d_1^2 + d_2^2 = 2(a^2 + b^2)$;
- $S = ab\sin\alpha$, $S = d_1 d_2 \sin\varphi$,

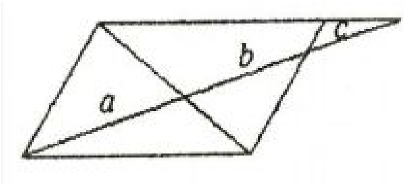
  $S = a \cdot h_a = b \cdot h_b$; $S = \dfrac{a^2 - b^2}{2} tg\varphi$;



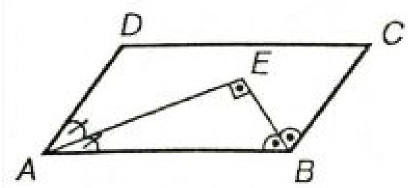
- $d_1^2 = a^2 + b^2 - 2ab\cos\alpha$, $d_2^2 = a^2 + b^2 - 2ab\cos\beta$.
- Parallelogramning ichidan olingan nuqtadan uning tomonlarigacha masofalar yig'indisi, bir uchidan chiqqan balandliklarni yig'indisiga teng.



- $S_1 = S_2;\ MN \parallel BC,\ PQ \parallel DC$;

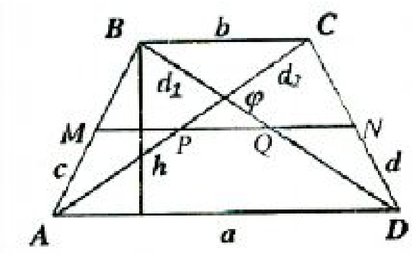


- $a^2 = b(b+c)$;

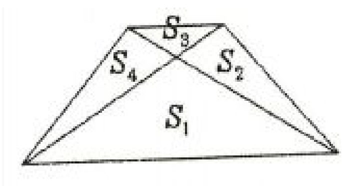


- parallelogramda bissektrisalar 90° burchak ostida kesishadi.

Trapetsiya

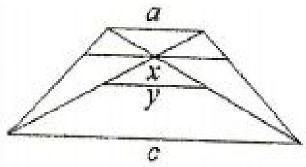


- $MN = \dfrac{a+b}{2}$ -o'rta chizig'i;
- $S = MN \cdot h = \dfrac{1}{2}d_1 d_2 \sin\varphi \quad S = \dfrac{(a+b)h}{2}$;
- $d_1^2 + d_2^2 = c^2 + d^2 + 2ab$;
- $MP = QN = \dfrac{b}{2}, \quad MQ = PN = \dfrac{a}{2},\ PQ = \dfrac{a-b}{2}$;
- agar $c = d,\ \ \varphi = 90^0,\ \ d_1 = d_2$ bo`lsa, $h = \dfrac{a+b}{2}, \quad S = h^2$;
- agar $c = d = c_0, \quad d_1 = d_2 = d_0$ bo`lsa, $a \cdot b = d_0^2 - c_0^2$;
- $a + b = c + d$ bo'lsa, trapetsiyaga ichki aylana chizish mumkin;
- $\boldsymbol{c = d}$ bo'lsa, unga tashqi aylana chizish mumkin;

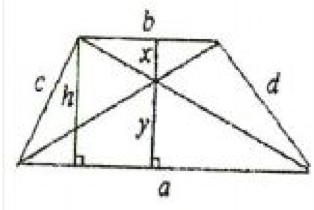


- $S_2 = S_4 = \sqrt{S_1 \cdot S_3}$;
- $S = \left(\sqrt{S_1} + \sqrt{S_3}\right)^2$;

- $x=\dfrac{2ac}{a+c}$ - chiziq;

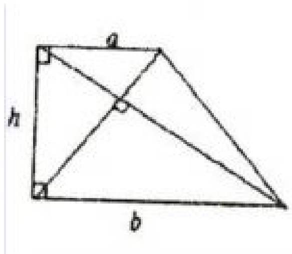


- $y=\dfrac{c-a}{2}$ – diagonallar o’rtasini tutashtiruvchi kesma;
- $h=x+y$;
- $y=\dfrac{a\cdot h}{a+b}$,
- $x=\dfrac{b\cdot h}{a+b}$;

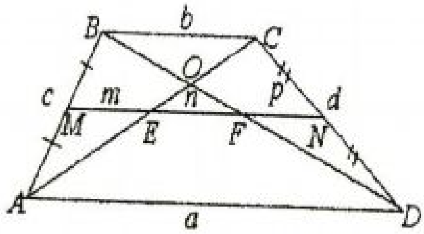


$h^2=a\cdot b$;

$2r=h=\dfrac{2ab}{a+b}$;

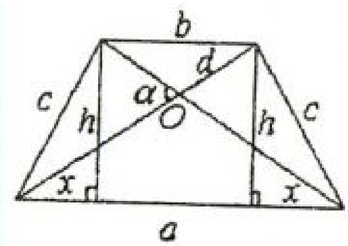


- $AE=EC,\ \ BF=FD$;
- $\dfrac{a+b}{2}=m+n+p=MN$;
- $n=\dfrac{a-b}{2},\ \ m=p=\dfrac{b}{2};\ S_{ABCD}=MN\cdot h$,



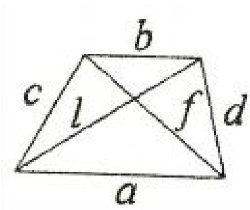
- $\dfrac{AO}{OC}=\dfrac{OD}{OB}=\dfrac{a}{b},\ S_{ABCD}=\dfrac{a+b}{2}h$; • $S_{MNCB}=S_{ADNM}\Rightarrow MN=\sqrt{\dfrac{a^2+b^2}{2}}$

- $x=\dfrac{b-a}{2},\ \ c^2=h^2+\dfrac{(b-a)^2}{4}$;
- $S=\dfrac{1}{2}d^2\sin\alpha=\dfrac{a+b}{2}h$;



- Teng yonli trapetsiyaga ichki chizilgan aylananing diametric trapetsiyaning balandligiga mos keladi va u quyidagiga teng: $h=\sqrt{ab}$, bu yerda ***a***, ***b*** – trapetsiyalarning asoslari;

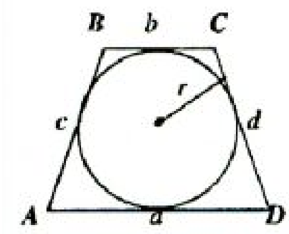
- $l=ab+\dfrac{c^2a-d^2b}{a-b}$;
- $f=ab+\dfrac{d^2a-c^2b}{a-b}$;



- To’g’ri burchakli trapetsiyada $l^2-f^2=a^2-b^2$.

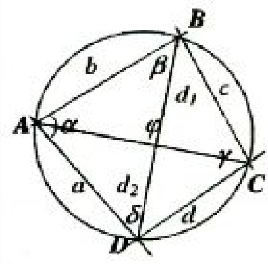
To'rtburchakka ichki chizilgan aylana

- $a+b=c+d$;
- $S=pr=(a+c)r=(b+d)r,\ 2p=a+b+c+d;$
- $S=\sqrt{(p-a)(p-b)(p-c)(p-d)}\ , S=\sqrt{a\cdot b\cdot c\cdot d}.$



To'rtburchakka tashqi chizilgan aylana

- $\alpha+\gamma=\beta+\delta=180^0;\quad a\cdot b+c\cdot d=d_1\cdot d_2;$
- $S=\sqrt{(p-a)(p-b)(p-c)(p-d)}\ , p=\dfrac{a+b+c+d}{2};$
- $R=\dfrac{1}{4S}\sqrt{(ab+cd)(ac+bd)(ad+bc)}.$

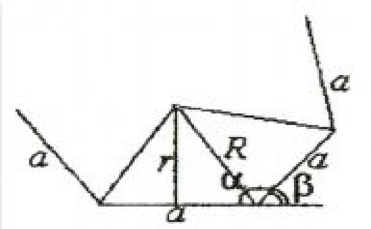


Ko'pburchaklar

- Qavariq ko'pburchak ichki burchaklarining yig'indisi -$(n-2)\pi$ ga teng. ***n*** - ko'pburchak tomonlarining soni;
- Ko'pburchakning bitta burchagining gradus o'lchovi - $\dfrac{\pi(n-2)}{n}$ ga teng;
- Tashqi burchaklar yig'indisi - 2π ga teng;
- Ko'pburchakning diagonallari soni — $\dfrac{n(n-3)}{2}$;
- Eyler formulasi: $U+Y=Q+2$, bu yerda U - qavariq ko’pburcgak uchlari soni, Y - yoqlari soni, Q - qirralari soni.

Muntazam ko'pburchaklar

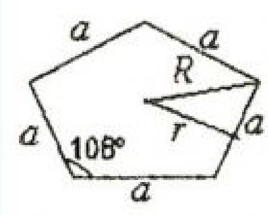
- Ichki burchagi - $\dfrac{(n-2)\pi}{n}$;
- Tashqi burchagi - $\dfrac{2\pi}{n}$;
- $r=\dfrac{1}{2}\sqrt{4R^2-a^2}$, $r=\dfrac{a}{2\cdot tg\dfrac{\pi}{n}}$, $R=\dfrac{a}{2\cdot sin\dfrac{\pi}{n}}$;



- $S=\frac{1}{2}R^2n\cdot sin\frac{360^\circ}{n}, S=p\cdot r=\frac{1}{2}a\cdot n\cdot r=\frac{n\cdot a\sqrt{4R^2-a^2}}{4}=\frac{R^2n\cdot a}{2}sin\frac{360^0}{n}$;
- $a=2R\cdot sin\frac{\pi}{n}=2r\cdot tg\frac{\pi}{n}$;
- $\alpha+\beta=180^0$.

Muntazam beshburchak

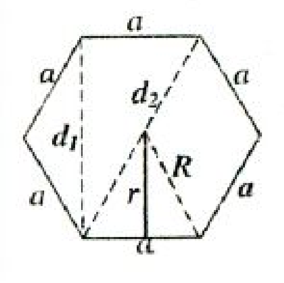
- Ichki burchaklar yig'indisi - 540^0;
- Ichki burchagi: 108^0;
- Tashqi burchagi: 72^0;



- $a=\frac{R}{2}\sqrt{10-2\sqrt{5}}=2r\sqrt{5-2\sqrt{5}}$;
- $R=\frac{a}{10}\sqrt{50+10\sqrt{5}}=r\left(\sqrt{5}-1\right)$; $r=\frac{a}{10}\sqrt{25+10\sqrt{5}}=\frac{R}{4}\left(\sqrt{5}+1\right)$;
- $d=\frac{1+\sqrt{5}}{2}\cdot a$, d – diagonal;
- $S=\frac{5}{8}R^2\sqrt{10+2\sqrt{5}}=\frac{a^2}{4}\sqrt{25+10\sqrt{5}}=5r^2\sqrt{5-2\sqrt{5}}$;

Muntazam oltiburchak

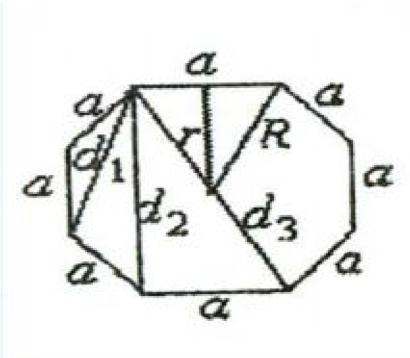
- Ichki burchaklar yig`indisi - 720°;
- Ichki burchagi: 120°;
- Tashqi burchagi: 60°;



- $a=R=\frac{2}{3}r\sqrt{3}$, $r=\frac{a\sqrt{3}}{2}$;
- $r=\frac{R}{4}\left(\sqrt{5}+1\right)=\frac{a}{10}\sqrt{25+10\sqrt{5}}$;
- $d_1=\sqrt{3}a$, $d_2=2R=2a$; $S=\frac{3\sqrt{3}}{2}\cdot a^2=\frac{3\sqrt{3}}{2}\cdot R^2=2\sqrt{3}\cdot r^2$.

Muntazam sakkizburchak

- Ichki burchaklar yig`indisi - 1080°;
- Ichki burchagi: 135°;
- Tashqi burchagi: 45°;



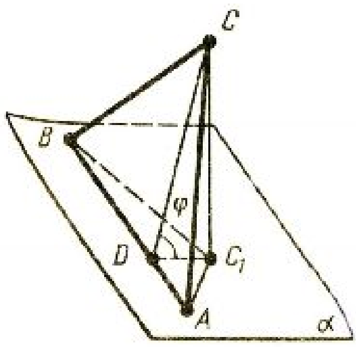
- $a = R\sqrt{2-\sqrt{2}} = 2r\left(\sqrt{2}-1\right)$;
- $R = r\sqrt{4-2\sqrt{2}} = \frac{a}{2}\sqrt{4+2\sqrt{2}}$;
- $r = \frac{R}{2}\sqrt{2+2\sqrt{2}} = \frac{a}{2}\left(\sqrt{2}+1\right)$;
- $d_1 = a\sqrt{2+\sqrt{2}}$, $d_2 = a\left(1+\sqrt{2}\right)$, $d_2 = 2R = \frac{a}{\sqrt{2-\sqrt{2}}}$;
- $S = 2\sqrt{2}\cdot R^2 = 2\cdot a^2\left(\sqrt{2}+1\right) = 8\cdot r^2\left(\sqrt{2}-1\right)$;

O'xshash ko'pburchaklar

- $\frac{S_1}{S_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{p_1}{p_2}\right)^2$, S_1 va S_2 - o'xshash ko'pburchak yuzlari, a_1, a_2 - mos tomonlari, p_1 va p_2 - perimetrlari.

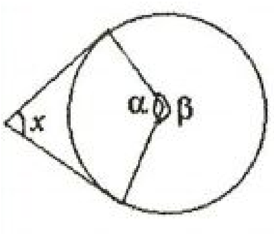
Ko'pburchak ortogonal poeksiyasining yuzi

- $S_{proek} \equiv S_{ABC_1} = S_{ABC}\cdot cos\varphi.$

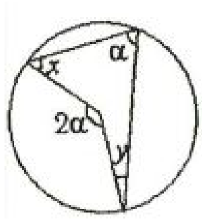


Aylanadagi burchaklar va vatarlar

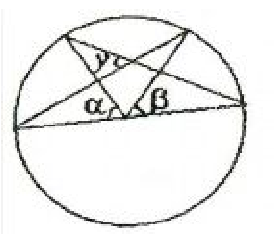
- $x = \frac{\beta-\alpha}{2}$;



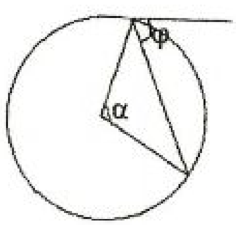
$\alpha = x + y$;



- $y = \frac{\alpha+\beta}{2}$;



$\varphi = \frac{\alpha}{2}$;



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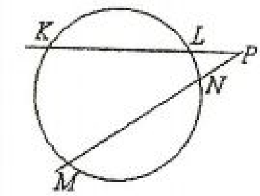
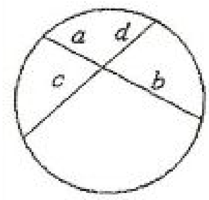
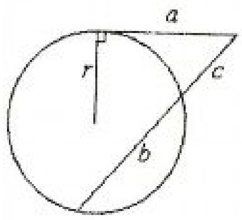
- $PL \cdot PK = PN \cdot PM$;



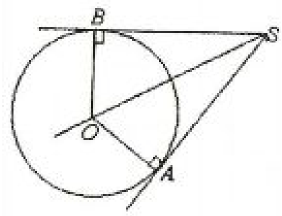
$ab = cd$;



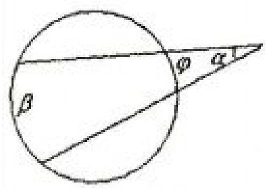
- $a^2(b+c)c$



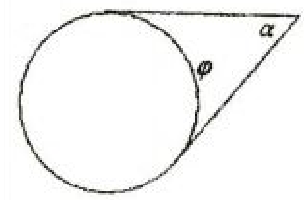
$AS = BS, \quad \angle ASO = \angle BSO$;



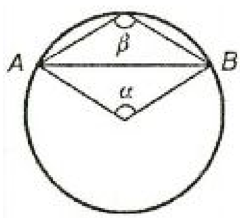
- $\alpha = \dfrac{|\beta - \varphi|}{2}$;

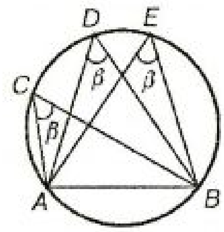


$\alpha + \varphi = 180^0$;

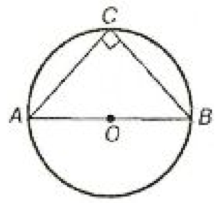


- $\beta = \dfrac{360^0 - \alpha}{2}$ AB – vatar



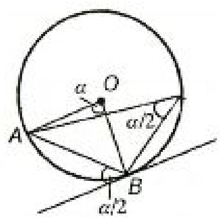


- $\angle C = 90^0$, $AB = d = 2R$;

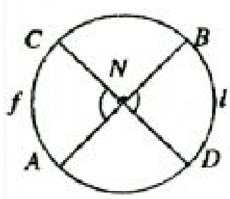


$\angle AOB = \alpha$

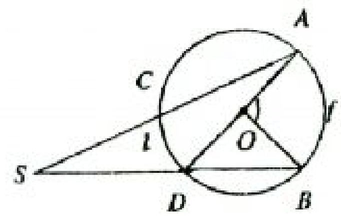
$\angle ACB = \angle ABD = \dfrac{\alpha}{2}$



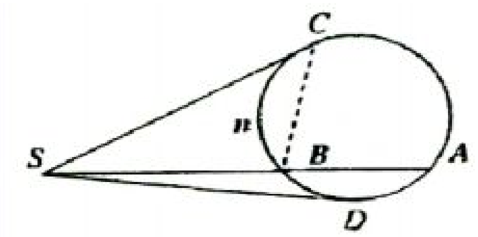
- $AN \cdot NB = CN \cdot ND$;
- $\angle ANC = \angle BND = \dfrac{1}{2}(\overset{\frown}{AfC} + \overset{\frown}{BlD})$;



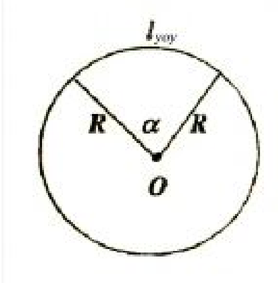
- $\angle ADB = \dfrac{1}{2}\overset{\frown}{AfB}$; $\angle AOB = \overset{\frown}{AfB}$;
- $\angle ASB = \dfrac{1}{2}(\overset{\frown}{AfB} - \overset{\frown}{ClD})$; $SC \cdot SA = SD \cdot SB$;



- $\angle BSC = \angle ASC = 0{,}5 \cdot \overset{\frown}{BnC}$;
- $\angle CSD = \dfrac{1}{2}(\overset{\frown}{CAD} - \overset{\frown}{CBD})$;
- $SC^2 = SA \cdot SB$, $CS = DS$



Aylana

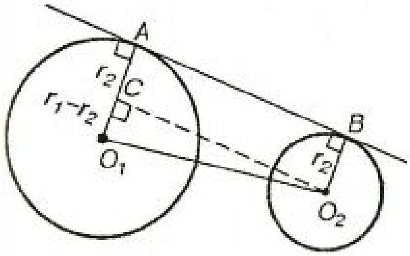


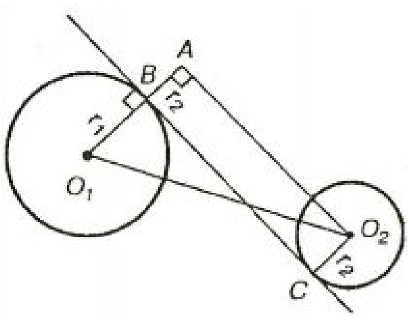
- $d = 2R, \quad C = 2\pi R = \pi d$ **- aylana uzunligi;**
- $l_{yoy} = \dfrac{\pi R \alpha}{180^\circ}$, $l_{yoy} = \alpha_{rad} R$, ***l*** **- yoy uzunligi;**
- $\alpha_{rad} = \dfrac{\pi\, \alpha^0}{180^\circ}$, α^0 - markaziy burchakning gradus o'lchovi,

  α_{rad} - radian o'lchovi;
- Markazi ***(a; b)*** nuqtada radiusi ***R*** ga teng aylana tenglamasi:

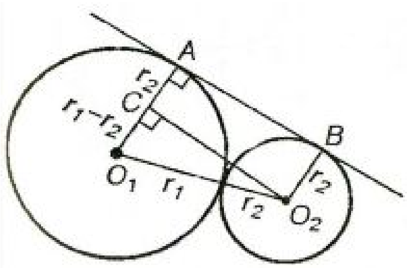
  $$(x-a)^2 + (y-b)^2 = R^2;$$
- Markazi koordinata boshida ***O(0; 0)*** radiusi ***R*** ga teng aylana tenglamasi: $x^2 + y^2 = R^2$
- Ikkita aylanaga o'tkazilgan urinma va kesuvchilar:

$$CO_2 \parallel AB, \quad CO_2 = AB,$$
$$|O_1O_2|^2 = (r_1 - r_2)^2 + (CO_2)^2$$





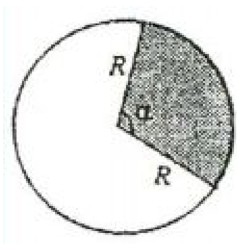
$$BC \parallel AO_2, \quad |BC| = |AO_2|,$$
$$|O_1O_2|^2 = (r_1 + r_2)^2 + |AO_2|^2$$



$$|AB| = |CO_2| = 2\sqrt{r_1 \cdot r_2}.$$

Doira va doiraviy figuralar

- **Doira yuzi:** $S = \pi R^2$, $S = \dfrac{1}{4}\pi d^2$;



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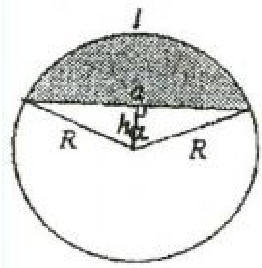
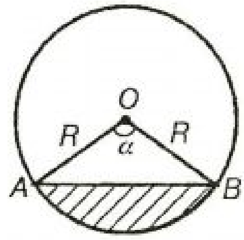
- **Sektor yuzi:** $S_{sek} = \dfrac{\pi R^2 \alpha}{360^\circ}$;



- **Segment yuzi:**

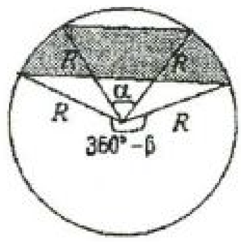
$$S_{seg} = \frac{\pi R^2 \alpha}{360^\circ} - \frac{1}{2} R^2 \sin\alpha =$$

$$= \frac{R(l-a) + ah}{2} = \frac{\pi R^2 \alpha}{360^\circ} \mp S_{\Delta AOB};$$

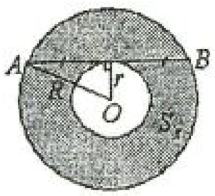


- **Kesim yuzi:**

$$S_{kes} = \frac{\pi R^2}{360^\circ}(\beta - \alpha) - \frac{1}{2} R^2 (\sin\beta - \sin\alpha);$$

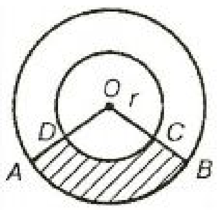


- **Halqa yuzi:** $S_{hal} = \pi\left(R^2 - r^2\right) = \pi\left(\dfrac{AB}{2}\right)^2$;

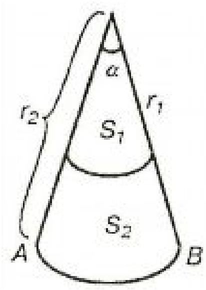


- $OC = r, \quad OB = R,$

$S_{ABCD} = \dfrac{\alpha\pi}{360}\left(R^2 - r^2\right)$;



$\dfrac{S_1}{S_1 + S_2} = \left(\dfrac{r_1}{r_2}\right)^2$.



Nuqtalar orasidagi masofa

- $A(x_1; y_1), \quad B(x_2; y_2): \quad |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$;
- $A(x_1; y_1; z_1), B(x_2; y_2; z_2): |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

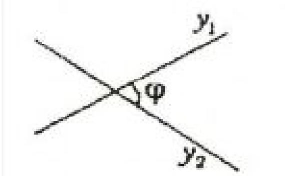
Kesma o'rtasining koordinatalari

- $A(x_1; y_1), \quad B(x_2; y_2): \quad x = \dfrac{x_1 + x_2}{2}, \quad y = \dfrac{y_1 + y_2}{2}$;
- $A(x_1; y_1; z_1), \; B(x_2; y_2; z_2): \; x = \dfrac{x_1 + x_2}{2}, \; y = \dfrac{y_1 + y_2}{2}, \; z = \dfrac{z_1 + z_2}{2}$.

To'g'ri chiziq

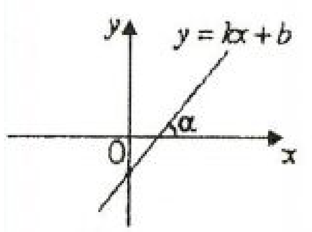
- $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1},\quad k=\frac{y_1-y_2}{x_1-x_2};$$



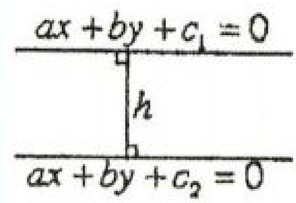
- $y_1=k_1x+b_1,\quad y_2=k_2x+b_2;$
- $A(x_1;y_1)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi:

$$y-y_1=k(x-x_1);$$

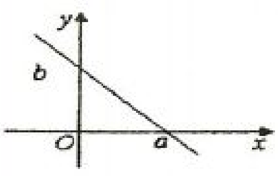


- $tg\varphi=\frac{|k_1-k_2|}{1+k_1\cdot k_2};$
- Parallellik alomati: $k_1=k_2;$
- Perpendikulyarlik alomati: $k_1\cdot k_2=-1;$
- Kesishish alomati: $k_1\neq k_2;$
- $y=kx+b,\quad k=tg\alpha;$
- 3 nuqtaning bir to'g'ri chiziqda yotish sharti: $\frac{x_0-x_1}{x_2-x_0}=\frac{y_0-y_1}{y_2-y_0};$
- $A(x_0;y_0)$ nuqtadan $ax+by+c=0$ to'g'ri chiziqqacha bo'lgan masofa: $d=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}};$
- Parallel to'g'ri chiziqlar orasidagi masofa:

$$h=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}};$$



- $\frac{x}{a}+\frac{y}{b}=1.$



- To'g'ri chiziqning umumiy ko'rinishdagi tenglamasi:

$$ax+by+c=0,\quad a,b,c\in R,\quad a^2+b^2\neq 0.$$

- $a_1x+b_1y+c_1=0$ va $a_2x+b_2y+c_2=0$ to'g'ri chiziqlar orasidagi burchaklar **bissektrisalarining tenglamalari**:

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}.$$

FAZODA TEKISLIK VA TO'G'RI CHIZIQ

1. Tekislikning umumiy ko'rinishdagi tenglamasi:

$$Ax+By+Cz+D=0,\quad A,B,C,D\in R,\quad A^2+B^2+C^2\neq 0;$$

- $M(x_0,y_0,z_0)$ nuqtadan o`tib $\vec{p}=(\alpha_1,\beta_1,\gamma_1)$ va $\vec{q}=(\alpha_2,\beta_2,\gamma_2)$ vektorlarga parallel bo`lgan tekislikning umumiy tenglamasi:

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \end{vmatrix}=0;$$

- Uchta $M_0(x_0,y_0,z_0)$, $M_1(x_1,y_1,z_1)$ va $M_2(x_2,y_2,z_2)$ nuqtalardan o`tuvchi tekislik tenglamasi:

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}=0;$$

- Tekislikning koordinata o'qlardan ajratgan kesmalarga nisbatan tenglamasi: $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1;$
- $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ tenglama bilan berilgan tekisliklar orasidagi φ burchakni topish formulasi:

$$cos\,\varphi=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\cdot\sqrt{A_2^2+B_2^2+C_2^2}};$$

- Parallellik sharti: $\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2};$
- Perpendikulyarlik sharti: $A_1A_2+B_1B_2+C_1C_2=0;$
- $M(x_0,y_0,z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikgacha bo'lgan masofa:

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}};$$

2. To'g'ri chiziqning kanonik tenglamasi:

$$l: \quad \frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p},$$

bu erda $\vec{s}=\{m,n,p\}-l$ to`g`ri chiziqning yo`naltiruvchi vektori.

- $M_0(x_0,y_0,z_0)$ va $M_1(x_1,y_1,z_1)$ nuqtalardan o`tuvchi to`g`ri chiziq tenglamasi:

$$\frac{x-x_0}{x_1-x_0}=\frac{y-y_0}{y_1-y_0}=\frac{z-z_0}{z_1-z_0};$$

- $\frac{x-x_0}{m_0}=\frac{y-y_0}{n_0}=\frac{z-z_0}{p_0}$ va $\frac{x-x_1}{m_1}=\frac{y-y_1}{n_1}=\frac{z-z_1}{p_1}$ to`g`ri chiziqlar orasidagi φ burchakni topish formulasi:

$$\cos\varphi=\frac{m_1m_0+n_1n_0+p_1p_0}{\sqrt{m_1^2+n_1^2+p_1^2}\cdot\sqrt{m_0^2+n_0^2+p_0^2}}.$$

3. Fazoda tekislik va to'g'ri chiziq:

Fazoda $l: \quad \frac{x-x_1}{m}=\frac{y-y_1}{n}=\frac{z-z_1}{p}$ to`g`ri chiziq va

$Q:\quad Ax+By+Cz+D=0$ tekislik berilgan bo`lib, $\vec{s}=\{m,n,p\}$ - $l$ to`g`ri chiziqning yo`naltiruvchi vektori; $\vec{n}=\{A,B,C\}$ - Q tekislikning normal vektori bo`lsin. **Unda:**

- Agar $\vec{s}\parallel\vec{n}$ bo`lib, $Q\perp l$ bo`lsa, $\frac{A}{m}=\frac{B}{n}=\frac{C}{p}$ bo`ladi;
- Agar $\vec{s}\perp\vec{n}$ bo`lib, $Q\parallel l$ bo`lsa, $Am+Bn+Cp=0$ bo`ladi;
- $\boldsymbol{l}$ to'g'ri chiziq va $\boldsymbol{Q}$ tekislik orasidagi burchak:

$$\sin\alpha=\frac{|A\cdot m+B\cdot n+C\cdot p|}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{m^2+n^2+p^2}};$$

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- $M_1(x_1, y_1, z_1)$ nuqta orqali o'tib $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ to'g'ri chiziqqa parallel bo'lgan to'g'qi chiziq tenglamasi:

$$\frac{x-x_1}{m}=\frac{y-y_1}{n}=\frac{z-z_1}{p};$$

- $M_1(x_1, y_1, z_1)$ nuqta orqali o'tib $Ax+By+Cz+D=0$ tenglamaga perpendikulyar bo'lgan to'g'ri chiziq tenglamasi:

$$\frac{x-x_1}{A}=\frac{y-y_1}{B}=\frac{z-z_1}{C};$$

- $M_1(x_1, y_1, z_1)$ nuqtadan va $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ to'g'ri chiziqdan o'tuvchi tekislik tenglamasi:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_0-x_1 & y_0-y_1 & z_0-z_1 \\ m & n & p \end{vmatrix}=0;$$

- $\frac{x-x_0}{m_1}=\frac{y-y_0}{n_1}=\frac{z-z_0}{p_1}$ va $\frac{x-x_0}{m_2}=\frac{y-y_0}{n_2}=\frac{z-z_0}{p_2}$ to'g'ri chiziqlarning bir tekislikda yotish sharti:

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix}=0;$$

- $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ to'g'ri chiziqning $Ax+By+Cz+D=0$ tekislikda yotish sharti:

$$\begin{cases} Am+Bn+Cp=0 \\ Ax_0+By_0+Cz_0=0 \end{cases};$$

- $M_1(x_1, y_1, z_1)$ nuqtadan o'tib $\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ to'g'ri chiziqqa perpendikulyar bo'lgan tekislik tenglamasi:

$$m(x-x_1)+n(y-y_1)+p(z-z_1)=0.$$

IKKINCHI TARTIBLI EGRI CHIZIQLAR

1. **Ikkinchi tartibli egri chiziqning umumiy ko'rinishdagi tenglamasi:**

$$Ax^2 + 2Bxy + Cy^2 + Dx + Ey + F = 0, \qquad (1)$$

bunda $A, B, C, D, E, F \in R, \quad A^2 + B^2 + C^2 \neq 0$.

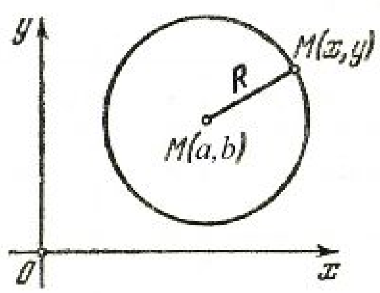
2. Agar $B = 0$ bo`lsa, u holda (1) tenglamadan **markaziy egri chiziq tenglamasini** olamiz:

$$Ax^2 + Cy^2 = \Delta, \quad \Delta = \frac{D^2}{4A} + \frac{E^2}{4C} - F. \qquad (2)$$

Aylana

1. Aylananing umumiy tenglamasi:

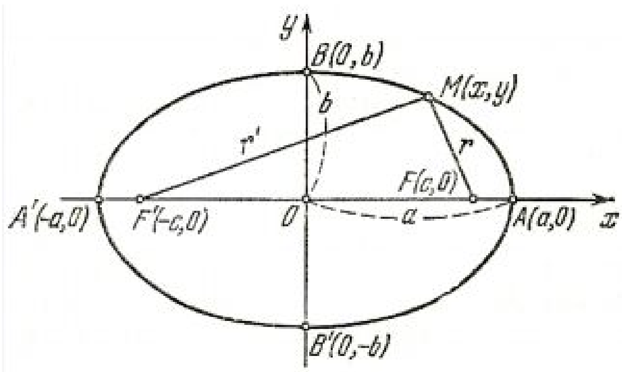
$$Ax^2 + Ay^2 + Dx + Ey + F = 0, \quad A \neq 0.$$



2. Markazi $M(a,b)$ nuqtada yotuvchi va radiusi R bo`lgan aylana tenglamasi: $(x-a)^2 + (y-b)^2 = R^2$.

Ellips

1. Agar $A > 0, \; C > 0, \; \Delta > 0$ bo`lsa, u holda (2) tenglamadan **ellips tenglamasini** olamiz: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, \quad a = \sqrt{\dfrac{\Delta}{A}}, \quad b = \sqrt{\dfrac{\Delta}{C}}$. (3)



2. (3) tenglama koordinata o`qlariga nisbatan simmetrik bo`lib, **ellipsning kanonik** tenglamasidir.

3. Ellipsning $F_1(c,0)$ va $F_2(-c,0)$ fokuslari orasidagi masofa:

   $\varepsilon = \dfrac{2c}{2a} = \dfrac{c}{a} < 1$, bunda $0 \le \varepsilon < 1$ - ellipsning **ekssentrisiteti**.

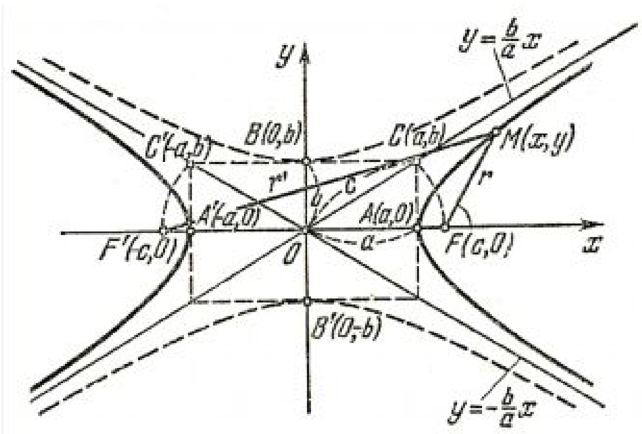
4. Ellipsning **direktrisalari** $d_1: \ x - \dfrac{a}{\varepsilon} = 0; \quad d_2: \ x + \dfrac{a}{\varepsilon} = 0$ tenglamalardan iboratdir.

5. Ellipsning **fokal radiuslari**: $r_1 = a - \varepsilon x; \quad r_2 = a + \varepsilon x \Rightarrow r_1 + r_2 = 2a$.

Giperbola

1. Agar $A > 0, \ C < 0, \ \Delta > 0$ bo`lsa, u holda (2) tenglamadan **giperbola tenglamasini** olamiz:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \quad a = \sqrt{\frac{\Delta}{A}}, \quad b = \sqrt{\frac{\Delta}{-C}} \qquad (4)$$



2. Giperbolaning **fokal radiuslari**:

   $r_1 = \pm(\varepsilon x - a); \quad r_2 = \pm(\varepsilon x + a) \Rightarrow |r_1 - r_2| = 2a, \ 1 < \varepsilon < +\infty, \ |x| \ge a$,

   bunda $x > 0$ da $+$ ishorasi, $x < 0$ da $-$ ishorasi olinadi.

3. Giperbolaning **asimtotasi:** $y = \pm \dfrac{b}{a} x$.

4. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ va $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = -1$ giperbolalar **qo`shma giperbolalardir**.

5. Giperbolaning **ekssentrisiteti:** $\varepsilon = \dfrac{c}{a}, \quad 1 < \varepsilon < +\infty$.

6. Giperbolaning $F_1(c,0), \quad F_2(-c,0)$ fokuslarga mos **direktrisalarning** tenglamalari $d_1: \ x - \dfrac{a}{\varepsilon} = 0; \quad d_2: \ x + \dfrac{a}{\varepsilon} = 0$ dan iboratdir.

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Parabola

1. Ox (Oy) o`qqa simmetrik bo`lgan **parabolaning tenglamasi:**

$$y^2 = 2px \quad (x^2 = 2py).$$

2. Parabolaning **direktrisalari:** $x = -\frac{p}{2}$ $\left(y = -\frac{p}{2}\right)$.

3. Parabolaning **fokal radiuslari**: $r = x + \frac{p}{2}$ $\left(r = y + \frac{p}{2}\right)$.

4. Parabolaning **ekssentrisiteti:** $\varepsilon = 1$.

Ellips, giperbola ва parabolaning qutib tenglamasi

$$r = \frac{p}{1 - \varepsilon \cos\varphi}, \qquad (*)$$

bu erda ε $-$**ekssentrisitet,** p $-$**parametr:** ellips va giperbola uchun $p = \frac{b^2}{a}$; parabola uchun $p = 1$. Bu $(*)$ tenglama $\varepsilon < 1$ bo`lganda ellipsni, $\varepsilon = 1$ bo`lganda parabolani, $\varepsilon > 1$ bo`lganda esa giperbolani tasvirlaydi.

V E K T O R L A R

- Boshi $A(x_1; y_1; z_1)$, oxiri $B(x_2; y_2; z_2)$ nuqtada bo'lgan $\overrightarrow{AB}$ **vektor koordinatasi:** $\overrightarrow{AB} = (x_2 - x_1;\ y_2 - y_1;\ z_2 - z_1)$;
- Uchlarining koordinatalari bilan berilgan $\overrightarrow{AB}$ **vektor uzunligi:**
$$\overrightarrow{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2};$$
- Vektor $\vec{a} = (a_1, a_2, a_3)$ ko'rinishda ham beriladi. a_1, a_2, a_3 - $\vec{a}$ vektoring koordinatalari;
- $\vec{a} = (a_1, a_2, a_3)$ **vektor uzunligi:** $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$;
- $\overrightarrow{AB} = \vec{a}$ bo'lsa, $x_2 - x_1 = a_1$, $y_2 - y_1 = a_2$, $z_2 - z_1 = a_3$ bo'ladi;

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- $A(x_1;y_1;z_1)$, $B(x_2;y_2;z_2)$ va $C(x_3;y_3;z_3)$ ABC uchburchakni uchlari bo`lsa, BD **medianasi va** AC **asosi orasidagi** $\varphi-$ **burchakni topish:**

$$x_D=\frac{x_1+x_3}{2},\quad y_D=\frac{y_1+y_3}{2},\quad z_D=\frac{z_1+z_3}{2};\ \overrightarrow{BD}=\left(x_D-x_2,\,y_D-y_2,\,z_D-z_2\right),$$

$$\overrightarrow{AC}=\left(x_3-x_1,\ y_3-y_1,\ z_3-z_1\right)\ \Rightarrow\ \cos\varphi=\frac{\overrightarrow{BD}\cdot\overrightarrow{AC}}{\left|\overrightarrow{BD}\right|\left|\overrightarrow{AC}\right|};$$

- $ABCD$ to`rtburchakning tomonlari $\overrightarrow{AB}$, $\overrightarrow{BC}$ va $\overrightarrow{CD}$ bo`lsa, uning $\overrightarrow{AC}$ va $\overrightarrow{BD}$ diagonallari uchun $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$, $\overrightarrow{BD}=\overrightarrow{BC}+\overrightarrow{CD}=\overrightarrow{BC}+\overrightarrow{BA}$ o`rinli bo'ladi;
- $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorlar parallelogrammning tomonlari bo`lsa, $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AB}+\overrightarrow{AD}$, $\overrightarrow{BD}=\overrightarrow{BA}+\overrightarrow{AD}=\overrightarrow{AD}-\overrightarrow{AB}$ lar **parallelogrammning diogonallari bo'ladi**;
- $\overrightarrow{AB}(x_1,y_1,z_1)$ va $\overrightarrow{BC}(x_2,y_2,z_2)$ vektorlar parallelogrammning qyshni tomonlari, $\overrightarrow{AB}$ va $\overrightarrow{BC}$ vektorlar parallelogrammning diogonallari bo`lsa,

$$\overrightarrow{AB}(x_1,y_1,z_1)+\overrightarrow{BC}(x_2,y_2,z_2)=\overrightarrow{AC}(x_1+x_2;y_1+y_2;z_1+z_2),$$

$$\overrightarrow{BC}(x_2,y_2,z_2)-\overrightarrow{AB}(x_1,y_1,z_1)=\overrightarrow{BD}(x_2-x_1;y_2-y_1;z_2-z_1),$$

$$\cos\varphi=\frac{\overrightarrow{BD}\cdot\overrightarrow{AC}}{\left|\overrightarrow{BD}\right|\cdot\left|\overrightarrow{AC}\right|}=\frac{x_2^2+y_2^2+z_2^2-(x_1^2+y_1^2+z_1^2)}{\sqrt{(x_1+x_2)^2+(y_1+y_2)^2+(z_1+z_2)^2}\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}}$$

bo'ladi, bu erda $\varphi-$ $\overrightarrow{AC}$ va $\overrightarrow{BD}$ **vektorlar orasidagi burchak.**

Birlik vektorlar

- **Tekislikda:** $\vec{i}=(1;0)$, $\vec{j}=(0;1)$, $|\vec{i}|=1$, $|\vec{j}|=1$, $\vec{i}\cdot\vec{j}=0$,

$$\vec{a}(x;y)=x\cdot\vec{i}+y\cdot\vec{j};$$

- $\vec{e}$ - birlik vektor, $\vec{e}=\left(\frac{x}{\sqrt{x^2+y^2}};\frac{y}{\sqrt{x^2+y^2}}\right)$;
- **Fazoda:** $\vec{i}=(1;0;0)$, $\vec{j}=(0;1;0)$, $\vec{k}=(0;0;1)$, $|\vec{i}|=|\vec{j}|=|\vec{k}|=1$,

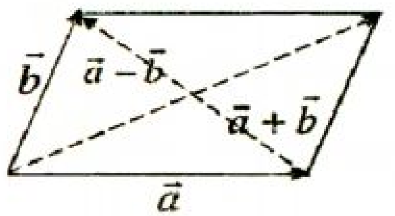
$$\left(\vec{i}\cdot\vec{j}\right)=\left(\vec{i}\cdot\vec{k}\right)=\left(\vec{k}\cdot\vec{j}\right)=0,\ \ \vec{a}=(x,y,z),\ \ \vec{a}=x\cdot\vec{i}+y\cdot\vec{j}+z\cdot\vec{k};$$

- $\vec{e}$ **- birlik vektorni toppish:**

$$\vec{e}=\left(\frac{x}{\sqrt{x^2+y^2+z^2}};\frac{y}{\sqrt{x^2+y^2+z^2}};\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

Vektorlar ustida amallar

- $\vec{a}=(a_1,a_2,a_3)$, $\vec{b}=(b_1,b_2,b_3)$, $\vec{c}=\vec{a}+\vec{b}$;
- $\vec{c}=\left\{\overrightarrow{a_1}\pm\overrightarrow{b_1};\ \overrightarrow{a_2}\pm\overrightarrow{b_2};\ \overrightarrow{a_3}\pm\overrightarrow{a_4}\right\}$;
- $\vec{a}\cdot\vec{b}=a_1\cdot b_1+a_2\cdot b_2+a_3\cdot b_3$;
- $\lambda\vec{a}=\left\{\lambda\overrightarrow{a_1};\lambda\overrightarrow{a_2};\lambda\overrightarrow{a_3}\right\}$.



Skalyar ko'paytma

- **Koordinatalari bilan berilgan bo'lsa:** $\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+a_3b_3$;
- **Modullari berilgan bo'lsa:** $\vec{a}\cdot\vec{b}=\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cdot\cos\varphi$

  bunda φ - $\vec{a}$ va $\vec{b}$ **orasidagi burchak;**
- $\lambda\left(\vec{a}\cdot\vec{b}\right)=\left(\lambda\vec{a}\right)\cdot\vec{b}=\vec{a}\cdot\left(\lambda\vec{b}\right)$, $\left(\vec{a}+\vec{b}\right)\cdot\vec{c}=\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}$;
- $\vec{a}\cdot\vec{a}=\left(\vec{a}\right)^2=\left|\vec{a}\right|^2$, $\left|\vec{a}\right|=\sqrt{\vec{a}\cdot\vec{a}}$;
- Ikki $\vec{a}$ va $\vec{b}$ vektor **orasidagi burchak:**

$$\cos\varphi=\frac{\vec{a}\cdot\vec{b}}{\left|\vec{a}\right|\cdot\left|\vec{b}\right|},\quad \cos\varphi=\frac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}};$$

- $\vec{a}\ ||\ \vec{b}$ bo'lsa, u holda ular orasidagi burchak $\varphi=0$ bo'ladi;
- Ikki $\vec{a}$ va $\vec{b}$ vektorning **perpendikulyarlik** sharti:

$$\vec{a}\cdot\vec{b}=0,\ \ a_1b_1+a_2b_2+a_3b_3=0;$$

- Ikki vektorning **parallellik** yoki kollinearlik sharti:

$$\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3};$$

- **Vektor ko'paytma:** $\vec{c} = \vec{a} \times \vec{b}$, $S = |\vec{a}| \cdot |\vec{b}| \cdot Sin\alpha$,

$$S = |\vec{a} \times \vec{b}| = \sqrt{\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}^2 + \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix}^2 + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}^2}.$$

- $\vec{a} = (a_1, a_2, a_3)$ vektorning **yo`naltiruvchi kosinuslari:**

$$\cos\alpha = \frac{a_1}{\sqrt{a_1^2 + a_2^2 + a_3^2}}; \; \cos\beta = \frac{a_2}{\sqrt{a_1^2 + a_2^2 + a_3^2}}; \; \cos\gamma = \frac{a_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}},$$

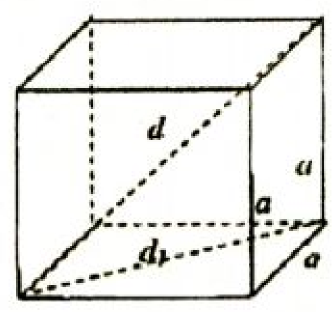
bundan $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

S T E R E O M E T R I Y A

Ko'pyoqlilar

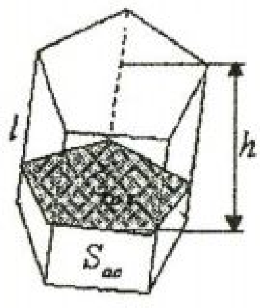
l - yon qirrasi uzunligi, ***P*** — asos perimetri uzunligi, ***S*** - asos yuzi, ***H*** – balandlik, P_{kes} - perpendikulyar kesim perimetri, S_{yon} - yon sin yuzi, S_t -to'la sirt yuzi, S_{kes} - perpendikulyar kesim yuzi, ***V*** - hajm.

Kub



- **Yon sirti:** $S_{yon} = 4a^2$;
- **To'la sirti:** $S_t = 6a^2$;
- **Hajmi:** $V = a^3$;
- $d = \sqrt{3}a$, $R = \frac{a\sqrt{3}}{2}$, $r = \frac{1}{2}a$;
- **9** ta simmetriya tekisligiga ega;
- **8** ta uch, **12** ta qirrasi, **6** ta yog'i bor.
- ***R*** va ***r*** - kubga tashqi va ichki chizilgan shar radiusi.

Ixtiyoriy prizma

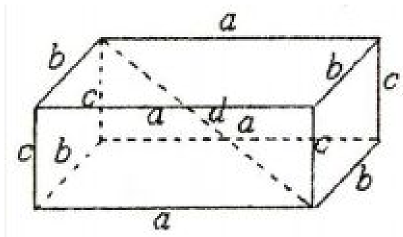


- **Yon sirti:** $S_{yon} = P_{kes} \cdot l$;
- **To'la sirti:** $S_t = S_{yon} + 2S_{asos}$;
- **Hajmi:** $V = S_{kes} \cdot l = S_{asos} \cdot h$;

- diagonallari soni: $n(n-3)$;

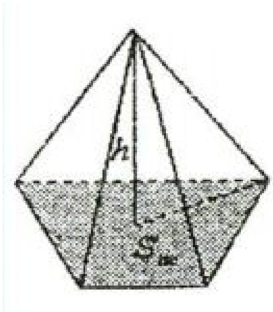
- ***n*** burchakli prizmaning ***3n*** ta qirrasi, ***n+2*** ta yog'i, ***2n*** ta uchi bor.

To'g'ri burchakli parallelepiped



- **Yon sirti:** $S_{yon} = P \cdot c = 2(a+b)c$;
- **To'la sirti:** $S_t = 2(ab + ac + bc)$;
- **Hajmi:** $V = a \cdot b \cdot c$;
- $d = \sqrt{a^2 + b^2 + c^2}$;
- **5** ta simmetriya tekisligiga ega; **8** ta uchi, **12** ta qirrasi bor.

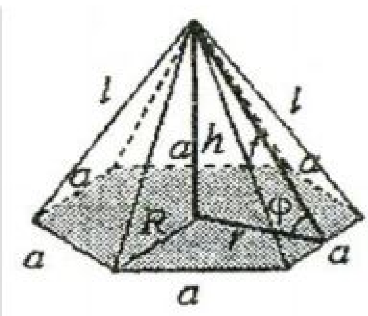
Ixtiyoriy piramida



- **To'la sirti:** $S_t = S_{asos} + S_{yon}$, $S_t = \dfrac{3V}{r}$;
- **Hajmi:** $V = S_{asos} \cdot h = \dfrac{1}{3} S_t \cdot r$;
- $S_{asos} = S_{yon} \cos\varphi$, φ - ikki yoqli burchak;
- ***n*** burchakli piramidaning ***2n*** ta qirrasi, ***n+1*** ta yog'i va uchi bor.

Muntazam piramida

l – yasovchi, ***f*** – apofema, R - tashqi va r-ichki radiuslar.

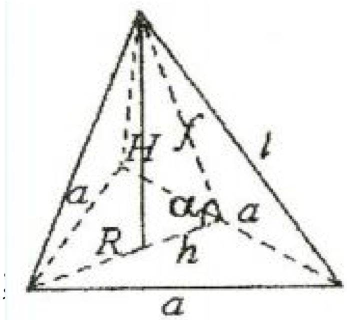


- $P_{asos} = n \cdot a$, $S_{asos} = n \cdot a \cdot r$;
- **Yon sirti:** $S_{yon} = \dfrac{P_{asos} \cdot f}{2} = \dfrac{S_{asos}}{\cos\varphi}$;
- **To'la sirti:** $S_t = S_{asos} + S_{yon}$;
- **Hajmi:** $V = \dfrac{1}{3} S_{asos} \cdot h$;
- $R^2 = \left(\dfrac{a}{2}\right)^2 + r^2$, $l^2 = R^2 + h^2$, $f^2 = r^2 + h^2$;

- Asosiga ichki chizilgan aylana radiusi r ga asosidagi ikki yoqli burchagi φ ga teng bo'lgan muntazam piramidaga r_{shar} radiusli shar ichki chizilgan bo'lsa: $r_{shar} = \dfrac{\sin\varphi}{1+\cos\varphi} \cdot r$.

Muntazam uchburchakli piramida

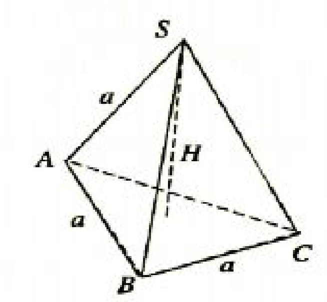
$\boldsymbol{l}$ – yon qirrasi, $\boldsymbol{f}$ – apofema, α - ikki yoqli burchak.



- **Yon sirti:** $S_{yon} = \dfrac{3}{2} a \cdot f$;
- **To’la sirti:** $S_t = \dfrac{a\sqrt{3}}{4}\left(a + \sqrt{a^2 + 12h^2}\right)$;
- **Hajmi:** $V = \dfrac{1}{3} S_{asos} H = \dfrac{a^2\sqrt{3}}{12} H$, $S_{asos} = \dfrac{a^2\sqrt{3}}{4}$
- $f = \sqrt{\dfrac{a^2}{12} + H^2} = \sqrt{r^2 + H^2}$, $l = \sqrt{\dfrac{a^2}{3} + H^2} = \sqrt{R^2 + H^2}$;
- $r = \dfrac{\sqrt{3} \cdot a}{6}$, $R = \dfrac{\sqrt{3} \cdot a}{3}$.

Muntazam tetraedr

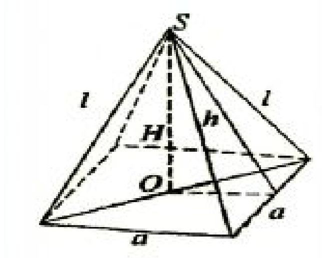
a - tetraedrning har bir qirrasi.



- **Yon sirti:** $S_{yon} = \dfrac{3\sqrt{3}}{4} a^2$;
- **To’la sirti:** $S_t = \sqrt{3} \cdot a^2$;
- **Hajmi:** $V = \dfrac{a^3\sqrt{2}}{12}$;
- $H = \dfrac{a\sqrt{2}}{\sqrt{3}}$, $R = \dfrac{3}{4} H$, $R = \dfrac{a\sqrt{6}}{4}$, $R = 3r$;
- $r = \dfrac{H}{4}$, $r = \dfrac{a\sqrt{6}}{12}$.

Muntazam to'rtburchakli piramida

l - yon qirrasi uzunligi, ***h*** - apofemasi, ***H*** - balandligi, ***a*** - asosining tomoni uzunligi, ***φ*** - ikki yoqli burchak.

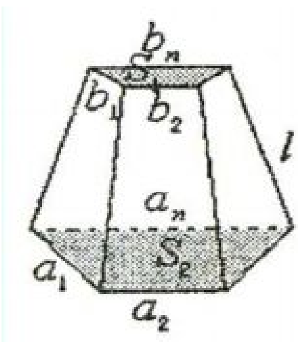


- **Yon sirti:** $S_{yon} = 2ah$, $S_{yon} = \dfrac{S_{asos}}{\cos\varphi}$;
- **To'la sirti:** $S_t = S_{asos} + S_{yon}$, $S_{asos} = a^2$;
- **Hajmi:** $V = \dfrac{1}{3}a^2H$, $H = \sqrt{l^2 - \dfrac{a^2}{2}}$;
- $h = \sqrt{\dfrac{a^2}{4} + H^2} = \sqrt{r^2 + H^2}$, $l = \sqrt{\dfrac{a^2}{2} + H^2} = \sqrt{R^2 + H^2}$, $r = \dfrac{\sqrt{3}\cdot a}{6}$, $R = \dfrac{\sqrt{2}\cdot a}{2}$.

Kesik piramida

- Piramidaning – ***H*** va kesik piramidaning - H_1 balandliklari;

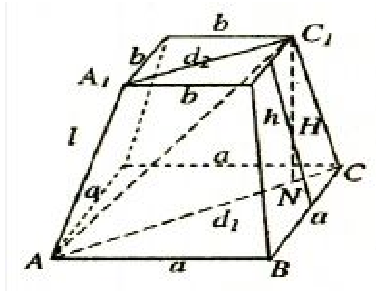
 l - yon qirrasi uzunligi, S_1 va S_2 - piramidaning asoslari yuzi, P_1 va P_2 - piramidaning asoslari piremetri, a_1 va a_2 - piramidaning asoslarnig uzunligi, hamda ***V***-piramida, V_1 -kesik piramida hajmi bo'lsin;
- Muntazam bo'lmagan kesik piramidaning yon sirti, alohida-alohida olingan yoqlari yuzining yig'indisiga teng;



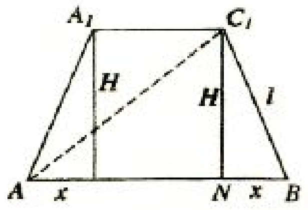
- **Yon sirti:** $S_{yon} = \dfrac{1}{2}(P_1 + P_2)\cdot l$;
- **To'la sirti:** $S_t = S_1 + S_2 + S_{yon}$;
- **Hajmi:** $V_1 = \dfrac{1}{3}\left(S_1 + \sqrt{S_1 \cdot S_2} + S_2\right)\cdot H_1$;
- $\dfrac{a_2}{a_1} = \dfrac{H_1}{H}$, $\dfrac{S_2}{S_1} = \left(\dfrac{H_1}{H}\right)^2 = \left(\dfrac{a_2}{a_1}\right)^2$, $\dfrac{V_1}{V} = \left(\dfrac{H_1}{H}\right)^2$.

Muntazam to'rtburchakli kesik piramida

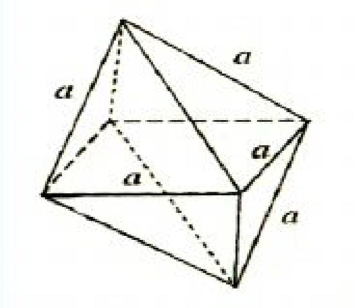
- **Yon sirti:** $S_{yon} = \dfrac{1}{2}(P_1 + P_2)\cdot h$; $P_1 = 4a$, $P_2 = 4b$;



- **To'la sirti:** $S_{to`la} = S^1_{asos} + S^2_{asos} + S_{yon}$;
- **Hajmi:** $V = \frac{1}{3}\left(S_1 + \sqrt{S_1 \cdot S_2} + S_2\right) \cdot H_1$;
- $AN = y,\ NC = x$;
- $A_1C_1 = d_2 = \sqrt{2}b$, $AC = d_1 = \sqrt{2}a$,
- $d = AC_1$, $d = \sqrt{y^2 + H^2}$
- $x = \frac{(a-b)\sqrt{2}}{2}$; $y = \frac{(a+b)\sqrt{2}}{2}$;
- $H = \sqrt{d^2 - y^2} = \sqrt{l^2 - x^2}$.



Oktaedr



a - har bir qirrasi uzunligi.

- **To'la sirti:** $S_t = 2a^2\sqrt{3}$;
- **Hajmi:** $V = \frac{a^3\sqrt{2}}{3}$. $R = \frac{a\sqrt{2}}{2}$; $r = \frac{a\sqrt{6}}{6}$.

Dodekaedr

Barcha **12** ta yoqlari muntazam beshburchakdan iborat ko'pyoq **dodekaendr** deyiladi.

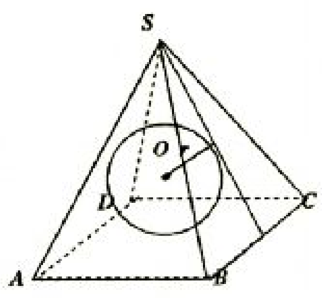
- **To'la sirti:** $S_t = 3a^2\sqrt{5\left(5+2\sqrt{5}\right)}$; **Hajmi:** $V = \frac{a^3\left(15+7\sqrt{5}\right)}{4}$;
- $R = \frac{a\sqrt{3}\left(1+\sqrt{5}\right)}{4}$, $r = \frac{a\sqrt{10\left(25+11\sqrt{5}\right)}}{20}$.

Ikosaedr

Barcha **20** ta yoqlari teng tomonli uchburchakdan iborat ko'pyoq **ikosaedr** deyiladi.

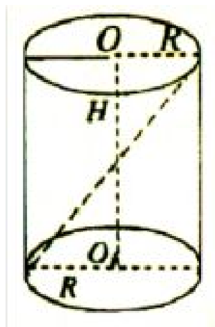
- **To'la sirti:** $S_t = 5a^2\sqrt{3}$; **Hajmi:** $V = \frac{5a^3\left(3+\sqrt{5}\right)}{12}$;
- $R = \frac{a\sqrt{2\left(5+\sqrt{5}\right)}}{4}$, $r = \frac{a\sqrt{3}\left(3+\sqrt{5}\right)}{12}$.

Ko'pyoqqa ichki chizilgan shar



- **V - ko'pyoq hajmi:** $V = \frac{1}{3} \cdot S_t \cdot r$;
- S_t **- ko'pyoq to'la sirti:** $S_t = \frac{3V}{r}$;
- ***r*** -ichki chizilgan **shar radiusi:** $r = \frac{3V}{S_t}$.

Silindr



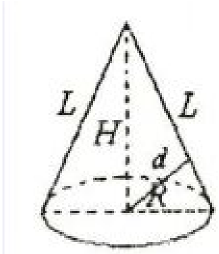
R - silindr asosining radiusi, ***H*** – balandligi.

- **Yon sirti:** $S_{yon} = 2\pi RH$;
- **To'la sirti:** $S_t = 2\pi R \cdot (R + H)$;
- Hajmi: $V = \pi R^2 H$;
- Diagonal yoki o'q kesim yuzi: $S_{o'q} = 2R\,H$;
- $S_{asos} = \pi R^2$, $a^2 = 4R^2 + H^2$;

Silindrning yoyilmasi

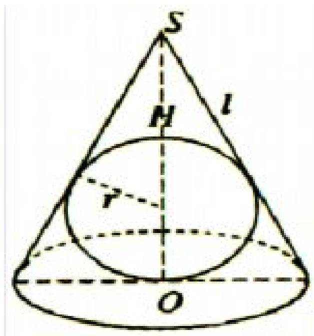
Konus

R- konus asosining radiusi, ***L***- yasovchisi, ***H*** - balandligi, φ - yasovchi va asos tekisligi orasidagi burchak.



- **Yon sirti:** $S_{yon} = \pi R\,L$, $S_{yon} = \frac{S_{asos}}{\cos\varphi} = \frac{\pi R^2}{\cos\varphi}$;
- **To'la sirti:** $S_t = \pi R \cdot (R + L)$;
- **Hajmi:** $V = \frac{l}{3}\pi R^2 \cdot H$;
- Konusga ichki chizilgan shar radiusi r ga teng bo'lsa:

$$r = \frac{R \cdot H}{l + R};$$



- $S_t^{sil} = S_t^{kon} \Rightarrow \frac{V_{kon}}{V_{sil}} = \sqrt{\frac{2}{3}}$,

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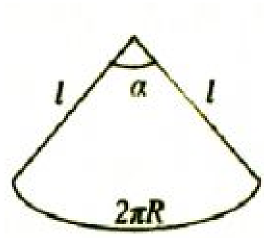
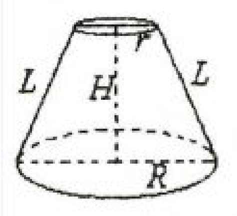
- $V_{kon} = V_{sil} \quad \Rightarrow \quad \frac{S_t^{sil}}{S_t^{kon}} = \sqrt[3]{\frac{3}{2}}$;
- $l_{yasovchi} = \sqrt{R^2 + H^2}$;
- Konus yoyilmasida radiusi ***l*** ga, yoy uzunligi **2πR** ga teng bo'lgan sektor hosil bo'ladi;
- Yoyilmasi uchidagi burchak uchun: $\alpha = \frac{360^\circ R}{l}$



Kesik konus

Kesik konus asoslarining radiuslari ***R*** va ***r***, balandligi ***H*** bo'lsin.

- **Yon sirti:** $S_{yon} = \pi L \cdot (R + r)$;
- **To'la sirti:** $S_t = \pi L \cdot (R + r) + \pi R^2 + \pi r^2$;
- **Hajmi:** $V = \frac{1}{3} \pi H (R^2 + R \cdot r + r^2)$;

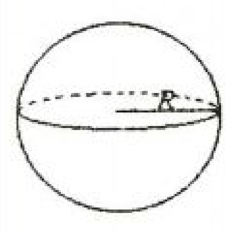


- Konus uchidan ***H₁*** masofada konusni ***S₁*** yuzali doira bo'ylab kesuvchi tekislik undan ***V₁*** hajmli konus ajratsin, konus hajmi ***V***, konus asosi yuzi ***S***. U holda

$$\frac{S_1}{S} = \left(\frac{H_1}{H}\right)^2; \quad \frac{H_1}{H} = \frac{r}{R}; \quad \frac{V_1}{V} = \left(\frac{H_1}{H}\right)^3.$$

Shar va sfera

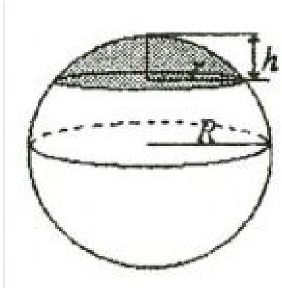
- **Shar sirti:** $S = 4\pi R^2$, $S = \pi d^2$;
- **Hajmi:** $V = \frac{4}{3} \pi R^3 = \frac{\pi}{6} d^3$;



- Shar kesimining radiusi ***r*** va shar markazidan kesimgacha masofa ***d*** uchun: $r^2 + d^2 = R^2$.

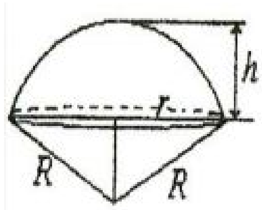
Shar segmenti

- **Yon sirti:** $S_{yon} = 2\pi R\,h = \pi\left(r^2 + h^2\right);$
- **To'la sirti:** $S_t = \pi\left(2R\,h + r^2\right);$
- **Hajmi:** $V = \pi h^2\left(R - \frac{1}{3}h\right) = \frac{\pi}{6}h\left(3r^2 + h^2\right).$



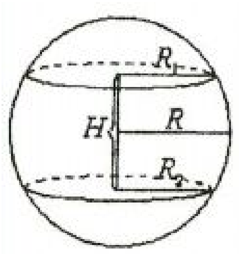
Shar sektori

- **To'la sirti:** $S_t = \pi R(2h + r);$
- **Hajmi:** $V = \frac{2}{3}\pi R^2 h = \frac{\pi}{6}d^2 h,\ d = 2R.$



Shar halqasi

- **Yon sirti:** $S_{yon} = 2\pi R\,H;$
- **To'la sirti:** $S_t = \pi\left(2R \cdot H + R_1^2 + R_2^2\right);$
- **Hajmi:** $V = \frac{1}{6}\pi H \cdot\left(3R_1^2 + 2R_2^2 + H^2\right).$



O'xshash ko'pyoqlar

- $\frac{S_1}{S_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{p_1}{p_2}\right)^2 = \left(\frac{H_1}{H_2}\right)^2;\ \frac{V_1}{V_2} = \left(\frac{a_1}{a_2}\right)^3 = \left(\frac{H_1}{H_2}\right)^3 = \left(\frac{p_1}{p_2}\right)^3 = \left(\frac{l_1}{l_2}\right)^3.$

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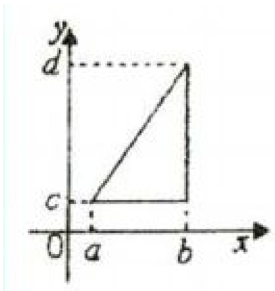
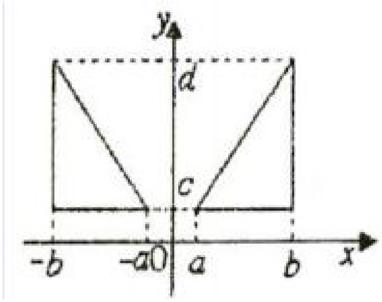
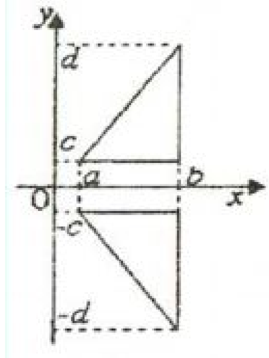
S I M M E R T I Y A

O'qqa nisbatan simmetriya

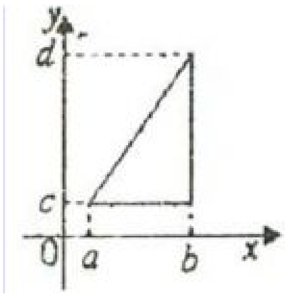


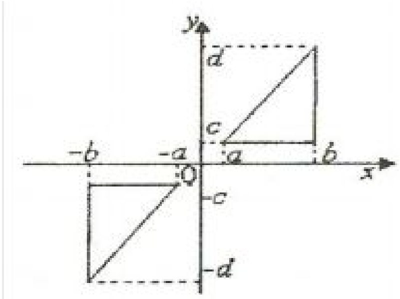




Berilgan *shakl* Oy **o'qiga nisbatan** Ox **o'qiga nisbatan.**

Nuqtaga nisbatan simmetriya





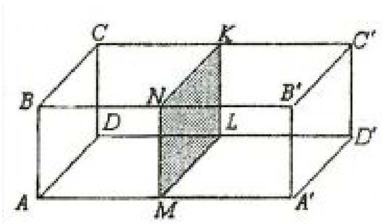
Berilgan *shakl* **Koordinata boshiga nisbatan simmetriya.**

Tekislikka nisbatan simmetriya

$A'B'C'D'$ to'rtburchak va $ABCD$ to'rtburchak $MNKL$ tekislikka nisbatan simmetriyadir.

$MA = MA'$, $NB = NB'$,

$KC = KC'$, $LD = LD'$.



1 dan 1000 gacha bo'lgan tub sonlar jadvali

2	79	191	311	439	577	709	857
3	83	193	313	443	587	719	859
5	89	197	317	449	593	727	863
7	97	199	331	457	599	733	877
11	101	211	337	461	601	739	881
13	103	223	347	463	607	743	883
17	107	227	349	467	613	751	887
19	109	229	353	479	617	757	907
23	113	233	359	487	619	761	911
29	127	239	361	491	631	769	919
31	131	241	373	499	641	773	929
37	137	251	379	503	643	787	937
41	139	257	383	509	647	797	941
43	149	263	389	521	653	809	947
47	151	269	397	523	659	811	953
53	157	271	401	541	661	821	967
59	163	277	409	547	673	823	971
61	167	281	419	557	677	827	977
67	173	283	421	563	683	829	983
71	179	293	431	569	691	839	991
73	181	307	433	571	701	853	997

10 dan 99 gacha bo'lgan natural sonlar kvadratlarinlng jadvali

Birlar O'nlar	0	1	2	3	4	5	6	7	8	9
1	100	121	144	169	196	225	256	289	324	361
2	400	441	484	529	576	625	676	729	784	841
3	900	961	1024	1089	1156	1225	1296	1369	1444	1521
4	1600	1681	1764	1849	1936	2025	2116	2209	2304	2401
5	2500	2601	2704	2809	2916	3025	3136	3249	3364	3481
6	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
7	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
8	6400	6561	6724	6889	7056	7225	7396	7569	7744	7921
9	8100	8281	8464	8649	8836	9025	9216	9409	9604	9801