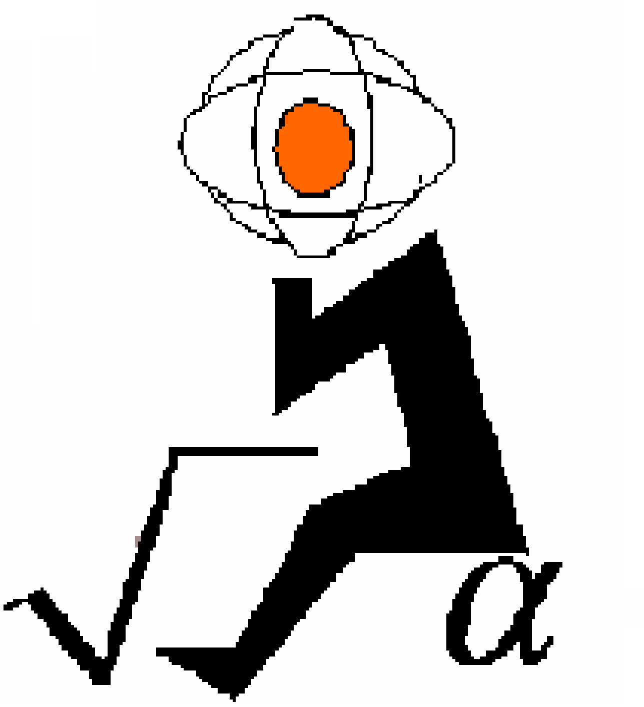
Elementar matematikaning **asosiy formulalari**

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$Sin^2 x + Cos^2 x = 1$$



Kitobcha elementar matematikaning barcha asosiy formulalari va qoidalarini o‘z ichiga olgan bo‘lib, umumiy o‘rta ta’lim maktablari o‘quvchilari, akademik litsey va kasb – hunar kollejlari talabalari, Oliy o‘quv yurtlariga kiruvchi abiturientlar va matematika o‘qituvchilari uchun mo‘ljallangan.

Mundarija

ALGEBRA

TRIGONOMERTIYA

ALGEBRA VA ANALIZ ASOSLARI

ELEMENTAR FUNKSIYALAR VA ULARNING ASOSIY XOSSALARI HOSILA

GEOMETRIYA

PLANIMETRIYA

STEREOMETRIYA

ALGEBRA.

To‘plamlar nazariyasi elementlari

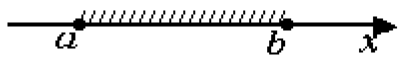
1. $a \in A$ — a element A to‘plamga tegishli.
2. $a \notin A$ — a element A to‘plamga tegishli emas.
3. $B \subset A$ — B to‘plam A ning qism to‘plami.
4. $B \cup A$ — B va A to‘plamlarning birlashmasi.
5. $B \cap A$ — B va A to‘plamlarning kesishmasi.
6. $A \Rightarrow B$ — A tasdiqdan B tasdiq kelib chiqadi.
7. $A \Leftrightarrow B$ — A va B tasdiqlar teng kuchli.
8. $\varnothing$ – bo‘sh to‘plam.

Sonli to‘plamlar

1. $N = \{1,2,3,...,n,...\}$ - **natural sonlar** to‘plami.
$N_1 = \{1,3,5,...,2n-1,...\}$ - **toq sonlar** to‘plami.
$N_2 = \{2,4,6,...,2n,...\}$ - **juft sonlar** to‘plami.
2. $Z = \{...,-n,...,-2,-1,0,1,2,...,n,...\}$ - **butun sonlar** to‘plami.
3. $P = \left\{ x : x = \frac{m}{n},\ m \in Z,\ n \in N \right\}$ - **ratsional sonlar** to‘plami.
4. $R = \{x : -\infty < x < +\infty\}$ - **haqiqiy sonlar** to‘plami.
5. $N \subset Z \subset P \subset R$.
6. Irratsional sonlar to‘plami: cheksiz, davriy bo‘lmagan o‘nli kasrlar.
Masalan: $\pi = 3{,}14...,\sqrt{2},\ \sqrt[3]{9}$,..

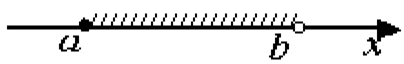
Sonli oraliqlar

1. Yopiq oraliq:
$[a,b] = \{x : a \le x \le b,\ x \in R\}$.

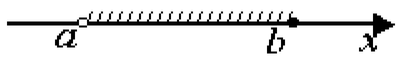


2. Ochiq oraliq:
$(a,b) = \{x : a < x < b,\ x \in R\}$.

3. Yarim ochiq oraliq:
$[a,b) = \{x : a \le x < b,\ x \in R\}$.



4. Yarim yopiq oraliq:
$(a,b] = \{x : a < x \le b,\ x \in R\}$.



Tub va murakkab sonlar

1. Faqat o‘ziga va 1 ga bo‘linadigan birdan katta natural sonlar **tub sonlar** deyiladi.

2. Uch va undan ortiq natural bo‘luvchiga ega bo‘lgan sonlar **murakkab sonlar** deyiladi.

3. Teorema. Agar n sonning $\sqrt{n}$ dan katta bo‘lmagan tub bo‘luvchisi mavjud bo‘lmasa, u holda n tub son bo‘ladi.

Natural sonlarning kanonik yoyilmasi

1. Har qanday $a \in N$ sonini $a = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} ... p_k^{\alpha_k}$ ko‘rinishida ifodalash mumkin, unga a sonining **kanonik yoyilmasi** deyiladi. Bunda p_1, p_2, p_3, ..., p_k – tub sonlar.

Masalan: $72=2^3 3^2$ ($p_1 = 2,\ p_2 = 3,\ \alpha_1 = 3,\ \alpha_2 = 2$).

2. $a = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} .... p_k^{\alpha_k}$ sonning **bo‘luvchilari soni**

$$n(a) = (\alpha_1 + 1) \cdot (\alpha_2 + 1) \cdot ... \cdot (\alpha_k + 1),$$

formula bilan, **bo‘luvchilar yig‘indisi** esa

$$S(a) = \frac{p_1^{\alpha_1+1} - 1}{p_1 - 1} \cdot \frac{p_2^{\alpha_2+1} - 1}{p_2 - 1} \cdot \frac{p_3^{\alpha_3+1} - 1}{p_3 - 1} \cdot ... \cdot \frac{p_k^{\alpha_k+1} - 1}{p_k - 1}$$

formula orqali hisoblanadi.

3. $n! = 1 \cdot 2 \cdot 3 \cdot ... \cdot n$ sonining kanonik yoyilmasida p tub son

$$\alpha_p = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + ...$$ daraja bilan qatnashadi.

4. $n!$ soni $k = \left[\frac{n}{5}\right] + \left[\frac{n}{5^2}\right] + \left[\frac{n}{5^3}\right] + ...$ ta nol bilan tugaydi.

Sonning butun va kasr qismi

1. a sonining **butun qismi** deb, a dan katta bo‘lmagan eng katta butun songa aytiladi va u $[a]$ orqali belgilanadi. Masalan: [2,4]=2, [–3,52]= –4.

2. Sonning **kasr qismi** deb (a-$[a]$) ga aytiladi va u $\{a\}$ orqali belgilanadi. Masalan: {-0,3}=0,7; {2,6}=0,6.

Tub sonlar jadvali (1000 gacha)

2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	857	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	
59	139	233	337	439	557	653	769	883	

Bo‘linish alomatlari

1. Agar sonning oxirgi raqami 0 yoki juft bo‘lsa, bu son 2 ga bo‘linadi.
2. Agar sonning raqamlari yig‘indisi 3 (9) ga bo‘linsa, bu son 3 (9) ga bo‘linadi.
3. Agar sonning oxirgi ikki raqamidan tashkil topgan ikki xonali son 4 (25) ga bo‘linsa, bu son 4 (25) ga bo‘linadi.
4. Agar sonning oxirgi raqami 0 yoki 5 bo‘lsa, bu son 5 ga bo‘linadi.
5. Agar sonning oxirgi raqami 0 bo‘lsa, bu son 10 ga bo‘linadi.
6. Agar sonning toq o‘rindagi raqamlari yig‘indisi bilan juft o‘rindagi raqamlari yig‘indisining ayirmasi 11 ga bo‘linsa, bu son 11 ga bo‘linadi.

Masalan: 9873424, (9+7+4+4)–(8+3+2)=11.

Eng katta umumiy bo‘luvchi (EKUB)

1 dan boshqa umumiy bo‘luvchilarga ega bo‘lmagan sonlar **o‘zaro tub** sonlar deyiladi.

Sonlarning **EKUB**i deb, shu sonlarning umumiy bo‘luvchilarining eng kattasiga aytiladi va u quyidagicha topiladi:

1) har bir sonning kanonik yoyilmasida qatnashgan umumiy ko‘paytuvchilar eng kichik darajasi bilan olinadi;

2) ajratib olingan sonlar ko‘paytiriladi.

Eng kichik umumiy karrali (EKUK)

Sonlarning **EKUK**i deb, shu sonlarga bo‘linadigan sonlarning eng kichigiga aytiladi va u quyidagicha topiladi:

1) har bir sonning kanonik yoyilmasida qatnashgan ko‘paytuvchilar eng katta darajasi bilan olinadi;

2) ajratib olingan sonlar ko‘paytiriladi.

Masalan: EKUB(28,144) va EKUK(28,144) ni toping.

$28 = 2^2 \cdot 7$, $144 = 2^4 \cdot 3^2 \Rightarrow \text{ЭКУБ}(28,144) = 2^2 = 4$,

$\text{ЭКУК}(28,144) = 2^4 \cdot 3^2 \cdot 7^1 = 1008$.

Qoldiqli bo‘lish

$$a = p \cdot q + r \quad (0 \le r < p),$$

bu yerda a–bo‘linuvchi, p–bo‘luvchi, q–bo‘linma, r–qoldiq.

O‘lchov birliklari

1. 1 km=1000m.

2. 1 m=10 dm=100sm.

3. 1 sm=10 mm.

4. 1 km^2=1000000m^2.

5. 1 m^2=100dm^2=10000sm^2.

6. 1 ga=100ar=10000m^2.

7. 1m^3=1000dm^3=1000000sm^3.

8. 1dm^3=1000sm^3.

9. 1l=1dm^3=1000sm^3.

10. 1t=10s=1000kg.

11. 1kg=1000g.

12. 1g=1000mg.

Oddiy kasrlar ustida amallar

$$\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}; \quad \frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}; \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}; \quad \frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}.$$

Proporsiya

Agar $a : b = c : d$ bo‘lsa, quyidagilar o‘rinli:

1) $ad = bc$;

2) $\frac{ma + nb}{pa + qb} = \frac{mc + nd}{pc + qd}$;

3) $\frac{a - b}{c - d} = \frac{b}{d}$;

4) $\frac{a - c}{b - d} = \frac{a}{b}$;

Davriy o‘nli kasrlarni oddiy kasrga aylantirish

$$\overline{a_0,a_1a_2a_3....a_k(b_1b_2b_3....b_n)} = \frac{\overline{a_0a_1a_2a_3....a_kb_1b_2b_3....b_n} - \overline{a_0a_1a_2a_3....a_k}}{\underbrace{999...999}_{n}\underbrace{000...000}_{k}}$$

Sonli tengsizliklar

1) $a > b \;\Leftrightarrow\; a - b > 0$, $\quad a > b \;\Leftrightarrow\; b < a$;

2) $a > b,\; b > c \;\Rightarrow\; a > c$, $\quad a > b \;\Leftrightarrow\; a \pm c > b \pm c$;

3) $a > b,\; c > 0 \;\Rightarrow\; ac > bc,\; \frac{a}{c} > \frac{b}{c}$;

4) $a > b,\; c < 0 \;\Rightarrow\; ac < bc,\; \frac{a}{c} < \frac{b}{c}$;

5) $a > b > 0 \;\Rightarrow\; a^n > b^n,\; \sqrt[n]{a} > \sqrt[n]{b} \;\; (n \in \mathbb{N})$;

6) $a > b > 0,\; c > 0 \;\Rightarrow\; \frac{a}{c} > \frac{b}{c},\; \frac{c}{a} < \frac{c}{b}$.

O‘rta qiymatlar

Agar $x_1, x_2, ..., x_n$ musbat sonlar bo‘lsa, ularning:

1) o‘rta arifmetigi: $A = \frac{x_1 + x_2 + ... + x_n}{n}$;

2) o‘rta geometrigi: $G = \sqrt[n]{x_1 \cdot x_2 \cdot ... \cdot x_n}$;

3) o‘rta garmonigi: $H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_n}}$;

4) o‘rta kvadratigi: $K = \sqrt{\frac{x_1^2 + x_2^2 + ... + x_n^2}{n}}$;

Koshi teoremasi: $H \le G \le A \le K$.

Protsentlar

1) a soning P protsentini topish: $x = \frac{a \cdot P}{100}$;

2) P protsenti a ga teng sonni topish: $x = \frac{a \cdot 100}{P}$;

3) a soni b soning $\frac{a}{b} \cdot 100\%$ ini tashkil etadi;

4) a soni $a + b$ yig‘indining $\frac{a}{a+b} \cdot 100\%$ ini tashkil etadi.

Daraja va uning xossalari

1) $a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n}$; **2)** $a^0 = 1, \quad a^1 = a$;

3) $a^{-p} = \dfrac{1}{a^p}$; **4)** $a^p \cdot a^q = a^{p+q}$;

5) $a^p : a^q = a^{p-q}$; **6)** $\left(a^p\right)^q = a^{p \cdot q}$;

7) $\left(a \cdot b\right)^p = a^p \cdot b^p$; **8)** $\left(\dfrac{a}{b}\right)^p = \dfrac{a^p}{b^p}$.

Bu yerda $n \in N, \; a > 0, \; b > 0; \; q, \; p \in \mathbb{R}$.

Haqiqiy sonning moduli

$$|a| = \begin{cases} a, & \text{agar} \quad a \ge 0 \quad \text{bo'lsa;} \\ -a, & \text{agar} \quad a < 0 \quad \text{bo'lsa.} \end{cases}$$

1) $|ab| = |a| \cdot |b|$; **2)** $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$;

3) $|a+b| \le |a| + |b|$; **4)** $|a-b| \ge |a| - |b|$; **5)** $|a|^2 = a^2$.

Arifmetik ildiz va uning xossalari

$$\left(\sqrt[n]{a}\right) \Leftrightarrow \begin{cases} a \ge 0, \\ \sqrt[n]{a} \ge 0, \\ \left(\sqrt[n]{a}\right)^n = a. \end{cases} \qquad \sqrt[n]{a^n} = \begin{cases} |a|, & n = 2k, \quad k \in N, \\ a, & n = 2k+1, \quad k \in N. \end{cases}$$

1) $\sqrt[m]{ab} = \sqrt[m]{a} \cdot \sqrt[m]{b}$; **2)** $\sqrt[m]{\dfrac{a}{b}} = \dfrac{\sqrt[m]{a}}{\sqrt[m]{b}}$;

3) $\sqrt[m]{a^n} = a^{\frac{n}{m}}$; **4)** $\sqrt[mp]{a^{np}} = \sqrt[m]{a^n}$;

5) $\sqrt[n]{a^{n+p}} = a\sqrt[n]{a^p}$; **6)** $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$;

7) $\sqrt[m]{a\sqrt[n]{b\sqrt[p]{c}}} = \sqrt[mnp]{a^{np}b^p c}$; **8)** $\sqrt[n]{a\sqrt[n]{a\sqrt[n]{a...}}} = \sqrt[n-1]{a}$;

9) $\sqrt{A \pm \sqrt{B}} = \sqrt{\dfrac{A+m}{2}} \pm \sqrt{\dfrac{A-m}{2}}; \quad m = \sqrt{A^2 - B}$.

Qisqa ko‘paytirish formulalari

1) $(a\pm b)^2 = a^2 \pm 2ab + b^2$;

2) $a^2 - b^2 = (a-b)(a+b)$;

3) $(a\pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$;

4) $a^3 \pm b^3 = (a\pm b)(a^2 \mp ab + b^2)$;

5) $a^4 - b^4 = (a-b)(a+b)(a^2+b^2)$;

6) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+ac+bc)$.

Kombinatorika elementlari

1. m ta elementdan n tadan barcha o‘rinlashtirishlar soni:

$$A_m^n = m(m-1)(m-2)...(m-n+1) = \frac{m!}{(m-n)!}.$$

2. n ta elementdan barcha o‘rin almashtirishlari soni:

$$P_n = n! = 1\cdot 2\cdot 3\cdot 4\cdot ...\cdot n.$$

3. m ta elementdan n tadan barcha gruppalashlar soni:

$$C_m^n = \frac{A_m^n}{P_n} = \frac{m(m-1)(m-2)...(m-n+1)}{1\cdot 2\cdot 3\cdot ....\cdot n} = \frac{m!}{(m-n)!\cdot n!}.$$

Chiziqli tenglama

$$ax + b = 0.$$

1. $a \neq 0,\ \ b \in R$ bo‘lsa, yagona yechimga ega: $x = -\frac{b}{a}$;

2. $a = 0,\ \ b \neq 0$ bo‘lsa, yechimi yo‘q;

3. $a = 0,\ \ b = 0$ bo‘lsa, yechimi cheksiz ko‘p: $x \in R$.

Kvadrat tenglama

$$ax^2 + bx + c = 0 \quad (a \neq 0).$$

1. $D = b^2 - 4ac > 0$ bo‘lsa, 2 ta har xil haqiqiy yechimi bor:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

2. $D = b^2 - 4ac = 0$ bo‘lsa, 1 ta ikki karrali haqiqiy yechimi bor:

$$x_{1,2} = \frac{-b}{2a}.$$

3. $D = b^2 - 4ac < 0$ bo‘lsa, haqiqiy yechimi yo‘q.

Yechimlarining xossalari:

Agar x_1 va x_2 sonlar $ax^2+bx+c=0$ tenglamaning ildizlari bo‘lsa, u holda:

1. $x_1+x_2=-\dfrac{b}{a}$, $\quad x_1\cdot x_2=\dfrac{c}{a}$ **(Viet teoremasi);**

2. $ax^2+bx+c=a(x-x_1)(x-x_2)$;

3. $x_1^2+x_2^2=\left(x_1+x_2\right)^2-2x_1x_2=\dfrac{b^2-2ac}{a^2}$;

4. $x_1^3+x_2^3=\left(x_1+x_2\right)^3-3x_1x_2(x_1+x_2)=-\left(\dfrac{b}{a}\right)^3+\dfrac{3bc}{a^2}$;

5. $\dfrac{1}{x_1^2}+\dfrac{1}{x_2^2}=\dfrac{b^2-2ac}{c^2}$; $\quad \dfrac{1}{x_1^3}+\dfrac{1}{x_2^3}=\dfrac{-b^3+3abc}{c^3}$;

6. to‘la kvadratga ajratish: $ax^2+bx+c=a\left(x+\dfrac{b}{2a}\right)^2-\dfrac{b^2-4ac}{4a}$.

Kub tenglama

$$x^3+ax^2+bx+c=0.$$

Agar x_1, x_2, x_3 lar bu tenglamaning ildizlari bo‘lsa, u holda

1. $x^3+ax^2+bx+c=(x-x_1)(x-x_2)(x-x_3)$.

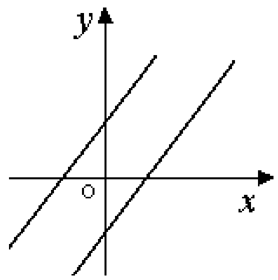
2. Viet teoremasi: $\begin{cases} x_1+x_2+x_3=-a, \\ x_1x_2+x_1x_3+x_2x_3=b, \\ x_1x_2x_3=-c. \end{cases}$

Ikki noma’lumli chiziqli tenglamalar sistemasi

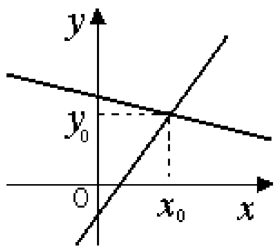
Umumiy ko‘rinishi: $\begin{cases} a_{11}x+a_{12}y=b_1, \\ a_{21}x+a_{22}y=b_2. \end{cases}$

Geometrik talqini

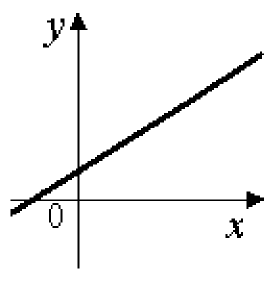
1. Agar $\dfrac{a_{11}}{a_{21}}=\dfrac{a_{12}}{a_{22}}\neq\dfrac{b_1}{b_2}$ bo‘lsa,
sistema **yechimga ega emas**.



2. Agar $\dfrac{a_{11}}{a_{21}}\neq\dfrac{a_{12}}{a_{22}}$ bo‘lsa,
sistema **yagona yechimga ega**.



3. Agar $\dfrac{a_{11}}{a_{21}}=\dfrac{a_{12}}{a_{22}}=\dfrac{b_1}{b_2}$ bo‘lsa,
sistema **cheksiz ko‘p yechimga ega**.



Arifmetik progressiya $a_{n+1} = a_n + d,\ n = 0, 1, 2, \ldots$ **.**

bu yerda a_1 – **birinchi hadi**, d –**ayirmasi.**

1. ayirmasini topish: $d = a_{n+1} - a_n = \dfrac{a_n - a_m}{n - m},\ n > m$;

2. n - hadini topish: $a_n = a_1 + (n-1)d$;

3. o‘rta hadini topish: $a_n = \dfrac{a_{n-k} + a_{n+k}}{2},\ k < n$;

4. dastlabki n ta hadining yig‘indisini topish:

$$S_n = \frac{a_1 + a_n}{2} \cdot n = \frac{2a_1 + (n-1)d}{2} \cdot n;$$

5. n - dan k - gacha bo‘lgan hadlar yig‘indisini topish:

$$S_n^k = a_n + dn(k-1) \qquad (k > n);$$

6. $a_n = S_n - S_{n-1}$.

Geometrik progressiya $b_{n+1} = b_n q,\quad n = 1, 2, 3, \ldots$**.**

bu yerda, b_1 – **birinchi hadi**, q – **maxraji**.

1. maxrajini topish: $q = \dfrac{b_{n+1}}{b_n}$;

2. n - hadini topish: $b_n = b_1 \cdot q^{n-1},\ b_n = b_{n-m} \cdot q^m$;

3. o‘rta hadini topish: $|b_n| = \sqrt{b_{n-k} \cdot b_{n+k}}$;

4. dastlabki n ta hadi yig‘indisini topish:

$$S_n = \frac{b_n q - b_1}{q - 1} = \frac{b_1(q^n - 1)}{q - 1};$$

1. $b_n = S_n - S_{n-1}$;

2. Cheksiz kamayuvchi geometrik progressiya hadlari yig‘indisi:

$$S = \frac{b_1}{1-q}, \qquad |q| < 1.$$

Tengsizliklarning xossalari

1. Agar $f(x) \ge g(x)$ bo‘lsa, $c > 0$ da $cf(x) \ge cg(x)$, $c < 0$ da $cf(x) \le cg(x)$ bo‘ladi.

2. Agar $f^{2n}(x) \ge g^{2n}(x)$ $\left(f^{2n}(x) \le g^{2n}(x)\right)$ bo‘lsa, u holda $|f(x)| \ge |g(x)|$ $\left(|f(x)| \le |g(x)|\right)$ bo‘ladi.

3. Agar $|f(x)| \le c$ $\left(|f(x)| \ge c\right)$ bo‘lsa, u holda $-c \le f(x) \le c$ $\left(f(x) \le -c\right.$ ёки $\left.f(x) \ge c\right)$ lar o‘rinli.

Kasr ratsional tenglama va tengsizliklar

1. $\dfrac{f(x)}{g(x)}=0 \quad \Leftrightarrow \quad \begin{cases} f(x)=0, \\ g(x)\neq 0. \end{cases}$

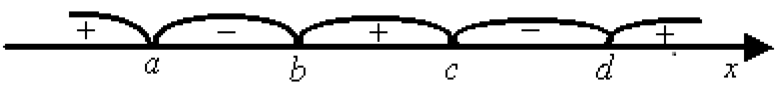
2. $\dfrac{f(x)}{g(x)}=\dfrac{h(x)}{v(x)} \quad \Leftrightarrow \quad \begin{cases} f(x)v(x)-g(x)h(x)=0, \\ g(x)v(x)\neq 0. \end{cases}$

3. $\dfrac{f(x)}{g(x)}\geq 0 \quad \Leftrightarrow \quad \begin{cases} f(x)g(x)\geq 0, \\ g(x)\neq 0. \end{cases}$

4. $\dfrac{f(x)}{g(x)}> h(x) \quad \Leftrightarrow \quad \begin{cases} \left(f(x)-g(x)h(x)\right)g(x)>0, \\ g(x)\neq 0. \end{cases}$

Oraliqlar usuli

Agar $a<b<c<d$ bo‘lsa, u holda



a) $(x-a)(x-b)(x-c)(x-d)>0$ tengsizlikning yechimi $(-\infty;a)\cup(b;c)\cup(d;+\infty)$ bo‘ladi.

b) $(x-a)(x-b)(x-c)(x-d)<0$ tengsizlikning yechimi $(a;b)\cup(c;d)$ bo‘ladi.
Bu usul **oraliqlar usuli** deyiladi.

Kvadrat tengsizlik

$$ax^2+bx+c\geq 0 \qquad \left(ax^2+bx+c\leq 0\right)$$

1) $a>0, D>0$ bo‘lsa, $x\in\left(-\infty;x_1\right]\cup\left[x_2;+\infty\right)$ $\qquad \left(x\in\left[x_1;x_2\right]\right)$;

2) $a>0, D=0$ bo‘lsa, $x\in\left(-\infty;+\infty\right)$ $\qquad \left(x=x_1\right)$;

3) $a>0, D<0$ bo‘lsa, $x\in\left(-\infty;\infty\right)$ $\qquad \left(x\in\varnothing\right)$;

4) $a<0, D>0$ bo‘lsa, $x\in\left[x_1;x_2\right]$ $\qquad \left(\left(-\infty;x_1\right]\cup\left[x_2;+\infty\right)\right)$;

5) $a<0, D=0$ bo‘lsa, $x=x_1$ $\qquad \left(x\in\left(-\infty;+\infty\right)\right)$;

6) $a<0, D<0$ bo‘lsa, $x\in\varnothing$ $\qquad \left(x\in\left(-\infty;\infty\right)\right)$.

Irratsianal tenglama va tengsizliklar

1. $\sqrt[2k]{f(x)}=0 \quad \Rightarrow \quad f(x)=0$.

2. $\sqrt[2k]{f(x)}=g(x) \quad \Leftrightarrow \quad \begin{cases} g(x)\geq 0, \\ f(x)=g^{2k}(x). \end{cases}$

3. $\sqrt[2k]{f(x)}=\sqrt[2k]{g(x)} \quad \Rightarrow \quad \begin{cases} f(x)\geq 0, \\ f(x)=g(x). \end{cases}$

4. $\sqrt[2k+1]{f(x)}=g(x) \quad \Rightarrow \quad f(x)=g^{2k+1}(x)$.

5. $\sqrt[2k]{f(x)}<g(x) \quad \Rightarrow \quad \begin{cases} f(x)\geq 0,\ g(x)>0, \\ f(x)<g^{2k}(x). \end{cases}$

6. $\sqrt[2k+1]{f(x)}<g(x) \quad \Rightarrow \quad f(x)<g^{2k+1}(x)$.

7. $\sqrt[2k]{f(x)}>g(x) \quad \Rightarrow \quad \begin{cases} g(x)\geq 0, \\ f(x)>g^{2k}(x). \end{cases}$ *ёки* $\begin{cases} g(x)<0, \\ f(x)\geq 0. \end{cases}$

8. $\sqrt[2k+1]{f(x)}>g(x) \quad \Rightarrow \quad f(x)>g^{2k+1}(x)$.

Ko‘rsatkichli tenglama va tengsizliklar

1. $a^{f(x)}=1 \ \Rightarrow \ f(x)=0.$

2. $[f(x)]^{g(x)}=1 \ \Rightarrow \ \begin{cases} f(x)=1, \\ g(x)\in R, \end{cases}$ yoki $\begin{cases} f(x)\neq 0, \\ g(x)=0. \end{cases}$

3. $a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad (a>0)$.

4. $a^{f(x)}=b^{g(x)} \Rightarrow f(x)=g(x)\log_a b \quad (a,b>0)$.

5. $a^{f(x)}>a^{g(x)} \Rightarrow \begin{cases} 0<a<1, \\ f(x)<g(x), \end{cases}$ yoki $\begin{cases} a>1, \\ f(x)>g(x). \end{cases}$

6. $a^{f(x)}>b,\ 0<a\neq 1 \ \Rightarrow \begin{cases} b>0,\ a>1, \\ f(x)>\log_a b, \end{cases}$ yoki $\begin{cases} 0<a<1,\ b>0, \\ f(x)<\log_a b. \end{cases}$

7. $a^{f(x)}<b \ \Rightarrow \begin{cases} b>0,\ a>1, \\ f(x)<\log_a b, \end{cases}$ yoki $\begin{cases} b>0,\ 0<a<1, \\ f(x)>\log_a b. \end{cases}$

Logarifm va uning asosiy xossalari

$$b=\log_a N \quad (a>0,\ a\neq 1,\ N>0) \quad \Leftrightarrow \quad N=a^b.$$

1) $\log_a 1=0$, $\log_a a=1$, $\quad a^{\log_a N}=N$;

2) $\log_a(bc)=\log_a b+\log_a c$, $\quad \log_a\left(\dfrac{b}{c}\right)=\log_a b-\log_a c$;

3) $\log_{a^n} b^m=\dfrac{m}{n}\log_a b$, $\quad \log_a b=\dfrac{1}{\log_b a}$;

4) $\log_{ba} c=\dfrac{\log_a c}{1+\log_a b}$, $\quad \log_a b=\dfrac{\log_c b}{\log_c a}$;

5) $\log_a b\cdot\log_c d=\log_a d\cdot\log_c b$, $\quad a^{\log_b c}=c^{\log_b a}$;

6) $\log_{10} x=\lg x$ **– o‘nli logarifm;**

7) $\log_e x=\ln x$ **– natural logarifm**;

8) Agar $a>1,\ 0<b<1$ yoki $0<a<1, b>1$ bo‘lsa $\log_a b<0$;

9) Agar $a>1,\ b>1$ yoki $0<a<1, 0<b<1$ bo‘lsa $\log_a b>0$;

10) Agar $b>a>1$ bo‘lsa, $\log_b p<\log_a p$ bo‘ladi ($p>0$);

11) Agar $0<a<b<1$ bo‘lsa, $\log_b p>\log_a p$ bo‘ladi ($p>0$);

12) $a>b>0$ bo‘lsin.

agar $0<p<1$ bo‘lsa, $\log_p a<\log_p b$ bo‘ladi,

agar $p>1$ bo‘lsa, $\log_p a<\log_p b$ bo‘ladi.

Logarifmik tenglama va tengsizliklar

1. $\log_a f(x)=b \quad \Leftrightarrow \quad \begin{cases} f(x)>0, \quad 0<a\neq 1, \\ f(x)=a^b. \end{cases}$

2. $\log_{f(x)} a=b \quad \Leftrightarrow \quad \begin{cases} f(x)\neq 1, \quad a>0, \\ f(x)=a^{\frac{1}{b}}. \end{cases}$

3. $\log_a f(x)=\log_a g(x) \quad \Leftrightarrow \quad \begin{cases} 0<a\neq 1, \quad f(x)>0, \\ f(x)=g(x). \end{cases}$

4. $\log_{f(x)} g(x)=b \quad \Leftrightarrow \quad \begin{cases} 0<f(x)\neq 1, \\ g(x)=f^b(x). \end{cases}$

5. $\log_{\varphi(x)} f(x) = \log_{\varphi(x)} g(x) \quad \Leftrightarrow \quad \begin{cases} 0 < \varphi(x) \neq 1, \quad f(x) > 0, \\ f(x) = g(x). \end{cases}$

6. $\log_{\varphi(x)} f(x) > \log_{\varphi(x)} g(x) \Rightarrow \begin{cases} 0 < \varphi(x) < 1, \\ f(x) > 0, \\ f(x) < g(x). \end{cases}$ yoki $\begin{cases} \varphi(x) > 1, \\ g(x) > 0, \\ f(x) > g(x). \end{cases}$

7. $\log_{f(x)} g(x) > b \quad \Rightarrow \begin{cases} 0 < f(x) < 1, \\ g(x) > 0, \\ g(x) < f^b(x). \end{cases}$ yoki $\begin{cases} f(x) > 1, \\ g(x) > f^b(x). \end{cases}$

TRIGONOMETRIYA

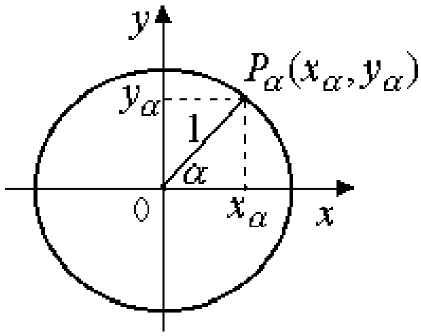
$$\alpha^\circ = \frac{180^\circ}{\pi} \cdot \alpha_{\text{рад}}, \qquad \alpha_{рад} = \frac{\pi}{180^\circ} \cdot \alpha^0$$

$Sin\alpha = y_\alpha, \qquad Cos\alpha = x_\alpha,$

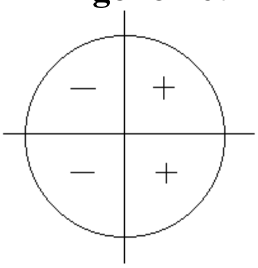
$tg\alpha = \frac{y_\alpha}{x_\alpha}, \qquad ctg\alpha = \frac{x_\alpha}{y_\alpha},$

$Sec = \frac{1}{x_\alpha}, \qquad Csc\alpha = \frac{1}{y_\alpha}.$

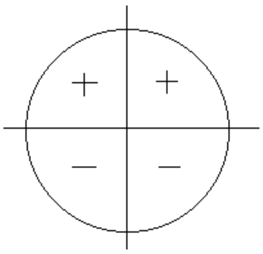
1 rad $\approx 57^0 17' 15''; \quad \pi = 3{,}141592\ldots$



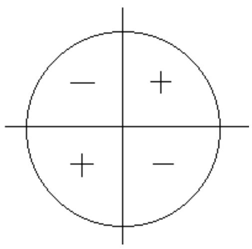
Trigonometrik funksiyalarning choraklardagi ishoralari



Cosx



Sinx



tgx, ctgx

Asosiy trigonometrik ayniyatlar

1. $Sin^2 x + Cos^2 x = 1.$

2. $tgx \cdot ctgx = 1.$

3. $1 + tg^2 x = \frac{1}{Cos^2 x}.$

4. $tgx = \frac{Sinx}{Cosx}.$

5. $ctgx = \frac{Cosx}{Sinx}.$

6. $1 + ctg^2 x = \frac{1}{Sin^2 x}.$

Qo‘shish formulalari

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta;$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta;$$

$$tg(\alpha \pm \beta) = \frac{tg\alpha \pm tg\beta}{1 \mp tg\alpha tg\beta}; \quad ctg(\alpha \pm \beta) = \frac{ctg\alpha ctg\beta \mp 1}{ctg\alpha \pm ctg\beta}.$$

Ikkilangan va uchlangan burchaklar

$$\sin 2x = 2\sin x\cos x; \qquad \sin 2x = 2\sin x\cos x;$$

$$tg2x = \frac{2tgx}{1 - tg^2x}; \qquad ctg2x = \frac{ctg^2x - 1}{2ctgx};$$

$$\sin 3x = 3\sin x\cos^2 x - \sin^3 x = 3\sin x - 4\sin^3 x;$$

$$\cos 3x = \cos^3 x - 3\cos x\sin^2 x = 4\cos^3 x - 3\cos x;$$

$$tg3x = \frac{tgx + tg2x}{1 - tgxtg2x} = \frac{3tgx - tg^3x}{1 - 3tg^2x}.$$

Yig‘indini ko‘paytmaga keltirish

1. $Sinx \pm Siny = 2Sin\frac{x \pm y}{2}Cos\frac{x \mp y}{2}.$

2. $Cosx + Cosy = 2Cos\frac{x + y}{2}Cos\frac{x - y}{2}.$

3. $Cosx - Cosy = -2Sin\frac{x + y}{2}Sin\frac{x - y}{2}.$

4. $Cosx + Sinx = \sqrt{2}Sin\left(\frac{\pi}{4} + x\right) = \sqrt{2}Cos\left(\frac{\pi}{4} - x\right).$

5. $Cosx - Sinx = \sqrt{2}Cos\left(\frac{\pi}{4} + x\right) = \sqrt{2}Sin\left(\frac{\pi}{4} - x\right).$

6. $pCosx + qSinx = rSin(z + x)$, $r = \sqrt{p^2 + q^2}$, $Sinz = \frac{p}{r}, Cosz = \frac{q}{r}$.

7. $tgx \pm tgy = \frac{Sin(x \pm y)}{CosxCosy}$, $\qquad ctgx \pm ctgy = \frac{Sin(x \pm y)}{SinxSiny}$.

8. $tgx + ctgy = \frac{Cos(x - y)}{CosxSiny}$, $\qquad tgx - ctgy = -\frac{Cos(x + y)}{CosxSiny}$.

Ko‘paytmani yig‘indiga keltirish

1. $SinxCosy = \frac{1}{2}\left(Sin\left(x+y\right)+Sin\left(x-y\right)\right).$

2. $CosxCosy = \frac{1}{2}\left(Cos\left(x+y\right)+Cos\left(x-y\right)\right).$

3. $SinxSiny = \frac{1}{2}\left(Cos\left(x-y\right)-Cos\left(x+y\right)\right).$

Yarim burchak formulalari

1. $Sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-Cos\alpha}{2}}, \qquad Cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+Cos\alpha}{2}}.$

2. $tg\frac{\alpha}{2} = \frac{1-Cos\alpha}{Sin\alpha} = \frac{Sin\alpha}{1+Cos\alpha} = \pm\sqrt{\frac{1-Cos\alpha}{1+Cos\alpha}}.$

3. $ctg\frac{\alpha}{2} = \frac{1+Cos\alpha}{Sin\alpha} = \frac{Sin\alpha}{1-Cos\alpha} = \pm\sqrt{\frac{1+Cos\alpha}{1-Cos\alpha}}.$

Darajani pasaytirish

$$Cos^2\alpha = \frac{1+Cos2\alpha}{2}, \qquad Sin^2\alpha = \frac{1-Cos2\alpha}{2}.$$

$$Cos^3\alpha = \frac{3Cos\alpha + Cos3\alpha}{4}, \qquad Sin^3\alpha = \frac{3\sin\alpha - \sin 3\alpha}{4}.$$

$Sinx, Cosx, tgx$ va $ctgx$ larni $tg\frac{x}{2}$ orqali ifodasi

$$Sinx = \frac{2tg(x/2)}{1+tg^2(x/2)}; \qquad tgx = \frac{2tg(x/2)}{1-tg^2(x/2)};$$

$$Cosx = \frac{1-tg^2(x/2)}{1+tg^2(x/2)}; \qquad ctgx = \frac{1-tg^2(x/2)}{2tg(x/2)}.$$

Trigonometrik funksiyalarni birini ikkinchisi orqali ifodalash

	$\sin x$	$\cos x$	tgx	$ctgx$
$\sin x$	$\sin x$	$\pm\sqrt{1-\cos^2 x}$	$\frac{tgx}{\pm\sqrt{1+tg^2x}}$	$\frac{1}{\pm\sqrt{1+ctg^2x}}$
$\cos x$	$\pm\sqrt{1-\sin^2 x}$	$\cos x$	$\frac{1}{\pm\sqrt{1+tg^2x}}$	$\frac{ctgx}{\pm\sqrt{1+ctg^2x}}$

tgx	$\frac{\sin x}{\pm\sqrt{1-\sin^2 x}}$	$\frac{\pm\sqrt{1-\cos^2 x}}{\cos x}$	tgx	$\frac{1}{ctgx}$
$ctgx$	$\frac{\pm\sqrt{1-\sin^2 x}}{\sin x}$	$\frac{\cos x}{\pm\sqrt{1-\cos^2 x}}$	$\frac{1}{tgx}$	$ctgx$
$\sec x$	$\frac{1}{\pm\sqrt{1-\sin^2 x}}$	$\frac{1}{\cos x}$	$\pm\sqrt{1+tg^2x}$	$\frac{\pm\sqrt{1+ctg^2x}}{ctgx}$
$\csc x$	$\frac{1}{\sin x}$	$\frac{1}{\pm\sqrt{1-\cos^2 x}}$	$\frac{\pm\sqrt{1+tg^2x}}{tgx}$	$\pm\sqrt{1+ctg^2x}$

Keltirish formulalari

α	$\frac{\pi}{2}\pm x$	$\pi\pm x$	$\frac{3\pi}{2}\pm x$	$2\pi\pm x$
$Sin\alpha$	$Cosx$	$\mp Sinx$	$-Cosx$	$\pm Sinx$
$Cos\alpha$	$\mp Sinx$	$-Cosx$	$\pm Sinx$	$Cosx$
$tg\alpha$	$\mp ctgx$	$\pm tgx$	$\mp ctgx$	$\pm tgx$
$ctg\alpha$	$\mp tgx$	$\pm ctgx$	$\mp tgx$	$\pm ctgx$

Trigonometrik funksiyalarning ayrim burchaklardagi qiymatlari

Gradus o‘lchovi	Radian o‘lchovi	$\sin x$	$\cos x$	tgx	$ctgx$	$\sec x$	$\csc x$
0	0	0	1	0	–	1	–
30^0	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
45^0	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60^0	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2}{\sqrt{3}}$
90^0	$\frac{\pi}{2}$	1	0	–	0	–	1
120^0	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2}{\sqrt{3}}$
135^0	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$

150^0	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	2
180^0	π	0	-1	0	–	-1	–
210^0	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	-2
225^0	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
240^0	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2}{\sqrt{3}}$
270^0	$\frac{3\pi}{2}$	-1	0	–	0	–	-1
360^0	2π	0	1	0	–	1	–

Gradus o‘lchovi	Radian o‘lchovi	$\sin x$	$\cos x$	tgx	$ctgx$
15^0	$\frac{\pi}{12}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$2-\sqrt{3}$	$2+\sqrt{3}$
18^0	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{5+\sqrt{5}}}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$	$\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$
36^0	$\frac{\pi}{5}$	$\frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$
54^0	$\frac{3\pi}{10}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$
75^0	$\frac{5\pi}{12}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$2+\sqrt{3}$	$2-\sqrt{3}$

Trigonometrik tenglamalar

1) $Sinx=a, \quad |a|\le 1, \quad x=(-1)^n arcSina+n\pi, \; n\in Z$;

2) $Cosx=a, \quad |a|\le 1, \quad x=\pm arcCosa+2n\pi, \; n\in Z$.

a	$Sinx=a$	$Cosx=a$
0	$x=\pi k, \; k\in Z$	$x=\pi/2+\pi k, \; k\in Z$
1	$x=\pi/2+2\pi k, \; k\in Z$	$x=2\pi k, \; k\in Z$

$\frac{1}{2}$	$x=(-1)^k \pi/6+\pi k,\ k\in Z$	$x=\pm\pi/3+2\pi k,\ k\in Z$
$-\frac{1}{2}$	$x=(-1)^{k+1} \pi/6+\pi k,\ k\in Z$	$x=\pm 2\pi/3+2\pi k,\ k\in Z$
-1	$x=-\pi/2+2\pi k,\ k\in Z$	$x=\pi+2\pi k,\ k\in Z$
$\frac{\sqrt{3}}{2}$	$x=(-1)^k \pi/3+\pi k,\ k\in Z$	$x=\pm\pi/6+2\pi k,\ k\in Z$
$-\frac{\sqrt{3}}{2}$	$x=(-1)^{k+1} \pi/3+\pi k,\ k\in Z$	$x=\pm 5\pi/6+2\pi k,\ k\in Z$
$\frac{\sqrt{2}}{2}$	$x=(-1)^k \pi/4+\pi k,\ k\in Z$	$x=\pm\pi/4+2\pi k,\ k\in Z$
$-\frac{\sqrt{2}}{2}$	$x=(-1)^{k+1} \pi/4+\pi k,\ k\in Z$	$x=\pm 3\pi/4+2\pi k,\ k\in Z$

3) $tgx=a,\quad x=arctga+n\pi,\ n\in Z$;

4) $ctgx=a,\quad x=arcctga+n\pi,\ n\in Z$.

a	$tgx=a$	$ctgx=a$
0	$x=\pi k,\ k\in Z$	$x=\pi/2+\pi k,\ k\in Z$
1	$x=\pi/4+\pi k,\ k\in Z$	$x=\pi/4+\pi k,\ k\in Z$
-1	$x=-\pi/4+\pi k,\ k\in Z$	$x=3\pi/4+\pi k,\ k\in Z$
$\sqrt{3}$	$x=\pi/3+\pi k,\ k\in Z$	$x=\pi/6+\pi k,\ k\in Z$
$-\sqrt{3}$	$x=-\pi/3+\pi k,\ k\in Z$	$x=5\pi/6+\pi k,\ k\in Z$
$\sqrt{3}/3$	$x=\pi/6+\pi k,\ k\in Z$	$x=\pi/3+\pi k,\ k\in Z$
$-\sqrt{3}/3$	$x=-\pi/6+\pi k,\ k\in Z$	$x=2\pi/3+\pi k,\ k\in Z$

Trigonometrik tengsizliklar

1) $Sinx \geq a \quad \left(|a| \leq 1\right), \quad \Leftrightarrow x \in \left[arcSina + 2n\pi; \pi - arcSina + 2n\pi\right];$

2) $Sinx \leq a \quad \left(|a| \leq 1\right), \Leftrightarrow \quad x \in \left[-\pi - arcSina + 2n\pi; arcSina + 2n\pi\right];$

3) $Cosx \geq a \quad \left(|a| \leq 1\right), \Leftrightarrow x \in \left[-arcCosa + 2n\pi; arcCosa + 2n\pi\right];$

4) $Cosx \leq a \quad \left(|a| \leq 1\right), \Leftrightarrow x \in \left[arcCosa + 2n\pi; 2\pi - arcCosa + 2n\pi\right];$

5) $tgx \geq a \quad \left(a \in R\right), \quad \Leftrightarrow \quad x \in \left[arctga + n\pi; \frac{\pi}{2} + n\pi\right);$

6) $tgx \leq a \quad \left(a \in R\right), \Leftrightarrow \quad x \in \left(-\frac{\pi}{2} + n\pi; arctga + n\pi\right];$

7) $ctgx \geq a \quad \left(a \in R\right), \Leftrightarrow \quad x \in \left(n\pi;\ arcctga + n\pi\right];$

8) $ctgx \leq a \quad \left(a \in R\right), \Leftrightarrow \quad x \in \left[arcctga + n\pi;\ n\pi\right)$. (Bu yerda $n \in Z$.)

Ba’zi trigonometrik ayniyatlar

1. $\sin x \sin(60 - x) \sin(60 + x) = \frac{1}{4} \sin 3x$.

2. $16Sin10^0 Sin30^0 Sin50^0 Sin70^0 Sin90^0 = 1$.

3. $16Cos80^0 Cos60^0 Cos40^0 Cos20^0 Cos0^0 = 1$.

4. $16Sin20^0 Sin40^0 Sin60^0 Sin80^0 = 3$.

5. $16Cos10^0 Cos30^0 Cos50^0 Cos70^0 = 3$.

6. $Cos\frac{\pi}{7} Cos\frac{4\pi}{7} Cos\frac{5\pi}{7} = \frac{1}{8}$.

7. $Cos\frac{\pi}{7} Cos\frac{2\pi}{7} Cos\frac{4\pi}{7} = -\frac{1}{8}$.

8. $Cos\frac{\pi}{7} Cos\frac{3\pi}{7} Cos\frac{5\pi}{7} = -\frac{1}{8}$.

9. $Cos\frac{2\pi}{7} Cos\frac{4\pi}{7} Cos\frac{6\pi}{7} = \frac{1}{8}$.

10. $Cos\frac{\pi}{5} Cos\frac{3\pi}{5} = -\frac{1}{4}$.

11. $Cos\frac{\pi}{5}+Cos\frac{3\pi}{5}=\frac{1}{2}$.

12. $Sin\frac{\pi}{4n}Sin\frac{3\pi}{4n}Sin\frac{5\pi}{4n}\cdot\cdots\cdot Sin\frac{(2n-1)\pi}{4n}=\frac{\sqrt{2}}{2^n}$.

13. $Cosx\cdot Cos2x\cdot Cos4x\cdot\ldots\cdot Cos2^n x=\frac{1}{2^{n+1}}\cdot\frac{Sin2^{n+1}x}{Sinx}$.

14. $Cosx+Cos2x+\ldots+Cosnx=\frac{Sin\frac{nx}{2}Cos\frac{(n+1)x}{2}}{Sin\frac{x}{2}}$.

Teskari trigonometrik funksiyalar orasidagi bog‘lanishlar

$$arcSinx+arcCosx=\frac{\pi}{2};\quad arctgx+arcctgx=\frac{\pi}{2}$$

1. $arcSin(-x)=-arcSinx; arcCos(-x)=\pi-arcCosx$.

2. $arctg(-x)=-arctgx; arcctg(-x)=\pi-arcctgx$.

3. $arcSinx=\frac{\pi}{2}-arcCosx=arctg\frac{x}{\sqrt{1-x^2}}$.

4. $arcCosx=\frac{\pi}{2}-arcSinx=arcctg\frac{x}{\sqrt{1-x^2}}$.

5. $arctgx=\frac{\pi}{2}-arcctgx=arcSin\frac{x}{\sqrt{1+x^2}}$

6. $arcctgx=\frac{\pi}{2}-arctgx=arcCos\frac{x}{\sqrt{1+x^2}}$.

Trigonometrik va teskari trigonometrik funksiyalar orasidagi bog‘lanishlar

1. $Sin(arcSinx)=x,\ x\in[-1;1]$.

2. $arcSin(Sinx)=x, x\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$.

3. $Cos(arcCosx)=x,\ x\in[-1;1]$.

4. $arcCos(Cosx)=x,\ x\in[0;\pi]$.

5. $arctg(tgx)=x,\quad x\in\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$.

6. $tg(arctgx)=x,\quad x\in R$.

7. $arcctg(ctgx) = x, \ \ x \in (0;\pi)$.

8. $ctg(arcctgx) = x, \quad x \in R$.

9. $Sin(arcCosx) = \sqrt{1-x^2}\ , x \in [-1;1]$.

10. $Cos(arcSinx) = \sqrt{1-x^2}\ , x \in [-1;1]$.

11. $Sin(arctgx) = \dfrac{x}{\sqrt{1+x^2}}$.

12. $Cos(arctgx) = \dfrac{1}{\sqrt{1+x^2}}$.

13. $Sin(arcctgx) = \dfrac{1}{\sqrt{1+x^2}}$.

14. $Cos(arcctgx) = \dfrac{x}{\sqrt{1+x^2}}$.

15. $tg(arcSinx) = \dfrac{x}{\sqrt{1-x^2}}, x \in [-1;1]$.

16. $tg(arcSinx) = \dfrac{x}{\sqrt{1-x^2}}, x \in [-1;1]$,

17. $tg(arcSinx) = \dfrac{x}{\sqrt{1-x^2}}, x \in [-1;1]$.

18. $tg(arcSinx) = \dfrac{x}{\sqrt{1-x^2}}, x \in [-1;1]$.

19. $Sin(arcctgx) = \dfrac{1}{\pm\sqrt{1+x^2}}$.

20. $Sin(arcctgx) = \dfrac{1}{\pm\sqrt{1+x^2}}$.

Teskari trigonometrik funksiyalar yig'indisi

1. $arcSinx + arcSiny = arcCosx(\sqrt{1-x^2}\sqrt{1-y^2} - xy)$.

2. $arcSinx + arcSiny = arcCosx(\sqrt{1-x^2}\sqrt{1-y^2} - xy)$.

3. $arcSinx + arcSiny = arcCosx(\sqrt{1-x^2}\sqrt{1-y^2} - xy)$.

4. $arcSinx + arcSiny = arcCosx(\sqrt{1-x^2}\sqrt{1-y^2} - xy)$.

5. $arcCosx - arcCosy = \begin{cases} -arcCos\left(xy + \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right), & \text{x} \geq \text{y}, \\ arcCos\left(xy + \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right), & \text{x} < \text{y}. \end{cases}$

6. $arctgx + arctgy = arctg\dfrac{x+y}{1-xy}, \ xy \neq 1$.

7. $arctgx - arctgy = arctg\dfrac{x-y}{1+xy}, \ xy \neq -1$.

8. $arcctgx \pm arcctgy = arcctg\dfrac{xy \mp 1}{x \pm y}, \ x \neq -y$.

Funksiya va uning asosiy xossalari

X sonlar to‘plamidan olingan x ning har bir qiymatiga biror qonuniyat yoki qoida yordamida Y sonlar to‘plamidan olingan yagona y qiymat mos kelsa, bunday moslik **funksiya** deyiladi va $y=f(x)$ ko‘rinishida belgilanadi.

X to‘plam funksiyaning **aniqlanish sohasi** deyiladi va $D(f)$ ko‘rinishida belgilanadi.

Y to‘plam esa funksiyaning **qiymatlari to‘plami** deyiladi va $E(f)$ ko‘rinishida belgilanadi.

Funksiyaning aniqlanish sohasini (a.s.) topishga doir misollar:

1) $y=\sqrt[2n]{f(x)}$ funksiyaning a.s. $f(x)\ge 0$ tengsizlikning yechimi bo‘ladi;

2) $y=\dfrac{1}{f(x)}$ funksiyaning a.s. $f(x)\ne 0$ tengsizlikning yechimi bo‘ladi;

3) $y=\log_{g(x)}f(x)$ funksiyaning a.s. $\begin{cases} f(x)>0, \\ g(x)>0, \\ g(x)\ne 1, \end{cases}$ sistemani yechimi bo‘ladi.

Funksiyaning qiymatlar sohasini topishga doir misollar:

1) $y=\sqrt{ax^2+bx+c}$ va $y_0=\dfrac{4ac-b^2}{4a}\ge 0$ bo‘lsin.

agar $a>0$ bo‘lsa, $E(y)=\left[\sqrt{y_0};+\infty\right)$;

agar $a<0$ bo‘lsa, $E(y)=\left[0;\sqrt{y_0}\right]$ bo‘ladi**;**

2) $y=a\cos kx+b\sin kx$ bo‘lsa, $E(y)=\left[-\sqrt{a^2+b^2};\sqrt{a^2+b^2}\right]$.

Juft, toqligi

Agar $\forall x\in D(f)$ uchun $-x\in D(f)$ va $f(-x)=f(x)$ bo‘lsa, $y=f(x)$ **funksiya juft**, $f(-x)=-f(x)$ bo‘lsa, $y=f(x)$ **funksiya toq** deyiladi, aks holda **juft ham emas, toq ham emas**.

Masalan: $y=x^2$–juft funksiya, $y=x^3$–toq funksiya.

Agar $f(x)$, $g(x)$– juft, $\varphi(x)$, $\phi(x)$ – toq funksiyalar bo‘lsa, u holda $f(x)\pm g(x)$ – juft, $\varphi(x)\pm\phi(x)$ – toq, $f(x)\cdot g(x)$ – juft, $\phi(x)\cdot\varphi(x)$ – juft,
$f(x):g(x)$ – juft, $g(x)\cdot\varphi(x)$ – toq, $g(x):\varphi(x)$ – toq, $\varphi(x):\phi(x)$ – juft.
$g(x)\pm\varphi(x)$ – juft ham, toq ham emas.

Juft funksiyaning grafigi Oy o‘qiga nisbatan simmetrik bo‘ladi.

Toq funksiyaning grafigi koordinatalar boshiga nisbatan simmetrik bo‘ladi.

Davriyligi

Agar ixtiyoriy $x\in D(f)$ uchun $(x+T)\in D(f)$ $(T>0)$ va $f(x\pm T)=f(x)$ *bo‘lsa,* $y=f(x)$ **davriy funksiya,** *T soni esa uning **davri** deyiladi.*

Agar $T>0$ soni $y=f(x)$ funksiyaning davri bo‘lsa, nT $(n\in\mathbb{Z})$ soni ham $y=f(x)$ funksiyaning davri bo‘ladi.

Agar $y=f(x)$ funksiyaning eng kichik musbat (e.k.m.) davri T bo‘lsa, u holda $y=f(kx+b)$ funksiyaning e.k.m. davri $\frac{T}{k}$ bo‘ladi.

Bir necha davriy funksiyalarning yig‘indisidan iborat funksiyaning e.k.m. davri qo‘shiluvchi funksiyalar e.k.m. davrlarining EKUK iga teng.

Funksiyaning o‘sishi va kamayishi (monotonligi)

$y=f(x)$ funksiya $(a;b)$ oraliqda aniqlangan va shu oraliqdan olingan ixtiyoriy x_1, x_2 $(x_1<x_2)$ lar uchun $f(x_1)<f(x_2)$ bo‘lsa, $y=f(x)$ funksiya $(a;b)$ oraliqda **o‘suvchi**; $f(x_1)>f(x_2)$ bo‘lsa, $y=f(x)$ funksiya $(a;b)$ oraliqda **kamayuvchi** deyiladi.

Teskari funksiyani topish

$y=f(x)$ funksiyaga teskari funksiyani topish uchun

1) $y=f(x)$ tenglamadan $D(f)$ ni hisobga olgan holda x topiladi;

2) hosil bo‘lgan tenglikda x va y larning o‘rni almashtiriladi.

Masalan: $y=\frac{2}{x-1}+3$ $(x\neq 1)$ ga teskari funksiyani topaylik:

$$\frac{2}{x-1}=y-3\Rightarrow x-1=\frac{2}{y-3}\Rightarrow x=\frac{2}{y-3}+1.$$

Endi x va y larning o‘rni almashtiriladi: $y=\frac{2}{x-3}+1.$

Elementar funksiyalar va ularning asosiy xossalari

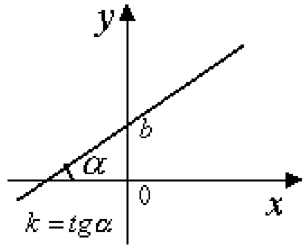
$y = kx + b$ – chiziqli funksiya

1. Aniqlanish sohasi: $D(y) = R$.

2. Qiymatlar sohasi: $E(y) = R$.

3. $k > 0$ bo‘lsa, son o‘qida o‘suvchi;
$k < 0$ bo‘lsa, son o‘qida kamayuvchi.

Bu yerda, k - to‘g‘ri chiziqning OX o‘qi bilan hosil qilgan burchak tangensi.



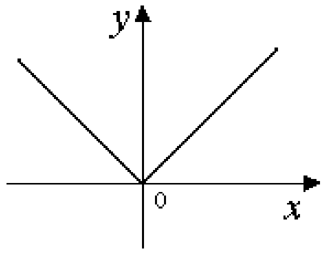
$y = |x|$ funksiya

1. Aniqlanish sohasi: $D(y) = R$.

2. Qiymatlar sohasi: $E(y) = [0; +\infty)$.

3. $[0;+\infty)$ oraliqda o‘suvchi, $(-\infty;0]$ oraliqda kamayuvchi.

4. Juft junksiya.

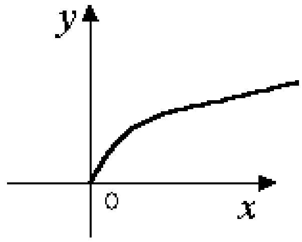


$y = \sqrt{x}$ funksiya

1. Aniqlanish sohasi: $D(y) = [0,+\infty)$.

2. Qiymatlar sohasi: $E(y) = [0,+\infty)$.

3. $[0;+\infty)$ oraliqda o‘suvchi,



$y = \frac{1}{x}$ funksiya

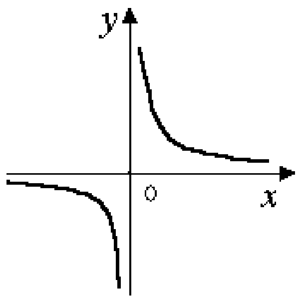
1. Aniqlanish sohasi: $D(y) = (-\infty;0) \cup (0;+\infty)$.

2. Qiymatlar sohasi: $E(y) = (-\infty;0) \cup (0;+\infty)$.

3. Toq funksiya.

4. $(-\infty;0)$ va $(0;+\infty)$ oraliqda kamayuvchi funksiya.

5. Asimptotalari: $x = 0$, $y = 0$.



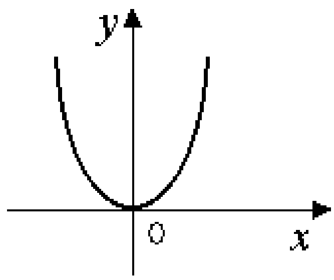
$y = x^2$ funksiya

1. Aniqlanish sohasi: $D(y) = (-\infty;+\infty)$.

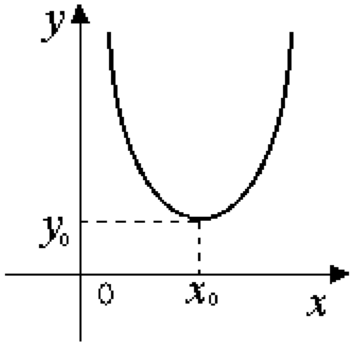
2. Qiymatlar sohasi: $E(y) = [0;+\infty)$.

3. Juft funksiya.

4. $[0;+\infty)$ oraliqda o‘suvchi, $(-\infty;0]$ oraliqda kamayuvchi.



Kvadratik funksiya $y = ax^2 + bx + c \quad (a \neq 0)$



1. Aniqlanish sohasi: $D(y) = (-\infty; +\infty)$.

2. Qiymatlar sohasi: $a > 0$ bo‘lsa, $E(y) = [y_0; +\infty)$,
$a < 0$ bo‘lsa, $E(y) = (-\infty; y_0]$.

3. Parabola uchining koordinatalari:

$$x_0 = -\frac{b}{2a}, \quad y_0 = \frac{4ac - b^2}{4a} \Rightarrow$$

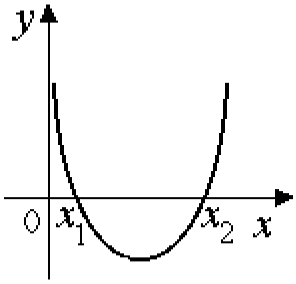
$$y = ax^2 + bx + c = a(x - x_0)^2 + y_0 .$$

4. Simmetriya o‘qi: $x = -\dfrac{b}{2a}$.

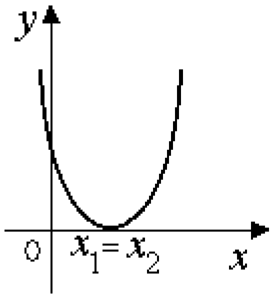
5. $a > 0$ bo‘lsa, $[x_0; +\infty)$ oraliqda o‘suvchi, $(-\infty; x_0]$ oraliqda kamayuvchi.
$a < 0$ bo‘lsa, $(-\infty; x_0]$ oraliqda o‘suvchi, $[x_0; +\infty)$ oraliqda kamayuvchi.

Parabolaning joylashishi:

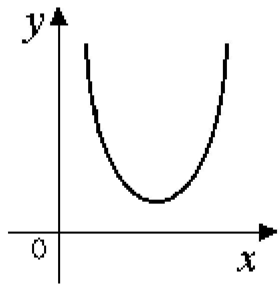
1. Agar $a > 0$ bo‘lsa:



$D > 0$

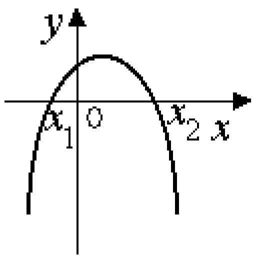


$D = 0$

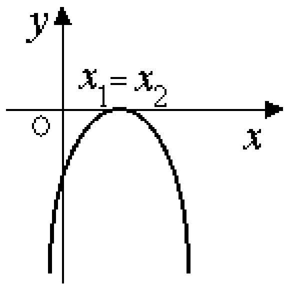


$D < 0$

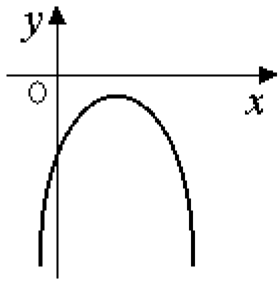
2. Agar $a < 0$ bo‘lsa:



$D > 0$

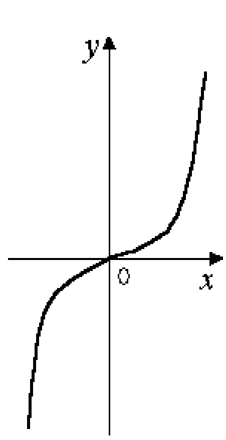


$D = 0$



$D < 0$

$y = x^3$ funksiya



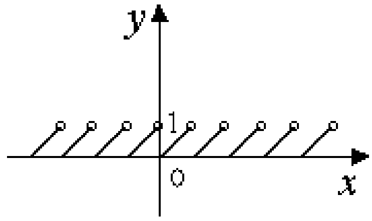
1. Aniqlanish sohasi: $D(y) = (-\infty; +\infty)$.

2. Qiymatlar sohasi: $E(y) = (-\infty; +\infty)$.

3. Toq funksiya.

4. $(-\infty; +\infty)$ oraliqda o‘suvchi.

$y=\{x\}$ **funksiya**

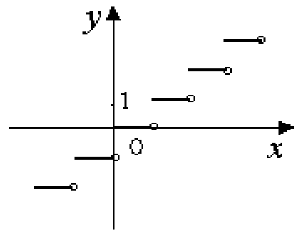


1. Aniqlanish sohasi: $D(y)=R$.

2. Qiymatlar sohasi: $E(y)=[0,1)$.

3. Davriy, e.k.m. davri 1 ga teng.

$y=[x]$ **funksiya**

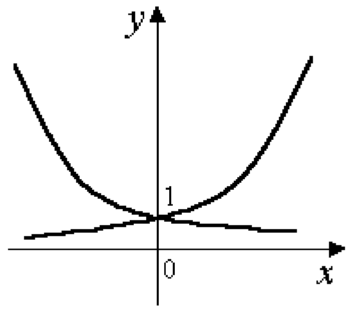


1. Aniqlanish sohasi: $D(y)=R$.

2. Qiymatlar sohasi: $E(y)=Z$.

3. Kamaymaydigan funksiya.

Ko‘rsatkichli funksiya $y=a^x \ (a>0, a\neq 1)$



1. Aniqlanish sohasi: $D(y)=(-\infty;+\infty)$.

2. Qiymatlar sohasi: $E(y)=(0;+\infty)$.

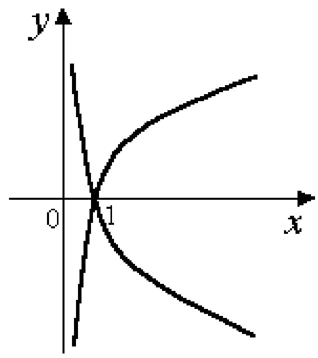
3. $a>1$ bo‘lsa, $(-\infty;+\infty)$ oraliqda o‘suvchi;

$0<a<1$ bo‘lsa, $(-\infty;+\infty)$ oraliqda kamayuvchi.

4. Grafigi (0;1) nuqtadan o‘tadi.

5. Asimptotasi: $y=0$.

Logarifmik funksiya $y=\log_a x \ (0<a\neq 1)$



1. Aniqlanish sohasi: $D(y)=(0;+\infty)$.

2. Qiymatlar sohasi: $E(y)=(-\infty;+\infty)$.

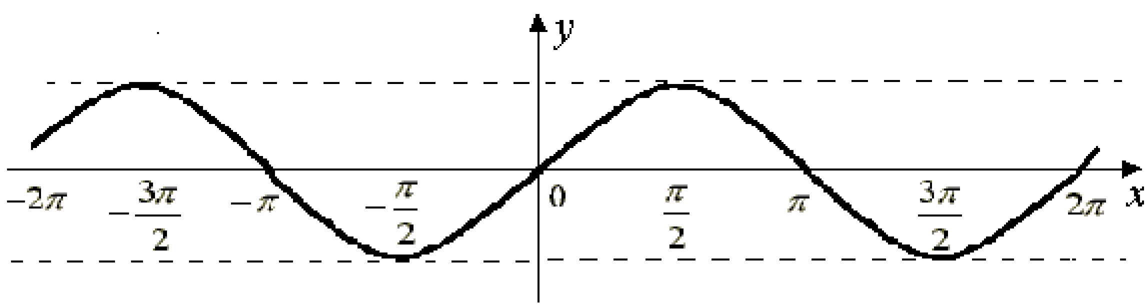
3. $a>1$ bo‘lsa, $(0;+\infty)$ oraliqda o‘suvchi;

$0<a<1$ bo‘lsa, $(0;+\infty)$ oraliqda kamayuvchi.

4. Grafigi (1;0) nuqtadan o‘tadi.

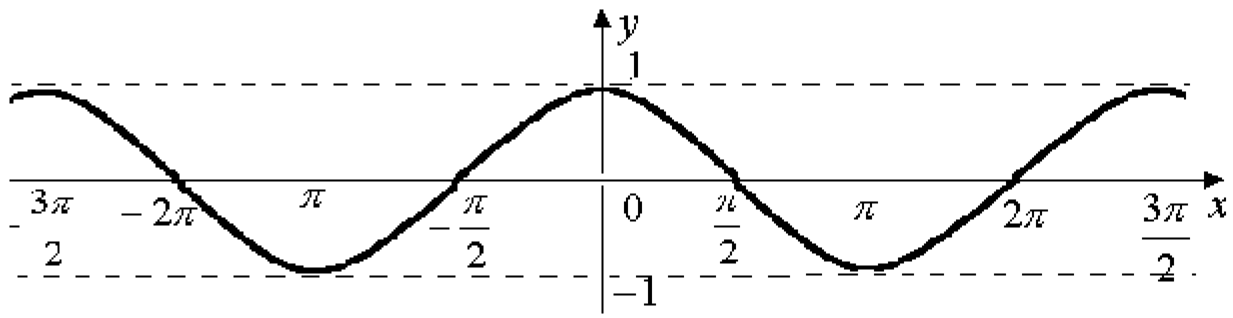
5. Asimptotasi: $x=0$.

$y=\sin x$ **funksiya**



1. Aniqlanish sohasi: $D(y) = R$.
2. Qiymatlar sohasi: $E(y) = [-1;1]$.
3. Toq funksiya: $\sin(-x) = -\sin x$.
4. Eng kichik musbat davri: $T = 2\pi$.
5. Nollari: $x_0 = \pi n, \quad n \in Z$.
6. Ishora o‘zgarmas oraliqlar:
   $x \in (2\pi n, \pi + 2\pi n)$ $(n \in \mathbb{Z})$ bo‘lsa, $Sinx > 0$;
   $x \in (-\pi + 2\pi n, 2\pi n)$ $(n \in \mathbb{Z})$ bo‘lsa, $Sinx < 0$.
7. $\left[-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n\right]$ $(n \in \mathbb{Z})$ oraliqlarda o‘suvchi;
   $\left[\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right]$ $(n \in \mathbb{Z})$ oraliqlarda kamayuvchi.
8. Eng katta qiymati 1: $Sinx = 1 \Rightarrow x = \frac{\pi}{2} + 2\pi n, \quad n \in Z$.
9. Eng kichik qiymati -1: $Sinx = -1 \Rightarrow x = -\frac{\pi}{2} + 2\pi n, n \in Z$.
10. $[2\pi n; \pi + 2\pi n]$ $(n \in \mathbb{Z})$ oraliqlarda qavariq;
    $[\pi + 2\pi n; \ 2\pi + 2\pi n]$ $(n \in \mathbb{Z})$ oraliqlarda botiq.

$y = \cos x$ funksiya



1. Aniqlanish sohasi: $D(y) = R$.
2. Qiymatlar sohasi: $E(y) = [-1;1]$.
3. Juft funksiya: $\cos(-x) = \cos x$.
4. Eng kichik musbat davri: $T = 2\pi$.
5. Nollari: $x_0 = \frac{\pi}{2} + \pi n, \quad n \in Z$.
6. Ishora o‘zgarmas oraliqlar:
   $x \in \left(-\frac{\pi}{2} + 2\pi n, \frac{\pi}{2} + 2\pi n\right)$ $(n \in \mathbb{Z})$ bo‘lsa, $Cosx > 0$;
   $x \in \left(\frac{\pi}{2} + 2\pi n, \frac{3\pi}{2} + 2\pi n\right)$ $(n \in \mathbb{Z})$ bo‘lsa, $Cosx < 0$.

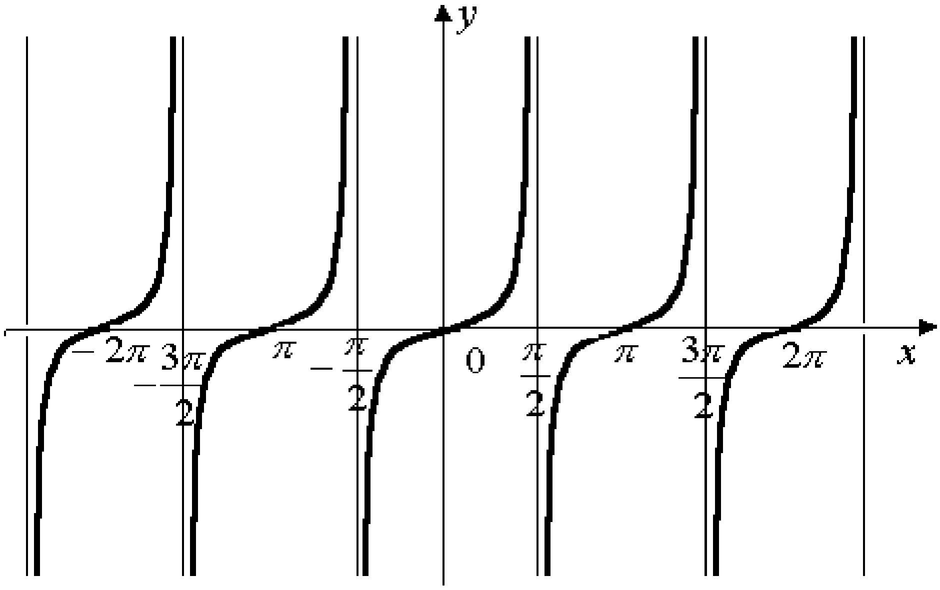
7. $[-\pi + 2\pi n;\ 2\pi n]$ $(n \in \mathbb{Z})$ oraliqlarda o‘suvchi;
$[2\pi n;\ \pi + 2\pi n]$ $(n \in \mathbb{Z})$ oraliqlarda kamayuvchi.

8. Eng katta qiymati 1, $Cosx = 1 \Rightarrow x = 2\pi n, \quad n \in Z$.

9. Eng kichik qiymati -1, $Cosx = -1 \Rightarrow x = \pi + 2\pi n, \ n \in Z$.

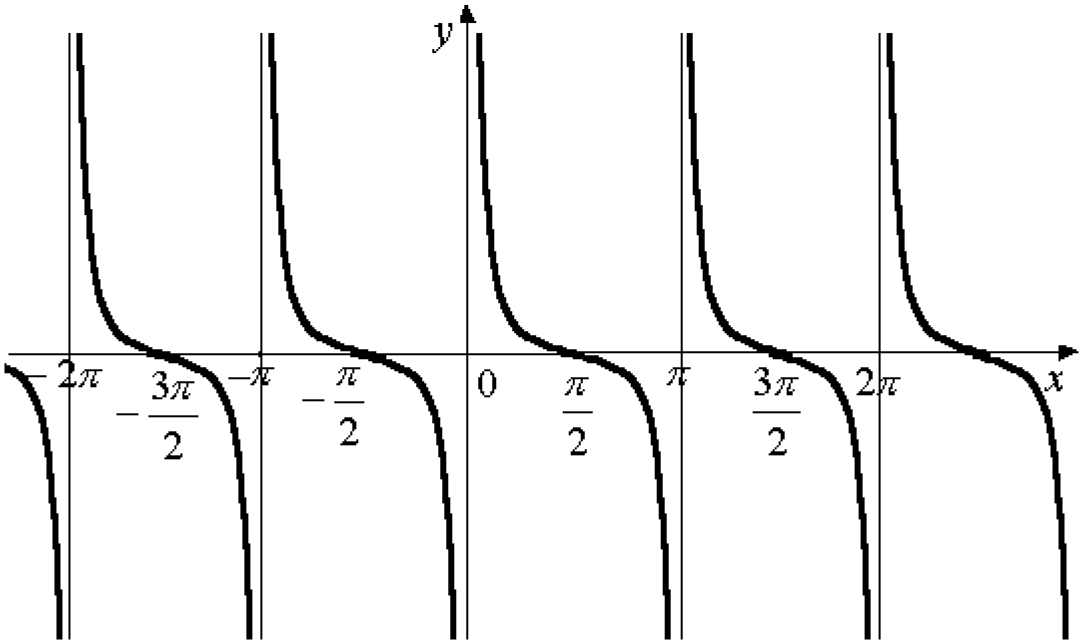
10. $-\frac{\pi}{2} + 2\pi n \le x \le \frac{\pi}{2} + 2\pi n$ $(n \in Z)$ oraliqlarda qavariq.
$\frac{\pi}{2} + 2\pi n \le x \le \frac{3\pi}{2} + 2\pi n$ $(n \in Z)$ oraliqlarda botiq.

$y = tgx$ **funksiya**



1. Aniqlanish sohasi: $x \ne \frac{\pi}{2} + \pi n, \quad n \in Z$.
2. Qiymatlar sohasi: $E(y) = R$.
3. Toq funksiya: $tg(-x) = -tgx$.
4. Eng kichik musbat davri: $T = \pi$.
5. Nollari: $x_0 = \pi n, \ n \in Z$.
6. Ishora o‘zgarmas oraliqlar:
$x \in \left(\pi n, \frac{\pi}{2} + \pi n\right)$ $(n \in \mathbb{Z})$ bo‘lsa, $tgx > 0$;
$x \in \left(-\frac{\pi}{2} + \pi n, \pi n\right)$ $(n \in \mathbb{Z})$ bo‘lsa, $tgx < 0$.
7. $\left(-\frac{\pi}{2} + \pi n;\ \frac{\pi}{2} + \pi n\right)$ $(n \in Z)$ oraliqlarda o‘suvchi.
8. Asimptotalari: $x = \frac{\pi}{2} + \pi n, \ n \in Z$.

$y = ctgx$ **funksiya**



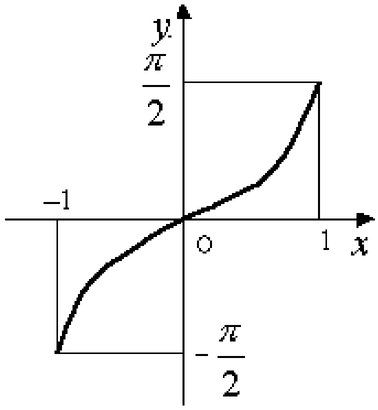
1. Aniqlanish sohasi: $x \neq \pi n,\quad n \in Z$.
2. Qiymatlar sohasi: $E(y) = R$.
3. Toq funksiya: $ctg(-x) = -ctgx$.
4. Eng kichik musbat davri: $T = \pi$.
5. Nollari: $x_0 = \frac{\pi}{2} + \pi n,\quad n \in Z$.
6. Ishora o‘zgarmas oraliqlar:

   $x \in \left(\pi n, \frac{\pi}{2} + \pi n\right)$ $(n \in \mathbb{Z})$ bo‘lsa, $ctgx > 0$;

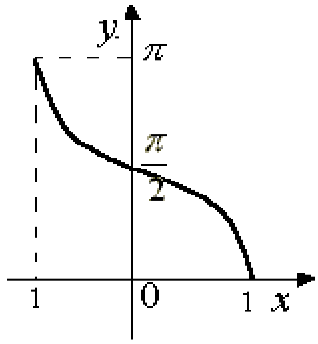
   $x \in \left(-\frac{\pi}{2} + \pi n, \pi n\right)$ $(n \in \mathbb{Z})$ bo‘lsa, $ctgx < 0$.
7. $[\pi n;\ \pi + \pi n],\ n \in Z$ oraliqlarda kamayuvchi.
8. Asimptotalari: $x = \pi n,\ n \in Z$.

$y = \arcsin x$ **funksiya**

1. Aniqlanish sohasi: $D(y) = [-1;1]$.
2. Qiymatlar sohasi: $E(y) = \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$.
3. Toq funksiya: $arcSin(-x) = -arcSinx$.
4. Aniqlanish sohasida o‘suvchi funksiya.

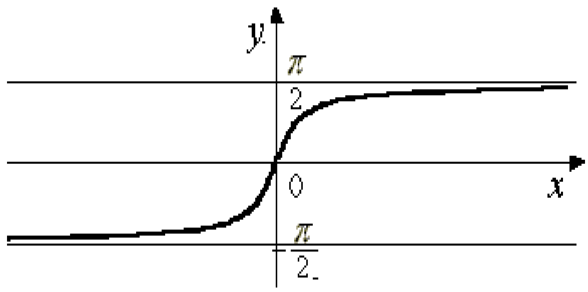


$y = \arccos x$ **funksiya**



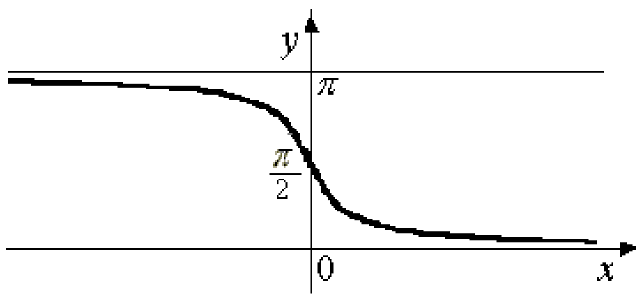
1. Aniqlanish sohasi: $D(y) = [-1;1]$.
2. Qiymatlar sohasi: $E(y) = [0;\pi]$.
3. Toq ham, juft ham emas: $arcCos(-x) = \pi - arcCosx$.
4. Aniqlanish sohasidakamayuvchi funksiya.

$y = arctgx$ **funksiya**



1. Aniqlanish sohasi: $D(y) = R$.
2. Qiymatlar sohasi: $E(y) = \left(-\frac{\pi}{2};\frac{\pi}{2}\right)$.
3. Toq funksiya: $arctg(-x) = -arctgx$.
4. Aniqlanish sohasida o‘suvchi funksiya.
5. Asimtotalari $y = \pm\frac{\pi}{2}$.

$y = arcctgx$ **funksiya**



1. Aniqlanish sohasi: $D(y) = R$.
2. Qiymatlar sohasi: $E(y) \in (0;\pi)$.
3. Toq ham emas, juft ham emas: $arcctg(-x) = \pi - arcctgx$.
4. Aniqlanish sohasida kamayuvchi funksiya.
5. Asimptotalari $y = 0, \quad y = \pi$.

Hosila

$y = f(x)$ funksiyaning $x = x_0$ nuqtadagi hosilasi

$$y' = f'(x_0) = \lim_{\Delta x \to x_0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to x_0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Hosilaning geometrik ma’nosi

$y = f(x)$ funksiya grafigiga $x = x_0$ nuqtada o‘tkazilgan urinmaning burchak koeffitsienti $k = tg\alpha = y'(x_0)$.

Urinma tenglamasi:

$$y = f(x_0) + f'(x_0)(x - x_0).$$

Hosilaning fizik ma'nosi

Harakatlanayotgan jismning t_0 vaqtda bosib o'tgan yo'li $S=f(t)$, uning t_0 vaqtdagi tezligi $V(t_0)$, tezlanishi esa $a(t_0)$ bo'lsa, u holda

$$V(t_0)=f'(t_0), \quad a(t_0)=V'(t_0).$$

Differensiallashning asosiy qoidalari

$$(u\pm v)'=u'\pm v'; \qquad (u\cdot v)'=u'v+uv';$$

$$\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}; \qquad \left(f\left(g(x)\right)\right)'=f'\left(g(x)\right)\cdot g'(x).$$

Funksiyani hosila yordamida tekshirish

$y=f(x)$ funksiya (a,b) oraliqda aniqlangan bo'lsin.

1) Agar (a,b) oraliqda $y'>0$ bo'lsa, u holda funksiya shu oraliqda o'suvchi;

2) Agar (a,b) oraliqda $y'<0$ bo'lsa, u holda funksiya shu oraliqda kamayuvchi;

3) Agar (a,b) oraliqda $y''>0$ bo'lsa, u holda funksiya shu oraliqda botiq, agar $y''<0$ bo'lsa, shu oraliqda qavariq bo'ladi.

Funksiyaning eng katta va eng kichik qiymatlari

$f'(x)=0$ tenglamaning ildizlari va hosila mavjud bo'lmagan nuqtalar $y=f(x)$ funksiyaning statsionar nuqtalari deyiladi.

$[a,b]$ kesmada uzluksiz bo'lgan $y=f(x)$ funksiyaning shu kesmadagi **eng katta** va **eng kichik** qiymatlari $f(x_1)$, $f(x_2)$, …, $f(x_n)$, $f(a)$, $f(b)$ sonlar orasida bo'ladi. Bu yerda $x_1, x_2,... x_n$ lar $y=f(x)$ funksiyaning (a,b) oraliqdagi **statsionar** nuqtalari.

Elementar funksiyalarning hosilalari

Funksiya	Hosilasi	Funksiya	Hosilasi
$y = C$	$y' = 0$	$y = x$	$y' = 1$
$y = x^m$	$y' = m \cdot x^{m-1}$	$y = ctgkx$	$y' = -\dfrac{k}{Sin^2 kx}$
$y = a^x$	$y' = a^x \cdot \ln a$	$y = \arcsin kx$	$y' = \dfrac{k}{\sqrt{1-(kx)^2}}$
$y = \log_a x$	$y' = \dfrac{1}{x \ln a}$	$y = \arccos kx$	$y' = -\dfrac{k}{\sqrt{1-(kx)^2}}$
$y = Sinkx$	$y' = k \cdot Coskx$	$y = arctgkx$	$y' = \dfrac{k}{1+(kx)^2}$
$y = Coskx$	$y' = -k \cdot Sinkx$	$y = arcctgkx$	$y' = -\dfrac{k}{1+(kx)^2}$
$y = tgkx$	$y' = \dfrac{k}{Cos^2 kx}$	$y = e^x$	$y' = e^x$
$y = [g(x)]^n$	$y' = n[g(x)]^{n-1} g'(x)$	$y = a^{g(x)}$	$y' = a^{g(x)} \ln a \cdot g'(x)$
$y = \dfrac{1}{g^n(x)}$	$y' = -\dfrac{ng'(x)}{g^{n+1}(x)}$	$y = tgf(x)$	$y' = \dfrac{g'(x)}{Cos^2 f(x)}$
$y = \sqrt[n]{g(x)}$	$y' = \dfrac{g'(x)}{n\sqrt[n]{g^{n-1}(x)}}$	$y = ctgg(x)$	$y' = -\dfrac{g'(x)}{Sin^2 g(x)}$
$y = \dfrac{1}{\sqrt[n]{g(x)}}$	$y' = -\dfrac{g'(x)}{n\sqrt[n]{g^{n+1}(x)}}$	$y = arcS\text{in} f(x)$	$y' = \dfrac{f'(x)}{\sqrt{1-f^2(x)}}$
$y = Sing(x)$	$y' = Cosg(x) \cdot g'(x)$	$y = arcCosf(x)$	$y' = -\dfrac{f'(x)}{\sqrt{1-f^2(x)}}$
$y = Cosf(x)$	$y' = -Sinf(x) \cdot f'(x)$	$y = arctgf(x)$	$y' = \dfrac{f'(x)}{1+f^2(x)}$
$y = \log_a g(x)$	$y' = \dfrac{g'(x)}{g(x)\ln a}$	$y = arcctgg(x)$	$y' = -\dfrac{g'(x)}{1+g^2(x)}$

Ajoyib limitlar

1. $\lim\limits_{x \to 0} \dfrac{Sinx}{x} = \lim\limits_{x \to 0} \dfrac{x}{Sinx} = 1.$

2. $\lim\limits_{x \to 0} x^x = 1.$

3. $\lim\limits_{x \to 0} \dfrac{Sinpx}{x} = p, \quad -\infty < p < +\infty.$

4. $\lim\limits_{x \to 0} (1+x)^{\frac{1}{x}} = e.$

5. $\lim\limits_{x\to 0}\frac{C^x-1}{x}=\ln C,\quad C>0$. **6.** $\lim\limits_{x\to 0}\frac{tgx}{x}=1$.

7. $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e=2,7183...$.

Boshlang'ich funksiya (integral)
Boshlang'ich funksiya va uni topishning sodda qoidalari

Agar berilgan oraliqdagi barcha x uchun $F'(x)=f(x)$ tenglik bajarilsa, u holda $F(x)$ funksiya shu oraliqda $f(x)$ funksiyaning **boshlang'ich funksiya**si deyiladi.

Agar $F(x)$ funksiya $y=f(x)$ funksiyaning boshlang'ich funksiyasi bo'lsa:

1) har qanday o'zgarmas S uchun $F(x)+C$ ham $f(x)$ ning boshlang'ich funksiyasi bo'ladi;

2) $\frac{1}{k}F(kx+b)$ funksiya $y=f(kx+b)$ funksiyaning boshlang'ich funksiyasi bo'ladi.

Elementar funksiyalarning boshlang'ichlari

Funksiya	Boshlang'ichi		Funksiya	Boshlang'ichi
$y=x^m$, $m\neq -1$	$Y=\frac{x^{m+1}}{m+1}$		$y=ctgkx$	$Y=\frac{1}{k}\ln\lvert Sinkx\rvert+C$
$y=a^x$	$Y=\frac{a^x}{\ln a}+C$		$y=\frac{1}{Sinx}$	$Y=\ln\left\lvert tg\frac{x}{2}\right\rvert+C$
$y=\frac{1}{x}$	$Y=\ln\lvert x\rvert+C$		$y=\frac{1}{Cosx}$	$Y=\ln\left\lvert tg\left(\frac{x}{2}+\frac{\pi}{2}\right)\right\rvert+C$
$y=Sinkx$	$Y=-\frac{1}{k}\cdot Coskx+C$		$y=\frac{1}{Sin^2x}$	$Y=-ctgx+C$
$y=Coskx$	$Y=\frac{1}{k}\cdot Sinkx+C$		$y=\frac{1}{Cos^2x}$	$Y=tgx+C$
$y=tgkx$	$Y=-\frac{1}{k}\ln\lvert Coskx\rvert+C$		$y=\frac{1}{a^2+x^2}$	$Y=\frac{1}{a}arctg\frac{x}{a}+C$
$y=\frac{1}{x^2-a^2}$	$Y=\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$		$y=\frac{1}{\sqrt{a^2-x^2}}$	$Y=\arcsin\frac{x}{a}+C$
$y=e^x$	$Y=e^x+C$		$y=\frac{1}{\sqrt{x^2\pm a^2}}$	$Y=\ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$

Aniq integralning asosiy xossalari

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx; \qquad \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx;$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx; \qquad \int_a^b f(kx+p)dx = \frac{1}{k}\int_{ka+p}^{kb+p} f(t)dt;$$

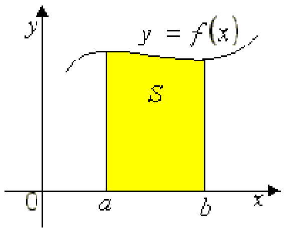
$$\int_a^b \big(f(x)\pm g(x)\big)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx.$$

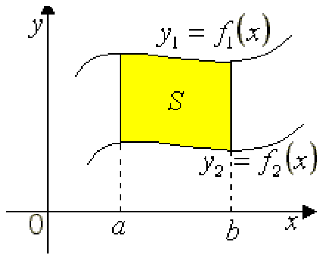
Nyuton-Leybnits formulasi

Agar $F(x)$ funksiya $y=f(x)$ funksiyaning boshlang‘ich funksiyasi bo‘lsa, u holda $\int_a^b f(x)dx = F(b)-F(a)$.

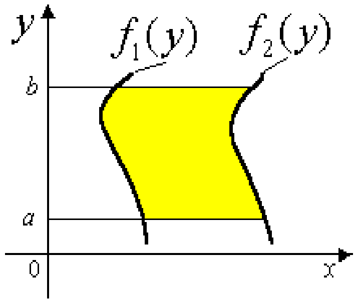
Egri chiziqli trapetsiyaning yuzi

1. $S=\int_a^b f(x)dx = F(x)\Big|_a^b = F(b)-F(a)$.





2. $S=\int_a^b [f_1(x)-f_2(x)]dx$.

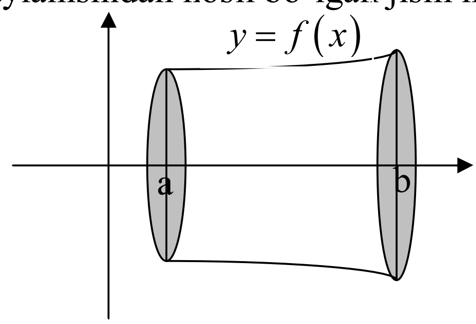


3. $S=\int_a^b [f_2(y)-f_1(y)]dy$.

Aylanma jismning hajmi

Egri chiziqli trapetsiyaning Ox o‘qi atrofida aylanishidan hosil bo‘lgan jism hajmi

$$V=\pi\int_a^b f^2(x)dx.$$



GEOMETRIYA
Planimetriya

Tekislikda istalgan uchtasi bir to‘g‘ri chiziqda yotmaydigan n $(n>2)$ ta nuqtalar berilgan bo‘lsa, ikkitasini o‘z ichiga oluvchi $\frac{n(n-1)}{2}$ ta to‘g‘ri chiziq o‘tkazish mumkin. Bu to‘g‘ri chiziqlar tekislikni $\frac{n^2+n+2}{2}$ ta qismga ajratadi.

To‘g‘ri chiziqni kichik lotin xarflari $a, b, c, d, \ldots$ va nuqtalarni katta lotin xarflari $A, B, C, D, \ldots$ bilan belgilanadi.

Burchaklar

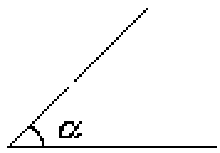
Boshi bir nuqtada bo‘lgan ikkita nurdan tashkil topgan shakl **burchak** deyiladi, berilgan nuqta burchakning uchi, nurlar esa burchakning tomonlari deyiladi.

Burchak kattaligi kichik yunon harflari $\alpha, \beta, \phi, \gamma, \ldots$ bilan belgilanadi.

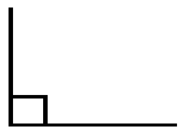
Burchakning uchidan chiqib, uni teng ikkiga bo‘lgan nur **bissektrisa** deyiladi.

Turlari:

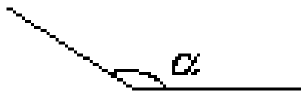
O‘tkir burchak: $0<\alpha<90°$.



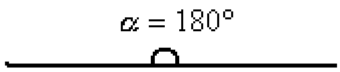
To‘g‘ri burchak: $\alpha=90°$.

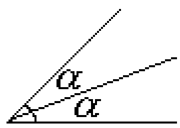


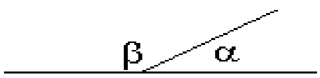
O‘tmas burchak: $90°<\alpha<180°$.

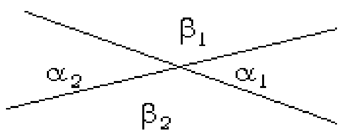


Yoyiq burchak: $\alpha=180°$.







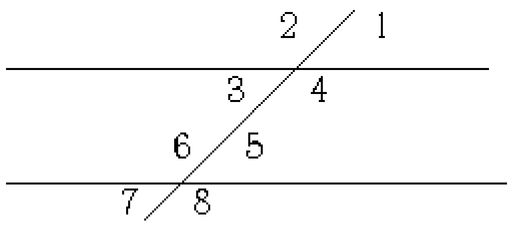


$\alpha+\beta=180°$ Qo‘shni burchaklar

$\alpha_1=\alpha_2$; $\beta_1=\beta_2$ Vertikal burchaklar.

Ikki paralel to‘g‘ri chiziqni uchinchi chiziq kesib o‘tganda xosil bo‘lgan burchaklar

Mos burchaklar: 1 va 5; 2 va 6; 3 va 7; 4 va 8.
Ichki almashinuvchi burchaklar: 3 va 5; 4 va 6.
Tashqi almashinuvchi burchaklar: 1 va 7; 2 va 8.
Ichki bir tomonli burchaklar: 3 va 6; 4 va 5.
Tashqi bir tomonli burchaklar: 1 va 8; 2 va 7.
$\angle 1=\angle 3=\angle 5=\angle 7$; $\angle 2=\angle 4=\angle 6=\angle 8$;
$\angle 4+\angle 5=\angle 3+\angle 6=180^0$.



Uchburchak

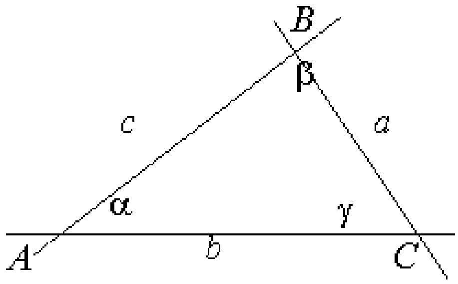
ABC uchburchakning mos tomonlari a, b, c va mos burchaklari α, β, γ orqali ifodalaymiz.

1) $\alpha+\beta+\gamma=180°$;
2) uchburchak tengsizligi

$a+b>c$; $b+c>a$; $a+c>b$;

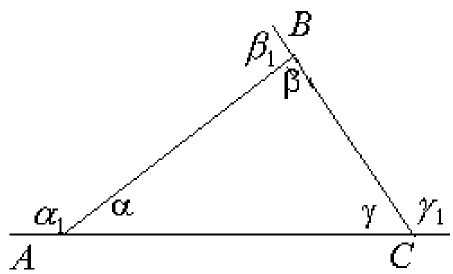
$a-b<c$; $b-c<a$; $a-c<b$;

3) uchburchakning katta burchagi qarshisida katta tomoni, kichik burchagi qarshisida kichik tomoni yotadi;



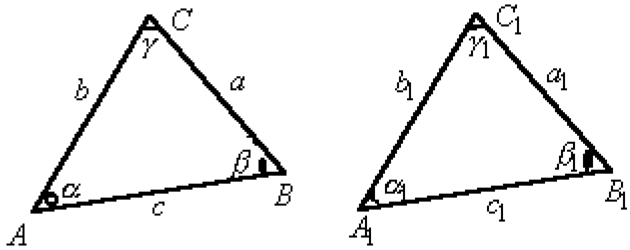
4) tashqi burchaklari: $\alpha_1, \beta_1, \gamma_1$: $\alpha_1+\beta_1+\gamma_1=360°$;

$\alpha_1=\beta+\gamma$; $\beta_1=\alpha+\gamma$; $\gamma_1=\alpha+\beta$.



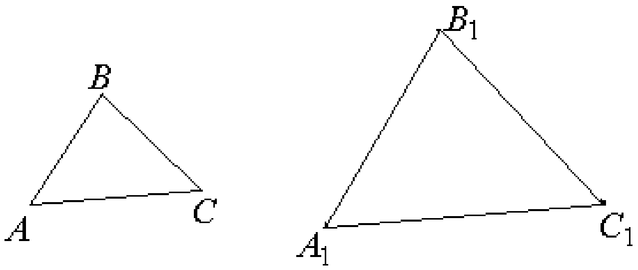
Uchburchaklarning tengligi

Agar ΔABC va $\Delta A_1B_1C_1$ larda:
1) $a=a_1$, $b=b_1$, $\angle\gamma=\angle\gamma_1$;
2) $a=a_1$, $\angle\beta=\angle\beta_1$, $\angle\gamma=\angle\gamma_1$;
3) $a=a_1$, $b=b_1, c=c_1$ bo‘lsa, u holda ΔABC va $\Delta A_1B_1C_1$ lar teng bo‘ladi.



O‘xshash uchburchaklar

1. $\Delta ABC \sim \Delta A_1B_1C_1 \quad \Leftrightarrow \quad \begin{cases} \angle A=\angle A_1,\ \angle B=\angle B_1,\ \angle C=\angle C_1, \\ \dfrac{AB}{A_1B_1}=\dfrac{AC}{A_1C_1}=\dfrac{BC}{B_1C_1}. \end{cases}$



2. $\angle A = \angle A_1,\ \angle B = \angle B_1 \ \Rightarrow\ \Delta ABC \sim \Delta A_1B_1C_1$.

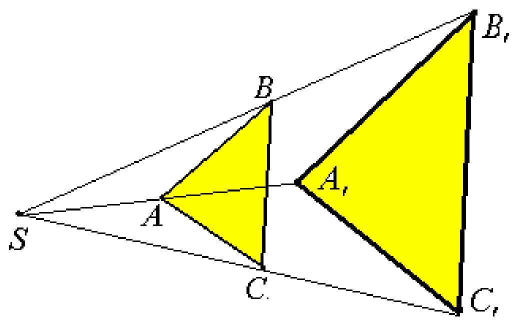
3. $\dfrac{AB}{A_1B_1} = \dfrac{AC}{A_1C_1},\quad \angle A = \angle A_1 \ \Rightarrow\ \Delta ABC \sim \Delta A_1B_1C_1$.

4. $\dfrac{AB}{A_1B_1} = \dfrac{AC}{A_1C_1} = \dfrac{BC}{B_1C_1} \ \Rightarrow\ \Delta ABC \sim \Delta A_1B_1C_1$.

5. $\Delta ABC \sim \Delta A_1B_1C_1 \ \Leftrightarrow\ \dfrac{S_{\Delta ABC}}{S_{\Delta A_1B_1C_1}} = \left(\dfrac{AB}{A_1B_1}\right)^2 = \left(\dfrac{P_{ABC}}{P_{A_1B_1C_1}}\right)^2$.

Gomotetiya

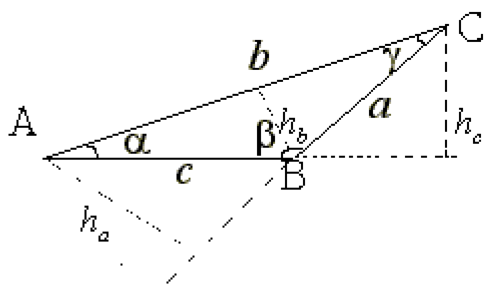
$\Delta ABC \sim \Delta A_1B_1C_1$.



Balandlik, bissektrisa, mediana
Balandlik

Uchburchak uchidan shu uch qarshisidagi tomonga tushirilgan perpendikulyar **balandlik** deyiladi.

$h_a,\ h_b,\ h_c$–uchburchak balandliklari bo‘lsin:

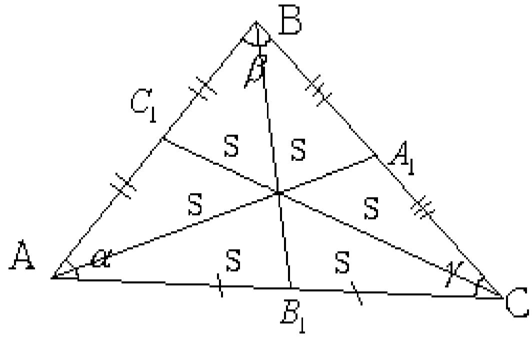


1) $h_a = \dfrac{2S}{a} = bSin\gamma = cSin\beta$;

2) $h_a : h_b : h_c = \dfrac{1}{a} : \dfrac{1}{b} : \dfrac{1}{c} = bc : ac : ab$;

3) $\dfrac{1}{r} = \dfrac{1}{h_a} + \dfrac{1}{h_b} + \dfrac{1}{h_c}$, r – ichki chizilgan aylana radiusi.

Mediana



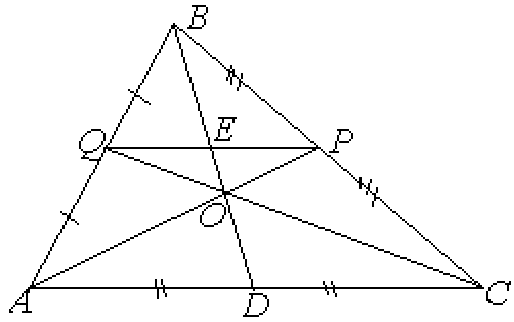
Uchburchak uchi bilan shu uch qarshisidagi tomon o‘rtasini tutashtiruvchi kesma **mediana** deyiladi.

Uchburchakning uchta medianasi bir nuqtada kesishadi va uchidan boshlab hisoblaganda shu nuqtada 2:1 nisbatda bo‘linadi.

Medianalar kesishish nuqtasi uchburchakning **og‘irlik markazi** deyiladi.

$m_a,\ m_b,\ m_c$ –uchburchakning medianalari bo‘lsin:

1) $AP = m_a = \frac{1}{2}\sqrt{2\left(b^2+c^2\right)-a^2} = \frac{1}{2}\sqrt{b^2+c^2+2bcCos\alpha}$;



2) $a = \frac{2}{3}\sqrt{2\left(m_b{}^2+m_c{}^2\right)-m_a{}^2}$;

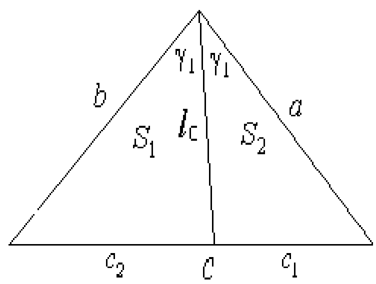
3) $OE = \frac{1}{6}BD$;

4) $S_{\Delta EOP} = S_{\Delta EOQ} = \frac{1}{24}S_{\Delta ABC}$;

$S_{\Delta BQE} = S_{\Delta BEP} = 1/8\ S_{\Delta ABC}$.

Bissektrisa

Uchburchak uchidan chiqib, shu uchidagi burchakni teng ikkiga bo‘luvchi va ikkinchi uchi shu burchak qarshisidagi tomonda yotuvchi kesma **bissektrisa** deyiladi.



Bissektrissalar $l_a,\ l_b,\ l_c$ – bilan belgilanadi.

1) $2\gamma_1 = \gamma$;

2) $\frac{a}{b} = \frac{c_1}{c_2}$; $\frac{S_1}{S_2} = \frac{b}{a}$;

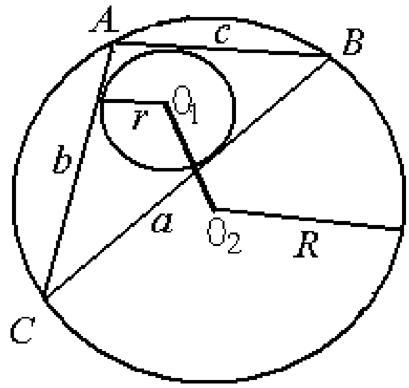
3) $l_c = \frac{1}{b+a}\sqrt{ab\left(a+b+c\right)\left(-c+b+a\right)} = \frac{2abCos\frac{\gamma}{2}}{b+a} = \sqrt{ba-c_1c_2}$.

Ichki va tashqi chizilgan aylanalar

Ichki chizilgan aylana markazi bissektrisalar kesishgan nuqtada bo‘ladi. Ichki chizilgan aylana radiusini r bilan belgilaymiz.

Tashqi chizilgan aylana markazi uchburchak tomonlarining o‘rta perpendikulyarlari kesishgan nuqtada bo‘ladi. Tashqi chizilgan aylana radiusini R orqali belgilaymiz.

1) $r = \frac{S}{p} = \frac{\sqrt{p(p-a)(p-b)(p-c)}}{p}$; $p = \frac{a+b+c}{2}$;

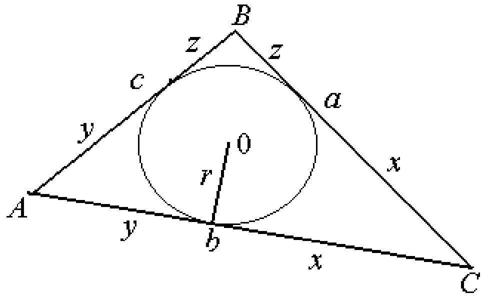


2) $R = \frac{abc}{4S} = \frac{abc}{4\sqrt{p(p-a)(p-b)(p-c)}}$;

3) $R = \frac{a}{2\sin\alpha} = \frac{b}{2\sin\beta} = \frac{c}{2\sin\gamma}$;

4) Ichki va tashqi chizilgan aylanalar markazlari orasidagi masofa $O_1O_2=\sqrt{R^2-2Rr}$;

5) $y=\dfrac{c+b-a}{2}$.



Uchburchak yuzi

1) $S=\dfrac{a\cdot h_a}{2}=\dfrac{b\cdot h_b}{2}=\dfrac{c\cdot h_c}{2}$, $\quad S=\dfrac{1}{2}abSin\gamma=\dfrac{a^2Sin\beta Sin\gamma}{2Sin\alpha}$;

2) $S=\sqrt{p(p-a)(p-b)(p-c)}$ **(Geron formulasi)**;

3) $S=\dfrac{abc}{4R}=pr$, $\quad p=\dfrac{a+b+c}{2}$;

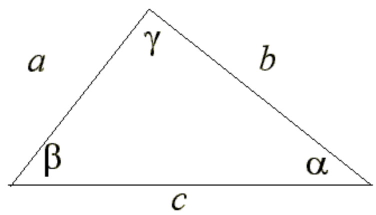
4) $S=\dfrac{4}{3}\sqrt{m(m-m_a)(m-m_b)(m-m_c)}$, $\quad m=\dfrac{m_a+m_b+m_c}{2}$.

Uchburchakdagi asosiy teoremalar

1. Sinuslar teoremasi:

$$\frac{a}{Sin\alpha}=\frac{b}{Sin\beta}=\frac{c}{Sin\gamma}=2R,$$

R – tashqi chizilgan aylana radiusi.



2. Kosinuslar teoremasi:

$$a^2=b^2+c^2-2bcCos\alpha.$$

3. $a=bCos\gamma+cCos\beta$.

4. Molveyde formulasi:

$$\frac{a+b}{c}=\frac{Cos\frac{\alpha-\beta}{2}}{Sin\frac{\gamma}{2}}.$$

5. Tangenslar teoremasi

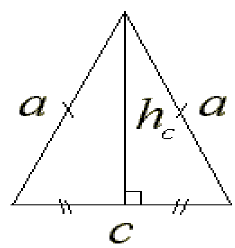
$$\frac{a+b}{a-b}=\frac{tg\frac{\alpha+\beta}{2}}{tg\frac{\alpha-\beta}{2}}=\frac{ctg\frac{\gamma}{2}}{tg\frac{\alpha-\beta}{2}}.$$

6. $Sin\dfrac{\alpha}{2}=\sqrt{\dfrac{(p-b)(p-c)}{bc}}$; $\quad Cos\dfrac{\alpha}{2}=\sqrt{\dfrac{p(p-a)}{bc}}$.

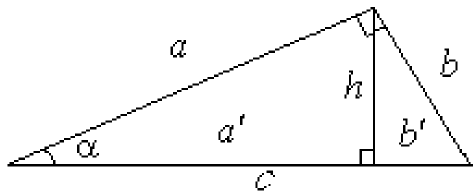
Teng yonli uchburchak

$$h_c=\frac{\sqrt{4a^2-c^2}}{2};\quad S=\frac{c\sqrt{4a^2-c^2}}{4};$$

$$R=\frac{a^2}{2h_c};\qquad r=\frac{c(2a-c)}{4h_c}.$$



To‘g‘ri burchakli uchburchak



1) $a^2 + \text{в}^2 = c^2$ **(Pifagor teoremasi)**;

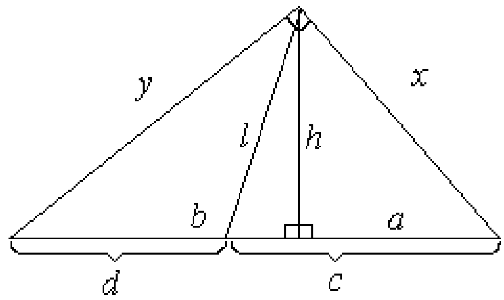
2) $Sin\alpha = \frac{b}{c}, \quad Cos\alpha = \frac{a}{c}; \quad tg\alpha = \frac{b}{a}, \quad ctg\alpha = \frac{a}{b};$

3) $a^2 = ca', \qquad b^2 = cb';$

$h^2 = a'b', \qquad h_c = \frac{ab}{c};$

4) $R = \frac{c}{2} = m_c, \quad r = \frac{a+b-c}{2}, \qquad r + R = \frac{a+b}{2};$

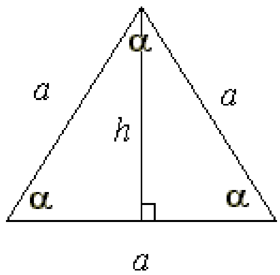
5) $S = \frac{ab}{2} = \frac{ch}{2} = \frac{a^2 tg\alpha}{2} = \frac{c^2 Sin2\alpha}{4} = r^2 + 2rR;$



6) $\left(\frac{d}{c}\right)^2 = \frac{b}{a}, \qquad \left(\frac{y}{x}\right)^2 = \frac{b}{a},$

l – bissektrisa.

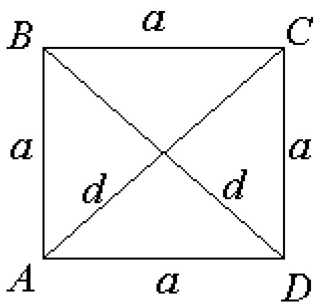
Teng tomonli uchburchak



$R = \frac{\sqrt{3}a}{3}, \quad r = \frac{\sqrt{3}a}{6}, \quad R = 2r;$

$h = 3r = 1{,}5R = r + R, \quad S = \frac{\sqrt{3}}{4}a^2.$

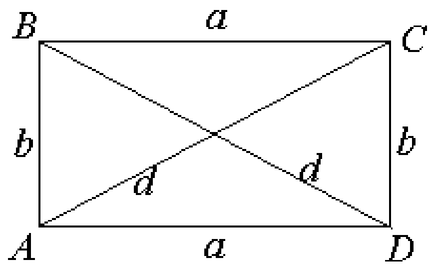
To‘rtburchaklar
Kvadrat



$P = 4a, \qquad S = a^2 = \frac{d^2}{2},$

$d = a\sqrt{2}, \qquad r = \frac{a}{2}, \qquad R = \frac{d}{2}.$

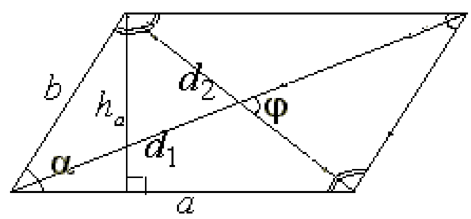
To‘g‘ri to‘rtburchak



$P = 2(a+b), \qquad S = ab,$

$d = \sqrt{a^2 + b^2}, \qquad R = \frac{d}{2}.$

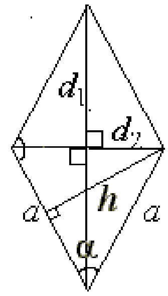
Parallelogramm



1) $P = 2(a+b);$ 2) $2a^2 + 2b^2 = d_1^2 + d_2^2;$

3) $S = a \cdot h_a = a \cdot b \cdot Sin\alpha, \qquad S = \frac{1}{2} d_1 \cdot d_2 \cdot Sin\varphi.$

Romb



1) $4a^2 = d_1^2 + d_2^2$; **2)** $S = a \cdot h = 2a \cdot r = \frac{1}{2} d_1 \cdot d_2$,

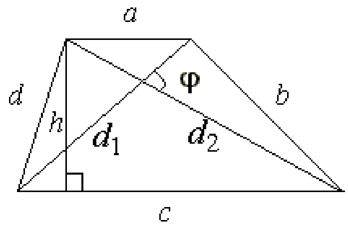
$S = a^2 \cdot Sin\alpha$; **3)** $h = 2r$.

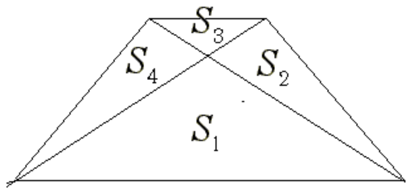
Trapetsiya

1) $S = mh = \frac{1}{2} d_1 \cdot d_2 Sin\varphi$;

2) $d_1 = ab + \frac{c^2 a - d^2 b}{a - b}$;

3) $S_2 = S_4 = \sqrt{S_1 \cdot S_3}$,



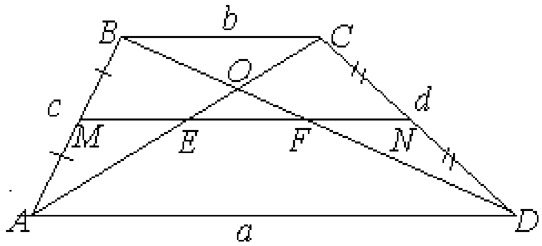


$S = \left(\sqrt{S_1} + \sqrt{S_3}\right)^2$.

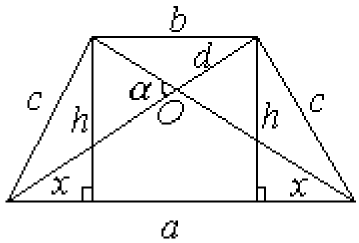
4) $AE = EC$, $BF = FD$;

$MN = \frac{a+b}{2}$ –o'rta chiziq;

$EF = \frac{a-b}{2}$, $ME = FN = \frac{b}{2}$; $\frac{AO}{OC} = \frac{OD}{OB} = \frac{a}{b}$.



Teng yonli trapetsiya



$x = \frac{|b-a|}{2}$, $c^2 = h^2 + \frac{(b-a)^2}{4}$;

$S = \frac{1}{2} d^2 Sin\alpha = \frac{a+b}{2} h$; $2r = h = \sqrt{ab}$.

To'g'ri burchakli trapetsiyada

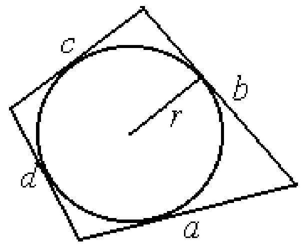
$d_1^2 - d_2^2 = a^2 - b^2$.

Aylanaga tashqi va ichki chizilgan to'rtburchaklar

1) Aylanaga tashqi chizilgan to'rtburchak

$a + c = b + d$;

$S = pr = (a+c)r = (b+d)r$, bu yerda $2p = a + b + c + d$.



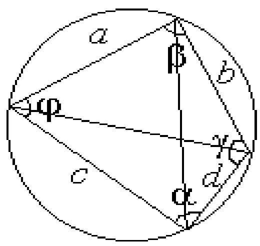
2) Aylanaga ichki chizilgan to'rtburchak

$\alpha + \beta = \gamma + \phi = 180°$;

$ad + bc = d_1 \cdot d_2$;

$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$;

$R = \frac{1}{4S}\sqrt{(ab+cd)(ac+bd)(ad+bc)}$.



Ko'pburchaklar

1) Ichki burchaklari yig'indisi: $(n-2)\pi$;

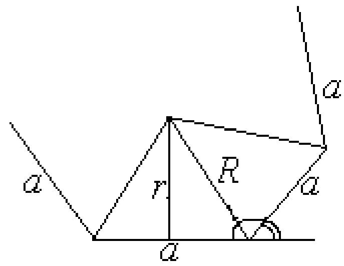
2) Tashqi burchaklari yig'indisi: 2π;

3) Diagonallari soni: $\frac{n(n-3)}{2}$ ta.

Muntazam ko'pburchaklar

1) Ichki burchagi: $\frac{(n-2)\pi}{n}$;

2) Tashqi burchagi: $\frac{2\pi}{n}$;



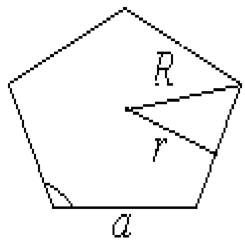
3) $r = \frac{1}{2}\sqrt{4R^2 - a^2}$;

4) $S_n = \frac{r \cdot n \cdot a}{2} = \frac{n \cdot a\sqrt{4R^2 - a^2}}{4}$;

5) $a = 2R \cdot Sin\frac{\pi}{n} = 2r \cdot tg\frac{\pi}{n}$.

Muntazam beshburchak, oltiburchak, sakkizburchak

Muntazam beshburchak



Ichki burchaklari yig'indisi: $540°$;

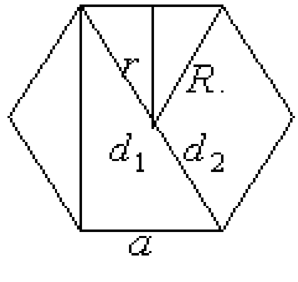
Ichki burchagi: $108°$; Tashqi burchagi: $72°$;

$a = \frac{R}{2}\sqrt{10-2\sqrt{5}} = 2r\sqrt{5-2\sqrt{5}}$;

$R = \frac{a}{10}\sqrt{50+10\sqrt{5}} = r\left(\sqrt{5}-1\right)$; $r = \frac{R}{4}\left(\sqrt{5}+1\right) = \frac{a}{10}\sqrt{25+10\sqrt{5}}$;

$d = \frac{1+\sqrt{5}}{2}a$, d – diagonal; $S = \frac{5}{8}R^2\sqrt{10+2\sqrt{5}} = \frac{a^2}{4}\sqrt{25+10\sqrt{5}} = 5r^2\sqrt{5-2\sqrt{5}}$.

Muntazam oltiburchak



Ichki burchaklari yig'indisi 720^0;

Ichki burchagi: $120°$; Tashqi burchagi: $60°$;

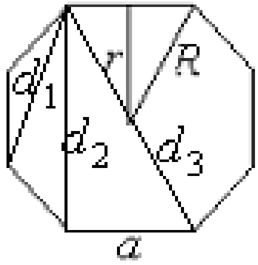
$a = R = \frac{2}{3}r\sqrt{3}$; $r = \frac{a\sqrt{3}}{2}$; $d_1 = a\sqrt{3}$, $d_2 = 2R = 2a$;

$S = \frac{3}{2}R^2\sqrt{3} = \frac{3}{2}a^2\sqrt{3} = 2r^2\sqrt{3}$.

Muntazam sakkizburchak

Ichki burchaklari yig‘indisi $1080°$;

Ichki burchagi: $135°$; Tashqi burchagi: $45°$;



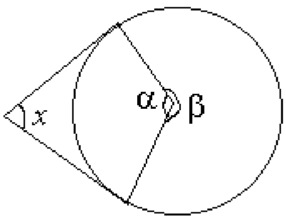
$a = R\sqrt{2-\sqrt{2}} = 2r\left(\sqrt{2}-1\right)$;

$R = \frac{a}{2}\sqrt{4+2\sqrt{2}} = r\sqrt{4-2\sqrt{2}}$; $r = \frac{R}{2}\sqrt{2+\sqrt{2}} = \frac{a}{2}\left(\sqrt{2}+1\right)$;

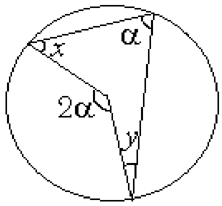
$d_1 = a\sqrt{2+\sqrt{2}}$; $d_2 = a\left(1+\sqrt{2}\right)$; $d_3 = 2R = a\sqrt{4-2\sqrt{2}}$;

$S = 2R^2\sqrt{2} = 2a^2\left(\sqrt{2}+1\right) = 8r^2\left(\sqrt{2}-1\right)$.

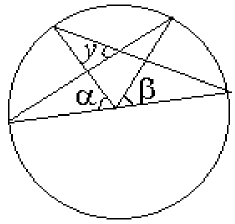
Aylanadagi burchaklar

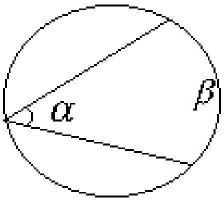


$$x = \frac{\beta-\alpha}{2}$$

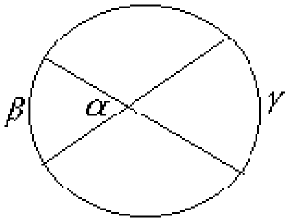


$$y = \frac{\alpha+\beta}{2}$$

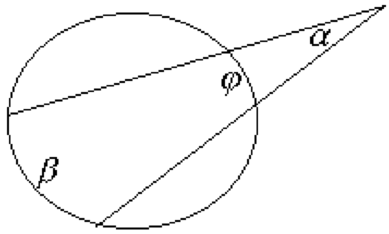




$$\alpha = \frac{\beta}{2}$$



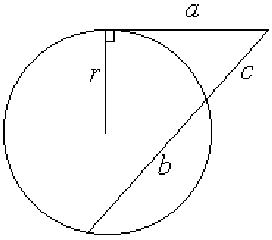
$$\alpha = \frac{\beta+\gamma}{2}$$



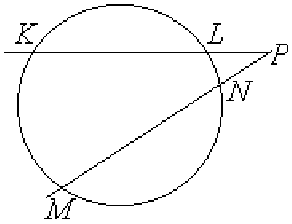
$$\alpha = \frac{|\beta-\phi|}{2}$$

Aylanadagi teoremalar

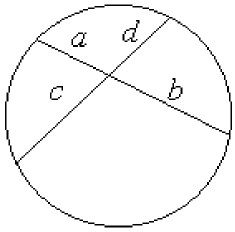
$$a^2 = (b+c)c$$



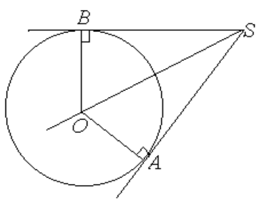
$$PL \cdot PK = PN \cdot PM$$



$$ab = cd$$

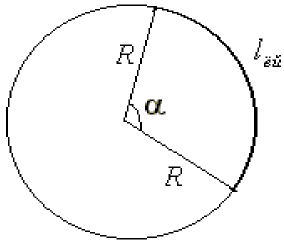


$$AS = BS,$$
$$\angle ASO = \angle BSO$$



Doira, sektor, segment, halqa

Aylana va doira



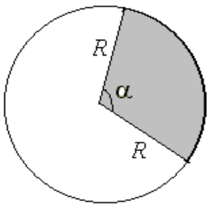
1) Aylana uzunligi: $l = 2\pi R$;

2) Doira yuzi: $S = \pi R^2 = \dfrac{\pi D^2}{4}$;

3) Yoy uzunligi: $l_{ёй} = \dfrac{\pi R \alpha}{180°}$.

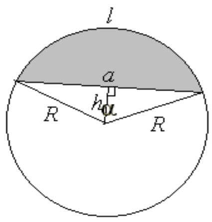
Sektor yuzi

$S_{сект} = \dfrac{\pi R^2 \alpha}{360°}$.

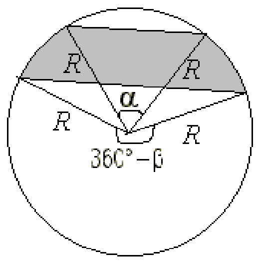


Segment yuzi

$S_{сегм} = \dfrac{\pi R^2 \alpha}{360°} - \dfrac{1}{2} R^2 Sin\alpha$.



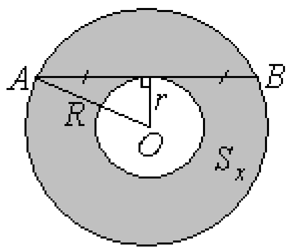
Aylananing ikki paralel vatarlari orasidagi bo‘lagi yuzi



$$S_{кес} = \frac{\pi R^2}{360°}(\beta - \alpha) - \frac{1}{2} R^2 (Sin\beta - Sin\alpha).$$

Halqa yuzi

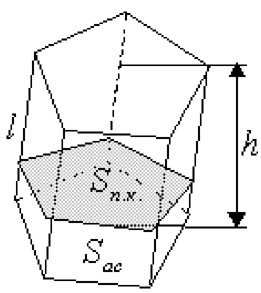
$$S_x = \pi\left(R^2 - r^2\right).$$



STEREOMETRIYA

Prizma

Ixtiyoriy prizma



Yon sirti: $S_{ён} = P_{пк} \cdot l$.

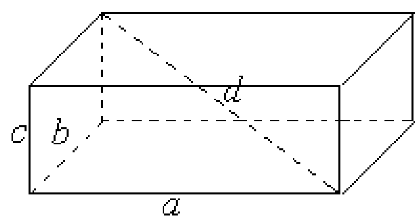
To‘la sirti: $S_T = S_{ён} + 2S_{ac}$.

Hajmi: $V = S_{п.к.} \cdot l = S_{ac} \cdot h$.

Diagonallar soni: $n(n-3)$.

Bu yerda $S_{п.к.}$ – perpendikulyar kesim yuzi, $P_{п.к.}$ - perpendikulyar kesim perimetri.

To‘g‘ri burchakli parallelepiped:



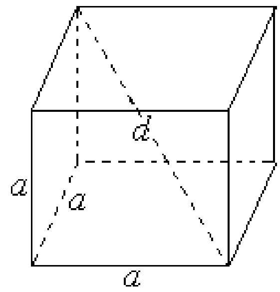
Yon sirti: $S_{ён} = 2(ac + bc)$.

To‘la sirti: $S_т = 2(ab + ac + bc)$.

Hajmi: $V = abc$.

$d^2 = a^2 + b^2 + c^2$.
3 ta simmetriya tekisligiga ega.
8 ta uchi, 12 ta qirrasi, 6 ta yoqi, 4 ta dioganali bor.

Kub



Yon sirti: $S_{ён} = 4a^2$.

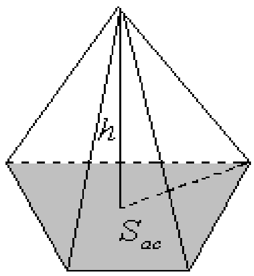
To‘la sirti: $S_T = 6a^2$.

Hajmi: $V = a^3$.

$d = a\sqrt{3}$; $r = \frac{a}{2}$; $R = \frac{a\sqrt{3}}{2}$. 9 ta simmetriya tekisligiga ega.

Piramida

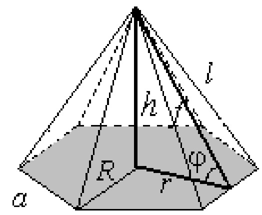
Ixtiyoriy piramida



To‘la sirti: $S_T = S_{ac} + S_{ён}$.

Hajmi: $V = \frac{1}{3} S_{ac} \cdot h = \frac{1}{3} S_T r$.

Muntazam piramida



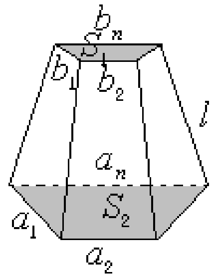
l –yon qirra, f – apofema.

$P_{ac} = n \cdot a$, $S_{ac} = \frac{nar}{2}$.

$S_{ён} = \frac{1}{2} P_{ac} \cdot f$. $S_{ac} = S_{ён} Cos\phi$, ϕ –asosidagi ikki yoqli burchak.

$R^2 = \left(\frac{a}{2}\right)^2 + r^2$; $l^2 = R^2 + h^2$; $f^2 = r^2 + h^2$.

Kesik piramida

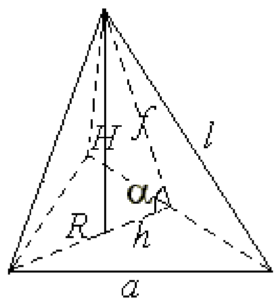


$S_m = S_1 + S_2 + S_{ён}$. $V = \frac{1}{3} h \left(S_1 + \sqrt{S_1 S_2} + S_2 \right)$.

Muntazam kesik piramida uchun:

$S_{ён} = \frac{1}{2}(P_1 + P_2) \cdot l$, l -apofema.

Muntazam uchburchakli piramida



l – yon qirra, f – apofema,

α – ikki yoqli burchak.

$f = \sqrt{\frac{a^2}{12} + H^2}$. $l = \sqrt{\frac{a^2}{3} + H^2}$.

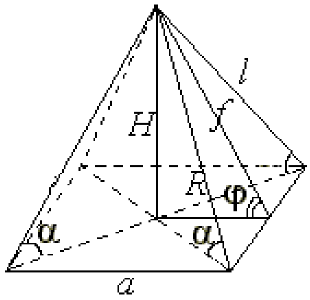
$S_{ён} = \frac{3}{2} af$, $S_{ac} = \frac{a^2\sqrt{3}}{4}$, $V = \frac{1}{3} S_{ac} \cdot H = \frac{a^2\sqrt{3}}{12} H$.

Muntazam to‘rtburchakli piramida

l – yon qirra, f – apofema. $f=\sqrt{\dfrac{a^2}{4}+H^2}$.

$l=\sqrt{\dfrac{a^2}{2}+H^2}$. $r=\dfrac{a\sqrt{3}}{6}$, $R=\dfrac{a\sqrt{3}}{3}$.

$S_{ён}=2af=\dfrac{S_{ас}}{Cos\phi}$, $S_{ас}=a^2$, $V=\dfrac{1}{3}S_{ас}\cdot H=\dfrac{a^2\sqrt{3}}{12}H$.



Silindr

R –asosining radiusi, H –balandlik.

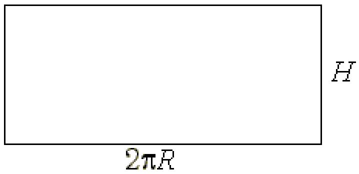
$S_{ас}=\pi R^2$.

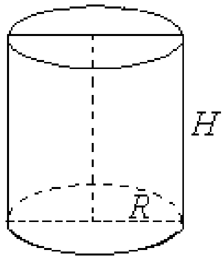
$S_{ён}=2\pi RH$.

$S_{т.с.}=2\pi R(R+H)$.

$V=\pi R^2 H$.

Yon sirti yoyilmasi:





Konus

L -yasovchi, R -asosinig radiusi, H -balandlik.

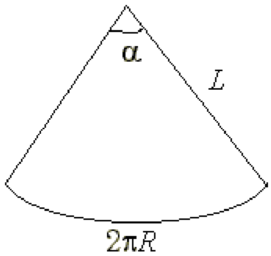
$L=\sqrt{R^2+H^2}$.

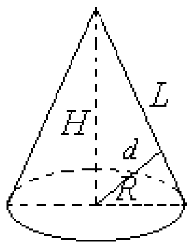
$S_{ён}=\pi RL$.

$S_{т.с.}=\pi R(R+L)$.

$V=\dfrac{1}{3}\pi R^2 H=\dfrac{1}{3}S_{ён}d$.

Yon sirti yoyilmasining uchidagi burchakni topish: $\alpha=2\pi R/L$.



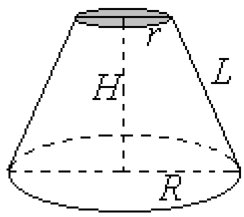


Kesik konus

L -yasovchi, R, r -asosining radiuslari, H -balandlik.

$L=\sqrt{(R-r)^2+H^2}$. $S_{ён}=\pi L(R+r)$.

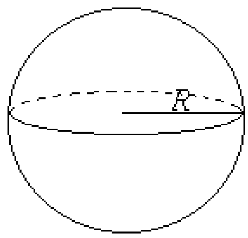
$S_{т.с.}=\pi\left(R^2+r^2+L(R+r)\right)$. $V=\dfrac{1}{3}\pi H\left(R^2+Rr+r^2\right)$.



Sfera va shar

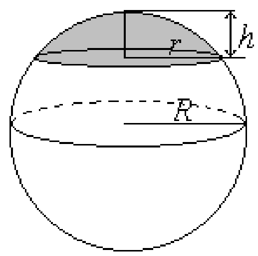
R -radiusi, d -diametr.

$S=4\pi R^2=\pi d^2$. $V=\dfrac{4}{3}\pi R^3=\dfrac{\pi}{6}d^3$.



Shar segmenti

R-sharning radiusi, h-segment balandligi.



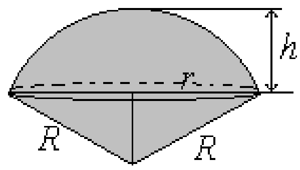
$r=\sqrt{h(2R-h)}$. $\qquad S_{\text{ён}}=2\pi Rh=\pi\left(r^2+h^2\right)$.

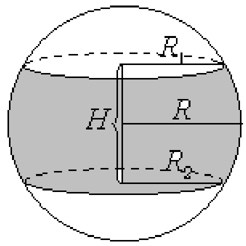
$S_{\text{т.с.}}=\pi\left(2Rh+r^2\right)$. $\qquad V=\dfrac{\pi h^2}{3}(3R-h)=\dfrac{\pi}{6}h\left(3r^2+h^2\right)$.

Shar sektori

$S_{\text{т.с.}}=\pi R(2h+r)$.

$V=\dfrac{2\pi}{3}R^2h=\dfrac{\pi}{6}d^2h$.





Shar halqasi

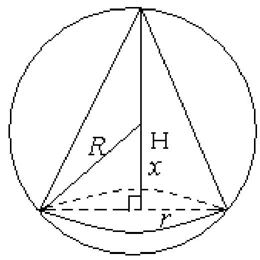
$S_{\text{ён}}=2\pi RH$.

$S_{\text{т.с.}}=\pi\left(2RH+R_1{}^2+R_2{}^2\right)$. $\qquad V=\dfrac{1}{6}\pi H\left(3R_1{}^2+3R_2{}^2+H^2\right)$.

Sharga ichki chizilgan konus

l-yasovchi, R-sharning radiusi,

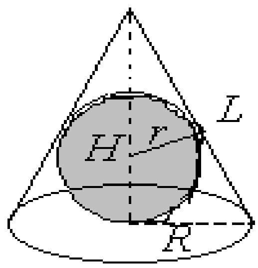
H-konusning balandligi, r-radiusi.



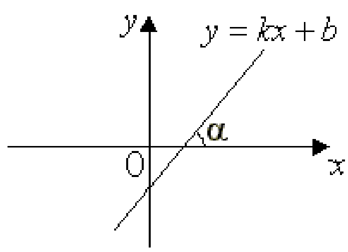
$R=\dfrac{r^2+H^2}{2H}$. $\qquad x=H-R$.

Konusga ichki chizilgan shar

$r=\dfrac{RH}{L+R}$.



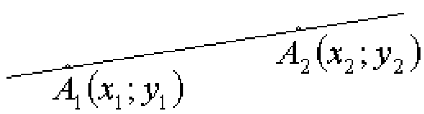
Tekislikda Dekart koordinatalar sistemasi
To‘g‘ri chiziq tenglamasi



1) To‘g‘ri chiziq tenglamasi $y=kx+b$, bu yerda k – burchak koeffitsienti $k=tg\alpha$, α – Ox o‘qi bilan hosil qilgan burchak;

2) $A_1(x_1;y_1)$ va $A_2(x_2;y_2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi:

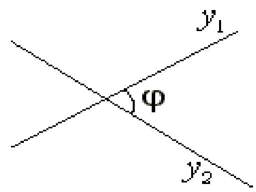
$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}; \qquad y=k(x-x_1)+y_1,$$



bunda $k=\dfrac{y_1-y_2}{x_1-x_2}$;

3) $A(x_1, y_1)$ nuqtadan o‘tuvchi to‘g‘ri chiziq: $y - y_1 = k(x - x_1)$;

4) Ikki to‘g‘ri chiziq orasidagi burchak tangensi:



$$tg\phi = \frac{k_1 - k_2}{1 + k_1 \cdot k_2};$$

5) Ikki to‘g‘ri chiziqning parallellik alomati: $k_1 = k_2$;

6) Ikki to‘g‘ri chiziqning perpendikulyarlik alomati: $k_1 \cdot k_2 = -1$;

7) Ikki to‘g‘ri chiziqning kesishish alomati: $k_1 \neq k_2$;

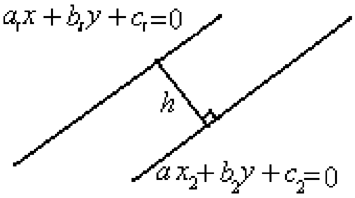
8) $\mathrm{A}(x_1, y_1)$, $\mathrm{B}(x_2, y_2)$ va $\mathrm{C}(x_3, y_3)$ nuqtalarning bir to‘g‘ri chiziqda yotish sharti:

$$\frac{x_0 - x_1}{x_2 - x_0} = \frac{y_0 - y_1}{y_2 - y_0};$$

9) $A(x_0, y_0)$ nuqtadan $ax + by + c = 0$ to‘g‘ri chiziqqacha bo‘lgan masofa:

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}};$$

10) Parallel to‘g‘ri chiziqlar orasidagi masofa:



$$h = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}};$$

11) Tekislikda uchlari $\mathrm{A}(\mathrm{x}_1, y_1)$, $\mathrm{B}(\mathrm{x}_2, y_2)$ va $\mathrm{C}(\mathrm{x}_3, y_3)$ nuqtalarda bo‘lgan ABC uchburchakning yuzi

$$S = \frac{1}{2}\left|(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)\right|.$$

Aylana tenglamasi

Markazi $(a;b)$ nuqtada, radiusi R ga teng aylana tenglamasi:

$$(x - a)^2 + (y - b)^2 = R^2.$$

Fazoda Dekart koordinatalar sistemasi

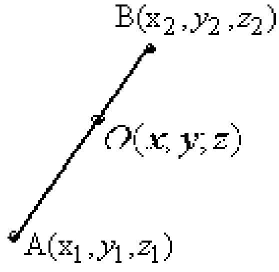
Fazoda $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ va $\mathrm{C}(\mathrm{x}_3, y_3, z_3)$ nuqtalar berilgan bo‘lsin.

1. AB kesma uzunligi $|AB| = \sqrt{(\mathrm{x}_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

2. AB kesma o‘rtasining koordinatasi

$$x = \frac{x_1 + x_2 + x_3}{2}, \quad y = \frac{y_1 + y_2 + y_3}{2}, \quad z = \frac{z_1 + z_2 + z_3}{2}.$$

3. AB kesmani $\frac{\lambda}{\mu}$ nisbatda bo‘lish



$$x = \frac{\mu x_1 + \lambda x_2}{\mu + \lambda}, \quad y = \frac{\mu y_1 + \lambda y_2}{\mu + \lambda}, \quad z = \frac{\mu z_1 + \lambda z_2}{\mu + \lambda}.$$

4. ABC uchburchak medianalari kesishgan $O(x; y; z)$ nuqta koordinatasi:

$$x=\frac{x_1+x_2+x_3}{3},\ y=\frac{y_1+y_2+y_3}{3},\ z=\frac{z_1+z_2+z_3}{3}.$$

5. Markazi $(a;b;c)$ nuqtada bo‘lgan R radiusli sfera tenglamasi:

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2.$$

Fazoda vektorlar

Boshi $A(x_0;y_0;z_0)$ va oxiri $B(x_1;y_1;z_1)$ nuqtalarda bo‘lgan vektor $\vec{AB}$ kabi belgilanadi. Vektorlar kichik lotin $\vec{a},\ \vec{b},\ \vec{c}\ \dots$ harflari bilan belgilanadi.

1. Koordinatalari: $\vec{AB}=\left(\overline{x_1-x_0;\ y_1-y_0;\ z_1-z_0}\right)$.

2. Moduli (uzunligi): $\left|\vec{AB}\right|=\sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2}$.

Birlik vektorlar

$$\vec{i}=(1,0,0),\ \vec{j}=(0,1,0),\ \vec{k}=(0,0,1);\quad |\vec{i}|=1,\ |\vec{j}|=1,\ |\vec{k}|=1;$$

$$\vec{i}\cdot\vec{j}=0,\ \vec{i}\cdot\vec{k}=0,\ \vec{k}\cdot\vec{j}=0;\quad \vec{a}(x;y;z)=x\vec{i}+y\vec{j}+z\vec{k}.$$

$$\vec{e}=\left(\frac{x}{\sqrt{x^2+y^2+z^2}};\frac{y}{\sqrt{x^2+y^2+z^2}};\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$ – birlik vektor.

Kollinear va komplanar vektorlar

Bir to‘g‘ri chiziqqa parallel bo‘lgan vektorlar, **kollinear vektorlar** deyiladi.

Kollinear vektorlar bir xil yo‘nalgan yoki qarama qarshi yo‘nalgan bo‘ladi.

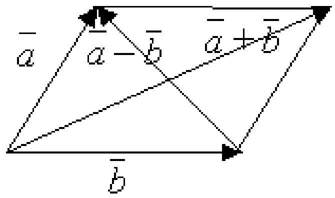
Bir xil yo‘nalgan vektorlarning uzunliklari teng bo‘lsa, ular **teng vektorlar** deyiladi.

Uzunligi nolga teng bo‘lgan vektor, **nol vektor** deyiladi.

Bir tekislikka parallel bo‘lgan uchta vektor **komplanar vektor** deyiladi.

Vektorlar ustida amallar

$\bar{a}(x_1;y_1;z_1),\ \bar{b}(x_2;y_2;z_2)$ vektorlar berilgan bo‘lsin.



1) $\bar{a}\pm\bar{b}=\left(\overline{x_1\pm x_2;y_1\pm y_2;z_1\pm z_2}\right)$;

2) $\lambda\bar{a}=\left(\overline{\lambda x_1;\lambda y_1;\lambda z_1}\right)\quad(\lambda\in R)$;

3) $\bar{a}\cdot\bar{a}=(\bar{a})^2=|\bar{a}|^2,\quad |\bar{a}|=\sqrt{\bar{a}\cdot\bar{a}}$;

4) Skalyar ko‘paytma

a) $\overline{a}\cdot\overline{b}=x_1\cdot x_2+y_1\cdot y_2+z_1\cdot z_2$; **b)** $\overline{a}\cdot\overline{b}=|\overline{a}|\cdot|\overline{b}|\cdot Cos\left(\overline{a}\,\hat{}\,\overline{b}\right)$;

5) Parallelik sharti: $\frac{x_1}{x_2}=\frac{y_1}{y_2}=\frac{z_1}{z_2}$;

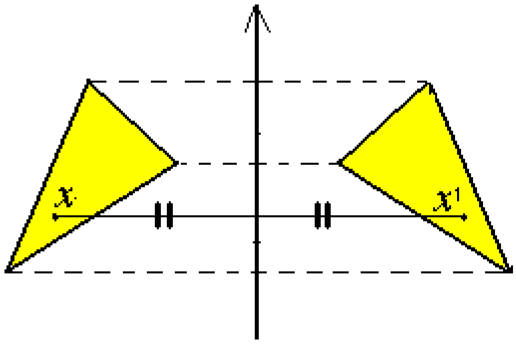
6) Perpendikulyarlik sharti: $\overline{a}\cdot\overline{b}=0$;

7) Vektorlar orasidagi burchak kosinusi:

$$Cos\left(\overline{a}\,\hat{}\,\overline{b}\right)=\frac{\overline{a}\cdot\overline{b}}{|\overline{a}|\cdot|\overline{b}|}=\frac{x_1x_2+y_1y_2+z_1z_2}{\sqrt{x_1^2+y_1^2+z_1^2}\sqrt{x_2^2+y_2^2+z_2^2}}.$$

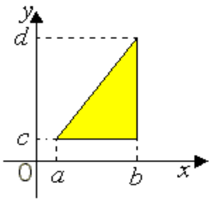
SIMMETRIYA

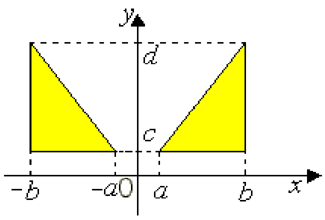
O‘qqa nisbatan simmetriya

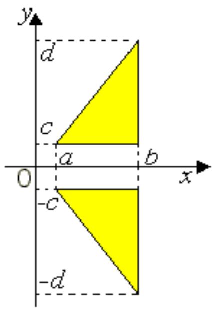


Hususiy xollarda

1) Berilgan *oy* o‘qiga nisbatan, *ox* o‘qiga nisbatan simmetriya shakllar







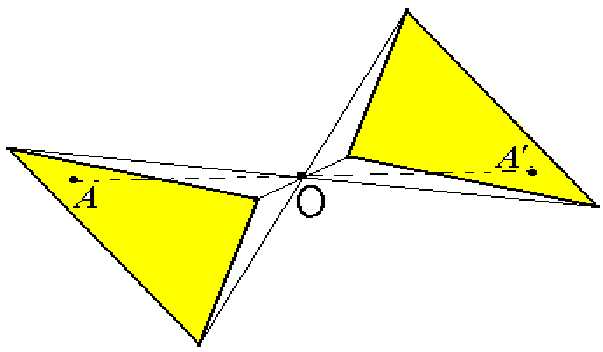
2) $y=kx+b$ va $y=-kx+b$ to‘g‘ri chiziqlar *oy* o‘qiga nisbatan simmetrik; $y=kx+b$ va $y=-kx-b$ lar *ox* o‘qiga nisbatan simmetrik;

3) O‘zaro teskari funksiyalar grafiklari $y=x$ to‘g‘ri chiziqqa nisbatan simmetrik bo‘ladi;

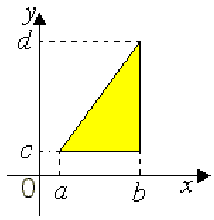
4) Juft funksiyaning grafigi *oy* o‘qiga nisbatan, toq funksiyaning grafigi esa koordinatalar boshiga nisbatan simmetrik bo‘ladi;

5) To‘g‘ri burchakli parallelepipedda 3 ta, kubda esa 9 ta simmetriya tekisligi mavjud.

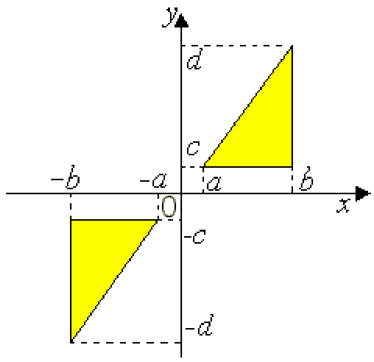
Nuqtaga nisbatan simmetriya



Berilgan shakl



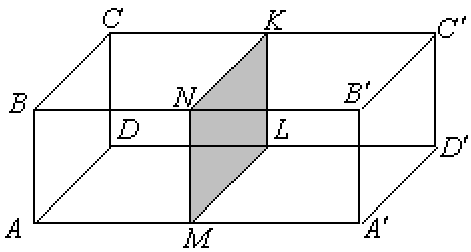
Koordinatalar boshiga nisbatan simmetriya



Tekislikka nisbatan simmetriya

$A'B'C'D'$ to‘rtburchak va $ABCD$ to‘rtburchak $MNKL$ tekislikka nisbatan simmetrik.
Bunda

$$MA = MA'; \quad NB = NB';$$
$$KC = KC'; \quad LD = LD'.$$



Muntazam ko‘pyoqlar haqida ma’lumotlar

	R	r	$Cos\alpha$	S	V
Tetraedr	$\frac{a\sqrt{6}}{4}$	$\frac{a\sqrt{6}}{12}$	$\frac{1}{3}$	$a^2\sqrt{3}$	$\frac{a^2\sqrt{2}}{12}$
Kub	$\frac{a\sqrt{3}}{2}$	$\frac{a}{2}$	0	$6a^2$	a^3
Oktaedr	$\frac{a\sqrt{2}}{2}$	$\frac{a\sqrt{6}}{6}$	$-\frac{1}{3}$	$2a^2\sqrt{3}$	$\frac{a^3\sqrt{2}}{3}$
Dodekaedr	$\frac{a\sqrt{3}}{\sqrt{5}-1}$	$\frac{a}{2}\sqrt{\frac{25+11\sqrt{5}}{10}}$	$-\frac{\sqrt{5}}{5}$	$3a^2\sqrt{25+10\sqrt{5}}$	$\frac{a^3}{4}\sqrt{15+7\sqrt{5}}$
Ikosaedr	$\frac{a}{4}\sqrt{10+2\sqrt{5}}$	$\frac{a(3+\sqrt{5})}{4\sqrt{3}}$	$-\frac{\sqrt{5}}{5}$	$5a^2\sqrt{3}$	$\frac{5a^3}{12}(3+\sqrt{5})$

Bu yerda α ikki yoqli burchak.

Ba'zi yig'indilar

1) $1+2+3+4+5+6+...+n=\frac{n(n+1)}{2}$;

2) $1+3+5+7+9+...+(2n-1)=n^2$;

3) $2+4+6+8+10+...+2n=n(n+1)$;

4) $1^2+2^2+3^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$;

5) $1^3+2^3+3^3+...+n^3=\left(\frac{n(n+1)}{2}\right)^2$;

6) $1^3+3^3+5^3+...+(2n-1)^3=n^2(2n^2-1)$;

7) $1^4+2^4+3^4+...+n^4=\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$;

8) $2^2+4^2+6^2+...+(2n)^2=\frac{2n(n+1)(2n+1)}{3}$;

9) $2^0+2^1+2^2+2^3+2^4+...+2^{n-1}=2^n-1$;

10) $2^2+6^2+10^2+...+(4n-2)^2=\frac{4n(2n-1)(2n+1)}{3}$;

11) $1^2+3^2+5^2+...+(2n-1)^2=\frac{n(n+1)(n+2)}{3}$;

12) $1\cdot4+2\cdot7+3\cdot10+....+n(3n+1)=n(n+1)^2$;

13) $1\cdot2\cdot3+2\cdot3\cdot4+3\cdot4\cdot5+....+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$;

14) $1\cdot2^2+2\cdot3^2+3\cdot4^2+....+(n-1)n^2=\frac{n(n^2-1)(3n+2)}{12}$;

15) $2\cdot1^2+3\cdot2^2+4\cdot3^2+....+(n+1)n^2=\frac{n(n+1)(n+2)(3n+1)}{12}$;

16) $a^0+a^1+a^2+a^3+a^4+....+a^n=\dfrac{a(a^n-1)}{a-1}, \qquad a\neq 1;$

17) $2\cdot 2^0+3\cdot 2^1+4\cdot 2^2+....+(n+1)2^{n-1}=n\cdot 2^n;$

18) $\dfrac{1}{1\cdot 2}+\dfrac{1}{2\cdot 3}+\dfrac{1}{3\cdot 4}+...+\dfrac{1}{n\cdot(n+1)}=\dfrac{n}{n+1};$

19) $\dfrac{1}{1\cdot 3}+\dfrac{1}{3\cdot 5}+...+\dfrac{1}{(2n-1)\cdot(2n+1)}=\dfrac{n}{2n+1};$

20) $\dfrac{1}{1\cdot 5}+\dfrac{1}{5\cdot 9}+\dfrac{1}{9\cdot 13}+...+\dfrac{1}{(4n-3)\cdot(4n+1)}=\dfrac{n}{4n+1};$

21) $\dfrac{1}{1\cdot 6}+\dfrac{1}{+6\cdot 11}+\dfrac{1}{11\cdot 16}+...+\dfrac{1}{(5n-4)\cdot(5n+1)}=\dfrac{n}{5n+1};$

22) $\dfrac{3}{1\cdot 2}+\dfrac{7}{2\cdot 3}+\dfrac{13}{3\cdot 4}+...+\dfrac{n^2+n+1}{n\cdot(n+1)}=\dfrac{n(n+2)}{n+1};$

23) $\dfrac{1^2}{1\cdot 3}+\dfrac{2^2}{3\cdot 5}+\dfrac{3^2}{5\cdot 7}...+\dfrac{n^2}{(2n-1)\cdot(2n+1)}=\dfrac{n(n+1)}{2(2n+1)};$

24) $\dfrac{7}{1\cdot 8}+\dfrac{7}{8\cdot 15}+\dfrac{7}{15\cdot 22}+...+\dfrac{7}{(7n-6)(7n+1)}=1-\dfrac{1}{7n+1};$

25) $\dfrac{1}{2}+\dfrac{2}{2^2}+\dfrac{3}{2^3}+...+\dfrac{n}{2^n}=2-\dfrac{n+2}{2^n};$

26) $\dfrac{1}{2}+\dfrac{3}{2^2}+\dfrac{5}{2^3}+...+\dfrac{2n-1}{2^n}=3-\dfrac{2n-3}{2^n};$

27) $\dfrac{0}{1!}+\dfrac{1}{2!}+\dfrac{2}{3!}+\dfrac{3}{4!}+...+\dfrac{n-1}{n!}=1-\dfrac{1}{n!};$

28) $\dfrac{1}{1\cdot 4}+\dfrac{1}{4\cdot 7}+\dfrac{1}{7\cdot 10}+...+\dfrac{1}{(3n-2)(3n+1)}=\dfrac{n}{3n+1};$

29) $\dfrac{1}{4\cdot 5}+\dfrac{1}{5\cdot 6}+\dfrac{1}{6\cdot 7}+...+\dfrac{1}{(n+3)(n+4)}=\dfrac{n}{4(n+4)};$

30) $\frac{1}{4\cdot 8}+\frac{1}{8\cdot 12}+\frac{1}{12\cdot 16}+...+\frac{1}{4n(4n+4)}=\frac{1}{16}-\frac{1}{16(n+1)}$;

31) $\frac{1}{1\cdot 10}+\frac{1}{10\cdot 19}+\frac{1}{19\cdot 28}+...+\frac{1}{(9n-8)(9n+1)}=\frac{n}{9n+1}$;

32) $\frac{1}{5\cdot 11}+\frac{1}{11\cdot 17}+\frac{1}{17\cdot 23}+...+\frac{1}{(6n-1)(6n+5)}=\frac{n}{5(6n+5)}$;

33) $\frac{1}{1\cdot 3}+\frac{2^2}{3\cdot 5}+\frac{3^2}{3\cdot 5}+...+\frac{n^2}{(2n-1)\cdot(2n+1)}=\frac{n(n+1)}{2(2n+1)}$;

34) $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}+...+\frac{1}{n\cdot(n+1)(n+2)}=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right)$;

35) $\frac{1}{1\cdot 3\cdot 5}+\frac{1}{3\cdot 5\cdot 7}+\frac{1}{5\cdot 7\cdot 9}+...+\frac{1}{(2n-1)(2n+1)(2n+3)}=\frac{n(n+1)}{2(2n+1)(2n+3)}$;

36) $\frac{1}{1\cdot 2\cdot 3\cdot 4}+\frac{1}{2\cdot 3\cdot 4\cdot 5}+...+\frac{1}{n\cdot(n+1)(n+2)(n+3)}=\frac{1}{3}\left(\frac{1}{6}-\frac{1}{(n+1)(n+2)(n+3)}\right)$;

37) $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)....\left(1-\frac{1}{n+1}\right)=\frac{1}{n+1}$;

38) $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)....\left(1-\frac{1}{(n+1)^2}\right)=\frac{n+2}{2n+2}$;

39) $1\cdot 1!+2\cdot 2!+3\cdot 3!+....+n\cdot n!=(n+1)!-1$;

40) $\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+...+\frac{n+2}{n!+(n+1)!+(n+2)!}=\frac{1}{2!}-\frac{1}{(n+2)!}$.