Bo'laklab integrallash
va
hosilalar jadvali.

Aniqmas integralni bo'laklab integrallash

$u(x)$ va $v(x)$ funktsiyalar biror x sohada uzluksiz va differentsiallanuvchi bo'lsin. Shu funktsiyalar ko'paytmasining differentsialini topamiz:

$$d(uv)u^{1}vdx+uv^{1}dx. \qquad (1)$$

Shartga asosan, $u^{1}v$ va uv^{1} funktsiyalar uzluksiz bo'lganligi sababli, (1) tenglikning ikkala tomonini integrallash mumkin:

$$\int d(uv)=\int u'vdx+\int uv'dx \text{ yoki } \int d(uv)=\int vdu+\int udv. \quad (2)$$

Aniqmas integralning 3-xossasiga asosan $\int d(uv)=uv+c$ edi. Bundan foydalanib, (2) tenglikni quyidagi ko'rinishda ifodalash mumkin:

$$\int udv=uv-\int vdu. \qquad (3)$$

(3) tenglikka aniqmas integralni bo'laklab integrallash formulasi deyiladi.

(3) formula yordamida integrallash uchun integral ostidagi ifodani u va dv ko'paytiruvchilar orqali ifodalash kerak. u ko'paytiruvchini shunday tanlash lozimki, uning u' hosilasi u ga nisbatan soddaroq bo'lsin.

Bo'laklab integrallash asosan uch bosqichdan iborat, ya'ni:

***1-bosqich*:** Integral ostidagi ifodani ikkita ko'paytiruvchiga ajratib, ulardan birinchi u va ikkitasini dv bilan belgilash.

***2-bosqich*:** u va dv ni topish.

***3-bosqich*:** Bo'laklab integrallash formulasi $\int udv=uv-\int vdu$ ni qo'llash.

Integrallashga bir necha misollar qaraymiz.

1-misol. $\int xe^{ax}dx$ integralni toping.

Yechilishi: Belgilashlarni quyidagicha amalga oshiramiz:

$$x=u \quad va \quad e^{ax}dx=dv.$$

$x=u$ belgilashning ikkala tomonini differentsiallaymiz, $e^{ax}dx=dv$ ning ikkala tomonini esa integrallaymiz:

$$dx=du \qquad \text{va} \qquad \frac{1}{a}e^{ax}=v.$$

Ushbularni (3) formulaga qo'yamiz:

$$\int xe^{ax}dx=\frac{x}{a}e^{ax}-\frac{1}{a}\int e^{ax}dx=\frac{x}{a}e^{ax}-\frac{1}{a}\cdot\frac{1}{a}e^{ax}+c=\frac{1}{a^{2}}(ax-1)e^{ax}+c.$$

$\int x\cdot e^{ab}\,dx=$

$u=x \qquad du=dx.$

$dv=e^{ax} \qquad v=\frac{1}{a}\cdot e^{ax}$

$x\cdot\frac{1}{a}\cdot e^{ax} \quad - \quad \frac{1}{a}$

1

9-misol. $\int x\sin^2\frac{x}{2}dx$ ni toping.

Yechilishi: 1) $x=u$; $\sin^2\frac{x}{2}dx=dv$.

2) $dx=du$; $v=\frac{1}{2}\int(1-\cos x)dx=\frac{x}{2}-\frac{1}{2}\sin x$

3) $\int x\sin^2\frac{x}{2}dx=\frac{x}{2}(x-\sin x)-\frac{1}{2}\int(x-\sin x)dx=\frac{x}{2}(x-\sin x)-\frac{x^2}{4}-\frac{1}{2}\cos x+c=$
$=\frac{x^2}{4}-\frac{x\sin x}{2}-\frac{1}{2}\cos x+c.$

Aniqmas integralni bo'laklab integrallashga doir misollar:

1. $\int x\sin x dx$; Javobi: $-x\cos x-\sin x+c.$
2. $\int x\cos 2x dx$; Javobi: $\frac{1}{2}x\sin 2x+\cos 2x+c.$
3. $\int(2x+1)\sin 3x dx$; Javobi: $(2x+1)(-\frac{1}{3}\cos 3x)+\frac{2}{9}\sin 3x+c.$
4. $\int x\,arctgx dx$; Javobi: $\frac{x^2}{2}arc\,tgx-\frac{x}{2}+\frac{1}{2}arctgx+c.$

5) $\int lnx dx$; Javob: $xln|x|-x+c$

6) $\int xln(x-1)dx$; $\frac{x^2}{2}ln|x-1|-\frac{1}{2}\left(\frac{x^2}{2}+x+ln|x-1|\right)+c$;

7) $\int xe^{2x}dx$; $\frac{1}{2}e^{2x}\left(x-\frac{1}{2}\right)+c$;

8) $\int x^2 cosxdx$; $x^2sinx+2xcosx-2sinx+c$;

9) $\int xarctgx\,dx$; $\frac{x^2+1}{2}arctgx-\frac{x}{2}+c$;

10) $\int(lnx)^2dx$; $x[(ln|x|-1)^2+1]+c$;

11) $\int\frac{xdx}{sin^2x}$; $-xctgx+ln|sinx|+c$;

12) $\int x^3e^{-x}dx$; $-e^{-x}(x^3+3x^2+6x+6)+c$;

13) $\int arcsinx\,dx$; $xarcsinx+\sqrt{1-x^2}+c$;

14) $\int\frac{arcsinxdx}{\sqrt{1+x}}$; $2\sqrt{1+x}\,arcsinx+4\sqrt{1-x}+c$;

15) $\int\frac{lnx}{x^2}dx$; $-\frac{ln|x|+1}{x}+c$;

16) $\int\frac{xdx}{cos^2x}$. $xtgx+ln|cosx|+c.$

u – hosila.
dv – integral.

4.Hosilalar jadvali

$u = u(x), v = v(x)$-differensiyallanuvchi funksiyalar deb hisoblab asosiy elementar funksiyalarning hosilalari jadvalini tuzamiz va differensiyallash qoidalarini keltiramiz:

1) $C' = 0;\ C = const$.
2) $x' = 1,\ x$ – erkli o'zgaruvchi.
3) $(u^{\alpha})' = \alpha u^{\alpha-1} \cdot u',\quad \alpha = const$.
4) Xususiy holda $\left(\sqrt{u}\right)' = \dfrac{1}{2\sqrt{u}} \cdot u'$.
5) Xususiy holda $\left(\dfrac{1}{u}\right)' = -\dfrac{1}{u^2} \cdot u'$.
6) $(a^u)' = a^u \cdot \ln a \cdot u',\ a = const,\ a > 0,\ a \neq 1$. 7) Xususiy holda $(e^u)' = e^u \cdot u'$.
8) $(\log_a u)' = \dfrac{1}{u \ln a} \cdot u',\ a = const,\ a > 0,\ a \neq 1$. 9) Xususiy holda $(\ln u)' = \dfrac{1}{u} \cdot u'$.
10) $(\sin u)' = \cos u \cdot u'$.
11) $(\cos u)' = -\sin u \cdot u'$.
12) $(tgu)' = \dfrac{1}{\cos^2 u} \cdot u'$.
13) $(ctgu)' = -\dfrac{1}{\sin^2 u} \cdot u'$.
14) $(\text{arcsinu})' = \dfrac{1}{\sqrt{1-u^2}} \cdot u'$.
15) $(\text{arccosu})' = -\dfrac{1}{\sqrt{1-u^2}} \cdot u'$.
16) $(arctgu)' = \dfrac{1}{1+u^2} \cdot u'$.
17) $(arcctgu)' = -\dfrac{1}{1+u^2} \cdot u'$.
18) $(shu)' = chu \cdot u'$.
19) $(chu)' = shu \cdot u'$.
20) $(thu)' = \dfrac{1}{ch^2 u} \cdot u'$.
21) $(cthu)' = -\dfrac{1}{sh^2 u} \cdot u'$.
22) $(u \pm v)' = u' \pm v'$.
23) $(u \cdot v)^1 = u' \cdot v + u \cdot v'$.
24) $(Cu)' = C \cdot u',\ \left(\dfrac{u}{C}\right)' = \dfrac{u'}{C},\ C = const$.

25) $\left(\frac{u}{v}\right)' = \frac{u'\cdot v - u\cdot v'}{v^2}$.

26) $y = f(u),\ u = u(x)$ murakkab funksiyani hosilasi uchun $y_x' = y_u' \cdot u_x'$ o'rinli.

27) $y = f(x)$ va $x = v(y)$ o'zaro teskari funksiyalar uchun $y_x' = \frac{1}{x_y'}$ o'rinli.

28) $\begin{cases} x = \varphi(t), \\ y = \psi(t) \end{cases}$ bo'lsa $y_x' = \frac{y_t'}{x_t'}$.

Izoh. $y = [u(x)]^{v(x)}$ ko'rinishdagi funksiyaning hosilasini topish talab etilsa, avval berilgan tenglikni *e* asosga ko'ra logarifmlab keyin tenglikni *x* bo'yicha differensiallash ma'qul.

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