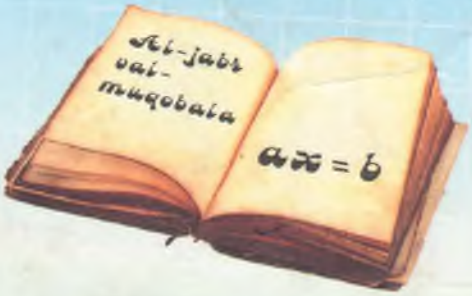
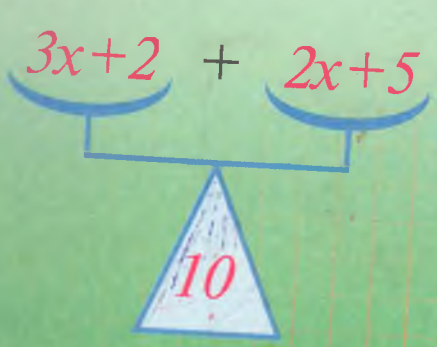
ALGEBRA

7



$$(a+b)^3=a^3+b^3+3ab(a+b)$$



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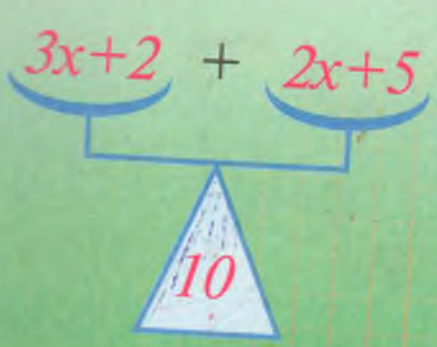

UO‘K 372.851(075)
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M54

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Mazkur darslik matematikadan uzviylashtirilgan va optimallashtirilgan ... tur hamda Davlat ta’lim standartlari asosida yozilgan bo‘lib, aniq fanlarga ... soslashtirilgan davlat umumta’lim maktablarining 7-sinfi uchun mo‘ljallan... Unda rivojlantiruvchi mashqlar (qiziqarli masalalar, mantiqiy masalalar, ... piada masalalari, matematik boshqotirmalar, murakkabroq va tarixiy masala... ham yetarlicha o‘rin berilgan.

UO‘K 372.851(...)
KBK 22.14ya7...

Darslikdagi shartli belgilar

□ – masalani yechish boshlandi
■ – masalani yechish tugadi
○ – matematik tasdiqni asoslash yoki formulani keltirib chiqarish boshlandi
● – asoslash yoki formulani keltirib chiqarish tugadi
| – bilish muhim va eslab qolish foydali matn
? – faollashtiruvchi savol va topshiriqlar
* – qiziqarli, murakkabroq, mantiqiy, olimpiada, tarixiy, noan’anaviy masala... rivojlantiruvchi mashqlar; matematik boshqotirmalar
– sinov (test) mashqlari
– tarixiy ma’lumotlar

O‘zbekiston Respublikasi Moliya vazirligi huzuridagi Respublika maqsadli kitob jamg‘armasi hi... ijara tizimi uchun chop etildi.

ISBN: 978-9943-07-334-0

5–6-SINFLARDA O‘TILGAN MAVZULARNI TAKRORLASH

1. Amallarni bajaring:

1) $7\frac{1}{2}\cdot 1\frac{1}{5}+4\frac{4}{5}\cdot\left(13\frac{1}{8}-2\frac{3}{4}+4\frac{1}{2}\right)$;

2) $\left(3\frac{8}{27}+4\frac{19}{27}\right):6\frac{2}{5}+14\frac{6}{7}:4\frac{4}{7}\cdot 3\frac{1}{5}$;

3) $1\frac{13}{22}\cdot 8\frac{4}{5}+3\frac{2}{3}:\left(9\frac{1}{6}-2\frac{2}{3}+4\frac{1}{2}\right)$;

4) $6\frac{3}{8}:2\frac{1}{8}\cdot 5\frac{2}{3}-\left(8\frac{10}{19}+5\frac{9}{19}\right):\left(8\frac{1}{9}-3\frac{2}{3}\right)$.

2. Amallarni bajaring:

1) $\dfrac{40,54\cdot(3,38+10,62)-(47,45+65,85):0,4}{22,38\cdot(424,6-422,1)+62,5-22,75:2,5}$;

2) $\dfrac{12,12-15,58:(6,38-4,48)+313,4}{292,4-2,3\cdot 6,6+60:1,5}$.

3. Tenglamani yeching:

1) $6\frac{1}{8}\cdot(2x-\frac{7}{15})=3\frac{4}{15}$;

2) $3\frac{2}{3}:(3x-\frac{1}{3})=8\frac{4}{5}$;

3) $5\frac{1}{4}\cdot(\frac{3}{7}x-\frac{4}{21})=6\frac{2}{3}$;

4) $7\frac{1}{3}x-4\frac{7}{10}\cdot 5=2\frac{2}{5}x-11\frac{1}{6}$;

5) $8\frac{1}{2}x-7\frac{7}{8}\cdot 4=3\frac{1}{4}x-17\frac{1}{2}$;

6) $37-2\frac{18}{43}-5\frac{39}{43}-2x-6\frac{27}{43}=20\frac{2}{43}$.

4. Karim velosipedda soatiga $11\frac{1}{3}$ km tezlik bilan $3\frac{3}{4}$ soat yurdi. Shu masofani $3\frac{1}{4}$ soatda bosib o‘tishi uchun tezlikni qanchaga orttirishi kerak?

5. To‘g‘ri to‘rtburchakning bo‘yi 9,5 sm. U enidan 1,9 marta uzun. To‘g‘ri to‘rtburchakning perimetri va yuzini toping.

6. To‘g‘ri to‘rtburchakning perimetri 20,4 sm. Bir tomoni ikkinchi tomonining 0,2 qismini tashkil qiladi. To‘g‘ri to‘rtburchakning yuzini toping.

7. *A* qishloqdan *B* qishloqqa qarab 5,4 km/soat tezlik bilan birinchi piyoda, *B* qishloqdan *A* qishloqqa qarab 6,1 km/soat tezlik bilan ikkinchi piyoda yo‘lga chiqdi. Ular *A* qishloqdan 16,2 km masofada uchrashdilar. Agar qishloqlar orasidagi masofa 28,4 km bo‘lsa, birinchi piyoda necha soat avval yo‘lga chiqqan?

8. Damin yo'lning $\frac{3}{8}$ qismini o'tdi. Yana 8 km yurib, yo'lning yarmini [illegible]
Damin jami necha kilometr yo'l yurishi kerak?

9. Toshkent va Samarqand shaharlari orasidagi masofa temiryo'l orqali [illegible] "Sharq" poyezdi bu masofani 3,5 soatda, "Afrosiyob" poyezdi esa [illegible] 10 minutda bosib o'tadi. Toshkentdan soat 8 da "Afrosiyob", 8.30 da "Sharq" poyezdlari yo'lga chiqsa, soat nechada ular orasidagi masofa 100 km bo'ladi?

10. 1) Mahsulotning narxi 15 % ga pasaydi. Ma'lum bir vaqt o'tgach, mahsulotga ehtiyoj ortgani sababli pasaygan narx 20 % ga oshdi. Dastlabki narxdan so'mdan bo'lgan mahsulot hozir necha so'mdan sotilyapti?
2) Mahsulotning narxi 25 % ga ortdi. Ma'lum bir vaqt o'tgach, mahsulotga ehtiyoj kamaygani sababli narx 10 % ga kamaydi. Dastlabki narx qanchaga o'zgargan?

11. Tarkibida 20 %; 30 %; 40 % tuz bo'lgan, mos ravishda, 300 g, [illegible] 600 g suyuqliklar aralashtirildi. Aralashmada tuz necha foizni tashkil etadi?

12. 7-sinf o'quvchilarining hammasi sport to'garaklariga qatnashadi. Ayrimlari futbol, ayrimlari tennis, ayrimlari esa ham futbol, ham tennisga qatnashadi. 65 % o'quvchi futbol, 72 % o'quvchi tennis bilan shug'ullansa, o'quvchilarning qancha foizi ham futbol, ham tennisga qatnashadi?

13. a soni b sonining 25 % ini, b son esa c sonining 25 % ini tashkil qiladi. a ni c ga nisbatini aniqlang.

14. To'g'ri to'rtburchak asosining uzunligi 15 % ga orttirildi. Balandligi esa [illegible] ga kamaytirildi. Uning yuzi qanday o'zgaradi?

15. To'g'ri to'rtburchakning eni 7 % ga orttirildi, bo'yi esa 13 % ga kamaytirildi. Uning yuzi qanday o'zgargan?

16. Amallarni bajaring:
1) $(-680):((-13)\cdot(-3)+25:(-5))-(-54):(-18)$;
2) $(-85)\cdot 6-216:(-12)+(-310):5$;
3) $(-60{,}125):(-3{,}65)-80{,}934:(-4{,}2)-(-8{,}5):(-4{,}6)$;
4) $6\frac{8}{21}\cdot(-0{,}63)+(-2\frac{16}{23})\cdot(-0{,}69)-9\frac{1}{7}\cdot 4\frac{3}{8}$.

17. Tenglamani yeching:
1) $-3\,(2x+3{,}2)=4-2\,(4x-1{,}9)$;
2) $-2{,}5x+10{,}136=3{,}2-4{,}1\cdot(-7{,}2+5{,}2x)$;
3) $18{,}1x+3\cdot(2{,}7x-2\frac{37}{75})=4{,}2x-1{,}6\,(x+3{,}2)$;
4) $4{,}5x-3\frac{1}{3}x+4{,}2=-5{,}1\cdot(2x-4)$.

18. 1) 7 ta sonning o‘rta arifmetigi 21,7 ga teng. Bu sonlarga yana bitta son qo‘shib, o‘rta arifmetik hisoblangan edi, u 23 ga teng chiqdi. Qo‘shilgan sonni toping.

2) Bir necha sonning o‘rta arifmetigi 45,3 ga teng. Bu sonlarga yana bitta 24,1 soni qo‘shilganidan so‘ng o‘rta arifmetik qiymat 40 ga teng bo‘ldi. Dastlab sonlar nechta bo‘lgan?

19. Otaning yoshi 85 da. Farzandlarining o‘rtacha yoshi 60 da. Farzandlari yoshiga otaning yoshi ham qo‘shilsa, o‘rtacha yosh 65 ga teng chiqdi. Otaning nechta farzandi bor?

20. Shahar va qishloq orasidagi masofa 32 km. Velosipedchi shahardan qishloqqa borishda 12,5 km/soat, qaytishda esa 10 km/soat tezlik bilan yurdi. Velosipedchining o‘rtacha tezligini toping.

21. Proporsiyani yeching (x ni toping):

1) $4\frac{1}{7}:2,5=3\frac{5}{8}:x$; 2) $\frac{5\frac{1}{2}}{1\frac{1}{2}}=\frac{3x+2}{2x-5}$; 3) $\left(\frac{3}{5}+2x\right):15=\left(1\frac{2}{5}-\frac{x}{3}\right):5$.

22. a) 540 sonini shunday 4 ta qismga ajratingki, bunda to‘rtinchi son: birinchi sondan $1\frac{5}{6}$ marta, ikkinchi sondan $4\frac{1}{3}$ marta, uchinchi sondan $1\frac{1}{2}$ marta kichik bo‘lsin.

b) 900 sonini 1; $\frac{1}{2};\frac{1}{3};\frac{1}{9}$ sonlariga: 1) to‘g‘ri proporsional; 2) teskari proporsional qilib qismlarga ajrating.

23. Birinchi idishda 30 litr, ikkinchi idishda 40 litr sut bor. Ikkinchi idishdan birinchi idishga qaraganda 2 marta ko‘p sut olingach, birinchisida ikkinchisiga qaraganda 5 litr ko‘p sut qoldi. Har bir idishdan necha litrdan sut olingan?

24. Sayyoh 5 km/soat tezlik bilan 3 soat-u 12 minut yo‘l bosdi. Keyin tezligini 1,5 km/soat ga kamaytirib, 2 soat-u 40 minut yo‘l yurdi. Sayyoh jami necha kilometr yo‘l bosgan?

I BOB. ALGEBRAIK IFODALAR

Siz 5- va 6-sinflarda butun sonlar va kasr sonlar ustida amallar b… tenglamalar yechishni o'rgandingiz. Geometrik shakllar, koordinat… chizig'i va koordinatalar tekisligi bilan tanishdingiz. Bularning … bitta fan – "Matematika" fanining kichik bir qismini tashkil qila… Aslida esa matematika alohida-alohida mustaqil fanlarga bo'linadi: … geometriya, matematik analiz, ehtimollar nazariyasi, matematik m… hokazo. Har bir fanning o'z o'rganish obyekti, hayotga tatbiqlari bor.

Aziz o'quvchi! Sizlar bilan *yangi o'quv fani – "Algebra"* fanini o'r… boshlaymiz. Bu fan juda qiziqarli va keng tatbiqlarga ega. Biz o'r… "Algebra" har xil amallarni, hisoblashlarni bajarish imkoniyatini be… qolmay, ularni iloji boricha tezroq bajarishni o'rgatadi.

"Algebra" so'zi buyuk o'zbek matematigi va astronomi, vatando… Abu Abdulloh Muhammad ibn Muso al-Xorazmiyning "Kitob al-m… fi hisob al-jabr val-muqobala" (qisqacha: "Al-jabr val-muqobala") … "aljabr" so'zidan olingan. Bu asari bilan Al-Xorazmiy algebra faniga as… gan, uni izchil bayon qilgan. Algebra fanining asosiy masalasi algebra… lar ustidagi matematik amallarni o'rganishdir. ***"Algebra" olamiga mar…***

1-§. Sonli ifodalar

"Sonli ifodalar", "ifodaning son qiymati" tushunchalari bilan 5–6… lardan tanishsiz. Eng sodda sonli ifodalarga ikkita son va bitta amal… zilgan ifodalar misol bo'la oladi:

$3{,}5 + 7{,}2$ (1); $\left| \right.$ $8{,}5 - \frac{1}{4}$ (2); $\left| \right.$ $(-6{,}2) \cdot \frac{1}{5}$ (3); $\left| \right.$ $-4{,}2 : (-3)$…

Bu yerda, 3,5 va 7,2 (1) ifodaning hadlari; 8,5 va $(-\frac{1}{4})$ … ifodaning hadlari ekanligini eslatib o'tamiz. (1) ifodaning qiymati … (2) ifodaning qiymati 8,25; (3) ifodaning qiymati (−1,24), (4) ifodaning … mati 1,4 ekani ravshan.

Bir necha son va ikki yoki undan ortiq amallardan hamda qavs… tuzilgan ifoda murakkabroq sonli ifodaga misol bo'la oladi:

$3{,}2 : 0{,}2 + \frac{1}{6}$; $\quad 8{,}7 \cdot 1{,}7 + 1{,}3 \cdot 1{,}7$; $\quad 4{,}8 : (1{,}2 + 0{,}4)$.

Bo‘lish amali o‘rniga kasr chizig‘i ishlatilishini bilasiz. Masalan, $3,2:0,2+\frac{1}{6}$ ifodani $\frac{3,2}{0,2}+\frac{1}{6}$; $4,8:(1,2+0,4)$ ifodani $\frac{4,8}{1,2+0,4}$ kabi yozish mumkin. Bu ifodalarning qiymati, mos ravishda, $16\frac{1}{6}$ va 3 dan iborat. $\frac{9-2,1}{75:3-5^2}$ ifoda esa ma’noga ega emas, chunki maxrajdagi ifodaning qiymati 0 ga teng.

0 ga bo‘lish esa mumkin emas!

Sonli ifodaning qiymatini to‘g‘ri aniqlashda amallar tartibi katta ahamiyatga ega. Qo‘shish va ayirish 1-bosqich amallari, ko‘paytirish va bo‘lish 2-bosqich amallari, darajaga ko‘tarish esa 3-bosqich amallari ekanligini eslatib o‘tamiz. Shularni e’tiborga olgan holda, quyidagi savollarga javob berishga harakat qiling:

1) Agar ifodada qavs bo‘lmasa, ifodaning qiymati qanday topiladi?
2) Agar ifodada qavslar bo‘lsa, uning qiymati qanday topiladi?
3) Agar ifoda kasr ko‘rinishida bo‘lsa, ifodaning qiymati qanday topiladi?
4) Agar ifodada qavslar ichida boshqa qavslar bo‘lsa, uning qiymati qanday topiladi?

Javobingizni sonli ifoda qiymatini topishning quyidagi qoidalari bilan taqqoslang:

1. Agar ifodada qavslar bo‘lmasa, u holda avval uchinchi bosqich amallar, keyin ikkinchi bosqich amallar va nihoyat birinchi bosqich amallar bajariladi. Shu bilan birga bir bosqich amallari ular qanday tartibda yozilgan bo‘lsa, xuddi shu tartibda bajariladi.
2. Agar ifodada qavslar bo‘lsa, u holda avval qavslar ichidagi sonlar ustida amallar, so‘ngra esa qolgan barcha amallar bajariladi, bunda qavs ichidagi va undan tashqaridagi barcha amallar birinchi bandda ko‘rsatilgan tartibda bajariladi.
3. Agar kasrning qiymati hisoblanadigan bo‘lsa, u holda kasrning suratidagi va maxrajidagi amallar bajariladi, so‘ngra birinchi natija ikkinchisiga bo‘linadi.
4. Agar ifodada qavslar ichida boshqa qavslar bo‘lsa, u holda avval eng ichkaridagi qavslar ichidagi amallar bajariladi.

1-misol. Amallarni bajaring: $20\frac{1}{4}:4,5+\left(8\frac{1}{3}\cdot 4,2-7,56:3,6\right)$.

□ 1) $8\frac{1}{3}\cdot 4,2=\frac{25}{3}\cdot 4\frac{1}{5}=\frac{25^{5}}{3_{1}}\cdot\frac{21^{7}}{5_{1}}=35$;

2) $75,6 \mid 36$ / $72 \mid 21$ / 36 / 36 / 0

3) $35,0 \mid 2,1$ / 0

4) $20\frac{1}{4} : 4,5 = \frac{81}{4} : \frac{9}{2} = \frac{\overset{9}{\cancel{81}}}{\underset{2}{\cancel{4}}} \cdot \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{9}}} = \frac{9}{2} = 4,5$

5) ...

Javob: 37,4. ■

2-misol. Amallarni bajaring: $\dfrac{2,14 - 0,4 \cdot 2,2}{3,45 - 3\frac{3}{20}}$.

□ 1) $2,2 \times 0,4$ / $0,88$

2) $2,14 - 0,88$ / $1,26$

3) $3,45 - 3\frac{3}{20} = 3,45 - 3,15 = 0,3$

4) $1,26 : 0,3 = 12,6 : 3 = 4,2$.

Javob: 4,2. ■

Sonli ifoda bitta sondan iborat bo'lishi ham mumkin, u holda uning qiymati shu sonning o'zi bo'ladi.

Ko'p masalalarni yechishda sonli ifodalarning qiymatlarini taqqoslash to'g'ri keladi. Taqqoslash natijasi to'g'ri tenglik yoki tengsizlik ko'rinishida yoziladi.

Masalan, $\frac{75+2}{7} = 11$ – to'g'ri tenglik, $\frac{5+1}{6} < 1\frac{1}{6}$ tengsizlik – to'g'ri tengsizlik.

Agar ikkita sonli ifoda "=" belgi bilan birl-ashtirilgan bo'lsa, hosil bo'lgan yozuv sonli tenglik deyiladi. Agar tenglikning chap va o'ng qismlarining qiymatlari bir xil bo'lsa, bu tenglik to'g'ri tenglik deyiladi.

Masalan: $\frac{75+2}{7} = 8+3$ to'g'ri tenglik, chunki uning ikkala qismining qiymati 11 ga teng.

?

1. 1) Sonli ifoda deb nimaga aytiladi?
2) Sonli ifodaning qiymati deganda nimani tushunasiz?
3) Sonli ifoda qiymatini hisoblashdagi amallar tartibini ayting.
4) Sonli tenglik deb nimaga aytiladi?
5) To'g'ri tenglik deb qanday tenglikka aytiladi?

2. Amallarni bajaring:

1) $\dfrac{(8-7{,}35):0{,}65+29{,}6}{\left(10{,}8:36+1{,}2:0{,}25-2\frac{1}{4}\right)\cdot\frac{13}{24}}$;

2) $\left(\left(\frac{7}{8}-\frac{47}{64}\right):1{,}5+\left(\frac{8}{9}-\frac{17}{27}\right):(0{,}475-0{,}205)\right)\cdot 1{,}6-\frac{1}{125}$;

3) $\left(1\frac{11}{24}+\frac{13}{36}\right)\cdot 1{,}44-\left(\frac{8}{15}\right)^2\cdot 0{,}5625-\left(10\frac{1}{8}-2\frac{9}{16}\right)\cdot 3\frac{1}{5}$;

4) $\left(7-8\frac{4}{5}\right)\cdot 2\frac{7}{9}-15:\left(\frac{1}{8}-\frac{3}{4}\right)^2+\left(9-3\frac{2}{5}\right)^2:3\frac{3}{5}$.

3. Sonli ifodaning qiymatini toping.

1) $32\,678\cdot 32\,676-32\,675\cdot 32\,679$;

2) $150\,014\cdot 150\,011-150\,012\cdot 150\,013$;

3) $\dfrac{65{,}3\cdot 78{,}6-13{,}3}{65{,}3+78{,}6\cdot 64{,}3}$; 4) $\dfrac{317{,}5\cdot 360{,}6-43{,}1}{635+721{,}2\cdot 316{,}5}$.

4. Qiymati: 1) $15\frac{1}{2}$; 2) $-13\frac{1}{7}$ 3) 8,75; 4) $-14{,}8$ ga teng ifoda tuzing.

5. Tenglik to‘g‘rimi:

1) $\left(-2{,}46:\frac{3}{4}+4\frac{4}{11}\cdot 4\frac{1}{8}\right)\cdot 5^2:\left(-5\frac{5}{6}\cdot 2\frac{4}{7}\right)=14{,}32$;

2) $(-7{,}2):\left(68{,}1:7\frac{1}{2}-8\frac{17}{20}+2\frac{1}{50}\right)+2\frac{7}{11}\cdot\frac{33}{58}=-1{,}7$;

3) $9{,}75:3\frac{3}{16}\cdot 3\frac{3}{4}-\left(5\frac{11}{19}+8\frac{8}{19}\right):2{,}8+(-3)^3\cdot 3\frac{2}{9}=-80\frac{9}{17}$;

4) $(-10{,}693+34{,}613):(-4{,}6)\cdot 4{,}2:(-7{,}2)-6{,}2:4\frac{3}{7}=1\frac{19}{30}$?

6. Quyidagi sonli ifodalarda qavslarni shunday qo‘yingki, javoblar to‘g‘ri bo‘lsin:

1) $5\,096+650:13-30\cdot 4=5\,026$; 2) $5\,096+650:13-30\cdot 4=20\,464$;

3) $5\,096+650:13-30\cdot 4=322$; 4) $5\,096+650:13-30\cdot 4=5\,176$.

7. Sonli ifoda tuzing va hisoblang:

1) 30 % i 120 ga teng bo‘lgan son bilan 120 ning 30 % i: ayirmasi, yig‘indisi, ko‘paytmasi, bo‘linmasi;

2) 15 % i 600 ga teng bo‘lgan son bilan 600 ning 35 % i: yig‘indisi, ayirmasi, ko‘paytmasi, bo‘linmasi;

3) 576 ning $\frac{5}{16}$ qismi bilan 720 ning $\frac{2}{15}$ qismi yig'indisi;

4) 930 ning $\frac{17}{31}$ qismi bilan 620 ning $\frac{7}{20}$ qismi yig'indisi;

5) 60 167 va 617 sonlari yig'indisi bilan shu sonlar ayirmasining nisb

6) 2 999 va 2 009 sonlar ayirmasi bilan shu sonlar yig'indisining nisb

7) 5 ning kvadrati bilan 6 ning kvadrati yig'indisidan shu sonlar ko'p uchlanganining ayirmasi;

8) $\frac{7}{15}$ va $\frac{7}{30}$ sonlar ayirmasi bilan shu sonlar yig'indisining ko'paytm

8. Sonli ifodaning qiymatini hisoblang:

1) $\underbrace{777...7}_{10\text{ ta}} - \underbrace{999...9}_{9\text{ ta}}$;

2) $\underbrace{1000...01}_{12\text{ ta}} - \underbrace{1000...01}_{11\text{ ta}}$;

3) $\underbrace{1000...01}_{17\text{ ta}} - \underbrace{9888...89}_{16\text{ ta}}$;

4) $\underbrace{6555...56}_{8\text{ ta}} - \underbrace{2666...62}_{7\text{ ta}}$.

2-§. Algebraik ifodalar

Sizlarga quyi sinflardan tanish bo'lgan "harfiy ifodalar" tush algebraik ifodalarga misol bo'la oladi. Masalan,

1) $(a+325)+2a$; 2) $15a(7a)$; 3) $4x+2(x+2)$ ifodalar algebraik ifodala

Sonlardan va harflardan tuzilib, amallar belgilari va qavslar bila lashtirilgan yozuv ***algebraik ifoda*** deyiladi.

Algebraik ifodada qatnashayotgan harfning biror qiymatida ko'rsatilgan amallar bajarilishi natijasida hosil bo'lgan son ***algebraik ning son qiymati*** deyiladi.

1-masala. $\frac{(8a+9b)\cdot 7}{2b-a}$ ifodaning $a=2$; $b=1{,}5$ bo'lgandagi qiymatini top

□ $\frac{(8\cdot 2+9\cdot 1{,}5)\cdot 7}{2\cdot 1{,}5-2}=\frac{(16+13{,}5)\cdot 7}{3-2}=\frac{29{,}5\cdot 7}{1}=206{,}5$ ■

2-masala. To'g'ri to'rtburchakning yuzi S dm², eni a dm ga teng. Shu to'rtburchakning perimetrini topish uchun algebraik ifoda tuzing. Per ning $a=60$ dm, $S=300$ dm² bo'lgandagi qiymatini toping.

□ To'g'ri to'rtburchakning bo'yi $b=\frac{S}{a}$ (dm) ga teng bo'ladi. U $P=2\cdot\left(a+\frac{S}{a}\right)$ (dm).

$$P = 2 \cdot \left(50 + \frac{300}{50}\right) = 2 \cdot 56 = 112 \text{ (dm)}.$$

Javob: 112 dm. ■

3-masala. a) $\frac{4a+5}{4(2a-5)}$; b) $\frac{3(a+7)}{8a+2(2a-9)}$ ifodalar a ning qanday qiymatlarida ma'noga ega?

□ a) $4(2a-5) \neq 0$, $2a-5 \neq 0$, $2a \neq 5$, $a \neq 2{,}5$;

b) $8a+2(2a-9) \neq 0$, $8a+4a-18 \neq 0$, $12a \neq 18$, $a \neq \frac{18}{12}$, $a \neq 1{,}5$.

Demak, a) ifodada a harf 2,5 dan boshqa barcha qiymatlarni qabul qilishi mumkin;

b) ifodada esa a harf 1,5 dan boshqa barcha qiymatlarni qabul qila oladi. ■

?

9. 1) Algebraik ifoda deb qanday ifodaga aytiladi?

2) Algebraik ifodaning son qiymati deb nimaga aytiladi?

3) 1-masalada harflar qanday qiymatlarni qabul qilishi mumkin?

4) 2-masalada harflar qanday qiymatlarni qabul qilmaydi?

10. Algebraik ifodaning son qiymatini toping:

1) $5\frac{1}{6} : a - 1\frac{2}{3}b + 15{,}5$, bunda $a = 2\frac{1}{5}$; $b = -\frac{9}{25}$;

2) $9a^2 - 3\frac{2}{3} : b - 16{,}2$, bunda $a = 1\frac{1}{3}$; $b = -7\frac{1}{3}$;

3) $\frac{3{,}5a - 2b + 0{,}5c}{-7a + b - 2c}$, bunda $a = \frac{1}{7}$; $b = 3\frac{3}{4}$; $c = -7$;

4) $\frac{(a^2 - 2b) \cdot c}{(a+c)^2}$, bunda $a = -1\frac{1}{2}$; $b = 3\frac{1}{2}$; $c = 5{,}6$.

11. Algebraik ifoda tuzing:

1) c va d sonlar kvadratlari yig'indisining shu sonlar kvadratlari ko'paytmasiga nisbati;

2) c va d sonlar kvadratlari ayirmasining shu sonlar ko'paytmasining ikkilanganiga nisbati;

3) m va n sonlar kublari ayirmasi bilan shu sonlar kvadratlari ikkil... yig'indisi;

4) m va n sonlar kublari yig'indisi bilan shu sonlar kublari ayirmasi.

12. Quyidagi algebraik ifodalarga olib keluvchi matnli masalalar tuzing:

1) a^2; 2) $2(a+b)$; 3) ah; 4) $6a$;
5) abc; 6) $2(ab+bc+ac)$; 7) $S:a$; 8) $V:(a...$

13. Algebraik ifodaning son qiymatini toping:

1) $(108\,a^2+225b+729):9-(12a^2+25b)$, bunda a) $a=1$; $b=2$;
b) $a=2$; $b=3$.

2) $\frac{2}{19}\cdot(76-19a)+55\frac{1}{19}+2a$, bunda: a) $a=-1$; b) $a=2$.

1) va 2) hol uchun ifodaning son qiymatiga e'tibor bering. Xulosa c...

14. Kilogrammlarda ifodalang:

1) m sentner; 2) m sentner n kilogramm; 3) m kg n gramm;
4) m sentner n kilogramm k gramm; 5) m tonna; 6) m tonna n s...
7) a tonna b sentner c kilogramm; 8) d gramm; 9) c tonna t s...
ner t kilogram p gramm; 10) a tonna b gramm; 11) c s...
n gramm.

15. Kvadrat metrlarda ifodalang:

1) a sm^2; 2) b sm^2 d mm^2; 3) a m^2 b sm^2; 4) a m^2 b sm^2 ...
5) x km^2 y m^2; 6) t km^2 x m^2 y sm^2; 7) a ga b m^2; 8) n ar p m^2 ...

16*. Kub metrlarda ifodalang.

1) a dm^3; 2) b m^3 a dm^3; 3) c sm^3; 4) k litr; 5) a km^3; 6) a km^3 b ...

17. a) Eni a ga, bo'yi b ga teng to'g'ri to'rtburchakning eni k % ga kamay... bo'yi esa t % ga uzaytirildi. Hosil bo'lgan to'g'ri to'rtburchak yuzini h... uchun algebraik ifoda tuzing.

b) Tomoni a ga teng bo'lgan kvadratning bir tomoni p % ga uzaytirilib, ... chi tomoni q % ga kamaytirilib, to'g'ri to'rtburchak hosil qilindi. Bu ... to'rtburchakning yuzini hisoblash uchun algebraik ifoda tuzing.

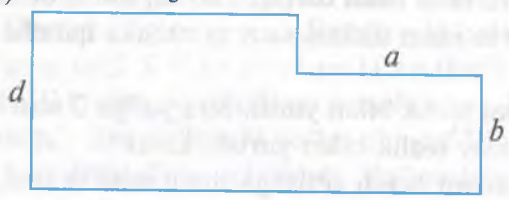
18. Mahsulotning narxi b so'm. Bu narx c % ga orttirildi. Ma'lum vaqt ... yangi narx d % ga kamaytirildi. Mahsulotning oxirgi narxini ifoda... gebraik ifoda tuzing.

19. Algebraik ifoda shaklida yozing:

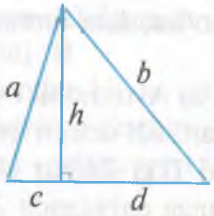
1) kichigi m ga teng bo'lgan 4 ta ketma-ket natural sonning o'rta a...
2) kattasi a ga teng bo'lgan uchta ketma-ket natural sonning ko'pay...
3) kichigi 3 k ga teng bo'lgan 2 ta ketma-ket natural sonning ko'pay...
4) kattasi $(4k+1)$ ga teng bo'lgan uchta ketma-ket natural sonning y...

20*. Shakllarning perimetri va yuzini hisoblash uchun ifoda tuzing:

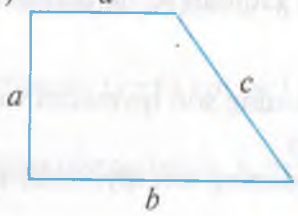
1)



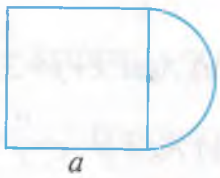
2)



3)



4)



21. Avtomobil avval v km/soat tezlik bilan t soat yurdi. So'ng tezligini 15 km ga oshirdi va avvalgiga qaraganda 15 minut ko'proq yurdi. 1) Avtomobil necha kilometr yo'l yurgan? 2) Uning o'rtacha tezligini aniqlang.

22. 1) A va B shaharlar orasidagi masofa s km. Bu shaharlardan bir vaqtda bir xil yo'nalishda ikkita avtomobil yo'lga chiqdi. Birinchi avtomobil oldinda, ikkinchisi orqada. Ikkinchi avtomobil birinchi avtomobilga t soatda va birinchi (A) shahardan 40 km uzoqlikda yetib oldi. Ularning tezligini aniqlang.

2) Bir shahardan ikkita avtomobil qarama-qarshi yo'nalishda bir vaqtda yo'lga chiqdi. Avtomobillardan birining tezligi v km/soat bo'lib, u ikkinchisidan 15 km/soat ortiq. t soatdan so'ng ular orasidagi masofa qancha bo'ladi?

3) Oralaridagi masofa s bo'lgan A va B shaharlardan bir-biriga qarab bir vaqtda ikkita avtomobil yo'lga chiqdi. Avtomobillardan birining tezligi v km/soat bo'lib, u ikkinchisidan 5 km/soat kam. t soatdan keyin ular orasidagi masofa qancha bo'ladi?

23. Xalqaro tennis musobaqalarini ko'rish uchun har biri 1 500 so'mdan n ta chipta va har biri 2 000 so'mdan m ta chipta sotildi. Bundan tushgan pulni hisoblash uchun algebraik ifoda tuzing. $n=1\,925$; $m=2\,314$ bo'lganida, uning son qiymati qancha bo'ladi?

24. 1) Bitta umumiy daftar 750 so'm, bitta rasm daftari 2 500 so'm, bitta ruchka 150 so'm turadi. 8 ta umumiy daftar, 4 ta rasm daftari va 9 ta ruchka qancha turadi?

2) Bitta umumiy daftar 750 so'm, bitta rasm daftari 2 500 so'm, bitta ruchka 150 so'm turadi. k ta umumiy daftar, m ta rasm daftari va p ta ruchka qancha turadi?

3) Bitta umumiy daftar a so'm, bitta rasm daftari c so'm, bitta ruchka b so'm turadi. 19 ta umumiy daftar, 30 ta rasm daftari va 15 ta ruchka qancha turadi?
4) Bitta umumiy daftar a so'm, bitta rasm daftari c so'm, bitta ruchka b so'm bo'lsa, k ta umumiy daftar, p ta rasm daftari va n ta ruchka qancha turadi?

25*. a) Avtomobil t soat v km/soat tezlik bilan yurdi. Shu yo'lga 2 soat kam sarflash uchun avtomobil qanday tezlik bilan yurishi kerak?
b) Ikki shahar orasidagi masofani bosib o'tishga avtomobil b soat, avtobus unga qaraganda 2 soat ko'proq vaqt sarflaydi. Agar ular bu shaharlardan biriga qarab bir vaqtda yo'lga chiqsa, qancha vaqtdan so'ng uchrashadi?

26. a) Agar $x+y=3$; $z=-9$ bo'lsa, $\dfrac{z}{x+y+z}$ ifodaning son qiymatini toping.

b) Agar $x-z=-7$; $y=12$ bo'lsa, $\dfrac{x+y-z}{y}$ ifodaning son qiymatini toping.

27. Quyidagi algebraik ifodalarda a qanday qiymatlarni qabul qiladi?

a) $\dfrac{a}{a-2}$; b) $\dfrac{5}{2a+1}$; d) $\dfrac{a+7}{3a+12}$; e) $\dfrac{3-a}{5a-1}$.

3-§. Algebraik tengliklar, formulalar

Ikki algebraik ifoda "=" belgisi bilan birlashtirilgan bo'lsa, uni *algebraik tenglik* deyiladi.

Amallar bo'ysunadigan qonunlar, turli xossa, munosabatlarning algebraik tenglik ko'rinishida yozilishi formuladir.

Quyidagi algebraik tengliklarni ko'rib o'taylik:

1) $a+b=b+a$; 2) $a\cdot b=b\cdot a$; 3) $a(b+c)=ab+ac$;
4) $a(b+c)=ab+ac$; 5) $a-(b+a)=a-b-c$; 6) $(a+b):c=a:c+b:c$.

Bu algebraik tengliklar nimani ifodalaydi? Ularni izohlang. Siz $P=2(a+b)$, $S=ab$ kabi tengliklarni bilasiz. Bu ikki tenglik nimani anglatishini eslab ko'ring. Bu yerda tomonlari a va b bo'lgan to'g'ri to'rtburchakning perimetri va yuzini topish formulalari berilgan.

$(y+7{,}25)\cdot 7=56{,}7$ tenglik ham algebraik tenglikka misol bo'la oladi.

Demak, ko'pgina amaliy masalalarni yechishda sonlar o'rniga harflar olish natijasida algebraik tengliklar va formulalar hosil bo'lar ekan.

a soni 3 ga karrali bo'lsa, uni formula ko'rinishida $a=3n$ kabi yozish mumkin. b sonni 3 ga bo'lganda 1 qoldiq qolsa, buni formula ko'rinishida $b=3n+1$ kabi yozish mumkin.

1-masala. Bog‘ to‘g‘ri to‘rtburchak shaklida bo‘lib, uning bo‘yi a km, eni esa undan 1 km kam. Yangi yer o‘zlashtirgandan keyin bog‘ning yuzi c km^2 ga ortdi. Bog‘ maydonining yuzi endi qancha bo‘ldi?

□ Dastlab bog‘ning yuzi $S=a(a-1)$ bo‘lgan. Yangi yer ochilganidan keyin bog‘ning yuzi $S_1=S+c=a(a-1)+c$ (km^2) ga teng bo‘ldi. ■

2-masala. Ikki qishloq orasidagi masofa s km. Har bir qishloqdan bir-biriga qarab bir vaqtda ikki yo‘lovchi yo‘lga chiqdi. Ular t soatdan keyin birinchi qishloqdan 6 km masofada uchrashdi. Yo‘lovchilarning tezligini toping. $s=15$ km; $t=3$ soat bo‘lgandagi yaqinlashish tezligini toping.

□ Birinchi yo‘lovchi 6 km yo‘l yurgan, uning tezligi $v_1=\frac{6}{t}$ km/soat. 2-yo‘lovchi $(s-6)$ km yo‘l yurgan, uning tezligi $v_2=\frac{s-6}{t}$ km/soat. s va t ning berilgan qiymatlarida $v_1=\frac{6}{3}=2$ (km/soat); $v_2=\frac{15-6}{3}=\frac{9}{3}=3$ (km/soat). ■

?

28. 1) Algebraik tenglik deb qanday tenglikka aytiladi?
2) Formula nima?
3) Qanday algebraik tengliklarni bilasiz?

29. Algebraik tenglik ko‘rinishida yozing:
a) m va n sonlari yig‘indisining yarmi shu sonlar yarimlarining yig‘indisiga teng;
b) a va b sonlar ayirmasining ikkilangani shu sonlar ikkilanganining ayirmasiga teng.
d) a va b sonlari yig‘indisi bilan ayirmasining ko‘paytmasi shu sonlar kvadratlarining ayirmasiga teng;
e) a va b sonlari yig‘indisining kvadrati a ning kvadrati, a va b sonlari ko‘paytmasining ikkilangani va b sonining kvadrati yig‘indisiga teng.

30. Masala shartiga mos algebraik tengliklar tuzing:
a) ikki son yig‘indisi 96 ga teng bo‘lib, ulardan biri a ga teng. Birinchi sonning 0,6 qismi ikkinchisining $\frac{3}{4}$ qismiga teng.
b) Ikki son ayirmasi 35 ga teng. Ulardan kattasi a ga teng. Katta sonning 0,4 qismi kichik sonning $\frac{5}{9}$ qismiga teng.

31. a) $0{,}4a-\frac{4}{9}b-\left(\frac{2}{5}a-\frac{1}{9}b\right)$ va $0{,}8a-\frac{5}{9}b-\left(\frac{4}{5}a-\frac{1}{6}b\right)$ algebraik ifodalarni soddalashtiring. $a=2\frac{1}{2}$; $b=-3$ bo‘lganida, ularning son qiymatini taqqoslang.

b) $0{,}6y-\frac{5}{7}x-\left(\frac{3}{5}y-\frac{3}{14}x\right)$ va $0{,}24x-\frac{5}{11}y-\left(\frac{6}{25}x-\frac{21}{22}y\right)$ algebraik ifodalar dalashtiring. $x=-6; y=2\frac{1}{2}$ bo'lganida, ularning son qiymatini taqqoslang.

32. "Captiva" avtomobili 100 km yo'lga b litr yonilg'i sarf qiladi. Bosib o'tilgan masofa s bo'lsa, sarf qilingan yonilg'ining miqdorini aniqlang. Jadvalni to'ldiring:

Bosib o'tilgan yo'l, km (s)	50	75	150	300			
Yonilg'i sarfi, l (litr)					$8b$	$9{,}5b$	$10{,}25b$

33. Birinchi qopda x kg, ikkinchi qopda undan y kg ko'p guruch bor. Ikkinchi qopdagi guruch massasi m ni aniqlash formulasini yozing. Formuladan foydalanib,
1) $x=60$ va $y=11$; 2) $x=55$ va $y=13$ hollar uchun masalani yeching.

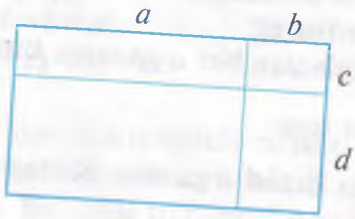
34. Do'konga har birida 50 kg dan un bo'lgan a ta qop va har birida 60 kg dan un bo'lgan b ta qop keltirildi. Do'konga keltirilgan barcha un massasini hisoblash formulasini tuzing.

35. Dehqonlar a kun 15 gektardan va b kun 20 gektardan yerga ishlov berishgan. Ular 1 kunda o'rtacha necha gektar yerga ishlov berishgan?

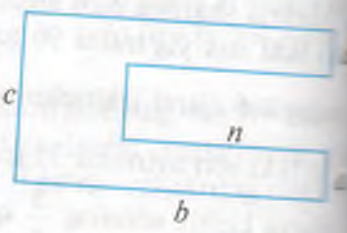
36. Do'konda har biri a kg dan 13 yashik uzum, har biri b kg dan 27 yashik olma, har biri c kg dan bo'lgan 31 yashik nok sotildi. Do'konda jami necha kilogramm meva sotilgan?

37*. Quyidagi shakllar yuzini va perimetrini hisoblash uchun formulalar tuzing.

1)



2)



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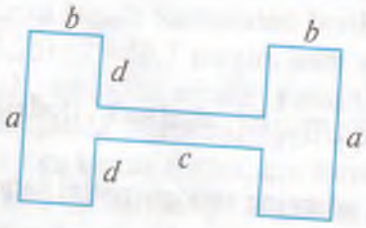
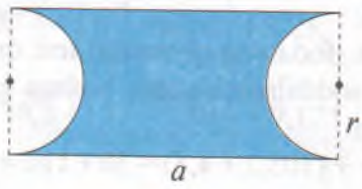
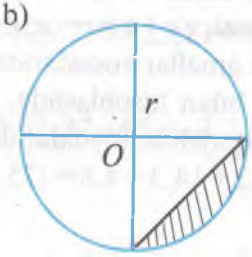


38*. 1) Bo‘yalgan va shtrixlangan shaklning yuzini toping.
2) *a*) shaklning perimetri (chegarasi uzunligi) formulasini yozing.

a)



b)



39. Quyidagi masalalarni yechish formulasini yozing:
a) Velosipedchi v km/soat tezlik bilan 2 soat, 10 km/soat tezlik bilan t soat yurdi. Velosipedchi jami necha kilometr yo‘l yurdi? Bu masofani o‘tishda uning o‘rtacha tezligi qanday bo‘lgan?
b) Hovuzga 1-quvurdan k soatda p litr suv, 2-quvurdan $(2k+1)$ soatda $(p-1)$ litr, 3-quvurdan $(k-1)$ soatda $(p+1)$ litr suv tushadi. Uchala quvur barobar ochib qo‘yilsa, t soatda hovuzga qancha suv tushadi?
d) Temperaturasi t °C bo‘lgan a litr suv bilan temperaturasi $(t-5)$ °C bo‘lgan $(a+2)$ litr suv aralashtirildi. Aralashmaning temperaturasini aniqlang.
e) Shahardan bir vaqtda bir xil yo‘nalishda ikki motosiklchi yo‘lga chiqdi. Birinchi motosiklchining tezligi a km/soatga teng bo‘lib, ikkinchisining tezligidan katta. 25 minutdan keyin motosiklchilar orasidagi masofa d km bo‘ldi. Ikkinchi motosiklchining tezligini toping.

40. Savdogar k ta kostyumning har birini p so‘mdan sotib oldi va ularning har birini bir xil narxda sotdi. Natijada u t so‘m foyda qildi. Savdogar kostyumni necha so‘mdan sotgan?

41. Bitta aerodromdan D shaharga ikkita samolyot bir vaqtda uchdi. Birinchi samolyotning tezligi v km/soat, ikkinchisiniki esa undan b km/soat kam. Agar birinchi samolyot D shaharga ikkinchi samolyotdan d soat oldin uchib kelgan bo‘lsa, aerodromdan D shahargacha bo‘lgan masofani toping.

4-§. Arifmetik amallarning xossalari

Arifmetik amallar deb, qo‘shish, ayirish, ko‘paytirish, bo‘lish amallariga aytiladi.

Amallarning asosiy xossalari qonunlar deb ataladi.

1. Qo‘shish va ko‘paytirish qonunlari.
1) *O‘rin almashtirish qonuni*: $a+b=b+a$; $ab=ba$;
2) *Guruhlash qonuni*: $(a+b)+c=a+(b+c)$; $(ab)c=a(bc)$;
3) *Taqsimot qonuni*: $a(b+c)=ab+ac$.

20

2. Ayirish qonunlari.

1) $a+(b-c)=a+b-c$; 2) $a-(b+c)=a-b-c$; 3) $a-(b-c)=$

Bu tengsizliklarda a, b, c – ixtiyoriy sonlar.

3. Bo'lish qonuni. $(a+b):c=a:c+b:c$, $c\neq 0$.

Yuqoridagi amallar xossalaridan algebraik ifodaning qiymatini tez, q
sional yo'l bilan hisoblashda, ifodalarni soddalashtirishda, boshqa
keltirib chiqarishda foydalaniladi. Masalan,

$25,7+105,2+14,3+4,8=(25,7+14,3)+\ +(105,2+4,8)=30+110=$

1-misol. Ifodani soddalashtiring: $3,2\cdot\left(\frac{7}{16}a+\frac{1}{2}b\right)+4,8\cdot\left(\frac{3}{4}a+\frac{5}{8}b\right)$.

□ $3,2\cdot\left(\frac{7}{16}a+\frac{1}{2}b\right)+4,8\cdot\left(\frac{3}{4}a+\frac{5}{8}b\right)=3\frac{1}{5}\cdot\left(\frac{7}{16}a+\frac{1}{2}b\right)+4\frac{4}{5}\cdot\left(\frac{3}{4}a+\frac{5}{8}b\right)=$

$=\frac{16^{1}}{5}\cdot\frac{7}{16_{1}}a+\frac{16^{8}}{5}\cdot\frac{1}{2_{7}}b+\frac{24^{6}}{5}\cdot\frac{3}{4}a+\frac{24^{3}}{5}\cdot\frac{5}{8}b=\frac{7}{5}a+\frac{8}{5}b+\frac{18}{5}a+3b=$

$=\left(\frac{7}{5}a+\frac{18}{5}a\right)+\left(\frac{8}{5}b+b\right)=\frac{25}{5}a+\left(1\frac{3}{5}b+3b\right)=5a+4\frac{3}{5}b=5a+4,6b.$ ■

2-misol. Arifmetik amallar xossalaridan foydalanib hisoblaylik:

□ *a)* $15,7+\left(24,3+7\frac{2}{5}\right)=(15,7+24,3)+7\frac{2}{5}=40+7\frac{2}{5}=47\frac{2}{5}=47,4;$

b) $72\frac{4}{5}-(6,5+2,8)=(72,8-2,8)-6,5=70-6,5=63,5;$

d) $189,4-(95,5-10,6)=(189,4+10,6)-95,5=200-95,5=104,5.$ ■

3-misol. Ifodani soddalashtiring:

□ $6,25\left(\frac{3}{5}a-4b\right)+17\frac{1}{2}\left(\frac{3}{7}a-2b\right)=6\frac{1}{4}\left(\frac{3}{5}a-4b\right)+17\frac{1}{2}\left(\frac{3}{7}a-2b\right)=$

$=\frac{25^{5}}{4}\cdot\frac{3}{5}a-\frac{25}{4_{1}}\cdot 4b+\frac{35^{5}}{2}\cdot\frac{3}{7_{1}}a-\frac{35}{2_{1}}\cdot 2b=\frac{15}{4}a-25b+\frac{15}{2}a-35b=$

$=\left(3\frac{3}{4}a+7\frac{1}{2}a\right)-(25b+35b)=11\frac{1}{4}a-60b=11,25a-60b.$ ■

?

42. 1) Arifmetik amallar deb qanday amallarga aytiladi?

2) Arifmetik amallarning asosiy xossalarini ayting.

3) Arifmetik amallar xossalaridan qayerda foydalaniladi?

Arifmetik amallar xossalarini qo'llab, sonli ifodaning qiymatini toping **(43–47)**:

43. 1) $37\frac{2}{5}+\left(6\frac{3}{4}+62,6\right)+3,25$;

2) $125,715+25,27+34,73+174,285$;

3) $130,5+19\frac{1}{8}-\left(30\frac{1}{2}+9,125\right)$;

4) $62,37-15,6-\left(2,37-5\frac{3}{5}\right)$.

44. 1) $-3\frac{1}{7}\cdot\left(\frac{2}{9}\cdot 1\frac{3}{11}\right)$;

2) $\left(9\frac{3}{8}\cdot\left(-\frac{2}{7}\right)\right)\cdot 2\frac{2}{5}$;

3) $\left(6\frac{5}{7}\cdot\left(-\frac{3}{7}\right)\right)\cdot\left(\left(-1\frac{5}{9}\right)\cdot\left(-\frac{7}{47}\right)\right)$;

4) $\left(-1\frac{1}{14}\cdot\frac{11}{36}\right)\cdot\left(-2\frac{5}{11}\cdot\left(-2\frac{1}{3}\right)\right)$.

45. 1) $1\frac{3}{7}+1\frac{3}{14}\cdot\left(2,8-\frac{14}{17}\right)$;

2) $2\frac{5}{6}+2\frac{7}{9}\cdot\left(1,08-1\frac{4}{5}\right)$;

3) $1\frac{7}{32}\cdot\left(2\frac{2}{13}-3\frac{19}{39}\right)+10\frac{5}{9}\cdot\left(2\frac{5}{19}-\frac{9}{76}\right)$;

4) $2\frac{7}{9}\cdot\left(5,4-1\frac{7}{20}\right)-5,75\cdot\left(2\frac{2}{23}-\frac{60}{69}\right)$.

46. 1) $2\frac{5}{9}\cdot 3\frac{1}{3}+6\frac{4}{9}\cdot 3\frac{1}{3}+3\frac{2}{3}\cdot 4,25+3\frac{2}{3}\cdot 4,75$;

2) $3\frac{4}{7}\cdot 5\frac{4}{19}+3\frac{4}{7}\cdot 1\frac{15}{19}+5\frac{7}{16}\cdot 4,375+1\frac{9}{16}\cdot 4,375$;

3) $\dfrac{0,215-1,6\cdot 0,215}{3,45-3\frac{3}{4}}+\dfrac{5,212\cdot 3,15+3,5\cdot 4,788}{3,125+\frac{3}{8}}$;

4) $\dfrac{0,15-0,15\cdot 6,4}{0,175-\frac{3}{8}}+\dfrac{9,65\cdot 1,5-1,5\cdot 0,5}{8,9-7\frac{2}{5}}$.

47. 1) $\left(3\frac{4}{5}+2\frac{8}{15}\right):1,9-\left(1\frac{1}{8}-2\frac{1}{12}\right):3\frac{3}{4}$;

2) $\left(6\frac{4}{7}-5\frac{3}{4}\right):\frac{23}{28}-\left(10\frac{5}{6}-3\frac{1}{12}\right):\frac{1}{6}$;

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3) $\left(4\frac{5}{12}-3\frac{13}{24}\right):1\frac{3}{4}-\left(3\frac{5}{12}+4\frac{11}{24}\right):1\frac{3}{4}+11\frac{1}{2}$;

4) $20+\left(57\frac{1}{3}-6\frac{7}{8}\right):1\frac{9}{20}-\left(8\frac{1}{3}+13\frac{1}{8}\right):1\frac{9}{20}$.

48*. Amallar xossalaridan foydalanib, algebraik ifodani soddalashtiring:

1) $\frac{4}{9}\cdot\left(0,75k-\frac{9}{16}p\right)-\frac{3}{7}\cdot\left(\frac{7}{9}k-\frac{7}{12}p\right)+15\cdot\left(1\frac{1}{5}k-2\frac{4}{15}p\right)$;

2) $\frac{3}{4}\cdot\left(0,8m-\frac{2}{3}n\right)-\frac{4}{7}\cdot\left(0,7m-\frac{7}{8}n\right)+32\cdot\left(4\frac{1}{8}m-1\frac{3}{16}n\right)$;

3) $0,6\cdot\left(\frac{5}{9}a-\frac{5}{6}b\right)-0,625\cdot\left(\frac{8}{15}a-\frac{4}{5}b\right)+24\cdot\left(2\frac{1}{6}a-1\frac{1}{12}b\right)$;

4) $\frac{2}{7}\cdot\left(\frac{7}{9}x-\frac{7}{8}y\right)-\frac{5}{8}\cdot(0,2x-0,45y)+18\cdot\left(3\frac{1}{9}x-2\frac{1}{18}y\right)$;

49*. Ifodalarni soddalashtiring, so‘ngra son qiymatini toping:

1) $9,5\cdot(6x-3y)+6\cdot\left(1,5x+\frac{1}{6}y\right)$, bunda $x=1,5;\ y=-2$;

2) $3,4\cdot(5x-1,5y)+6,5\cdot(7x-3,4y)$, bunda $x=-\frac{1}{7};\ y=-0,5$;

3) $6,2\cdot(15x-0,5y)-(0,5x-1,5y)$, bunda $x=\frac{1}{2};\ y=-2$;

4) $9,6\cdot(3,2x-2,5y)-1,8\cdot(2,5x-5y)$, bunda $x=\frac{1}{7};\ y=0,5$.

50. O‘xshash hadlarni ixchamlang:

1) $9,3a+8,8b-3,3a-2,8b-5a+2b$; 2) $8,9a-9,1b+3,1a-8,9b+4a-5$

3) $8\frac{1}{3}a-15\frac{4}{7}b+7\frac{2}{15}a+12\frac{9}{14}b+5\frac{1}{2}a-3\frac{1}{14}b$;

4) $2\frac{3}{4}x+2\frac{5}{6}y+4\frac{2}{3}x-2\frac{7}{12}x-3\frac{4}{9}y+1\frac{7}{15}y$.

5-§. Qavslarni ochish qoidalari

1. ***Algeraik yig'indi.***

1-masala. Avtobusda 60 nafar yo'lovchi bor edi. 2-bekatda 10 kishi avtobusdan tushdi, 15 kishi avtobusga chiqdi. 3-bekatda 8 nafar yo'lovchi tushdi, 4 nafar yo'lovchi chiqdi. Natijada, avtobusda necha nafar yo'lovchi bo'ldi?

□ Avtobusda nechta yo'lovchi bo'lganini bilish uchun ushbu ifodani tuzamiz: $60-10+15-8+4$. Bu ifodaning qiymati 61 ga tengligi ravshan.

Javob: Avtobusda 61 nafar yo'lovchi bo'ldi. ■

VI sinf "Matematika" kursidan $60-10+15-8+4$ ifodani $60+(-10)+15+(-8)+4$ kabi yig'indi ko'rinishida yozish mumkin ekanligini bilasiz. Bu kabi yig'indini *algebraik yig'indi* deb ataymiz.

"+" va "–" ishoralari bilan birlashtirilgan bir nechta algebraik ifodalardan tuzilgan yozuv *algebraik yig'indi* deb ataladi.

Masalan, $-2a+(-7b)-(-7a^2)+9$; $15a^2b-(-21a^2b^2)+(-8a^2b^3)$ algebraik yig'indilardir.

Ularni quyidagicha yozish mumkin:

1) $-2a+(-7b)-(-7a^2)+9=-2a-7b+7a^2+9$;

2) $15a^2b^2-(-21a^2b^2)+(-8a^2b^2)=15a^2b+21a^2b^2-8a^2b^3$;

1-algebraik yig'indidagi hadlar $-2a$; $-7b$; $7a^2$; 9 – qo'shiluvchilar bo'ladi, chunki $-2a-7b+7a^2+9=(-2a)+(-7b)+7a^2+9$.

2-algebraik yig'indida esa $15a^2b$; $21a^2b^2$; $-8a^2b^2$ hadlar qo'shiluvchilar bo'ladi, chunki $15a^2+21a^2b^2-8ab^3=15a^2+21a^2b^2+(-8ab^3)$.

2. ***Qavslarni ochish va qavs ichiga olish.***

Qo'shishning guruhlash qonunini eslab o'taylik: $a+(b+c)=a+b+c$. Bu tenglikda $c=-d$ deb olsak, $a+(b-d)=a+b-d$ tenglik hosil bo'ladi. Bundan qavslarni ochishning quyidagi qoidasi kelib chiqadi.

1-qoida. *Algebraik ifodaga qavs ichiga olingan algebraik yig'indini qo'shish uchun algebraik yig'indidagi har bir qo'shiluvchining ishoralarini saqlagan holda, "+" belgisi va qavslarni tushirib qoldirish mumkin.*

Masalan: 1) $67ab+(-17a^2b+5b^3-65c^5)=67ab-17a^2b+5b^3-65c^2$;

2) $-18+(-25-7b+82c^2)=-18-25-7b+82c^2=-43-7b+82c^2$;

3) $25+(75a-92ab+c)=25+75a-92ab+c$.

Ayirish amalining quyidagi xossalarini ko'rib o'taylik:

$-(-a)=a$; $-(a+b)=-a-b$; $a-(b+c)=a-b-c$; $a-(b-c)=a-b+c$.

Bu tengliklardan qavslarni ochishning quyidagi qoidasi kelib chiqadi:

2-qoida. *Algebraik ifodadan qavs ichiga olingan yig'indini ayirish uchun algebraik yig'indining har bir qo'shiluvchisi ishorasini qarama-qarshisiga o'zgartirib, qavs oldidagi "–" belgisi va qavslarni tushirib qoldirish mumkin.*

Misollar: 1) $72b^2-(18a+7)=72b^2-18a-7$;
2) $105-(61a^2-55b^2+9c)=105-61a^2+55b^2-9c$;
3) $-(2b-4ac)-(7k^2+13a)=-2b+4ac-7k^2-13a$;
4) Qavslarni ochib soddalashtiring:
$18a-(15a-(12a+7b))=18a-(15a-12a-7b)=18a-(3a-7b)=18a-3a+7b==15a+7b$.

51. 1) Algebraik yig'indi deb nimaga aytiladi?
2) Qavs oldida "+" ishora turgan bo'lsa, qavslar qanday ochiladi?
3) Qavs oldida "–" ishora turgan bo'lsa, qavslar qanday ochiladi?

52. Algebraik yig'indini qavslarsiz yozing:

1) $(+815)+(-7)-(+800)-(-9)$;
2) $(-32)-(-15)+(-75)-(+82)$;
3) $(-15a)+(-82b)-(-3\frac{1}{3})$;
4) $(+25\frac{1}{4}a)-(-16b)-(+8\frac{2}{9})$.

53 *(Og'zaki)*. Algebraik qo'shiluvchilarni ayting:

1) $8\frac{1}{2}a+5\frac{1}{7}b-9$;
2) $19-72\frac{1}{2}a+9\frac{5}{7}b$;
3) $-15a+2,3-3,5b$;
4) $-13\frac{1}{9}-1,5a-12,7b$.

54. Algebraik yig'indi shaklida yozing:

1) $3a-\frac{2}{7}b+13\frac{1}{2}c-1,6$;
2) $-18a+12,5b-4\frac{2}{5}c-6$;
3) $2a-5\frac{1}{4}-8\frac{1}{7}b+9c$;
4) $-15a+6\frac{1}{4}b-17c-5,5$.

Qavslarni oching **(55–56)**:

55. 1) $-b+\left(3a-3\frac{1}{2}c\right)-(-d)$;
2) $(+2c)+\left(4,5a+18\frac{3}{4}d\right)+\left(-12\frac{1}{15}b\right)$;
3) $1,8a-\left(3b-5\frac{3}{5}c\right)+(-7,9d)$;
4) $11,1a-\left(6,7b+3\frac{1}{4}c\right)-(+8,2d)$;
5) $-\left(1,2c-5b+6\frac{5}{7}d\right)$;
6) $+\left(-12\frac{7}{8}a+1\frac{2}{3}b-5\frac{3}{5}c\right)$.

56. 1) $(+15a)+(11,7b-(7\frac{1}{3}c-5d))$;
2) $8a-(19\frac{1}{9}b-(3\frac{3}{5}c-6d$

3) $13a-((8{,}7b-c)+15d)$; 4) $2{,}5a-((8{,}5b-9c)-12\frac{5}{7}d)$;

5) $3{,}18a-(4b+(3{,}2c-(9{,}5d-5\frac{2}{9}k)))$; 6) $5\frac{1}{17}a-(8b-(4c-(8d+4\frac{4}{5}k)))$.

57. Qavslarni oching va soddalashtiring:

1) $8x-(5{,}7x-(3{,}7x+1))$; 2) $-9t-(18{,}5t-(13{,}2t-5))$;

3) $3a-(5{,}7a+(4{,}3a-(9b+1{,}7a)))$; 4) $7x+(18{,}2a-(8{,}2a+(6\frac{1}{13}x-5a)))$;

5) $14{,}3y-(-(-(15{,}2y+1)+2y)-18{,}5y)$;

6) $3{,}7a+(-(15a+3b)-(-(8{,}2a-5\frac{1}{2}b)+15a))$.

58. n yoki $(-n)$ sonlaridan boshlab, barcha qo'shiluvchilarni qavs oldiga:

a) "+"ishorasini;

b) "–" ishorasini qo'ygan holda qavs ichiga oling:

1) $4a+5b+n-2c$; 2) $-3a+8b+n+4c$; 4) $b-n-3a+4b$; 5) $k-n+6b-8a$.

59. 1) $-7a+2b-1$ ifodani biri $2b$ ga teng bo'lgan ikkita qo'shiluvchining yig'indisi shaklida yozing;

2) $8a+3b+7$ ifodani kamayuvchisi $3b$ ga teng bo'lgan ayirma shaklida yozing;

3) $-2a-4b+4$ ifodani kamayuvchisi $-2a$ bo'lgan ayirma shaklida yozing;

4) $13a-8b+9$ ifodani bittasi 9 ga teng bo'lgan ikkita qo'shiluvchining yig'indisi shaklida yozing.

60*. $9a^2+3a^2b-4ab^2-9b^3$ ifodada

1) oxirgi uchta qo'shiluvchi oldiga "–" ishorasini qo'yib, qavslar ichiga oling;

2) oxirgi ikkita qo'shiluvchi oldiga "+" ishorasini qo'yib, qavslar ichiga oling;

3) ikkinchi va uchinchi qo'shiluvchilar oldiga "–" ishorasini qo'yib, qavslar ichiga oling;

4) birinchi va ikkinchi qo'shiluvchilar oldiga "–" ishorasini qo'yib, qavslar ichiga oling.

61. Ifodalarni soddalashtiring va harflarning berilgan qiymatlarida ifodaning qiymatini toping:

1) $\frac{2}{3}\cdot(x-6y)-\frac{1}{3}\cdot(2x-y)$, bunda $x=15; y=-\frac{3}{4}$;

2) $\frac{5}{6}\cdot(12c+a)+\frac{2}{3}\cdot(3c-2a)$, bunda $c=-3\frac{1}{4}; a=-12$;

3) $0{,}5\cdot(a-4b)+0{,}1\cdot(5a+10b)$, bunda $a=7{,}5; b=-3$;

4) $0{,}4\cdot(x-5y)+1{,}5\cdot(2x-y)$, bunda $x=-10; y=1\frac{2}{3}$.

62. Tenglamani yeching:

1) $2x+7=3x-2(4x-1)$;

2) $5-2(3x+3)=4(x-5)-2\frac{1}{2}$;

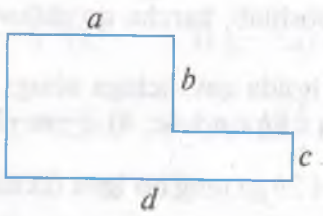
3) $5x+4(2x-5)=-5(2x+1)+7x$;

4) $2(0{,}5x-8)-5(1{,}2x+6)=3$.

63. 1) m va n – natural sonlar. $18m+2n$ son bilan $4m-5n$ son ayirmasining 7 ga bo'linishini ko'rsating;

2) k va p – natural sonlar. $17k-5p$ son bilan $3k-15p$ son yig'indisining 4 ga bo'linishini isbot qiling.

64*. Rasmda ko'rsatilgan o'lchamlar bo'yicha shaklning yuzini uch xil usulda hisoblash uchun ifodalar tuzing. Har biriga geometrik sharh bering.



65. 1) Teng yonli uchburchakning perimetri p sm ga teng. Yon tomoni asosidan 2,5 sm qisqa. Shu uchburchakning tomonlari uzunligini toping.

2) To'g'ri to'rtburchakning perimetri p sm ga teng. Asosining uzunligi a sm. Shu to'g'ri to'rtburchakning yuzini toping.

3) To'g'ri to'rtburchakning yuzi S sm^2 ga, balandligi esa h sm ga teng. Shu to'g'ri to'rtburchakning perimetrini toping.

4) To'g'ri to'rtburchakning bo'yi 30 sm, eni 20 sm. Uning bo'yini 8 sm ga kamaytirib, enini 5 sm ga uzaytirishdi. Yangi to'g'ri to'rtburchakning yuzi qanchaga o'zgardi? Tahlil qiling: qachon yangi to'g'ri to'rtburchakning yuzi avvalgisidan: a) katta; b) kichik; d) avvalgisiga teng bo'ladi?

Masalani umumiy holda hal qilishga harakat qiling.

Masalalar yechish

Algebraik ifodaning son qiymatini toping (66–68):

66. 1) $a+bc+ab$, bunda $a=-2$, $b=3{,}5$, $c=0{,}5$;

2) $a+bc-ac$, bunda $a=-3$, $b=-1\frac{1}{7}$, $c=7$;

3) $(2a+b)c+b$, bunda $a=-1{,}7$, $b=2{,}5$, $c=-6{,}7$;

4) $(3a-b)c-a$, bunda $a=2{,}1$, $b=-3$, $c=5$.

67. 1) $-4(2x-3y)^2+3xy$;

2) $3(x+4y)^2-2\frac{2}{3}xy$;

3) $3{,}5(5x-y)^2-2\frac{1}{7}xy$; 4) $-(3x+y)^2+2{,}5xy$, bunda $x=1{,}5$; $y=2{,}5$.

68. 1) $1{,}5((a-2b)^2+3\frac{1}{3})-(a-b)$; 2) $-2(5-(3a+1)^2)+(2a+b)$;

3) $((a-3b)b-9):2+(a^2-2b):3$; bunda $a=5$; $b=-1$. 4) $(3-a(4a-2{,}5b)):4+(3a-b^2):5$,

69. Harflarning qanday qiymatida ifoda ma'noga ega bo'lmaydi:

1) $\frac{a}{a-5}$; 2) $\frac{7}{a+5}$; 3) $\frac{10}{6y+3}$; 4) $\frac{a}{15a-45}$; 5) $\frac{a-2}{a^2+4a}$;

6) $\frac{4c-5}{11c^2-33c}$; 7) $\frac{5}{5a-10}$; 8) $\frac{7c-7}{7c+49}$; 9) $\frac{c}{4c-1{,}2}$; 10) $\frac{15}{1{,}5t-60}$?

70. Harflarning qanday qiymatlarida quyidagi ifodalar o'zaro teng bo'ladi:

1) $5(2a-1)$ va $12a-4$; 2) $0{,}2x+5$ va $3x-5(x-1)$;

3) $1{,}2y+5(y-6)$ va $9y-7(y-8)$; 4) $4a-10$ va $0{,}3a+11$.

71. 1) t ning qanday qiymatida $2t+5$ ifoda qiymati $5t-10(t-2)$ ifoda qiymatidan 16 ta kam bo'ladi?

2) a ning qanday qiymatida $3a-1{,}3$ ifoda qiymati $9(a+7)+3a$ ifoda qiymatidan 10 ta ortiq bo'ladi?

3) x ning qanday qiymatida $9x-5$ ifoda qiymati $4(x-5)+2{,}5x$ ifoda qiymatidan 4 marta kam bo'ladi?

4) y ning qanday qiymatida $1{,}8y+7$ ifoda qiymati $1{,}3(y-3)+7y$ ifoda qiymatidan 2 marta ortiq bo'ladi?

72. Hisoblang:

1) $\frac{(t-3)^2}{4t^2+2t-1}$; bunda $t=-2$; 2) $\frac{3a^2-4a+2}{(a+2)^2}$; bunda $a=-4$;

3) $\frac{x^2-2x+1}{0{,}2x^2+4x}$; bunda $x=-10$; 4) $\frac{k^2-6k+9}{0{,}5k^2-3k}$; bunda $k=5$.

73. Muqaddas kilosi a so'mdan yong'oq sotib oldi. Yong'oqlar qobig'idan tozalangach, umumiy massasining b % i qoldi. Muqaddas 1 kg tozalangan yong'oq uchun necha so'm sarflagan bo'ladi?

I bobga doir sinov mashqlari (testlar)

1. Sonli ifodaning qiymatini toping: $\frac{0{,}3\cdot 5^2-15}{3{,}5\cdot 5-10}$.

A) 2; B) -1; C) 1; D) 0.

2. Sonli ifodaning qiymatini toping: $\dfrac{\left(4{,}2:7-10\frac{2}{3}\cdot\frac{3}{16}\right)^2\cdot 5^3}{8{,}5:0{,}17}$.

A) 4,9; B) 3,92; C) 5; D) 6.

3. Algebraik ifodaning son qiymatini toping: $\dfrac{-4mk(k+2m)}{3m-2k}$, bunda $m=$ [illegible] $k=0{,}5$.

A) 25; B) 0,4; C) 0,9; D) $-0{,}9$.

4. $S=\dfrac{h}{2}(a+b)$ va $a=3{,}2$; $b=4{,}4$; $h=3$ bo'lsa, $\dfrac{1}{4}S$ ni toping:

A) 2,85; B) $-2{,}85$; C) 10,8; D) $-10{,}8$.

5. To'g'ri to'rtburchakning bo'yi a sm, u kvadratning tomonidan 6 sm uzun [illegible] esa shu kvadrat tomonidan 3 sm qisqa. To'g'ri to'rtburchakning perim[illegible] topish uchun ifoda tuzing.

A) $2(2a-9)$ sm; B) $2a-18$ sm; C) $4(a-6)$ sm; D) $2(a-3$ [illegible]

6. Bir qishloqdan qarama-qarshi yo'nalishda ikki velosipedchi yo'lga ch[illegible] Birinchi velosipedchining tezligi v bo'lib, u ikkinchisidan 1,5 marta k[illegible] Ular orasidagi masofa necha soatdan so'ng 45 km bo'ladi?

A) $\dfrac{16}{v}$; B) $\dfrac{18}{v}$; C) $2{,}5v$; D) $\dfrac{27}{v}$.

7. Ifodani soddalashtiring: $3{,}2(x+7)-4(3x+2{,}5)+7{,}5$.

A) $-12x+9$; B) $-8{,}8x+22{,}4$; C) $-8{,}8x+19{,}9$; D) $15{,}2x+$ [illegible]

8. Ifodani soddalashtiring: $-11{,}2(5{,}5c-2d)+4{,}2c+(-2{,}5d+c)$.

A) $-56{,}4c+19{,}9d$; B) $66{,}8c+19{,}9d$; C) $65{,}8c+22{,}4d$; D) $65{,}8c-$ [illegible]

9. Arifmetik amallar xossalaridan foydalanib hisoblang:

$\dfrac{2}{15}\cdot(0{,}75-0{,}6+3{,}5\cdot 0{,}15)$.

A) 0,15; B) 0,525; C) 0,09; D) 0,675.

10. Arifmetik amallar xossalaridan foydalanib hisoblang: $\left(21\frac{3}{7}+8\frac{4}{15}+3\frac{1}{3}\right)\cdot 5$ [illegible]

A) $170\frac{1}{5}$; B) $173\frac{2}{5}$; C) 172; D) $171\frac{3}{5}$.

11. Qavslarni oching va soddalashtiring: $-3\frac{1}{2}a-\left(4{,}3a-\left(2{,}7a-(-5{,}2a-1)\right)\right)$.

A) $a-1$; B) $-0{,}1a+1$; C) $0{,}1a+1$; D) $a+1$.

12. Ifodani soddalashtiring va uning $a=-3{,}2$; $b=-2{,}7$ bo'lgandagi qiymatini toping:

$$7{,}2\cdot\left(\frac{1}{4}a-\frac{5}{15}b\right)+4\cdot(b-a).$$

A) 2,72; B) 9,74; C) −4,34; D) −9,74.

13. Ifodaning qiymatini toping:

$$45{,}06:1{,}5-\left(2\frac{1}{3}\cdot 4{,}5-2{,}5\cdot 2\frac{1}{2}\right):4\frac{1}{4}.$$

A) 290,4; B) 29,04; C) 30,04; D) 300,4.

14. Ifodaning qiymatini toping:

$$\left(-\frac{1}{14}-\frac{1}{7}\right):(-3)-6\frac{1}{13}\cdot\left(-\frac{26}{79}\right)+9\frac{5}{8}\cdot\left(-3\frac{3}{7}\right).$$

A) $-31\frac{1}{14}$; B) $30\frac{1}{14}$; C) $-30\frac{13}{14}$; D) $30\frac{13}{14}$.

15. Uchburchak bir tomonining uzunligi a sm, u ikkinchi tomonidan 2 marta uzun, uchinchi tomonidan esa 2 sm qisqa. Uchburchak perimetrini hisoblash uchun ifoda tuzing.

A) $2{,}5a+2$; B) $4{,}5a$; C) $2{,}5a-2$; D) $2a+2{,}5$.

16. Avtobus t soatda s km yo'l bosadi. Avtomobil xuddi shu yo'lni avvalgisidan 2 soat kam vaqtda bosib o'tishi uchun qanday tezlikka ega bo'lishi kerak?

A) $s(t+2)$; B) $\frac{s}{t+2}$; C) $\frac{s}{t-2}$; D) $s(t-2)$.

17. Bo'yi a dm, perimetri p dm bo'lgan to'g'ri to'rtburchakning yuzini hisoblash uchun ifoda tuzing.

A) $a(0{,}5p-2a)$; B) $0{,}5a(p-a)$; C) $a(p-2a)$; D) $a(0{,}5p-a)$.

Rivojlantiruvchi mashqlar*

1. Doskaga 1, 2, 3, ..., 2012 sonlar yozilgan. Ular ustida 2011 marta quyidagi ishlar amalga oshiriladi:

1-ish: Ixtiyoriy ikkita a va b son o'chirilib, berilgan sonlar qatoriga $\frac{ab}{1}$ soni yoziladi.

2-ish: Ixtiyoriy ikkita c va d son o'chirilib, qatorga $\frac{cd}{2}$ soni yoziladi va h.k.

2011-ish: Oxirgi ikkita e va f son o'chirilib, qatorga $\frac{ef}{2011}$ soni yoziladi. Oxirida qolgan bitta sonni toping.

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2. Ikki son kvadratlari yig'indisi 3 ga bo'linishidan sonlar har birining 3 ga bo'linishi kelib chiqishini isbotlang.

3. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{199}-\frac{1}{200}=\frac{1}{101}+\frac{1}{102}+\ldots+\frac{1}{200}$ tenglik to'g'ri ekanini ko'rsating.

4. Qandaydir 4 xonali sonni shu sonning raqamlari teskari tartibda yozilganiga ko'paytirilganda, oxirida 3 ta nol bo'lgan 8 xonali son hosil bo'ldi. Shu sharni qanoatlantiruvchi barcha 4 xonali sonlarni toping.

5. 2002 ta sonlar yig'indisi musbat bo'lsin. Agar ularning ixtiyoriy 23 tasining yig'indisi musbat ekanligi ma'lum bo'lsa, u holda barchasining yig'indisi ham musbat bo'lishini isbotlang.

Ko'rsatma: Berilgan sonlardan kamida bittasini musbat ekanligi ravshan. $2\,002-1=2\,001=23\cdot 87$ ekanidan foydalaning.

6. Tomoni 10 sm bo'lgan kvadratdan diametrlarining yig'indisi 5 metrdan kam bo'lgan bir qancha doiralar qirqib olish mumkinmi?

Ko'rsatma: Berilgan kvadratni $n\cdot n$ dona o'zaro teng kvadratchalarga ajratamiz. Har bir kichik kvadratga ichki aylana chizing. Aylananing diametri $\frac{1}{10\cdot n}$ metrga teng bo'ladi. Diametrlar yig'indisi $n^2\cdot\frac{1}{10\cdot n}=0{,}1\cdot n$ metrga teng, bundan $0{,}1\cdot n>5$, $n>50$.

7. Agar $a_1+a_2+\ldots a_{2014}=0$ bo'lsa, ushbu:
$1\cdot(a_1-a_2)+2\cdot(a_2-a_3)+3\cdot(a_2-a_3)+\ldots+2013\cdot(a_{2013}-a_{2014})+2014\cdot a_{2014}$
ifodaning son qiymatini toping.

8. Agar soat 30 sekundda 6 marta bong ursa, uning 12 marta bong urishi uchun necha sekund kerak bo'ladi?

9. Birinchi hadi 7 ga teng bo'lgan sonlar ketma-ketligi quydagicha tuzilgan: ikkinchi hadidan boshlab keyingi har bir hadi oldingi son kvadrati raqamlari yig'indisining 1 bilan qo'shilganiga teng. Ketma-ketlikning 2014-hadi qanday songa teng?

10. *Professor Abdullajon A'zamov masalasi*:
1) Raqamlari turli ixtiyoriy ikki xonali son uchun quyidagi xossani isbotlang: bu sonning raqamlari o'rni almashtirilib, kattasidan kichigi ayirilsa va ayirmaning raqamlari almashtirilib, ayirmaga qo'shilsa, doim 99 hosil bo'ladi.
2) Yuqoridagi kabi xossani 4 xonali sonlar uchun tatbiq qiling.

11. *Azizxo'ja G'anixo'jaev masalalaridan* (**11–15**):
Sinfda ixtiyoriy 12 nafar o'quvchidan kamida biri qiz, ixtiyoriy 20 nafar o'quvchidan kamida biri o'g'il bola. Agar sinfda 30 nafar o'quvchi bo'lsa, ulardan nechtasi qiz, nechtasi o'g'il bola bo'ladi?

12. Uchta talaba-arxeolog *A.*, *B.*, *C.* qazilma ishlari davrida bir ko‘za topishgan. Ularning har biri ko‘za to‘g‘risida ikkitadan taxmin qildi.
A.: Ko‘za Samarqandda XV asrda yasalgan.
B.: Ko‘za Buxoroda XII asrda ishlangan.
C.: Ko‘za XV asrda yasalgan, lekin Buxoroda emas.
Ammo mutaxassislarning tekshirishi natijasida har uchala talaba taxminlaridan biri to‘g‘ri, ikkinchisi esa noto‘g‘ri ekanligi aniqlandi. Ko‘za qaysi shaharda va qachon ishlangan?

13. Turli beshta son bir qatorga yozilgan. Bu sonlardan shunday uchtasi topiladiki, ular qatorda turish tartibiga ko‘ra o‘suvchi yoki kamayuvchi bo‘lishini isbotlang.

14. 16 ta nuqta 4×4 kvadrat shaklida joylashtirilgan. Bu nuqtalarni ikki xil rangga shunday bo‘yangki, natijada uchlari bir xil rangda bo‘lgan:
a) kvadrat; b) to‘g‘ri to‘rtburchak; d) teng yonli uchburchak; e) to‘g‘ri burchakli uchburchak mavjud bo‘lmasin.

15. Qog‘ozchalarning bir tomoniga harf, ikkinchi tomoniga esa son yozilgan. “Unli harfning orqa tomoniga juft son yozilgan”. Shu jumlaning to‘g‘riligini tekshirish uchun nechta va qaysi qog‘ozchalarni ag‘darib ko‘rish zarur?

A	7	B	U	6	5

16. 1) $\frac{1}{7}$; 2) $\frac{1}{17}$ ning o‘nli kasr yozuvida verguldan keyingi 1001-xonada turgan raqamni toping?

17. Pallali tarozining massalari har xil bo‘lgan 3 ta toshi bor. Ular yordamida 1 kg, 2 kg, ...,10 kg massalarni o‘lchash mumkin bo‘lsa, toshlarning har biri necha kilogrammdan bo‘lishi mumkin? Zarur bo‘lsa, toshlarni tarozining ikkala pallasiga qo‘yish mumkin.
Javob: Masalan, 1 kg, 3 kg va 6 kg. Chindan ham, $10=6+3+1$; $9=6+3$; $8=6+3-1$; $7=6+1$; $6=6$; $5=6-1$; $4=3+1$; $3=3$; $2=3-1$; $1=1$. (Tortilayotgan mahsuloti bor pallaga qo‘yilgan tosh massasini ko‘rsatuvchi son oldiga minus ($-$) ishorasi qo‘yilgan.)

18. Rebusni hal qiling:
1) BDCE+BDCE=ADCBE; 2) $M^3=KUB$;
3) KNIGA+KNIGA+KNIGA=NAUKA.
Eslatib o‘tamizki, turli harflarga turli raqamlar, bir xil harflarga bir xil raqamlar mos keladi.

19. 12 yil avval Feruzaning yoshi kichik singlisi Barnoning yoshidan 5 marta katta edi. 16 yildan so‘ng Feruzaning yoshi singlisining yoshidan 1,5 marta katta bo‘ladi. Opa-singillar yoshini toping.

20. Kanalda uchta kema chapdan o'ngga suzib kelardi. Ularga qarab uchta ke... o'ngdan chapga suzib kelardi. Kanalning eniga faqat bitta kema sig'adi, am... kanalda bitta kema sig'adigan bo'g'oz bor. Kemalarning o'z yo'lini dav... ettirishiga yordam bering.

21. Jamila xola bir nechta olmalarni kvadrat shaklidagi stol tomonlariga 4 ta... qo'yib chiqdi. Qizi kelib, o'sha olmalarni xuddi shu stol tomonlariga 5 ta... qo'yib chiqdi. O'g'li kelib, o'sha olmalarni yana o'sha stol tomonlariga ... tadan qo'yib chiqdi. Buni xola, uning qizi va o'g'li qanday amalga oshir... gan?

22. 39 ta ketma-ket natural sonlar orasida raqamlari yig'indisi 11 ga bo'linadi... son albatta topilishini isbotlang.

23. Dilafro'z koptok rasmini chizyapti: avval ko'k, keyin yashil, keyin qizil, ke... qora. Yana: ko'k, yashil, qizil, qora. Chizish shu tartibda davom etave... 1) 32-; 2) 41-; 3) 91-; 4) 2015-koptok qaysi rangda bo'ladi?

24. 1) Aylanada 6 ta nuqta belgilangan. Har bir nuqta qolgan barcha nuqta... bilan tutashtirilsa, jami nechta kesma hosil bo'ladi? Mos rasm chizing.
2) Aylanada belgilangan nuqtalar soni: a) 100 ta; b) 1000 ta; d) n ta bo'lsa-ch...

25. 1, 2, 3, 4, 5, 6 raqamlari yordamida hammasi bo'lib: 1) raqamlar takrorla... masa; 2) raqamlar takrorlanishi mumkin bo'lsa, nechta uch xonali son tuz... mumkin?

26. Uch to'p yong'oq bor. Ularning ixtiyoriy bittasiga qolgan ikkitasida nech... yong'oq bo'lsa, shuncha yong'oqni qo'shish yoki undan shuncha yong'oq... olib qo'yish mumkin.
Masalan: birinchi to'pda 30 ta, ikkinchisida 7 ta, uchinchisida 9 ta yong'o... bo'lsin, buni (30,7,9) kabi yozaylik. U holda (30, 7, 9) → (30, 46, 9) yo... (30, 7, 9) → (14, 7, 9) bo'lishi mumkin.
To'plardagi yong'oqlar soni (239, 117, 43) bo'lsa, yuqorida aytilgan amalni bir ne... cha bor qo'llab, to'plardan birini "bo'shatish" (ya'ni undagi yong'oqlar so... 0-nol qilish) mumkinmi? Agar yong'oqlar soni (240, 118, 50) bo'lsa-chi?

27. Ketma-ket kelgan 4 ta natural sonning ko'paytmasi:
a) 34 570 800; b) 73 188 024 ga teng. Shu sonlarni toping?

28. $1 \cdot 2 \cdot 3 \cdot \ldots \cdot 2\,000 \cdot 2\,001 + 2\,002 \cdot 2\,003 \cdot \ldots \cdot 4\,001 \cdot 4\,002$ yig'indi 4 003 ga bo'linishini isbotlang.
Ko'rsatma: $2\,002 \cdot 2\,003 \cdot \ldots \cdot 4\,001 \cdot 4\,002 = (4\,003 - 2\,001) \cdot (4\,003 - 2\,000) \cdot (4\,003 - 1999) \cdot \ldots \cdot (4\,003 - 2) \cdot (4\,003 - 1) = 4\,003 \cdot n - 1 \cdot 2 \cdot 3 \cdot \ldots \cdot 2\,000 \cdot 2\,001$, bunda n – natural son.

29. Doskada 4 ta son yozilgan: 7 956, 3 923, 5 827, 9 725. Quyidagi amal... bajariladi:
1-ish: Berilgan sonlar yig'indisi 10 000 ga bo'linadi;

2-ish: Qoldiq berilgan sonlardan keyin yoziladi;
3-ish: Birinchi son (7 956) o'chiriladi.
Qolgan 4 ta son ustida yana ishlar shu ketma-ketlikda amalga oshiriladi va h.k. Qachondir doskada 4 ta son: 1 989, 1 989, 1 989, 1 989 paydo bo'lishi mumkinmi?

30. Tenglamani natural sonlarda yeching:
1) $x-y-z=x:y:z=2$.
2) Biror butun son n kubga ko'tarildi. n^3+n yoki n^3-n sonlardan kamida bittasi (hech bo'lmaganda bittasi) 10 ga bo'linishini ko'rsating.
3) Uch xonali son 7 raqami bilan boshlanadi. Shu 7 raqamni sonning oxiriga yozib, boshqa bir uch xonali son hosil qilindi. Bu son berilgan sondan 117 taga kam chiqdi. Dastlabki sonni toping.

31. 1, 2, ..., 9 raqamlarni har bir katakka bittadan shunday yozingki, natijada ko'paytma eng katta bo'lsin.

32. Arifmetik rebusni yeching, bunda bir xil harflarga bir xil raqamlar, turli harflarga turli raqamlar mos keladi:
TATIR + ATIR + TIR + IR + R = 55 550.

33. 1) Do'konda olmaning narxi 20 % ga oshdi. Sotuvchi olmaning yangi narxini qog'ozchaga yozib o'tirmadi, u olma narxi yozilgan avvalgi qog'ozchadagi dastlabki ikkita raqam o'rnini almashtirib qo'ydi, xolos. Qimmatlashishidan avval olmaning narxi (bu narx 500 so'mdan kam) necha so'm edi?

2) Do'konda olmaning narxi $16\frac{2}{3}\%$ ga arzonlashdi. Sotuvchi olmaning yangi narxini qog'ozchaga yozib o'tirmadi, u olma narxi yozilgan avvalgi qog'ozchadagi dastlabki ikkita raqam o'rnini almashtirib qo'ydi, xolos. Arzonlashmasdan avval olmaning narxi (bu narx 600 so'mdan kam) necha so'm edi?

34. Sonning 13 ga bo'linishining ushbu alomatini avval bir nechta misollarda tekshirib ko'ring, so'ngra isbotlashga harakat qiling:
1) berilgan sonning oxirgi raqamini o'chiramiz;
2) qolgan songa o'chirilgan raqamning 4 barobarini qo'shamiz.
Hosil bo'lgan son 13 ga bo'linsa, berilgan son ham 13 ga bo'linadi.
Misol: 4 225 soni 13 ga bo'linadimi?
1) $4\,225 \to 422$;
2) $422+4\cdot 5=442$;
3) $442 \to 44$;
4) $44+2\cdot 4=52$.
Ammo $52=4\cdot 13$. Demak, alomatga ko'ra, 4 225 son ham 13 ga bo'linadi: $4\,225=13\cdot 325$.

35. Sanjar va Sirojiddin 3 kunda 18 ta kitobcha xarid qilishdi. Sirojiddin bugun va kecha qancha kitobcha xarid qilgan bo'lsa, Sanjar bugun shuncha kitobcha xarid qildi. Sanjar o'tgan kun va kecha qancha kitobcha xarid qilgan bo'lsa, Sirojiddin o'tgan kuni undan 2 ta ortiq kitob xarid qildi. Sanjar va Sirojiddinning har biri qanchadan kitob xarid qilgan?

36. Arifmetik rebusni hal qiling: BIR + BIR + BIR + BIR = TO'RT. (Bir xil harflarga bir xil raqamlar, turli harflarga turli raqamlar mos keladi.)

37. Otabek bitta daftar, ikkita qalam va bitta o'chirg'ich uchun 1 200 so'm to'ladi. Nargiza ikkita daftar, uchta qalam va uchta o'chirg'ich uchun 2 700 so'm to'ladi. Saidkomil ikkita daftar, beshta qalam va bitta o'chirg'ich uchun necha so'm to'laydi?

38. Kurash musobaqasida qatnashib, Karim, Rahim, Salim, Alim birinchi 4 ta o'rinni egallashdi. Karim, Rahim, Salim egallagan o'rinlar yig'indisi 6 ga teng. Rahim va Alim egallagan o'rinlar yig'indisi ham 6 ga teng. Rahim Karimga qaraganda yuqori o'rinda bo'lsa, har bir kurashchi qanday o'rinni olgan?

39. a, b, c – to'g'ri burchakli uchburchakning tomonlari. Agar a, b, c – natural sonlar bo'lsa, abc ko'paytmaning 30 ga bo'linishini isbotlang.

40. Har qanday uch xonali son uchun quyidagi tasdiqlardan hech bo'lmaganda bittasi o'rinlidir:
1) bu son uchga bo'linadi;
2) bu sonning raqamlaridan biri 3 ga bo'linadi;
3) bu son raqamlaridan tuzilgan biror ikki xonali son 3 ga bo'linadi. Isbotlang.

41. Doskada 5 ta son yozilgan. Ularni qo'shib, ushbu 10 ta son hosil qilindi: 0, 2, 4, 4, 6, 8, 9, 11, 13, 15. Doskada qanday sonlar yozilgan bo'lishi mumkin?

42. A – 2 015 xonali natural son, a – uning raqamlari yig'indisi, b esa a sonning raqamlari yig'indisi, c esa b sonning raqamlari yig'indisi bo'lsin. c ni toping.

43. 75 ta butun son berilgan. Ularning qandaydir 56 tasiga bir vaqtning o'zida 1 raqamini qo'shishga ruxsat beriladi. Bu ishni bir necha marta takrorlab, hamma sonlarning teng bo'lishiga erishish mumkinligini isbotlang.

TARIXIY MASALALAR

1. *Al-Xorazmiy masalasi.* Sondan uning uchdan biri va to'rtdan biri ayirilsa, 8 qoladi. Sonning o'zini toping.

2. *Al-Xorazmiy masalasi.* Sen o'nni ikki qismga ajratding, keyin ulardan birini boshqasiga bo'lding, bo'linmada to'rt chiqdi. Sen o'nni qanday qismlarga ajratding?

3. *Geron masalasi* (eramizning I asri). Hovuzga ikkita quvurdan suv keladi. Birinchi quvurdan 1 soatda 1 m^3, ikkinchi quvurdan 1 soatda 4 m^3 suv tu-

shadi. Hovuzning hajmi 12 m^3. Ikkala quvur baravar ochib qo'yilsa, bo'sh hovuz qancha vaqtda to'ladi?

4. *Nyuton masalasi.* Oralaridagi masofa 59 mil bo'lgan ikki qishloqdan A va B kishilar bir-biriga qarab yo'lga chiqdi. B kishi A ga qaraganda 1 soat kech yo'lga chiqdi. A kishi 2 soatda 7 mil, B esa 3 soatda 8 mil bosadi. A kishi B bilan uchrashguncha necha mil yo'l yuradi? (Mil uzunlik o'lchov birliklaridan bo'lib, 1 mil $\approx$ 1,852 km.)

TARIXIY MA'LUMOT

Yurtdoshimiz buyuk matematik va astronom olim Abu Abdulloh Muhammad ibn Muso al-Xorazmiy (783–850) ning arifmetik ("Algorizmi hind hisobi haqida") va algebraik ("Al-jabr val-muqobala") asarlari matematikaning rivojiga nihoyatda kuchli ta'sir ko'rsatdi. Bu asarlar ko'p tillarga tarjima qilindi, asrlar davomida matematikadan asosiy qo'llanma bo'lib xizmat qildi. "Algorizmi hind hisobi haqida" risolasining XII asr boshidagi lotincha tarjimasi Angliyaning Kembrij universitetida saqlanadi. Al-Xorazmiyning bu asari tufayli Yevropaga o'nli sanoq sistemasi kirib borgan.

Xorazmiy algebrasi — "Al-jabr val-muqobala hisobi haqida qisqacha kitob" asarining arabcha nusxasi Oksford universitetining Bodleyan kutubxonasida saqlanadi. Risola uch qismdan iborat: 1) algebraik qism; 2) geometrik qism; 3) vasiyatlar haqidagi qism (Xorazmiy uni "Vasiyatlar kitobi" deb atagan).

Al-Xorazmiy risolasida barcha masalalarning bayoni va yechimlari so'zlar bilan beriladi, hech qanday belgilashlar, harfiy ifodalar ishlatilmaydi. Al-Xorazmiy yozadi: "...Men arifmetikaning oddiy va murakkab masalalarini o'z ichiga oluvchi "Al-jabr val-muqobala hisobi haqida qisqacha kitob"ni ta'lif qildim, chunki meros taqsim qilishda, vasiyatnoma tuzishda, mol taqsimlashda va adliya ishlarida, savdoda va har qanday bitimlarda va, shuningdek, yer o'lchashda, ariqlar o'tkazishda, muhandislikda va boshqa shunga o'xshash turlicha ishlarda kishilar uchun bu zarurdir". Binobarin, olim o'zining bu asarini kundalik hayot talabi va ehtiyojlarini hisobga olgan holda yozgan.

II BOB. BIR NOMA'LUMLI BIRINCHI DARAJALI TENGLAMALAR

1-§. Tenglama va uning yechimlari

Tenglama tushunchasiga olib keluvchi masalalar bilan 5–6-sinflard tanishgansiz. Bitta masala yechaylik.

Masala. Nodira va Nozima bog'dan birgalikda 156 kg uzum uzishdi Nodira Nozimadan 18 kg ortiq uzum uzdi. Har bir qiz bog'dan necha kilogramm uzum uzgan?

□ Nodira x kg uzum uzdi, deylik. U holda Nozima $(x - 18)$ kg uzum uzgan bo'ladi. Masala shartiga ko'ra, $x + (x - 18) = 156$, bundan $2x - 18 = 156$, $2x = 156 + 18$, $2x = 174$, $x = 87$.

Demak, Nodira bog'dan 87 kg uzum uzgan. Nozima esa $x - 18 = 87 - 18 = 69$ (kg) uzum uzgan.

Javob: Nodira 87 kg, Nozima 69 kg uzum uzgan. ■

$x + (x - 18) = 156$ tenglikda x harfi bilan noma'lum – qidirilayotgan, izlanayotgan son belgilangan.

Harf bilan belgilangan noma'lum son qatnashgan tenglik *tenglama* deyiladi.
Tenglik belgisidan chapda turgan ifodalar *tenglamaning chap qismi* deyiladi.
Tenglik belgisidan o'ngda turgan ifodalar *tenglamaning o'ng qismi* deyiladi.
Tenglamaning chap yoki o'ng qismidagi har bir qo'shiluvchi *tenglamaning hadi* deyiladi.

$$2x - 18 = 156 \qquad (1)$$

tenglamaning chap qismi $2x - 18$, o'ng qismi esa 156 dan iborat. $x = 87$ bo'lganda, (1) tenglamaning chap qismi $2 \cdot 87 - 18 = 174 - 18 = 156$ ga teng; o'ng qismi ham 156 ga teng. Demak, $x = 87$ bo'lganda bu tenglama to'g'ri tenglikka aylanadi: $2 \cdot 87 - 18 = 156$. Ayni shu 87 soni (1) tenglamaning ildizi deyiladi.

Noma'lumning tenglamani to'g'ri tenglikka aylantiradigan qiymati *tenglamaning ildizi* deyiladi.

Masalan, 5 soni $3x - 4 = 11$ tenglamaning ildizi, chunki $3 \cdot 5 - 4 = 11$ – to'g'ri tenglik. 5 dan farqli birorta ham son bu tenglamaning ildizi bo'la olmaydi.

Tenglama faqat bitta ildizga ega bo'lishi shart emas: u ikkita, uchta, ..., yuzta, ... ildizlarga ega bo'lishi ham mumkin.

Masalan, $(x + 2)(x + 1)(x - 2) = 0$ tenglama uchta ildizga ega: -2; -1; 2.

Chindan ham, $x = -2$, $x = -1$, $x = 2$ bo'lganda, tenglama to'g'ri tenglikka aylanadi. $(x-4)(x-3,5)(x-\frac{1}{2})(x+0,75)=0$ tenglama esa to'rtta ildizga ega: $4;\ 3,5;\ \frac{1}{2};\ -0,75$.

Tenglama ildizlarga ega bo'lmasligi ham mumkin.

Masalan, $5x - 3 = 5x + 7$ tenglamaning ildizlari yo'q, chunki x ning ixtiyoriy qiymatida bu tenglamaning o'ng qismi chap qismidan katta.

Tenglama ildizlarining soni cheksiz ko'p bo'lishi ham mumkin.

Masalan, $7(x+1) = 7x+7$ tenglamaning ildizlari soni cheksiz ko'p: x ning xohlagan qiymati bu tenglamaning ildizi bo'ladi, chunki ixtiyoriy x da tenglamaning chap va o'ng tomonlari o'zaro tengdir.

Tenglamani yechish – uning barcha ildizlarini topish yoki ularning yo'qligini ko'rsatish demakdir.

Amaliyotning ko'pgina masalalarini yechish (sodda almashtirishlardan so'ng)

$$ax = b \qquad (2)$$

ko'rinishdagi tenglamaga keltiriladi, bunda a va b – berilgan sonlar, x – noma'lum son.

(2) tenglama chiziqli tenglama deyiladi.

Masalan, $7x = 14$, $-3x = 24$, $\frac{4}{5}x = -1$, $0,65x = 1,3$ kabi tenglamalar chiziqli tenglamalardir.

(2) chiziqli tenglamada:

1) $a \neq 0$ bo'lsa, tenglama bitta (yagona) $x = \frac{b}{a}$ ildizga ega;

2) $a = 0$, $b \neq 0$ bo'lsa, tenglama ildizga ega emas;

3) $a = 0$, $b = 0$ bo'lsa, tenglama cheksiz ko'p ildizga ega bo'ladi.

?

1. 1) Tenglama deb nimaga aytiladi? Misollar keltiring.
2) Tenglamaning chap, o'ng qismi, hadi nima? Misollarda tushuntiring.
3) Tenglamaning ildizi deb nimaga aytiladi?
4) Tenglamani yechish deganda nimani tushunasiz?

2. 1) Chiziqli tenglama deb nimaga aytiladi? Misollar keltiring.
2) Qachon chiziqli tenglama: a) bitta ildizga ega bo'ladi? b) ildizga ega bo'lmaydi? d) cheksiz ko'p ildizga ega bo'ladi? Har bir holga uchtadan misol tuzing.

3 (*Og'zaki*). x ning har qanday qiymatlarida tenglama to'g'ri tenglikka aylanadi:
1) $2x + 4 = 6$; 2) $5 - x = 4$; 3) $8x = 4$; 4) $2x + 5 = 0$;

$-x-19=-8x-9$, bundan $7x=10$ tenglamaga kelamiz va uni yechamiz: $x=\frac{10}{7}=1\frac{3}{7}$. *Javob*: $x=1\frac{3}{7}$. ■

3-masala. $\frac{2x+3}{2}-\frac{2-3x}{4}=-2+2,5x$ tenglamani yeching.

□ Tenglamaning chap va o'ng qismlarini kasrlarning umumiy maxraji 4 ga ko'paytiramiz:

$4\cdot\frac{2x+3}{2}-4\cdot\frac{2-3x}{4}=-4\cdot 2+4\cdot 2,5x$, bundan $2(2x+3)-(2-3x)=-8+10x$.

Qavslarni ochamiz va o'xshash hadlarni ixchamlaymiz: $4x+6-2+3x=-8+10x$, $7x+4=-8+10x$, bundan $3x=12$, ya'ni $x=12:3$, $x=4$.
Javob: $x=4$. ■

Demak, tenglamani yechishda uning asosiy xossalaridan foydalaniladi. Shu xossalarni keltiramiz:

1-xossa. Tenglamaning istalgan hadi ishorasini qarama-qarshisiga o'zgartirib, uning bir qismidan ikkinchi qismiga o'tkazish mumkin.

2-xossa. Tenglamaning ikkala qismini nolga teng bo'lmagan bir xil songa ko'paytirish yoki bo'lish mumkin.

Bu xossalar bir noma'lumli istalgan tenglamani yechish imkonini beradi. Buning uchun:

1) noma'lum qatnashgan hadlarni tenglikning chap qismiga, noma'lum qatnashmagan hadlarni esa o'ng qismiga o'tkazish lozim (1-xossa);
2) o'xshash hadlarni ixchamlash kerak;
3) tenglamaning ikkala qismini noma'lum oldida turgan koeffitsiyentga (agar u nolga teng bo'lmasa) bo'lish (2-xossa) kerak.

Ko'rilgan 3 ta masalada har bir tenglama bittadan ildizga ega edi. Bir noma'lumli tenglama: a) ildizga ega bo'lmasligi; b) cheksiz ko'p ildizga ega bo'lishi ham mumkin. Shu hollarga misollar keltiramiz.

4-masala. $\frac{3x+2}{9}-\frac{x+8}{6}=\frac{x+11}{6}+\frac{x}{3}$ tenglama ildizlarga ega emasligini ko'rsating.

□ 1) Tenglamaning ikkala qismini kasrlarning umumiy maxraji 18 ga ko'paytiramiz:

$18\cdot\frac{3x+2}{9}-18\cdot\frac{x+8}{6}=18\cdot\frac{x+11}{6}+18\cdot\frac{x}{3}$. Bundan $2(3x+2)+3(x+8)=3(x+11)+6x$.

2) Qavslarni ochamiz va o'xshash hadlarni ixchamlaymiz:
$6x+4+3x+24=3x+33+6x$, $9x+28=9x+33$, ya'ni $9x-9x=33-28$, $0\cdot x=5$.
Bu tenglamaning chap qismi ixtiyoriy x da nolga teng, o'ng qismi esa 5 ga teng, $0\neq 5$. Demak, berilgan tenglama ildizlarga ega emas. ■

5-masala. $\frac{2x+3}{2}=\frac{3x-2}{3}+2\frac{1}{6}$ tenglama cheksiz ko‘p yechimlarga ega ekanini ko‘rsating.

□ 1) Tenglamaning ikkala qismini kasrlarning umumiy maxraji 6 ga ko‘paytiramiz:

$6\cdot\frac{2x+3}{2}=6\cdot\frac{3x-2}{3}+6\cdot\frac{13}{6}$, bundan $6x+9=6x-4+13$, $6x+9=6x+9$.

Oxirgi tenglik x ning ixtiyoriy qiymatida to‘g‘ridir. Demak, istalgan x son berilgan tenglamaning ildizi bo‘ladi. ■

(?)

11. 1) “Al-jabr” qoidasi (usuli, almashtirishi) nimani bildiradi? Misollarda tushuntiring.
2) “Val-muqobala” qoidasi (usuli, almashtirishi) nimani bildiradi? Misollarda tushuntiring.
3) To‘g‘ri tenglikning ikkita asosiy xossasini ayting. Misollarda izohlang.
4) Tenglamaning ikkita asosiy xossasini ayting va misollarda tushuntiring.
5) Bir noma’lumli chiziqli tenglama qanday yechiladi? Yechish algoritmini ayting.
6) Chiziqli tenglama ildizga ega bo‘lishi shartmi? Fikringizni asoslang.
7) Chiziqli tenglama cheksiz ko‘p ildizga ega bo‘lishi shartmi? Misollarda izohlang.

Tenglamani yeching **(12–24)**:

12 (*Og‘zaki*). 1) $5x=50$; 2) $9x=81$; 3) $7x=77$; 4) $10x=150$.

13. 1) $6x=6{,}6$; 2) $11x=12{,}1$; 3) $7{,}1x=14{,}2$; 4) $0{,}9x=2{,}7$.

14. 1) $3{,}5x=-7$; 2) $-2{,}5x=10$; 3) $-8x=4{,}8$; 4) $1{,}7x=-5{,}1$.

15. 1) $\frac{3}{4}x=\frac{9}{16}$; 2) $\frac{7}{8}x=1$; 3) $\frac{10}{11}x=7$; 4) $\frac{4}{7}x=3{,}2$.

16. 1) $10x-8=x-1$; 2) $5x-3=x+5$;
3) $1{,}7x-1=0{,}7x+3$; 4) $5-x=7-3x$; 5) $7x+3=2x-7$.

17. 1) $7:\left(\frac{1}{3}+x\right)=9:\left(x+\frac{3}{4}\right)$; 2) $\left(1{,}7:\left(1\frac{2}{3}x-3{,}75\right)\right):\frac{8}{25}=1\frac{5}{12}$;

3) $\left(4\frac{3}{8}x+5\frac{1}{16}\right)\cdot\frac{4}{15}=\frac{5}{12}x+2{,}4$; 4) $(3+2{,}5x):3{,}5=(3x+7):0{,}7$.

18. 1) $4(x+1)+5(x+2)=7(x+3)-9$; 2) $7(3x+2)+5(x-3)-2(x-6)=8$;
3) $6(2x-1)-7(3x+11)+5(10-x)=7$; 4) $5(8x-1)-7(3x+1)+9(6-5x)=16$.

19. 1) $\frac{3x+2}{6}-\frac{5x-8}{8}=\frac{2x+1}{4}-\frac{x-1}{2}-\frac{2}{3}$;

2) $\frac{x+10}{3}+\frac{16x-3}{20}-\frac{7x-6}{4}=0,5(x-3)+0,3(x-3)$.

20.* 1) $\frac{3x+4}{2}-\frac{5x+22}{5}=0,2(9x+44)+0,75(3x+10)$;

2) $\frac{5(3x-2)}{4}+1,5x=22\frac{5}{6}+x+\frac{9-x}{18}$.

21. 1) $5(4x-1)-2,7x+0,2x=6,5-0,5x$;

2) $0,72x-6,8=0,6(0,4x-1,2)$;

3) $2,4x-5,375=0,25x-1,53x-5,425+3,7x$;

4) $11,4x+14,4-1,71x=68,377+6,9x-36,4$.

22. 1) $\frac{x-5}{3}+\frac{x-4}{4}+2,5=\frac{x+3}{6}$; 2) $\frac{x+3}{3}+\frac{x+4}{4}-3,5=\frac{x-3}{6}$;

3) $\frac{3-x}{3}+\frac{4-x}{4}+4,5=\frac{2-x}{6}$.

23.* 1) $\left\{\frac{10}{11}\cdot\left[\frac{4}{5}\cdot\left(\frac{1}{4}\cdot(x-1)+5\right)+3\right]-2\right\}-x=0$;

2) $\frac{1}{10}\cdot\left\{\frac{1}{9}\cdot\left[\frac{1}{8}\cdot\left(\frac{1}{7}(x+2)+7\right)+8\right]+9\right\}=1$.

24.* 1) $-2x-\{2,5x-[3x-(3,5x-(4x-9))]\}=-10$;

2) $-x-\left\{\frac{5}{4}x-\left[1,5x-\left(1,75x-(2x-2,25)\right)\right]\right\}=-2,5$.

25. 1) x ning qanday qiymatida $3\cdot(x-4)-2\cdot(3-x)-8$ ifodaning son qiymati 7 ga teng bo‘ladi?

2) x ning qanday qiymatida $\frac{3x-1}{5}-\frac{5x+1}{6}$ va $\frac{x+1}{8}-3$ ifodalarning son qiymatlari teng bo‘ladi?

Masalalar yechish

26. x ning istalgan qiymati tenglamaning ildizi bo'la olishini ko'rsating:

1) $4\cdot(2{,}5-3x)-1{,}7x=5\cdot(2-2{,}74x)$;
2) $2\cdot(1{,}3x+0{,}25)-1{,}65x=0{,}95x+0{,}5$;
3) $3\cdot(1{,}2x-0{,}5)-0{,}65x=2{,}95x-1{,}5$;
4) $2{,}8\cdot(5x+3)-6{,}5x=3\cdot(2{,}5x+2{,}8)$.

27. Tenglamaning ildizga ega emasligini ko'rsating:

1) $\frac{5x+7}{12}-\frac{4-x}{3}=\frac{8+3x}{4}$;
2) $\frac{x-6}{5}+\frac{7x-8}{15}=\frac{2x+11}{3}$;
3) $\frac{3x-7}{11}+\frac{x+5}{22}=\frac{19+7x}{22}$;
4) $0{,}8\cdot(4x-2{,}5)+5\cdot(2{,}4x+1{,}8)=(4\cdot3{,}8x-7)$.

28. Tenglamadan noma'lum x ni toping. a ning qanday qiymatlarida tenglama yechimga ega bo'ladi:

1) $2x-3\cdot(x-a)=3+a$;
2) $5a+7\cdot(x-2)=3a+2x$;
3) $\frac{3ax-4}{2}=\frac{3-4ax}{5}$;
4) $\frac{7-2ax}{3}=\frac{6-3ax}{4}$;
5) $3ax-4\cdot(2+x)=7$;
6) $14-3ax=3\cdot(2+3x)$?

29.* a sonning tenglama cheksiz ko'p ildizga ega bo'ladigan qiymatini toping:

1) $2\frac{3}{7}x-4=2\frac{3}{7}x-a$;
2) $x-(7-4x)=(5x-a)$;
3) $\frac{a}{2}-\frac{x}{5}=0{,}6x-0{,}8(x-1)$;
4) $\frac{x}{17}+\frac{a}{8}=(x+12)-\frac{16}{17}x$.

30.* a ning qanday qiymatlarida tenglama yechimiga ega emas:

1) $3x-7=ax$;
2) $2x\cdot(5-3a)=1{,}8$;
3) $\frac{2-x}{ax+3}=\frac{3}{4}$;
4) $\frac{5+ax}{3-x}=-1{,}5$?

31. Tenglamani yeching:

1) $\frac{5x+7}{4}=\frac{7x+8}{5}$;
2) $\frac{3-2x}{4-5x}=2\frac{6}{17}\cdot\frac{17}{72}$;
3) $7\frac{9}{13}:5{,}1=3\frac{11}{13}:x$.

32. 1) $\frac{x}{5}+3{,}4=\frac{3x-15}{4}$ tenglamaning ildizi x_0 bo'lsa, x_0^2+31 ni toping.

2) $\frac{4x}{5}+1{,}2-\frac{2-3x}{2}=16+\frac{5x+30}{2}$ tenglamaning ildizi x_0 bo'lsa, $612:(-x_0)$ ni toping.

3) $\frac{3x-1}{5}-\frac{x}{8}-0{,}125=\frac{5x-11}{6}-1$ tenglamaning ildizi x_0 bo'lsa, $x^2_0+7x_0+2$ ni toping.

33. 1) $\frac{3x-2}{5}+\frac{4-x}{3}=\frac{2-x}{15}+1$ tenglamaning ildizi x_0 bo'lsa, $10x_0:3$ ni toping.

2) $3\cdot(2{,}5x-2)-4{,}5x+1{,}8x=0{,}5\cdot(3x-2)$ tenglamaning ildizi x_0 bo'lsa, $33\cdot x_0-49$ ni toping.

3) $8\frac{2}{3}:(4x-5)=4\frac{1}{3}:(4-3x)$ tenglamaning ildizi x_0 bo'lsa, $7x_0+0{,}9$ ni toping.

34.* 1) $\frac{x}{2}=\frac{y}{4}=\frac{z}{3}$ bo'lsa, $\frac{x^2+y^2+z^2}{x^2+yz}$ ifodaning son qiymatini toping.

2) $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ bo'lsa, $\frac{x\cdot y\cdot z}{x^3+y^3+z^3}$ ifodaning son qiymatini toping.

3) $\frac{x+y}{x-y}=\frac{3}{2}$ bo'lsa, $\frac{y}{x}$ ni toping.

35.* 1) a, b, c moddalardan $\frac{b}{a}=\frac{2}{3}, \frac{c}{b}=\frac{5}{8}$ proporsiyada qo'shilgan 50 gramm aralashmada c moddasi necha gramm bor?

2) $ax=by=cz=6$ va $x+y+z=36$ bo'lsa, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ ifodaning son qiymatini toping.

3) a, b, c – natural sonlar va $2a=5b=10c$ bo'lsa, $a+b+c$ yig'indi 316 ga teng bo'la oladimi? 312 ga-chi?

4) $\frac{1}{x}=\frac{1}{y}+\frac{1}{z}$ ifodada y va z 20 % ga kamaytirilsa, x necha foiz kamayadi?

36. *Al-Xorazmiy tenglamalarini yeching:*

1) $110-x+\frac{1}{3}\cdot(20+x)-x=4x$;

2) $300-x+\frac{12}{33}\cdot(100-10-x)-20=2x$;

3) $500-x+100-\frac{x}{5}-\frac{3}{4}x=2\cdot(100+x+\frac{3}{4}x)$;

4) $300-x-\frac{x}{3}+100-\frac{x}{3}-x-\frac{x}{3}=4\cdot(x+\frac{x}{3})$.

37. Tenglamani yeching:

1) $\left((x-6):\frac{3}{16}\right):\left(0{,}3\cdot 3\frac{1}{3}+7\right)=1$; 2) $\left(3{,}4:\left(3\frac{1}{3}\cdot x-7{,}5\right)\right):\frac{16}{25}=3\frac{1}{8}$;

3) $\frac{0{,}1(6)+0{,}(6)}{0{,}(3)+1{,}1(6)}\cdot(x-1)=0{,}3(8)x$; 4) $x:2{,}0(6)=0{,}(27):0{,}4(09)$.

38. Tenglamani yeching:

1) $\frac{x}{3}+\frac{x}{15}+\frac{x}{35}+\frac{x}{63}+\frac{x}{99}+\frac{x}{143}=12$; 2) $\frac{x}{4}+\frac{x}{12}+\frac{x}{20}+\frac{x}{30}+\frac{x}{42}+\frac{x}{56}+\frac{x}{72}=34$.

39. a ning qanday qiymatlarida tenglama yechimga ega emas:

1) $ax+1=a$; 2) $ax=2x+3$; 3) $ax-3=a+2x$; 4) $ax+5=a-2x$?

40.* a ning qanday qiymatlarida tenglama cheksiz ko'p yechimga ega:

1) $ax-a=x-1$; 2) $ax+1=a+x$;

3) $10(ax-1)=2a-5x-9$; 4) $ax-7\frac{1}{3}\cdot\frac{9}{22}=x-8\frac{1}{9}\cdot\frac{27}{73}$?

3-§. Masalalarni tenglamalar yordamida yechish

Masalalarni tenglamalar yordamida yechish ikki bosqichdan iborat:
1) masala shartiga mos tenglama tuzish;
2) tuzilgan tenglamani yechish.

Masala. Olmalarni har bir qutiga 12,5 kg dan solish mo'ljallangan edi. Bu qutilar o'rniga har biriga 15,5 kg olma sig'adigan boshqa qutilar olindi. Shunda mo'ljaldagidan 12 ta kam quti talab qilindi. Qutilarga hammasi bo'lib necha kilogramm olma joylangan?

□ ***1-usul. 1-bosqich:*** 1) Qutilarga jami x kg olma joylashgan bo'lsin;

2) olmalar kichik qutilarga joylanganda, $\frac{x}{12{,}5}$ dona quti kerak bo'lardi;

3) olmalar katta qutilarga joylanganda, $\frac{x}{15{,}5}$ dona quti kerak bo'ladi;

4) $\frac{x}{12{,}5}-\frac{x}{15{,}5}$ – qutilar soni orasidagi farq;

5) masala shartiga ko'ra, bu farq 12 ga teng:

$$\frac{x}{12{,}5}-\frac{x}{15{,}5}=12.$$

Mana shu tenglama masala mazmuniga mos tenglamadir, u masala shartlarini o'zida aks ettiradi.

2-bosqichda tenglamani yechamiz: $15{,}5x-12{,}5x=12\cdot12{,}5\cdot15{,}5$;
$3x=12\cdot12{,}5\cdot15{,}5$; $x=(4\cdot12{,}5)\cdot15{,}5$; $x=50\cdot15{,}5=5\cdot155=775$ (kg).
Javob: 775 kg.

2-usul. 1-bosqich:
1) x – katta qutilar soni bo'lsin;
2) jami olmalar $15{,}5x$ (kg) bo'ladi;
3) $(x+12)$ – kichik qutilar soni;
4) $12{,}5\cdot(x+12)$ (kg) – jami olmalar;
5) masala mazmunini aks ettiruvchi tenglama:
$15{,}5x=12{,}5\cdot(x+12)$.

2-bosqich: tenglamani yechamiz:
$15{,}5x=12{,}5\cdot x+12{,}5\cdot12$; $15{,}5x-12{,}5x=150$; $3x=150$; $x=50$ (ta).
Katta qutilar soni 50 ta ekan. Har bir katta qutiga 15,5 kilogrammdan olma joylashgan, demak, jami olmalar $15{,}5\cdot50=775$ (kg).
Javob: 775 kg.

3-usul. 1-bosqich:
1) x – kichik qutilar soni bo'lsin;
2) $(x-12)$ – katta qutilar soni bo'ladi;
3) jami olmalar $12{,}5\,x$ (kg) dan iborat;
4) ikkinchi tomondan, jami olmalar $15{,}5\,(x-12)$ (kg);
5) masala mazmuniga mos tenglama:
$15{,}5\,(x-12)=12{,}5\cdot x$.

2-bosqichda tenglamani yechamiz:
$15{,}5\cdot x-15{,}5\cdot12=12{,}5\cdot x$; $15{,}5\cdot x-12{,}5\,x=15{,}5\cdot12$; $3x=15{,}5\cdot12$;
$x=15{,}5\cdot4$; $x=62$ (ta).
Kichik qutilar soni 62 dona ekan. Har bir kichik qutiga 12,5 kilogrammdan olma joylashgan, demak, jami olmalar $12{,}5\cdot62=775$ (kg).
Javob: 775 kg.

4-usul (arifmetik usul).
1) 12 ta kichik qutiga necha kilogramm olma joylanadi?
$12{,}5\cdot12=150$ (kg);
2) katta va kichik qutilar sig'imi orasidagi farq necha kilogramm?
$15{,}5-12{,}5=3$ (kg);
3) Har bir katta qutiga 12,5 kg dan olma solinsa, 150 kg olma ortib qolar ekan. Har bir katta qutiga yana 3 kg dan olma solinishi kerak. Unda 150 kg olmani joylash uchun $150:3=50$ (ta) katta quti kerak bo'ladi. Demak, katta qutilar soni 50 ta ekan. U holda jami olmalar 775 (kg) ni tashkil qiladi.
Javob: 775 kg.
Javobning to'g'riligini masala shartidan foydalanib tekshirish mumkin.

1) Har bir qutiga 12,5 kilogrammdan olma joylansa, 775 olmani joylash uchun nechta quti kerak?
775 : 12,5 = 62 (ta);
2) Har bir qutiga 15,5 kilogrammdan olma joylansa, 775 kg olmani joylash uchun nechta quti kerak?
775 : 15,5 = 50 (ta);
3) Olmalar faqat katta qutilarga joylansa, nechta quti kam talab etiladi?
62 − 50 = 12 (ta), ya'ni masala shartidagi 12 soni hosil bo'ldi, bu esa masalada izlanayotgan miqdor − jami olmalar necha kilogrammligi to'g'ri topilganini bildiradi. ■

?

41. 1) Masalalarni tenglamalar yordamida yechish nechta bosqichdan iborat?
2) Shu bosqichlarni ayting.
3) Ikkita qopda 90 kg shakar bor. Birinchi qopdagi shakarning 20 % i ikkinchi qopdagi shakarning 25 % iga teng. Har bir qopda necha kilogrammdan shakar bor?
Masalani: a) tenglama tuzib; b) arifmetik usulda yeching. Yechish bosqichlarini yozing.
Masalalarni tenglama tuzib yeching (**42–57**):

42. Ma'lum bir miqdor pul 5 kishiga bo'lib berilganida har bir kishiga a so'mdan tegadi. Xuddi shu pul 8 kishiga bo'lib berilsa, kishi boshiga tushgan pul qanchaga kamayadi?

43. Fotima pulining 0,2 qismini Zuhraga berganida ularning pullari teng bo'ladi. Zuhra pulining $\frac{1}{3}$ qismini Fotimaga bersa, Fotimaning jami puli 6 000 so'm bo'ladi. Zuhraning puli necha so'm?

44. Bir kilogrammining narxi 3 000 so'm va 4 200 so'm bo'lgan ikki xil mahsulot aralashtirilib, 1 kilogrammi 3 300 so'm bo'lgan 32 kg mahsulot hosil qilindi. Har bir mahsulotdan necha kilogrammdan olingan?

45.* Uchta idish bor: A, B, C. A idishda biror moddaning 8 l 20 % li eritmasi bor. O'sha moddaning B idishda 4 l 40 % li, C idishda esa 4 l 60 % li eritmasi bor. A idishdagi eritmaning yarmi B ga solindi. B idishdagi aralashmaning yarmi C ga quyildi. C idishda endi moddaning necha foizli eritmasi bor?

46.* Silindr shaklidagi katta idish (sisterna)ni III quvur yopiq bo'lganda I quvur 12 soatda, II quvur 18 soatda to'ldiradi. Idish balandligining yarmida o'rnatilgan III quvur to'la idishning yarmini 36 soatda bo'shatadi. Uchala quvur baravar ochib qo'yilsa, idish necha soatda to'ladi?

47.* I va II quvurlar baravar ochilib, III quvur yopilsa, sisterna 36 soatda to'ladi. Sisterna balandligining (tubidan hisoblaganda) $\frac{1}{3}$ qismida o'rnatilgan III

quvur to'la sisternaning $\frac{1}{3}$ qismini 18 soatda bo'shatadi. Uchala quvur bir vaqtda ochilsa, idish necha soatda to'ladi?

48. Idishda ma'lum miqdor suv bor. Agar idishga *a l* suv quyilsa, idishning $\frac{1}{4}$ qismi to'ladi. Agar idishdagi dastlabki suvdan *a l* olib tashlansa, idishning $\frac{3}{20}$ qismi to'la bo'ladi. Dastlab idishning qancha qismi to'la bo'lgan?

49. Koptok tashlangan balandligining $\frac{3}{5}$ qismiga qadar sakraydi. Bu koptokning 2 marta sakragandan keyingi balandligi 27 sm bo'lsa, koptok necha santimetr balandlikdan tashlangan?

50. 1 kg i 200 so'mdan olingan uzumning 3 kg idan 1 kg sharbat olinib, 720 so'mga sotildi. Uzumning narxi 50 so'mga arzonlashsa, avvalgi foydani saqlab qolish uchun sharbatning yangi narxi dastlabkisidan necha so'm arzon bo'lishi kerak?

51.* Tijoratchi donasini 1 550 so'mdan olgan buyumlar $\frac{1}{3}$ qismining donasini 1 500 so'mdan, $\frac{1}{4}$ qismining donasini 2 000 so'mdan, $\frac{1}{6}$ qismining donasini 1 800 so'mdan va qolganining donasini 1 200 so'mdan sotib, 30 000 so'm foyda qildi. Necha dona buyum sotilgan?

52. 80 yoshli bir ota o'g'lining yoshida bo'lganida, o'g'lining yoshi otasi yoshining $\frac{2}{9}$ qismiga teng edi. O'g'li hozir necha yoshda?

53. 36 yoshdagi otaning yoshi 3 nafar farzandining yoshlari yig'indisidan 3 marta ortiq. Necha yildan so'ng otaning yoshi farzandlari yoshlarining yig'indisiga teng bo'ladi?

54. 5 yil oldin otaning yoshi o'g'li yoshidan 7 marta ortiq edi. Otaning hozirgi yoshidan 5 yil keyin otaning yoshi o'g'li yoshidan 3 marta ortiq bo'ladi. Ota hozir necha yoshda?

55.* 1) Ota 40 yoshda, o'g'li 12 yoshda. Necha yil avval otaning yoshi o'g'li yoshidan 5 marta katta bo'lgan?

2) Ona qizidan 39 yosh katta. 7 yildan so'ng onaning yoshi qizi yoshidan 4 marta katta bo'ladi. Ona hozir necha yoshda? Qizi-chi?

56. Mahsulotning 20 % i 20 % foyda bilan, qolganining 25 % i 5 % foyda bilan, undan qolganining $\frac{2}{3}$ qismi 10 % zarar bilan sotildi. Ortib qolgan mahsulot necha foiz foyda bilan sotilsa, o'rtacha foyda 15 % bo'ladi?

57. Idishning $\frac{1}{3}$ qismi suv bilan to'la. Bu suvning $\frac{1}{4}$ qismi ishlatilganidan keyin unga 45 litr suv solinsa, idishning $\frac{1}{8}$ qismi bo'sh bo'ladi. Idishga jami necha litr suv sig'adi?

Masalalar yechish

58. Sinovda 60 ta savol berildi, har bir to'g'ri javob 5 ballga baholandi. 4 ta noto'g'ri javob uchun jarima sifatida bitta to'g'ri javob bekor qilindi. Bu sinovda hamma savollarni belgilagan bir o'quvchi 225 ball olgan bo'lsa, u nechta savolga to'g'ri javob bergan?

59. Arava oldingi g'ildiragining aylanasi uzunligi keyingi g'ildiragi aylanasidan 0,5 m qisqa. 45 m masofada oldingi g'ildirak necha marta aylansa, 54 m masofada keyingi g'ildirak shuncha marta aylanadi. Har bir g'ildirak aylanasi uzunligini toping.

60. Aravaning oldingi g'ildiragi aylanasi uzunligi 1,5 *m* ga, keyingi g'ildiragi aylanasi uzunligi esa 2 *m* ga teng. Qanday masofada oldingi g'ildirak keyingisiga qaraganda 50 ta ko'p aylanadi?

61. 1) Bir son ikkinchisidan 7 taga ko'p. Agar sonlardan kichigini 4 ga, kattasini 5 ga bo'lsak, birinchi bo'linma ikkinchisidan 2 taga ortiq bo'ladi. Shu sonlarni toping.

2) Ikki sonning ayirmasi 68 ga teng. Agar sonlardan kattasini kichigiga bo'lsak, to'liqsiz bo'linma 5 ga va qoldiq 8 ga teng bo'ladi. Shu sonlarni toping.

62.* Mis va ruxdan iborat qotishmaning og'irligi 24 N bo'lib, u suvga cho'ktirilganda o'z og'irligining $2\frac{8}{9}N$ ini yo'qotadi. Mis suvda o'z og'irligining $11\frac{1}{9}$ % ini, rux esa $14\frac{2}{7}$ % ini yo'qotishi ma'lum bo'lsa, qotishmada mis va rux og'irliklari qanchadan?

63. 1 kilogrammining narxi 6 000 so'm va 8 400 so'm bo'lgan ikki xil mahsulot aralashtirilib, 1 kilogrammi 6 600 so'm bo'lgan 32 kg mahsulot hosil qilindi. Har bir mahsulotdan necha kilogrammdan olingan?

64.* Birinchi soatda sayyohlar 3,5 km/soat tezlik bilan yurishdi. Hisoblab ko'rishsa, bunday tezlik bilan yurilganda manzilga mo'ljaldan 1 soat kech borilar ekan. Ular tezlikni 1,5 km/soat ga oshirishdi va manzilga mo'ljaldan yarim soat oldin yetib kelishdi. Sayyohlar necha kilometr yo'l yurishgan?

65. Ikki qishloq orasidagi masofa 9 km. Yo'l qiyalik va tekislikdan iborat. Piyoda qiyalikdan tepaga 4 km/soat tezlik bilan ko'tarildi, tekis yo'lda 5 km/soat tezlik bilan yurdi, qiyalikdan pastga esa 6 km/soat tezlik bilan tushdi. Piyoda bir qishloqdan ikkinchisiga borish va kelishga 3 soat-u 41 minut sarflagan bo'lsa, yo'lning tekis qismi necha kilometrni tashkil qiladi?

66. Poyezd 40 km/soat tezlik bilan ketyapti. Haydovchi ro'paradan kelayotgan va uzunligi 75 m bo'lgan poyezd uning poyezdi yonidan 3 sekundda o'tib ketganini aniqladi. Ro'paradan kelayotgan poyezdning tezligi qancha bo'lgan?

67. Mashina a km yo'lni v km/soat tezlik bilan 1 soatda bosib o'tdi. U tezlikni 2 barobar oshirib, o'sha yo'lning uchdan birini necha soatda bosib o'tadi?

68. Sirojiddin buvisinikiga borish uchun avtobus, yengil mashina va poyezddan foydalanadi. U uyidan vokzalgacha bo'lgan s masofani avtobusda bosib o'tdi. Uning poyezdda yurgan yo'li bu masofaning 100 baravarini tashkil qiladi. U poyezddan tushgach, s masofaning 4 baravarini yengil mashinada bosib o'tdi. Sirojiddinning uyidan buvisinikigacha bo'lgan masofa 1 050 km bo'lsa, uning: a) yengil mashinada; b) poyezdda yurgan yo'li jami masofaning qancha qismini tashkil qilishini toping.

69. Ali bilan Valining pullari nisbati $\frac{8}{15}$. Alining pullari 25 % ga ko'payib, Valiniki 25 % kamaysa, pullarning nisbati qanday bo'lib qoladi?

70. Sotuvchi molining 20 % ini 40 % foyda bilan sotdi. Jami sotuvdan 32 % foyda ko'rish uchun u qolgan molini necha foiz foyda bilan sotishi kerak?

71. Bir-biriga tegib turadigan tishli uchta g'ildirakdan birinchisi 5 marta aylanganida, ikkinchi g'ildirak 6 marta, uchinchisi 10 marta aylanadi. Hamma tishlar soni 56 ta bo'lsa, kichik g'ildirakning tishi nechta?

72. Uchburchakning bir tomoni ikkinchisidan 4 sm uzun, uchinchisidan esa 3 sm qisqa. Agar uchburchakning perimetri 50 sm bo'lsa, uning tomonlari uzunliklarini toping.

73. Uchburchakning bir tomoni ikkinchisining 60 % ini tashkil qiladi, uchinchisidan esa 4 sm qisqa. Agar uchburchakning perimetri 48 sm bo'lsa, uning tomonlari uzunliklarini toping.

74. Ikki shahar orasidagi masofani yo'lovchi poyezdi yuk poyezdiga nisbatan t soat tez bosib o'tadi. Agar yo'lovchi poyezdining tezligi v_1 km/soat, yuk poyezdiniki v_2 km/soat, $v_1 > v_2$ bo'lsa, shaharlar orasidagi masofani toping.

75. Kichik korxona "Sog'lom bola yili"da bolalar aravachasi ishlab chiqarish bo'yicha buyurtmani 20 kunda bajarishi kerak edi. Korxona har kuni rejadan tashqari 3 ta aravacha ishlab chiqarib, buyurtmani muddatidan 3 kun avval bajaribgina qolmay, rejadan ortiq yana 9 ta aravacha ishlab chiqardi. Korxona reja bo'yicha nechta aravacha ishlab chiqarishi kerak edi?

76. Beshik ustasi Ahmad ota boshliq korxona buyurtmani n kunda bajarishi kerak edi. U har kuni rejadan tashqari k donadan beshik tayyorladi va m kunda topshiriqni bajarib, bundan tashqari yana d ta beshik tayyorladi. Korxona bir kunda nechta beshik tayyorlashi kerak edi?

77. Birinchi qopda a kg, ikkinchi qopda b kg guruch bor edi. Ikkinchi qopdan birinchi qopdan olinganiga qaraganda n marta ko'p guruch olishdi. Natijada birinchi qopda ikkinchisidagiga qaraganda k marta ko'p guruch qoldi. Har bir qopdan necha kilogrammdan guruch olishgan?

76- va **77-**masalalardagi javoblarni tahlil qiling. Harflarning qanday qiymatlarida masala ma'noga ega?

4-§. Masala yechishning turli usullari

Masala yechishning uchta usuli bilan tanishamiz.

1. ***Algebraik usul.*** Bu usulda masalada topilishi talab etilgan noma'lum miqdor biror harf bilan belgilanadi. Masala mazmunidan kelib chiqib, bu noma'lum va masalada berilgan miqdorlar orasidagi bog'lanish o'rnatiladi. Bu bog'lanish masala shartlariga mos keladigan tenglamadir. Tuzilgan tenglama yechiladi va bu yechim, zarur bo'lsa, masala shartini qanoatlantirishi tekshiriladi. Demak, masalani algebraik usulda yechish shu masalaga mos tenglamani yechishga keltiriladi. Algebraik usulning afzalligi shundaki, bunda mazmun jihatidan turlicha bo'lgan masalalar bitta tenglamaga keltirilishi mumkin.
2. ***Arifmetik usul.*** Bu usulda tenglama tuzilmaydi. Har bir masalaga o'ziga xos yondoshiladi, masalada berilgan miqdorlar orasidagi bog'lanish mulohazalar yordamida, yo'naltiruvchi savollar berish yo'li bilan o'rnatiladi.
3. Avvalgi paragrafda bitta masalani algebraik (tenglama tuzib) va arifmetik usulda yechgan edik. Masala yechishning e'tiborga loyiq yana bir usuli bor, bu usul ham qadimdan ma'lum. Uni **"masalada keltirilgan amallar tartibini va amallarning o'zini ham teskarisiga o'zgartirish"** usuli deyish mumkin.

1-masala. Alijon bir qop kartoshkani bozorga olib keldi. Agar u 1 kg kartoshkani 1 650 so'mdan sotsa, 8 140 so'm zarar ko'radi. Agar 1 kilogrammini 1980 so'mdan sotsa, 16280 so'm foyda ko'radi. Qopda necha kilogramm kartoshka bor?

□ ***Algebraik usul.***

Qopda x kg kartoshka bor, deylik. U holda $1\ 650 \cdot x$ – kartoshkaning 1 kilogrammi 1 650 so'mdan sotilganda tushadigan pul;

$1\ 980 \cdot x$ – kartoshkaning 1 kilogrammi 1 980 so'mdan sotilganda tushadigan pul.

Bir qop kartoshkaning narxi $1\ 650 \cdot x + 8\ 140$ so'm bo'lsa, Alijon zarar ko'rmaydi. $1\ 980 \cdot x - 16\ 280$ so'm bo'lsa, u foyda ko'rmaydi.

Masala shartiga muvofiq, $1\,980 \cdot x - 16\,280 = 1\,650 \cdot x + 8\,140$ tenglamani hosil qilamiz. Bundan $1\,980 \cdot x - 1\,650 \cdot x = 16\,280 + 8\,140$; $330 \cdot x = = 24\,420$; $x = 24\,420 : 330$; $x = 74$ (kg).

Javob: Qopda 74 kg kartoshka bor. ■

□ ***Arifmetik usul.***

1-savol. Qimmatga sotilgan 1 kg kartoshka bilan arzonga sotilgan 1 kg kartoshkaning narxida qancha farq bor? $1980 - 1650 = 330$ (so'm).

2-savol. Olinadigan foyda va ko'riladigan zarar orasidagi farq necha so'm?

$16\,280 - (-8\,140) = 16\,280 + 8\,140 = 24\,420$ (so'm).

3-savol. Foyda – zarar orasidagi bu farqlar necha kilogramm kartoshka sotilganda hosil bo'ladi?

$24\,420 : 330 = 74$ (kg). ■

2-masala. Men bir son o'yladim. Uning 3 baravariga 10 ni qo'shdim va natijani 10 ga bo'ldim. Bo'linmaning $\frac{4}{5}$ qismiga 3 ni qo'shgandim, 11 hosil bo'ldi. Men o'ylagan sonni toping.

□ *1-usul.* (***Algebraik usul***). O'ylangan sonni x deb belgilaymiz. O'ylangan son x ustida bajarilgan amallarni masalada aytilish tartibida yozib chiqaylik:

1-ish – o'ylangan sonning uch baravari: $3x$;

2-ish – natijaga 10 ni qo'shish: $3x + 10$;

3-ish – yig'indini 10 ga bo'lish: $(3x + 10) : 10$;

4-ish – bo'linmaning $\frac{4}{5}$ qismini topish: $((3x + 10) : 10) \cdot \frac{4}{5}$;

5-ish – natijaga 3 ni qo'shish: $((3x + 10) : 10) \cdot \frac{4}{5} + 3$.

Masala shartiga ko'ra shu 5 ta "ish" natijasi 11 ga teng: $((3x + 10) : 10) \cdot \frac{4}{5} + 3 = 11$.

Masalaga mos chiziqli tenglama hosil qildik. Uni o'zingiz mustaqil yeching, shunda o'ylangan son $x = 30$ ekanini topasiz.

Javob: o'ylangan son 30 ga teng. ■

2-usul. Bu usulda masalani yechish, unda aytilgan oxirgi amalga teskari amaldan boshlanadi. Masalada oxirgi 5-ishda bajariladigan amal qo'shish amalidir, buning natijasida mos ifodaga 3 ni qo'shib, 11 hosil bo'lgani aytilgan. Qo'shish amaliga teskari amal ayirish bo'lgani uchun biz 11 dan 3 ni ayiramiz va bu masalani hal etishdagi 1-ish bo'ladi:

1-ish: $11 - 3 = 8$.

2-ish: Masaladagi 4-ish $\frac{4}{5}$ ga ko'paytirish edi: Biz 8 ni $\frac{4}{5}$ ga bo'lamiz: $8 : \frac{4}{5} = 8 \cdot \frac{5}{4} = 10$.

3-ish: Masalada 3-ish 10 ga bo'lish edi. Biz natijani 10 ga ko'paytiramiz: $10 \cdot 10 = 100$.

4-ish: Masalada 2-ish 10 ni qo'shish edi. Biz avvalgi natijadan 10 ni ayiramiz: $100 - 10 = 90$.

5-ish: Masaladagi 1-ish o'ylangan sonni 3 ga ko'paytirish edi. Biz chiqqan natijani 3 ga bo'lamiz: $90 : 3 = 30$.

Demak, o'ylangan son 30 ekan. Masalaning bu usulda yechilishi chizmada yaqqolroq tasvirlanishi mumkin. Shunday chizmani o'zingiz chizib, bunga ishonch hosil qiling.

(?)

78. 1) Masala yechishning qanday usullarini bilasiz?
2) Algebraik usul, arifmetik usul deganda nimani tushunasiz?
3) Bu usullarning bir-biridan farqi nimada?
4) Ushbu masalani ikki usulda yeching:
Feruza bir son o'yladi. Uni 6 ga ko'paytirib, 9 ga bo'ldi. Natijadan 5 ni ayirdi. Hosil bo'lgan sonning 40 % ini 2 ga bo'lgan edi, 5 chiqdi. Feruza o'ylagan sonni toping.
5) 4-banddagi masala yechilishini chizmada tasvirlang.

Masalalarni turli usullarda yeching **(79–99)**:

G'iyosiddin Jamshid al-Koshiyning "Hisob ilmi kaliti" asaridagi masalalar **(79–80)**:

79. Oltin va durdan yasalgan bezakning og'irligi 3 misqol, bahosi 24 dinor. 1 misqol oltin 5 dinor, 1 misqol dur 15 dinor bo'lsa, bezakda necha misqoldan oltin va dur bor? (1 misqol $\approx 4{,}5$ gramm).

80. O'ylangan sonni 2 ga ko'paytirib, hosil bo'lgan songa 1 qo'shilsa, yig'indini 3 ga ko'paytirib, ko'paytmaga 2 qo'shilsa, so'ng hosil bo'lgan son 4 ga ko'paytirilib, bu ko'paytmaga 3 qo'shilsa, 95 hosil bo'ladi. O'ylangan sonni toping.

81. 1) 20 % li sharbat hosil qilish uchun necha litr qaynagan suvga 200 gramm shakar qo'shish kerak?

2) Idishning $\frac{1}{5}$ qismi bo'sh. Idishdagi suvning qancha qismi qadar suv quyilsa, idish to'ladi?

82. Simning uchidan $\frac{1}{3}$ qismi kesilsa, simning o'rta nuqtasi boshlang'ich holdagidan 4 sm ga o'zgaradi. Dastlab simning uzunligi qancha bo'lgan?

83. Sonning o'ndan biridan 10 ni olib, ayirmaning 0,1 qismini hisoblasak, 10 hosil bo'ldi. Shu sonni toping.

84*. Sotuvchi bir to'p gazlamaning dastlab $\frac{4}{9}$ qismi va 10 metrini, keyin $\frac{1}{3}$ qismidan 8 m kamini va so'ngra qolgan 18 metrini sotdi. Ikkinchi galda necha metr gazlama sotilgan?

85. Rasul avval pulining $\frac{2}{5}$ qismidan 250 so'm ortig'ini, keyin qolganining $\frac{3}{5}$ qismidan 250 so'm kamini ishlatdi. Shunda o'zida 3 750 so'm pul qoldi. Rasulning jami puli dastlab necha so'm bo'lgan?

86*. (*Qadimgi masala.*) Ota katta o'g'liga tangalarining $\frac{1}{13}$ qismini, kichik o'g'liga esa qolgan tangalarining $\frac{1}{17}$ qismini berdi. Shundan so'ng otada 384 ta tanga qoldi. Dastlab unda qancha tanga bo'lgan?

87. (*Qadimgi hind masalasi*). Asalarilarning beshdan biri oq gullarga, uchdan biri qizil gullarga qo'ndi. Ular ayirmasining uch baravari sariq gullardan bol yig'ishmoqda. Faqat bittagina asalari gullar iforidan bahra olib, uchib yuripti. Qani ayting-chi, gulzorda qancha asalari bor edi?

88. Arqonning yarmi va yana $\frac{1}{2}$ dm, keyin qolgan arqonning yarmi va yana $\frac{1}{2}$ dm, so'ngra bundan qolgan arqonning yarmi va yana $\frac{1}{2}$ dm qirqib olindi. Shundan so'ng jami arqondan 12 dm qoldi. Dastlab arqonning uzunligi qancha bo'lgan?

89*. Said otada qo'y va tovuqlar bor. Ular boshlarining umumiy soni 172 ta, oyoqlarining soni esa 434 ta. Said otaning nechta qo'yi, nechta tovug'i bor?

90*. 5 tonna va 10 tonna yuk ko'taradigan 58 ta mashina bilan 465 t yukni tashishdi. Yuk tashishda qancha 5 tonnalik va qancha 10 tonnalik mashinalar band bo'lgan?

91*. Ikki sonning yig'indisi 120 ga teng. Sonlardan birini 3 ga, ikkinchisini 2 ga ko'paytirib, natijalarni qo'shsak, 293 hosil bo'ladi. Shu sonlarni toping.

92. Ikki sonning ayirmasi 150 ga teng. Sonlarning kattasini 2 ga, kichigini 3 ga ko'paytirib, natijalarni ayirsak, 201 hosil bo'ladi. Shu sonlarni toping.

93. Tadbirkor bir shahardan ikkinchi shaharga 700 metr gazlama olib keldi. Gazlama ikki xil bo'lib, birinchi xilining 1 metri 17 500 so'm, ikkinchi xilining 1 metri 12 000 so'm turadi. Tadbirkor hamma gazlamalarini sotib, 9 775 000 so'mga ega bo'ldi. Birinchi va ikkinchi xil gazlamalar necha metrdan bo'lgan?

94. Feruzadagi 200 so'mlik va 500 so'mliklarning umumiy soni 863 ta va ular 394 000 so'mni tashkil qiladi. Feruzada nechta 200 so'mlik va nechta 500 so'mlik pul bor?

95. Bitta daftar va bitta ruchka birgalikda 210 so'm turadi. 4 ta daftar va 3 ta ruchka birgalikda 720 so'm turadi. Bitta daftar va bitta ruchkaning narxini toping.

96. Bitta qurut va bitta xo'rozqand birgalikda 55 so'm turadi. 6 ta qurutning puliga 5 ta xo'rozqand sotib olish mumkin. Bitta qurut necha so'm turadi? Bitta xo'rozqand-chi?

97. Sondan uning beshdan biri va yettidan biri ayirilsa, 115 qoladi. Sonning o'zini toping.

98*. Doira 8 ta qism (sektor)ga shunday bo'linganki, har bir sektorning burchagi unga qo'shni bo'lgan har bir sektor burchagidan 10° ga farq qiladi. Sektorning burchaklarini toping. (Masala 8 xil yechimga ega. Shu yechimlarni toping). Mos shakllar chizing.

99*. Savdogar bir to'p matoni 9 ta xaridorga sotdi. 1-xaridorga matoning $\frac{1}{9}$ qismini, 2-xaridorga qolgan matoning $\frac{1}{8}$ qismini, 3-xaridorga 2-sidan qolgan matoning $\frac{1}{7}$ qismini, 4-xaridorga avvalgisidan qolgan matoning $\frac{1}{6}$ qismini, 5-xaridorga 4-sidan qolgan matoning $\frac{1}{5}$ qismini, 6-xaridorga 5-sidan qolgan matoning $\frac{1}{4}$ qismini, 7-xaridorga 6-sidan qolgan matoning $\frac{1}{3}$ qismini, 8-xaridorga 7-sidan qolgan matoning $\frac{1}{2}$ qismini sotdi. 9-xaridorga esa oxirida qolgan 40 m matoni sotdi. To'pda necha metr mato bo'lgan? Har bir xaridor necha metrdan mato olgan?

100. Don quritilgach, uning namligi p % dan q % ga tushdi ($p>q$). Donning massasi necha foizga kamaygan?

□ Donning dastlabki massasini a kg desak, undagi quruq modda miqdori $a-\frac{a\cdot p}{100}=\frac{a(100-p)}{100}$ kg bo'ladi. Bu esa quritilgan don massasining $(100-q)$ % i ga teng. U holda donning quritilgandan keyingi massasi $\frac{a(100-p)}{100}:\frac{100-q}{100}=\frac{a(100-p)}{100-q}$ kilogrammga teng bo'ladi. Demak, donning dastlabki massasi $a-\frac{a\cdot(100-p)}{100-q}=\frac{a(p-q)}{100-q}$ kilogrammga kamayadi. Bu esa dastlabki don massasining $\frac{(p-q)}{100-q}\cdot 100\,\%$ ini tashkil qiladi. ■

101. Donning namligi 24 % edi. U quritilgach, namligi 14 % bo'ldi. Donning massasi necha foizga kamaydi?

102*. Yangi uzilgan bodringdagi suv miqdori 99 % ga teng. Ma'lum muddat o'tgach, bodringlarning nami qochdi va hozir undagi suv miqdori 98 % ni tashkil etadi. 1) Avval 1 tonna bo'lgan bodring hozir necha kilogramm bo'lib qoldi? 2) Hozir 300 kg bo'lgan bodring uzilganida necha kilogramm bo'lgan?

103*. (*Qadimgi masala.*) Kulol 25 ta ko'zani bozorga olib borish uchun arava yolladi. Bozorga olib kelingan har bir butun ko'za uchun kulol aravakashga 15 tanga beradi. Sindirib qo'yilgan har bir ko'za uchun aravakash kulolga 50 tanga to'laydi. Yo'lda bir necha ko'za sinib qoldi va kulol aravakashga 18 tanga berdi. Nechta ko'za bozorga sinmay kelgan?

Masalalar yechish

1-masala. Mashina v_1 km/soat tezlik bilan t_1 soat, v_2 km/soat tezlik bilan t_2 soat yurdi. Mashina jami necha kilometr yo'l yurdi? Bu masofani o'tishda uning o'rtacha tezligi qanday bo'lgan?

□ Yo'l formulasi $s = v \cdot t$ ga asosan:

1) mashina t_1 soatda $s_1 = v_1 \cdot t_1$ (km) yo'lni;

2) t_2 soatda $s_2 = v_2 \cdot t_2$ (km) yo'lni bosib o'tdi. $t_1 + t_2$ soatda mashina jami $(s_1 + s_2)$ (km) yo'l yurgan. Uning o'rtacha tezligi $\frac{s_1 + s_2}{t_1 + t_2} = \frac{v_1 \cdot t_1 + v_2 \cdot t_2}{t_1 + t_2}$ (km/soat) bo'lgan.

Javob: $v_1 \cdot t_1 + v_2 \cdot t_2$; $\frac{v_1 \cdot t_1 + v_2 \cdot t_2}{t_1 + t_2}$. ■

2-masala. Fermer xo'jaligi maydoni a ga bo'lgan 1-dalaning har gektaridan k tonna, maydoni b ga bo'lgan 2-dalaning har gektaridan m tonna, maydoni c gektar bo'lgan 3-dalaning har gektaridan n tonna g'alla hosili oldi. Xo'jalik bir gektar yerdan o'rtacha qancha hosil oldi?

□ Savollar berib, ularga javob topamiz:

1) Birinchi daladan qancha hosil olingan? $a \cdot k$ (tonna);

2) Ikkinchi daladan qancha hosil olingan? $b \cdot m$ (tonna);

3) Uchinchi daladan qancha hosil olingan? $c \cdot n$ (tonna);

4) Uchala daladan necha tonna hosil olingan? $(a \cdot k + b \cdot m + c \cdot n)$ (tonna);

5) Uchala dala maydoni necha gektar? $(a + b + c)$ (ga);

6) Xo'jalik 1 ga yerdan o'rtacha qancha hosil olgan?

$(a \cdot k + b \cdot m + c \cdot n) : (k + m + n)$ (tonna);

Javob: $\frac{(a \cdot k + b \cdot m + c \cdot n)}{k + m + n}$ tonna. ■

Masalalarni yechish jarayonida topilgan $\frac{v_1 \cdot t_1 + v_2 \cdot t_2}{t_1 + t_2}$, $\frac{(a \cdot k + b \cdot m + c \cdot n)}{k + m + n}$ kabi miqdorlar *o'rta vaznli* qiymat deyiladi. Xususan, $k = m = n = 1$ bo'lsa, sizlarga 5–6 sinflardan ma'lum bo'lgan o'rta qiymat $\frac{a+b+c}{3}$ ni hosil qilamiz.

3-masala. Temperaturasi t_1°C bo'lgan a litr suvga, temperaturasi t_2°C bo'lgan b litr suv qo'shildi. Aralashmaning o'rtacha temperaturasi necha gradus bo'ldi?

Javob: $t_{o'rta} = \left(\frac{a \cdot t_1 + b \cdot t_2}{a + b} \right)$ °C.

Bu yerda ma'lum sonlar (a, b, t_1, t_2) ga ko'ra beshinchi son $t_{o'rta}$ topilyapti. Shu 5 ta miqdorning ixtiyoriy 4 tasi berilganda beshinchisini topish mumkin. Xususan, yuqoridagi 3-masalani: 1) t_1 ni; 2) t_2 ni; 3) a ni; 4) b ni topishga moslab tuzing va uni yeching.

104*. A va B shaharlar orasidagi masofa a km. Poyezd A dan B ga v_1 km/soat tezlik bilan, B dan A ga esa v_2 km/soat tezlik bilan yurdi. Borish va kelishdagi jami yo'lni poyezd o'rtacha qanday tezlik bilan o'tgan?

□ *1-savol:* A dan B ga borish uchun poyezd necha soat sarflagan?

Poyezdning A dan B ga borish uchun ketgan vaqtini t_1 bilan belgilaylik. $s = v \cdot t$ yo'l formulasidan $t = \frac{s}{v}$. Bu formulaga asosan, $t_1 = \frac{a}{v_1}$ (soat).

2-savol: B dan A ga kelish uchun poyezd necha soat vaqt sarflagan?

Poyezdning B dan A ga kelish uchun ketgan vaqtini t_2 bilan belgilaylik. $t = \frac{s}{v}$ formulaga ko'ra, $t_2 = \frac{a}{v_2}$ (soat).

3-savol: Hammasi bo'lib poyezd necha soat yo'l yurgan?

$t_1 + t_2 = \frac{a}{v_1} + \frac{a}{v_2} = \frac{av_1 + av_2}{v_1 \cdot v_2}$ (soat).

4-savol: Poyezd borish va kelishda hammasi bo'lib necha kilometr yo'l yurgan?

$a + a = 2a$ (km).

5-savol: Poyezd jami yo'lni o'rtacha qanday tezlik bilan o'tgan?

$\frac{2a}{t_1 + t_2} = \frac{2a}{\frac{av_1 + av_2}{v_1 \cdot v_2}} = \frac{2v_1v_2}{v_1 + v_2}$ (km/soat). *Javob*: $\frac{2v_1v_2}{v_1 + v_2}$ km/soat. ■

Umuman, a va b musbat sonlar berilsa, ular yordamida tuzilgan ushbu

$\frac{2ab}{a+b}$ *ifoda a va b sonlarning o'rta garmonik qiymati deyiladi.* Bu tushunchaga olib keluvchi masalani yuqorida (104*-masala) ko'rdik.

105. Yuk poyezdi *A* shahardan *B* shaharga 60 km/soat tezlik bilan yurdi. *B* dan *A* ga qaytishda u soatiga 40 km/soat tezlik bilan yurdi. Poyezd borish va kelishdagi jami yo'lni o'rtacha qanday tezlik bilan yurgan?

106. Harflar yordamida *n* ta sonning:
1) o'rta arifmetik qiymati;
2) o'rta vaznli qiymatini hisoblash formulasini yozing.

107*. 1) Temperaturasi 36° bo'lgan 6 litr suvga temperaturasi 15° bo'lgan suv qo'shilganda aralashtirilgan suvning temperaturasi 24° bo'ldi. Necha litr suv qo'shilgan?
2) 40° li 8 litr suvga 10 litr suv qo'shilgach, aralashtirilgan suvning temperaturasi 30° bo'ldi. Qo'shilgan suvning temperaturasi necha gradus ekan?
3) Temperaturasi 25° bo'lgan 10 litr suvga temperaturasi 30° litr bo'lgan 15 litr suv qo'shildi. Aralashtirilgan suvning temperaturasini toping.

108. 1 litr qatiq 2 400 so'm. Ayron qilish uchun 2 litr qatiq 8 litr suv bilan aralashtirildi. Ayronning bir litri qancha turadi?

109. Hind va seylon choylarini aralashtirishdi. Hind choyi jami aralashmaning 30 % ini tashkil qiladi. Agar aralashmaga yana 120 gramm hind choyi qo'shilsa edi, u aralashmaning 45 % ini tashkil qilar edi. Dastlab aralashmada necha gramm hind choyi bo'lgan?

Ko'rsatma: Aralashmaning massasi dastlab *x* gramm bo'lgan deb, $\frac{0,3x+120}{x+120}=0,45$ tenglamaga keling va uni yeching.

110. (*Ulug' rus yozuvchisi Lev Tolstoy masalasi.*) O'roqchilar biri ikkinchisidan ikki marta katta bo'lgan ikki maydondagi pichanni o'rishlari kerak. Ular kunning birinchi yarmida katta maydonda ishladilar. Peshindan keyin o'roqchilar teng ikkiga bo'linib, birinchi yarmi katta maydonda kechgacha ishladi va pichanni o'rib tamomladi. O'roqchilarning ikkinchi yarmi esa kichik maydonga o'tdi, lekin ishni kechgacha tugatolmadi. Qolgan pichanni ertasiga bir o'roqchi kun bo'yi o'rib tugatdi. O'roqchilar necha nafar bo'lgan?

□ ***1-usul*** (mantiqiy mulohaza yuritish usuli). Kichik maydonda qolgan pichanni ertasiga 2 nafar o'roqchi yarim kunda o'rishi mumkin. U holda kichik maydonning hammasini o'roqchilarning yarmi yarim kun (kunning birinchi yarmi) va 2 o'roqchi yarim kunda o'radi. Demak, katta maydonni jami o'roqchilar yarim kun va 4 o'roqchi yarim kunda o'rib bitirishi kerak. Bu xulosani masala sharti bilan solishtirib, o'roqchilar sonining yarmi 4 kishi ekan degan natijaga kelamiz.
Javob: O'roqchilar 8 nafar. ■

***2-usul** (algebraik usul).*

□ O‘roqchilar x nafar bo‘lsin. Katta maydonni x kishi yarim kun, $\frac{x}{2}$ kishi yarim kunda o‘rib tamomlagan. Demak, $x+\frac{x}{2}$ nafar kishi katta maydonni yarim kunda o‘ra oladi. Kichik maydon kattasidan 2 baravar kichik. Shunday qilib, uni $\frac{x+\frac{x}{2}}{2}$ kishi yarim kunda o‘ra oladi. Kichik maydonda $\frac{x}{2}$ o‘roqchi yarim kun va 1 o‘roqchi bir kun ishlagan, ya’ni kichik maydonni $\frac{x}{2}+2$ kishi yarim kunda o‘ra oladi. U holda, masala mazmuniga mos tenglama: $\frac{x+\frac{x}{2}}{2}=\frac{x}{2}+2$ bo‘ladi.

Bundan $x+\frac{x}{2}=x+4$, $x=8$ natijani olamiz.

Javob: O‘roqchilar 8 nafar.■

111. (*Qadimgi masala*).

Qo‘nishmoqchi bo‘lishdi,
Gul shoxiga bulbullar.
Bir shoxga ikki bulbul
Qo‘nib aylasa navo,
Ortib qolar bitta shox,
Bulbulsiz chekar u oh.
Bitta shoxda bir bulbul,
Yolg‘iz aylasa nola.
Bir bulbulga yetmas shox,
Ilg‘ab olgin, ey bola.
Nechta edi bulbullar
Va atirgul shoxlari?
Fikr qilib yecha qol,
Mayli, sinab topa qol!

Rivojlantiruvchi mashqlar*

1. (*Qadimgi masala.*) 21 ta bir xil sig‘imli idishning 7 tasiga to‘ldirib, 7 tasiga yarim qilib yog‘ solingan, 7 tasi esa bo‘sh. Uch nafar tijoratchi yog‘ni va idishlarni teng bo‘lib olishmoqchi. Yog‘ni idishdan idishga quymasdan, qanday qilib yog‘ va xumlarni teng taqsimlash mumkin?

2. 24 ta bir xil sig‘imli xumning 5 tasiga to‘ldirib, 11 tasiga yarim qilib un solingan, 8 tasi esa bo‘sh. Uch kishi unni va xumlarni teng bo‘lib olishmoqchi. Unni idishdan idishga solmasdan, qanday qilib un va xumlarni teng taqsimlash mumkin?

3. Bir kishi bozorga tuxum olib keldi. Birinchi xaridorga u jami tuxumlarning yarmini va yana 1 dona tuxum, ikkinchi xaridorga qolgan tuxumlarning yarmini va yana 1 ta tuxum, uchinchi xaridorga qolgan tuxumlarning yarmini va yana 1 ta tuxum sotdi. Shundan so'ng uning savatida 10 dona tuxum qoldi. Kishi bozorga jami nechta tuxum olib kelgan edi?

4. (*Qadimgi masala.*) Sayyoh bir shaharga kelib karvonsaroyda bir hafta tunamoqchi bo'ldi. Unga har kuni shomda bir tangadan to'lashini aytishdi. Sayyoh karvonsaroy egasiga bu yurt tangalariga ega emasligini, ammo har bir halqasi bir tangalik kumush zanjiri borligini aytib, tanga o'rniga kuniga bittadan halqa olishni taklif qildi. Karvonsaroy sohibi rozi bo'ldi-yu, faqat har kuni bittadan halqa qirqilsa, zanjir buziladi deb, bittadan ortiq halqani qirqmaslikni shart qilib qo'ydi. Agar zanjirda 7 ta halqa bo'lsa, sayyoh bu shartga amal qilib, karvonsaroyda 7 kun tura oladimi?

5. (*Evklid masalasi.*) Ot va eshakka bug'doy yuklashdi. Yo'lda ot eshakka qarab dedi: "Agar sendagi yukning bir qopini menga yuklashganda edi, men sendan ikki marta ko'p bug'doy ko'targan bo'lar edim. Agar mendagi yukning bir qopini senga yuklashganda edi, har ikkimiz bir xil yuk ko'targan bo'lar edik". Ot va eshakning har biriga necha qopdan bug'doy yuklangan?

6. Bir gala kaptar daraxt yoniga uchib kelishdi. Ularning bir qismi daraxt shoxiga, bir qismi daraxt tagiga qo'ndi. Daraxt shoxidagi kaptarlar pastdagi kaptarlarga shunday deyishdi: "Agar sizlardan biringiz bizning yonimizga o'tib qo'nsangiz, biz sizlardan uch marta ko'p bo'lamiz, agar bizdan bir kaptar sizga qo'shilsa, bizning to'da sizning to'dangizga tenglashadi". Daraxt shoxida va daraxt ostida nechtadan kaptar bor?

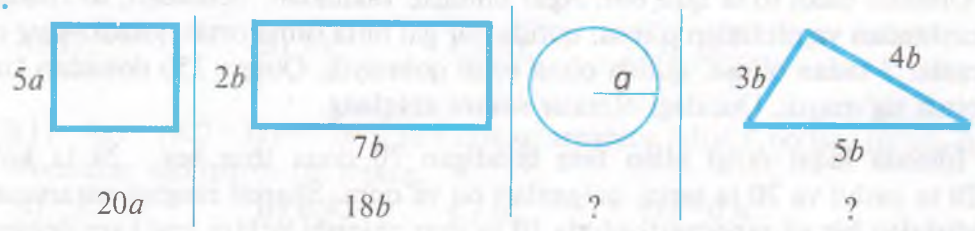
7. Yunon matematigi Pifagordan "Nechta shogirdingiz bor?" deb so'rashganida, u shunday deb javob qaytargan ekan: "Shogirdlarimning yarmi matematika o'rganmoqda, choragi tabiatni o'rganishyapti, yettidan biri xayol surish bilan band, yana uch nafari qizlar". Pifagorning nechta shogirdi bo'lgan?

8. Uchib borayotgan g'ozlar galasiga bir g'oz duch kelib: "Salom, yuz g'oz!" – deb murojaat qiladi. "Bizlar yuzta emasmiz, – deb javob qildi g'ozlar yo'lboshchisi. – Agar bizga hozir nechta bo'lsak, yana shuncha, yana uning yarmicha, yana yarmining yarmicha va sen qo'shilsang, yuzta g'oz bo'lar edik". Galada nechta g'oz bor?

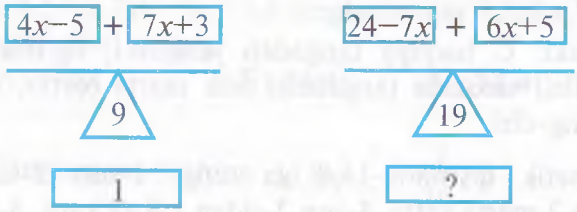
So'roq belgisi o'rniga qanday sonni qo'yish kerak? (9–11):

9. x^3-5x 12 D x^5-3x ? B

10.



11.



12. So‘roq belgisi o‘rnida qanday shakl bo‘lishi kerak **(12–13)**:

$\frac{3x-1}{5}=\frac{5x-7}{3}$ ⊖ $\frac{5x-4}{4}=\frac{9x-8}{7}$?

13.

$\frac{3x-2}{5}=\frac{7x-15,4}{4}$ ⋔ $\frac{6x-11}{5}=\frac{9x-28}{5,2}$?

14. Bir nechta muntazam uchburchakning uchlariga 1, 2, 3 raqamlari ixtiyoriy tartibda yozildi. Uchburchaklarni ularning uchlari ustma-ust tushadigan qilib taxlashdi. Har bir uchdagi raqamlar yig‘indisi 2013 ga teng bo‘lishi mumkinmi? 2016 ga-chi?

15. Bukinistik (eski kitoblar bilan savdo qiluvchi) do‘kon kitobni muqovada ko‘rsatilgan narxdan 35 % arzonga oldi va o‘sha narxdan 25 % arzonga sotdi. Do‘kon necha foiz foyda qildi?

16. Ixtiyoriy natural son n da $\frac{2n+1}{2n\cdot(n+1)}$ kasr qisqarmas ekanini isbotlang.

17. Parallel yo‘llardan ikkita poyezd bir-biriga qarab kelmoqda. Birining tezligi 60 km/soat, ikkinchisiniki 80 km/soat. Ikkinchi poyezdda o‘tirgan yo‘lovchi birinchi poyezd uning yonidan 6 sekundda o‘tib ketganini payqadi. Birinchi poyezdning uzunligini toping.

18. 500 ta qutida olma bor. Har bir qutidagi olmalar soni 240 tadan ko‘p emas. Hech bo‘lmaganda 3 ta qutidagi olmalar soni bir xil (o‘zaro teng) ekanini isbotlang.

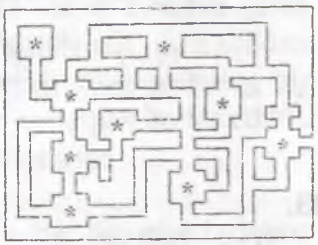
19. *Olmalar bilan to'la quti bor. Agar olmalar: ikkitadan, uchtadan, to'rttadan,* beshtadan va oltitadan olinsa, qutida har gal bitta olma ortib qoladi. Agar olmalar 7 tadan olinsa, qutida olma ortib qolmaydi. Qutiga 350 donadan ko'p olma sig'maydi. Qutidagi olmalar sonini aniqlang.

20. Idishda faqat rangi bilan farq qiladigan 70 dona shar bor: 20 ta ko'k, 20 ta yashil va 20 ta sariq, qolganlari oq va qora. Sharlar rangiga qaramasak, idishdan bir xil rangdagi kamida 10 ta shar chiqishi uchun eng kam deganda nechta dona shar olish kerak?

21. 8 ta tanganing bittasi qalbaki. U haqiqiy tangadan yengilmi, og'irmi – noma'lum. Toshlari yo'q pallali tarozida tangalarni uch marta tortib, ular ichidan qalbakisini topib bering-chi.

22. 5 ta sonning o'rta arifmetik qiymati 13,8 ga teng. 1-son 2-sidan 0,9 ga ortiq, 3-son esa 1-sidan 2 marta katta. 4-son 2-sidan 1,8 ga kam, 5-son esa 3-sidan 1,2 marta kam. Shu sonlarni toping.

23. 9 ta konvertdagi xatni chizmada yulduzcha (*) bilan belgilangan joylarga olib borish kerak. Xat tashuvchi chizmaga qaraboq, ayni bir yo'ldan ikki marta yurmasdan xatlarni belgilangan joylarga yetkazishi mumkinligini aytdi. U qanday yo'l bilan yuradi?



24. To'rt nafar qiz: Barno, Lola, Feruza, Oydinlarning har biri pianino, dutor, skripka, rubob cholg'u asboblaridan faqat bittasini chala oladi. Qizlarning har biri ingliz, fransuz, nemis, koreys tillaridan faqat bittasini biladi. Quyidagilar ma'lum:

1) skripka chaladigan qiz koreys tilini biladi;
2) Lola dutor ham, skripka ham chalmaydi, ingliz tilini bilmaydi,
3) Barno dutor ham, rubob ham chalmaydi, nemis tilini ham ingliz tilini ham bilmaydi;
4) nemis tilida gaplashadigan qiz dutor chalishni bilmaydi;
5) Feruza fransuz tilini biladi, ammo u rubob chalishni bilmaydi.

Kim qaysi cholg'u asbobida chala oladi va qaysi tilni biladi?

II bobga doir sinov mashqlari (testlar)

1. $\frac{17+x}{5}+2=\frac{3x-7}{4}$ tenglamaning ildizi x_0 bo'lsa, x_0^2+31 ifoda son qiymatining 42 % ini toping.

A) 84; B) 94; C) 8,4; D) 73.

2. $\frac{4x+7}{5}-\frac{2-3x}{2}=32+\frac{5x-2}{2}$ tenglamaning ildizi x_0 bo'lsa, $47-x_0$ ifodaning $\frac{1}{5}$ qismi 30 dan qancha ko'p?

A) 17; B) 10; C) 20; D) $30\frac{1}{5}$.

3. $5(11-4x)-10(2-3x)=25+3(5x+2)$ tenglamaning ildizi x_0 bo'lsa, $10\cdot x_0^2-0,5x_0$ ifodaning son qiymatini toping.

A) -6; B) 6,4; C) 6; D) $-0,4$.

4. $2(4,5+2,7x)=9x+5,4$ tenglamaning ildizi x_0 bo'lsa, $21\cdot x_0^2+19$ ifodaning son qiymatini toping.

A) -40; B) 24,4; C) 14,4; D) 40.

5. Tenglamani yeching: $13,8x-\frac{7}{2}x+\frac{7}{10}x=\frac{x}{2}-1$.

A) $-\frac{2}{21}$; B) $\frac{4}{21}$; C) $\frac{13}{20}$; D) $\frac{3}{7}$.

6. Tenglamani yeching: $(3x-7):2=(5x-7):8$.

A) -3; B) 3; C) 14; D) 8.

7. Tenglamani yeching: $8\frac{4}{7}x:2\frac{1}{7}=4x:1$.

A) $\frac{5}{7}$; B) 4; C) 1; D) -1.

8. Tenglamani yeching: $6,5\cdot(3-2x)-8=2\cdot(3-6,5x)$.

A) 0; B) -1; C) cheksiz ko'p yechimi bor; D) ildizga ega emas.

9. Tenglamani yeching: $3,8\cdot(4-x)+4,8+0,8x=5\cdot(4-0,6x)$.

A) cheksiz ko'p yechimga ega; B) ildizga ega emas;
C) 0 va 1; D) to'g'ri javob keltirilmagan.

10. Uchta javonda hammasi bo'lib 425 ta kitob bor. Birinchi javonda ikkinchisiga qaraganda 6 ta ko'p, ammo uchinchisiga qaraganda 8 ta kam kitob bor. Har bir javonda nechtadan kitob bor?

A) 140, 134, 151; B) 141, 135, 149;
C) 142, 133, 150; D) 143, 137, 145.

11. Uchburchakning perimetri 49 sm. Uning birinchi tomoni ikkinchisidan 1 sm uzun, ammo uchinchisidan 2 sm qisqa. Shu uchburchak tomonlarini toping.

A) 15, 18, 19; B) 13, 14, 22; C) 16, 15, 18; D) 15, 16, 19.

12. Ikkala to'pda birgalikda 120 m adras bo'lib, birinchi to'pda ikkinchisiga qaraganda 24 m adras kam edi. Ikkinchi to'pdan birinchisiga qaraganda 2 marta ko'p adras sotildi. Natijada, birinchi to'pda ikkinchisiga qaraganda 1,5 marta ko'p adras qoldi. Har bir to'pdan necha metrdan adras sotilgan?

A) 14 m, 21 m; B) 20 m, 30 m; C) 24 m, 36 m; D) 30 m, 60 m.

13. Teng yonli uchburchakning perimetri 66 sm ga teng. Asosi yon tomonidan 3 sm uzun. Shu uchburchak yon tomonining asosiga nisbatini toping.
A) 0,875; B) 0,8; C) 1:33; D) 3:4.

14. Qayiq oqim bo'yicha 1,2 soat va oqimga qarshi 1,6 soat suzdi. Qayiqning oqim bo'yicha bosib o'tgan yo'li oqimga qarshi bosib o'tgan yo'lidan 6,6 km ortiq bo'ldi. Agar daryo oqimining tezligi 3,5 km/soat bo'lsa, qayiqning turg'un suvdagi tezligini toping.
A) 4,1 km/soat; B) 3,8 km/soat; C) 3,1 km/soat; D) 2,8 km/soat.

15. Eni 100 metr bo'lgan to'g'ri to'rtburchak shaklidagi maydonning chegarasi bo'ylab ariq qazildi. Ariqning uzunligi 500 metr bo'lsa, maydonning yuzi necha gektarga teng?
A) 5 ga; B) 3 ga; C) 1,5 ga; D) 2 ga.

TARIXIY MASALALAR

Quyida keltirilgan 1–11-tenglamalar al-Xorazmiyning ,,Al-jabr val-muqobala hisobi haqida qisqacha kitob" asarining ,,Vasiyatlar kitobi" bobidagi masalalar mazmunini aks ettiruvchi tenglamalardir. Shu tenglamalarni yeching:

1. $\frac{10-x}{x}=4.$

2. $\frac{x}{x+2}=\frac{1}{2}.$

3. $10:6=x:4;$ 2) $10:8=4:x;$ 3) $30:10=6:x.$

4. $\frac{10+x}{3}=x.$

5. 1) $\frac{\frac{4}{5}\cdot(10+x)+1}{3}=x;$ 2) $\frac{\frac{4}{5}\cdot(10+x)-1}{2}=x.$

6. $2\frac{1}{2}+20-x=\frac{2}{3}\cdot 50.$

7. 1) $90-x+\frac{1}{2}(10+x)-x=4-x;$ 2) $90-x+\frac{x}{3}=2x.$

8. $300-x+\frac{x}{2}=2x.$

9. $300-x+\frac{1}{2}[500-(300-x)]+300-x=4x.$

10. $500-x+100-\frac{x}{5}=2(100+x)$.

11. 1) $300-x+100-\frac{x}{3}=2x$; 2) $300-x+\frac{x}{3}+100-\frac{x}{3}=2x$.

TARIXIY MA'LUMOT

Muhammad ibn Muso al-Xorazmiy "Al-jabr val-muqobala hisobi haqida qisqacha kitob" asarida kiritilgan "al-jabr", "val-muqobala" qoidalarini biz tenglamaning asosiy xossalari sifatida bayon qildik, xolos.

"Algebrada uch xil sonlar bilan ish ko'riladi", – deydi al-Xorazmiy. Ular:
– ildiz yoki narsa (tenglamadagi noma'lum son – x);
– kvadrat (mol) (noma'lumning kvadrati – x^2);
– oddiy son (bunda natural son nazarda tutiladi).

Xorazmiy shu uch xil miqdorlar orasidagi turli bog'lanishlarni tahlil qiladi va ushbu ko'rinishdagi tenglamalarni yechish usullarini ko'rsatadi:
1) $cx^2=bx$ — kvadratlar ildizlarga teng;
2) $cx^2=a$ — kvadratlar sonlarga teng;
3) $bx=a$ — ildizlar songa teng;
4) $cx^2+bx=a$ — kvadratlar va ildizlar sonlarga teng;
5) $cx^2+a=bx$ — kvadratlar va son ildizlarga teng;
6) $bx+a=cx^2$ — ildizlar va son kvadratlarga teng.

Biz 7-sinfda faqat chiziqli tenglamalarni o'rganamiz [3) banddagi $bx=a$ tenglama]. Qolganlari 8-sinfda o'rganiladi. Har qanday chiziqli yoki kvadrat tenglama "al-jabr", "val-muqobala" almashtirishlari natijasida yuqoridagi 6 ta tenglamaning biriga keltirilishi mumkin.

Buyuk vatandoshimiz al-Xorazmiy haqida Yurtboshimiz Islom Karimov "Yuksak ma'naviyat – yengilmas kuch" asarida shunday durdona fikr aytgan: "Muhammad ibn Muso Xorazmiyning o'nli sanoq sistemasi, algoritm va algebra tushunchalarini dunyoda birinchi bo'lib ilm-fan sohasiga joriy etgani va shu asosda aniq fanlar rivoji uchun o'z vaqtida mustahkam asos yaratgani umuminsoniy taraqqiyot rivojida qanday katta ahamiyatga ega bo'lganini barchamiz yaxshi bilamiz".[1]

[1] *I. Karimov.* Yuksak ma'naviyat – yengilmas kuch. – T.: Ma'naviyat, 2008. – 41-bet.

III BOB. BIRHADLAR VA KO'PHADLAR

1-§. Natural ko'rsatkichli daraja

Qo'shiluvchilari bir xil sonlardan iborat bo'lgan yig'indini ko'paytma bilan almashtirish mumkinligini bilasiz.

Masalan: $5+5+5+5=5\cdot 4$; umumiy holda $\underbrace{a+a+\ldots+a}_{n\text{ ta}}=a\cdot n$.

Ko'paytuvchilari bir xil sonlar bo'lgan ko'paytmani qisqa yozuv bilan ifodalash mumkin.

Masala: a) tomoni 6 ga teng kvadrat yuzini; b) qirrasi uzunligi 6 ga teng kubning hajmini aniqlang.

□: a) $S=a\cdot a=a^2$ ekanligini e'tiborga olsak, $S=6\cdot 6=6^2=36$.

b) $V=a\cdot a\cdot a=a^3$, bundan $V=6\cdot 6\cdot 6=6^3=216$.

Javob: a) 36 yuz birligi; b) 216 hajm birligi. ■

Bu yerda 6^2 ni "oltining kvadrati" yoki "oltining 2-darajasi", 6^3 ni "oltining kubi" yoki "oltining 3-darajasi" kabi o'qilishi ham sizga ma'lum.

Har biri a ga teng bo'lgan n ta ko'paytuvchining ko'paytmasi a sonining *n-darajasi* deyiladi va $\underbrace{a\cdot a\cdot\ldots\cdot a}_{n\text{ ta}}=a^n,\ n\in N$ kabi yoziladi.

Bu yerda, a^n – natural ko'rsatkichli daraja; a – darajaning asosi; n – daraja ko'rsatkichi.

a^n – yozuv "a ning n-darajasi" kabi o'qiladi.

a^n darajani hisoblash – darajaga ko'tarish amali bo'lib, a^n darajani hisoblashdan hosil bo'lgan songa *darajaning qiymati* deyiladi.

Masalan: $2^7=2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2=128$, bunda 2 – daraja asosi, 7 – daraja ko'rsatkichi; 128 – daraja qiymati.

a sonining 1-darajasi deb shu sonning o'ziga aytiladi: $a^1=a$; $6^1=6$; $\left(\frac{1}{8}\right)^1=\frac{1}{8}$.

Darajaning asosi istalgan son bo'lishi mumkin.

Misollar: 1) $1^5=1\cdot 1\cdot 1\cdot 1\cdot 1=1$;

2) $(-1)^4=(-1)\cdot(-1)\cdot(-1)\cdot(-1)=1$;

3) $(-1)^5=(-1)\cdot(-1)\cdot(-1)\cdot(-1)\cdot(-1)=-1$;

4) $0^7=0\cdot 0\cdot 0\cdot 0\cdot 0\cdot 0\cdot 0=0$;

5) $10^6=10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10=1\ 000\ 000$.

6) $13^2 = 13 \cdot 13 = 169$;

7) $(-5)^4 = (-5) \cdot (-5) \cdot (-5) \cdot (-5) = 625$;

8) $\left(\frac{2}{3}\right)^5 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{32}{243}$;

9) $\left(-\frac{3}{7}\right)^3 = \left(-\frac{3}{7}\right) \cdot \left(-\frac{3}{7}\right) \cdot \left(-\frac{3}{7}\right) = -\frac{27}{343}$;

10) $\left(-\frac{1}{2}\right)^4 = \left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right) = \frac{1}{16}$;

11) $0{,}3^3 = 0{,}3 \cdot 0{,}3 \cdot 0{,}3 = 0{,}027$.

Yuqoridagi misollardan shunday xulosaga kela olamiz:

1) $1^n = \underbrace{1 \cdot 1 \cdot \ldots \cdot 1}_{n \text{ ta}} = 1$; 2) $(-1)^{2k} = 1$; $(-1)^{2k-1} = -1$;

3) $0^n = \underbrace{0 \cdot 0 \cdot \ldots \cdot 0}_{n \text{ ta}} = 0$; 4) $10^n = \underbrace{10 \cdot 10 \cdot \ldots \cdot 10}_{n \text{ ta}} = \underbrace{1000\ldots0}_{n \text{ ta}}$;

5) $(-a)^{2k} = a^{2k}$; $(-a)^{2k-1} = -a^{2k-1}$.

Darajaga ko'tarish amali – uchinchi bosqich amal. Agar ifodada qavslar bo'lmasa, u holda avval uchinchi bosqich amallar, keyin ikkinchi bosqich amallar va, nihoyat, birinchi bosqich amallar bajarilishini eslatib o'tamiz.

1-misol. Hisoblang: $8^1 \cdot 5^3 \cdot (-3)^4$.

□ 1) $8^1 = 8$; 2) $5^3 = 5 \cdot 5 \cdot 5 = 125$; 3) $(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81$;
4) $(8 \cdot 125) \cdot 81 = 1000 \cdot 81 = 81\,000$.

Javob: 81 000. ■

2-misol. $-6^3 + 128 : (-2)^3$ ifodaning son qiymatini toping:

1) $-6^3 = -6 \cdot 6 \cdot 6 = -216$; 2) $(-2)^3 = (-2) \cdot (-2) \cdot (-2) = -8$;
3) $128 : (-8) = -16$; 4) $-216 + (-16) = -232$.

3-misol. Hisoblang: $7 \cdot (-1)^{n+3} + 5$, bu yerda $n \in N$.

□ a) n juft; b) n toq bo'lgan hollarni qaraymiz:

a) n juft, deylik. U holda $(n+3)$ – toq bo'ladi va $7 \cdot (-1)^{n+3} + 5 = 7 \cdot (-1) + 5 = -2$.

b) n toq bo'lsin deylik. U holda $(n+3)$ – juft bo'ladi va $7 \cdot (-1)^{n+3} + 5 = 7 \cdot 1 + 5 = 12$.

Javob: a) -2; b) 12. ■

4-misol. $a = 2$, $b = -1$ bo'lganda $\frac{-5a^5}{b-a}$ ifodaning son qiymatini toping:

$$\frac{-5 \cdot 2^5}{-1-2} = \frac{-5 \cdot 32}{-3} = \frac{160}{3} = 53\frac{1}{3}.$$

Sonlarni daraja yordamida yozishdan juda ko'p hollarda foydalaniladi:

a) natural sonlarni xona birliklari yig'indisi ko'rinishida yozish uchun:
$4\,271 = 4 \cdot 1\,000 + 2 \cdot 100 + 7 \cdot 10 + 1 = 4 \cdot 10^3 + 2 \cdot 10^2 + 7 \cdot 10 + 1$.
b) fizika, kimyo darslarida $3{,}84 \cdot 10^5$; $4{,}17 \cdot 10^8$ kabi sonlarni uchratgansiz.

Sonlarning $a \cdot 10^n$ ko'rinishdagi yozuvi (bunda $1 \le a < 10$, $n \in N$) sonning *standart shakli* deyiladi.

1) Yerdan Quyoshgacha bo'lgan masofa taxminan 150 000 000 km bo'lib, uni $1{,}5 \cdot 10^8$ km kabi yozish mumkin.
2) Yer sharining radiusi taqriban 6,37 mln metr ga teng, u $6{,}37 \cdot 10^6$ m kabi yoziladi.
3) $8\,725 = 8{,}725 \cdot 10^3$; 4) $78{,}15 = 7{,}815 \cdot 10$; 5) $10\,030\,000 = 1{,}003 \cdot 10^7$.

?

1. 1) Sonning natural ko'rsatkichli darajasi deb nimaga aytiladi?
2) 3^4 son qanday yozilishi mumkin? 5^4 yozuv nimani bildiradi?
3) Daraja, darajaning asosi, darajaning ko'rsatkichi, darajaning qiymati tushunchalari nima?
4) Darajaga ko'tarish amali nechanchi bosqich amal?
5) Agar ifodada to'rt amal va darajaga ko'tarish amali qatnashgan bo'lsa, amallar qanday tartibda bajariladi?
6) Sonning birinchi darajasi nimaga teng?
7) Sonning standart shakli deb nimaga aytiladi?
8) Sonning standart shakli qayerlarda ko'p qo'llaniladi?

Yig'indini ko'paytma shaklida yozing **(2–3)**:

2. 1) $8+8+8+8+8$; 2) $9+9+9+9$;
3) $a+a+a+a+a+a$; 4) $b+b+b+b+b+b+b$.

3.
1) $(-3n)+(-3n)+(-3n)$; 2) $(19mn)+(19mn)+(19mn)$; 3) $(2a-3b)+(2a-3b)$;
4) $(4a-c)+(4a-c)+(4a-c)$; 5) $\underbrace{7+7+\ldots+7}_{31\text{ ta}}$; 6) $\underbrace{11+11+\ldots+11}_{43\text{ ta}}$;
7) $\underbrace{k+k+\ldots+k}_{p\text{ ta}}$; 8) $\underbrace{a+a+\ldots+a}_{k\text{ ta}}$; 9) $\underbrace{b+b+\ldots+b}_{n\text{ ta}}$.

Ko'paytmani daraja shaklida yozing **(4–5)**:

4. 1) $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$; 2) $\left(-\frac{1}{7}\right) \cdot \left(-\frac{1}{7}\right) \cdot \left(-\frac{1}{7}\right)$; 3) $\frac{5}{8} \cdot \frac{5}{8} \cdot \frac{5}{8} \cdot \frac{5}{8}$;
4) $(-5{,}9) \cdot (-5{,}9) \cdot (-5{,}9) \cdot (-5{,}9) \cdot (-5{,}9)$; 5) $t \cdot t \cdot t \cdot t \cdot t$;
6) $k \cdot k \cdot k \cdot k \cdot k \cdot k \cdot k$; 7) $(3a) \cdot (3a) \cdot (3a)$;
8) $(-4b) \cdot (-4b) \cdot (-4b) \cdot (-4b)$.

5. 1) $(2x-y)\cdot(2x-y)\cdot(2x-y)$; 2) $(a+3b)\cdot(a+3b)$;

3) $\frac{3a}{4}\cdot\frac{3a}{4}\cdot\frac{3a}{4}$; 4) $\frac{l}{k}\cdot\frac{l}{k}\cdot\frac{l}{k}\cdot\frac{l}{k}\cdot\frac{l}{k}\cdot\frac{l}{k}$.

Ko'paytmaning daraja shaklidagi yozuvidan foydalanib, ifodani soddalashtiring **(6–9)**:

6. 1) $3\cdot3\cdot3\cdot7$; 2) $5\cdot5\cdot5\cdot11$; 3) $4\cdot4\cdot7\cdot7\cdot7\cdot3\cdot3$;

4) $5\cdot5\cdot8\cdot8\cdot3\cdot3\cdot3\cdot3$; 5) $3{,}4\cdot3{,}4\cdot3\cdot3\cdot3\cdot7\cdot7$;

6) $7{,}2\cdot7{,}2\cdot7{,}2\cdot5\cdot5\cdot8\cdot8\cdot8$; 7) $1{,}5\cdot1{,}5\cdot\frac{1}{9}\cdot\frac{1}{9}\cdot\frac{1}{9}\cdot\frac{1}{9}$; 8) $\frac{5}{8}\cdot\frac{5}{8}\cdot5{,}2\cdot5{,}2\cdot5{,}2$.

7. 1) $11\cdot11\cdot11\cdot b\cdot b\cdot b$; 2) $y\cdot y\cdot y\cdot y\cdot6\cdot6$;

3) $\frac{m}{n}\cdot\frac{m}{n}\cdot\frac{m}{n}\cdot(m-n)\cdot(m-n)$; 4) $\frac{p}{q}\cdot\frac{p}{q}\cdot(9p-q)\cdot(9p-q)\cdot(9p-q)$.

8. 1) $a\cdot a\cdot a\cdot a\cdot a+b\cdot b\cdot b$; 2) $x\cdot x\cdot x+y\cdot y\cdot y\cdot y$;

3) $a\cdot a\cdot a+a\cdot a\cdot a+a\cdot a\cdot a$;

4) $k\cdot k\cdot k\cdot k+k\cdot k\cdot k\cdot k+k\cdot k\cdot k\cdot k$.

9. 1) $\underbrace{t\cdot t+t\cdot t+\ldots+t\cdot t}_{n\text{ ta}}$; 3) $\underbrace{x\cdot x\cdot\ldots\cdot x}_{m\text{ ta}}+\underbrace{y\cdot y\cdot\ldots\cdot y}_{m\text{ ta}}$;

2) $\underbrace{b\cdot b\cdot b+b\cdot b\cdot b+\ldots+b\cdot b\cdot b}_{n\text{ ta}}$; 4) $\underbrace{b\cdot b\cdot\ldots\cdot b}_{a\text{ ta}}+\underbrace{a\cdot a\cdot\ldots\cdot a}_{b\text{ ta}}$.

10 (*Og'zaki*). Ifodani o'qing, darajaning asosini, daraja ko'rsatkichini ayting:

1) 13^3; 2) $\left(4\frac{5}{4}\right)^5$; 3) $\left(-\frac{2}{7}\right)^4$; 4) $(-5{,}7)^{25}$; 5) $\left(\frac{3x}{4y}\right)^{15}$.

Hisoblang **(11–15)**:

11. 1) 3^5; 2) $0{,}6^3$; 3) $\left(\frac{1}{4}\right)^2$; 4) $(-1)^{10n-1}$. $(n\in N)$.

12. 1) $(-7)^2$; 2) $(-0{,}3)^3$; 3) $\left(-\frac{1}{3}\right)^4$; 4) 0^{16}.

13. 1) $\frac{(-0{,}3)^3}{(0{,}1)^5}$; 2) $\frac{(0{,}2)^4}{(-0{,}1)^3}$; 3) $\frac{(3{,}6)^2}{(1{,}8)^2}$; 4) $\frac{(4{,}6)^2}{(2{,}3)^3}$.

14. 1) $5\cdot(-2)^3$; 2) $-5\cdot(-2)^4$; 3) $-\frac{1}{4}\cdot(-2)^4$; 4) $-\frac{2}{3}\cdot(-9)^2$.

15. 1) $(-6)^2+\left(-\frac{5}{6}\right)$; 2) $(-4)^3+\left(-\frac{3}{4}\right)$; 3) $-(-8)^2\cdot3^2$; 4) $-(-9)^2\cdot(-2)^3$.

16. $-a^2$; $(-a)^2$; $(-a)^3$ ifodaning qimatini toping, bunda $a=1\frac{2}{3}$; -4.

17. Jadvalni to'ldiring:

b	-5	-4	-3	-2	-1	0	1	2	3	4	5
b^2											
b^3											

18. Ikki xonali son $\overline{ab}=10a+b$; uch xonali son esa $\overline{abc}=10^2a+10b+c$ kabi yozilishi mumkin, bunda a, b, c – raqamlar, $a\neq 0$. To'rt xonali son $\overline{abcd}$ ni qanday yozish mumkin? Misollar keltiring.

19. Sonni xona qo'shiluvchilari yigindisi shaklida yozing:
1) 345 727; 2) 4 532 186; 3) 200 570 607; 4) 52 070 507.

20. Tasvirlangan sonni xona qo'shiluvchilari yig'indisi shaklida yozing:
1) $3\cdot 10^5+5\cdot 10^4+8\cdot 10^3+9\cdot 10^2+2\cdot 10+3$;
2) $4\cdot 10^6+6\cdot 10^5+3\cdot 10^4+3\cdot 10^3+5\cdot 10+8$;
3) $9\cdot 10^5+2\cdot 10^3+3\cdot 10^2+9$;
4) $1\cdot 10^6+1\cdot 10^4+1\cdot 10^2+1$.

Ifodaning qiymatini toping (21–22):

21. 1) $0{,}5\cdot 3^n-0{,}2\cdot 3^{n+1}-3{,}6$, bunda $n=2$;
2) $0{,}9\cdot 3^{n+1}-(0{,}3\cdot 2)^{n+1}-3\cdot 2^n$, bunda $n=3$;

22. 1) $0{,}7\cdot 3^{n-1}+(2{,}5\cdot 0{,}4)^{2n+11}-0{,}3\cdot 5^{2n}$, bunda $n=2$;
2) $0{,}75\cdot 4^{2n-1}+(2{,}8\cdot 0{,}5)^{n-1}-0{,}5\cdot 4^{2n}$, bunda $n=2$.

2-§. Natural ko'rsatkichli darajaning xossalari

Quyidagi masalalarni ko'rib chiqaylik.

1-masala. Darajalarni ko'paytiring: $a^n\cdot a^m$.

$$a^n\cdot a^m=\underbrace{(a\cdot a\cdot\ldots\cdot a)}_{n\text{ ta ko'paytuvchi}}\cdot\underbrace{(a\cdot a\cdot\ldots\cdot a)}_{m\text{ ta ko'paytuvchi}}=\underbrace{(a\cdot a\cdot\ldots\cdot a)}_{(n+m)\text{ ta ko'paytuvchi}}=a^{n+m}.$$

2-masala. Darajalarni bo'ling: $a^n:a^m$, $(n>m)$,

$$a^n:a^m=\frac{a^n}{a^m}=\frac{\overbrace{(a\cdot a\cdot\ldots\cdot a)}^{n\text{ ta ko'paytuvchi}}}{\underbrace{(a\cdot a\cdot\ldots\cdot a)}_{m\text{ ta ko'paytuvchi}}}=\frac{\overbrace{(a\cdot a\cdot\ldots\cdot a)}^{m\text{ ta ko'paytuvchi}}\cdot\overbrace{(a\cdot a\cdot\ldots\cdot a)}^{(n-m)\text{ ta ko'paytuvchi}}}{\underbrace{(a\cdot a\cdot\ldots\cdot a)}_{m\text{ ta ko'paytuvchi}}}=\underbrace{(a\cdot a\cdot\ldots\cdot a)}_{(n-m)\text{ ta ko'paytuvchi}}=a^{n-m}.$$

3-masala. Darajani darajaga ko'taring: $(a^m)^n$.

$$(a^m)^n=\underbrace{(a^m\cdot a^m\cdot\ldots\cdot a^m)}_{n\text{ ta}}=a^{\overbrace{m+m+\ldots+m}^{n\text{ ta}}}=a^{mn}.$$

4-masala. Ko‘paytmani darajaga ko‘taring: $(ab)^n$.

$$(ab)^n=\underbrace{(ab)\cdot(ab)\cdot\ldots\cdot(ab)}_{n\text{ ta}}=\overbrace{a\cdot a\cdot\ldots\cdot a}^{n\text{ ta}}\cdot\underbrace{b\cdot b\cdot\ldots\cdot b}_{n\text{ ta}}=a^n\cdot b^n.$$

5-masala. Kasrni darajaga ko‘taring: $\left(\frac{a}{b}\right)^n$.

$$\left(\frac{a}{b}\right)^n=\underbrace{\frac{a}{b}\cdot\frac{a}{b}\cdot\ldots\cdot\frac{a}{b}}_{n\text{ ta}}=\frac{\overbrace{a\cdot a\cdot\ldots\cdot a}^{n\text{ ta}}}{\underbrace{b\cdot b\cdot\ldots\cdot b}_{n\text{ ta}}}=\frac{a^n}{b^n}.$$

Bu masalalardan darajaning shunday xossalari kelib chiqadi:

1. Asoslari bir xil bo‘lgan darajalarni ko‘paytirishda asos o‘zgarmasdan qoladi, daraja ko‘rsatkichlari esa qo‘shiladi, ya’ni $a^n\cdot a^m=a^{n+m}$.
2. Asoslari bir xil bo‘lgan darajalarni bo‘lishda asos o‘zgarmasdan qoladi, daraja ko‘rsatkichlari esa ayriladi, ya’ni $a^n:a^m=a^{n-m}$, bunda $n>m$.
3. Darajani darajaga ko‘tarishda asos o‘zgarmasdan qoladi, daraja ko‘rsatkichlari esa ko‘paytiriladi, ya’ni $(a^m)^n=a^{mn}$.
4. Ko‘paytmani darajaga ko‘tarishda har bir ko‘paytuvchi shu darajaga ko‘tariladi, ya’ni $(a\cdot b)^n=a^n\cdot b^n$.
5. Kasrni darajaga ko‘tarishda uning surat va maxraji xuddi shu darajaga ko‘tariladi, ya’ni $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$.

6-masala. Hisoblang: $\frac{\left(5^3\cdot5^6\right)^3}{\left(5^4\cdot5^8\right)^2}$.

□ 1) $5^3\cdot5^6=5^{3+6}=5^9$; *(1-xossa)*

2) $(5^9)^3=5^{9\cdot3}=5^{27}$; *(3-xossa)*

3) $5^4\cdot5^8=5^{4+8}=5^{12}$; *(1-xossa)*

4) $(5^{12})^2=5^{12\cdot2}=5^{24}$; *(3-xossa)*

5) $\frac{5^{27}}{5^{24}}=5^{27-24}=5^3=125$. *(2-xossa)*

Javob: 125. ■

Shunga o‘xshash:

$$\frac{17^8\cdot5^7\cdot2^5}{17^7\cdot5^5\cdot2^3}=\frac{17^8}{17^7}\cdot\frac{5^7}{5^5}\cdot\frac{2^5}{2^3}=17^1\cdot5^2\cdot2^2=17\cdot25\cdot4=1\,700.$$

7-masala. $a^n:a^n$ ni hisoblang

□ Darajaning 2-xossasiga ko‘ra $a^n:a^n=a^{n-n}=a^0$.

0 dan farqli 2 ta bir xil sonni bir-biriga bo‘lsak, natija 1 chiqadi, ya’ni $a^n:a^n=1$ ekanidan $a^0=1$ tenglikni hosil qilamiz. ■

?

23. 1) Bir xil asosli darajalarni ko'paytirish xossasini ayting va formulasini yozing.
2) Bir xil asosli darajalarni bo'lish xossasini ayting va formulasini yozing.
3) Darajani darajaga ko'tarish xossasini ayting.
4) Ko'paytmani darajaga ko'tarish xossasini ayting.
5) Kasr qanday qilib darajaga ko'tariladi?
6) Nolga teng bo'lmagan sonning 0-darajasi nimaga teng?

Ko'paytmani daraja shaklida yozing **(24–33)**:

24. 1) $13^6 \cdot 13^7$; 2) $27^3 \cdot 27^5$; 3) $16^4 \cdot 16$; 4) $8 \cdot 8^{18}$.

25. 1) $a^5 \cdot a^8$; 2) $b^{21} \cdot b^9$; 3) $d \cdot d^{15}$; 4) $c^6 \cdot c$.

26. 1) $(0{,}5b)^5 \cdot (0{,}5b)^3$; 2) $(0{,}7c)^3 \cdot (0{,}7c)^2$;
3) $\left(\frac{3}{4}k\right)^{11} \cdot \left(\frac{3}{4}k\right)^{7}$; 4) $\left(\frac{5}{7}t\right)^{12} \cdot \left(\frac{5}{7}t\right)^{11}$.

27. 1) $(-6{,}2)^{15} \cdot (-6{,}2)^7$; 2) $(-7{,}4)^5 \cdot (-7{,}4)^3$;
3) $\left(-1\frac{2}{3}\right)^3 \cdot \left(-1\frac{2}{3}\right)^5$; 4) $\left(-2\frac{3}{5}\right)^7 \cdot \left(-2\frac{3}{5}\right)^5$.

28. 1) $(2,3)^4 \cdot (2,3) \cdot (2,3)^8$; 2) $(5,8)^7 \cdot (5,8)^2 \cdot (5,8)$;
3) $\left(\frac{5}{9}\right)^3 \cdot \left(\frac{5}{9}\right)^{13} \cdot \left(\frac{5}{9}\right)$; 4) $\left(\frac{7}{12}\right) \cdot \left(\frac{7}{12}\right)^3 \cdot \left(\frac{7}{12}\right)^5$.

29. 1) $a^5 \cdot a^8 \cdot a^3$; 2) $b^{11} \cdot b^5 \cdot b^{13}$; 3) $x \cdot x^3 \cdot x^{21}$; 4) $y^8 \cdot y \cdot y^6$.

30. 1) $(-3{,}5a)^5 \cdot (-3{,}5a) \cdot (-3{,}5a)^7$; 2) $(-7{,}9b)^3 \cdot (-7{,}9b)^8 \cdot (-7{,}9b)$;
3) $\left(-\frac{7}{15}x\right)^4 \cdot \left(-\frac{7}{15}x\right) \cdot \left(-\frac{7}{15}x\right)^{12}$; 4) $\left(-\frac{2}{11}y\right) \cdot \left(-\frac{2}{11}y\right)^2 \cdot \left(-\frac{2}{11}y\right)^7$.

31. 1) $(a-b)^3 \cdot (a-b)^7$; 2) $(c+d)^8 \cdot (c+d)^5$; 3) $(2a+b)^3 \cdot (2a+b)$;
4) $(4c-d) \cdot (4c-d)^7$; 5) $(a+b)^2 \cdot (a+b)^3 \cdot (a+b)^4$.

32. 1) $5^{12} \cdot 5^n$; 2) $6^{17} \cdot 6^k$; 3) $12^a \cdot 12^5$; 4) $18^{15} \cdot 18^c$, $(a, n, k, c \in N)$.

33. 1) $a^n \cdot a^8$; 2) $b^{12} \cdot b^m$; 3) $(2x)^{25} \cdot (2x)^n$; 4) $(3y)^n \cdot (3y)^{12}$, $(m, n \in N)$.

Sonlarni asosi 2 bo'lgan daraja shaklida yozing **(34–37)**:

34. 1) 8; 2) 16; 3) 2; 4) 64.

35. 1) $2^{12} \cdot 128 \cdot 4$; 2) $256 \cdot 2^{10} \cdot 512$; 3) $64 \cdot 1024 \cdot 2^7$; 4) $256 \cdot 64 \cdot 2^9$.

36. 1) $2^{15} \cdot 256 \cdot 32$; 2) $2^{13} \cdot 1024 \cdot 16$; 3) $2^k \cdot 2^5 \cdot 8$; 4) $2^k \cdot 64 \cdot 32$, $(k \in N)$.

37. 1) $2^{3n} \cdot 2^n \cdot 2^{2n}$; 2) $2^{5m} \cdot 2^{6m} \cdot 2^{3m}$; 3) $2^5 \cdot 2^n \cdot 8$;
4) $2^{5k} \cdot 2 \cdot 16$, $(m, n \in N)$.

Sonlarni asosi 3 bo'lgan daraja shaklida yozing **(38–41)**:

38. 1) 27; 2) 9; 3) 3; 4) 243.

39. 1) 81; 2) $3 \cdot 3^{12}$; 3) $3^{17} \cdot 27$; 4) $3^{15} \cdot 3^8 \cdot 9$.

40. 1) $3^{15} \cdot 3 \cdot 3^2$; 2) $3 \cdot 3^{16} \cdot 3^4$; 3) $3^8 \cdot 81 \cdot 3$; 4) $3^9 \cdot 243 \cdot 27$.

41. 1) $3^m \cdot 3^{2m} \cdot 3$; 2) $3 \cdot 3^{8n} \cdot 3^n$; 3) $3^{m+2} \cdot 3^m \cdot 27$; 4) $3^n \cdot 3^{2n+1} \cdot 729$, $(m, n \in N)$.

Bo'linmani daraja shaklida yozing **(42–45)**:

42. 1) $18^{10} : 18^2$; 2) $25^{13} : 25^5$; 3) $(0{,}5)^{11} : (0{,}5)^7$; 4) $(5{,}2)^{13} : (5{,}2)^5$.

43. 1) $a^{19} : a^5$; 2) $x^{21} : x^{20}$; 3) $b^{18} : b^{12}$; 4) $c^7 : c$.

44. 1) $\left(-\frac{5}{6}a\right)^5 : \left(-\frac{5}{6}a\right)^3$; 2) $\left(-\frac{3}{17}b\right)^{12} : \left(-\frac{3}{17}b\right)^4$;

3) $\left(\frac{3y}{5}\right)^9 : \left(\frac{3y}{5}\right)^8$; 4) $\left(\frac{5}{21}c\right)^{28} : \left(\frac{5}{21}c\right)^{10}$.

45. 1) $(c-d)^{15} : (c-d)^7$; 2) $(a+b)^{17} \cdot (a+b)^3$; 3) $(2m+n)^3 : (2m+n)$;
4) $(3x-5)^{35} : (3x-5)^{32}$.

46. Sonlarni asosi 2 bo'lgan daraja shaklida yozing:

1) $16 : 2^3$; 2) $512 : 2^4$; 3) $\frac{2^8}{2^3}$; 4) $\frac{2^{15}}{2^{13}}$.

47. Sonlarni asosi 3 bo'lgan daraja shaklida yozing:

1) $729 : 34$; 2) $38 : 243$; 3) $\frac{2^8}{2^3}$; 4) $\frac{3^7}{3}$.

Hisoblang **(48–50)**:

48. 1) $\frac{2^2 \cdot 5^4}{5^3}$; 2) $\frac{3^4 \cdot 6}{3^3}$; 3) $\frac{2^5 \cdot 2^6}{2^3 \cdot 2^4}$; 4) $\frac{3^8 \cdot 3^5}{3^9 \cdot 3^2}$.

49. 1) $\frac{5^6 \cdot 5^7}{5 \cdot 5^3 \cdot 5^8}$; 2) $\frac{6^3 \cdot 6^4 \cdot 6^8}{6^9 \cdot 6^4}$; 3) $\frac{7^9 \cdot 7 \cdot 7^5}{7^{12} \cdot 7^2}$; 4) $\frac{8^4 \cdot 8^{14} \cdot 8^5}{8 \cdot 8^{20}}$.

50. 1) $\frac{(-9)^8}{9^6}$; 2) $\frac{12^{16}}{(-12)^8}$; 3) $\frac{(-11)^{13}}{11^{11}}$; 4) $\frac{15^4}{(-15)^3}$.

Bo'linmani daraja shaklida yozing **(51–52)**:

51. 1) $\frac{17^n}{17 \cdot 17^5}$; 2) $\frac{21^m}{21^2 \cdot 21^3}$; 3) $\frac{6^{8n}}{6^{2n} \cdot 6}$; 4) $\frac{9^{21m}}{9^{12m} \cdot 9^5}$.

52. 1) $\frac{5^{2k} \cdot 5^{8k}}{5^k \cdot 5^4}$; 2) $\frac{11^m \cdot 11^{12m}}{11^{11m}}$; 3) $\frac{6^{8n+7} \cdot 6^{3n-2}}{6^{10n-1}}$; 4) $\frac{13^{9n+5} \cdot 13^{6n-5}}{13^{10n}}$.

Tenglamani yeching **(53–55)**:

53. 1) $x : 5^3 = 5$; 2) $x : 8 = 8^2$; 3) $x \cdot 6^{12} = 6^{14}$; 4) $x \cdot 7^{18} = 7^{20}$.

54. 1) $13^8 \cdot x = 13^{10}$; 2) $18^5 \cdot x = 18^7$; 3) $21^3 : x = 21$; 4) $43^7 : x = 43^5$.

55. 1) $\frac{x}{7} = 7^2$; 2) $\frac{x}{5^2} = 5$; 3) $\frac{24^5}{x} = 24^3$; 4) $\frac{19^8}{x} = 19^7$.

Ifodani asosi b bo'lgan daraja shaklida yozing **(56–59)**:

56. 1) $(b^8)^3$; 2) $(b^3)^7$; 3) $(b^n)^5$; 4) $(b^m)^6$.

57. 1) $(b^8)^2 \cdot b^3$; 2) $(b^3)^4 \cdot b^2$; 3) $b^7 \cdot (b^3)^2$; 4) $b^9 \cdot (b^3)^6$.

58. 1) $(b^3)^3 \cdot b \cdot b^4$; 2) $b^4 \cdot (b^3)^6 \cdot b^7$; 3) $b^3 \cdot (b^2)^4 \cdot b \cdot (b^3)^6$; 4) $(b^5)^2 \cdot b \cdot (b^3)^7 \cdot b$.

59. 1) $\left(b^8\right)^5 : \left(b^3\right)^{13}$; 2) $\left(b^4\right)^5 : \left(b^5\right)^3$; 3) $\dfrac{\left(b^4\right)^8 \cdot b^{13}}{b^{25}}$; 4) $\dfrac{b^{15} \cdot \left(b^3\right)^5}{\left(b^3\right)^6 \cdot b}$.

60. m ning qanday qiymatlarida tenglik o'rinli bo'ladi?

1) $5^m = 125$; 2) $3^m = 243$; 3) $512 = 2^m$;

4) $216 = 6^m$; 5) $(3^2)^m = 81$; 6) $(4^5)^m = 1024$.

Sonlarni ko'rsatkichi 2 bo'lgan daraja shaklida yozing **(61–63)**:

61. 1) $0,09$; 2) $0,0324$; 3) $\dfrac{49}{81}$; 4) $\dfrac{121}{144}$; 5) $1\dfrac{27}{169}$; 6) $2\dfrac{7}{81}$.

62. 1) 9^4; 2) 17^8; 3) $(-0,9)^{16}$; 4) $\left(-\dfrac{2}{3}\right)^{26}$.

63. 1) b^8; 2) a^{14}; 3) k^{28}; 4) y^{36}.

Ko'paytmani darajaga ko'taring **(64–69)**:

64. 1) $(2 \cdot 7)^4$; 2) $(8 \cdot 5)^{12}$; 3) $(2,4 \cdot 17)^8$; 4) $(6,1 \cdot 19)^{14}$; 5) $\left(8 \cdot \dfrac{2}{3}\right)^6$; 6) $\left(\dfrac{2}{7} \cdot 5\right)^{11}$.

65. 1) $(5b)^4$; 2) $(13a)^7$; 3) $(-9x)^6$; 4) $(-15y)^9$.

66. 1) $(xy)^5$; 2) $(mn)^6$; 3) $(-2,5mn)^5$; 4) $(-4cd)^3$.

67. 1) $(xyz)^5$; 2) $(abc)^4$; 3) $(8 \cdot 11 \cdot 9)^{11}$; 4) $(3 \cdot 4 \cdot 5)^3$.

68. 1) $(a^2 \cdot b)^3$; 2) $(kt^3)^2$; 3) $(4b^5)^4$; 4) $(0,2a^3)^5$.

69. 1) $(20n^5 m^4)^3$; 2) $(3a^5 b^8)^2$; 3) $(-5,2m^4 n^5)^4$; 4) $(-2,5b^2 c^3 d)^5$.

Ko'paytmani daraja shaklida yozing **(70–72)**:

Namuna: $2^5 \cdot k^5 = (2k)^5$

70. 1) $5^2 \cdot a^2$; 2) $6^5 \cdot b^5$; 3) $9^6 \cdot 5^6$; 4) $15^8 \cdot 3^8$.

71. 1) $\left(\dfrac{2}{13}\right)^3 \cdot a^3$; 2) $\left(\dfrac{18}{19}\right)^8 \cdot b^8$; 3) $(-1,9)^{12} \cdot y^{12}$; 4) $(-2,1)^{13} \cdot c^{13}$.

72. 1) $25a^2$; 2) $8k^3$; 3) $11^5 \cdot a^5 \cdot b^5$; 4) $27^3 \cdot c^3 \cdot d^3$.

Ifodani ko'rsatkichi 2 bo'lgan daraja shaklida yozing **(73–75)**:

73. 1) $a^4 \cdot a^8$; 2) $b^{12} \cdot c^{14}$; 3) $64\,a^3$; 4) $81\,x^4$.

74. 1) $a^{10} \cdot b^{14} \cdot c^8$; 2) $x^2 \cdot y^{12} \cdot z^{24}$; 3) $144 \cdot m^4 \cdot n^6 \cdot k^2$; 4) $81 \cdot a^8 \cdot b^{10} \cdot c^{16}$.

75. 1) $0,81 \cdot a^8 \cdot b^4$; 2) $0,64 \cdot x^6 \cdot y^{12}$; 3) $\dfrac{25}{49} \cdot x^6 \cdot y^8$; 4) $\dfrac{16}{121} \cdot m^{18} \cdot n^{16}$.

Masalalar yechish

Ifodani ko‘rsatkichi 3 bo‘lgan daraja shaklida yozing **(76–79)**:

76. 1) a^9; 2) c^{18}; 3) 17^{21}; 4) 11^{12}.

77. 1) $(-0,7)^{15}$; 2) $0,081$; 3) $\left(-\frac{3}{7}\right)^{27}\cdot y^9$; 4) $-0,008$.

78. 1) $a^6\cdot b^9$; 2) $m^{27}\cdot n^{18}$; 3) $x^6\cdot y^{30}\cdot z^{15}$; 4) $b^{33}\cdot c^{21}\cdot d^9$.

79. 1) $-8\cdot b^6$; 2) $-64\cdot c^3$; 3) $0,027\cdot m^3\cdot n^9$; 4) $0,729\cdot x^6\cdot y^{12}$.

Hisoblang **(80–84)**:

80. 1) $(0,125)^{15}\cdot 8^{15}$; 2) $(-0,4)^8\cdot(2,5)^8$; 3) $\left(\frac{7}{8}\right)^{12}\cdot\left(\frac{8}{7}\right)^{12}$; 4) $\left(1\frac{2}{5}\right)^{20}\cdot\left(\frac{5}{7}\right)^{20}$.

81. 1) $\left(-\frac{2}{5}\right)^{25}\cdot(2,5)^{25}$; 2) $\left(-\frac{2}{9}\right)^{13}\cdot(4,5)^{13}$; 3) $\left(\frac{3}{88}\right)^2\cdot 11^2$; 4) $\left(\frac{5}{24}\right)^3\cdot 12^3$.

82. 1) $\frac{5^9\cdot 3^9}{15^7}$; 2) $\frac{2^5\cdot 9^5}{18^4}$; 3) $\frac{21^4}{7^2\cdot 3^2}$; 4) $\frac{12^8}{3^5\cdot 4^5}$.

83. 1) $\frac{6^{12}\cdot 5^{12}}{10^{10}\cdot 3^{10}}$; 2) $\frac{8^9\cdot 6^9}{12^8\cdot 4^8}$; 3) $\frac{21^5}{3^2\cdot 7^4\cdot 9}$; 4) $\frac{8^{16}}{16^{12}}$.

84. 1) $\frac{16\cdot 27^4}{3^{10}}$; 2) $\frac{5\cdot(2^5)^2}{512}$; 3) $\frac{6^8\cdot(3^4)^2}{18^6}$;

4) $\frac{25^2\cdot 3^5}{15^4}$; 5) $\frac{3^9\cdot(3^2)^5}{(3^5)^3}$; 6) $\frac{6^8\cdot(6^2)^5}{(6^5)^3}$.

Kasrni darajaga ko‘taring **(85–88)**:

85. 1) $\left(\frac{2}{9}\right)^3$; 2) $\left(\frac{3}{7}\right)^2$; 3) $\left(\frac{7}{8}\right)^3$; 4) $\left(\frac{2}{5}\right)^4$.

86. 1) $\left(-\frac{k}{12}\right)^2$; 2) $\left(-\frac{m}{4}\right)^4$; 3) $\left(-\frac{k}{3}\right)^3$; 4) $\left(-\frac{6}{a}\right)^4$.

87. 1) $\left(\frac{d}{4b}\right)^3$; 2) $\left(\frac{2b}{3c}\right)^4$; 3) $\left(\frac{5^4}{6^5}\right)^3$; 4) $\left(\frac{7^2}{8^3}\right)^5$.

88. 1) $\left(\frac{c+d}{5}\right)^4$; 2) $\left(\frac{3}{5+d}\right)^3$; 3) $\left(\frac{a+b}{c-3d}\right)^5$; 4) $\left(\frac{m+2k}{m-n}\right)^5$.

Kasrni daraja shaklida yozing **(89–91)**:

89. 1) $\frac{4^5}{5^8}$; 2) $\frac{2^9}{3^9}$; 3) $\frac{a^5}{2^5}$; 4) $\frac{7^8}{6^8}$.

90. 1) $\frac{a^{12}}{b^{12}}$; 2) $\frac{t^{11}}{k^{11}}$; 3) $\frac{16}{49}$; 4) $\frac{121}{196}$.

91. 1) $\frac{(3a)^5}{(5b)^5}$; 2) $\frac{(4x)^{12}}{(7y)^{12}}$; 3) $\frac{1}{-27}$; 4) $\frac{-1}{729}$.

Hisoblang **(92–93)**:

92. 1) $\left(\frac{3}{7}\right)^5 \cdot \frac{7^3}{3^4}$; 2) $\frac{5^5}{8^7} \cdot \left(\frac{8}{5}\right)^6$; 3) $\left(\frac{3}{8}\right)^5 \cdot \left(\frac{8}{3}\right)^4$; 4) $\left(\frac{7}{12}\right)^{12} \cdot \left(\frac{12}{7}\right)^{10}$.

93. 1) $\left(-\frac{36}{49}\right)^4 \cdot \left(\frac{7}{6}\right)^6$; 2) $\left(\frac{16}{25}\right)^5 \cdot \left(\frac{5}{8}\right)^5 \cdot (2{,}5)^3$;

3) $\left(\frac{5^3}{7^2}\right)^4 \cdot \left(\frac{14}{5^3}\right)^6 \cdot \left(\frac{5^2}{2}\right)^5$; 4) $\left(\frac{11^4}{18^3}\right)^3 \cdot \left(\frac{6}{11}\right)^6 \cdot \left(\frac{3}{11}\right)^5$.

94. Ifodaning son qiymatini toping.

1) $\frac{5-a^3}{2a^2}$; bu yerda $a=-3$; 2) $\frac{4a^3}{a^3-4}$; bu yerda $a=-5$.

95. Ifodani daraja shaklida yozing.

1) $8^{2n+3} \cdot 8^{5n-1} : 8^{3n+5}$; 2) $7^{5n+7} \cdot 7^{2n-5} \cdot 7^{n-2}$;

3) $\frac{x^{6n-5} \cdot x^{8n+7}}{x^{9n-3}}$; 4) $\frac{y^{3n+3} \cdot y^{4n-5}}{y^{5n-7}}$.

96. n ning qanday qiymatlarida tenglik to‘g‘ri bo‘ladi?

1) $(7^n)^5 = 7^{20}$; 2) $(3^3)^n = 3^{27}$; 3) $5^{3n} = 5^9$;

4) $(2^2)^n = 2^{18}$; 5) $(9^3)^n = 3^{12}$; 6) $(6^2)^{2n} = 36^3$.

97. Ko‘paytmani darajaga ko‘taring.

1) $(9a^3b^2c^4)^3$; 2) $(12x^4y^3z)^2$;

3) $(-6{,}2a^3b^7z^3)^4$; 4) $(-9{,}5x^2y^4z^5)^5$.

98. Ifodani asosi b bo‘lgan daraja shaklida yozing:

1) $\frac{b^3 \cdot b^9}{b \cdot b^5}$; 2) $\frac{b^{15} \cdot b^3}{b^2 \cdot b^4}$; 3) $\frac{(b^3)^5 \cdot (b^2)^5}{b^{10} \cdot b^3}$; 4) $\frac{b \cdot (b^3)^4 \cdot b^5}{(b^5)^2 \cdot b^3}$.

99. Sonlarni taqqoslang:

1) 34^4 va 11^{12}; 3) 25^5 va 5^{25};

2) 22^{11} va 11^{22}; 4) 8^{20} va 4^{40}.

100. Hisoblang:

1) $\frac{3 \cdot 6^{12} - 4 \cdot 6^{11}}{36^5}$; 2) $\frac{8 \cdot 7^{18} - 5 \cdot 7^{17}}{49^8}$;

3) $\frac{(5 \cdot 4^{22} + 7 \cdot 4^{21}) \cdot 54}{(27 \cdot 16^4)^2}$; 4) $\frac{6 \cdot (5 \cdot 8^{15} - 29 \cdot 8^{14})}{8^{16} + 3 \cdot 8^{15}}$.

101. Tenglamani yeching:

1) $y:3,75=12,125-5\frac{1}{8}$; 3) $\frac{5}{18}+\frac{1}{12}=\frac{7}{8}x$;

2) $24,6:x=0,0003\cdot 1000$; 4) $\frac{19}{24}+\frac{5}{16}=\frac{15}{16}\cdot(x+3,2)$.

102. Sonni standart shaklda yozing:
1) 39 000; 2) 98 710 000; 3) 162 000 000; 4) 250 700 000.

103. Hisoblang:

1) $-(-3)^2\cdot(-2)^3+(-1)^4\cdot(-2)^5+4^3\cdot(-5)$; 2) $(-0,3)^3:(-0,1)^4+(3,4)^3:(-1,7)^2$;

3) $(-0,2)^3:(0,1)^4-\frac{2}{5}\cdot(-5)^2+\frac{1}{7}\cdot(-7)^2$; 4) $\left(-3\frac{1}{7}\right)^2\cdot\frac{49}{121}-(-1,2)^{40}:(-0,6)^{39}:2^{39}$;

5) $\left((-1)^5\right)^6+\left((-1)^5\right)^5+\left(-(-1)^6\right)^7-\left((-1)^3\right)^5$; 6) $\frac{8^2\cdot 4^5}{16^2\cdot 8}+\frac{16^3}{4^3\cdot 64}+\frac{10^8\cdot 4^5}{4^3\cdot 10^7}$;

7) $\frac{120^6\cdot 6^4\cdot 11^4}{60^8\cdot 132^2}+\frac{9^6\cdot 8^6\cdot 24^4}{36^8\cdot 8^3}$; 8) $\left((-25)^4\right)^6:\left((-25)^{13}\right)^2-\left(\frac{1}{12}\right)^2$.

104. 1) 3^{69+3n} soni 9^{24+n} dan necha marta katta?

2) (14^{n+7}) soni (2^{n+1}) sonidan necha marta katta?

3) Agar $\frac{x^m-x^n}{\frac{1}{x^n}-\frac{1}{x^m}}=4^m\cdot 2^{2n}$ bo'lsa, x ni toping.

4) Agar $2^m\cdot 5^n=200$; $2^n\cdot 5^m=500$ bo'lsa, $m+n$ nimaga teng?

3-§. Birhad va uning standart shakli

Qirralari uzunligi $5a$, b, c bo'lgan to'g'ri burchakli parallelepipedning hajmi $5abc$ ga, qirrasining uzunligi a ga teng bo'lgan kub sirti $6a^2$, qirralari uzunliklari yig'indisi $12a$ ga teng. $5abc$ ifoda to'rtta ko'paytuvchining ko'paytmasidan iborat bo'lib, birinchi ko'paytuvchi 5 sonidan, qolganlari esa a, b, c harflaridan; $6a^2$ ifoda esa 2 ta ko'paytuvchili ko'paytma bo'lib, birinchi ko'paytuvchi 6 sonidan, ikkinchisi esa a^2 dan iborat; $12a$ ifoda 2 ta ko'paytuvchidan iborat, birinchi ko'paytuvchi 12, ikkinchi ko'paytuvchi a.

Sonli va harfiy ko'paytuvchilarning ko'paytmasidan tuzilgan algebraik ifoda birhad deyiladi.

Masalan, abc; $-3a$; $4ab$; $\frac{1}{3}a(-0,5)b\cdot ab$; $-2a^2b$; $\frac{2}{5}a^2b\cdot\frac{5}{4}ac$ ifodalar birhaddir.

Birhadlarga a; 2; $\frac{3}{8}$ lar ham misol bo'la oladi, chunki ularni $a=1a$; $2=2a^0$; $\frac{3}{8}=\frac{3}{8}a^0$ ko'rinishda yozish mumkin.

Agar birhad birinchi o'rinda turgan faqat bitta son ko'paytuvchidan va har xil asosli harfiy darajalardan tuzilgan bo'lsa, uni *standart shaklidagi birhad* deyiladi. Bu yerda son ko'paytuvchini *birhadning koeffitsiyenti* deyiladi. Masalan, $3ab$ birhadning koeffitsiyenti 3 ga teng;

$\frac{7}{9}abc^3$ birhadning koeffitsiyenti $\frac{7}{9}$ ga teng;

$-3{,}7a^3b$ birhadning koeffitsiyenti $(-3{,}7)$ ga teng;
$bc^2=1\cdot bc^2$, ya'ni bc^2 birhadning koeffitsiyenti 1 ga teng;
$-a^5b^2c=-1\cdot a^5b^2c$, ya'ni $-a^5b^2c$ ifodaning koeffitsiyenti (-1) ga teng.

Masala. $9ab\cdot\frac{1}{3}\cdot a^2\cdot\frac{5}{27}c$ birhadning $a=\frac{2}{3}$; $b=2{,}5$; $c=8{,}1$ bo'lgandagi qiymatini hisoblang.

□ ***1-usul.*** Harflarning o'rniga berilgan qiymatlarni qo'yamiz:

$$9\cdot\frac{2}{3}\cdot 2{,}5\cdot\frac{1}{3}\cdot\left(\frac{2}{3}\right)^2\cdot\frac{5}{27}\cdot 8{,}1=\frac{9}{1}\cdot\frac{2}{3}\cdot\frac{5}{2}\cdot\frac{1}{3}\cdot\frac{4}{9}\cdot\frac{5}{27}\cdot\frac{81}{10}=\frac{10}{3}=3\frac{1}{3}.$$

Javob: $3\frac{1}{3}$. ■

□ ***2-usul.*** Berilgan birhadni standart shaklga keltirib olamiz:

$$9\cdot\frac{1}{3}\cdot\frac{5}{27}a^3bc=\frac{5}{9}a^3bc.$$

Endi harflarning qiymatlarini standart shaklga keltirilgan birhadga qo'yamiz:

$$\frac{5}{9}\cdot\left(\frac{2}{3}\right)^3\cdot 2{,}5\cdot 8{,}1=\frac{5}{9}\cdot\frac{8}{27}\cdot\frac{5}{2}\cdot\frac{81}{10}=\frac{10}{3}=3\frac{1}{3}.$$

Javob: $3\frac{1}{3}$. ■

"Birhadning darajasi" tushunchasini kiritamiz.

Standart shaklga keltirilgan birhaddagi harflar darajalarining yig'indisi birhadning darajasi deyiladi.

Masalan, $5a^2b^3c$ birhadning darajasi 6, chunki $2+3+1=6$;
$-7x^2y$ birhadning darajasi 3, chunki $2+1=3$;
5,42 esa nolinchi darajali birhad hisoblanadi.
0 soni – darajasi aniqlanmagan birhaddir.

105. 1) Birhad deb nimaga aytiladi?
2) Birhadning standart shakli nima?
3) Birhadning koeffitsiyenti deb nimaga aytiladi?
4) Birhadning darajasi deb nimaga aytiladi?
5) Birhad faqat sondan iborat bo'lsa, uning darajasi nimaga teng?

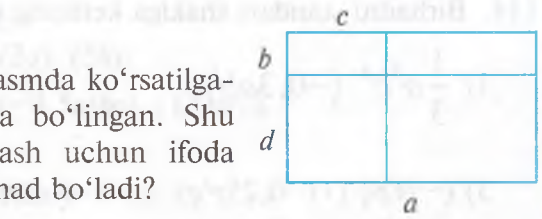
106. Quyidagi ifodalardan qaysi biri birhad?

1) $8x^4y$; 2) $-3ttyy$; 3) $-(m+n^4k^2)^3$;
4) $6{,}2a(a+b)$; 5) y; 6) $-a+b$;
7) $-b$; 8) $3{,}5y^ny^m$; 9) 5,7.

107. So'z orqali aytilgan fikrni algebraik ifoda yordamida yozing va ular ichidan birhadlarni aniqlang:

1) a son bilan b son kubining ko'paytmasi;
2) x son kvadrati bilan y son kubi ko'paytmasining uchlangani;
3) a va b sonlar kvadratlarining yig'indisi;
4) a va b sonlar kvadratlari ko'paytmasiga qarama-qarshi son;
5) t son kubi bilan p son kvadrati ayirmasining yarmi;
6) m sutkadagi minutlar soni;
7) k metrdagi detsimetrlar soni.

108. To'g'ri to'rtburchak o'lchamlari rasmda ko'rsatilganidek, to'rtta to'g'ri to'rtburchakka bo'lingan. Shu to'rtburchaklarning yuzlarini aniqlash uchun ifoda tuzing. Bu ifodalardan qaysi biri birhad bo'ladi?



109. Birhadning son qiymatini toping:

1) $27a^2b$, bunda $a=\frac{1}{3}$; $b=-6$; 2) $20a^3$, bunda $a=0{,}5$;

3) $-\frac{1}{8}b^2$, bunda $b=100$; 4) $-3p^3q^2$, bunda $p=\frac{1}{3}$; $q=-9$;

5) $\frac{1}{12}c^3\cdot(-16)$, bunda $c=0{,}3$; 6) $\frac{5}{6}y^2x^5$, bunda $y=-3$; $x=-1$.

110. Birhadning son qiymatini toping:

1) $-0{,}09x^6y^3$, bunda $x=-1$; $y+1=1$;
2) $-6{,}5ab^2$, bunda $a=-4$; $ab=4$;
3) p^2q^3t, bunda $p=-4$; $q=0{,}5$; $p+q+t=3$;
4) $\frac{2}{5}a^4b^3c$, bunda $a=-1$; $b=2$; $a+b+c=16$.

111. Birhadni standart shaklda yozing va uning koeffitsiyentini ko'rsating:

1) $6x^2x(-4x)$; 2) $0{,}8b^3(-5b^2)$; 3) $-65a^2b\cdot\left(\frac{3}{13}b^3\right)$;

4) $\frac{3}{4}y^3yy^2y(-6,4)$; 5) $pq\cdot 1,5q\cdot\frac{2}{3}$; 6) $(-3)^3tp^3\cdot 4tp$;

7) $-7aaabbbb\cdot(-2)ab$; 8) $-2,4x^2xxy\cdot\frac{5}{6}yy$.

112. Birhadni standart shaklda yozing:

1) $-2,5a^2b^4(-4b^n)$; 2) $\frac{7}{16}p^3\cdot q^k\cdot(-64p)$;

3) $0,001\ y^{m+1}x^n(-1000y^3x^5)$; 4) $t^2\cdot(-aab^n)(-0,2)b^4$.

113. Birhadlarni standart shaklda yozing va son qiymatini toping:

1) $xy5y$, bunda $x=-\frac{1}{25}$, $y=15$; 3) $x^2\cdot 0,25x$, bunda $x=-2$;

2) $\frac{7}{9}a^2\cdot 12\cdot b\cdot\frac{3}{49}$, bunda $a=-3$; $b=4,9$; 4) $y^3\cdot\frac{15}{169}y$, bunda $y=-13$.

114. Birhadni standart shaklga keltiring va uning darajasini aniqlang:

1) $\frac{1}{3}a^3b^4\cdot(-0,3ab)$; 2) $(-0,2x^2y^4c^5)\cdot(-0,4xc)$;

3) $(-16pq^2)\cdot(-0,25p^2q)$; 4) $\frac{2}{11}txxxt\cdot 1\frac{1}{8}$.

4-§. Birhadlarni ko'paytirish

Masala. To'g'ri to'rtburchakning tomonlari a va b ga teng. Uning tomonlari, mos ravishda, 6 marta, $4k$ marta uzaytirildi. Yangi to'g'ri to'rtburchakning yuzini aniqlang.

□ Yangi to'g'ri to'rtburchakning o'lchamlari $6a$ va $4kb$ dan iborat va yuzi $S_1=(6a)(4kb)$ bo'ladi. $(6a)(4kb)$ ifoda $(6a)$ va $(4kb)$ birhadlarning ko'paytmasidan iborat. Sonlarni ko'paytirish qoidalariga ko'ra, bu ifodani quyidagicha yozish mumkin:

$(6a)\cdot(4kb)=(6\cdot 4)\ (a\cdot k\cdot b)=24akb$.■

Hosil bo'lgan ifoda birhaddir.

Demak, birhadlarni ko'paytirish natijasida yana birhad hosil bo'ladi va uni soddalashtirib standart shaklda yozish lozim.

Misollar. 1. Birhadlarni ko'paytiring:

a) $(7a^3b^2c^4)\cdot(\frac{5}{49}a^2b)$; b) $(-3ab)\cdot\left(\frac{7}{9}a^2b^3\right)$.

2. Birhadlarni darajaga ko‘taring:

a) $(3a^2bc^3)^2$; b) $\left(\frac{2}{5}ab^2c^4\right)^3$.

□ 1. a) $\left(7a^3b^2c^4\right)\cdot\left(\frac{5}{49}a^2b\right)=\left(\frac{7}{1}\cdot\frac{5}{49}\right)\cdot\left(a^3\cdot b^2\cdot c^4\cdot a^2b\right)=\frac{5}{7}a^5b^3c^4$;

b) $(-3ab)\cdot\left(\frac{7}{9}a^2b\right)=\left(-3\cdot\frac{7}{9}\right)\left(ab\cdot a^2b\right)=-\frac{7}{3}a^3b^2=-2\frac{1}{3}a^3b^2$.

2. a) $3a^2bc^3$ ifoda 3; a^2; b; c^3 birhadlarning ko‘paytmasidir.

Ko‘paytmani darajaga ko‘tarish xossasiga ko‘ra:

$(3a^2bc^3)^2=3^2(a^2)b^2(c^3)^2=9a^4b^2c^6$. ■

?

115. 1) Birhadlar qanday ko‘paytiriladi?

2) Birhadning darajasi qanday topiladi?

3) Birhadlar ko‘paytmasi yana birhad bo‘ladimi?

Birhadlarni ko‘paytiring **(116–120)**:

116. 1) $(4x)\ (12y)$; 2) $(3a)\ (5b)$;

3) $(-0,5p^2y)\cdot(-8py)$; 4) $(-4,5a^2bc)\ (13ab^2c)$.

117. 1) $\frac{2}{3}a\cdot12ab^3$; 2) $0,8x^2y\cdot(-xy)$;

3) $-\frac{7}{9}pt^3\cdot(-1p)$; 4) $-\frac{5}{24}a^4b^2\cdot2\ 4a^2b^4$.

118. 1) $14y\cdot0,5y^2$; 2) $8x^2\cdot\left(-\frac{3}{4}x\right)$; 3) $-6b^3\cdot1,3b^2$; 4) $-12a^4\ 0,3a^2$.

119. 1) $\frac{3}{4}ay^4\cdot16y$; 2) $1,6x^2c\cdot(-2xc^3)$;

3) $-a^3b^4\cdot1,4a^4b^5$; 4) $-0,1t^8x\cdot100tx^3$.

120. 1) $10ax^4\cdot(-0,1a^5)\cdot(-0,5a^2x^8)$; 2) $-20x^4\cdot0,5xy^2\cdot(-0,3x^2y^3)$;

3) $-\frac{1}{3}a^2bc\cdot\left(-15ab^2c\right)\cdot\left(0,2abc^2\right)$; 4) $12x^2y^2z\cdot\left(-\frac{3}{4}xy^2z^2\right)\cdot\left(0,1x^2yz^2\right)$.

Birhadlarni darajaga ko‘taring **(121–124)**:

121. 1) $(9x)^2$; 2) $(6y)^2$; 3) $\left(\frac{1}{7}a^2\right)^2$; 4) $\left(\frac{1}{2}a^3\right)^3$;

5) $(0,5y^3)^4$; 6) $(0,1c^5)^4$; 7) $(1,6a^3)^2$; 8) $(2,4c^5)^2$.

122. 1) $(4xy)^2$; 2) $(5ax)^3$; 3) $(8a^2b)^2$;
4) $(4ac^4)^3$; 5) $(2a^2c^3)^3$; 6) $(5x^5y^3)^2$.

123. 1) $\left(-\frac{1}{2}ab\right)^4$; 2) $\left(-\frac{1}{3}xy\right)^3$; 3) $(-10a^3b^2)^3$;
4) $(-11a^2x^6)^4$; 5) $(-xy^2z^3)^4$; 6) $(-a^2b^3c^4)^5$.

124. 1) $-(2ax^2)^2$; 2) $-(3a^2b)^3$; 3) $-(-5x^3c)^3$;
4) $-(-2ab^4)^3$; 5) $-(-a^2b^3c^3)^3$; 6) $-(-a^3b^2c)^4$.

125. Birhadlarni kvadrat ko'rinishida ifodalang:
1) $81a^2y^4$; 2) $49a^6b^4$; 3) $-100x^4y^8$; 4) $-25x^2y^6$;
5) $-5x^3y^5\cdot\left(-\frac{1}{5}x^5y^3\right)$; 6) $-0{,}1a^4t^2\cdot(-10a^2t^4)$;
7) $-(-3xy)^3 27y^6$; 8) $-(-2a^4)32b^8$.

126. Quyidagi ifodalarni birhadning:
a) kvadrati shaklida ifodalang: $\frac{1}{4}x^4$; $0{,}36a^6b^8$ $\frac{1}{9}a^6$; $0{,}16a^4b^{10}$;
b) kubi shaklida ifodalang: $0{,}001x^6$; $-125a^3c^9$; $0{,}008x^9$; $-27a^3b^{12}$.

127. Ifodani soddalashtiring:
1) $35a\cdot(2a)^2$; 6) $(-4y^2)\cdot 3y^5$;
2) $20a^3\cdot(5a)^3$; 7) $\left(3x^6y^3\right)^4\cdot\left(-\frac{1}{81}xy^2\right)$;
3) $-4x^3\cdot(5x^2)^3$; 8) $\left(-\frac{1}{8}x^2y^3\right)\cdot\left(4x^6y\right)^2$;
4) $-0{,}4x^5\cdot(2x^3)^3$; 9) $\left(-\frac{2}{3}ab^5\right)^3\cdot 18a^5b$;
5) $(-a^3)^2\cdot 12a^6$; 10) $90a^4b^3\cdot\left(-3\frac{1}{3}ab^6\right)^2$.

128. Ifodani soddalashtiring:
1) $2\frac{2}{3}a^2y^8\cdot\left(-1\frac{3}{2}ay^3\right)^4$; 2) $3\frac{3}{7}x^5y^6\cdot\left(-2\frac{1}{3}x^5y\right)^2$;
3) $4\frac{1}{6}a^8b^5\cdot\left(-1\frac{1}{5}a^5b\right)^3$; 4) $\left(-2\frac{1}{2}a^3b\right)^4\cdot 3\frac{1}{5}a^8b^5$;
5) $x^{n-2}\cdot x^{3-n}\cdot x$; 6) $a^{m+1}\cdot a\cdot a^{3-m}$;
7) $-2ab^2\cdot 3a^3\cdot b^4\cdot(-2a^5b^2)^2$; 8) $-4x^5y^2\cdot 3xy^4\cdot(3x^2y^3)^2$;
9) $3a^2b\cdot(-2a^3b^4)\cdot(-3a^3b^2)^3$; 10) $5x^4y\cdot(-3x^2y^3)\cdot(-2xy^4)^4$.

129. Birhadlarni standart ko'rinishga keltiring:

1) $(10a^2y)^2\cdot(3ay^2)^3$; 2) $(4ab^2)^3\cdot(0{,}5a^3b)^2$;

3) $\left(-\frac{1}{2}xy^3\right)^3\cdot\left(4y^5\right)^2$; 4) $\frac{2}{3}\left(x^2y^3\right)^3\cdot\left(-9x^4\right)^2$;

5) $-(3x^6y^2)^2\cdot(-x^2y)^4$; 6) $-(-a^2y^4)^4\cdot(6a^4y)^2$;

7) $(-5ab^6)^4\cdot(0{,}2a^6b)^4$; 8) $(-10x^3y^2)^5\cdot(-0{,}2xy^2)^5$.

5-§. Ko'phadlar

Bir nechta birhadlarning algebraik yig'indisi ko'phad deyiladi. Ko'phadlarni tashkil etuvchi birhadlar ko'phadning hadlari deyiladi.

Masalan: $9x^2+6xy-3{,}6y^2+3{,}1y$ ko'phad hadlari $9x^2$, $6xy$, $-3{,}6y^2$, $3{,}1y$ bo'lgan birhadlardan iborat.

$-2{,}8a^2b+9\frac{1}{7}b^2c-6\frac{5}{8}c$ ko'phadning hadlari esa $-2{,}8a^2b$, $9\frac{1}{7}b^2c$, $-6\frac{5}{8}c$ birhadlar bo'ladi.

Agar ko'phadning ba'zi hadlari standart shaklda yozilmagan bo'lsa, ularni standart shaklga keltirilib, soddalashtiriladi.

Masala. Ko'phadni soddalashtiring:

□ $4b^2\,5a-11abacc+\cancel{18}^{6}ab\frac{1}{\cancel{3}^{1}}b^2=20ab^2-11a^2bc^2+6ab^3$. ■

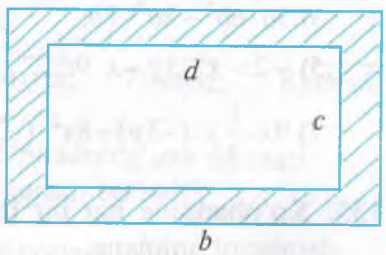
Ko'phadlar harfli ko'paytuvchilari bir xil bo'lmagan hadlari soniga qarab ikkihad, uchhad va hokazo deyiladi.

Ikkihadga misollar: a^3-b^2; $2a^2b+7bc$.

Uchhadga misollar: $3{,}1a+\frac{1}{2}a^2b-4c$; $\frac{2}{5}a+0{,}5b-2$.

Birhad ham hadlari soni bitta bo'lgan ko'phad hisoblanadi.

Masala. Shtrixlangan shaklning yuzini aniqlovchi ifoda – ko'phadni yozing va uning $a=12$; $b=10$; $c=3$; $d=6$ bo'lgandagi son qiymatini toping.



□ Ravshanki, $S=ab-cd$.

Bunga harflarning qiymatini qo'yamiz:

$S=12\cdot10-3\cdot6=120-36=84$. ■

Masala. Ko'phadni soddalashtiring va $x=-5$; $y=0{,}5$ bo'lganda uning son qiymatini toping: $-0{,}2xy^2x^2-0{,}2y\cdot10+8x$.

□ $-0{,}2xy^2x^2-0{,}2y\cdot10+8x=-0{,}2x^3y^2-2y+8x$.

$-0{,}2\cdot(-5)^3\cdot(0{,}5)^2-2\cdot0{,}5+8\cdot(-5)=-0{,}2\cdot(-125)\cdot0{,}25-1-40=$

$=6{,}25-41=-34{,}75$. ■

Ko‘phadni tashkil etgan birhadlarning eng katta darajasi ko‘phadning darajasi deyiladi.

Masalan, $9a^2b^2c^2-2,1ab^2c+4abc$ ko‘phad 6-, 4-, 3- darajali birhadlardan tashkil topgan. Ta’rifga ko‘ra, bu ko‘phad 6-darajali ko‘phaddir.

$3xy^2-2x^2y-\frac{1}{2}$ – ko‘phad 3 darajali;

$8b-7,5$ – ko‘phad esa 1-darajali ko‘phaddir.

?

130. 1) Ko‘phad deb nimaga aytiladi?
2) Ko‘phadning hadlari nima?
3) Birhadni ham ko‘phad deyish mumkinmi?
4) Ko‘phadning darajasi deb nimaga aytiladi?

131. (*Og‘zaki*). Ko‘phadni tashkil qiluvchi birhadlarni va ularning darajalarini ayting:

1) $-7a^3+3a^2-4a+1$; 2) $5a^2-0,5a+6$; 3) $-9x+0,7x-x^2$.

132. Ko‘phadni birhadlarning yig‘indisi shaklida yozing:

1) $3a^5+7a^4-1.2a-8$; 2) $2.2b^5+3b^4-$ [illegible] -17;

3) $8,5x^4y-4,1x^2y^2+1,5xy^3-y^4$; 4) $3.5x^4-1,8x^3y-1.2x^2y-4xy^2$.

133. Birhadlardan ko‘phadlar tuzing:

1) $-3x^2$; $4x$ va 8; 2) $2,1x^2$; $6x$ va 5;

3) x^3; $-x^2$ va $-x$; 4) b^6; $3b^5$ va $-2b^3$;

5) $9a^2$; $-3ab$; $-5ab^2$ va b^3; 6) $-5a^4b^2$; $-5,2a^3b^3$; $7ab^3$ va $-3,5a^2b^2$.

134. Ko‘phadning har bir hadini standart shaklga keltiring va soddalashtiring:

1) a^2b+bab; 2) $y\cdot xy+x^2y$;

3) $5a\cdot 6b^2-3a^2\cdot 7b$; 4) $3x\cdot 4y^2-5x^2\cdot 2x$;

5) $-2x\cdot x^2\cdot 5y+x\cdot 9c$; 6) $4c\cdot 3b\cdot c^2-c\cdot 5c$;

7) $9x\cdot\frac{1}{3}y\cdot(-3y)-8x^2\cdot(-3y)$; 8) $2x\cdot 5,5y\cdot(-3x^2)-(-4x)\cdot 5y^2$.

135. Ko‘phadning har bir hadini standart shaklga keltiring, soddalashtiring va darajasini aniqlang:

1) $15a^3\cdot\frac{1}{3}ba-7ab\cdot\frac{1}{4}a^2b+13aba$; 2) $2ac^2\cdot 0,4x-$ [illegible] $10aba-5abab^3$;

3) $2,5pq^3\cdot(-2)pqt-12abc\cdot\frac{1}{3}\cdot a^2bc$; 4) $3pp^3t\cdot\left(-\frac{5}{9}tx\right)-$ [illegible].

136. Ko‘phadni standart shaklga keltiring va darajasini aniqlang:

1) $2xxxx\cdot\left(-2\frac{3}{4}xyy\right)+5ppp\cdot 4t$; 2) $4,5ttt\cdot\left(-5\frac{2}{9}x+c\right)-12abc\cdot 3a^2bc^2$;

3) $(0,5xy)\cdot(-4x^2y^2)-9x^2y\cdot\left(\frac{5}{27}y\right)$; 4) $(-0,3a)\cdot(4ab^2)-(8b^2)\cdot\left(\frac{1}{4}a^2b\right)$.

137. Ko‘phadning son qiymatini toping:

1) $3a^2+0,5ab+0,1b^2$, bunda $a=4;\ b=-10$;

2) $2a^5-0,01ab+2b^2$, bunda $a=-1;\ b=-0,4$;

3) $y^3-2yt+t^2$, bunda $y=t=-0,2$;

4) $0,1\,x^2+3xy+4y^2$, bunda $x=y=-10$.

138. Ko‘phadni soddalashtiring va uning son qiymatini toping:

1) $a^2aba4b+a^3bbb3a+5$, bunda $a=-2;\ b=0,5$;

2) $a^24aa\ bb5b+2a5ba^2+2$, bunda $a=-1;\ b=3$;

3) $-0,5aa\,2bbb^2+aa^2ba-ab^2$, bunda $a=5;\ b=-1$;

4) $-0,2\ ab^2a^2b-abab-5$, bunda $a=-5;\ b=\frac{1}{2}$.

139. Ko‘phadni harflarning darajalarini kamayib borish tartibida yozing:

1) $5x^6+7-3x^8+x^5-x^2$; 2) $2y^8-y^{10}-4y^3-12$;

3) $0,5a^9+2a^5-3,5a^{10}-2+7a^3$; 4) $5x^6+0,5x^8-x+x^2+7$.

140. Ko‘phadni: 1) x ning; 2) y ning darajalarini ortib borish tartibida yozing:
$18x^7-x^2y-5x^{10}y^8-3x^{13}y^7+y^{10}$.

141. Quyidagi sonlarni ko‘phad ko‘rinishida yozing:

1)$\overline{ab}$; 2)$\overline{xy}$; 3)$\overline{abc}$; 4)$\overline{xyz}$; 5)$\overline{abcd}$; 6)$\overline{xyzt}$; 7)$\overline{mnfkl}$; 8)$\overline{abcde}$.

142. a ning qanday qiymatida $5a^6+15^3+25a+7$ ko‘phadning son qiymati
a) juft; b) toq; c) 5 ga karrali bo‘ladi?

143. Ko‘phadlarning qiymati manfiy bo‘lmasligini isbotlang:

1) a^2+b^2+4; 2) $x^4+3x^2y^2+y^4+y^2+1$;

3) $x^6+y^6+x^6y^6+x^{12}y^{16}$; 4) $a^8+a^4b^4+a^{10}+b^2+b^8$.

144. a^2+b^2; x^2+y^2+6; $4x^2y^2$; y^8+3y^4-8, $1+y^3$ ko‘phadlar ichida harflarning ixtiyoriy qiymatlarida musbat qiymat qabul qiladiganlarini aniqlang.

6-§. O'xshash hadlarni ixchamlash

Siz 5- va 6- sinflarda $5a+0{,}3a+5$ (1) va $4x-6x+7-3x$ (2) kabi ifodalar va ularning o'xshash hadlarini soddalashtirish-ixchamlash bilan tanishgansiz:

$$5a+0{,}3a+5=5{,}3a+5; \quad 4x-6x+7-3x=-5x+7.$$

Bu misollar ko'phadlarning o'xshash hadlarini ixchamlashga misol bo'la oladi. (1) ko'phadda $5a$; $0{,}3a$ hadlar, (2) ko'phadda $4x$; $-6x$; $-3x$ hadlar o'xshash bo'lib, ular faqat koeffitsiyentlari bilangina farq qiladi.

Endi $15a^2b+8+9ab-12a^2b-15$ (3) ko'phadni ko'rib chiqamiz. Bu ko'phadda 1- va 4- hadlarning harfiy qismlari bir xil.

Agar ko'phadda bir xil harfiy qismlarga ega bo'lgan hadlar mavjud bo'lsa, ular *o'xshash hadlar* deyiladi.

Masalan, yuqoridagi (3) ko'phadda $15a^2b$ va $-12a^2b$ hadlar o'xshash.

Harfiy qismlari bo'lmagan 8 va -15 sonlar ham o'xshash deb qabul qilinadi, chunki ularni bu sonlar bilan harflarning nolinchi darajasi ko'paytmasi ko'rinishida ifodalash mumkin: $8=8a^0b^0$ va $-15=-15a^0b^0$.

Misol. $15a^2b+8+9ab-12a^2b-15$ ko'phadning o'xshash hadlarini ixchamlaymiz.

□ $15a^2b+8+9ab-12a^2b-15=(15a^2b-12a^2b)+9ab+(8-15)=$
$=(15-12)a^2b+9ab+(-7)=3a^2b+9ab-7.$ ■

Ko'phadlarni o'xshash hadlarining algebraik yig'indisi bitta bo'lgan birhad bilan almashtirish orqali soddalashtirish *o'xshash hadlarni ixchamlash* deyiladi.

$3a^2b+ab-7$ ko'phadda har bir hadi standart shaklda bo'lib, ular orasida o'xshash hadlar yo'q.

Har bir hadi standart shaklga keltirilgan va o'xshash hadlari bo'lmagan ko'phadlar *standart shaklda yozilgan ko'phadlar* deyiladi.

Har qanday ko'phadni standart shaklda yozish mumkin. Buning uchun avval ko'phadning har bir hadini standart shaklda yozish va so'ngra o'xshash hadlarini ixchamlash kerak.

Masala. Ko'phadni standart shaklga keltiring:

$$15xy\cdot\frac{1}{5}xz+4xyx-9x^2\cdot\frac{1}{3}y-xyx+x^2yz.$$

□ $15xy\cdot\frac{1}{5}xz+4xyx-9x^2\cdot\frac{1}{3}y-4xyx+x^2yz=3x^2yz+4x^2y-3x^2y-4x^2y+x^2yz=$
$=4x^2yz-3x^2y.$ ■

145. 1) Qanday birhadlar o'xshash deyiladi?
2) O'xshash hadlar qanday ixchamlanadi?
3) Ko'phadning standart shakli nima?
4) Ko'phad standart shaklga qanday keltiriladi?

146. O'xshash hadlarni ixchamlang:

1) $0,5x^2 - x^2 + 0,8x^2 + 3x^2$;
2) $0,8a^2 - a^2 + 2,2a^2 + 5a^3$;
3) $y^3 - \frac{1}{3}y^3 + \frac{1}{3}y^3 + 4y^3$;
4) $3b^3 - \frac{3}{4}b^3 + \frac{3}{4}b^3 + b^3$;
5) $-2xy^2 - xy^2 + 0,8xy^2 - 1,9xy^2$;
6) $-5ab^3 - ab^3 - 0,2ab^3 + 9,2ab^2$;
7) $\frac{1}{3}xy + xy - \frac{1}{6}xy + 4xy$;
8) $\frac{1}{9}ab + ab - \frac{2}{9}ab + 3ab$.

147. Ko'phadni standart shaklga keltiring:

1) $18a^2 - 5a - a^2 - 3a$;
2) $4b^2 - 3b + 2b - 5b^2$;
3) $0,8c^2 - 0,1c^2 - 0,6c^3$;
4) $3,2m^2 + 2,4m^3 - 1,8m^2$.

148. Ko'phadning son qiymatini toping:

1) $0,2a^4 + a^2 + 0,1a^4 - 8 - 1,3a^4 + 0,2a^2$, bunda $a = 12$;
2) $\frac{1}{8}b^5 + \frac{1}{12} - \frac{1}{8}b^5 - b^4 - \frac{1}{3}b^4$, bunda $b = -2$;
3) $9x^5 + \frac{1}{3}x^3 + 0,5x^4 - \frac{1}{2}x + 1$, bunda $x = -1$;
4) $-3y^3 - \frac{5}{6}y + \frac{3}{4}y^2 + 8$, bunda $y = 2$.

149. O'xshash hadlarni ixchamlang va ko'phadning darajasini aniqlang:

1) $13t^2 - 2t^2 - 10t - 3t^2 + 4t + 11$;
2) $x^2 + 8x - 21 - x^3 \cdot 12x^2 + 8x - 18$;
3) $n^3 + n^2 + n + 1 - n^4 - n^3 - n^2 - n - 1$;
4) $-x^2 - x^3 - x^4 - 1 + x^2 + x^3 + 9$;
5) $2a^2 - 7ab - 5a^2 - 11ab + 13c^2$;
6) $4x^2 + y^2 + 6xy - 12y^2 - 8xy$;
7) $11p^2q + 8pq^2 + 5pq^2 - 13p^2q - 11p^3$;
8) $15a^3b^3 - 13ab^3 - 2a^2b^2 + 15a^3b^3 + ab - a^2b^2$.

150. Ifodani soddalashtiring va uning qiymatini toping:

1) $-a - 3b - 4 + 2b$, bunda $a = -14$, $y = -5$;
2) $-15x - y - 2 + 14x$, bunda $x = -19$, $y = -2$;
3) $2ab - 2a - b + 2b$, bunda $a = -3$, $b = 7$;
4) $ab - 6a + a + 7b$, bunda $a = b = -3$;
5) $3ab^3 - a^2 + b^2 - 2ab^2 + a^3b - b^4$, bunda $a = 1$, $b = -1$;
6) $n^4 - 3n^3m + n^2m^2 + n^3m - 4nm^3$, bunda $n = -1$, $m = 1$.

151. * o'rniga shunday birhad yozingki, natijada 6-darajali ko'phad hosil bo'lsin:

1) $x^3-2x^2+3x-1+*$; 3) y^6-2y^4-3y+*;

2) $2a^3-a^5+2a^4+*$; 4) $x^3y^3-x^2y^2+xy+*$.

152. Quyudagi ko'phadlardan, undagi harflarning har qanday qiymatida a) musbat; b) manfiy bo'ladiganini ko'rsating:

1) a^5+a^3+a; $-3a^2-1$; $a^6+a^4+a^2+3$;

2) a^2+b^2+a; $-a^2-b^2-a^2b^2-16$; $-a-b-8$.

153. Ko'phadni standart shaklga keltiring:

1) $18x^3-13xxy+3x^3-2x^2-8y+4$; 2) $3y\cdot y^3-3y^2\cdot 4y^2+6y^2-8y-1$

3) $5a\cdot\left(-4b^4\right)-3x^2x^3+27x^3-x^6$; 4) $18x\left(-x^2y\right)+16x^3y-14xxy+12$

154. Ifodani soddalashtiring:

1) $15a\cdot a^3-6a^2\cdot a^2-16a^4+4a^2-a^2$;

2) $0,3x\cdot 4y^2-1,2xy\cdot y+4,8xy^3-6x\cdot 0,8y^3$;

3) $5a^2\cdot 2b-10ab\cdot 4a-14a^2b+18a^2b$;

4) $\frac{2}{9}p\cdot 3q^2-\frac{7}{13}pq\cdot 26q+\frac{1}{3}pq\cdot 4q+1,2pq\cdot 5pq$;

5) $0,8a^nb^2-0,01b\cdot 12b^{n+1}-1,6b^{n+2}$;

6) $3,2t^2t^nt-3,4t^{n+1}\cdot 2t^2-4,8t^{n+2}\cdot 0,1t+t^{n+3}$.

7-§. Ko'phadlarni qo'shish va ayirish

Misol. Ifodalarni soddalashtiring:

1) $(3x^2+44x-8)+(5x+2)$;

3) $(x^3+2y^3+3x-4y+7)-(x^3-2y^3-4x-5y+8)$;

2) $(3a^2+4ab-b^2)+(2a^3-a^2+4ab-b^2+7)+(a^2-2ab+b^2-3)$.

□ Qavslarni ochish qoidasiga ko'ra o'xshash hadlarni e'tiborga olib, ifodani quyidagicha soddalashtirish mumkin:

1) $\left(3x^2+4x-8\right)+\left(5x+2\right)=3x^2+4x-8+5x+2=3x^2+9x-6$;

2) $\left(3a^2+4ab-b^2\right)+\left(2a^3-a^2+4ab-b^2+7\right)+\left(a^2-2ab+b^2-3\right)=$

$=3a^2+4ab-b^2+2a^3-a^2+4ab-b^2+7+a^2-2ab+b^2-3=3a^2+6ab-b^2+2a^3-$

3) $(x^3+2y^3+3x-4y+7)-(x^3-2y^3-4x-5y+8)=x^3+2y^3+3x-4y+7-x^3+2y^3+$
$+4x+5y-8=4y^3+7x+y-1$. ■

1-, 2-, 3-misollarda qavs ichidagi har bir ifoda ko'phad ekani ravshan. 1-, 2-misollarda ko'phadlarni qo'shishni; 3-misolda esa ko'phadlarni ayirishni bajardik. Natijada, yana ko'phad hosil bo'ldi.

Demak, bir nechta ko'phadning algebraik yig'indisini standart shaklidagi ko'phad ko'rinishida yozish uchun qavslarni ochish va o'xshash hadlarni ixchamlash kerak.

Ko'phadlarning yig'indisi yoki ayirmasini sonlarni qo'shish va ayirishga o'xshash "ustun" usulida topish qulay bo'ladi. Bunda o'xshash hadlar birining ostiga ikkinchisi turadigan qilib yoziladi.

Masalan:

1) $\begin{array}{r} 6a-3bc+2ac \\ + \\ 4bc-9ac \\ \hline 6a+bc-7ac \end{array}$;

2) $\begin{array}{r} 15xyz-3xy+9xz-11yz \\ - \\ 6xyz-5xy-7xz+5yz \\ \hline 9xyz+2xy+16xz-16yz \end{array}$.

?

155. 1) Ko'phadlar qanday qo'shiladi va ayriladi?
2) Ko'phadlar yig'indisi yana ko'phad bo'ladimi? Ayirmasi-chi?
3) Ko'phadning yig'indisi va ayirmasi "ustun" usulida qanday aniqlanadi?
4) Ko'phadlar yig'indisi standart shaklga qanday keltiriladi?

Ko'phadlar yig'indisi va ayirmasini toping hamda ularni standart ko'rinishga keltiring **(156–157)**:

156. 1) $8x^2-6x+4$ va $8x^2-6$;
2) $4a^2+8a-6$ va $4a^2+2$;
3) $6a+4$ va $-3a^2+2a+8$;
4) $4x+2$ va $-4x^2-4x+1$;
5) $a+4b$ va $4a-4b$;
6) $x+7y$ va $3-7y$;
7) $a^2-6ab-b^2$ va a^2+b^2;
8) $x^2-4xy+y^2$ va x^2-y^2.

157. 1) $3y^2+9y-12$ va $4y^2-7y+4$;
2) $6y^2-4y+1$ va $9y^2+3y-12$;
3) $10a^3-a-4$ va $9a^2+a-5$;
4) $3a^2+4a-3$ va $6a^3-4a+3$;
5) $5m^4+5m^2-14$ va $5m^5-5m^2+14$;
6) $x^2-4x+16$ va $x^3+4x-16$;
7) $3p^2+4pq+9q^2$ va $7p^2-pq-9q^2$;
8) $9x^2+3px-4p^2$ va $3x^2+4px-4p^2$.

158. Ifodani soddalashtiring:
1) $(4a+6b)+(10a-8b)+(-6a+12b)$;
2) $(3a+6b)+(9a-12b)+(10b-6a)$;
3) $(3x-12y)-(6x+13y)+(4x-18y)$;
4) $(4x+11y)-(7x+4y)+(7x-9y)$;
5) $(4b^2+3b)+(3b^2-4b-5)-(-b^2+19)$;

6) $(9c^2+4c)+(-8c^2-12c+4)-(-4c^2-5)$;
7) $(2a-b+c)+(3a-2c)-(a-2b-c)$;
8) $(v-2b-k)+(2v-a)-(v-4+2k)$.

159. Avtobus 4 soat yo'l yurdi. U 1-soatda x km va har bir keyingi soatlarda avvalgisidan 5 km ko'p yo'l o'tdi. Avtobusning
1) 2-soatda;
2) 3-soatda;
3) dastlabki 2 soatda;
4) oxirgi 2 soatda;
5) butun yo'l davomida qancha masofa bosib o'tganini aniqlang.

160. Velosipedchi 5 soat yo'l yurdi. U 1 soatda y km va keyingi har bir soatda avvalgisiga qaraganda 1 km kam yo'l bosib o'tdi. Velosipedchi: 1) 2 soatda; 2) 3 soatda; 3) dastlabki 3 soatda; 4) oxirgi 2 soatda; 5) butun yo'l davomida necha kilometr yo'l bosib o'tdi?

161. Ko'phadlarning algebraik yig'indisini toping:

1) $\left(5\frac{2}{5}a-\frac{1}{4}a^2\right)+\left(\frac{3}{4}a^2-2\frac{3}{5}a\right)$;

2) $\left(9\frac{5}{7}x^2-\frac{5}{6}x\right)+\left(-\frac{1}{6}x+1\frac{2}{7}x^2\right)$;

3) $(0,5b-0,6b^2)-0,5b-0,7b^2$;
4) $(3,6k^3+1,5k)-(1,5k-4,4k^3)$;
5) $(21,2a-6,2b+11,1c)-(11,2a-3,2+1,9c)$;
6) $(8,1x+9,7y-3,9z)-(15,1x-0,3y+1,1z)$.

162. 1) $(9k^2-5pk-p^2)-(3k^2-pk+p^2)$;
2) $(6p^2-12pk+t^2)+(-13t^2-9p^2+6pt)$;
3) $21ac+23bc+27b^2-(20ac+20bc-13b^2)$;
4) $51z+23az+36az^2-(26z+23az-14az^2)$.

163. 1) $\left(\frac{1}{3}b+\frac{1}{4}c\right)-\left(\frac{1}{3}b-\frac{3}{4}c\right)+(b+c)$; 2) $\left(\frac{7}{9}p-\frac{2}{3}q\right)-\left(2\frac{7}{9}p-\frac{4}{5}q\right)+(p-q)$;

3) $(0,7a-1,3b)+(1,2a-b)-(-1,3a+0,7b)$;
4) $(8,1k-9,3n)+2m-n-(-3,2k+0,3n)$.

164. 1) $(-4a^3+ab^2)+(3a^2b-2)+(a^2b-ab^2+5a^3)$;
2) $(13b^2+15bc+12b^2c)-(15bc+13b^2)-(12b^2c-13b^2)$;
3) $-xy-(x^2+xy)+(2xy-x^2)$;
4) $0,6x^2y+(0,4x^2y-x)-(x^2y+y)$.

165. Ko‘phadlar yig‘indisini ustun usulida toping:

1) $5ab^2+a^3-4b^3$ va $3a^3-4ab^2$;

2) $4p^2+5pq-9q^2$ va $9q^2-5pq+4p^2q^2-p^3$;

3) $6{,}5p^2+3pq-4{,}7q^2$ va $-0{,}8p^2+pq-0{,}8q^2$;

4) $1\frac{5}{8}x^2-3\frac{5}{6}xy+5\frac{5}{6}y^2$ va $4\frac{3}{8}x^2-4\frac{1}{6}xy+3\frac{1}{6}y^2$.

166. Ko‘phadlar ayirmasini ustun usulida toping:

1) $2y^2+6y-2$ va $1+6y-3y^2$;

2) t^4-2t^2+3t va $t+t^2+t^4$;

3) $8\frac{2}{5}a^2+5\frac{2}{7}ab-4\frac{1}{3}b^2-7$ va $-2\frac{3}{5}a^2-3\frac{5}{7}ab+2\frac{2}{3}b^2+8$;

4) $8{,}2x^2+4{,}5xy-2{,}7y^2-9$ va $-6{,}5x^2-3xy+0{,}3y^2+8$.

167.* 1) Agar $A=6a^2+3a+b$ va $B=-5a^2-2a^2-b$ bo‘lsa, $A+B$ ifoda nimaga teng?

2) Agar $A=3p^2-4q^3+5$ va $B=3p^2-5q^3-4$ bo‘lsa, $A+B$ ifoda nimaga teng?

3) Agar $A=-9x^3-3x+2x^2+1$ va $B=3x^2+3x+9x^3+7$ bo‘lsa, $A-B$ ifoda nimaga teng?

4) Agar $A=8k-15k^2+9$ va $B=9+8k+12k^2$ bo‘lsa, $A-B$ ifoda nimaga teng?

5) Agar $A=x^2-3y^2+2xy$; $B=4x^2-xy-11b^2$ va $C=-5x^2-3xy+4$ bo‘lsa, $A+B-C$ ifoda nimaga teng?

6) Agar $A=3p^2-2pq+8q^2$; $B=9p^2-5pq-5q^2$ va $C=p^2+3pq+2q^2$ bo‘lsa, $A-B+C$ ifoda nimaga teng?

168. Sonlar yig‘indisini ko‘phad ko‘rinishida tasvirlang:

1) $\overline{ab}+\overline{bc}$; 2) $\overline{cd}+\overline{ab}$; 3) $\overline{abc}+\overline{bc}$;

4) $\overline{bcd}+\overline{cd}$; 5) $\overline{abc}+\overline{bca}+\overline{acb}$; 6) $\overline{abc}+\overline{bcd}+\overline{acd}$.

169. Qavslarni oching va o‘xshash hadlarni ixchamlang:

1) $9{,}5a-(2{,}5a-(0{,}4a-(0{,}5a+2{,}1a)))$;

2) $3{,}4b-(1{,}6b+(8{,}2b-(1{,}25b-8{,}5b)))$;

3) $4{,}2t-(6{,}5c+(1{,}38k-5{,}4t-(2{,}1t+3{,}6c))+1{,}8t-(3{,}2c+6k))$;

4) $1{,}8p-(4{,}8q+(8z-1{,}2p-(3{,}4p+2{,}1z)))-(4{,}2p-(8q+5{,}2z))$.

170.* $15b^4-3b^2+2b-8+b^3$ ko‘phadni:

1) 1 ta qo‘shiluvchisi $(15b^4+2b)$ bo‘lgan 2 ta ko‘phad yig‘indisi;

2) 1 ta qo‘shiluvchisi $(15b^4-8)$ bo‘lgan 2 ta ko‘phad yig‘indisi;

3) kamayuvchisi $(15b^4-3b^2)$ bo‘lgan 2 ta ko‘phad ayirmasi;

4) kamayuvchisi $(2b^2-8)$ bo‘lgan 2 ta ko‘phad ayirmasi;

5) birhad va to‘rthad ayirmasi;

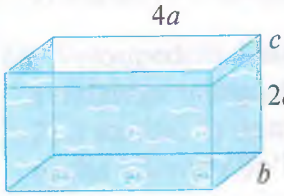
6) ikkihad va uchhad ayirmasi shaklida ifodalang.

171. Isbotlang:

1) uchta ketma-ket natural sonlar yig'indisi 3 ga bo'linadi;
2) beshta ketma-ket natural sonlar yig'indisi 5 ga bo'linadi;
3) to'rtta ketma-ket natural sonlar yig'indisi 4 ga bo'linmaydi;
4) oltita ketma-ket natural sonlar yig'indisi 6 ga bo'linmaydi.

8-§. Ko'phadni birhadga ko'paytirish

Rasmda ko'rsatilgan akvariumga to'lguncha suv quyildi. Akvariumning o'lchamlari detsimetrlarda ifodalansa, unga qancha suv ketadi?



□ Rasmdagi akvarium to'g'ri burchakli parallelepiped shaklida. Bu parallelepipedning hajmi asos yuzi va balandligi ko'paytmasiga teng: $V=(4ab)(2a+c)$ (dm^3).

Bu ifoda $2a+c$ ko'phad bilan $4ab$ birhadning ko'paytmasidir.

Ko'paytirishning taqsimot qonunini qo'llab, quyidagilarni yozish mumkin: $(4ab)(2a+c)=4ab\cdot 2a+4ab\cdot c=8a^2b+4abc$. Demak, akvariumning sig'imi $V=8a^2b+4abc$ (litr). ■ Bundan shunday natijaga kelamiz:

Ko'phadni birhadga ko'paytirish uchun ko'phadning har bir hadini shu birhadga ko'paytirish va hosil bo'lgan ko'paytmalarni qo'shish kerak.

Misollar. Ko'phadni birhadga ko'paytiring:

1) $(5x^2-y-3xy^2)\cdot(-7xy)$; 2) $\left(-4n^2m+3n^2-2m^2\right)\cdot\left(-\frac{1}{2}nm\right)$;

3) $5tz\left(\frac{1}{5}t^2-\frac{3}{10}z+3\right)$.

□ 1) $(5x^2-y-3xy^2)\cdot(-7xy)=5x^2\cdot(-7xy)-y\cdot(-7xy)-3xy^2\cdot(-7xy)=$
$=-35x^3y+7xy^2+21x^2y^3$.

2) Ko'phadning har bir hadini birhadga ko'paytmasini yozmasdan og'zaki ko'paytirib, birdaniga javobini yozish mumkin:

$$\left(-4n^2m+3n^2-2m^2\right)\cdot\left(-\frac{1}{2}nm\right)=2n^3m^2-\frac{3}{2}n^2m+nm^3.$$

3) Misolni yechishda ko'paytirishning o'rin almashtirish xossasini e'tiborga olsak, birhadni ko'phadga ko'paytirish ham yuqoridagiga o'xshash bajariladi:

$$5tz\cdot\left(\frac{1}{5}t^2-\frac{3}{10}t+3\right)=t^3z-\frac{3}{2}t^2z+15tz.$$ ■

?

172. 1) Ko'phad birhadga qanday ko'paytiriladi? Birhad ko'phadga-chi?
2) Ko'phadning birhadga ko'paytmasi yana ko'phad bo'ladimi?

Ko'paytmani toping **(173–175)**:

173. 1) $-0,7\cdot(8a+3b)$;

2) $-1,2\cdot(1,8x+6y)$;

3) $\left(5,5y-\frac{4}{5}\right)\cdot\left(-\frac{2}{5}\right)$;

4) $\left(9,6t-\frac{5}{8}\right)\cdot\left(-\frac{5}{12}\right)$;

5) $\frac{1}{8}\cdot(16k^2-8k+8)$;

6) $\frac{7}{11}\cdot\left(1\frac{4}{7}c^2+11c\right)$;

7) $-7,2\cdot\left(0,5x^3-1\frac{1}{2}x^2+x+5\right)$;

8) $-3,6\cdot\left(2y^2-1\frac{1}{6}x^3+7-5x\right)$.

174. 1) $3a^3\cdot(-9a+8b)$;

2) $8x^2\cdot(3x+7k)$;

3) $-0,7ab\cdot(a^3+b^3)$;

4) $(-0,8bc)\cdot(9c-2b)$;

5) $(-4a^2b+3ab-8a)\cdot(-ab)$;

6) $(x^5y-x^4y^2+y+1)\cdot(-x)$;

7) $(4y^4+3y^3-2y+6)\cdot\left(-\frac{1}{12}y^2\right)$;

8) $(6x^3+4x^2-2)\cdot\left(-\frac{5}{6}x^2y\right)$.

175. 1) $-0,5a\cdot(a^3+1,6a-0,8a^2)\cdot a^2$;

2) $-8,2x^3\cdot(3x+x^2-5)\cdot x^3$;

3) $\frac{5}{6}ab\cdot(0,2a^2-1,8ab+1)\cdot a^3$;

4) $\frac{7}{9}xy^2\cdot(1,8x^2-2,7xy+15)\cdot y$;

5) $-\frac{2}{11}\cdot\left(1,1a^2b^2-1,21ab+5\frac{1}{2}\right)\cdot(-a-b)$;

6) $t^3\cdot\left(5,2t^2p+13t^2p^2-6\frac{1}{2}\right)\cdot\frac{2}{13}t^3p^3$.

176. Ko'phad ko'rinishida tasvirlang:

1) $-0,7a\cdot\left(a^3-a\cdot(2,1a^2-5,1)\right)\cdot a^3$;

2) $\frac{2}{5}xy\cdot\left(0,5x^2-1,5\cdot\left(xy-\frac{2}{3}\right)\right)\cdot x^3$;

3) $-\frac{1}{8}p\cdot(1,6pq\cdot(pq-2)+0,8)\cdot(-4q)$;

4) $-t^3\cdot(-0,2tp(2t-1)+5)\cdot\frac{1}{8}t^2p^2$.

Ifodani soddalashtiring **(177–178)**

177. 1) $5a^2+3a\cdot(4-a)$;
2) $9x^3-2x\cdot(0{,}5+3x^2)$;
3) $-12y^3-4y\cdot(1-4y^2)$;
4) $3k\cdot(k^2-t^2)-1{,}5k^2$;
5) $1{,}6p\cdot(p^2-1)-2p^2(4-p)$;
6) $2{,}5a\cdot(4a^2+a)+1{,}2a^2(5-a)$;
7) $-1{,}5n\cdot(n+m^2)+0{,}5n\cdot(3n-m^2)$;
8) $4t\cdot(t^2-p)-4p\cdot(p^2-t)$.

178. 1) $6a^2-2a(a-4)+5a(1-a)$;
2) $15m^4-3m^3(m-3)+(m^2+3m)\cdot 8m^2$;
3) $-4y^3(2+5y^2)-(y^2+3)\cdot 5y^3+(3y-1)\cdot 2y^4$;
4) $-5{,}5p+(-p^2+1{,}2p^2-6)\cdot(-1{,}5p)-3{,}5p^3\cdot p$.

179. Harfning qanday qiymatida tenglik to‘g‘ri bo‘ladi?
1) $2x(1-3x)+1{,}5x(4x-2)=-5$;
2) $5x(2x+4)-2{,}5x(4x+2)=10$;
3) $-x^2(5-4x^2)-4x(x^3-1{,}25x)+5x=8$;
4) $-2x\left(3-x-5x^2\right)+3x\left(-\frac{2}{3}x-1-3\frac{1}{3}x^2\right)=27$.

180. Ifodaning son qiymatini toping:
1) $8(0{,}2x^2-0{,}4x+1{,}5)-0{,}4(3x^2+5x-3)$, bunda $x=-21$;
2) $5(0{,}4y^2-3{,}6y+1{,}6)-2(y^2-0{,}9y-0{,}6)$, bunda $y=-5$;
3) $\left(\frac{3}{4}x+\frac{2}{3}y\right)\cdot 12xy-24xy\cdot\left(\frac{3}{8}x+\frac{7}{12}y\right)$, bunda $x=-1{,}5;\ y=2{,}3$;
4) $-6ab\left(\frac{2}{3}ab-2\right)+5a^2\left(\frac{2}{5}b^2-3\right)$, bunda $a=-\frac{1}{3};\ b=-6$.

181. Berilgan ifodalardan b ga bog‘liq bo‘lmaganini aniqlang:
1) $8b(5+b)-2b(5b-1)-25(b+3)-b+5$;
2) $3(6b^2-2)-4(2b+5)-5(b^2+2)-b^2+7$;
3) $-b(4c^2+b)-2b(b-2c^2)+3b(b+1)-(3+3b)$;
4) $9b(b+y)+3b(4b-y)-3b(b+2y)-6y$.

182. Ko‘phad ko‘rinishida ifodalang:
1) $3a^kb\left(\frac{2}{3}a-\frac{5}{6}b^k\right)$;
2) $-36a^kb^n\left(\frac{5}{6}ab^3-\frac{3}{4}a^4b\right)$;
3) $\left(\frac{3}{4}a^{3n}b^n-\frac{1}{8}a^nb^{2n}\right)(-12ab)$;
4) $\left(16a^{3n}b-12a^{2n}\right)\left(-\frac{1}{4}a^nb^2\right)$.

183. * o‘rniga shunday birhad qo‘yingki, natijada to‘g‘ri tenglik hosil bo‘lsin:
1) $*\cdot(x^3+3xy+2xy^2+y^3)=1{,}2x^4y+3{,}6x^3y^2+2{,}4x^2y^3+1{,}2xy^4$;
2) $*\cdot(ab^2+0{,}1a^2b-4ab+5b^2)=2a^3b^3+0{,}2a^4b^2-8a^2b^2+10a^2b^3$;
3) $*\cdot(5a^4+10a^3+15a^2+20a)=*+*+*+4a^2$;
4) $*\cdot(0{,}5ab-0{,}2a^2b-b^2)=2{,}5a^2b^3-*-*$.

184. Ko‘phadni standart shaklga keltiring va uning darajasini aniqlang:
1) $a^5b(b^4+ab^5-a^2b^6+a^3b^7)$;
2) $2x^2y(3y^3+2y^2-x-x^2)$;
3) $(p^4-p^3q+p^2q^2-pq^3)pqt$;
4) $(2a^2+3a^2-b-b^2)abc$.

5) $3t(5t^3-2t-pt+p^3)\cdot p$;
6) $2k(6k^3-k^2-6k+k^3)\cdot k$;
7) $-pq(p^2q^2-pq-5)a$;
8) $-xy(x^2y-xy^2-x^3y^3)\cdot z$.

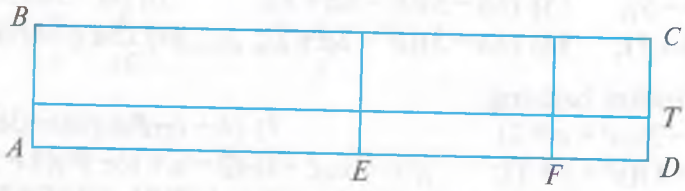
185. Agar $m>1$ bo'lsa, ifodani soddalashtiring:

a) $\left(0,8b^{m+3}-\frac{1}{6}b^m-0,5b^{m-1}\right)\cdot 1,2b^m-9b^m\cdot\left(\frac{1}{9}b^{m-1}-b^m+0,2b^{m+1}\right)$;

b) $\left(-1\frac{1}{3}c^{m-1}+\frac{1}{9}c^m+3c^3\right)\cdot 0,3c^{m+1}-0,4c^m\cdot\left(\frac{3}{4}c^m-c^{m+1}-1\frac{1}{4}c^4\right)$.

9-§. Ko'phadni ko'phadga ko'paytirish

Masala. $ABCD$ to'g'ri to'rtburchakda $AE=3a$; $EF=2b$; $FD=c$; $CT=k$; $TD=0,5d$ bo'lsa, uning yuzini aniqlash uchun ifoda tuzing.



□ To'g'ri to'rtburchakning tomonlarini aniqlaymiz:
$AD=AE+EF+FD=3a+2b+c$; $CD=CT+TD=k+0,5d$, u holda $ABCD$ to'g'ri to'rtburchakning yuzi $S=AD\cdot CD=(3a+2b+c)(k+0,5d)$.

Bu ifoda $(3a+2b+c)$ va $(k+0,5d)$ ko'phadlarning ko'paytmasidan iborat. Ko'paytirishning taqsimot qonunini, ko'phadlarni birhadlarga ko'paytirish qoidalarini qo'llab quyidagiga ega bo'lamiz:
$S=(3a+2b+c)(k+0,5d)=(3a+2b+c)\cdot k+(3a+2b+c)\cdot 0,5d=3ak+2bk+ck+$
$+1,5ad+bd+0,5cd$. ■

Demak, ko'phadni ko'phadga ko'paytirish uchun birinchi ko'phadning har bir hadini ikkinchi ko'phadning har bir hadiga ko'paytirish va hosil bo'lgan ko'paytmalarni qo'shish kerak.

Ko'phadni ko'phadga ko'paytirish natijasida yana ko'phad hosil bo'ladi. Hosil bo'lgan ko'phadni standart shaklida yozish kerak.

Misollar. Ko'paytmani bajaring:

□ 1) $(3a-4b)(0,2a+5b)=0,6a^2-0,8ab+15ab-20b^2=0,6a^2+14,2ab+2b^2$.

2) $(x-3y+4z)(-7y+5z)=-7xy+21y^2-28yz+5xz-15yz+20z^2=-7xy+21y^2-43yz+5xz+20z^2$.

3) Bir nechta ko'phadlarni ko'paytirish uchun ko'paytirishni navbatma-navbat bajarish kerak.
$(m-n)(m-3n)(2m+5n)=(m^2-mn-3mn+3n^2)(2m+5n)=(m^2-5mn+3n^2)$
$(2m+5n)=2m^2-10m^2n+6mn^2+5m^2n-25mn^2+15n^3=2m^3-5m^2n-19mn^2+15n^3$. ■

186. 1. Ko‘phadni ko‘phadga qanday ko‘paytirish kerak? Misollarda tushuntiring.
2. Ko‘phadlar ustun usulida qanday ko‘paytiriladi deb o‘ylaysiz? Misollar tuzing.
3. Ko‘phad ko‘phadga ko‘paytirilgach, ko‘paytma standart shaklda qanday yoziladi?

187. Ko‘phadlarni ko‘paytiring:

1) $(x+5)(y-8)$;
2) $(x-2)(y-6)$;
3) $(a-9)(11-b)$;
4) $(a-3)(8-b)$;
5) $(-15-a)(b+4)$;
6) $(-8-b)(a-9)$;
7) $(2x+3)(2y-5)$;
8) $(5a-1)(2b+7)$;
9) $(4a-3b)(5a-2b)$;
10) $(12x+7)(5x-3)$;
11) $(8x^2+1)(4x-1)$;
12) $(3x^2-1)(5x+1)$;
13) $(x-3)(x^2-x-5)$;
14) $(m+n)(m^2-2m-4)$;
15) $(6b-5)(b^2-6b+8)$;
16) $(8a-3)(a^2-2a+5)$;
17) $3(b-5)(b+3)$;
18) $4(x+8)(x-7)$;
19) $-9(y-1)(y+5)$;
20) $-5(2-a)(a+7)$;
21) $3b(2b+1)(3b-4)$;
22) $5n(n-m)(n+3m)$;
23) $(2m-5n)(5m^2-6mn+7n^2)$;
24) $(8a-3b)(9a^2-2ab+b^2)$.

188. Ko‘paytirishni bajaring:

1) $(a^2-a-2)(a^2+a+2)$
2) $(y^2-y-1)(y^2+y+1)$;
3) $(3m^2-2m+1)(-3m^2+2m-1)$;
4) $(-2n^2+3k+1)(2n^2+3n-1)$;
5) $(a+1)(a^4-a^3+a^2-a+1)$;
6) $(x-1)(x^4+x^3-x^2+x-1)$;
7) $(4+t-t^2+t^3)(t-3)$;
8) $(3-y+y^2-y^3)(1-y)$;
9) $(x+5)(x-2)(x^2+3y-10)$;
10) $(a+2)(a-3)(a^2-a-6)$;
11) $(t+1)(t^2-t+1)(t^6-t^3+1)$;
12) $(x-1)(x^2+x+1)(x^6+x^3+1)$.

189. Ifodani ko‘phad ko‘rinishida yozing:

1) $(3t-3)(2t+8)+7$;
2) $(2a-16)(8a-1)+2$;
3) $2x^2+(4-3x)(x+12)$;
4) $4b^2+(3-4b)(b-2)$;
5) $4a-(a+3)(2a-1)$;
6) $13p-(p-2)(6p-5)$;
7) $(x+3)(x-5)-(x^2-3x)$;
8) $(t+11)(t-1)-(t^2-9t)$;
9) $(2x+1)x-(x-4)(x+3)$;
10) $(5y-7)y-(y-3)(2y+1)$;
11) $a(3a+5)+(a+2)(a-5)$;
12) $t(2t-3)+(t+3)(t-7)$;
13) $(q+3p)q-(3p+q)(p-q)$;
14) $(8p-q)\cdot p-(2p-5q)\cdot(p-2q)$.

190.* x ning qanday qiymatlarida quyidagi ifodalar teng bo‘ladi?

1) $(1,5x+2,5)(2x-0,5)$ va $(3x-1,5)(x+3,5)$;
2) $(10x+2)(4x-6)$ va $(5x-2)(8x+2)$;
3) $(10x-2)(4-2x)$ va $(2x-6)(4-10x)$;
4) $(3,5x-0,5)(0,5x+2,5)$ va $(1,5+3,5x)(0,5x-1,5)$.

191. Ifodani soddalashtiring:

1) $ab(a+b)-(a^2+b^2)(2a-b)$;
2) $pq(p+q)-(p^2+q^2)(p-2q)$;
3) $(8p+3k)(3p-8k)-(3p+8k)(8p-3k)$;
4) $(3c-4p)(4c+3p)-(4c-3p)(3c+4p)$;
5) $(p^3-3q)(p^2+3q)-(p^2-3q)(p^3+3q)$;
6) $(a^3+2b)(a^2-2b)-(a^2+2b)(a^3-2b)$.

192*. a) Sinfda 2 ta akvarium bo‘lib, ular to‘g‘ri burchakli parallelepiped shaklida. Birinchi akvariumning bo‘yi enidan 20 sm uzun, ikkinchisining bo‘yi va enining har biri birinchisinikiga qaraganda 20 sm uzun. Agar ikkala akvariumga 25 sm balandlikda suv quyilsa, ikkinchi akvariumga birinchisiga qaraganda 30 litr ko‘p suv quyish kerak bo‘ladi. Kichik akvariumning eni va bo‘yini toping.

b) Xonaning polini gilam bilan qoplash (to‘shash) kerak. Xonaning eni bo‘yidan 2 metr kichik. Eni va bo‘yi xonaning eni va bo‘yidan 30 sm kichik bo‘lgan gilam sotib olindi. Uning narxi pol yuzini to‘liq qoplaydigan gilam narxidan 20 550 so‘m arzon. Agar 1 kv.metr gilam 5 000 so‘m bo‘lsa, xonaning eni va bo‘yini toping.

193. Ifodaning qiymatini toping:

1) $\left(2a+\frac{1}{3}\right)\cdot\left(4a^2-\frac{2}{3}a+\frac{1}{9}\right)$, bunda $a=-4\frac{1}{2}$;

2) $\left(3a-\frac{3}{4}\right)\cdot\left(9a^2+\frac{9}{4}a+\frac{9}{16}\right)$, bunda $a=3\frac{1}{3}$.

194. Ko‘phadlarni ko‘paytiring:

1) $(3a^4-2a^3x+4a^2x^2+ax^3)(a^3-2a^2x+5ax^2-x^3)$;
2) $(7a^5-3a^4b^2+a^3b^2+3ab^5)(2a^3-4ab^2+3ab^3-12b^5)$;
3) $(a^4-a^{2n}b^n+b^{2n})(a^n-b^k)$;
4) $(a^{mn}+a^m+a^{m-1})(a^{m+1}-a^m)$.

195. Ko‘phadlarni “ustun” usulida ko‘paytiring:

1) $(b+2)(b^2+3b-4)$;
2) $(3a+4)(2a-3)$;
3) $(x-5y)(x^2+5xy+25y)$;
4) $(5x^2-2y)(-x^2+3y+2)$.

196. 1) Avtomobil v km/soat tezlik bilan 6 soat yurdi. Keyin tezligini 20 km/soat oshirib 3 soat yurdi. Avtomobil jami necha kilometr yurgan?

2) Velosipedchi k km/soat tezlik bilan 2 soat yurdi. Keyin tezligini n km/soat oshirib, 2,5 soat yurdi. U jami necha kilometr yo‘l bosgan?

197.* Kater daryo oqimi bo‘yicha 5 soat, oqimga qarshi 6 soat yurdi. Katerning turg‘un suvdagi tezligi m km/soat, daryo oqimining tezligi n km/soat bo‘lsa, kater bu vaqt davomida necha kilometr yo‘l bosgan?

10-§. Birhad va ko‘phadni birhadga bo‘lish

1. *Birhadni birhadga bo‘lish.*

Birhadlarni qo‘shish, ayirish, ko‘paytirish va natural ko‘rsatkichli darajaga ko‘tarishni o‘rganib oldik. Bu amallarni bajarish natijasida yana birhad hosil bo‘ldi.

Endi birhadni birhadga bo‘lishni ko‘rib o‘taylik.

Misol. Bo‘lishni bajaring:

1) $(12b):3$; 2) $(15ab):(5b)$; 3) $(48a^4b^7):(12a^2b^6)$;

4) $(-\frac{8}{9}a^5b^3c):(-4a^5b^2c)$; 5) $(20z^8):(4xz^6)$; 6) $y^5:y^6$.

□ Bo‘linmalarni aniqlashda bo‘lish amali o‘rniga kasr chizig‘ini qo‘yamiz va bir xil asosli darajalarni bo‘lish qoidalaridan foydalanamiz.

1) $12b:3=\frac{12b}{3}=4b$;

2) $(15ab):(5b)=\frac{15ab}{5b}=\frac{15}{5}\cdot\frac{a}{1}\cdot\frac{b}{b}=3a$;

3) $(48a^4b^7):(12a^2b^6)=\frac{48a^4b^7}{12a^2b^6}=\frac{48}{12}\cdot\frac{a^4}{a^2}\cdot\frac{b^7}{b^6}=4a^2b$;

4) $\left(\frac{8}{9}a^5b^3c\right):(-4a^5b^3c)=\left(\frac{8}{9}:(-4)\right)\cdot\frac{a^5b^3c}{a^5b^3c}=\frac{8}{9}\cdot\frac{1}{-4}\cdot\frac{a^5}{a^5}\cdot\frac{b^3}{b^3}\cdot\frac{c}{c}=-\frac{2}{9}\cdot1\cdot1\cdot1=-\frac{2}{9}$;

5) $(20z^8):(4xz^6)=\frac{20z^8}{4xz^6}=\frac{20}{4}\cdot\frac{1}{x}\cdot\frac{z^8}{z^6}=5\cdot\frac{1}{x}\cdot z^2=\frac{5z^2}{x}$. ■

1–4-misollar natijalarida birhadlar hosil bo‘ldi.
5-misolda esa birhad hosil bo‘lmadi, ya’ni maxrajda harf qatnashyapti. Bunday ifodalarni algebraik kasrlar deyiladi. Algebraik kasrlar bilan V bobda to‘liq tanishamiz.

Yuqoridagi misollardan shunday xulosaga kelish mumkin:
Birhadni birhadga bo‘lganda yana birhad hosil bo‘lishi uchun
a) ikkala birhad ham standart shaklida yozilgan bo‘lishi kerak;
b) bo‘luvchida ham, bo‘linuvchida ham bir xil harf bo‘lib, bo‘luvchidagi harfning ko‘rsatkichi bo‘linuvchidagi harf ko‘rsatkichidan katta bo‘lmasligi kerak;
d) bo‘luvchi birhadning koeffitsiyenti “0” bo‘lmasligi kerak.

Bo‘lish natijasini ko‘paytirish bilan tekshirish mumkin: bo‘linuvchi bo‘luvchining bo‘linmaga ko‘paytmasiga teng bo‘lishi kerak.
Masalan, $(81x^3y^7z^2):(9x^2y^3z)=9xy^4z$ bo‘lish to‘g‘ri bajarilgan, chunki $(9x^2y^3z)\cdot(9xy^4z)=81x^3y^7z^2$.

2. *Ko‘phadni birhadga bo‘lish*

Ko‘phadlarni qo‘shish, ayirish, ko‘paytirish natijasida yana ko‘phad hosil bo‘lishi avvalgi mavzularda ko‘rsatildi. Birhadni birhadga bo‘lganda har doim ham birhad hosil bo‘lavermaydi. Bundan ko‘phadni birhadga bo‘lganda doimo ko‘phad hosil bo‘lmasligi mumkin, degan xulosaga kelamiz.

Masala. Bo‘lishni bajaring:

1) $(3x^3y+6xy^3-0{,}75xyz):(3xy)$; 2) $(54p^3q^2-72p^2q^4):(9p^2q^2)$;

3) $(16mn-3m^2n):(5mn)$; 4) $(12ab+11ac):(ab)$.

□ Bu bo'linmalarni aniqlashda sonlarning bo'lish xossasidan foydalanamiz: yig'indini songa bo'lish uchun har bir qo'shiluvchini shu songa bo'lish kerak:

1) $(3x^3y+6xy^3-0,75xyz):(3xy)=(3x^3y):(3xy)+(6xy^3):(3xy)-(0,75xyz):(3xy)=$

$=\frac{3x^3y}{3xy}+\frac{6xy^3}{3xy}-\frac{0,75xyz}{3xy}=x^2+2y^2-0,25z;$

2) $(54p^3q^2-72p^2q^4):(9p^2q^2)=(54p^3q^2):(9p^2q^2)-(72p^2q^4):(9p^2q^2)=$

$=\frac{54p^3q^2}{9p^2q^2}-\frac{72p^2q^4}{9p^2q^2}=6p-8q^2;$

3) $(16mn-3m^2n+5mn^2):(5mn)=\frac{16mn}{5mn}-\frac{3m^2n}{5mn}+\frac{5mn^2}{5mn}=\frac{16}{5}-\frac{3}{5}m+n;$

4) $(12ab+11ac):(ab)=\frac{12ab}{ab}+\frac{11ac}{ab}=12+\frac{11c}{b}.$ ■

Ko'rilgan misollarning 1–3-sida ko'phadni birhadga bo'lish natijasida ko'phad hosil bo'ldi. Bunday hollarda ko'phad birhadga qoldiqsiz bo'linadi deyiladi. 4-misolda ko'phadni birhadga bo'lish natijasida ko'phad hosil bo'lmadi. Ko'phadni birhadga bo'lishda bo'luvchi birhad nolga teng bo'lmasligi kerak.

?

198. 1) Birhad birhadga qanday bo'linadi? Misollarda tushuntiring.
2) Birhadni birhadga bo'lganda doimo birhad hosil bo'ladimi? Misollar keltiring.
3) Ko'phad birhadga qanday bo'linadi? Misollarda tushuntiring.
4) Ko'phadni birhadga bo'lganda doimo ko'phad hosil bo'ladimi? Misollar keltiring.

Bo'lishni bajaring **(199–209)**:

199. 1) $(18a):(3a)$; 2) $(15x):(2x)$; 3) $(-16xy):(4xy)$; 4) $(-26ab):(2ab)$.

200. 1) $(9a):\left(-\frac{1}{3}a\right)$; 2) $(8c):\left(-\frac{4}{5}c\right)$; 3) $\left(\frac{3}{4}b\right):\left(-\frac{1}{2}b\right)$; 4) $\left(\frac{7}{8}n\right):\left(-1\frac{3}{4}n\right)$.

201. 1) $(10xyz):(-3x)$; 2) $(-18pt):(3t)$;
3) $(8,1ab):(-9a)$; 4) $(5,12pqt):(-0,25qt)$.

202. 1) $(25a^5):(5a^2)$; 2) $(-72x^8):(9x^3)$;
3) $(-0,8a^8):(-0,5a^8)$; 4) $(-8,5y^5):(-1,7y)$.

203. 1) $\left(1\frac{7}{25}a^3b^2p^2\right):\left(-3\frac{4}{7}a^2bp\right)$; 2) $\left(-3\frac{1}{2}x^2y^5z\right):\left(1\frac{3}{4}xy^3\right)$;

3) $(2{,}25p^2q^2y^3):(-1{,}5p^2y^3)$; 4) $(1{,}69a^7b^3c^2):(-1{,}3a^4b^2c)$.

204. 1) $(1{,}9a^2b^5c):(3{,}61ab^5c)$; 2) $(14x^5y^2z^3):(7x^5yz)$;

3) $\left(-\frac{8}{5}a^8b^3y^3\right):\left(-1\frac{1}{4}a^6b^2y^3\right)$; 4) $\left(-\frac{7}{9}a^6b^4y\right):\left(-2\frac{1}{3}a^3b^3y\right)$.

205. 1) $(3a^4b^3)^3:(4a^2b)^2$; 2) $(5a^3x^2)^3:(15a^2x)^2$; 3) $(-xyz^2)^6:(-$

206. 1) $(8{,}4a^6b^{n+2}):(0{,}7a^5b^{n-2})$; 2) $(12{,}5x^5y^n):(2{,}5x^3y^{-}$

3) $(-3{,}9a^{k+n}b^{k+n}c^2):(-0{,}3a^{k-1}b^{n-1}c^2)$; 4) $(-1{,}5x^ny^5z^k):(-0{,}5x^{-}$

5) $\left(-22b^mc^{k+2}d^3\right):\left(3\frac{1}{7}b^5c^2d\right)$; 6) $\left(8\frac{1}{4}a^{n+k}x^5y^k\right):\left(2\frac{3}{4}a^kx^2y^3\right)$.

207. 1) $(15x+10):5$; 2) $(23b+26):13$; 3) $(8t-12):4$;

4) $(18m-12):6$; 5) $(-15+6x):(-3)$; 6) $(-85+34a):(-17$

208. 1) $(8ab-6ap):a$; 2) $(9x^2-12xy):3$; 3) $(p-t):t$; 4) $(mn-$

209. 1) $(13x^2b-26xb^2):(13xb)$; 6) $(-85p^8q^7+34p^9q^6):(-17p^{2}$

2) $(28a^5t^4-14a^5t^3):(4at)$; 7) $(-54a^2b^4-62a^5b^2):(-4a^{2}b^{}$

3) $(65y^8z^3-26yz^7):(-13yz)$; 8) $(3p^9q^3-5p^8q^5):(-2p^3q^3)$;

4) $(35a^6b^4-25a^7b^3):(-5ab)$; 9) $(x^3z^4-3x^5z^7):(-5x^2z^2)$;

5) $(-32x^3y^4+24x^5y^4):(-8x^2y^2)$; 10) $(p^2t^3-8p^{10}t^{12})(-13p^3t^3)$.

210. Ifodani soddalashtiring:

1) $(3a^{21}-2a^3):a^2+(18a^3-27a^2):(9a)$;

2) $(15a^3-18a^2):(3a^2)+(12a^2-8a):(4a)$;

3) $(13a^5-26a^3):(13a^3)-(25a^3-15a^2):(5a^2)$;

4) $(45a^6-30a^5):(15a^4)-(64a^4-48a^3):(4a^2)$.

211.* Ifodaning qiymatini toping:

1) $(8a^4+12a^2):(4a^2)-(10a^2):(5a^2)$, bunda $a=7$;

2) $(27a^3-9a^2):(9a)-(54a^3):(18a^2)$, bunda $a=3$;

3) $(15x^3y^2+10x^3y):(5x^2y)-(18x^2y^3-12xy^3):(6xy^2)$, bunda $x=-12{,}5$. $y=$

4) $(21a^2b^2-14a^4b^3):(7a^2b^2)-(12a^4b^2-6a^2b^2):(6a^2b^2)$, bunda $a=2$, $b=-$

212.* Bo‘lishni bajaring:

1) $(25a^8+4a^7):(-5a^6)+(12{,}5a^5-7{,}5a^2):(2{,}5a)$;

2) $(18b^9-17b^6):(-2b^5)+(15b^7-12b^3):(-3b^2)$;

3) $(0{,}9y^6-0{,}6y^5):(-0{,}3y^3)-(9{,}3y^7-6{,}2y^3):(3{,}1y)$;

4) $(-3{,}5t^3+1{,}4t^2):(0{,}7t^2)-(2{,}4t^8-1{,}6t^6):(0{,}8t^6)$.

Masalalar yechish

213. Algebraik ifoda ko‘rinishida yozing:

1) m sonining kvadrati bilan n soni yig‘indisi;

2) a soni kubi bilan b soni kvadrati ayirmasi;

3) $2x$ va 5 sonlari ayirmasining kvadrati;

4) $\frac{1}{3}$ va c sonlari kvadratlarining ayirmasi.

214. To'g'ri burchakli parallelepipedning eni n sm ga, balandligi esa k sm ga teng, bo'yi enidan 2 sm uzun. Uning qirralari uzunliklari yig'indisini, sirti yuzini va hajmini topish uchun ifoda tuzing.

215. Quyidagi sonlarni standart shaklda yozing:
1) 310 000 000 000; 2) 8 010 000 000; 3) 87 100.

216. Birhadlarni ko'paytiring:

1) $(\frac{5}{9}m^2n^3)^2 \cdot (-1{,}62mn^3k)$; 2) $(\frac{3}{7}a^3b^4)^3 \cdot (-34{,}3a^2b)$;

3) $(0{,}25xy^2)\cdot(2x^2y)^3$; 4) $(0{,}3a^4b^5)\cdot(3\frac{1}{3}a^2b)^3$.

217. Amallarni bajaring:

1) $\frac{5^n \cdot 25^3}{125^2}$; 2) $\frac{8^n \cdot 16^2}{64^3}$; 3) $\frac{a^9 \cdot (b^5)^5}{(b^2)^3 \cdot (a^3)^2 \cdot (a^2b)^2}$; 4) $\frac{(x^3)^6 \cdot (y^4)^5 \cdot (x^2y)^4}{(x^2)^4 \cdot (y^5)^2}$.

218. Ko'phadni standart shaklga keltiring:
1) $(3x)(0{,}5y) - 9a(0{,}4b) + (1{,}5b)(-0{,}2a) - 1{,}2xy + 4ab$;
2) $18pt - 0{,}3ab - (4p)\cdot(3t) + (4a)\cdot(-5{,}2b) + 3p\cdot(-4)t$;
3) $7xyz\frac{5}{14}x + 3\frac{5}{12}x^2\cdot\frac{24}{41}yz - 2\frac{2}{5}xy\cdot\left(-\frac{5}{12}\cdot x\right)$;
4) $11ppqm3q - \frac{3}{7}pm2\frac{1}{3}qm + \frac{2}{3}p^2q\cdot\left(-4\frac{1}{2}m\right)$.

219. Ko'phadlarning son qiymatini toping:

1) $-1{,}3a + 28ab^2 - 18bab$, bunda $a = 1\frac{1}{3}$, $b = \frac{3}{5}$;
2) $-8{,}1x^2y + 3\frac{1}{3}yxy - 2\frac{1}{3}yxy$, bunda $x = 6\frac{2}{3}$; $y = 0{,}75$;
3) $15q^3 - 4q^2 + 11q - 9q - 8q^2 + 18q^2 + q$; $q = -\frac{1}{3}$;
4) $9b^2 - 15{,}5b^3 + 3b - 8b^2 + 5{,}5b^3 + 3b^2 - 8b$, bunda $b = 1{,}5$.

220.* Ko'phadlarning algebraik yig'indisini toping:
1) $(-3x^4 + 2x^2y^3) + (x^3y^2 - 2) + (x^3y^2 - xy^3 + 4x^4)$;
2) $(4x^3 + 6x^2y^2 + 8x^3y^2) - (6x^2y^2 + 4x^3) - (8x^3y^2 - 4x^3)$;
3) $(9a - 11a^2b^2 - b^3) + (-7a^3 + 2a^2b^2 - b^3) - (a^2 - 9a^2b^2 + 5b^3)$;
4) $(5a^3 - 3a^2b^2 + b^2) - (-a^3 + b^3 - 3a^2b^2) + (4a^3 + b^3 - a^2b^2)$.

Ko'phadlarni ko'paytiring **(221–222)**:

221. 1) $(0{,}2a + 0{,}2b - 0{,}1c)(5a - 10z)$; 2) $(0{,}5p - 0{,}5pq + 0{,}4t)(2p - 2{,}5t)$;
3) $\left(\frac{3}{8}x - \frac{3}{8}y + \frac{1}{9}x\right)(72x + 8)$; 4) $\left(\frac{3}{5}k^2 - \frac{5}{6}t + \frac{4}{15}n\right)(30k - 0{,}15)$.

222. 1) $(3a-b)(3a+b)(a-b)$; 2) $(2a-1)(2a+1)(a+1)$;
3) $(m+2)(2m+1)(3m-1)$; 4) $(m-n)(3n+5)(2n-3)$.

223. Bo'lishni bajaring:
1) $(0{,}02x^5+0{,}1x^4+0{,}03x^3+0{,}002x^2-0{,}04x):(0{,}01x)$;
2) $(-0{,}07x^3-0{,}009x^2+0{,}08x+0{,}0001):0{,}1$;
3) $(-0{,}3a^3n^2-\frac{7}{8}a^4n^5+\frac{2}{9}a^3n^6):(\frac{5}{36}a^3n^2)$;
4) $(0{,}75p^7x^4+1{,}2p^4x^5-0{,}9p^2x^2):(\frac{3}{5}px^2x^4)$.

III bobga doir sinov mashqlari (testlar)

1. Hisoblang: $\frac{4^8\cdot 16^2}{64^3}$.

A) 16; B) 64; C) $\frac{1}{16}$; D) $\frac{1}{64}$.

2. $\frac{2\frac{2}{3}\cdot a^2b^5\cdot\left(-1\frac{1}{2}ab^3\right)^4}{a^2b^4}$.

A) $-4a^4b^{13}$; B) $4a^4b^{13}$; C) $\frac{27}{2}$ [illegible]; D) $-\frac{27}{4}a^4b^{13}$.

3. Birhadlarning son qiymatini toping: $\frac{1}{8}$ [illegible] bunda $x=-3$; $y=-1{,}6$; $z=10$.

A) -54; B) 54; C) [illegible]; D) 86,4.

4. Birhadlarni standart shaklda yozing: $3^5a^{\square}\cdot\left(-\frac{2}{5}\right)$ [illegible]

A) $48a^3b^4$; B) $96a^3b^4$; C) [illegible]; D) $-96a^3b$.

5. Birhadlarni ko'paytiring: $\left(-\frac{5}{18}p^4q^2t^5\right)\cdot(0{,}9pq^3t)$.

A) $4p^5p^5t^6$; B) $-\frac{1}{4}p^5q^5t^6$; C) [illegible]; D) $\frac{1}{4}p^5q^5t^3$.

6. Ko'phadlarning algebraik yig'indisini toping: $\left(4\frac{2}{7}p-3\frac{1}{2}p-5\frac{5}{7}q\right)+3(p-q)$.

A) $-8p+2q$; B) $10pq$; C) $8p-8q$; D) $6pq$.

7. Ko'phadni birhadga ko'paytiring: $\left(3\frac{5}{14}a-1\frac{1}{7}b\right)\cdot(-28b)$.

A) $-94a+32b^2$; B) $94a-32b^2$; C) $54a-32b^2$; D) $54a+32b^2$.

8. Soddalashtiring: $-8x(0{,}3x-y)-3x(4\frac{1}{3}x-y)$.

A) $-15{,}4x^2-11xy$; B) $10{,}6x^2+5xy$; C) $10{,}6x^2+11xy$; D) $-15{,}4x^2+11xy$.

9. Ko‘phadni standart shaklga keltiring: $0,4xyz\,5x-3\frac{3}{7}y^2\,4\frac{1}{12}xyz-4\frac{4}{9}xy\cdot\left(-2\frac{1}{4}xz\right)$.

A) $-12x^2y^2z$; B) $12x^2yz-14xy^3z$; C) $8x^2yz-14xy^2z$; D) $12x^2yz-14xy^2z$.

10. Ko‘phadlarni ko‘paytiring: $(2x-1)(2x+1)(4x^2+1)$.

A) $8x^2-1$; B) $16x^4-1$; C) $16x^4-8x^2+1$; D) $16x^4+1$.

11. Bo‘lishni bajaring: $(21x^3y^2z-14xy^2z^3+x^2y^2z^2):(-7xy^2z)$.

A) $-3x^2+2z^2-\frac{1}{7}xz$; B) $3x+2z-\frac{1}{7}y$; C) $-3x^2-2z^2-\frac{1}{7}$; D) $3x^2+2z-7x$.

12. Algebraik ifodaning son qiymatini toping: $(25a^3-5a^2):(5a^2)-24a^3:(-8a)$, bunda $a=-2,5$.

A) 30,25; B) $-30,25$; C) 7,5; D) 5,25.

13. Ifodani soddalashtiring va hosil bo‘lgan ko‘phadning darajasini aniqlang:

$(36a^6+15a^5-12a^3):(-3a)-8a^2(3a^2+\frac{1}{a^2})$.

A) 5; B) 4; C) 2; D) 6.

14. Ko‘phadlarni ko‘paytiring va koeffitsiyentlar yig‘isindini toping:

$(3x+2y)(3x-2y)(9x^2-6xy+4x^2)$.

A) 70; B) 35; C) 40; D) 81.

15. Ifodani soddalashtiring: $\frac{3}{7}\left(1\frac{2}{5}a+2,1\right)+\frac{3}{5}\left(\frac{2}{3}a-\frac{1}{2}\right)$.

A) $1\frac{2}{3}a-\frac{1}{5}$; B) $2\frac{1}{3}a+1\frac{1}{6}$; C) $a+\frac{2}{5}$; D) $0,6+a$.

16. Hisoblang: $-(-9)^3\cdot\left(-\frac{1}{9}\right)^2$.

A) 27; B) 81; C) 9; D) -81.

17. Hisoblang: $\frac{(-7,5)^4}{2,5^3}$.

A) $-202,5$; B) 202,5; C) 22,5; D) $-20,25$.

18. Hisoblang: $\frac{14^5}{2^6\cdot 7^5}$.

A) 0,5; B) 2; C) $\frac{1}{32}$; D) 7.

19. Hisoblang: $\frac{(-0,9)^5}{(-0,1)^3}$.

A) 0,729; B) 590,49; C) 65,61; D) 0,81.

20. Hisoblang: $\frac{3^{n+1}\cdot 5^{3n+2}\cdot 3^{2n+1}}{(15^3)^n}$.

A) 225; B) 3375; C) $\frac{1}{225}$; D) $\frac{1}{15}$.

21. Birhadning son qiymatini toping: $\frac{1}{35}a(-0,7)\cdot b^2$, bunda $a=3\frac{1}{2}$; $b=10$.

A) -7; B) 3,5; C) 14; D) 7.

22. Birhadni standart shaklda yozing: $\left(-\frac{2}{9}\right)^3 p^2q(-63)\cdot q^2p$.

A) $-\frac{56}{81}p^3q^3$; B) $\frac{4}{81}p^2q^3$; C) $\frac{7}{81}p^3q^2$; D) $\frac{56}{81}p^3q^3$.

23. Ifodani soddalashtiring: $(a+3b)(a+b+2)-(a+b)(a+3b+2)$.

A) $2a-b$; B) $a-2b$; C) $4a+2b$; D) $4b$.

24. Ifodani soddalashtiring: $(a+3b)(a+b-1)-(a+b)(a+3b-1)$.

A) $4a+2b$; B) $6ab$; C) $2a-b$; D) $-2b$.

Rivojlantiruvchi mashqlar*

1. Hisoblang **(1–3)**:

1) $-(-2)^2\cdot(-3)^3+(-1)^5\cdot(-2)^3+4^2\cdot(-3)$;

2) $(0,2)^3:(-0,1)^4+(3,8)^3:(-1,9)^2$;

3) $(-0,3)^4:(0,1)^3-\frac{2}{3}\cdot(-9)^2+\frac{1}{2}\cdot(-4)^2$;

4) $\left(-3\frac{1}{7}\right)^2\cdot\frac{49}{121}-(-1,2)^{40}:(-0,6)^{39}:2^{39}$.

2. 1) $((-1)^3)^4+((-1)^3)^3+(-(-1)^4)^5-((-1)^5)^3$;

2) $\left((-16)^4\right)^6:\left((-16)^{13}\right)^2-\left(\frac{1}{16}\right)^2$.

3. 1) $\frac{5\cdot\left(3\cdot8^{15}-19\cdot8^{14}\right)}{8^{16}+3\cdot8^{15}}$;

2) $\frac{5\cdot3^{32}-4\cdot3^{30}}{21\cdot9^{16}}$;

3) $\frac{(-2)\cdot(-4)^{17}-(-4)^{16}}{4^{16}\cdot24}$;

4) $\frac{2\cdot4^9\cdot27^3+15\cdot2^{18}\cdot9^4}{6^9\cdot4^5+2^{10}\cdot6^{10}}$;

5) $\frac{(-3)\cdot(-3)^{15}-(-3)^{14}}{3^{14}\cdot8}$;

6) $\frac{(-7)(-7)^{12}-(-7)^{11}}{7^{11}\cdot48}$.

4. Jadvalni to‘ldiring:

x	0	$-\frac{1}{2}$	$\frac{1}{2}$	-2	2	-3	3	$-0,3$	$0,3$	-5	5
$(-x)^2$											
$-x^2$											
x^3											
$(-x)^3$											

5. Ifodani daraja shaklida yozing:

1) $73^{n+4} \cdot 72^{n-1} : 7^{n+2}$;
2) $6^{4n+3} \cdot 6^{3n-2} : 6^{2n-1}$;
3) $a^7 \cdot a^{2n} \cdot a^{3n-2}$;
4) $x^{n+2} \cdot x^8 \cdot x^{4n-1}$;
5) $a^{12n-8} \cdot a^{8n+2} : a^{10n-4}$;
6) $b^{10n-6} \cdot b^{6n+4} : b^{8n-2}$.

6. Ifodani ko‘rsatkichi 3 bo‘lgan daraja ko‘rinishida yozing:

1) a^9b^6;
2) $a^{12} \cdot b^{15}$;
3) $-1000b^9$;
4) $x^{18} \cdot y^{15} \cdot z^{12}$;
5) $-0,008 \cdot x^6y^3$;
6) $-0,001 \cdot a^{12}b^{21}$;
7) $0,027 \cdot x^{18}y^{27}$;
8) $-27 \cdot a^{15}b^{24}$.

7. Tenglamani yeching:

1) $(-0,3)^2 \cdot (x+100) - 1 = (0,2)^2 \cdot (x-50) + 1,5$;
2) $(-1,2)^2 \cdot (100-x) + 3 = (x+25) \cdot (-0,44)$.

8. n ning qanday qiymatlarida tenglik to‘g‘ri bo‘ladi?

1) $(5^n)^n = 5^{16}$;
2) $(3^n)^2 = 3^{18}$;
3) $2^{3n} = 4^9$;
4) $9 \cdot (9^2)^n = 9^{23}$;
5) $2^n \cdot 4^6 = 2^{17}$;
6) $5^{2n} \cdot 2^{2n} = 10\,000$;
7) $(1,3)^{2n} \cdot (1,69)^3 = (1,3)^{16}$;
8) $(1,7)^n \cdot (2,89)^4 = (1,7)^{18}$.

9. 1) $8^{18} \cdot 5^{55}$ ko‘paytma necha xonali son bo‘ladi?

2) n butun son bo‘lsa, $(n+2)^2 + (n+3)^2 + (n+4)^2$ yig‘indining eng kichik qiymati nechaga teng?

10. Birhadni standart shaklda yozing:

1) $3\frac{1}{7}a \cdot 1\frac{3}{11}b^3\left(-1\frac{5}{8}a^2\right) \cdot \frac{8}{13}c^2b$;
2) $2\frac{14}{15}bc^2 \cdot \left(-1\frac{5}{22}\right)a^2 \cdot b(-2,5)cab$;
3) $x^2 \cdot (-2)^3 yz^2x \cdot (-1)^6 zy^2x \cdot \left(-\frac{7}{8}\right)yz$;
4) $(-3)^3 \cdot z^3 \cdot \left(-\frac{8}{9}\right)^2 y^2x^2 \cdot \left(-3\frac{3}{8}\right)zyx$.

11. Birhadlardan ko‘phad tuzing:

1) $12a^2$, $-7ab$, $-8ab^2$ va $9b^3$;
2) $2,7a^2$, $5a^2b$, $-3,3ab^2$ va $-5b^3$.

12. Ko‘phadni birhadlar yig‘indisi shaklida yozing:

1) $8a^3 - 5a^2 - 7ab - 4b^2 + b^3$;

2) $-3{,}8a^5 + 2{,}9a^4b - 1{,}4a^3b - a^2b^2 + 4{,}2ab^3 + 8$.

13. Ko‘phadni, uning har bir hadini standart shaklga keltirib, soddalashtiring:

1) $18a^3 3abb^2 - 2a^2b^2 a4ba + 8ab^2a$;

2) $1{,}8a^4b^2 5ab - 3a^2b^2 - 2a^2b3a^3b^2$;

3) $1{,}5xy^2 \cdot (-4)x^2yz + 3yxz2x^2y^2 - 4{,}5xyz$;

4) $3\frac{1}{6}x^3y^4\left(-\frac{6}{19}\right)x + 2x^2y^2y^2x^2 - 3x^3y^3$.

14. Ko‘phadni soddalashtiring:

1) $6ab - 5bc + 8ac - 3ab + 2bc + 4ab - 3ac$;

2) $3a^2b - 9b^2 + 5a^2b + 5c^2 - 3b^2 - 4c^2$;

3) $6xy^2 + 8x^3 - 7x^2y - 4x^3 + 5x^2y - 10xy^2$;

4) $1\frac{2}{7}ab + \frac{3}{8}a^2 - 6{,}4b^3 + \frac{5}{7}ab - 0{,}375a^2 + 5{,}4b^3$.

15. O‘xshash hadlarni ixchamlang:

1) $3\frac{5}{12}xy^2 + 1{,}8x^2y - 2\frac{5}{6}xy^2 + 2{,}2x^2y - 1{,}7x^2y$;

2) $1{,}74a^2b - 0{,}625a^2b + 1{,}8a^2b - 7{,}5ab^2 + 2{,}5ab^2$;

3) $4m - 3n - 1{,}8m - 4{,}1n + 3\frac{2}{7}m + 8\frac{5}{9}n$;

4) $1{,}28c^2 + 3{,}72d^2 + 2{,}62c^2 - 1{,}22d^2 + 3c^2 - 2d^2$.

16. Ko‘phadlarni qo‘shing:

1) $(4a^4 - 5a^3b + 8a^2b^2 + 2ab^3) + (-3a - 7ab^3 + 2a^3b + 2b^4) + (4a^3b - 7a^2b^2 + 6ab^3)$;

2) $(2x^4 + 4ax^3 - bx^2 + 4cx - 2d) + (5x^4 - 7ax^3 + 6bx^2 - 4cx + 3d) +$
$+ (-6x^4 - 7ax^2 - 6bx^3 - 4cx - 3d)$;

3) $\left(1\frac{2}{3}a^2 + 1{,}2a^2b^2 - 1{,}25ab + \frac{5}{12}b^2\right) + \left(-\frac{3}{2}a^2 - 0{,}4ab + 0{,}75b^2 - 0{,}4a^2b^2\right)$;

4) $\left(15\frac{5}{6}a^3 - 8\frac{2}{3}a^2b + 7{,}8ab^2 + 12\frac{1}{3}b^3\right) + \left(-8{,}5a^3 + 15\frac{5}{7}a^2b - 4\frac{5}{9}ab^2 - 18{,}2b^3\right)$.

17. Qavslarni oching va ixchamlang:

1) $3a - \{4a - [5a - (6a + 7a)]\}$;

2) $8x - \{6x + [8x - (11x - 3x)]\}$;

3) $2a - \{6b + [4c - 4a - (a + b)] + 3a - (2b + 3c)\}$;

4) $3x - \{4y + [3z - 4x - (2x + 3z)]\} - [3x - (2y + 5z)]$.

18. Ko‘phadlarni ayiring:

1) $(1{,}8a^2-4{,}47ab-18{,}25ac+4{,}75bc)-(-1{,}2a^2-1{,}47ab+1{,}75ac+6{,}75bc)$;

2) $(4a^4-2{,}5a^3b+8a^2b^2-7{,}1ab^3+5b^4)-(3{,}2a^4-3a^3b+7a^2b^2-8{,}1ab^3+3{,}4b^4)$;

3) $(2{,}5x^3+4x^2y-3{,}8xy^2+4{,}3y^3)-(1{,}2x^3+2{,}5x^2y-1{,}8xy^2+2{,}3y^3)$;

4) $(3{,}6a^3b^2c-1{,}5a^4b^3c^2+1{,}8a^5b^4c^3)-(-0{,}4a^3b^2c-6{,}5a^4b^3c^2+0{,}8a^5b^4c^3)$.

19. Ko‘phad va birhad ko‘paytmasini toping:

1) $(8a^n-4a^{n-1}+3a^{n-2}\cdot b^m)\cdot(-0{,}5a^{n+2}\cdot b^3)$;

2) $\left(1\frac{2}{3}x^{m+2}\cdot y^{2n-3}-2{,}4x^m y^n-1{,}2y^{3-2n}\right)\cdot(-3x^{4-m}y^{2n})$;

3) $2\frac{2}{3}a^k b^n\cdot(9a^5-12b^3+9a^3b^2-18a^2b^3-3)$;

4) $5x^{3k-1}\cdot y^{2n-3}\cdot(0{,}8x^{1-2k}+y^{3-n}-0{,}4x^{2-3k}\cdot y^{5-2n}+0{,}2y^n)$.

20. Avval ifodani soddalashtiring, so‘ng son qiymatini toping:

1) $7a^3\cdot(4a^2-3ab)-2a^4\cdot(14a+10{,}5b)$, bunda $a=-\frac{1}{2}$, $b=\frac{10}{21}$;

2) $6a^2\cdot(3a-7b)+5a\cdot(-3{,}6a^2+10ab)$, bunda $a=1{,}5$, $b=-\frac{7}{8}$.

21. Amallarni bajaring:

1) $\left[2a^2b+(-2{,}5a^2b)-(-a^2b)\right]\cdot(6ab^4c^3-2ab^4c^3)$;

2) $\left[-16{,}6x^{12}\cdot y^4+13{,}4x^{12}y^4\right]\cdot(5x^2y^3-2{,}5x^2y^3)$;

3) $\left[2a^3b^4-\left(6\frac{1}{8}a^3b^4-8\frac{1}{24}a^3b^4\right)\right]\cdot(3a^2b^2-2{,}5a^2b^2)$;

4) $\left[-7{,}5x^8y^5+(-2{,}5x^8y^5)-(-4{,}5x^8y^5)\right]\cdot(5{,}9x^2y^3-3{,}9x^2y^3)$.

22. Ko‘phadlarni ko‘paytiring:

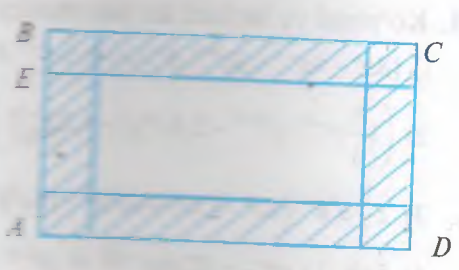
1) $(a+b)(a+b)$;
2) $(a+b)(a-b)$;
3) $(a^2+ab+b^2)(a-b)$;
4) $(a^2-ab+b^2)(a+b)$;
5) $(a^3-b^3)(a^3+b^3)$;
6) $(a^n+b^n)(a^n-b^n)$.

23. Ko‘phadlarni ko‘paytiring:

1) $(2a-3b)(4a^2+6ab+9b^2)$;
2) $(2a+3b)(4a^2-6ab+9b^2)$;
3) $(3x+4y)(9x^2-12xy+16y^2)$;
4) $(5x-4y)(25x^2+20xy+16y^2)$;
5) $\left(2a+\frac{1}{3}\right)\left(4a^2-\frac{2}{3}a+\frac{1}{9}\right)$;
6) $\left(3a+\frac{1}{4}\right)\left(9a^2-\frac{3}{4}a+\frac{1}{16}\right)$.

24. To'g'ri to'rtburchakning tomonlari a va b ga teng.
1) Agar tomonlardan birini c ga, ikkinchisini d ga orttirsak, uning yuzi qanchaga ortadi? Qachon yuzi 2 marta (4 marta, k marta) ortadi?
2) Agar tomonlardan birini k ga, ikkinchisini n ga kamaytirsak, uning yuzi qanchaga kamayadi? Qachon yuzi 3 marta (m marta) kamayadi?
3) Agar tomonlardan birini c ga orttirsak, ikkinchisini d ga kamaytirsak, uning yuzi qanday o'zgaradi? Qachon yuzi ortadi? Qachon kamayadi? Tahlil qiling.

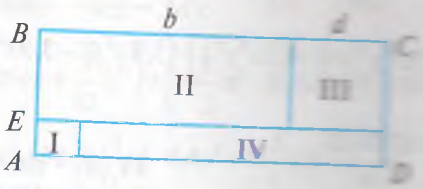
25. Berilgan o'lchamlar bo'yicha shtrixlangan yuzni hisoblash formulasini chiqaring;



1) $2bc + 2c(a - 2c) = 2ac + 2c(b - 2c)$ tenglikning to'g'riligini shakl yordamida ko'rsating, bunda $a = AB$, $b = BC$, $c = BE$;
2) shtrixlangan yuzni ikkita to'g'ri to'rtburchak yuzlarining ayirmasi sifatida tasvirlang. Bundan foydalanib, $ab - (b - 2c)(a - 2c) = 2ac + 2c \cdot (b - 2c)$ tenglikni isbotlang.

26. Tengliklarning to'g'riligini tekshiring, ularga geometrik sharh bering. Mos shakllar chizing:
1) $(a + b)(c + d) = ac + bc + ad + bd$;
2) $(a + b)(c - d) = ac + bc - ad - bd$;
3) $(a + b + c)(d + l) = ad + bd + cd + al + bl + cl$.

27. 1) Tenglikning to'g'riligini isbotlang:
$c^2 + b(a - c) + (b + d - c)c + d(a - c) = a(b + d)$.
2) $ABCD$ to'g'ri to'rtburchak yuzini hisoblash uchun ikkita usul tuzing, $AB = CD = a$, $BC = b + d$, $AE = c$.



$ABCD$ to'g'ri to'rtburchakning yuzi I, II, III, IV to'g'ri to'rtburchaklar yuzlari yig'indisiga tengligidan foydalaning va 1) tenglikka geometrik talqin bering.

28. Bo'lishni bajaring:
1) $6{,}2a^8b^{k+1} : (-0{,}2a^6b^{k-1})$;
2) $-7{,}5a^{k+1}b^{k+1}c^2 : (-2{,}5a^kb^kc)$;
3) $8{,}4x^5(a + 2b)^{11} \cdot c^3 : (-2{,}4x^3(a + 2b)^{10}c^2)$;
4) $-1{,}5x^ky^3z^n : (-0{,}5x^4y^2z^5)$.

29. *(Al-Xorazmiy masalalaridan).*

a) Ko‘phadlarni ko‘paytiring va soddalashtiring:

1) $(10 + x)(10 + x)$;
2) $(10 - x)(10 - x)$;
3) $(10 - x)(10 + x)$;
4) $\left(10+\frac{x}{2}\right)\left(\frac{1}{2}-5x\right)$.

b) Ko‘phadlar yig‘indisini toping:

1) $(100 + x^2 - 20x) - (50 + 10x - 2x^2)$;
2) $(100 + x^2 - 20x) + (50 + 10x - 2x^2)$.

30. Ko‘phadni birhadga bo‘ling:

1) $\left[5x^3(a+b)^4 - 3x^2y(a+b)^3 + 2(a+b)^2xy\right] : 2x(a+b)^2$;
2) $\left(36a^{3k-1} - 24a^{3k} + 12a^{3k+1} - 6a^{3k+2}\right) : 12a^{3k-1}$;
3) $\left[15a^4(x-y)^2 - 8a^3(x-y)^3 + 6a^2(x-y)^4\right] : \left[-10a^2(x-y)^2\right]$;
4) $\left[12(a-b)^k - 6(a-b)^n + 15(a-b)^m\right] : \left[-12(a-b)^m\right]$.

31. Bo‘lishni bajaring:

1) $\left(12\cdot5^{2n+1} - 8\cdot5^{2n} + 4\cdot5^{2n-1}\right) : \left(4\cdot5^{2n-2}\right)$;
2) $\left(36\cdot18^n - 8\cdot2^{n-1}\cdot9^n - 3^{n+1}\cdot6^{n+1}\right) : 18^{n-1}$, bunda n – natural son;
3) $\left(8a^5 + 3a^3\right) : \left(-2a^2\right) + \left(7{,}5a^3 + 8a^2\right) : (2{,}5a)$;
4) $\left(16b^7 - 15b^5\right) : \left(2b^4\right) - \left(18b^3 - 3b^2\right) : (2b)$.

32. Tengliklarning to‘g‘riligini ko‘rsatuvchi masalalar tuzing, ularga talqin toping:

1) $x - (y + z + t) = x - y - z - t$;
2) $x - (y - z) = x - y + z$;
3) $(x - y)z = xz - yz$;
4) $(x + z) - (y + z) = x - y$;
5) $(xy) : z = x \cdot (y : z)$;
6) $x : (yz) = (x : y) : z$.

33. Firma 143 dona kompyuter sotib oldi va ularni bo‘limlarga teng miqdorda tarqatishni rejalashtirdi. Ammo yana 2 ta bo‘lim ochishga to‘g‘ri keldi. Natijada, har bir bo‘lim 2 tadan kam kompyuter oldi. Firmada bo‘limlar soni nechta bo‘ldi?

Javob: 13 ta.

34. Feruza bir natural son o‘yladi, uni kvadratga oshirib, o‘ylagan sonini qo‘shgan edi, natija 240 ga teng bo‘ldi. Feruza qanday son o‘ylagan edi?

35. Tenglamaning natural ildizini toping:

1) $x(x-1) = 6$;
2) $x^2 + x = 90$;
3) $\frac{6}{x-1} + \frac{6}{x} + \frac{6}{x+1} = 11$;

4) $\frac{8}{x-1}+\frac{8}{x+1}=6$; 5) $\frac{9}{x-1}+\frac{9}{x+5}=4$.

Javob: 1) 3; 2) 9; 3) 2.

36. Firma ishlab chiqarishda yangi texnologiyadan foydalanib, mart oyida fevral oyiga nisbatan mahsulot ishlab chiqarishni $\frac{1}{5}$ hissa, aprel oyida mart oyiga nisbatan $\frac{1}{20}$ hissa, may oyida esa aprel oyiga nisbatan $\frac{1}{10}$ hissa oshirdi. Firma may oyida 11 088 ta mahsulot ishlab chiqargan bo'lsa, u aprel, mart, fevral oylarida qanchadan mahsulot ishlab chiqargan?

37. *Al-Xorazmiy masalalaridan.*

1) Bir kishi 4 o'lcham bug'doy va 6 o'lcham arpa oldi. 1 o'lcham arpaning narxi 1 o'lcham bug'doy narxining yarmiga teng. Xarid uchun to'langan pul arpa va bug'doy o'lchamlari ayirmasi bilan narxlar ayirmasining yig'indisiga teng. 1 o'lcham bug'doyning va bir o'lcham arpaning narxi topilsin;

Javob: Bug'doyning bir o'lchami $\frac{4}{13}$ dirham, arpaning bir o'lchami $\frac{2}{13}$ dirham.

2) Biri ikkinchisidan 2 ta ortiq sonlarning nisbati $\frac{1}{2}$ ga teng. Shu sonlarni toping.

Javob: 2 va 4.

3) Bir odam shunday vasiyat qildi: naqd 10 dirham (pul birligi) pulim bor. Bir kishiga qarz ham berganman. Qarzning miqdori o'g'lim oladigan merosga teng. Ikkala o'g'lim barobar meros olsin. Ukamga jami merosning $\frac{1}{5}$ qismini va yana 1 dirham beringlar. U kishining o'g'illari va ukasi necha dirhamdan olishgan?

Javob: O'g'illari $5\frac{5}{6}$ dirhamdan, ukasi $4\frac{1}{6}$ dirham.

38. *Buyuk matematik Leonard Eyler masalasi.*

Keksa bir kishi hamma to'plagan pullarini o'g'illariga teng bo'lib bermoqchi bo'libdi va shunday vasiyatnoma tuzibdi: "To'ng'ich o'g'lim 1000 dona kumush pul va qolgan pullarning $\frac{1}{8}$ qismini, ikkinchi o'glim 2000 dona kumush pul va qolgan pullarning $\frac{1}{8}$ qismini, uchinchi o'glim 3000 dona kumush pul va qolgan pullarning $\frac{1}{8}$ qismini, qolgan o'g'illarimga ham shu tariqa pul taqsimlansin". U kishining necha nafar o'g'li va necha dona kumush puli bo'lgan?

Javob: 7 ta o'g'il; 49 000 dona kumush pul.

39. Uzunliklari 1 sm, 2 sm, ..., 9 sm bo‘lgan 9 ta cho‘p bor. Ulardan tomonining uzunligi nechaga teng bo‘lgan kvadratlarni necha xil usul bilan yasash mumkin? (Cho‘plarni sindirish mumkin emas, biror kvadrat qurishda cho‘plarning hammasi qatnashishi shart emas. Turli cho‘plardan tuzilgan kvadratlar turli usulda qurilgan hisoblanadi).

Javob: Tomoni 7 sm, 8 sm, 10 sm, 11 sm bo‘lgan kvadratlarni 1 ta usulda; tomoni 9 sm bo‘lgan kvadratni 5 xil usulda yasash mumkin. Bu cho‘plardan boshqa kvadratlar yasab bo‘lmaydi.)

40. 1, 2, 3, 4, 5, 6 sonlarni doirachalarga shunday joylashtiringki, uchburchakning har bir tomonidagi raqamlar yig‘indisi o‘zaro teng bo‘lsin.

41. 7^a va 7^b sinf o‘g‘il bolalari bayramda qizlarga berish uchun 58 ta oq va qizil atirgullar olishdi. 7^a dagi har bir qizga 5 tadan oq, 7^b dagi har bir qizga 7 tadan qizil atirgullar berishdi. 7^a da necha, 7^b da necha nafar qizlar bor?
Javob: 7^a sinfda 6, 7^b sinfda 4 nafar qiz bor.

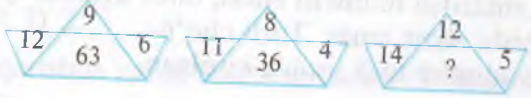
42. Bir dona daftar 70 so‘m, bir dona qalam 40 so‘m turadi. 530 so‘mga nechta daftar va nechta qalam sotib olish mumkin?
Javob: 3 ta daftar va 8 ta qalam yoki 7 ta daftar va 1 ta qalam.

43. Tojiniso aya nabiralariga 50 dona qurutni bo‘lib berdi. Har bir kichik yoshdagi nabira 5 tadan, har bir katta yoshdagi nabira 6 tadan qurut oldi. Ayaning kichik yoshdagi va katta yoshdagi nabiralari necha nafar bo‘lishi mumkin?

44. 1, 2, 3, 4, 5, 6, 7 raqamlarini doirachalarga shunday joylashtiringki, bitta to‘g‘ri chiziqdagi 3 ta doirachaga yozilgan raqamlar yig‘indisi teng bo‘lsin.

45. Sonlarning joylashishidagi qonuniyatni toping va “?” belgisi bor doirachalarni to‘ldiring.

46. Sonlarning joylashishidagi qonuniyatni toping va so‘roq (?) belgisi o‘rniga mos sonni qo‘ying:



47. Qalamni qog‘ozdan uzmasdan va bir chiziq ustidan ikki marta yurmasdan quyidagi shaklni chizing:

TARIXIY MASALALAR

1. *Nyuton masalasi.* Bitta xattot 8 kunda 15 varaq yoza oladi. 405 varaqni 9 kunda yozib tugatish uchun nechta xattot kerak bo‘ladi?
2. Yem otning o‘ziga 14 kunga, ot bilan toychoqqa esa 10 kunga yetadi. Shu yem toychoqning o‘ziga necha kunga yetadi?
3. Asalarilarning oltidan biri oq gullarga, uchdan biri esa qizil gullarga qo‘ndi. Ular ayirmasining ikki baravari esa sariq gullardan bol yig‘moqda. Beshtagina asalari gullar iforidan rohatlanib, uchib yuribdi. Qani menga ayt-chi, gulzorda qancha asalari bor?

TARIXIY MA’LUMOT

Noma’lum kattaliklarni harflar bilan belgilash mashhur yunon matematigi Diofant (III asr) asarlaridayoq uchraydi. Koeffitsiyentlarni ham, ma’lum miqdorlarni ham harflar bilan belgilashni F. Viyet (1540–1603) birinchilardan bo‘lib qo‘llagan. Algebraik tenglamalarni umumiy holda tadqiq qilish harfiy koeffitsiyentlar kiritilgandan keyingina mumkin bo‘ldi. F. Viyet undosh bosh lotin harflari — *B, G, D,* ... bilan koeffitsiyentlarni, unli harflari — *A, E, I,* ... bilan esa noma’lumlarni belgilagan. Mashhur fransuz matematigi va faylasufi R. Dekart (1596 – 1650) koeffitsiyentlarni belgilash uchun lotin alifbosining dastlabki (kichik) harflari *a, b, c, d,* ... dan, noma’lumlarni belgilash uchun esa alifboning oxirgi harflari *x, y, z* lardan foydalangan. Darajaning hozirgi zamonaviy belgilanishi a^2, a^3, ..., a^n (n – natural son)ni ham Dekart kiritgan (1637-yil).

„Al-jabr val muqobala” asarining „Ko‘paytirish haqida bob”iga al-Xorazmiy birhadlarni ko‘paytirishga, ikkihadni ikkihadga ko‘paytirishga hamda soddalashtirishga doir misollarni kiritadi.

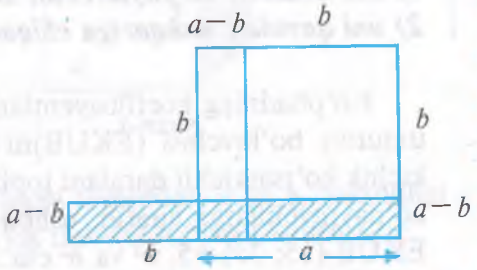
Al-Xorazmiy, Ahmad Farg‘oniy, Beruniy, al-Koshiy asarlarida algebraik simvolika bo‘lmagan. Matematik Abu Hasan Ali ibn Muhammad al-Kalasadiy (XV asr) asarida algebraik simvolika elementlarini uchratish mumkin. Al-Kalasadiy tenglamalarda noma’lumning birinchi darajasini “shay” so‘zining birinchi harfi bilan, kvadratini “mol” so‘zining, kubini “ka’b” so‘zining birinchi harflari bilan belgilagan. Tenglik “=” belgisi o‘rniga “adala” (tenglik) so‘zidagi a harfini ishlatgan. Biz o‘rganayotgan “Algebra” kursining simvolikasi (belgilashlar tizimi) XIV–XVII asrlarda shakllangan.

1-§. Umumiy ko'paytuvchini qavsdan tashqariga chiqarish

$a^2-b^2=(a+b)(a-b)$ tenglikning to'g'riligi shakldan ravshan. a^2-b^2 ayirma tomoni a bo'lgan kvadrat yuzidan tomoni b ga teng bo'lgan kvadrat yuzining ayirmasiga teng. Ayni shu yuz shtrixlangan yuz $(a-b)(a+b)$ ga teng (1-*rasm*).

a^2-b^2 ifodani unga teng bo'lgan $(a-b)(a+b)$ ifoda bilan almashtirilsa, hisoblash ancha soddalashadi. Chindan ham, $a=287$, $b=213$ bo'lganda

$a^2-b^2=(287-213)(287+213)=74\cdot 500=37000.$



1-rasm

Ko'phadni ikkita yoki bir nechta ko'phadlar ko'paytmasi ko'rinishida ifodalash ko'phadni ko'paytuvchilarga ajratish (yoyish) deyiladi.

1-masala. $ab-ac+ad$ ifodaning $a=40$, $b=21$, $c=15$, $d=19$ bo'lgandagi son qiymatini toping.

□ Hisoblashlarni quyidagicha olib boramiz:

$40\cdot 21-40\cdot 15+40\cdot 19=40\cdot(21-15+19)=40\cdot 25=1000.$ ■

Bunda ko'paytirishning taqsimot qonunidan foydalanilgan:

$$ab-ac+ad=a(b-c+d).$$

$40\cdot 21-40\cdot 15+40\cdot 19$ sonli ifodada umumiy ko'paytuvchi 40 soni bo'ladi; $ab-ac+ad$ algebraik ifodada esa umumiy ko'paytuvchi a bo'ladi.

Agar ko'phadning barcha (son yoki harfiy) hadlari umumiy ko'paytuvchiga ega bo'lsa, u holda shu ko'paytuvchini qavsdan tashqariga chiqarish mumkin. Qavs ichida berilgan ko'phadni shu umumiy ko'paytuvchiga bo'lish natijasida hosil bo'lgan ko'phad qoladi.

2-masala. Ko'phadni ko'paytuvchilarga ajrating: $4ab-10b+14bc$.

□ Bu ko'phadning barcha hadlari $2b$ umumiy ko'paytuvchiga ega:

$4ab=2b\cdot 2a$, $-10b=2b\cdot(-5)$, $14bc=2b\cdot 7c$.

Shuning uchun $4ab-10b+14bc=2b(2a-5+7c)$. ■

Masalaning mazmuniga qarab, umumiy ko'paytuvchini qavsdan tashqariga "+" ishorasi bilan ham, "−" ishorasi bilan ham chiqarish mumkin.

Misollar:

1) $ab-bc=b(a-c)=-b(c-a)$;

2) $2ab^2-6a^2b=2ab(b-3a)=-2ab(3a-b)$.

Ko'phadni umumiy ko'paytuvchini qavsdan tashqariga chiqarish yo'li bilan ko'paytuvchilarga ajratish uchun:

1) shu umumiy ko'paytuvchini aniqlab olish;

2) uni qavsdan tashqariga chiqarish kerak.

Ko'phadning koeffitsiyentlari natural sonlar bo'lganda ularning eng katta umumiy bo'luvchisi (EKUB)ni topish, bir xil asosli darajalar orasidan esa kichik ko'rsatkichli darajani topish lozim. Masalan, $15x^3b^2-35x^2b^3$ ko'phadni ko'paytuvchilarga ajratib, ushbu $5x^2b^2(3x-7b)$ ifodani hosil qilamiz. Bunda EKUB (15; 35)=5, x^2 va b^2 esa x va b ning eng kichik ko'rsatkichli darajalaridir.

Agar ko'phad ko'paytuvchilarga ajratilgan bo'lsa, shu jarayon to'g'ri bajarilganligini tekshirish uchun ko'paytuvchilarni o'zaro ko'paytirib chiqish kerak. Chindan ham,

$$5x^2b^2(3x-7b)=15x^3b^2-35x^2b^3,$$

ya'ni berilgan ko'phad hosil bo'ldi.

Umumiy ko'paytuvchi ko'phad bo'lishi ham mumkin. Masalan:

1) $3(m-n)+x(m-n)=(m-n)(3+x)$;

2) $4x(2a-b)-6y(2a-b)+9(2a-b)=(2a-b)(4x-6y+9)$.

Ba'zi hollarda $m-n=-(n-m)$ kabi tenglikdan foydalanish qulayroq bo'ladi. Masalan:

1) $(3-a)x-(a-3)y=(3-a)x+(3-a)y=(3-a)(x+y)$;

2) $16mn^2(x-y^2)+20m^2n(x-y^2)+24mn(y^2-x)=16mn^2(x-y^2)+20m^2n(x-y^2)-24mn\cdot(x-y^2)=(x-y^2)(16mn^2+20m^2n-24mn)=4mn\cdot(x-y^2)(4n+5m-6)$.

Ko'phadning koeffitsiyentlari oddiy yoki o'nli kasrlar bo'lishi mumkin. Masalan:

1) $\frac{1}{3}ab-\frac{2}{3}bc=\frac{1}{3}b(a-2c)$;

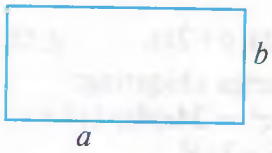
2) $1{,}6mn+0{,}4ma-0{,}8mb=0{,}4m(4n+a-2b)$.

?

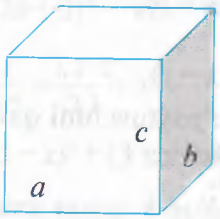
1. 1. Ko'phadni ko'paytuvchilarga ajratish deb nimaga aytiladi?
2. Umumiy ko'paytuvchini qavsdan tashqariga chiqarish deyilganda nima tushuniladi?
3. Umumiy ko'paytuvchini qavsdan tashqariga chiqarish uchun nimalar qilish kerak?

2. Agar to'g'ri to'rtburchakning bo'yi a sm, eni b sm bo'lsa, uning perimetrini toping (2-rasm).

□ To'g'ri to'rtburchakning qarama-qarshi tomonlari o'zaro teng bo'lgani uchun uning perimetri $P=2a+2b$ bo'ladi. Bu ifodada 2 umumiy ko'paytuvchi. Uni qavsdan tashqariga chiqaramiz: $P=2a+2b=2(a+b)$. ■



2-rasm



3-rasm

3. Agar to'g'ri burchakli parallelepipedning asosi tomonlari a va b, balandligi c bo'lsa, uning barcha qirralari uzunliklari yig'indisini toping (3-rasm).

Umumiy ko'paytuvchini qavsdan tashqariga chiqaring **(4–9)**:

4. 1) $12a^3b-18a^2b^2-36a^4b^3+42a^2b$;
2) $15a^4b^3-9a^3b^2+12a^2b^3-21ab$;
3) $28x^5y^4-21x^4y^3+14x^3y^2-35x^2y^2$.

5. 1) $81a^6b^5+72a^5b^4-63a^4b^3-54a^3b^2+45a^2b$;
2) $0{,}01x^8y^5-0{,}04x^7y^6-0{,}09x^6y^7-0{,}03x^5y^8$;
3) $\frac{1}{16}u^6v^4+\frac{1}{8}u^5v^5-\frac{1}{4}u^4v^6+\frac{1}{2}u^3v^7$.

6. 1) $(5a-4b)(3x-2y)+(3b-a)(3x-2y)$;
2) $(6a-7b)(4x-3y)-(4a-5b)(4x-3y)$;
3) $(8a-5x)(3c-4d)+(6a-7x)(4d-3c)$;
4) $(7a-5b)(5c+3d)-(6a-7b)(5c+3d)+(3a-2b)(5c+3d)$.

7. 1) $3x(2a+4b)+y(-a-2b)+(3ab+6b^2)$;
2) $a(2b-c)+(4b-2c)-(2bc-c)$;
3) $a(u-v)-(4v-4u)b-(3u-3v)c$.

8. 1) $a(x+5y)-(bx+5by)-(3x+15y)$;
2) $a(t-5)-(10-2t)b+(4t-20)c$;
3) $(ab-2a)+(4c-2bc)-(3b-6)$.

9. 1) $(a-b)\,a^2+ab(b-a)$;
2) $(3x+6y)y+(x+2y)x$;
3) $(6a-4b)(a-b)+(10b-15a)(a+b)$;
4) $(a+b)^2\cdot(c+d)+2a\cdot(a+b)\cdot(c+d)+(a+b)\cdot(c+d)^2$.

10. Ko'paytmani ko'phad shaklida yozing:
1) $(a+1)(a-1)$; 2) $3x(1-x)$;
3) $5b^2(2b+3)$; 4) $a^2(a+3b)+b^2(3a+b)$.

11. Ko'paytmani ko'phad shaklida yozing:

1) $(a+2b)(2a-b)$; 2) $(\frac{1}{2}x+y)(\frac{1}{3}x-y)$;

3) $(2y+x)(2x-y)$; 4) $(a-b)(a+2b)$.

12. Umumiy ko'paytuvchini qavsdan tashqariga chiqaring:
1) $7a^2bc-14ab^2c$; 2) $18xyz^2-24xy^2z$;
3) $12x^5-4x^3$; 4) $6a^2b^2-3ab^3$.

Masalalar yechish

13. Velosipedchi A shahardan B shaharga tomon 20 km/soat tezlik bilan yo'lga chiqdi. Oradan ikki soat o'tgandan keyin A dan B ga tomon 24 km/soat tezlik bilan ikkinchi velosipedchi jo'nadi. Ikkala velosipedchi ham B shaharga bir vaqtda yetib keldi. A va B shaharlar orasidagi masofani toping?

14. A shahardan B shaharga tomon 70 km/soat tezlik bilan, B shahardan A shaharga tomon 60 km/soat tezlik bilan yengil mashinalar yo'lga chiqdi. Agar shaharlar orasidagi masofa 260 km bo'lsa, ular necha soatdan keyin uchrashadi?

15. Ko'paytmani taqsimot qonunidan foydalanib hisoblang:
1) $17\cdot312+17\cdot188$; 3) $58\cdot129+58\cdot171$;

2) $119\cdot425+119\cdot375$; 4) $27\frac{1}{4}\cdot1-23\frac{1}{4}\cdot1$.

Umumiy ko'paytuvchini qavsdan tashqariga chiqaring (16–19):

16. 1) $3a+3b$; 2) $4m-4n$; 3) $3x-6$; 4) $9+3a$;

5) $4a+8b-4$; 6) $8m-12n-16$; 7) $\frac{1}{3}a-\frac{2}{3}b+\frac{1}{3}$; 8) $0{,}8m+0{,}2b-0{,}4c$.

17. 1) $mx-my$; 2) $ac+bc$; 3) $x+3xy$; 4) $2xy-3x$;
5) $7ab+7b$; 6) $2bd-2ab$; 7) $42z-14xz$; 8) $5p-6pk$.

18. 1) $ab+ac-a^2$; 2) $xy+yz-y^2$; 3) $6b^2-3b+12ab$; 4) $a^2b-2ab+b$;
5) a^3+3a; 6) a^4+4a^3; 7) $a^2b^2-a^2b$; 8) $x^3y^4-x^4y^3$.

19. 1) $15m^6+5m^2$; 2) $7x^4-28x^2$; 3) $4a^2-2a$;
4) $9xyz^2-54xy^2z$; 5) $15x^2y^3+5xy^2$; 6) $7a^2bc-49ab^2c$.

20. Umumiy ko'paytuvchini qavsdan tashqariga chiqarib hisoblang:
1) $136\cdot2+136\cdot64$; 2) $0{,}6\cdot2+0{,}6\cdot0{,}4$;
3) $129\cdot2-129\cdot29$; 4) $0{,}51\cdot2+0{,}51\cdot0{,}49$.

Ko'paytuvchilarga ajrating (21–22):

21. 1) $m(a+b)+n(a+b)$; 2) $a(b+4)-b(b+4)$;

3) $a(b+2)-(b+2)$; 4) $a(y-3)+b(y-3)$.

22. 1) $m^2(x-1)-2mn(x-1)+n^2(x-1)$; 2) $x(p-q)+y(p-q)-z(p-q)$;
3) $a(x^2+y^2)+b(x^2+y^2)-z(x^2+y^2)$; 4) $m^2(x^2+2)-2mn(x^2+2)+n^2(x^2+2)$.

23. Tenglamani yeching:
1) $5+(x-2)(x+2)=(x-3)^3+4$; 2) $(2x+3)^2-(2x-3)^2=24$;
3) $x:15=2\frac{1}{2}:12,5$; 4) $\frac{x}{3,5}=\frac{1,2:2}{2,1}$;
5) $(x-4)2-(x+4)2=16$; 6) $(3-x)^2+(3+x)^2=18$.

24. It bilan tulki orasidagi masofa 100 m. Tulkining tezligi 4 m/sek, itning tezligi 6 m/sek bo'lsa, it tulkini necha sekunddan keyin quvib yetadi?

25. O'lchamlari (eni, bo'yi va balandligi) mos ravishda a, b va c bo'lgan to'g'ri burchakli parallelepipedning to'la sirtini toping.

□ To'g'ri burchakli parallelepipedning 6 ta yog'i bor, har biri to'g'ri to'rtburchakdan iborat. Qarama-qarshi yoqlarining yuzi o'zaro teng. Har bir yoq to'g'ri to'rtburchak bo'lgani uchun $S_{to'la}=2ab+2bc+2ca$. Bu algebraik ifodada 2 soni umumiy ko'paytuvchi. Uni qavsdan tashqariga chiqaramiz: $S_{to'la}=2\cdot(ab+bc+ca)$. ■

26. Zavodda o'rtacha mehnat unumdorligi 20 % ga ortgan va yil oxirida 12000 birlik mahsulot ishlab chiqarilgan. Yil boshida ishlab chiqarilgan mahsulot hajmini toping.

□ Dastlabki mahsulot miqdori q_0, oxirgisi q_1, o'sish foizi p % bo'lsin. Unda dastlabki mahsulot miqdoriga, masalaning shartiga ko'ra, uning p % ini qo'shamiz: $q_1=q_0+q_0\frac{p}{100}$ bunda, q_0 – umumiy koeffitsiyent, uni qavsdan tashqariga chiqaramiz: $q_1=q_0\cdot\left(1+\frac{p}{100}\right)$. Agar $q_0=12\,000$, $p=20$ % bo'lsa,

$$q_1=12\,000\cdot\left(1+\frac{20}{100}\right)=12\,000\cdot\left(1+\frac{1}{5}\right)=12\,000\cdot\frac{6}{5}=\frac{72\,000}{5}=12\,400.$$

Javob: $q_1=12\,400$. ■

27. Firma a dona mahsulot ishlab chiqardi. Uning $\frac{3}{5}$ qismini b_1 so'mdan, qolgan qismini b_2 so'mdan sotdi. Firma a dona mahsulotni necha so'mga sotdi? Agar $a=20\,000$, $b_1=150$ so'm, $b_2=180$ so'm bo'lsa,

$$\frac{20\,000}{5}\cdot(3\cdot150+2\cdot180)=4\,000\cdot810=3\,240\,000.$$

Javob: Firma 20 000 dona mahsulotni 3 240 000 so'mga sotgan.

2-§. Guruhlash usuli

Ko‘phadning hamma hadlari uchun umumiy ko‘paytuvchi mavjud bo‘lmasligi mumkin. Bunday hollarda guruhlash usuli qo‘llaniladi.

Ba’zi hollarda ko‘phadning bir nechta hadini qavs ichiga olib, ko‘phad ko‘rinishidagi umumiy ko‘paytuvchini hosil qilish mumkin. Masalan:

1) $m(2a+b)-2a-b=m(2a+b)-(2a+b)=(2a+b)(m-1)$;
2) $2(a+b)+ca+cb=2(a+b)+(ca+cb)=2(a+b)+c(a+b)=(a+b)(2+c)$.

Birinchi misolda oxirgi ikki hadi “–” ishorasi bilan, ikkinchi misolda oxirgi ikki hadi “+” ishorasi bilan olindi. Natijada ko‘phad ko‘rinishidagi umumiy ko‘paytuvchi hosil bo‘ldi. Birinchi misolda hosil qilingan umumiy ko‘paytuvchi $2a+b$ qavsdan tashqariga chiqarildi, ikkinchi misolda esa hosil qilingan umumiy ko‘paytuvchi $a+b$ qavsdan tashqariga chiqarildi.

Ba’zan ko‘phad hadlarini turli usullar bilan guruhlash mumkin bo‘ladi. Masalan, $3ap+3aq-bp-bq$ ko‘phadni ikki usul bilan ko‘paytuvchilarga ajratish mumkin:

1-usul: $3ap+3aq-bp-bq=(3ap+3aq)-(bp+bq)=3a(p+q)-b(p+q)=(p+q)(3a-b)$;

2-usul: $3ap+3aq-bp-bq=(3ap-bp)+(3aq-bq)=p(3a-b)+q(3a-b)=(3a-b)(p+q)$.

Bu misolda ko‘phad hadlarini ikkitadan guruhladik. Ko‘phad hadlarini ikkitadan ham, uchtadan ham guruhlash mumkin bo‘lgan hollarga misol keltiramiz:

1-misol: $ax+bx+ay+by-az-bz$ ko‘phad berilgan bo‘lsa, uni quyidagicha guruhlaymiz:

$ax+bx+ay+by-az-bz=(ax+bx)+(ay+by)-(az+bz)=x(a+b)+y(a+b)-z(a+b)=(a+b)(x+y-z)$.

Bunda hadlar ikkitadan guruhlandi. Shu ko‘phad hadlarini uchtadan guruhlasa ham bo‘ladi:

$ax+bx+ay+by-az-bz=(ax+ay-az)+(bx+by-bz)=a(x+y-z)+b(x+y-z)=(x+y-z)(a+b)$.

2-misol: $ax^3+bx^3+bx^2+ax^2+ax+bx$ ko‘phadning hadlarini avval ikkitadan, keyin uchtadan guruhlaymiz:

$ax^3+bx^3+bx^2+ax^2+ax+bx=(ax^3+bx^3)+(bx^2+ax^2)+(ax+bx)=x^3(a+b)+x^3(a+b)+x(a+b)=(a+b)(x^3+x^2+x)=(a+b)\cdot x\,(x^2+x+1)$;

$ax^3+bx^3+bx^2+ax^2+ax+bx=(ax^3+ax^2+ax)+(bx^3+bx^2+bx)=a\cdot x\cdot(x^2+x+1)+b\cdot x\cdot(x^2+x+1)=(x^2+x+1)\cdot(ax+bx)=(x^2+x+1)\cdot x\cdot(a+b)$.

Ko‘phadni guruhlash usuli bilan ko‘paytuvchilarga ajratish uchun:
1) ko‘phadning hadlarini, ular ko‘phad ko‘rinishidagi umumiy ko‘paytuvchiga ega bo‘ladigan qilib, guruhlarga ajratiladi;
2) bu umumiy ko‘paytuvchini qavsdan tashqariga chiqariladi.

28. 1) Ko‘phad hadlarini guruhlash nima?
2) Guruhlash usullarini tushuntirib bering.
3) Ko‘phadni guruhlash usuli bilan ko‘paytuvchilarga ajratish uchun nimalar qilish kerak?

Guruhlash usulidan foydalanib ko‘paytuvchilarga ajrating **(29–31)**:

29. 1) $m+n+a(m+n)$;
2) $a-b+p(a-b)$;
3) $x+2m(x+y)+y$;
4) $x+2m(x-y)-y$.

30. 1) $x-y+(x-y)^2$;
2) $(m-n)^2+m-n$;
3) $3ap-3aq+(p-q)^2$;
4) $4ab-4b+a-1$.

31. 1) $a(x-c)+bc-bx$;
2) $4p(q-1)+q-1$;
3) $2a(x-y)-y+x$;
4) $3q(p-1)+2-2p$.

Guruhlashlar usuli bilan ko‘paytuvchilarga ajrating **(32–34)**:

32. 1) $ac+bc-2ad-2bd$;
2) $2bx-3ay-6by+ax$;
3) $ac-3bd+ad-3bc$;
4) $5ay-3bx+ax-15by$.

33. 1) $18a^2-27ab+14ac-21bc$;
2) $10x^2+10xy+5x+5y$;
3) $35ax+24xy-20ay-42x^2$;
4) $48xz^2+32xy^2-15yz^2-10y^3$.

34. 1) $16ab^2-5b^2c-10c^3+32ac^2$;
2) $6mnk^2+15m^2k-14n^3k-35mn^2$;
3) $xy^2-by^2-ax+ab+y^2-a$;
4) $ax^2-ay-bx^2+cy+by-cx^2$.

35. Qulay usul bilan hisoblang:

1) $127\cdot170+182\cdot15+127\cdot130+118\cdot15$;

2) $125\cdot75-216\cdot15+125\cdot25-216\cdot35$;

3) $13{,}6\cdot15-6\cdot13{,}6+14\cdot5{,}3-5\cdot5{,}3$;

4) $5\frac{1}{3}\cdot4\frac{2}{5}+3\frac{3}{4}\cdot7\frac{2}{3}+5\frac{1}{3}\cdot5\frac{1}{5}+3\frac{3}{4}\cdot2\frac{1}{3}$.

36*. Guruhlash usulidan foydalanib tenglamani yeching:

1) $2x(x-3)+4x-12=0$;
2) $y(y+6)-2y-12=0$;
3) $3\cdot(2x-5)+4x^2-10x=0$;
4) $\frac{1}{2}\cdot(3x-4)-1+\frac{3}{4}x=0$;
5) $x(x+2)-9x-18=0$;
6) $x(2x-3)-18x+27=0$.

Ko‘phadni ko‘paytuvchilarga ajrating **(37–39)**:

37*. 1) $18a^4b^3c^2-24a^3b^4c+12a^2b^2c^2-16ab^3c$;

2) $ax^3+bx^3+bx^2+ax^2+ax+bx+a+b$;

3) $15a^2b^3-6ab^2cd-15abc^2d+6c^3d^2$;

4) $x^3+ax^2+abx+bx^2+bcx+acx+cx^2+abc$.

38. 1) $ac+ad+2bc+2bd$;
2) $2ax-2ay-3by+3bx$;
3) $x^2y-z^2x+y^2x-z^2y$;
4) $x^2+xy-xy-yz$;
5) $a^3+2+a+2a^2$;
6) $y^2+3-y-3y^3$.

39. 1) $x^3+x-3xy+2+2x^2-6y$;
2) $ab-a^2+5-5b-5a^2+a^3$;
3) $4ax+2ay-az-4bx-2by+bz$;
4) $6ax+3by-3x+6ay+3by-3y$.

Ko‘phadni, uning ba’zi hadlarini o‘xshash hadlar yig‘indisi ko‘rinishida tasvirlab, ko‘paytuvchilarga ajrating **(40–41)**:

40.* 1) $a^2+3ab+2b^2$;
2) $x^2-4xy+3y^2$;
3) $x^2-5ax+4a^2$;
4) $a^2+7ab+6b^2$;
5) $a^2+5ab+6b^2$;
6) $x^2+7xy+12y^2$;
7) $x^2+8x+15$;
8) $a^2+7a+10$.

41. 1) $y^2+by-2b^2$;
2) $x^2-xy-2y^2$;
3) $6x^2-7xy+4y^2$;
4) $18x^2+39xy+20y^2$;
5) $14x^2-30ax-5a^2$;
6) $2x^2+5xy+2y^2$;
7) $x^2-\frac{3}{2}xy-3y^2$;
8) $0,3x^2-0,85xy-0,5y^2$.

Namuna: 1) $a^2-5a-6=a^2-6a+a-6=(a^2+a)+(-6a-6)=$
$=a(a+1)-6(a+1)=(a+1)(a-6)$.
2) $0,2x^2+0,9xy-0,5y^2=0,2x^2+xy-0,1xy-0,5y^2=(0,2x^2+xy)+$
$+(-0,1xy-0,5y^2)=x\cdot(0,2x+y)-0,5(0,2x+y)=(0,2x+y)(x-0,5y)$.

42. Guruhlash usulidan foydalanib, ko‘paytuvchilarga ajrating:
1) $20a^2+3bc-15ab-4ac$;
2) $-15x^2+4yz-10xy+6xy$;
3) $9a^3-2bc^2-ac^2+18a^2b$;
4) $pq-20k^3-4k^2p-5qk$;
5) $12z^2-9yz-4az-3ay$;
6) $15x^2+2yz-5xz-6xy$;
7) $3a^2-2bc-ac+6ab$;
8) $mp+10n^2-2mn-5np$.

43*. 41-mashqni yechish usulidan foydalanib, tenglamani hal qiling:
1) $x^2-2x+1=0$;
2) $x^2+2x+1=0$;
3) $x^2-5x+6=0$;
4) $x^2+5x+6=0$;
5) $x^2-x-20=0$;
6) $x^2+x-20=0$;
7) $x^2-7x+12=0$;
8) $x^2+7x+12=0$.

44. 1) $x^2+5x-6=0$;
2) $x^2+x-12=0$;
3) $6x^2-11x+3=0$;
4) $4x^2-5x+1=0$;
5) $2x^2-3x-2=0$;
6) $3x^2+11x-4=0$.

Namuna: $10x^2-13x-3=0$ tenglamani yeching.
$10x^2-13x-3=10x^2-15x+2x-3=(10x^2-15x)+(2x-3)=5x(2x-3)+$
$+(2x-3)=(2x-3)(5x+1)=0$.
Bundan:

$2x-3=0$; $2x=3$; $x=\frac{3}{2}$; $5x+1=0$; $5x=-1$; $x=-\frac{1}{5}$.

Javob: $x=\frac{3}{2}$; $x=-\frac{1}{5}$.

45. Hisoblang:

1) $318^2 - 318 \cdot 57 + 261 \cdot 682$;

2) $(89,4)^2 + 89,4 \cdot 13,7 - 103,1 \cdot 49,4$;

3) $41,8 \cdot 41 - 30 \cdot 41,8 + 71 \cdot 58,2 - 60 \cdot 58,2$;

4) $13\frac{11}{18} \cdot 40\frac{1}{5} + 40,2 \cdot 36\frac{1}{3} + 58,8 \cdot 17\frac{11}{17} + 58,8 \cdot 32\frac{5}{17}$.

3-§. Yig'indining kvadrati. Ayirmaning kvadrati

Ikki son yig'indisining kvadrati $(a+b)^2$ ni ko'ramiz.

Bu algebraik ifoda $a+b$ va $a+b$ ko'phadlarning ko'paytmasini anglatadi. Shuning uchun ko'paytirishni bajarib $(a+b)^2=(a+b)(a+b)=a^2+ab+ab+b^2=$ $=a^2+2ab+b^2$ ko'phadni hosil qilamiz. Biz ushbu formulani chiqardik:

$$(a+b)^2=a^2+2ab+b^2. \qquad (1)$$

Bu formula so'z bilan quyidagicha aytiladi:

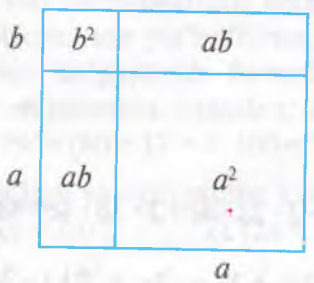
Ikki son yig'indisining kvadrati – birinchi sonning kvadrati qo'shuv birinchi son bilan ikkinchi son ko'paytmasining ikkilangani qo'shuv ikkinchi sonning kvadratiga teng bo'ladi.

(1) formulani geometrik usul bilan ham osongina isbotlash mumkin. Buning uchun tomonining uzunligi $a+b$ bo'lgan kvadratni ko'raylik (4-rasm). Shu kvadratning yuzi tomoni a bo'lgan kvadratdan, tomonlari a va b bo'lgan ikkita to'g'ri to'rtburchakdan, tomoni b bo'lgan kvadratdan iborat. Bundan kvadrat yuzi $a^2+2ab+b^2$ ga tengligi kelib chiqadi.

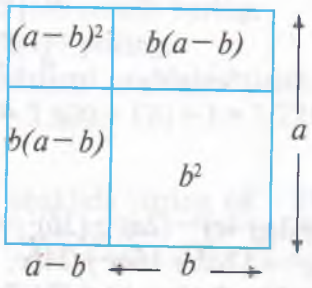
Endi ikki son ayirmasining kvadratini ko'raylik:
$(a-b)^2=(a-b)(a-b)=a^2-ab-ab+b^2=a^2-2ab+b^2$, ya'ni

$$(a-b)^2=a^2-2ab+b^2. \qquad (2)$$

Bu formulani ham geometrik usul bilan osongina isbot qilish mumkin. 5-rasmdan ko'rinadiki, tomoni a bo'lgan kvadrat yuzi a^2 bo'lib, $a^2=(a-b)^2+2b(a-b)+b^2$.



4-rasm



5-rasm

Bu ifodani soddalashtirib (2) formulani hosil qilamiz.
(2) formulani so‘z bilan quyidagicha aytish mumkin:

Ikki son ayirmasining kvadrati – birinchi son kvadrati ayiruv birinchi son bilan ikkinchi son ko‘paytmasining ikkilangani qo‘shuv ikkinchi sonning kvadratiga teng bo‘ladi.

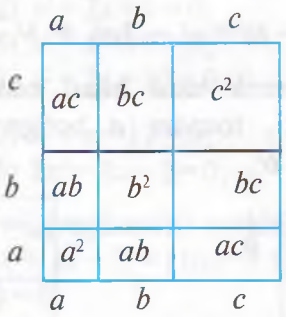
Endi uchta son yig‘indisining kvadrati uchun formula chiqaramiz: $(a + b + c)^2$ ni ko‘raylik. Uni $((a + b) + c)^2$ kabi yozib olamiz. Ikki son yig‘indisi kvadratining formulasidan foydalanish uchun $(a+b)$ ni birinchi had, c ni ikkinchi had deb qaraymiz va $((a + b) + c)^2$ ni yozib chiqamiz:
$(a + b + c)^2=((a + b) + c)^2=(a + b)^2 + 2(a + b)c + c^2=a^2 + 2ab + b^2+ + 2ac + 2bc + c^2$, ya’ni

$$(a + b + c)^2=a^2 + b^2 + c^2 + 2ab + 2ac + 2bc \qquad (3)$$

formulani hosil qilamiz.

Bu formulani ham geometrik usul bilan isbotlash mumkin. Tekislikda tomoninguzunligi $a+b+c$ bo‘lgan kvadrat chizamiz (6-rasm). Uchta, tomonlari mos ravishda a, b, c lardan iborat bo‘lgan kvadratdan, tomoni a va b bo‘lgan 2 ta, tomoni b va c bo‘lgan 2 ta, tomoni c va a bo‘lgan 2 ta to‘g‘ri to‘rtburchakdan iborat. 3 ta kvadrat va 6 ta to‘g‘ri to‘rtburchaklarning yuzlari topilgan formulaning o‘ng tomonini ifodalaydi (4.6-rasm).

(1), (2) va (3) formulalarning qo‘llanilishiga doir misollar ko‘ramiz:
1) $(3a+4b)^2=(3a)^2+2\cdot 3a\cdot 4b+(4b)^2=9a^2+24ab+16b^2$;
2) $(2a^2-5)^2=(2a^2)^2-2\cdot 2a^2\cdot 5+5^2=4a^2-20a^2+25$;
3) $(-2m-n)^2=((-1)(2m+n))^2=(-1)^2(2m+n)^2=(2m+n)^2=(2m)^2+2\cdot 2m\cdot n+ + n^2=4m^2+4mn+n^2$;



6-rasm

4) $(2a+3b+4c)^2=(2a)^2+(3b)^2+(4c)^2+2\cdot 2a\cdot 3b+2\cdot 2a\cdot 4c+2\cdot 3b\cdot 4c=4a^2+9b^2+ + 16c^2+12ab+16ac+24bc$;
5) $(a-2b+3c)^2=a^2+(-2b)^2+(3c)^2+2a\cdot(-2b)+2\cdot a\cdot 3c+2\cdot(-2b)\cdot 3c= =a^2+4b^2+9c^2-4ab+6ac-12bc$.

Yig'indi yoki ayirmaning kvadrati formulalari qisqa ko'paytirish formulalari deyiladi.

(1), (2) va (3) formulalarni guruhlash usuli bilan isbotlash mumkin:

1) $(a+b)^2=a^2+2ab+b^2$ ni isbotlaymiz:

$a^2+2ab+b^2=a^2+ab+ab+b^2=(a^2+ab)+(ab+b^2)=a(a+b)+b(a+b)=$
$=(a+b)(a+b)=(a+b)^2$;

2) $(a-b)^2=a^2-2ab+b^2$ ni ham shunga o'xshash isbotlanadi:

$a^2-2ab+b^2=a^2-ab+b^2-ab=a(a-b)+b(b-a)=a(a-b)-b(a-b)=$
$=(a-b)(a-b)=(a-b)^2$;

3) $(a+b+c)^2=a^2+b^2+c^2+2ab+2ac+2bc$ tenglikni isbot qilamiz:

$a^2+b^2+c^2+2ab+2ac+2bc=(a+b)^2+2c(a+b)+c^2=(a+b+c)^2$.

Ushbu

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3 \qquad (4)$$

formulani isbotlaylik.

□ $(a+b)^3=(a+b)(a+b)^2=(a+b)(a^2+2ab+b^2)=a^3+2a^2b+ab^2+a^2b+2ab^2+b^3)=$
$=a^3+3a^2b+3ab^2+b^3$. ■

Shu yo'l bilan

$$(a-b)^3=a^3-3a^2b+3ab^2-b^3 \qquad (5)$$

formulani ham isbot qilish mumkin.

(4),(5) formulalarni $(a+b)^3=a^3+b^3+3ab(a+b)$, $(a-b)^3=a^3-b^3-3ab(a-b)$ kabi yozib olish ko'pgina hisoblashlarda qo'l keladi.

(4), (5) formulalar, mos ravishda, yig'indining kubi va ayirmaning kubi deyiladi.

(1), (2), (4), (5) formulalar qisqa ko'paytirish formulalari hisoblanadi.

46. 1) Ikki son yig'indisining kvadrati, ikki son ayirmasining kvadrati formulasini so'z bilan aytib bering.

2) Ikki son yig'indisining va ayirmasining kvadrati formulasini geometrik usul bilan isbotlang.

3) Ikki son yig'indisining va ayirmasining kubi formulasini yozing.

4) Shu formulalarning geometrik ma'nosini tushuntirib bering.

5) Uchta son yig'indisining kvadrati formulasini yozing.

Qisqa ko'paytirish formulalaridan hisoblashlarni soddalashtirishda foydalanish mumkin. Masalan: 1) $61^2=(60+1)^2=3\,600+120+1=3\,721$;

2) $79^2=(80-1)^2=6\,400-160+1=6\,241$.

47. Yig'indi va ayirmaning kvadratini ko'phad shaklida yozing **(47–51)**:

1) $(q+2p)^2$; 2) $(2x+3y)^2$; 3) $(4m-6n)^2$; 4) $(7a-b)^2$.

48. 1) $\left(a-\frac{1}{2}\right)^2$; 2) $\left(m-\frac{1}{3}\right)^2$; 3) $\left(\frac{a}{3}-\frac{b}{2}\right)^2$; 4) $\left(\frac{m}{2}+\frac{n}{3}\right)^2$.

49. 1) $(0{,}1m+0{,}3n)^2$; 2) $(0{,}7c-0{,}4d)^2$; 3) $\left(\frac{1}{3}y^3-\frac{1}{4}\right)^2$; 4) $\left(\frac{2}{3}x^3-\frac{3}{4}\right)^2$.

50. 1) $(a+b-4)^2-(a^2+b-1)(a^2+b-8)$;
2) $(2a^2+a-5)^2-(2a^2+a)(2a^2+a-1)+9(2a+3)(a-1)$.

51. 1) $2x(x+3)^2-3x(x-1)(x+8)-3x\cdot(2-3x)$;
2) $y(2y-1)(2y-2)-4y(y+1)^2+2y(7y+1)$.

Isbotlang **(52–53)**:

52. 1) $(a-1)^2+2(a-1)+1=a^2$; 2) $(1-a)^2+2a(1-a)+a^2=1$.

53. 1) $(x+y)^2-2(x+y)(x-y)+(x-y)^2=4y^2$;
2) $(a^2+b^2)(c^2+d^2)=(ac+bd)^2+(ad-bc)$.

54. Qirrasining uzunligi a va b bo'lgan ikkita kub yasang. So'ngra o'lchamlari a, a, b va a, b, b bo'lgan 3 tadan to'g'ri burchakli parallelepiped yasang. Ularni taxlab, qirrasining uzunligi $a+b$ bo'lgan kubni hosil qiling.
Bu bilan $(a+b)^3=a^3+3a^2b+3ab^2+b^3$ formulaning geometrik ma'nosini ochgan bo'lasiz.

55.* To'g'ri to'rtburchak shaklidagi oynaning bir tomoni ikkinchisidan 30 sm uzun. Uni deraza romiga solish uchun bo'yi va enidan 10 sm dan kesishdi. Oynaning kesib tashlangan bo'laklarining yuzi 1400 sm^2 ga teng. Oynaning dastlabki o'lchamlarini toping.

Ifodani soddalashtiring **(56–57)**:

56. 1) $(x-2y)^2+(x+2y)^2$; 3) $(x+2y)^2-(x-2y)^2$;
2) $(2x-y)^2-(2x+y)^2$; 4) $(3x+y)^2-(3x-y)^2$.

57. 1) $(1-3a)^2+2(1+a)^2$; 3) $(x+1)^2-(x-1)^2$;
2) $3(2-a)^2+4(a-5)^2$; 4) $(1-x)^2-(3+x)^2$.

4-§. Kvadratlar ayirmasining formulasi

Kvadratlar ayirmasining formulasini chiqarish uchun ikki son yig'indisining ularning ayirmasiga ko'paytmasini ko'ramiz:

$$(a+b)(a-b)=a^2-ab+ab-b^2=a^2-b^2,$$

ya'ni $(a+b)(a-b)=a^2-b^2$. Demak,

$$a^2-b^2=(a+b)(a-b). \qquad (1)$$

Ikki son kvadratlarining ayirmasi shu sonlar yig'indisi bilan ular ayirmasining ko'paytmasiga teng.

(1) formulani geometrik usulda ham isbotlash mumkin.

Tomoni a bo'lgan kvadrat ichiga, tomoni b bo'lgan ($b<a$) kvadrat 7-rasmdagidek joylashtirilgan bo'lsin.

Unda katta kvadrat yuzi a^2, kichik kvadrat yuzi esa b^2 bo'ladi.

Tomoni $a-b$ bo'lgan kvadrat yuzi $(a-b)^2$, tomonlari $(a-b)$ va b bo'lgan ikki-

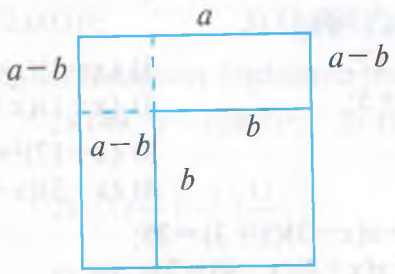
ta to‘g‘ri to‘rtburchak yuzlari yig‘indisi $2b(a-b)$ bo‘ladi. Bundan ko‘rinadiki, $a^2=b^2+(a-b)^2+2b(a-b)$ tenglik o‘rinli. Shu tenglikda b^2 ni chap tomonga o‘tkazib, o‘ng tomonda $(a-b)$ umumiy ko‘paytuvchini qavsdan tashqariga chiqarsak, $a^2-b^2=(a-b)(a-b+2b)=(a-b)(a+b)$, ya’ni

$$a^2-b^2=(a-b)(a+b)$$

kelib chiqadi.

Ushbu formulani guruhlash usuli bilan ham isbotlasa bo‘ladi. Buning uchun ifodaga ab ni qo‘shib, ab ni ayiramiz: $a^2-ab+ab-b^2=a(a-b)+b(a-b)$. Bunda $(a-b)$ umumiy ko‘paytuvchi, uni qavsdan tashqariga chiqarsak, $a^2-b^2=(a-b)(a+b)$ kelib chiqadi.

(1) formula *qisqa ko‘paytirish formulasi* deyiladi.



7-*rasm*

(1) formuladan hisoblashlarni soddalashtirish uchun foydalaniladi. Masalan:

1) $72\cdot 68=(70+2)(70-2)=4900-4=4\ 896$;
2) $97\cdot 103=(100-3)(100+3)=10\ 000-9=9\ 991$;
3) $998\cdot 1\ 002=(1\ 000-2)(1\ 000+2)=1\ 000\ 000-4=999\ 996$;
4) $58\cdot 62=(60-2)(60+2)=3\ 600-4=3\ 596$;
5) $101\cdot 99=(100+1)(100-1)=10\ 000-1=9\ 999$.

(1) tenglik *ikki son kvadratlari ayirmasi formulasi* deyiladi.

Ko‘paytuvchilarga ajrating **(58–59)**:

58. 1) $9a^2-4b^2$; 2) $4m^2-25n^2$; 3) $16p^2-9q^2$; 4) $36a^2-9b^2$.

59. 1) $(a+b)^2-c^2$; 2) $m^2-(n-q)^2$; 3) $(a+b)^2-4(a-b)^2$; 4) $\frac{1}{9}m^2-9n^2$.

60. Quyidagi tenglikni isbotlang:

1) $(10a^2+5)^2=100a(a+1)+25$;
2) $(10a^2+b)(10a+b+2)=(10a+b+1)^2-1$.

61. Ko‘paytuvchilarga ajrating:

1) $36a^4-49b^4$; 2) $36a^4b-64b^5$; 3) $(a^2-5a)^2-25$; 4) $(6a-7d)^2-49d^2$.

62. Ko‘paytirishni bajaring:

1) $(2+x)(2-x)(4+x^2)$; 2) $(x^2+16)(x-4)(x+4)$;
3) $(2x-3y)(2x+3y)(4x^2+9y^2)$; 4) $(3x-\frac{1}{2}y)(3x+\frac{1}{2}y)(9x^2+\frac{1}{4}y^2)$.

ta to‘g‘ri to‘rtburchak yuzlari yig‘indisi $2b(a-b)$ bo‘ladi. Bundan ko‘rinadiki, $a^2=b^2+(a-b)^2+2b(a-b)$ tenglik o‘rinli. Shu tenglikda b^2 ni chap tomonga o‘tkazib, o‘ng tomonda $(a-b)$ umumiy ko‘paytuvchini qavsdan tashqariga chiqarsak, $a^2-b^2=(a-b)(a-b+2b)=(a-b)(a+b)$, ya’ni

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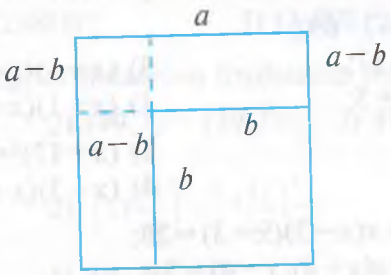

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1) $(10a^2+5)^2=100a(a+1)+25$;
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1) $(2+x)(2-x)(4+x^2)$; 2) $(x^2+16)(x-4)(x+4)$;

3) $(2x-3y)(2x+3y)(4x^2+9y^2)$; 4) $(3x-\frac{1}{2}y)(3x+\frac{1}{2}y)(9x^2+\frac{1}{4}y^2)$.

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63. Tenglamani yeching:
1) $(2x+1)^2-4(x-1)(x+1)=17$; 2) $(2x-3)^2-(2x-3)(2x+3)=18$.

64. Tenglamani yeching:
1) $16x^2-9=0$; 2) $0{,}02x^2-0{,}08=0$; 3) $1{,}44x^2-1{,}21=0$; 4) $9x^2+6x+1=0$.

65. Ko'paytuvchilarga ajrating:
1) $(2a+3b)^2-(3a+4b)^2$;
2) $(5a-2b)^2-(3a+b)^2$;
3) $(3+7a)^2-(3a-3)^2$;
4) $(a^2-3)^2-(a^2+3)^2$.

Masalalar yechish

66. Kvadratga oshiring:
1) $(a-b+c)^2$; 2) $(a-b-c)^2$; 3) $(a+b+c+d)^2$; 4) $(a+b-c+d)^2$.

Tenglamani yeching **(67–69)**:

67. 1) $x(x-1)-x(x-3)=12$;
2) $(x-5)(x+1)-x=x^2+5$;
3) $(x-1)(x-3)=(x-2)(x-4)$;
4) $(x+1)(x+2)-x^2=5x+4$.

68. 1) $(x-4)^2-x^2=16$;
2) $(x+1)^2=x^2+1$;
3) $(x+12)^2=x(x+8)$;
4) $(x-3)(x+1)=(x-2)^2$.

69. 1) $(x-2)(x^2-4x+4)-x(x-3)(x+3)=26$;
2) $(x-3)(x^2+3x+9)-x(x+4)(x-4)=21$;
3) $(2x-1)(4x^2+2x+1)-4x(2x^2-3)=23$;
4) $(4x+1)(16x^2-4x+1)-16x(4x^2-5)=17$.

x ni shunday birhadga almashtiringki, natijada tenglik to'g'ri bo'lsin:.

70. 1) $(x-5b^2)^2=9x^2-30b^2x+25b^4$;
2) $(x-3b^2)^2=16a^2b^2-24ab^4+9b^6$;
3) $(7a^3-x)^2=49a^6-70a^3b+25b^2$;
4) $(3a^4-x)^2=9a^8-42a^4b^2+49b^4$.

x ni shunday birhadga almashtiringki, natijada ikkihadning kvadrati hosil bo'lsin:

71. 1) $9a^2+24ab+x$; 2) $x-12ab+9b^2$; 3) $49a^2-x+16b^2$.

4) $0{,}01a^2-x+100b^2$; 5) $49a^2-x+16b^2$; 6) $\frac{4}{9}a^2+x+\frac{9}{4}b^2$.

Qisqa ko'paytirish formulalaridan foydalanib hisoblang:

72. 1) $1\,999^2$; 2) $1\,001^2$; 3) 81^2; 4) 198^2; 5) 59^2;
6) 79^2; 7) 38^2; 8) 71^2; 9) 101^2; 10) $2\,001^2$.

Ifodani ikkihadning kvadrati shaklida tasvirlang:

73. 1) $a^6-8a^3b^5+16b^{10}$; 2) $16a^2+40ab+25b^2$; 3) $81x^2-144xy+64y^2$;

4) $49x^2-42xy+9y^2$; 5) $0{,}25x^2+0{,}4xy+0{,}16y^2$; 6) $\frac{1}{9}a^2-ab+9b^2$;

7) $36a^2-ab+\frac{1}{36}b^2$; 8) $4x^2+1\frac{1}{3}xy+\frac{1}{9}y^2$.

$(a+b)^2=a^2+2ab+b^2$ formulada $a=1$, b esa moduli 1 ga nisbatan kichik bo'lsa (masalan, $b=0{,}0032$ yoki $b=-0{,}0021$), u holda b^2 son yanada kichik

bo'ladi. Shu sababli $(a+b)^2=a^2+2ab+b^2$ tenglikni $a=1$ bo'lganda $(1+b)^2\approx1+2b$ taqribiy tenglik bilan almashtirish mumkin.

Masalan: $(1{,}002)^2=(1+0{,}002)^2\approx1+2\cdot0{,}002=1{,}004$;

$(0{,}997)^2=(1-0{,}003)^2\approx1-2\cdot0{,}003=1-0{,}006=0{,}994$.

74. Taqribiy formula yordamida hisoblang:

1) $(1{,}001)^2$; 2) $(1{,}003)^2$; 3) $(0{,}998)^2$; 4) $(0{,}999)^2$;

5) $(1{,}0001)^2$; 6) $(1{,}0003)^2$; 7) $(0{,}9998)^2$; 8) $(0{,}9999)^2$.

Agar $a=2$ bo'lib, b son moduli bo'yicha kichik bo'lsa, masalan: $b=0{,}001$; $b=0{,}002$, ushbu $(2+b)^2\approx4+4b$ taqribiy formuladan foydalansa bo'ladi. Masalan, $(2{,}001)^2=(2+0{,}001)^2\approx4+4\cdot0{,}001=4+0{,}004=4{,}004$.

75. Taqribiy formula $(2+b)^2\approx4+4b$ yordamida hisoblang:

1) $(2{,}002)^2$; 2) $(2{,}003)^2$; 3) $(1{,}998)^2$; 4) $(1{,}999)^2$.

Kvadratlar ayirmasining formulasidan foydalanib hisoblang **(76–78)**:

76. 1) 2011^2-2001^2; 2) $(49{,}7)^2-(39{,}7)^2$; 3) 1999^2-1^2; 4) $(2003)^2-3^2$.

77. 1) $(349{,}5)^2-(249{,}5)^2$; 2) $\left(17\frac{11}{19}\right)^2-\left(7\frac{11}{19}\right)^2$; 3) $799\cdot1001$;

4) $28\cdot32$; 5) $49\cdot51$; 6) $4{,}5\cdot5{,}5$.

78. 1) $\dfrac{(1{,}7)^2-(1{,}3)^2}{(2{,}8)^2-(2{,}2)^2}$; 3) $\dfrac{3^4\cdot0.328-3^3\cdot0{,}628\cdot3}{27\cdot0.09}$;

2) $\dfrac{(1{,}2)^2-(0{,}3)^2}{(0{,}8)^2-(0{,}7)^2}$; 4) $\dfrac{7^5\cdot0{,}143-7^4\cdot0{,}843\cdot7}{0{,}49\cdot2401}$.

(1) formuladan foydalanib ko'paytirishni bajaring **(79–80)**:

79. 1) $(a+b)(b-a)$; 2) $(m-2)(2+m)$; 3) $(p-q)(p+q)$; 4) $(4+q)(4-q)$;

5) $(2a-b)(2a+b)$; 7) $(2d-1)(1+2d)$; 6) $(3a+b)(3a-b)$; 8) $(3+d)(3-d)$.

80. 1) $\left(y-\frac{1}{3}x\right)\left(y+\frac{1}{3}x\right)$; 5) $(2+x)(2-x)(4+x^2)$;

2) $\left(2m-\frac{1}{2}n\right)\left(2m+\frac{1}{2}n\right)$; 6) $(3x+y)(3x-y)(9x^2+y^2)$;

3) $\left(2d-\frac{1}{2}\right)\left(2d+\frac{1}{2}\right)$; 7) $(y^2+9)(y+3)(y-3)$;

4) $\left(a-\frac{3}{4}b\right)\left(\frac{3}{4}b+a\right)$; 8) $(2a+3b)(2a-3b)\,(4a^2+9b^2)$.

Ko'paytuvchilarga ajrating **(81–84)**:

81. 1) $(5a+4b)^2-(2a+3b)^2$; 3) $(a^2+5)^2-(a^2+7)^2$;

2) $(7a-3b)^2-(5a+6b)^2$; 4) $(7+5a)^2-(3a-2)^2$.

82. 1) $(2a+5b)^2-(2a-3c)^2$; 2) $(4a-3b)^2-(3a+2c)^2$;
3) $(3x-1)^2-(5-2x)^2$; 4) $(a^2-2)^2-(a^2+2)^2$;

83. 1) $49a^4-81b^4$; 3) $(a^2+10a)^2-625$;
2) $49a^4b-64b^5$; 4) $(4a-5d)^2-25d^2$.

84. 1) $(2a+5b)^2-9(2b-a)^2$; 2) $(2a-3b)(4a^2-c^2)-(2a-c)(4a^2-9b^2)$;
3) $8a^3-60a^2b+15ab^2-125b^3$; 4) $27x^3-27x^2y+9xy^2-y^3$.

85. Ko'paytirishni bajaring:
1) $(5+x)(5-x)(25+x^2)$; 4) $(5x-4y)(5x+4y)(25x^2+16y^2)$;
2) $(x^2+9)(x+3)(x-3)$; 5) $(36x^2+\frac{1}{9}y^2)(6x-\frac{1}{3}y)(6x+\frac{1}{3}y)$;
3) $(16x^2+9y^2)(4x+3y)(4x-3y)$; 6) $(0,01x^2+0,09y^2)(0,1x+0,3y)(0,1x-0,3y)$.

Tenglamani yeching **(86–87)**:

86. 1) $(3x+2)^2-9(x-1)(x+1)=27$; 2) $(4x+3)(4x-3)-(4x-1)2=6$;
3) $(2x-5)^2-(2x-1)(2x+1)=30$; 4) $(3x-2)2-(4-3x)^2=12$.
5) $9x^2-16=0$; 6) $0,01x^2-0,04=0$; 7) $25x^2-81=0$; 8) $1,69x^2-2,25=0$.

87. 1) $4x^2+4x+1=0$; 2) $16x^2+40x+25=0$; 3) $4x^2-4x+1=0$;
4) $16x^2-40x+25=0$; 5) $\frac{1}{9}x^2-2x+9=0$; 6) $0,01x^2+0,04x+0,04=0$;
7) $0,09x^2-3x+25=0$; 8) $\frac{25}{36}x^2-3\frac{1}{3}x+4=0$.

88. Agar kvadratning ikki qarama-qarshi tomonining har biri 6 sm ga qisqartirilsa hamda qolgan ikki tomonining har biri 6 sm ga uzaytirilsa, kvadratning yuzi qanday o'zgaradi?

89. Kubning qirrasi 4 sm. Agar uning har bir qirrasi 1 sm ga qisqartirilsa, hajmi qanchaga kamayadi?

90. Ifodaning son qiymatini toping:
1) $14m-(m+2)^2+(m-2)(m+2)$, bunda $m=2,5$;
2) $(2x+3)^2-5x-(x-3)(x+3)$, bunda $x=-1,5$;
3) $3(n-5)(n+3)-(n-3)^2-(n-5)(5+n)$, bunda $n=9$;
4) $(b+2)^2+(b-2)(b+2)-2(b+1)(b-3)$, bunda $b=-\frac{1}{3}$.

5-§. Ko'phadni ko'paytuvchilarga ajratishning bir nechta usullarini qo'llash

Ko'phadni ko'paytuvchilarga ajratishda uning berilishiga qarab turli usullardan foydalaniladi: umumiy ko'paytuvchini (agar u bor bo'lsa) qavsdan tashqariga chiqarish, kvadratlar ayirmasi formulasidan foydalanish, guruhlashning har ikki usulini qo'llash, qisqa ko'paytirish formulalarini ishlatish shular jumlasidandir. Misollar keltiramiz:

1) a^3-4a ko'phadni ko'paytuvchilarga ajrating:

□ $a^3-4a=a(a^2-4)=a(a^2-2^2)=a(a+2)(a-2)$. ■

Bunda avval qavsdan tashqariga chiqarish, so'ngra esa kvadratlar ayirmasi formulasini qo'llash bajarilgan.

2) $(a^2+4)^2-16a^2$ ko'phadni ko'paytuvchilarga ajrating:

□ $(a^2+4)^2-16a^2=(a^2+4)^2-(4a)^2=((a^2+4)-4a)((a^2+4)+4a)=(a^2-4a+4)\cdot$ $\cdot(a^2+4a+4)=(a-2)^2(a+2)^2$. Berilgan ko'phad umumiy ko'paytuvchiga ega emas. Unga kvadratlar ayirmasi formulasini qo'llash mumkin, keyin yig'indi va ayirma kvadratlari uchun formuladan foydalansa bo'ladi. Natijada, ko'phad ko'paytuvchilarga ajraladi:

$$(a^2-4)^2-16a^2=(a-2)^2(a+2)^2. \blacksquare$$

3) $x^2-9y^2+2(x+3y)$ ko'phadni ko'paytuvchilarga ajrataylik:

$x^2-9y^2+2(x+3y)=(x^2-9y^2)+2(x+3y)=(x+3y)(x-3y)+2(x+3y)=$ $=(x+3y)(x-3y+2)$.

Bunda avval guruhlash usuli qo'llanildi, so'ngra kvadratlar ayirmasi formulasidan foydalanildi, nihoyat umumiy ko'paytuvchini qavsdan tashqariga chiqarish bajarildi.

Ko'rib chiqilgan misollardan ko'phadni ko'paytuvchilarga ajratish uchun quyidagi tartibga rioya qilish lozimligi kelib chiqadi:

1) *umumiy ko'paytuvchini (agar u bor bo'lsa) qavsdan tashqariga chiqarish;*

2) *ko'phadni qisqa ko'paytirish formulalari bo'yicha ko'paytuvchilarga ajratishga urinib ko'rish;*

3) *agar oldingi usullar maqsadga olib kelmasa, guruhlash usulini qo'llashga harakat qilish.*

Masala: Tenglikni isbotlang:

$$a^3+b^3=(a+b)(a^2-ab+b^2). \qquad (1)$$

(1) tenglikning o'ng tomonidagi qavslarni ochamiz:

$(a+b)(a^2-ab+b^2)=a^3-a^2b+ab^2+a^2b-ab^2+b^3=a^3+b^3$.

Shunga o'xshash

$$a^3-b^3=(a-b)(a^2+ab+b^2) \qquad (2)$$

formulani ham isbot qilish mumkin. Bunda ham qavslarni ochish kerak bo'ladi.

Misollar:

1) $(2a)^3+(3b)^3=(2a+3b)(4a^2-6ab+9b^2)$;

2) $(\frac{1}{3}a)^3+(3b)^3=(\frac{1}{3}a+3b)(\frac{1}{9}a^2-ab+9b^2)$;

3) $(3a)^3-(2b)^3=(3a-2b)(9a^2+6ab+4b^2)$;

4) $(\frac{1}{2}a)^3-(\frac{2}{3}b)^3=(\frac{1}{2}a-\frac{2}{3}b)(\frac{1}{4}a^2+\frac{1}{3}ab+\frac{4}{9}b^2)$.

(1) va (2) formulalar mos ravishda *kublar yig'indisi va kublar ayirmasi formulasi* deyiladi. Bu formulalar ham ko'phadni ko'paytuvchilarga ajratishda qo'llaniladi.

Ko'phadni ko'paytuvchilarga ajratishda uni ikkihadga bo'lish haqidagi Bezu teoremasi muhim ahamiyatga ega:

Bezu teoremasi: $P(x)$ ko'phadni $(x-a)$ ikkihadga bo'lganda chiqadigan qoldiq $P(a)$ ga teng (ya'ni qoldiq $P(x)$ ko'phadning $x=a$ bo'lgandagi son qiymatiga teng).

Bezu teoremasining natijasi: a son $P(x)$ ko'phadning ildizi bo'lishi uchun $P(x)$ ning $(x-a)$ ga qoldiqsiz bo'linishi zarur va yetarlidir.

Agar $P(x)$ ko'phad $x-a$ ga qoldiqsiz bo'linib, bo'linma $q(x)$ bo'lsa, u holda $P(x)$ ko'phadni ko'paytuvchilarga ajratish va

$$P(x)=(x-a)q(x)$$

ko'rinishda yozish mumkin. Masalan, $8a^3+27b^3$ ko'phad $2a+3b$ ga qoldiqsiz bo'linishini ko'rsatamiz. Bo'lishni burchak usulida bajaramiz:

$$\begin{array}{r|l} 8a^3+27b^3 & 2a+3b \\ \hline 8a^3+12a^2b^3 & 4a^2-6ab+9b^2 \\ \hline -12a^2b^2+27b^3 & \\ -12a^2b^2-18ab^2 & \\ \hline 18ab^2+27b^3 & \\ 18ab^2+27b^3 & \\ \hline 0 & \end{array}$$

Demak, berilgan ko'phadni quyidagicha ko'paytuvchilarga ajratish mumkin:

$$8a^3+27b^3=(2a+3b)(4a^2-6ab+9b^2).$$

91. Quyidagi ko'phadlar ikkihadga qoldiqsiz bo'linishini ko'rsating. Bo'lishni burchak usuli bilan bajaring:

1) $(a^4+ab^3-a^2b^2+a^3b):(a^2-b^2)$;
2) $(2a^3-15b^2+6ab^2-5a^2b):(2a-5b)$;
3) $(3a^5+a^4-3a^3-3a^2+2):(-a^2+1)$.

92*. Ko'phadni Bezu teoremasi yordamida ko'paytuvchilarga ajrating:

1) x^n-a^n ko'phad $(x-a)$ ga qoldiqsiz bo'linishini;
2) x^n+a^n ko'phad $(x+a)$ ga qoldiqsiz bo'linishini isbotlang, bunda n – natural son.

(**Ko'rsatma**: Bezu teoremasining natijasidan foydalaning).

3) $3x^5-5x^4+2x^3-x^2+4x-3$ ko'phad $(x-1)$ ga, $(x-2)$ ga qoldiqsiz bo'linishini ko'rsating.
4) $x^5-5x^4+3x^3+13x^2-8x-3$ ko'phadni $(x+1)$ga, $(x-2)$ ga, $(x-3)$ ga qoldiqsiz bo'linishini ko'rsating.

93. $2x^5+3x^4-5x^3-4x^2+3x+1$ ko'phadni $(x-1)$ ga, $(x+1)$ ga, $(x-2)$, $(x+2)$ ga bo'lgandagi qoldiqni toping.

?

94. 1) Ko'phadni ko'paytuvchilarga ajratishni qanday tartibda amalga oshirish kerak?

2) Ikki son kublari yig'indisi, kublari ayirmasi formulalarini yozing va misollar keltiring.

3) Ko'phadni ko'paytuvchilarga ajratishning burchak usuli nima? Misollar keltiring.

Ko'paytuvchilarga ajrating **(95–97)**:

95. 1) $ax-by-bx-x-by-y$;
2) $2a^2-a+2ab-b-2ac+c$;
3) $a^5-a^4b+a^3b^2-a^2b^2+ab^4-b^5$;
4) $ax^2+bx+b^2y+abxy+a^2bx+ab^2$.

96. 1) $xyz+4xz+3xy+12x$;
2) $2a+a^2+2a^3+a^4$;
3) $a^3+a^2b-a^2x-abx$;
4) $a^4-a^3+a^2-a$.

97*. 1) $3xyz+x^2y+x^2z+y^2x+y^2z+z^2x+z^2y$;
2) $x^2+y^2+z^2+2xy+2xz+2yz$;
3) $3abc+ab(a+b)+bc(b+c)+ac(a+c)+ab+bc+ac$;
4) $xy(x-y)-xz(y-z)-xz(x-y)+yz(y-z)$.

98. Tenglikni isbotlang:
1) $a^3+b^3=(a+b)(a^2-ab+b^2)$;
2) $a^3-b^3=(a-b)(a^2+ab+b^2)$;
3) $(a^3+b^3):(a+b)+ab=a^2+b^2$;
4) $(a^3-b^3):(a-b)+ab=(a+b)^2$;
5) $(a^3+b^3):(a+b)-ab=(a-b)^2$;
6) $(a+b)^3=a^3+b^3+3ab(a+b)$;
7) $(a-b)^3=a^3-b^3-3ab(a-b)$.

99*. Isbotlang:

1. 1) $x^{16}-y^{16}=(x-y)(x+y)(x^2+y^2)(x^4+y^4)(x^8+y^8)$;
   2) $x^{64}-y^{64}=(x-y)(x+y)(x^2+y^2)\cdot\ldots\cdot(x^{32}+y^{32})$.
2. $a=b+1$ bo'lsa, ifodani soddalashtiring:
   $(a+b)(a^2+b^2)(a^4+b^4)(a^8+b^8)(a^{16}+b^{16})(a^{32}+b^{32})$.
3. $a+b+c=12$ va $ab+bc+ac=-15$ bo'lsa, $a^2+b^2+c^2$ ni; $a-b+c=8$ va $a^2+b^2+c^2=110$ bo'lsa, $ac-ab-bc$ ni toping.

Hisoblang **(100–102)**:

100. 1) $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{2^2}\right)\left(1+\frac{1}{2^4}\right)\ldots\left(1+\frac{1}{2^{16}}\right)\left(1+\frac{1}{2^{32}}\right)$;

2) $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\ldots\left(1-\frac{1}{100^2}\right)$.

100. *1-misolning ishlanishi:* $M=(1+\frac{1}{2})(1+\frac{1}{2^2})(1+\frac{1}{2^4})\ldots(1+\frac{1}{2^{16}})(1+\frac{1}{2^{32}})$ sonli ifodani $1-\frac{1}{2}=\frac{1}{2}$ ga ko'paytiramiz va bo'lamiz. Unda $(1-\frac{1}{2})(1+\frac{1}{2})=(1-\frac{1}{2^2})$ ekanligini hisobga olsak,

$$M=2\left(1-\frac{1}{2^2}\right)\left(1+\frac{1}{2^2}\right)\left(1+\frac{1}{2^4}\right)\ldots\left(1+\frac{1}{2^{16}}\right)\left(1+\frac{1}{2^{32}}\right)=2\left(1-\frac{1}{2^4}\right)\left(1+\frac{1}{2^4}\right)\ldots\left(1+\frac{1}{2^{16}}\right)\left(1+\frac{1}{2^{32}}\right)=2\left(1-\frac{1}{2^{64}}\right).$$

Demak, $M=2\left(1-\frac{1}{2^{64}}\right)$.

100. *2-misolning ishlanishi:*

$$N_{100}=\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\ldots\left(1-\frac{1}{100^2}\right)=\frac{3}{4}\cdot\frac{8}{9}\cdot\frac{15}{16}\ldots\frac{100^2-1}{100^2}=\frac{1}{2}\cdot\frac{101}{100}.$$

$$N_{100}=\frac{1}{2}\cdot\frac{101}{100}.$$

Umumiy formula: $N_n=\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\left(1-\frac{1}{n^2}\right)=\frac{1}{2}\cdot\frac{n+1}{n}$.

101. 1) $\left(1-\frac{1}{5^2}\right)\left(1-\frac{1}{6^2}\right)\ldots\left(1-\frac{1}{103^2}\right)$; 2) $\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\left(1-\frac{1}{101^2}\right)$.

102.* 1) $\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{5^2}\right)\ldots\left(1-\frac{1}{102^2}\right)$; 2) $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\ldots\left(1-\frac{1}{2000^2}\right)$.

Hisoblang **(103–105)**:

103. 1) $\frac{(3,7^2-6,3^2)\cdot(13^2-12,6^2)}{(4,2^2-5,8^2)\cdot(2,3^2-0,3^2)}$; 2) $\frac{(7,2^2-6,8^2)(17,3^2-7,3^2)}{(14^2-13,4^2)(5,4^2-6,5^2)}$.

104. 1) $\frac{27,4^4-12,9^4}{(27,1+12,9)^2-27,1\cdot25,8}$; 2) $\frac{2,7\cdot(1,7^3-1,5^3)}{5,1^2+5,1\cdot4,5+4,5^2}$.

105. 1) $\frac{(25^2-21^2)\cdot(25^2+21\cdot25+21^2)}{25^3-21^3}$; 2) $\frac{(73^2-37^2)(73^2-73\cdot37+37^2)}{73^3+37^3}$.

106. 1) Agar $x=7,5$ va $y=6,5$ bo'lsa, $x^3-x^2y-xy^2+y^3$ ni;
2) Agar $x=59,3$ va $y=58,3$ bo'lsa, $x^3-y^3-2y^2-3y+x^2-2xy-1$ ni hisoblang.

107. Ko'phadning eng kichik qiymatini toping:
1) $x^2-16xy+64y^2-10$; 2) $(2x-1)(2x+1)+3y(3y-4x)$;
3) $9x^2-24xy+16y^2-25$; 4) $(2x-5y+3)\cdot(2x-5y-3)$.

108. a va b ning qanday qiymatlarida $2a^2-2ab+b^2-2a+2$ ko'phad eng kichik qiymatga erishadi?

□ Berilgan ko'phad ko'rinishini o'zgartiramiz: $2a^2-2ab+b^2-2a+2=$
$=a^2+(a^2-2ab+b^2)-2(a-1)=(a^2-2ab+b^2)+(a^2-2a+1)+1=(a-b)^2+$
$+(a-1)^2+1$. Ushbu ifodada $(a-b)^2\geq0$, $(a-1)^2\geq0$. Shuning uchun $(a-b)^2+$

$+(a-1)^2+1>0$ bo'ladi. Ko'phad eng kichik qiymatga erishishi uchun $(a-b)^2=0$, $(a-1)^2=0$, ya'ni $a=b$, $a=1$ bo'lishi kerak. Demak, berilgan ko'phad $a=b=1$ bo'lganda eng kichik qiymatga erishadi. Ko'phadning eng kichik qiymati 1 ga teng. ■

109. Ko'phadning eng katta qiymatini toping:

1) $4y(5x-y)-(5x-2)(5x+2)$;
2) $3y(4x-3y)-(2x-3)(2x+3)$;
3) $100-9x^2+30xy-25y^2$;
4) $25-4x^2-4xy-y^2$.

4-misolni yechib ko'rsatamiz:

□ Ko'phadning ko'rinishini o'zgartiramiz:

$25-4x^2-4xy-y^2=25-(4x^2+4xy+y^2)=25-(2x+y)^2$.

Hosil bo'lgan ifoda eng katta qiymatga erishishi uchun 25 dan ayiriluvchi son nolga teng bo'lishi kerak. Demak, berilgan ko'phadning eng katta qiymatiga $2x+y=0$ bo'lganda erishiladi.

Javob: 25. ■

110*. Ko'paytuvchilarga ajrating:

1) $x^3+y^3+z^3-(x+y+z)^3$;
2) $(x-2y)^3-(32-2y)^3-(x-32)^3$;
3) $(3x-2y)^3-(2y-2)^3-(3x-2)^3$;
4) $(3z-x)^3-(2y-x)^3-(3z-2y)^3$.

Oxirgi 4-misolni ishlab ko'rsatamiz:

□ $(3z-x)^3-(2y-x)^3-(3z-2y)^3=(3z-x-2y+x)[(3z-x)^2-(3z-x)(2y-x)+$
$+(2y-x)^3]-(3z-2y)^3=(3z-2y)[(3z-x)^2-(3z-x)(2y-x)+(2y-x)^2]-(3z-2y)^3=$
$=(3z-2y)[(3z-x)^2-(3z-x)(2y-x)+(2y-x)^2-(3z-2y)^2]=$
$=(3z-2y)(9z^2-6zx+x^2+6yz-3xz-2xy+x^2+4y^2-4xy+x^2-9z^2+12yz-4y^2)=$
$=(3z-2y)(3x^2-6xy+18yz-9xz=3(3z-2y)(x(x-2y)+3z(2y-x))=$
$=3(3z-2y)(2y-x)(-x+3z)=3(3z-x)(2y-x)(3z-2y)$. ■

4) ko'phadni ko'paytuvchilarga ajratishda ikki son kublarining formulasidan, ikki son ayirmasining formulasidan va guruhlash usulidan foydalanildi.

111. Tenglamani yeching:

1) $\left(\frac{1}{x}-\frac{3}{7}\right)\left(\frac{4}{11}-\frac{1}{x}\right)=0$;
2) $\left(\frac{3}{8}+\frac{2}{x}\right)\left(\frac{5}{3}-\frac{4}{x}\right)=0$;
3) $\left(\frac{7}{x}+8\right)\left(\frac{3}{x}-4\right)=0$;
4) $\left(\frac{5}{3x}-\frac{1}{9}\right)\left(\frac{2}{3x}-\frac{5}{6}\right)=0$.

Tenglamalarni ko'paytuvchilarga ajratib yeching **(112–115)**:

112. 1) $2x^3+24x^2+72x=0$;
2) $x^3+4x^2+4x=0$;
3) $x^3-4x^2-4x+16=0$;
4) $4x^3-4x^2-9x+9=0$.

113*. 1) $\left(\frac{x}{3}-\frac{x}{4}\right)^2-16=0$;
2) $25-\left(x-\frac{x}{9}\right)^2=0$;
3) $1-\left(\frac{x}{4}-\frac{x}{5}\right)^2=0$;
4) $\left(\frac{2x}{3}-\frac{1}{3}\right)^2-\frac{1}{9}=0$.

114. 1) $3(x-2)^2+(x^2-4)=0$; 2) $(x+3)^2-4(9-x^2)=0$;
3) $5(16-x^2)=x(x-4)$; 4) $x(2x+1)=(2x+1)2$.

115*. 1) $(x+4)(x^2-4x+16)-4(x+6)=x^3+14$;
2) $(x-3)(x^2+3x+9)-x(x-2)(x+2)+5x=0$;
3) $(1-3x)(1+3x+9x^2)+9x(3x^2-5)+46x=8$;
4) $(1-5x)(1+5x+25x^2)+25x(5x^2-1)+27x=11$.

116. Ko'paytuvchilarga ajrating:
1) $(x-y)^3-(2-y)^3+(2-x)^3$; 2) $(x+y)(x+y+2)-(x-y)(x-y-2)$;
3) x^2+xy-y^2-x+y; 4) $(x+y+2)(x+y)-(x-y)^2+1$.

Masalalar yechish

Umumiy ko'paytuvchini qavsdan tashqariga chiqarib, ko'paytuvchilarga ajrating **(117–119)**:

117. 1) $5(a+b)^2+(a+b)$; 2) $3(x-y)^2+7(x-y)$;
3) $2(a-b)-(b-a)^2$; 4) $3(a-b)^2+(b-a)^2$.

118. 1) $2(x+y)^2+(x+y)(x-y)$; 3) $3(x-y)^2-(x+y)(x-y)$;
2) $(x+y)^3-2x(x+y)$; 4) $(3x^2+2)x-(3x^2+2)y+(3x^2+2)y^2$.

119. 1) $(a+b)(7x^2+3x)+(a-b)(7x^2+3x)$; 2) $(a-b)(7x^2-3x)+(a-b)(7x^2+3x)$;
3) $(3x^2-2)+4x(3x^2-2)-2y(3x^2-2)$; 4) $(3x^2+2)-(3x^2+2)y+(3x^2+2)y^2$.

Guruhlash usulidan foydalanib, ko'paytuvchilarga ajrating **(120–121)**:

120. 1) $18a^2-24ab+6ac-8bc$; 3) $-36x^2+42xy+3ax-42ay$;
2) $12x^2+12xy+6x+6y$; 4) $48xz^2+32xy^2-15yz^2-10y^3$.

121. 1) $8ab^2-3b^2c-6c^3+16ac^2$; 2) $6mnk^2+15m^2k-14n^3k-35mn^2$;
3) $-4ac+5c^2-10cx+8ax$; 4) $-24bx-15c^2+4bc+9cx$.

122. Ifodani soddalashtiring va uning son qiymatini toping:

1) $\left(4c+\frac{1}{4}x\right)\left(4c-\frac{1}{4}x\right)+\left(4c-\frac{1}{4}x\right)^2$, bunda $c=\frac{1}{2}$; $x=2$.

2) $(0{,}1a-0{,}2b)^2+(0{,}1a-0{,}2b)(0{,}1a+0{,}2b)$, bunda $a=-50$; $b=-1\frac{2}{3}$.

123. Umumiy ko'paytuvchini qavsdan tashqariga chiqarish usuli bilan tenglamani yeching:
1) $3(x-2)^2+(x^2-4)=0$; 3) $5(16-x^2)=x(x-4)$;
2) $(x+3)^2-4(9-x^2)=0$; 4) $x(2x+1)=(2x+1)^2$.

124. Ko'paytuvchilarga ajrating:
1) $(a+b+c)^3-a^3-b^3-c^3$;
2) $(x+y+z)^3-(x+y-z)^3-(x-y+z)^3-(y+z+x)^3$;
3) $ab(a-b)+bc(b-c)+ca(c-a)$;
4) $4x^2y^2(2x+y)+4y^2z^2(z-y)-4z^2x^2(2x+z)$.

125. Ko'paytuvchilarga ajrating:

1) $(2a+3b+5c)(6ab+15bc+10ac)-30abc$;
2) $(3a+2b)(3a+2b-2)-(3a-2b)(3a-2b-2)$;
3) $(a+b)(a+b+2)-(a-b)(a-b-2)$;
4) $a(a+2)+b(b+2)-2(a+1)(b+1)+1$.

126. Agar:

1) $(7x-8)(x-2)=0$, bo'lsa, $7x-8$; 2) $(3x-1)(x-35)=0$ bo'lsa, $3x-1$;

3) $(x-9)\left(\frac{1}{9}x+5\right)=0$ bo'lsa, $\frac{1}{9}x+5$; 4) $(6x-5)\left(x-\frac{1}{6}\right)=0$ bo'lsa, $6x-5$

qanday qiymatlar qabul qilishi mumkin?

Namuna sifatida 4-masalani yechib ko'rsatamiz.

□ Tenglamani yechish uchun ko'paytmaning har bir qavsini nolga tenglashtiramiz: $6x-5=0$, $x-\frac{1}{6}=0$, bundan $x_1=\frac{5}{6}$, $x_2=\frac{1}{6}$.

Endi $6x-5$ ifodaning qiymatini $x_1=\frac{5}{6}$, $x_2=\frac{1}{6}$ da hisoblaymiz:

$$6x_1-5=6\cdot\frac{5}{6}-5=0;\quad 6x_2-5=6\cdot\frac{1}{6}-5=1-5=-4.$$

Javob: 0 yoki -4. ■

127. Motorli qayiqning oqim bo'yicha tezligi 20 km/soat, oqimga qarshi tezligi 16 km/soat. Daryo oqimining tezligini va qayiqning turg'un suvdagi tezligini toping.

128. Hisoblang:

1) $\frac{(2,5+1,5)\cdot(0,8^2-0,8\cdot1,7+1,7^2)}{1,6^3+3,4^3}$;

2) $\frac{0,6^2-0,6\cdot0,2+0,1^2}{2,5-2,5^2}$;

3) $\frac{1,6^2-1,6\cdot0,8+0,4^2}{1,4^2-0,2^2}$;

4) $\frac{0,2^2+0,3\cdot0,4+0,3^2}{0,5\cdot0,6-0,5\cdot0,8}$.

129. Ko'paytuvchilarga ajrating:

1) a^4+a^2+1; 3) a^8+a^4+1; 5) $x^{12}-a^{12}$;
2) a^4-a^2+1; 4) a^8-a^4+1; 6) a^8-b^8.

130. 1) $a^5x^2-a^2x^8$; 2) $x^3-6x^2+11x-6$; 3) a^3-5a^2+9a-5;
4) $x^4-4x^2y^2-5y^4$; 5) x^4+x^2-2; 6) $y^8-y^6-4y^2-16$;

131*. 1) a va b ning qanday qiymatlarida $2a^2-2ab+b^2-2a+2$ ko'phad eng kichik qiymatga erishadi?

2) 1) a va b ning qanday qiymatlarida $1-2a^2+2ab-b^2+2a$ ko'phad eng katta qiymatga erishadi?

132. Agar:

1) $(3x+4)(x-3)=0$ bo'lsa, $3x+4$;

2) $(4x-3)(x+4)=0$ bo'lsa, $4x-3$;

3) $(\frac{1}{5}-x:3)(x+3)=0$ bo'lsa, $\frac{1}{5}-x:3$;

4) $(7x+4)(x-\frac{1}{7})$ bo'lsa, $7x+4$ qanday qiymatlar qabul qilishi mumkin?

Rivojlantiruvchi mashqlar*

1. Ixtiyoriy natural son n uchun n^3-1 son 2 ning darajasi bo'la olmasligini isbotlang.

○ $n^3-1=2^k$ deylik, ammo $n^3-1=(n-1)(n^2+n+1)$. U holda n^2+n+1 ko'paytuvchi 2 ning darajasi bo'lishi kerak. Lekin n^2+n+1 son – toq son va ixtiyoriy natural son n da 1 ga teng emas. Zidlikka keldik. Demak, n^3-1 son 2 ning darajasi bo'la olmaydi. ●

2. n^3+1 son 3 ning darajasi bo'ladigan barcha natural n larni toping.

□ ***1-usul***: $n^3+1=3^k$ deylik. U holda $n^3+1=(n+1)(n^2-n+1)$ ekanidan $n+1=3^m$, $n^2-n+1=3^d$, bunda m va d – manfiy bo'lmagan butun sonlar. n son 3 ga bo'linmaydi (agar $m=0$ bo'lsa, $n=0$), shu bilan birga $3n=(n+1)^2-(n^2-n+1)=3^{2m}-3^d$, $n=3^{2m-1}-3^{d-1}$.

Bu tenglik esa $d-1=0$ bo'lganda o'rinlidir, aks holda n son 3 ga bo'linar edi. Bundan $n^2-n+1=3$, ya'ni $n=2$.

2-usul: n^3+1 sonning 3 ga bo'linishidan, $n=3k-1$ sonning natural son ekani kelib chiqadi. U holda $n^3+1=(3k-1)^3+1=27k^3-27k^2+9k=9k(3k^3-3k+1)$. Qavs ichidagi ifoda 3 ga bo'linmaydi, u 3 ning nolinchi darajasi bo'lishi mumkin. Demak, $k=1$, bundan esa $n=2$ ekani ma'lum bo'ladi.

Javob: $n=2$. ■

3. a,b,c – nolga teng bo'lmagan sonlar. Agar $a^2-b^2=bc$, $b^2-c^2=ac$ bo'lsa, $a^2-c^2=ab$ ekanini isbotlang.

○ Har bir tenglikni c^2 ga bo'lib va $\frac{a}{c}=x$, $\frac{b}{c}=y$ deb, ularni shunday yozib olamiz:

$$x^2-y^2=y, \quad (1) \qquad y^2-1=x, \quad (2) \qquad x^2-1=xy. \quad (3)$$

(1) va (2) tengliklardan (3) tenglikni keltirib chiqarishimiz kerak. (1) va (2) tengliklarni hadma-had qo'shib, $x^2-1=x+y$ tenglikni hosil qilamiz, ya'ni $xy=x+y$, ekanligini ko'rsatish kifoya. (1) va (2) tengliklarni quyidagicha yozib olamiz:

$$x\cdot\frac{x}{y+1}=y\ \ (4),\qquad \frac{x}{y+1}=y-1\ \ (5).$$

(4) va (5) dan: $x(y-1)=y$, bundan esa $xy=x+y$ tenglikni hosil qilamiz. Demak, $x^2-1=xy$, ya'ni $a^2-c^2=ab$. Da'vo isbotlandi. ●

4. Hisoblang:

1) $2015\frac{1999}{9991}\cdot 2016\frac{1999}{9991}-2014\frac{1999}{9991}\cdot 2017\frac{1999}{9991}$;

2) $2015\cdot 2016-2014\cdot 2017$.

Ko'rsatma: 1) $(a+1)(a+2)-a(a+3)=a^2+3a+2-a^2-3a=2$ ayniyatdan foydalaning.

5. Birinchi raqami o'chirilganda 57 marta kamayadigan natural son mavjudmi?

□ Berilgan son $(k+1)$ xonali bo'lsin, deylik. Uning 1-raqamini x orqali, qolgan qismini y orqali belgilaymiz. U holda masala shartiga ko'ra $x\cdot 10^k+y=57y$ bo'ladi. Bundan $x\cdot 10^k=56\cdot y$ yoki $x\cdot 5^k\cdot 2^k=56y$ kelib chiqadi. Ammo $56=7\cdot 8$ bo'lgani uchun $x=7$, shuningdek, $8=2^3$ bo'lgani uchun $k=3$ va $y=125$ bo'lishini topamiz.

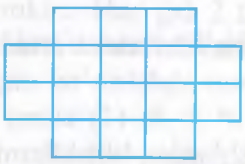
Javob: Mavjud. Bu son 7 125 ga teng. ■

6. Ixtiyoriy n natural son uchun n^5-5n^3+n ifodaning 120 ga qoldiqsiz bo'linishini isbotlang.

○ $n^5-5n^3+4n=n(n^4-5n^2+4)=n(n^2-1)(n^2-4)=(n-2)(n-1)n(n+1)(n+2)$ – bu ifoda ketma-ket kelgan 5 ta natural sonlar ko'paytmasidan iborat ekan. Bu sonlardan biri 5 ga, albatta, bo'linadi; 3 ta ketma-ket kelgan sonlardan biri 3 ga bo'linadi; 4 ta ketma-ket kelgan sonlardan biri 4 ga, yana biri esa 2 ga bo'linadi. Demak, ko'paytma $5\cdot 4\cdot 3\cdot 2=120$ ga bo'linadi. Da'vo isbotlandi. ●

7. $\overline{ABBA}+A+B=CDDA$ rebusni yeching. Bir xil harflar bilan bir xil raqamlar, turli harflar bilan turli raqamlar belgilangan.

8. O'lchamlari 4 x 5 bo'lgan to'g'ri to'rtburchakning uchlaridagi 4 ta 1 x 1 o'lchamli kvadratchalar kesib tashlangach, rasmdagi shakl hosil bo'ldi. Bu shaklni kvadrat bo'lmagan 3 ta qismga shunday ajratingki, ulardan kvadrat yasash mumkin bo'lsin.



9. O'lchamlari 10x10 bo'lgan kvadrat jadval kataklariga ixtiyoriy ravishda 1, 2, ..., 99, 100 sonlari yozilgan. Jadvalning ustunlaridagi sonlar yig'indisi S_1, S_2, ..., S_{10} bo'lsin. S_1, S_2, ..., S_{10} sonlar orasida ixtiyoriy ikkita qo'shni son 1 ga farq qilishi mumkinmi?

□ Sonlar kataklarga shunday joylashganki, S_1, S_2, ..., S_{10} sonlar orasida ixtiyoriy ikkita qo'shni son 1 ga farq qiladi, deb faraz qilaylik. U holda qo'shni ustunlardagi sonlar yig'indisi 1 ga farq qilgani uchun 5 ta ustundagi sonlar yig'indisi toq, qolgan 5 ta ustundagi sonlar yig'indisi esa juft bo'ladi. Demak, jadvaldagi barcha sonlar yig'indisi toq bo'lishi kerak, ammo $1+2+...+100=$ $=5\,050$ – juft son. Ziddiyatga keldik.

Javob: mumkin emas. ■

10. 1) Matematikadan shahar olimpiadasida Hasan va Husan qatnashishdi. "Sizlarning akangiz ham bormi va u necha yoshda" degan savolga ular: "Ha, bitta akamiz bor, uning yoshi ikkita bir xil raqam bilan yoziladigan ikki xonali son, uchalamizning yoshlarimiz yig'indisi esa 2-raqami 1-raqamidan 2 marta katta bo'lgan ikki xonali son", – deb javob berishdi. Hasan, Husan va akasining yoshi nechada ekan?

2) Maktabimizning 7-sinflarini 102 nafar o'quvchi bitirdi. Ularning 25 nafari matematikadan, 28 nafari tarixdan, 30 nafari geografiyadan 5 baho olgan. Tarixdan 5 olganlarning 8 nafari geografiyadan, 7 nafari matematikadan ham 5 olgan. Geografiyadan 5 olganlarning 6 nafari matematikadan ham 5 olgan. 3 nafar o'quvchi matematika, tarix, geografiya fanlaridan 5 olgan. Bu o'quv fanlarining birortasidan ham 5 olmagan o'quvchilar necha nafarni tashkil etadi?

Ko'rsatma: Masalada aytilgan 3 ta o'quv fanidan hech bo'lmaganda 1 ta 5 olgan o'quvchilar soni $16+19+15+4+5+3+3=65$ nafar, demak, izlanayotgan son $102-65=37$.

Javob: 37 ta o'quvchi. ■

11. Ixtiyoriy toq natural son n uchun $N=n^3+3n^2-n-3$ sonning 48 ga bo'linishini isbotlang.

○ $N=n^3+3n^2-n-3=n^2.(n+3)-(n+3)=(n^2-1)(n+3)=(n-1)(n+1)(n+3)$, n – toq bo'lgani uchun $n=2k+1$ deb yozish mumkin, $k\geq 0$. U holda $N=2k.(2k+2)(2k+4)=8k(k+1)(k+2)$. 8 soni 3 ta ketma-ket kelgan natural sonlar ko'paytmasiga ko'paytirilyapti. 3 ta ketma-ket natural sonning kamida bittasi juft son; bittasi esa 3 ga karrali son bo'ladi. U holda $k(k+1)(k+2)$ ko'paytma 6 ga, albatta, bo'linadi. Shunday qilib, N son $8\cdot 6=48$ ga bo'linadi. Da'vo isbotlandi. ●

12. Agar natural son a 5 ga bo'linmasa, u holda $A=a^8+3a^4-4$ sonning 100 ga bo'linishini isbotlang.

○ $A=(a^2+1)(a^2-1)(a^4+4)$ (1) ekani ravshan. a son 5 ga bo'linmagani uchun uni ushbu ko'rinishlardan biri bilan ifodalash mumkin: $a=5n\pm 1$ yoki $a=5m\pm 2$, bunda n va m – butun sonlar.

1) $a=5m\pm 1$ bo'lsa, a^2-1 va a^4+4 sonlarning har biri 5 ga bo'linadi; demak, (1) ifoda bu holda 25 ga bo'linadi. Ixtiyoriy butun a sonda (1) ifoda 4 ga bo'linadi (isbotlang!). U holda (1) ifoda $25\cdot 4=100$ ga ham bo'linadi.

2) $a=5m\pm 2$ bo'lsa, a^2+1 va a^4+4 sonlarning har biri 5 ga bo'linadi, ularning ko'paytmasi esa 25 ga bo'linadi. Demak, bu holda ham (1) son $25\cdot 4=100$ ga bo'linadi. Da'vo isbotlandi. ●

13. a va b ning qanday qiymatlarida $M=2a^2-8ab+17b^2-16a-4b+2\,082$ ifoda eng kichik qiymatga erishadi? Bu eng kichik qiymat nechaga teng?

□ Berilgan ifodani quyidagicha yozib olish mumkin: $M=2a^2+8b^2+32-8ab-16a+32b+9b^2-36b+36+2\,014=2(a^2+4b^2+16-4ab-8a+16b)+9(b^2-4b+4)+2\,014=2(a-2b-4)^2+9(b-2)^2+2\,014$, demak, $M\geq 2\,014$. Agar $b=2$, $a-2b-4=0$, ya'ni $a=8$ bo'lsa, $M=2\,014$ bo'ladi.

Javob: $a=8$, $b=2$ bo'lsa, berilgan ifoda eng kichik qiymatga erishadi va bu eng kichik qiymat 2 014 ga tengdir. ■

14. $n_1, n_2, n_3, \ldots, n_{2014}$ turli natural sonlar bo'lsin. 2 sonini $2=\frac{1}{n_1}+\frac{1}{n_2}+\ldots+\frac{1}{n_{2014}}$ ko'rinishda tasvirlash mumkinmi?

□ $2=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$ (1) ekanligi ravshan. $\frac{1}{6k}=\frac{1}{12k}+\frac{1}{20k}+\frac{1}{30k}$ (2)

tenglikning to'g'riligiga osongina ishonch hosil qilamiz. (1) da oxirgi qo'shiluvchi $\frac{1}{6}$ ning o'rniga (2) da $k=1$ bo'lganda hosil qilinadigan $\frac{1}{6}=\frac{1}{12}+\frac{1}{20}+\frac{1}{30}$ yig'indini qo'yamiz.

$2=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}$ (3); (3) tenglikdagi $\frac{1}{30}$ o'rniga (2) da $k=5$ da hosil bo'ladigan $\frac{1}{30}=\frac{1}{60}+\frac{1}{100}+\frac{1}{150}$ yig'indini qo'yamiz:

$$2=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{12}+\frac{1}{20}+\frac{1}{60}+\frac{1}{100}+\frac{1}{150} \quad (4)$$

Biz (2) dagi k ning o'rniga $k=1=5^0$; $k=5=5^1$ qiymatlarni berdik.

Endi $k=5^2$, $k=5^3$,... qiymatlarni berib, 2 sonini tasvirlashning bu jarayonini davom ettiraverishimiz mumkin. Har gal 2 ta yangi (turli) qo'shiluvchi qo'shilaveradi. Bu ishni 1005 marta takrorlab talab qilingan natijaga kelamiz.

Javob: 2 sonini masalada aytilgan ko'rinishda tasvirlash mumkin. ■

15. a, b, c – biror uchburchak tomonlarining uzunliklari bo'lsin.

$a^2\cdot(b+c-a)+b^2(c+a-b)+c^2\cdot(a+b-c)\ \square\ 3abc$ ekanini isbotlang.

○ Ixtiyoriy a, b, c lar uchun $(a-b)^2\geq 0$, $(b-c)^2\geq 0$, $(c-a)^2\geq 0$ ekani ravshan. a, b, c uchburchak tomonlarining uzunliklari bo'lgani uchun: $b+c-a>0$, $c+a-b>0$, $a+b-c>0$. U holda $(b-c)^2(b+c-a)\geq 0$, $(c-a)^2(c+a-b)\geq 0$,

$(a-b)^2(a+b-c) \geq 0$. Bu tengsizliklarni hadma-had qo'shib, $6abc - 2a^2(b+c-a) - 2b^2(a+c-b) - 2c^2(a+b-c) \geq 0$ tengsizlikka kelamiz. Bundan esa isbotlash talab etilgan tengsizlikni hosil qilamiz. ●

16. 400 sonining o'ng tomoniga yozilganda natural sonning kvadratini beruvchi barcha to'rt xonali sonlarni toping.

17. Raqamlari yig'indisining 4-darajasiga teng bo'lgan 4 xonali sonni toping.

18. Dalahovli to'g'ri to'rtburchak shaklida bo'lib, hovlining etagidan daryo o'tadi. Hovlini 3 tomondan umumiy uzunligi 100 metr bo'lgan devor bilan o'rashmoqchi. Dalahovlining o'lchamlari qanday bo'lganda uning yuzi eng katta bo'ladi?

19. Agar k juft natural son bo'lsa, 13^k+6 sonning 7 ga bo'linishini isbotlang.
Ko'rsatma: $k=2n$, $13^{2n}=(14-1)^{2n}=14A+1$ ko'rinishda yozilishi mumkin.

20. 2013 sonini ikkita natural son kublarining yig'indisi ko'rinishida tasvirlash mumkinmi?

□ $2013=a^3+b^3$ bo'lsin, deb faraz qilaylik, bunda a va b – natural sonlar. U holda $(a+b)^3=a^3+b^3+3a^2b+3ab^2=2013+3ab(a+b)$ son 3 ga bo'linadi, chunki 2013 soni ham uchga bo'linadi. 3 – tub son, demak, $a+b$ yig'indi ham 3 ga bo'linishi kerak. Shu bilan birga, $2013=(a+b)^3-3ab(a+b)=(a+b)((a+b)^2-3ab)$. Bu tenglikning o'ng qismidagi ko'paytma 9 ga bo'linadi, chunki har bir ko'paytuvchi 3 ga bo'linadi. Ammo $2013=9\cdot 223+6$ son 9 ga bo'linmaydi.
Javob: 2013 sonni ikkita natural son kublarining yig'indisi ko'rinishida tasvirlash mumkin emas. ■

21. Ko'paytuvchilarga ajrating: $x^4+2014x^2+2013x+2014$.

22. Ko'paytuvchilarga ajrating: x^8+x^7+1.

□ $x^8+x^7+1=x^6(x^2+x+1)-x^4(x^2+x+1)+x^3(x^2+x+1)-x(x^2+x+1)+(x^2+x+1)=(x^2+x+1)(x^6-x^4+x^3-x+1)$.

23. 10 ta tanga bor: 2 ta 2 tiyinlik, 2 ta 3 tiyinlik, 2 ta 5 tiyinlik, 2 ta 10 tiyinlik, 1 ta 15 tiyinlik, 1 ta 20 tiyinlik. Ularni yulduzdagi doirachalarga shunday joylashtiringki, uning har bir tomonidagi tanga qiymatlari yig'indisi o'zaro teng bo'lsin.

24. Rebusni yeching:
KVANT + KVANT + KVANT = JURNAL,
bir xil harflarga bir xil raqamlar, turli harflarga turli raqamlar mos keladi.

25. Karim bobo chevarasi Feruzadan 100 yosh katta. Feruza hisoblab ko'rsa, o'zining yoshi bilan katta bobosining yoshining ko'paytmasi 1989 ga teng ekan. Feruza necha yoshda?

26. Do'konga 1 tonna meva keltirildi: olma, nok, olxo'ri, olcha. Har bir qutiga: 48 kg olma, 20 kg nok, 14 kg olxo'ri, 10 kg olcha joylandi. Olma solingan qutilar soni nokli qutilardan 2 baravar ko'p, olxo'ri solingan qutilar soni olchali qutilar soniga teng. Do'konga har bir xil mevadan qanchadan keltirilgan?

27. Ifodani soddalashtirgandan keyin hosil bo‘lgan ko‘phadning hadlari sonini toping:

1) $(a^4-a^2b^2+b^4)(a^2+b^2)-(a^3+b^2)(a^3-b^2)$; 2) $(a^3-b^3)^2+(a^2+b^2)(a^4-a^2b^2+b^4)$.

28. $a+\frac{1}{a}=d$ bo‘lsa, 1) $a^3+\frac{1}{a^3}$; 2) $a^2+\frac{1}{a^2}$ ifodani hisoblang.

29. $a-\frac{1}{a}=c$ bo‘lsa, 1) $a^3-\frac{1}{a^3}$; 2) $a^2-\frac{1}{a^2}$ ifodani hisoblang.

30. Ko‘paytuvchilarga ajrating:

1) $a^4-4a^3+8a^2-16a+16$;
2) $(a+b+c)(ab+bc+ac)-abc$;
3) $(a+b)(a+b+2)-(a-b)(a-b-2)$;
4) $(a+b+2)(a+b)-(a-b)^2+1$;
5) $b^2+ab-2a^2+a-b$;
6) $a^4-20a^3+110a^2+20a-111$;
7) $a^5+a^4-2a^3-2a^2+a+1$.

31. $a=2015,9$, $b=2014,9$ bo‘lsa, $a^3-b^3-2b^2+a^2-2ab-3b-1$ ifodaning son qiymatini toping.

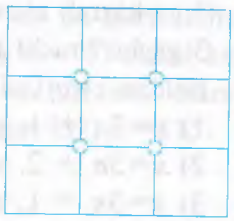
32. Bir bolada 15 va 20 tiyinlik tangalardan bir nechtasi bor. Uning 20 tiyinlik tangalari soni 15 tiyinlik tangalari sonidan ko‘p. Bola tangalardan ikkitasini sarfladi. Bu jami pulning beshdan bir qismini tashkil etadi. Qolgan pulning yarmiga daftar sotib oldi. Bunda u 3 ta tangani ishlatdi. Dastlab bolada har bir tangadan nechtadan bo‘lgan?

33. 2, 3, 4, 5, 6, 7, 8 raqamlari yordamida to‘la kvadrat bo‘lgan to‘rt xonali son tuzing.

34. Kvadratni 3 ta qismga shunday ajratingki, ulardan o‘tmas burchakli uchburchak tuzish mumkin bo‘lsin.

35. XX asrning turli yillarida tug‘ilgan 4 nafar kishi yoshlarining raqamlari yig‘indisi o‘zaro teng. Shu to‘rt kishi yoshlarining yig‘indisi esa aniq kubni beradi. Bu kishilarning kattasi necha yoshda?

36. *(Professor Abdullajon A’zamov masalasi).* 3×3 kvadrat kataklariga 1, 2, 3, ..., 9 raqamlari yozildi. So‘ng har bir doirachaga uni o‘rab turgan 4 ta sonning o‘rta arifmetigi yozildi. Shundan so‘ng hosil bo‘lgan 4 ta sonning (doirachalardagi sonlarning) o‘rta arifmetigi hisoblandi. Buning natijasida qanday eng katta son hosil bo‘lishi mumkin?



37. “Samarqand Darvoza” gipermarketida olma va noklar uchburchak shaklda terilgan: 1-qatorda 1 ta olma, 2-qatorda 2 ta nok, 3-qatorda 3 ta olma, 4-qatorda 4 ta nok va h.k. Jami qatorlar soni n ta bo‘lsa, nechta olma va nechta nokdan bunday “mazali uchburchak” yasalgan? ($n=18$; 21).

38.* 100 000 dan kichik barcha natural sonlar ichida 11 ga bo‘linadiganlarining nechtasining raqamlari yig‘indisi 11 ga teng bo‘ladi?

39. Buyuk Britaniyada har yili o'tkaziladigan "Qovoqlar tanlovi"da navbatdagi "g'olib"ning massasi 109 kg va "ekvatori" 2,5 metr bo'ldi. Bu qovoq suvda suza oladimi yoki cho'kib ketadimi? Agar shu qovoqqa massasi 20 kg bolani o'tqizib qo'yilsa, bola solda suzgandek suza oladimi?

Savolga javob topishda hisob-kitob ishlarini bajarishga to'g'ri keladi. Qovoqni shar deb hisoblang. R radiusli shar hajmi esa $V=\frac{4\pi}{3}R^3$ formulaga ko'ra aniqlanadi.

40. 1) 1, 2, 3, 4, 7, 9; 2) 1, 2, 4, 6, 7, 8, 9 sonlarni bo'sh doirachalarga shunday joylashtiringki, har bir qator va ustundagi sonlar yig'indisi to'rtburchaklardagi songa teng bo'lsin.

1)

○	+ ○	+ 6	=	14
+	+	+		
○	+ 5	+ ○	=	12
+	+	+		
8	+ ○	+ ○	=	19
=	=	=		
19	8	18		

2)

○	+ 3	+ ○	=	10
+	+	+		
○	+ ○	+ ○	=	19
+	+	+		
5	+ ○	+ ○	=	16
=	=	=		
15	12	18		

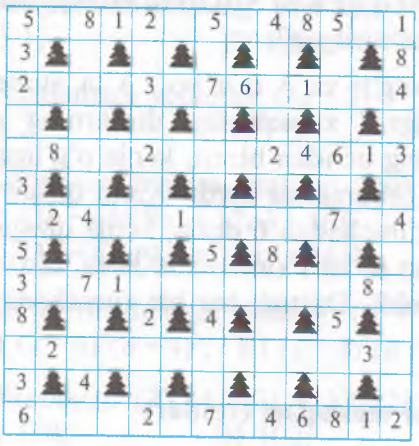
41. Ikkita natural sonning ko'paytmasini ularning kvadratlari ayirmasiga ko'paytirsak, 3 ga karrali son hosil bo'lishini isbotlang.

○ x va y – natural sonlar, deylik. $K=xy\cdot(x^2-y^2)=xy(x+y)(x-y)$ sonning 3 ga karrali ekanini ko'rsatamiz. Agar x yoki y 3 ga bo'linsa, u holda K ning 3 ga bo'linishi ravshan. Agar x ham, y ham 3 ga bo'linmasa, quyidagi 4 ta holdan birortasi bajariladi:

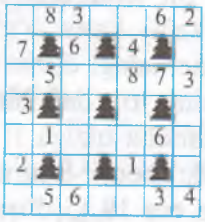
1) $x=3n+1$, $y=3m+1$, $n, m\in N$, bu holda $-y$ son 3 ga bo'linadi;
2) $x=3n+2$, $y=3m+1$, $n, m\in N$, bu holda $x+y$ son 3 ga bo'linadi;
3) $x=3n+1$, $y=3m+2$, $n, m\in N$, bu holda $x+y$ 3 ga bo'linadi;
4) $x=3n+2$, $y=3m+2$, $n, m\in N$, bu holda $x-y$ son 3 ga bo'linadi.

Har bir holda K sonning biror ko'paytuvchisi 3 ga bo'linyapti, demak, K soni ham 3 ga bo'linadi. Isbotlandi. ●

42. Har bitta archa atrofiga 1 dan 8 gacha raqamlarni shunday joylashtiringki, natijada har bir raqam archa atrofida bir marta qatnashsin.



Namuna



43. Agar EKUK $(m, n)=975$ va $\frac{m}{EKUB(m,n)}+\frac{n}{EKUB(m,n)}=18$ bo'lsa, m va n sonlarni toping.

□ EKUB$(m, n)=k$ deylik. U holda $m=k\cdot m_1$, $n=k\cdot n_1$, bunda m_1 va n_1 – o'zaro tub natural sonlar, EKUK $(m, n)=km_1n_1$. Masala shartiga ko'ra, $m_1+n_1=18$, $km_1\ n_1=\ 975$. Bu tengliklardan m_1 va n_1 sonlarining har biri 17 dan katta emasligi kelib chiqadi. Shu bilan birga, m_1 va n_1 sonlar 975 ning bo'luvchilaridir, $975=35513$. Demak, $m_1=5$, $n_1=13$, $k=15$. U holda $m=k\cdot m_1=15\cdot 5=75$, $n=k\cdot n_1=15\cdot 13=195$.
Javob: $m=75$, $n=195$. ■

44. Agar $m+n=667$ va EKUK (m, n):EKUB $(m, n)=120$ bo'lsa, m va n sonlarni toping.

□ EKUB $(m, n)=k$ deylik. U holda: $m=km_1$ va $n=kn_1$ hamda m_1 va n_1 o'zaro tub sonlar, EKUK $(m, n)=km_1n_1$.
Masala shartiga ko'ra, $k(m_1+n_1)=667$ va $m_1n_1=120$.
$667=12329$ va $120=358$ hamda m_1 va n_1 sonlar o'zaro tub bo'lgani uchun, bulardan $m_1=\ 8$ (u holda $n_1=15$) yoki $m_2=\ 24$ (u holda $n_2=\ 5$) ekani kelib chiqadi. $k\ (m_1+n_1)=\ 667$ tenglikdan $k_1=\ 29$, $k_2=\ 23$ yechimlarni olamiz. Shunday qilib, izlanayotgan sonlar: $m=232$, $n=435$ va $m=552$, $n=115$.
Javob: $m=232$, $n=435$; $m=552$, $n=115$. ■

45. (*Qadimiy rebus*). Turli harflarga turli raqamlar, bir xil harflarga bir xil raqamlar mos kelsa, ushbu rebusni yeching: *send*+*more*=*money*.

46. Har birida 100 grammdan choy bo'lgan 3 ta quticha bor. Qutichalardagi choylar I, II navli va bu ikkala navning aralashmasidan iborat. I navli choyning bir qutichasi (100 grammi) 600 so'm, II navli choyning bir qutichasi esa

450 so'm. Agar aralashma choyning bir qutichasi 510 so'm bo'lsa, bunda I va II navli choylar qanday nisbatda aralashtirilgan?

47. (*Qadimgi masala*). Bir kishining 3 o'g'li va 3 qizi bor. 3 ta xumchada u dur saqlar edi. U kishi katta o'g'liga I xumchadagi durlarning yarmini, o'rtanchasiga II xumchadagi durlarning uchdan birini, kenja o'g'liga esa III xumchadagi durlarning to'rtdan biri (choragi)ni berdi. Katta qiziga I xumchadan 4 dona, o'rtancha qiziga II xumchadan 6 dona, kenja qiziga esa III xumchadan 2 dona dur berdi. Shundan so'ng I xumchada 38 ta, II xumchada 12 ta, III xumchada esa 19 ta dur qoldi. Dastlab har bir xumchada qancha dur bo'lgan?

IV bobga doir sinov mashqlari (testlar)

1. Umumiy ko'paytuvchini qavsdan tashqariga chiqaring: $20a^2b^3-15a^3b^2$.
A) $5a^2b^2(4b-3a)$; B) $5ab(4ab^2-3a^2b)$; C) $5a^2(4ab^3-3ab^2)$; D) $5b^2(4a^2-3a^3)$.

2. Umumiy ko'paytuvchini qavsdan tashqariga chiqaring: $36x^3y^4+12x^4y^3$.
A) $12x^3y^3(3y+x)$; B) $6x^2y^2(6y^2+2x^2y)$;
C) $6xy(6xy^2+2x^2y)$; D) $12x^3y(3y^3+xy^2)$.

3. Ko'paytuvchilarga ajrating: $2(a+b)+a^2(a+b)-3(a+b)$.
A) $(a+b)(a^2-1)$; B) $(a+b)(a-1)$; C) $(a+b)(a^2+1)$; D) $(a+b)(a^2-2)$.

4. Ko'paytuvchilarga ajrating: $2(a-b)-a^2(a-b)+3(b-a)$.
A) $(a-b)(-1-a^2)$; B) $(a-b)(1-a^2)$; C) $(a-b)(-2-a^2)$; D) $(a-b)(-1+a^2)$.

5. Ko'paytuvchilarga ajrating: $2a(x-y)+2az+3b(y-x+z)$.
A) $(x-y+z)(2a-3b)$; B) $(x-y+z)(2a+3b)$;
C) $(x-y+z)(3a-2b)$; D) $(x-y+z)(3a+2b)$.

6. Hisoblang: $23{,}7\cdot 31{,}8-23{,}7\cdot 21{,}8+63$.
A) 200; B) 300; C) 2 370; D) 218.

7. Hisoblang: $46{,}7\cdot 1{,}54+46{,}7\cdot 8{,}46-167$.
A) 300; B) 467; C) 84; D) 154.

8. Ko'paytuvchilarga ajrating: $2ax+2bx-3ay-3by$.
A) $(a+b)(2x-3y)$; B) $(a+b)(2x-y)$; C) $(a+b)(x-3y)$; D) $(a+b)(2x+y)$.

9. Ko'paytuvchilarga ajrating: $5a(3a-2b)-9a+6b$.
A) $(3a-2b)(5a-3)$; B) $(3a-2b)(5a-2)$;
C) $(3a-2b)(5a-1)$; D) $(3a-2b)(5a-4)$.

10. Ifodaning son qiymatini toping:
$2a^3-2a^2b-3a+3b$, bunda $a=1{,}5$; $b=-1{,}5$.
A) 4,5; B) 4,6; C) 4,4; D) 4,7.

11. Ifodaning son qiymatini toping:
$2(a^2+b^2)(a^2-2ab+b^2)$, bunda $a=1{,}2$; $b=0{,}2$.
A) 2,96; B) 2,95; C) 2,97; D) 2,98.

12. Tenglamani yeching: $(4+3x)^2-(4x-3)^2=48$.
A) 1; B) 1,5; C) 0,5; D) 1,2.

13. Tenglamani yeching: $(4x-3)^2-(3x-5)=3$.
A) 2; B) 1,5; C) 2,5; D) 2,3.

14. Ko'paytuvchilarga ajrating: $27a^3-64b^3$.
A) $(3a-4b)(9a^2+12ab+16b^2)$; B) $(3a-4b)(9a^2-12ab+16b^2)$;
C) $(3a-4b)(9a^2+24ab+16b^2)$; D) $(3a-4b)(9a^2-24ab+16b^2)$.

15. Ko'paytuvchilarga ajrating: $(a^2+16)^2-64a^2$.
A) $(a-4)^2(a+4)^2$; B) $(a-4)^2(a+4)$; C) $(a-4)(a+4)^2$; D) $(a-4)(a+4)$.

16. Hisoblang: $(233+773):(232-2377+772)$.
A) 100; B) 102; C) 50; D) 200.

TARIXIY MASALALAR

1. *(Abu Ali ibn Sino masalalaridan)*:
1) Agar sonni 9 ga bo'lganda 2 yoki 7 qoldiq qolsa, bunday sonning kvadratini 9 bo'lganda 4 qoldiq chiqadi;
2) Agar sonni 9 ga bo'lganda 4 yoki 5 qoldiq qolsa, bunday sonning kvadratini 9 bo'lganda 7 qoldiq chiqadi;
3) Agar sonni 9 ga bo'lganda 1 yoki 8 qoldiq qolsa, bunday sonning kvadratini 9 bo'lganda 1 qoldiq chiqadi;
4) Agar sonni 9 ga bo'lganda 3 yoki 6 qoldiq qolsa, bunday sonning kvadrati 9 ga qoldiqsiz bo'linadi;
5) Agar sonni 9 ga bo'lganda qoldiq 1, 4 yoki 7 qolsa, u holda bunday sonning kubini 9 ga bo'lganda 1 qoldiq bo'ladi;
6) Agar sonni 9 ga bo'lganda qoldiq 2, 5 yoki 8 qolsa, u holda bunday sonning kubini 9 ga bo'lganda 8 qoldiq bo'ladi;
7) Agar sonni 9 ga bo'lganda qoldiq 3 yoki 6 qolsa, u holda bunday sonning kubi 9 ga qoldiqsiz bo'linadi;
8) Kubdan qirra ayrilsa, bu 6 ga karrali son bo'ladi, ya'ni n^3-n shaklidagi son 6 ga qoldiqsiz bo'linadi, bunda n – natural son.

2. (*Diofant masalasi*). Quyidagi tenglikning to'g'riligini ko'rsating:
$(a^2+b^2)(c^2+d^2)=(ac\pm bd)^2+(bc\pm ad)^2$.

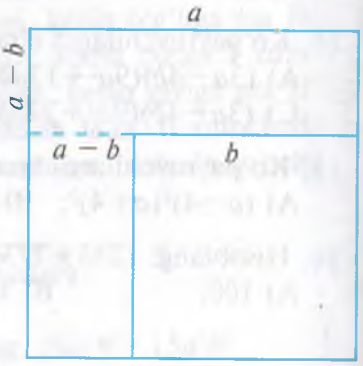
3. (*L.Eyler masalasi*). Quyidagi tenglikning to'g'riligini ko'rsating:
$(a^2+b^2+c^2+d^2)(m^2+n^2+p^2+q^2)=(an+bm+cq+dp)^2+(am-bn+cp-dq)^2+$
$+(-ap-bq+cm+dn)^2+(aq-bp-cn+dm)^2$.

TARIXIY MA'LUMOT

Al-Koshiyning "Arifmetika kaliti" asarida ikkihadni ixtiyoriy natural darajaga ko'tarish qoidalari berilgan.

Turli algebraik formulalarni isbotlashda, tenglamalarni yechishda geometrik mulohazalardan foydalanish qadimgi Xitoy, Yunoniston, Hindiston, O‘rta Osiyo matematiklari asarlarida uchraydi.

Ular $(a+b)^2 = a^2+2ab+b^2$, $(a-b)^2 = a^2-2ab+b^2$, $a^2-b^2=(a-b)(a+b)$ (yoki $a^2-b^2=(a-b)^2+2b(a-b)$) kabi ayniyatlarni geometrik usulda isbotlaganlar.



Masalan, $a^2-b^2=(a-b)(a+b)$ formulani isbotlashga shunday yondashilgan: tomoni a ga teng kvadratdan tomoni b ga teng kvadratni qirqib olinsa, qolgan shaklning yuzi:
$a\ (a-b)+b\ (a-b)=(a-b)\ (a+b)$ ga yoki $(a-b)^2+2b(a-b)$ ga teng bo‘lishi rasmdan ravshan ko‘rinib turibdi.

Demak, $a^2-b^2=(a-b)(a+b)$ formula to‘g‘ri.

To‘g‘ri burchakli uchburchakning tomonlarini butun (yoki ratsional) sonlarda ifodalash uchun xitoy matematiklari miloddan avvalgi birinchi ming yillardayoq

$$\left(\frac{p^2-q^2}{2}\right)^2+(pq)^2=\left(\frac{p^2+q^2}{2}\right)^2$$

tenglikdan foydalanganlar.

Al-Koshiyning $(a+b)^n$ (n – natural son) uchun yoyilmasidan, xususan, $n=3$ bo‘lganda sizlarga tanish

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3$$

formulani olamiz. $n=4$ bo‘lganda esa

$$(a+b)^4=a^4+4a^3b+6a^2b^2+4ab^3+b^4$$

formulani hosil qilamiz.

Al-Koshiy, shuningdek,

$$1^2+2^2+3^2+\ldots+n^2=\frac{1}{6}\cdot n(n+1)(2n+1);$$

$$1^3+2^3+3^3+\ldots+n^3=\frac{1}{4}\cdot n^2(n+1)^2$$

formulalarni ham isbotlagan.

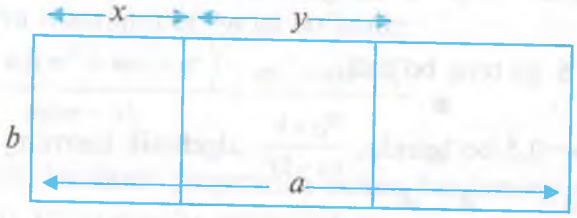
Prezidentimiz Islom Karimov 2014-yil Samarqandda bo‘lib o‘tgan “O‘rta asrlar Sharq allomalari va mutafakkirlarining tarixiy merosi, uning zamonaviy sivilizatsiya rivojidagi roli va ahamiyati” mavzusidagi Xalqaro konferensiyada so‘zlagan nutqida al-Koshiy ijodiga yuksak baho berdi.

V BOB. ALGEBRAIK KASRLAR

1-§. Algebraik kasr. Kasrlarni qisqartirish

Algebraik kasr tushunchasiga olib keladigan ikkita masala ko'ramiz.

1-masala. Tomonlari a va b bo'lgan to'g'ri to'rtburchak shaklidagi taxtadan eni x va y bo'lgan ikkita taxtacha arralab olindi (*5.1-rasm*).



5.1-rasm

Berilgan taxta yuzi bu taxtachalar yuzlari yig'indisidan necha marta katta?

□ Berilgan taxtaning yuzi $a \cdot b$ ga, arralab olingan taxtachalar yuzlari bx va by ga, ular yig'indisi $(bx+by)$ ga teng. U holda berilgan taxta yuzi arralab olingan taxtachalar yuzlari yig'indisidan necha marta kattaligini topish uchun $a \cdot b$ ni $(bx+by)$ ga bo'lish kerak, ya'ni masalaning javobi $\frac{a \cdot b}{bx+by}$ kasr bo'ladi.■

Shu bilan birga, arralab olingan taxtachalar yuzi $a \cdot b$ yuzning qancha qismini tashkil etishini bilish uchun esa $\frac{bx+by}{a \cdot b}$ kasrni hisoblash kerak. Bu kasrlar algebraik kasrlarga misol bo'la oladi.

2-masala. Velosipedchining shamol yo'q bo'lgandagi tezligi a km/soat ga, shamolning tezligi b km/soat ga teng. Velosipedchining shamol yo'nalishi bo'yicha tezligi uning shamol yo'nalishiga qarshi harakat tezligidan necha marta ortiq?

□ Velosipedchining shamol yo'nalishi bo'yicha tezligi $(a+b)$ km/soat ga, shamol yo'nalishiga qarshi tezligi $(a-b)$ km/soat ga teng. Shunga ko'ra, velosipedchining shamol yo'nalishi bo'yicha tezligi shamol yo'nalishiga qarshi harakat tezligidan $a>b$ bo'lganda $\frac{a+b}{a-b}$ marta ortiq bo'ladi. ■

$\frac{a+b}{a-b}$ ifoda algebraik kasr deyiladi, unda $a+b$ kasrning surati, $a-b$ esa uning maxraji.

Agar kasrning surat va maxraji algebraik ifodalar bo'lsa, u *algebraik kasr* deyiladi.

Misollar:

$$\frac{x}{y};\ \frac{x+y}{3};\ \frac{m-n}{p};\ \frac{a(x+y)}{b(a-x)};\ \frac{a^2+a+1}{a+1}.$$

Agar kasrning surat va maxraji o‘zgarmas son bo‘lsa, baribir, *algebraik kasr deyilaveradi*. Surat va maxrajlar o‘zgarmas bo‘lganda oddiy kasrga ega bo‘lamiz.

Agar algebraik kasrga kirgan harflar o‘rniga sonlar qo‘yilsa, zarur hisoblashlar bajarilgandan keyin biror son hosil bo‘ladi. Shu son *algebraik kasrning son qiymati* deyiladi.

Masalan: 1) $x=7$, $y=5$ bo‘lganda, $\frac{x+y}{x-y}$ algebraik kasrning son qiymati $\frac{7+5}{7-5}=\frac{12}{2}=6$ ga teng bo‘ladi;

2) $a=3{,}5$, $b=0{,}5$ bo‘lganda, $\frac{a+b}{(a-b)^2}$ algebraik kasrning son qiymatini topaylik: $\frac{3{,}5+0{,}5}{(3{,}5-0{,}5)^2}=\frac{4}{3^2}=\frac{4}{9}$.

$\frac{a+b}{a-b}$ algebraik kasrda a va b o‘rniga $a\neq b$ bo‘lgan istalgan sonni qo‘yish mumkin. Aks holda, $a-b=0$ bo‘ladi, nolga esa bo‘lish mumkin emas. Shuning uchun, har bir berilgan ko‘phadga kiruvchi harflar kasrning maxraji nolga aylanmaydigan qiymatlarni – *joiz qiymatlarni* qabul qiladi, deb kelishib olamiz.

Algebraik kasrning ham oddiy kasrning asosiy xossasiga o‘xshash xossasi bor.

Algebraik kasrning asosiy xossasi $\frac{a}{b}=\frac{na}{nb}$ kabi yoziladi, bu yerda $b\neq 0$, $n\neq 0$.

Bu xossa kasrning surat va maxraji bir xil algebraik ifodaga ko‘paytirilsa yoki bo‘linsa, berilgan kasrga teng kasr hosil bo‘lishini bildiradi.

Masalan: $\frac{a+b^2}{b}=\frac{(a+b^2)c}{bc}$; $\frac{ab-2}{a}=\frac{(ab-2)c}{ac}$.

Kasrning asosiy xossasi kasrning surati va maxraji bir xil umumiy ko‘paytuvchiga ega bo‘lsa, surat va maxrajni shu umumiy ko‘paytuvchiga bo‘lish mumkinligini anglatadi.

Masalan:

1) $\frac{a(2b+c)}{a(2b-c)}=\frac{2b+c}{2b-c}$;

2) $\frac{a^2-b^2}{a-b}=\frac{(a-b)(a+b)}{a-b}=a+b$;

3) $\frac{4a+4b}{a+b}=\frac{4(a+b)}{a+b}=4$.

Ikkinchi va uchinchi misollardan kasrni qisqartirish uchun avval kasrning surati va maxrajining umumiy ko'paytuvchisini ajratish kerakligi kelib chiqadi.

3-masala. Kasrni qisqartiring: 1) $\frac{24a^2b}{8ab^2}$; 2) $\frac{m^3-n^3}{m^2-mn}$.

□ 1) $24a^2b$ va $8ab^2$ ifodalar $8ab$ umumiy ko'paytuvchiga ega. Kasrning surat va maxrajini $8ab$ ga bo'lamiz: $\frac{24a^2b}{8ab^2}=\frac{8ab\cdot 3a}{8ab\cdot b}=\frac{3a}{b}$.

2) m^3-n^3 va m^3-mn ko'phadlar $m-n$ umumiy ko'paytuvchiga ega, chunki

$$m^3-n^3=(m-n)(m^2+mn+n^2),\ m^2-mn=n(m-n).$$

Kasrning surat va maxrajini $m-n$ ga bo'lamiz:

$$\frac{m^3-n^3}{m^2-mn}=\frac{(m-n)\left(m^2+mn+n^2\right)}{m(m-n)}=\frac{m^2+mn+n^2}{m}.\ \blacksquare$$

Shunday qilib, kasrlarni qisqartirish uchun kasrlarning surat va maxrajini ularning umumiy ko'paytuvchisiga bo'lish kerak.

Agar $\frac{a}{b}$ kasrning surati yoki maxrajidagi ishorani qarama-qarshisiga o'zgartirilsa, u holda berilgan kasrga ishorasi qarama-qarshi bo'lgan kasr hosil bo'ladi: $\frac{a}{-b}=-\frac{a}{b}$; $\frac{-a}{b}=-\frac{a}{b}$.

Misollar: $\frac{-3}{8}=-\frac{3}{8}$; $\frac{-a}{a-1}=-\frac{a}{a-1}=\frac{a}{1-a}$; $\frac{-(ax+b)}{x^2+4}=-\frac{ax+b}{x^2+4}$.

4-masala. $\frac{5m(x-y)}{m^2(y-x)}$ kasrni qisqartiring:

□ $\frac{5m(x-y)}{m^2(y-x)}=\frac{5m(x-y)}{-m^2(x-y)}=\frac{5m}{-m^2}=\frac{5}{-m}=-\frac{5}{m}$. ■

?

1. 1. Algebraik kasr deb nimaga aytiladi?
   2. Algebraik kasr qiymati deyilganda nima tushuniladi?
   3. Algebraik kasrni qisqartirish uchun nima qilish kerak?

2. Surati x va y sonlarining yig'indisiga, maxraji shu sonlarning ko'paytmasiga teng algebraik kasrni yozing.

3. Surati x va y sonlarining ayirmasiga, maxraji shu sonlarning yig'indisiga teng algebraik kasrni yozing.

4. Surati a va b sonlar kvadratlari yig'indisiga, maxraji shu sonlar ayirmasiga teng algebraik kasrni yozing.

5. Surati a va b sonlar kublarining ayirmasiga, maxraji shu sonlar kvadratlari yig'indisiga teng algebraik kasrni yozing.

6. Surati a va b sonlar yig'indisining kvadratiga, maxraji shu sonlar ayirmasining kvadratiga teng algebraik kasrni yozing.

7. Algebraik kasrning son qiymatini toping:

1) $\frac{1}{a+b}$, bunda $a=1\frac{2}{3}$, $b=3\frac{1}{3}$; 2) $\frac{b-a}{2a+b}$, bunda $a=1\frac{1}{3}$, $b=4\frac{2}{3}$;

3) $\frac{a+b}{a-b}$, bunda $a=2{,}5$, $b=1{,}5$; 4) $\frac{a^2+3b}{a^2-b}$, bunda $a=1{,}5$, $b=0{,}25$.

8. Agar:

1) $\frac{x}{y}=4$ bo'lsa, $\frac{x^2+xy-y^2}{x^2-xy+y^2}$; 2) $\frac{y}{x}=3$ bo'lsa, $\frac{3y^2-2xy+x^2}{x^2+xy+y^2}$;

3) $\frac{x}{y}=2$ bo'lsa, $\frac{4y^2-3xy+x^2}{x^2+xy+y^2}$; 4) $\frac{x+y}{x-y}=\frac{5}{2}$ bo'lsa, $\frac{x^2-y^2}{xy}$

ifodaning son qiymatini toping.

9. Agar $\frac{a+4b}{5a-7b}=2$ bo'lsa,

1) $\frac{4a-5b}{3a+b}$; 2) $\frac{3a^2-2ab+b^2}{5a^2+2b^2}$; 3) $\frac{a^3-3ab^2}{4a^2b+3b^2}$; 4) $\frac{a^2-ab+b^2}{a^2+b^2}$

ifodaning son qiymatini toping.

Kasrni qisqartiring **(10 – 26)** :

10. 1) $\frac{-16}{48}$; 2) $\frac{-64}{-128}$; 3) $\frac{-169}{26}$; 4) $\frac{144}{-12}$.

11. 1) $\frac{14a}{22}$; 2) $\frac{3m}{8m}$; 3) $\frac{18n}{72n}$; 4) $\frac{5ab}{15ac}$; 5) $\frac{2a^3}{8a^2}$; 6) $\frac{3x^2}{x^3y}$.

12. 1) $\frac{6a}{4ab}$; 2) $\frac{15c}{48b}$; 3) $\frac{4a^2b}{16a^3}$; 4) $\frac{12a^4b^2}{18a^3b^3}$.

13. 1) $\frac{2(m-n)}{3(m-n)}$; 2) $\frac{3(m+n)}{27(m+n)(m+n)}$; 3) $\frac{4(a-b)}{3(b-a)}$; 4) $\frac{6(x-y)}{18(y-x)}$.

14. 1) $\frac{a+b}{(a+b)^2}$; 2) $\frac{(m-n)}{(m-n)^2}$; 3) $\frac{2m(1+x)^2}{4m^2(1+x)}$; 4) $\frac{3n(1-x)^2}{8n^2(x-1)^2}$.

15. 1) $\frac{m+n}{(m+n)^3}$; 2) $\frac{(3x-2y)^2}{(2y-3x)}$; 3) $\frac{10m^2n(m-n)}{5m^3n(n-m)^2}$; 4) $\frac{9mn^2(m+n)}{12m^2n^3(m+n)^2}$.

16. 1) $\frac{4x-4y}{8c}$; 2) $\frac{3a+3b}{6a-6b}$; 3) $\frac{ac+bc}{ac-bc}$; 4) $\frac{m+mn}{m-mn}$.

17. 1) $\frac{m^2}{m^2-mn}$; 2) $\frac{pq^2}{p^2q+pq^2}$; 3) $\frac{4a+6b}{2a+3b}$; 4) $\frac{3a-6b}{12b-6a}$.

18. 1) $\frac{a}{a^2+ab}$; 2) $\frac{p^2q}{p^2q-pq^2}$; 3) $\frac{2m^2-mn}{n^2-2mn}$; 4) $\frac{2xy-x^2}{2y^2-xy}$.

19. 1) $\frac{8x^2-20xy}{20x^2-8xy}$; 2) $\frac{2a^2-ab}{b^2-2ab}$; 3) $\frac{a-2b}{12b-6a}$; 4) $\frac{x^2-2xy}{2y^2-xy}$.

20. 1) $\frac{12x^2+30xy}{30x^2+12xy}$; 2) $\frac{36a^2-24ab}{24a^2-36ab}$; 3) $\frac{m^3+3m^2n}{3m^2n-3m^3}$; 4) $\frac{a-2a^2b}{2a^3b^2-a^4b}$.

21. 1) $\frac{a^2-b^2}{a-b}$; 2) $\frac{a+b}{a^2-b^2}$; 3) $\frac{4x^2-9y^2}{2x-3y}$; 4) $\frac{16-x^2}{4+x}$.

22. 1) $\frac{2a(a-b)}{6a^2(b-a)}$; 2) $\frac{4a(b-5)}{16a^2(3+b)}$; 3) $\frac{3a(b^2-4)}{6a^2(b+4)}$; 4) $\frac{7b^2(4+b)}{14b(b-3)}$.

23. 1) $\frac{5-2c}{4c^2-25}$; 2) $\frac{d^2-9}{d+3}$; 3) $\frac{16-y^2}{2y-8}$; 4) $\frac{b^2+c^2}{b^4n-c^4n}$.

24. 1) $\frac{d^2+6d+9}{d+3}$; 2) $\frac{b-6}{b^2-12b+36}$; 3) $\frac{a^2-4a+4}{a-2}$; 4) $\frac{2p+1}{4p^2+4p+1}$.

25. 1) $\frac{4y^2+4y+1}{2y+1}$; 2) $\frac{4a^2-1}{4a^2+8a+1}$; 3) $\frac{3a^2+6ab+3b^2}{6a^2-6b^2}$; 4) $\frac{50m^2-100mn+50n^2}{12m^2-12n^2}$.

26. 1) $\frac{a^2-1}{(a+1)^2}$; 2) $\frac{(m+n)^2}{m+n}$; 3) $\frac{1-4y+4y^2}{3-6y}$; 4) $\frac{9-2x}{4x^2-36x+81}$.

27. Kasrni qisqartiring:

1) $\frac{4c^2-9}{9-12c+4c^2}$; 2) $\frac{9x^2-24xy+16y^2}{16y^2-9x^2}$; 3) $\frac{16y^2-8xy+x^2}{4y-x}$.

28. Agar:

1) $2a-b=5$ bo‘lsa, $8a^3-b^3-30ab$ ifoda;

2) $a+2b=4$ bo‘lsa, a^3+8b^3+24ab ifoda nimaga teng?

29.* a va b ning shunday qiymatlarini topingki, x ning barcha joiz qiymatlari uchun: 1) $\frac{ax^2-3x-b}{x-1}=2x-1$; 2) $\frac{ax^2+x-b}{2-x}=2x+3$ tenglik o‘rinli bo‘lsin.

Masalalar yechish

30. To‘g‘ri to‘rtburchak va kvadrat berilgan. To‘g‘ri to‘rtburchak asosi kvadratning tomoni uzunligidan a sm ortiq, balandligi esa kvadrat tomonidan b sm qisqa ($a>b$). Shu to‘gri to‘rtburchak va kvadratning yuzlari teng bo‘lsa, kvadratning tomoni uzunligini toping.

31. Bir qotishma tarkibida 60 %, ikkinchisida esa 40 % kumush bor. Ikkala qotishmani eritib, tarkibida 45 % kumush bo‘lgan 2 kg massali qotishma olindi. Birinchi va ikkinchi qotishmalarning massasini toping.

32. Birinchi kvadratning tomoni ikkinchi kvadrat tomonidan 3 sm uzun, birinchi kvadrat yuzi ikkinchisinikidan 21 sm^2 ortiq. Birinchi kvadrat perimetrining ikkinchi kvadrat perimetriga nisbatini toping.

Kasrni qisqartiring **(33–35)**:

33. 1) $\dfrac{25b-49b^3}{49b^3-70b^2+25b}$; 2) $\dfrac{49a^2-a^4}{a^4+14a^3+49a^2}$; 3) $\dfrac{27a^3+b^3}{3ab^2+b^3}$; 4) $\dfrac{2ab-b}{8a^3-1}$.

34. 1) $\dfrac{x^5-x^4y-xy^4+y^5}{x^4-x^3y-x^2y^2+xy^3}$; 3) $\dfrac{a^2-(x-y)a-xy}{a^3+a^2y+ax+xy}$;

2) $\dfrac{3a^5-81a^2}{(2a^2+6a+18)(a^4-3a^3)}$; 4) $\dfrac{9a^2+6a+2}{(3a+1)^4-1}$.

35.* 1) $\dfrac{x^2-4a^2}{x^2+4ax+4a^2}$; 3) $\dfrac{(5a-4)^2+2(5a-4)\cdot(4-3a)+(3a-4)^2}{(2a+5)^2-2(2a+5)(5-3a)+(3a-5)^2}$;

2) $\dfrac{x^4-x^2-12}{x^3+x^2-4x-4}$; 4) $\dfrac{a^{33}+1}{a^{11}-a^{22}+a^{33}}$.

36. a va b ning shunday qiymatlarini topingki, x ning barcha joiz qiymatlari uchun 1) $\dfrac{ax^2-x+b}{x-1}=2x+1$; 2) $\dfrac{ax^2+4x-2b}{3-x}=x-1$ tenglik o‘rinli bo‘lsin.

37. Hisoblang:

1) $\left(2010\dfrac{1997}{1999}\right)^2-\left(2011\dfrac{1997}{1999}\right)\cdot\left(2009\dfrac{1997}{1999}\right)$;

2) $\left(2008\dfrac{2007}{2008}\right)^2-\left(2007\dfrac{2007}{2008}\right)\cdot\left(2009\dfrac{2007}{2008}\right)$;

3) $(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)(2^{64}+1)-\frac{1}{3}\cdot 2^{128}$;

4) $(3^2+1)(3^4+1)(3^8+1)(3^{16}+1)(3^{32}+1)-\frac{1}{8}\cdot 3^{64}$.

□ 2- va 4-misollarni hisoblab ko'rsatamiz:

$$2)\left(2008\frac{2007}{2008}\right)^2-\left(2007\frac{2007}{2008}\right)\cdot\left(2009\frac{2007}{2008}\right)=\left(2007\frac{2007}{2008}+1\right)^2-\left(2007\frac{2007}{2008}\right)\cdot\left(2009\frac{2007}{2008}\right)=$$

$$=\left(2007\frac{2007}{2008}\right)^2+2\cdot\left(2007\frac{2007}{2008}\right)+1-\left(2007\frac{2007}{2008}\right)\cdot\left(2009\frac{2007}{2008}\right)=$$

$$=\left(2007\frac{2007}{2008}\right)\left(2007\frac{2007}{2008}-2009\frac{2007}{2008}\right)+2\cdot\left(2007\frac{2007}{2008}\right)+1=$$

$$=(-2)\cdot 2007\frac{2007}{2008}+2\cdot 2007\frac{2007}{2008}+1=1.$$

Javob: 1.■

□ 4) $(3^2+1)(3^4+1)(3^8+1)\ (3^{16}+1)(3^{32}+1)-\frac{1}{8}3^{64}=$

$$=\frac{(3^2-1)}{3^2-1}\cdot(3^2+1)(3^4+1)(3^8+1)\ (3^{16}+1)(3^{32}+1)-\frac{1}{8}3^{64}=$$

$$=\frac{1}{8}\cdot(3^{64}-1)-\frac{1}{8}3^{64}=-\frac{1}{8}.$$ *Javob*: $-\frac{1}{8}$. ■

38. Kasrni qisqartiring:

1) $\frac{x^{16}-x^8+1}{x^{24}+1}$; 2) $\frac{x^3-1}{x^4+x^2+1}$; 3) $\frac{m^4-16}{m^4-4m^3+8m^2-16m+16}$.

39. Hisoblang:

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots+\frac{1}{97\cdot 99}.$$

40. Hisoblang:

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\left(1-\frac{1}{19^2}\right).$$

Sonli ifodaning qiymatini hisoblang **(41–43)**:

41. $$\frac{(7-6{,}35):6{,}5+9{,}9}{\left(1{,}2:36+1{,}2:0{,}25-1\frac{5}{16}\right):\frac{169}{24}}.$$

42. $\left(\dfrac{(2,7-0,8)\cdot 2\frac{1}{3}}{(5,2-1,4):\frac{3}{70}}+0,125\right):2\frac{1}{2}+0,43.$

43. $\dfrac{\frac{5}{11}\cdot 0,0006\cdot 2\frac{1}{5}+1\frac{1}{8}\cdot 0,9954\cdot\frac{8}{9}}{25\cdot 0,0009+0,0001\cdot 25}.$

44. Soddalashtiring:

$$M=\frac{1}{1-x}+\frac{1}{1+x}+\frac{2}{1+x^2}+\frac{4}{1+x^4}+\frac{8}{1+x^8}+\frac{16}{1+x^{16}}.$$

□ Avval birinchi va ikkinchi kasrlarni qo'shamiz. $\overset{1+x}{\frac{1}{1-x}}+\overset{1-x}{\frac{1}{1+x}}=\frac{1+x+1-x}{1-x^2}=\frac{2}{1-x^2}$.

Hosil bo'lgan ifodaga ketma-ket $\frac{2}{1+x^2}$ ni, $\frac{4}{1+x^4}$ ni, $\frac{8}{1+x^8}$ ni, $\frac{16}{1+x^{16}}$ ni qo'shib boramiz:

$$M=\frac{2}{1-x^2}+\frac{2}{1+x^2}+\frac{4}{1+x^4}+\frac{8}{1+x^8}+\frac{16}{1+x^{16}}=\frac{4}{1-x^4}+\frac{4}{1+x^4}+\frac{8}{1+x^8}+\frac{16}{1+x^{16}}=$$

$$=\frac{8}{1-x^8}+\frac{8}{1+x^8}+\frac{16}{1+x^{16}}=\frac{8}{1-x^{16}}+\frac{16}{1+x^{16}}=\frac{32}{1-x^{32}}.$$

Shunday qilib, $M=\frac{32}{1-x^{32}}$. ■

45.* Soddalashtiring:

$$\frac{1}{b(abc+a+c)}-\frac{1}{a+\dfrac{1}{b+\dfrac{1}{c}}}:\frac{1}{a+\dfrac{1}{b}}.$$

Soddalashtiring **(46–47)**:

46. $\dfrac{2b+a-\dfrac{4a^2-b^2}{a}}{b^3+2ab^2-3a^2b}\cdot\dfrac{a^3b+2a^2b^2+ab^3-4a^2b^2}{a^2-b^2}.$

47. $P=\dfrac{\left(\dfrac{1}{a}+\dfrac{1}{b}-\dfrac{2c}{ab}\right)(a+b+2c)}{\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{2}{ab}-\dfrac{4c^2}{a^2b^2}}.$

□ Berilgan kasr ifodaning suratini va maxrajini alohida-alohida soddalashtiramiz:

$$\left(\frac{1}{a}+\frac{1}{b}-\frac{2c}{ab}\right)(a+b+2c)=\frac{(b+a-2c)(a+b+2c)}{ab}=\frac{(a+b)^2-4c^2}{ab};$$

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{2}{ab}-\frac{4c^2}{a^2b^2}=\frac{b^2+a^2+2ab-4c^2}{a^2b^2}=\frac{(a+b)^2-4c^2}{a^2b^2}.$$

Endi surat va maxraj uchun topilgan ifodalarni bo'lamiz:

$$P=\frac{(a+b)^2-4c^2}{ab}:\frac{(a+b)^2-4c^2}{a^2b^2}=\frac{(a+b)^2-4c^2}{ab}\cdot\frac{a^2b^2}{(a+b)^2-4c^2}=ab.$$

Javob: $P=ab$. ■

Hisoblang **(48–49)** :

48. $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)\ldots\left(1+\frac{1}{3^{32}}\right)$.

49. $(1+a)(1+a^2)(1+a^4)\ldots(1+a^{32})$.

50. Ixtiyoriy natural son n da $\frac{n^3}{6}+\frac{n^2}{2}+\frac{n}{3}$ ifoda butun songa teng bo'lishini isbotlang.

2-§. Kasrlarni umumiy maxrajga keltirish

Oddiy kasrlarni qo'shish yoki ayirishda dastlab ularni umumiy maxrajga keltirib olinadi.

Jumladan, $\frac{1}{2}, \frac{1}{5}, \frac{7}{9}$ oddiy kasrlar uchun umumiy maxraj 90; $\frac{1}{3}, \frac{1}{12}, \frac{1}{18}$ oddiy kasrlar uchun umumiy maxraj 36 bo'ladi. Birinchi misolda 2, 5, 9 maxrajlar o'zaro tub, shuning uchun EKUK (2, 5, 9) $=2\cdot5\cdot9=90$ bo'ladi. Ikkinchi misolda esa 3, 12, 18 maxrajlar o'zaro tub emas.

Hisoblashlardan EKUK (3, 12, 18) $=36$ ekani kelib chiqadi. Bu mulohazalar algebraik kasrlarni umumiy maxrajga keltirish jarayonida qo'llaniladi.

Algebraik kasrlarning umumiy maxraji shu kasrlar maxrajlarining umumiy karralisidir.

Kasrlarni umumiy maxrajga keltirish uchun kasrning asosiy xossasidan foydalaniladi.

1-masala: $\frac{m}{2ab^2}, \frac{n}{3a^2b}$ va $\frac{p}{6ac}$ algebraik kasrni umumiy maxrajga keltiring.

□ Berilgan kasrning umumiy maxraji har bir kasrning maxrajiga bo'linishi kerak. Avvalo u 2, 3, 6 ga bo'linishi uchun EKUK (2, 3, 6) $=6$ ga, ab^2, a^2b va ac ga bo'linishi kerak. Demak, umumiy maxraj o'rnida $6a^2b^2c$ ni olish mumkin. Bu umumiy maxrajni birinchi kasrning maxrajiga bo'lib, natijani uning

surat va maxrajiga ko‘paytiramiz, bu ko‘paytuvchi birhad bo‘ladi. U birinchi kasrning qo‘shimcha ko‘paytuvchisi deyiladi. Birinchi kasr uchun qo‘shimcha ko‘paytuvchi $3ac$ bo‘ladi, ikkinchi va uchinchi kasrlar uchun esa $2bc$ va ab^2 bo‘ladi. Jarayonni quyidagicha yozish mumkin: $\frac{3ac\,/\,m}{2ab^2}, \frac{2bc\,/\,n}{3a^2b}, \frac{ab^2\,/\,p}{6ac}$.

Kasrlarning surat va maxrajlarini mos qo‘shimcha ko‘paytuvchilarga ko‘paytirib, ularni $6a^2b^2c$ umumiy maxrajga keltiramiz:

$$\frac{m}{2ab^2}=\frac{3acm}{6a^2b^2c};\quad \frac{n}{3a^2b}=\frac{2bcn}{6a^2b^2c};\quad \frac{p}{6ac}=\frac{ab^2p}{6a^2b^2c}.$$

2-masala. Kasrlarni umumiy maxrajga keltiring:

$$\frac{a}{x^2-y^2};\quad \frac{b}{2x^2+4xy+2y^2};\quad \frac{c}{5x^2-10xy+5y^2}.$$

Kasrlarning maxrajini ko‘paytuvchilarga ajratamiz:
$x^2-y^2=(x-y)(x+y)$; $2x^2+4xy+2y^2=2(x^2+2xy+y^2)=2(x+y)^2$;
$5x^2-10xy+5y^2=5(x^2-2xy+y^2)=5(x-y)^2$.
Umumiy maxraj shunday ifodalarning ko‘paytmasidan iborat bo‘lishi kerakki, uning tarkibida $(x-y)(x+y)$, $2(x+y)^2$ va $5(x-y)^2$ ifodalar bo‘lishi lozim. Bundan umumiy maxraj $10(x-y)^2(x+y)^2$ ga teng bo‘lishi kelib chiqadi.
Endi berilgan kasrlarni quyidagicha yozib olish mumkin:

$$\frac{a}{x^2-y^2}=\frac{a\cdot 10(x-y)(x+y)}{10(x-y)^2(x+y)^2};\qquad \frac{b}{2x^2+4xy+2y^2}=\frac{b\cdot 5(x-y)^2}{10(x-y)^2\cdot(x+y)^2};$$

$$\frac{c}{5x^2-10xy+5y^2}=\frac{2(x+y)^2\cdot c}{10(x-y)^2\cdot(x+y)^2}.\ \blacksquare$$

Shunday qilib, algebraik kasrni umumiy maxrajga keltirish uchun:
1. Berilgan kasrlarning umumiy maxrajini topish;
2. Har bir kasr uchun qo‘shimcha ko‘paytuvchini topish;
3. Har bir kasrning suratini uning qo‘shimcha ko‘paytuvchisiga ko‘paytirish;
4. Har bir kasrni topilgan surat va umumiy maxraji bilan yozish kerak.

(?)

51. 1. Algebraik kasrlarni umumiy maxrajga keltirish uchun nimalar qilish kerak?
2. Kasrlarni umumiy maxrajga keltirishda muhim tushuncha “qo‘shimcha ko‘paytuvchi” nima? U qanday topiladi?
Misollar ko‘raylik. Kasrlarni umumiy maxrajga keltiring:

1) $\frac{1}{3}$ va $\frac{3}{4}$. Umumiy maxraj 12. Shuning uchun $\frac{1\cdot4}{3\cdot4}$ va $\frac{3\cdot3}{4\cdot3}$, ya’ni $\frac{4}{12}$ va $\frac{9}{12}$;

2) $\frac{4}{7}$ va $\frac{5}{21}$. Umumiy maxraj 21. Shuning uchun $\frac{4\cdot3}{7\cdot3}$ va $\frac{5}{21}$, ya'ni $\frac{12}{21}$ va $\frac{5}{21}$;

3) $\frac{2m}{n}$ va $\frac{m}{2n}$. Umumiy maxraj $2n$. Shuning uchun $\frac{4m}{2n}$ va $\frac{m}{2n}$;

4) $\frac{x}{y}$ va $\frac{2x}{3y}$. Umumiy maxraj $3y$. Shuning uchun $\frac{3\cdot x}{3\cdot y}$ va $\frac{2x}{3y}$.

Kasrlarni umumiy maxrajga keltiring **(52–61)**:

52. 1) $\frac{2}{3a}, \frac{7}{15b}$ va $\frac{8}{45ab}$; 2) $\frac{2x}{3y}, \frac{5}{xy}$ va $\frac{5y}{4x}$; 3) $\frac{3}{a}$ va $\frac{4}{a^2}$; 4) $\frac{a}{3x}$ va $\frac{b}{9x^3}$.

53. 1) $2m$ va $\frac{n^2}{m}$; 2) $4b$ va $\frac{a^2}{5b}$; 3) $2m^2$ va $\frac{c}{3mn}$; 4) $\frac{a}{2b}, \frac{2c}{3a}$ va ab.

54. 1) $\frac{1}{3p^2}, \frac{1}{9pk}$ va $\frac{1}{4k^2}$;

2) $\frac{1}{4b^2}, \frac{a^2+b^2}{6a^2b^2}$ va $\frac{a-3}{24ab^2}$;

3) $\frac{2b}{a^2}, \frac{5}{12a^2b}$ va $\frac{3}{16a^3b^4}$;

4) $\frac{3}{20x^3y}, \frac{11}{6xy^3}$ va $\frac{17}{3x^2y^3}$.

55. 1) $\frac{5a}{2a-3b}$ va $\frac{3b}{2a+3b}$;

2) $\frac{4x}{3x-2y}$ va $\frac{5y}{3x+2y}$;

3) $\frac{2c}{2c-5d}$ va $\frac{3d}{2c+5d}$;

4) $\frac{3a}{3a-4b}$ va $\frac{4b}{3a+4b}$.

56. 1) $\frac{2a}{16a^2-25}$ va $\frac{3a}{5-4a}$;

2) $\frac{2a}{81-25a^2}$ va $\frac{3a}{5a+9}$;

3) $\frac{5a}{27-8a^3}$ va $\frac{4a}{2a-3}$;

4) $\frac{3a}{9a^2-12a+16}$ va $\frac{2a}{3a+4}$.

57. 1) $\frac{15x}{9x^2-4}, \frac{9x+2y}{9x^2+12x+4}$ va $\frac{2y-3x}{9x^2-12x+4}$;

2) $\frac{8b}{4b^2-12bc+x^2}, \frac{2a}{3c-2b}$ va $\frac{1}{12a+12ab}$.

58. 1) $\frac{1}{4x^2-9y^2}, \frac{1}{4x^2y+12xy^2+9y^3}$ va $\frac{1}{3y-2x}$;

2) $\frac{2x}{8y^3-x^3}, \frac{3}{2x^2y-4xy}$ va $\frac{1}{2x^2y+4xy^2+4y^2}$.

59. 1) $\frac{1}{a-b}$, $\frac{1}{30a^2-30b^2}$ va $\frac{1}{5a+5b}$;

2) $\frac{1}{4a^2-9b^2}$, $\frac{1}{6ab-9b^2}$ va $\frac{1}{4a^2+6ab}$.

60. 1) $\frac{1}{64a^3-27b^3}$, $\frac{1}{a(4a-3b)}$ va $\frac{1}{16a+12ab+9b^2}$;

2) $\frac{1}{a^2b+6ab^2+9b^3}$, $\frac{1}{b^2(a+3b)^3}$ va $\frac{1}{2ab+6b^2}$.

61. 1) $\frac{4c}{5a^3b^2}$, $\frac{3d}{10a^2b^2}$ va $\frac{2k}{15ab}$; 2) $\frac{2}{35a^4b^5}$, $\frac{3}{49a^5b^4}$ va $\frac{1}{28a^3b^6}$;

3) $\frac{1}{36-12a+a^2}$, $\frac{1}{36-a^2}$ va $\frac{1}{a+6}$; 4) $\frac{1}{27a^3-b^3}$, $\frac{1}{9a^2+3ab+6^2}$ va $\frac{1}{b-3a}$.

(?)

62. 1. Kasrlardan birortasining maxraji umumiy maxraj bo'lib qolishi mumkinmi?
2. Umumiy maxraj berilgan barcha kasrlar maxrajlarining ko'paytmasi bo'lib qolishi mumkinmi?
3. Umumiy maxrajda kasrlar maxrajlaridagi harflar qanday darajada qatnashadi?
Har bir holat uchun 3 tadan misol tuzing.

3-§. Algebraik kasrlarni qo'shish va ayirish

Oddiy kasrlarni bir xil maxrajli va turli maxrajli bo'lgan hollarda qo'shish va ayirish qoidalarini algebraik kasrlarni qo'shish va ayirishga nisbatan ham aytish mumkin. Farqi shundaki, kasrlarning suratida va maxrajida ko'phadlar turgan bo'ladi. Bir xil maxrajli kasrlarni qo'shish va ayirish qoidalarini bunday yozish mumkin:

$$\frac{a}{k}+\frac{b}{k}=\frac{a+b}{k};$$

$$\frac{a}{k}-\frac{b}{k}=\frac{a-b}{k}$$

1-masala: $\frac{a-b}{a+b}, \frac{a+3b}{a+b}, \frac{a-8b}{a+b}$ kasrlarni qo‘shing.

□ $\frac{a-b}{a+b}+\frac{a+3b}{a+b}+\frac{a-8b}{a+b}=\frac{a-b+a+3b+a-8b}{a+b}=\frac{3a-6b}{a+b}=\frac{a(a-2b)}{a+b}$. ■

2-masala: $\frac{a^2}{a+2b}$ va $\frac{4b^2}{a+2b}$ kasrlarning ayirmasini toping.

□ $\frac{a^2}{a+2b}-\frac{4b^2}{a+2b}=\frac{a^2-4b^2}{a+2b}=\frac{(a+2b)(a-2b)}{a+2b}=a-2b$. ■

Har xil maxrajli algebraik kasrlarni qo‘shish va ayirish uchun shu kasrlarni umumiy maxrajga keltirish va bir xil maxrajli kasrlarni qo‘shish yoki ayirish qoidasini qo‘llash kerak.

3-masala: $\frac{2}{a^3}, \frac{1}{2ab^2}$ va $\frac{3}{4a^2b}$ kasrlarni qo‘shing.

□ Berilgan kasrlarning umumiy maxraji $4a^3b^2$ ko‘paytma bo‘ladi.

$$\frac{2}{a^3}+\frac{1}{2ab^2}+\frac{3}{4a^2b}=\frac{2\cdot 4b^3+2a^2b+ab^2}{4a^3b^3}=\frac{8b^3+2a^2b+ab^2}{4a^3b^3}=\frac{b(8b^2+2a^2+ab)}{4a^3b^3}.$$ ■

4-masala: $\frac{b}{2a^2c}$ va $\frac{c}{8ab^2}$ kasrlar ayirmasini toping.

□ $\frac{b}{2a^2c}-\frac{c}{8ab^2}=\frac{4b^2/b}{2a^2c}-\frac{ac/c}{8ab^2}=\frac{4b^3-ac^2}{8a^2b^2c}$. ■

Bu misolda qo‘shimcha ko‘paytuvchilar, mos ravishda, $4b^2$ va ac bo‘ladi.

5-masala: $\frac{2}{x^2-x}$ va $\frac{3}{x^2-1}$ kasrlarni qo‘shing.

□ Avvalo kasrlarning maxrajlarida turgan ko‘phadlarni ko‘paytuvchilarga ajratamiz:

$$x^2-x=x(x-1);\ x^2-1=(x-1)(x+1).$$

Berilgan kasrlarning umumiy maxraji $x(x-1)(x+1)$ ko‘paytma bo‘ladi. Kasrlarni umumiy maxrajga keltirib, qo‘shishni bajaramiz:

$$\frac{2}{x^2-x}+\frac{3}{x^2-1}=\frac{2}{x(x-1)}+\frac{3}{(x-1)(x+1)}=\frac{2(x+1)}{x(x-1)(x+1)}+\frac{3x}{x(x-1)(x+1)}=$$

$$=\frac{2(x+1)+3x}{x(x-1)(x+1)}=\frac{2x+2+3x}{x(x-1)(x+1)}=\frac{5x+2}{x(x-1)(x+1)}.$$ ■

Shunday qilib, turli maxrajli kasrlarni qo‘shish va ayirishni ushbu tartibda bajarish lozim:

1. Kasrlarning umumiy maxraji topiladi;
2. Kasrlar umumiy maxrajga keltiriladi;
3. Hosil bo‘lgan kasrlar qo‘shiladi;
4. Mumkin bo‘lsa natija soddalashtiriladi.

6-masala: $\frac{3}{(m+3)^2}+\frac{4}{m^2(m^2+6m+9)}-\frac{3}{m(m+3)}$ ifodaning qiymatini $m=\frac{1}{3}$ bo‘lganda hisoblang.

□ Berilgan ifodani quyidagicha almashtirish mumkin:

$$\frac{3}{(m+3)^2}+\frac{4}{m^2(m^2+6m+9)}-\frac{3}{m(m+3)}=\frac{3}{(m+3)^2}+\frac{4}{m^2(m+3)^2}-\frac{3}{m(m+3)}=$$

$$=\frac{3m^2+4-3m^2-9m}{m^2(m+3)^2}=\frac{4-9m}{m^2(m+3)^2}.$$

Endi ifodaning son qiymatini topamiz: $\frac{4-9\cdot\frac{1}{3}}{\left(\frac{1}{3}\right)^2\left(\frac{1}{3}+3\right)^2}=\frac{4-3}{\frac{1}{9}\cdot\frac{10^2}{9}}=\frac{9\cdot9}{10^2}=\frac{81}{100}$. ■

(?)

44. 1) Algebraik kasrlarni qo‘shish va ayirish amallarini qanday tartibda bajarish lozim?

2) Qo‘shish va ayirish qoidalarini misollarda tushuntirib bering.

Kasrlarning yig‘indisini (ayirmasini) toping **(45–54)**:

45. 1) $\frac{2a}{b^2}+\frac{3a}{b^2}$; 2) $\frac{4m}{3n^2}-\frac{2m}{3n^2}$; 3) $\frac{a}{a-b}+\frac{c}{a-b}$; 4) $\frac{x}{m+n}-\frac{y}{m+n}$.

46. 1) $\frac{c+d}{3a}+\frac{2c-d}{3a}$; 2) $\frac{m+2b}{2a^2}+\frac{5m-b}{2a^2}$; 3) $\frac{m+n}{2a}-\frac{m-n}{2a}$.

47. 1) $\frac{(a+1)^2}{4d}+\frac{(a-1)^2}{4d}$; 2) $\frac{(a+2)^2}{a^2b}-\frac{(2-a)^2}{a^2b}$.

48. 1) $\frac{8}{11}+\frac{3}{7}$; 2) $\frac{9}{11}-\frac{3}{7}$; 3) $\frac{3}{4a}+\frac{1}{a}$; 4) $\frac{c}{12a}+\frac{d}{4}$.

49. 1) $\frac{2}{b}-\frac{3}{7b}$; 2) $\frac{a}{5}-\frac{b}{15d}$; 3) $\frac{4}{a}+\frac{b}{5}$; 4) $11+\frac{3}{a}$.

50. 1) $7-\frac{1}{a}+\frac{3}{a^2}$; 2) $4+\frac{2}{c}-\frac{4}{c^2}$; 3) $b+\frac{c}{d}-\frac{c^2}{d^2}$; 4) $\frac{a}{b}-k+\frac{a^2}{b^2}$.

51. 1) $\frac{1}{ab}-\frac{1}{bc}$; 2) $\frac{1}{mn}+\frac{1}{mk}$; 3) $\frac{a}{bc}+\frac{a}{bd}$; 4) $\frac{b}{ac}-\frac{b}{cd}$; 5) $\frac{4}{m^2}-\frac{5}{mn}$; 6) $\frac{3}{mn}+\frac{2}{n^3}$.

52. 1) $\frac{2a}{7b^4d}+\frac{5c}{2bd^3}$; 2) $\frac{2a}{5b^4}-\frac{6c}{7a^3b}$; 3) $\frac{3}{3y^3}+\frac{1}{6x^2y}-\frac{5}{12xy^2}$; 4) $\frac{3}{8x^2y}-\frac{3}{5xy^2}+\frac{13}{14x^2y^2}$.

53. 1) $\frac{a}{c^2}+\frac{b}{a^2}+\frac{c}{b^2}$; 2) $\frac{a}{c}+\frac{a}{c^2d}+\frac{a}{cd^2}$; 3) $\frac{a}{c}-\frac{b}{a}+\frac{c}{b}$; 4) $\frac{b}{c}-\frac{b}{cd^2}+\frac{b}{c^2d}$.

54. 1) $\frac{m}{p^2}+\frac{n}{q^2}+\frac{k}{n^2}$; 2) $\frac{a}{p}-\frac{b}{q}+\frac{c}{k}$; 3) $\frac{m}{p^2}+\frac{m}{pd}+\frac{m}{pd^2}$; 4) $\frac{a}{bc}-\frac{a}{cd}+\frac{a}{bd}$.

Algebraik kasrlarni qo'shing va ayiring **(55–61)**:

55. 1) $\frac{x}{2(a-b)}+\frac{3x}{a-b}$; 2) $\frac{5x}{3(x-1)}-\frac{x}{x-1}$; 3) $\frac{a^2}{2(a-1)}+\frac{4a^3}{5(a-1)}$; 4) $\frac{4y}{5(y-3)}+\frac{5x}{2(y-3)}$.

56. 1) $\frac{5}{x-1}+\frac{3}{2x-2}$; 2) $\frac{a}{2a+2b}-\frac{2a}{4a+4b}$; 3) $\frac{7}{5b+5}+\frac{3}{10b+10}$; 4) $\frac{4x}{3x+3y}-\frac{x}{6x+6y}$.

57. 1) $\frac{2}{a^2+a}+\frac{a}{ab+b}$; 2) $\frac{5b}{ax+ay}+\frac{3a}{bx+by}$; 3) $\frac{a+y}{b^2+ab}-\frac{y-b}{ab+a^2}$; 4) $\frac{y+b}{a^2-ab}-\frac{y+a}{ab-b^2}$.

58. 1) $\frac{2}{x}-\frac{3}{x+y}$; 2) $\frac{5}{a-1}-\frac{4}{a}$; 3) $\frac{1}{x(x+3)}-\frac{1}{x(x-3)}$; 4) $\frac{3}{5(a+b)}-\frac{1}{8(a-b)}$.

59. 1) $\frac{a}{1+b}+\frac{b}{1-b^2}$; 2) $\frac{3}{x+3}+\frac{2}{x^2-9}$; 3) $\frac{5+p^2}{6+p}-\frac{p}{p^2-36}$; 4) $\frac{2x}{x+4}-\frac{5x-2}{x^2-16}$.

60. 1) $\frac{2x}{x+4}+\frac{5x-2}{x^2-16}$; 2) $\frac{c^2-8}{2c-3}+\frac{16c-2c^3}{9-4c^2}$; 3) $\frac{12n+5}{n^2-49}-\frac{6}{7-n}$; 4) $\frac{21y^2+1}{1-9y^2}-\frac{1}{1-3y}$.

61. 1) $\frac{2y+8}{y^2+4y+y}-\frac{7}{y+2}$; 2) $\frac{4}{(m+n)^2}-\frac{7}{n+m}$;

3) $\frac{4+5x}{1+6x+9x^2}-\frac{2}{3x+1}$; 4) $\frac{2a}{25+10a+a^2}-\frac{10}{a^2-25}$.

Amallarni bajaring **(61–73)**:

62. 1) $\frac{10b-1}{12b^2-3}+\frac{2b+2}{4b+2}-\frac{2b+1}{2b-1}$; 2) $\frac{18a}{81a^2-1}+\frac{9a+1}{3-27a}+\frac{9a-1}{18a+2}$;

3) $\frac{4a+1}{64a^3-1}+\frac{1}{16a^2+4a+1}-\frac{2}{4a-1}$;

4) $\frac{12a^2b^2-2ab+3}{8a^3b^3-1}-\frac{2ab+-1}{4a^2b^2+2ab+1}+\frac{2}{1-2ab}$.

63. 1) $\frac{x^3}{2(x+1)^3}-\frac{x^2}{(x+1)^2}+\frac{x}{2(x+1)}$;

2) $\frac{2}{2x+3}+\frac{3}{3-2x}+\frac{2x+15}{4x^2-9}$.

64. 1) $\frac{1}{3a-2}+\frac{3}{3a+2}+\frac{6a}{(3a+2)^2}$;

2) $\frac{2}{4a-3}+\frac{3}{4a+3}-\frac{16a-6}{16a^2-9}$.

65. 1) $\frac{a+2b}{3a-3b}+\frac{3c-a}{2c-2a}-\frac{a^2-bc}{ab+ac-bc-a^2}$;

2) $\frac{x+2}{x^3-3x^2-4x+12}-\frac{3-x}{x^2-5x+6}$.

66. 1) $\frac{2}{a+4}-\frac{a-3}{a^2-4a+16}-\frac{a^2-9a}{a^3+64}$;

2) $\frac{1}{2a-3b}-\frac{2a+3b}{4a^2+6ab+9b^2}-\frac{6ab}{8a^3-27b^3}$.

67. 1) $\frac{1}{a^2-7a+12}+\frac{2a-1}{a^2-4a+3}-\frac{2a-5}{(a^2-5a+4)(a-3)}$;

2) $\frac{4}{2a+3b}+\frac{5}{2a-3b}-\frac{30b}{4a^2-9b^2}$.

68.* 1) $\frac{a^2-(b-c)^2}{(a+c)^2-b^2}+\frac{b^2-(a-c)^2}{(a+b)^2-c^2}+\frac{c^2-(a-b)^2}{(b+c)^2-a^2}$;

2) $\frac{1}{(a-b)(a-c)}+\frac{1}{(b-a)(b-c)}+\frac{1}{(c-a)(c-b)}$.

69.* 1) $\frac{1}{2a(2a-3b)(2a-c)}+\frac{1}{3b(3b-2a)(3b-c)}$;

2) $\frac{x-y}{x+y}+\frac{y-z}{y+z}+\frac{z-x}{z+x}+\frac{(x-y)(y-z)(2-x)}{(x+y)(y+z)(z+x)}$.

70. 1) $\frac{3a+2}{9a^2-6a+4}-\frac{18a}{27a^3+8}-\frac{1}{3a+2}$;

2) $\frac{m^2-mn}{m^2n+n^3}-\frac{2m^2}{n^3-mn^2+m^2n-m^3}$.

71. 1) $\frac{7}{2a+3b}+\frac{8}{2a-3b}-\frac{48b}{4a^2-9b^2}$;

2) $\frac{6}{3a-2b}+\frac{5}{3a+2b}-\frac{30a}{9a^2-4b^2}$.

72. 1) $\frac{5}{3x-5}-\frac{50}{9x^2-25}-\frac{4}{3x+5}$;

2) $\frac{7}{3x-4}-\frac{6}{3x+4}-\frac{48}{9x^2-16}$.

73. 1) $4x^2-\frac{8x^3+27y^3}{2x-3y}-9y^2$;

2) $\frac{2x^2-8x+8}{x^4-4x^3+16x-16}-\frac{x}{x^2-4}$.

74. Hovuzni bir quvur a soatda, ikkinchi quvur esa b soatda to‘ldiradi. Agar ikkala quvur bir vaqtda ochib qo‘yilsa, hovuz necha soatda to‘ladi?

75. Ikkita ishchi birgalikda ishlab vazifani a soatda bajardi. Birinchi ishchi ayni shu vazifani bir o‘zi ishlab k soatda bajardi. Ikkinchi ishchining bir o‘zi shu vazifani necha soatda bajargan?

76. 1) $\frac{a-ab+c-cb}{1-3b+3b^2-b^3}$ ifodani soddalashtiring va uning $a=\frac{3}{4}$, $b=\frac{1}{2}$, $c=-\frac{1}{4}$ bo'lgandagi son qiymatini toping.

2) $\frac{3a^3+ab^2-6a^2b-2b^3}{9a^5-ab^4-18a^4b+2b^2}$ ifodani soddalashtiring va uning $a=\frac{1}{3}$, $b=-\frac{1}{2}$ bo'lgandagi son qiymatini toping.

77. 1) $\frac{ac+ax+bx+bc}{ay+2ax+2bx+by}$ ifodani soddalashtiring va uning $x=\frac{1}{2}, y=-\frac{1}{3}$, $c=-\frac{1}{4}$ bo'lgandagi son qiymatini toping.

2) $1+3a+\frac{9a^2}{1+3a}+\frac{1}{3a-1}+\frac{6a}{1-9a^2}$ ifodani soddalashtiring va uning $a=1$ bo'lgandagi son qiymatini toping.

78*. 1) $\frac{72a^2}{27a^3-1}+\frac{3a+1}{9a^2+3a+1}$ ifodani soddalashtiring va uning $a=2$ bo'lgandagi son qiymatini toping.

2) $\frac{12a^2-2a+3}{8a^3-1}-\frac{2a-1}{4a^2+2a+1}+\frac{2}{1-2a}$ ifodani soddalashtiring va uning $a=1\frac{1}{2}$ bo'lgandagi son qiymatini toping.

Tenglikning to'g'riligini ko'rsating **(79–80)**:

79.* 1) $\frac{1}{a(a+1)}+\frac{1}{(a+1)(a+2)}+\ldots+\frac{1}{(a+2012)(a+2013)}=\frac{2013}{a(a+2013)}$;

2) $\frac{a^3(c-b)+b^3(a-c)+c^3(b-a)}{a^2(c-b)+b^2(a-c)+c^2(b-a)}=a+b+c.$

80.* 1) $\frac{a^2-bc}{(a+b)(a+c)}+\frac{b^2-ac}{(b+c)(a+b)}+\frac{c^2-ab}{(a+c)(b+c)}=0$;

2) $a(b^2-c^2)+b(c^2-a^2)+c(a^2-b^2)=(a-b)(b-c)(c-a)$.

Masalalar yechish

Kasrlarni umumiy maxrajga keltiring **(81–92)**:

81. 1) $\frac{3}{4a}, \frac{5}{8b}$ va $\frac{7}{72ab}$; 2) $\frac{3x}{4y}, \frac{2}{xy}$ va $\frac{2y}{3x}$; 3) $\frac{4}{a^2}$ va $\frac{5}{a}$; 4) $\frac{a}{4x}$ va $\frac{b}{16x^3}$.

82. 1) $3a$ va $\frac{b^2}{2a}$; 2) $2a$ va $\frac{a^2}{4b}$; 3) $\frac{3b}{a^2}; \frac{5}{8a^2b}$ va $\frac{7}{12a^3b^4}$; 4) $\frac{7}{10x^3y}; \frac{9}{4xy^3}$ va $\frac{11}{3x^2y^3}$.

83. 1) $\frac{1}{4p^2}; \frac{1}{12pk}$ va $\frac{1}{3k^2}$;

2) $\frac{1}{3b^2}; \frac{a^2+b^2}{4a^2b^2}$ va $\frac{b-3}{12ab^2}$;

3) $\frac{3b}{a,}; \frac{5}{8ab^2}$ va $\frac{7}{12a^2b}$;

4) $\frac{a}{2b}, \frac{3c}{4a}$ va $5ab$.

84. 1) $\frac{3a}{a-2b}$ va $\frac{4b}{a+2b}$;

2) $\frac{5x}{2x-y}$ va $\frac{4y}{2x+y}$;

3) $\frac{3c}{5c-2d}$ va $\frac{4d}{5c+2d}$;

4) $\frac{4a}{4a-5b}$ va $\frac{3b}{4a+5b}$.

85. 1) $\frac{a}{4a^2-9}$ va $\frac{a}{3-2a}$;

2) $\frac{a}{64-27a^3}$ va $\frac{a}{4-3a}$;

3) $\frac{3a}{25-81a^2}$ va $\frac{4a}{5-9a}$;

4) $\frac{5a}{4a^2-12a+9}$ va $\frac{2a}{2a-3}$.

86. 1) $\frac{7x}{4x^2-9}, \frac{3x+4y}{4x^2+12x+9}$ va $\frac{3y-4x}{4x^2-12x+9}$,

2) $\frac{8b}{9b^2-24bc+16c^2}, \frac{3a}{3b-4c}$ va $\frac{b}{3b+4c}$.

87. 1) $\frac{4}{9x^2-4y^2}, \frac{1}{9x^2-12xy-4y^2}$ va $\frac{1}{3x-2y}$;

2) $\frac{3y}{27y^3-x^3}, \frac{2}{4y^2+2xy+2x^2}$ va $\frac{1}{6y-2x}$.

88. 1) $\frac{1}{a+b}, \frac{1}{8a^2-8b^2}$ va $\frac{1}{a+b}$;

2) $\frac{a}{(2a+3b)^2}, \frac{1}{3(2a+3b)^2}$ va $\frac{b}{2(2a-3b)}$.

89. 1) $\frac{1}{64b^3+27a^3}, \frac{1}{b(4b+3a)}$ va $\frac{1}{16b^2+12ab+9a^2}$;

2) $\frac{1}{64b^3+27a^3}, \frac{1}{b(4b+3a)}$ va $\frac{1}{16b^2-12ab+9a^2}$.

90. 1) $\frac{3c}{8a^2b^3}, \frac{4d}{16a^2b^2}$ va $\frac{2k}{24ab}$;

2) $\frac{4}{24a^4b^5}, \frac{3}{35a^5b^4}$ va $\frac{1}{14a^3b^6}$;

3) $\frac{1}{a^2-10a+25}, \frac{1}{25-a^2}$ va $\frac{1}{5+a}$;

4) $\frac{1}{8a^3-b^3}, \frac{1}{4a^2+8ab+b^2}$ va $\frac{1}{b-2a}$.

91. 1) $\frac{a}{4a^2-b^2}, \frac{b}{2a+b}$ va $\frac{1}{2a-b}$;

2) $\frac{m+n}{(m+n)^2-p^2}, \frac{m-n}{m+n+p}$ va $\frac{1}{m+n-p}$.

92. 1) $\frac{a}{a+b}, \frac{b}{4(a-b)}$ va $\frac{ab}{3(a^2-b^2)}$; 2) $\frac{4}{5-5x}, \frac{5x}{1-x^2}$ va $\frac{1}{4x^2+4x}$;

3) $\frac{5x}{x-y}, \frac{6y}{x+y}$ va $\frac{1}{x(x^2-y^2)}$; 4) $\frac{2}{a^3}, \frac{3}{2a^2b}$ va $\frac{4}{5ab^2}$.

Bir xil maxrajli kasrlarning yig'indisini (ayirmasini) toping **(93–94)**:

93. 1) $\frac{3a}{b^2}+\frac{b-3a}{b^2}$; 2) $\frac{3m}{2n^2}+\frac{2m}{2n^2}$; 3) $\frac{a}{a-b}+\frac{b}{a-b}$; 4) $\frac{2a}{m+n}-\frac{a}{m+n}$.

94. 1) $\frac{2c-3d}{3x}+\frac{c+3d}{3x}$; 2) $\frac{2m+b}{2a^2}+\frac{4m-2b}{2a^2}$; 3) $\frac{a+b}{2x}-\frac{a-b}{2x}$; 4) $\frac{6a-b}{n^3}-\frac{5a-b}{n^3}$.

Turli maxrajli kasrlarning yig'indisini (ayirmasini) toping **(95–99)**:

95. 1) $\frac{7}{9}+\frac{7}{2}$; 2) $\frac{7}{11}+\frac{2}{7}$; 3) $\frac{2}{3a}+\frac{1}{a}$; 4) $\frac{c}{6a}+\frac{d}{2a}$.

96. 1) $\frac{3}{b}-\frac{2}{9b}$; 2) $\frac{a}{3}-\frac{b}{9d}$; 3) $\frac{3}{a}+\frac{b}{4}$; 4) $8+\frac{2}{a}$.

97. 1) $5-\frac{1}{m}+\frac{2}{m^2}$; 2) $4+\frac{1}{c}-\frac{2}{c^2}$; 3) $b+\frac{2c}{d}-\frac{3c^2}{d^2}$; 4) $\frac{a}{b}-k+\frac{2a^2}{b^2}$.

98. 1) $\frac{3a}{4b^3d}+\frac{6c}{5bd^3}$; 2) $\frac{3a}{4b^4}-\frac{5cb}{7a^3}$; 3) $\frac{3}{2y^3}+\frac{1}{4x^2y}-\frac{5}{8xy^2}$; 4) $\frac{4}{9x^2y}-\frac{2}{5xy^2}+\frac{11}{14x^2y^2}$.

99. 1) $\frac{2a}{p^2}+\frac{3b}{q^2}+\frac{k}{r^2}$; 2) $\frac{a}{p}+\frac{2b}{q}+\frac{c}{2k}$; 3) $\frac{m}{p^2}+\frac{m}{2pq}+\frac{m}{pq^2}$; 4) $\frac{m}{bc}-\frac{m}{ca}+\frac{m}{ab}$.

Algebraik kasrlarni qo'shing va ayiring **(100–102)**:

100. 1) $\frac{2x}{3(a+b)}+\frac{x}{a+b}$; 2) $\frac{x}{2(x-1)}-\frac{4x}{x-1}$; 3) $\frac{a^2}{a-1}+\frac{4a^2}{5(a-1)}$; 4) $\frac{2x}{3(y-3)}+\frac{3x}{2(y-3)}$.

101. 1) $\frac{3}{x-1}+\frac{4}{3x-3}$; 2) $\frac{a}{a+b}-\frac{2a}{3a+3b}$; 3) $\frac{4}{3b+3}+\frac{5}{6b+6}$; 4) $\frac{3x}{2x+3y}-\frac{5x}{4x+4y}$.

102. 1) $\frac{3}{a^2+a}+\frac{b}{ab+b}$; 2) $\frac{3}{ax+by}+\frac{2a}{bx+by}$; 3) $\frac{b+x}{b^2+ab}-\frac{x-a}{ab+a^2}$; 4) $\frac{x+b}{a^2-ab}-\frac{x+a}{ab-a^2}$.

Amallarni bajaring **(103–107)**:

103. 1) $\frac{7b}{2b-3}+\frac{b+1}{2b+1}-\frac{2b+1}{2b-1}$; 2) $\frac{1}{a^2-81}+\frac{9a+1}{27-3a}+\frac{9a-1}{18a+2}$;

3) $\frac{1}{2a-3}+\frac{2}{2a+3}-\frac{4a}{(2a+3)^2}$; 4) $\frac{3}{2a-3}+\frac{2}{2a+3}-\frac{4a-6}{4a^2-9}$.

104. 1) $\frac{3}{a+3}-\frac{a-2}{a^2-6a+9}-\frac{a^2-9a}{a^3+27}$; 2) $\frac{1}{3a+2b}-\frac{1}{9a^2+6ab+4b^2}-\frac{6ab}{27a^3-8b^3}$.

105. 1) $\frac{a^2-(b+c)^2}{(a-c)^2-b^2}+\frac{b^2+(a-c)^2}{(a-b)^2-c^2}+\frac{c^2-(a+b)^2}{(b-c)^2-a^2}$;

2) $\frac{1}{(a+b)(a+c)}+\frac{1}{(b+a)(b+c)}+\frac{1}{(c+a)(c+b)}$.

106. 1) $\frac{8}{2a-3b}+\frac{5}{2a+3b}-\frac{48b}{4a^2-9b^2}$; 2) $\frac{7}{4a-3b}+\frac{6}{4a+3b}-\frac{40a}{16a^2-9b^2}$.

107. 1) $\frac{1}{4x-3}-\frac{6}{16x^2-9}-\frac{1}{4x+3}$; 2) $\frac{7}{3x-4}-\frac{6}{3x+4}-\frac{3x}{9x^2-16}$.

108. Bir ishni usta 6 soatda, shogirdi esa o'sha ishni 8 soatda bajaradi. Agar ular birgalikda ishlasa, ishni necha soatda bajaradilar?

109. Ikkita ishchi birgalikda ishlab vazifani 8 soatda bajaradi. Birinchi ishchi ayni shu vazifani bir o'zi ishlab 10 soatda bajaradi. Ikkinchi ishchining bir o'zi shu vazifani necha soatda bajaradi?

110.* 1) $\frac{3a^2}{8a^3-1}+\frac{2a+1}{4a^2+2a+1}$ ifodani soddalashtiring va uning $a=2$ bo'lgandagi son qiymatini toping.

2) $\frac{2a}{a^3-27}+\frac{1}{a^2+3a+9}$ ifodani soddalashtiring va uning $a=2$ bo'lgandagi son qiymatini toping.

111.* 1) $\frac{a+ab+c+cb}{1+3b+3b^2+b^3}$ ifodani soddalashtiring va uning $a=\frac{3}{4}$, $b=\frac{1}{2}$, $c=-\frac{1}{4}$ bo'lgandagi son qiymatini toping.

2) $\frac{ac-ax+bc-bx}{ay+by+2ax+2bx}$ ifodani soddalashtiring va uning $x=\frac{1}{2}$, $y=-\frac{1}{3}$, $c=-\frac{1}{4}$ bo'lgandagi son qiymatini toping.

4-§. Algebraik kasrlarni ko'paytirish va bo'lish

Algebraik kasrlarni ko'paytirish va bo'lish ham oddiy kasrlarni ko'paytirish va bo'lish qoidalari bo'yicha bajariladi:

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd};\quad \frac{a}{b}:\frac{c}{d}=\frac{ad}{bc}.$$

Bir nechta masala ko'ramiz.

1-masala. Kasrlarni ko‘paytiring:

$$\frac{2}{3xy},\ \frac{6x^3y^2}{5z},\ \frac{15z^2}{3x^3}.$$

□ $\frac{2}{3xy}\cdot\frac{6x^3y^2}{5z}\cdot\frac{15z^2}{3x^3}=\frac{2\cdot 6x^3y^2\cdot 15z^2}{3xy\cdot 5z\cdot 3x^3}=\frac{4yz}{x}$. ■

2-masala. Kasrlarni ko‘paytiring:

$$\frac{a+b}{a^2-ab};\ \frac{b^2-ab}{(a+b)^2}.$$

□ $\frac{a+b}{a^2-ab}\cdot\frac{b^2-ab}{(a+b)^2}=\frac{(a+b)\cdot(b^2-ab)}{(a^2-ab)\cdot(a+b)^2}=\frac{(a+b)\cdot b\cdot(b-a)}{a\cdot(a-b)\cdot(a+b)^2}=$

$=-\frac{b\cdot(a-b)}{a\cdot(a-b)\cdot(a+b)}=-\frac{b}{a+b}$. ■

3-masala. $\frac{a-b}{4a^2b^3}$ va $\frac{a^2-b^2}{12ab^2}$ kasrlarni bo‘ling.

□ Kasrlarni bo‘lish deyilganda birinchi kasrni ikkinchi kasrga bo‘lish tushuniladi.

Berilishi bo‘yicha $\frac{a-b}{4a^2b^3}$ – birinchi, $\frac{a^2-b^2}{12ab^2}$ – ikkinchi kasrdir. Endi bo‘lishni bajaramiz:

$$\frac{a-b}{4a^2b^3}:\frac{a^2-b^2}{12ab^2}=\frac{(a-b)\cdot 12ab^2}{4\cdot a^2b^3(a^2-b^2)}=\frac{3ab^2\cdot(a-b)}{a^2b^3\cdot(a+b)(a-b)}=\frac{3}{ab\cdot(a+b)}.$$ ■

Agar birinchi va ikkinchi ko‘paytuvchi bir xil bo‘lsa, u holda ko‘paytirishni bajarish uchun birinchi ko‘paytuvchini kvadratga oshirish yetarli bo‘ladi.
Masalan: $a^3a^3=(a^3)^2=a^6$; $(a+b)(a+b)=(a+b)^2$.
Agar bo‘linuvchi va bo‘luvchi bir xil darajada bo‘lsa, ularning nisbatini shu darajaga ko‘tarib qo‘yish mumkin.
Algebraik kasrni darajaga ko‘tarishda ushbu formuladan foydalaniladi:

$$\boxed{\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}}$$

Masalan: $\left(\frac{4a^3}{b^2}\right)^2=\frac{16a^6}{b^4}$; $\left(\frac{a+b}{5c}\right)^4=\frac{(a+b)^4}{625c^4}$.

?

112. 1. Kasrlarni ko‘paytirish va bo‘lish qoidalarini aytib bering.
2. Kasrlarni darajaga ko‘tarish qanday formula yordamida bajariladi?

Kasrlarni ko‘paytiring **(113–118)**:

113. 1) $\frac{95}{16}\cdot\frac{64}{19}$; 2) $\frac{169}{121}\cdot\frac{11}{26}$; 3) $\frac{144}{24}\cdot\frac{49}{14}$; 4) $25\cdot\frac{6}{125}$.

114. 1) $\frac{5}{52}\cdot 13$; 2) $17\cdot\frac{3}{68}$; 3) $\frac{8}{25}\cdot 625$; 4) $729\cdot\frac{3}{27}$.

115. 1) $\frac{ab^3}{c}\cdot\frac{c^2}{b^4}$; 2) $\frac{m^2b}{k^2}\cdot\frac{k^3}{m^3b^2}$; 3) $\frac{3a}{4b}\cdot\frac{12c}{a}$; 4) $\frac{4m}{9n}\cdot\frac{36k}{20d}$.

116. 1) $\left(\frac{3a}{4b}\right)^2\cdot\frac{8b^2}{27a^3}$; 2) $\left(\frac{3a^2}{2b}\right)^3\cdot\frac{8b^3}{21a^4}$; 3) $\left(\frac{ak}{bc}\right)^3\cdot abc$; 4) $a^2\cdot b\cdot c^2\left(\frac{ab}{cd}\right)^2$.

117. 1) $\frac{4a^2b}{3c}\cdot\frac{27c^3}{5a^3b}$; 2) $\frac{4x^2y}{5ab^2}\cdot 5a^2b^2$; 3) $\frac{3-x}{a-b}\cdot\frac{a-b}{3-x}$; 4) $\frac{x+y}{4a}\cdot\frac{2b}{x+y}$.

118. 1) $\frac{a-1}{b}\cdot\frac{4b^2}{a^2-1}$; 2) $\frac{1+a}{4b^2}\cdot\frac{b^3}{1-a^2}$; 3) $\frac{a^2-b^2}{9b^2}\cdot\frac{3b}{a+b}$; 4) $\frac{5m}{m^2-n^2}\cdot\frac{m+n}{20m^3}$.

Kasrlarni bo‘ling **(119–122)**:

119. 1) $\frac{3}{4}:\frac{3}{5}$; 2) $\frac{7}{11}:\frac{2}{11}$; 3) $\frac{10}{9}:\frac{1}{3}$; 4) $\frac{15}{13}:\frac{30}{39}$.

120. 1) $\frac{a}{7}:\frac{1}{3}$; 2) $\frac{5}{c}:\frac{m}{13}$; 3) $\frac{a}{2}:\frac{5}{6}$; 4) $\frac{8}{35}:\frac{b}{14}$.

121. 1) $\frac{a}{b}:\frac{a}{b}$; 2) $\frac{2a}{5b}:\frac{a}{3b}$; 3) $\frac{3a}{2b}:\frac{6a^2}{bc}$; 4) $\frac{5m}{n^2}:\frac{15m^3}{n}$.

122. 1) $\frac{ab^2}{c^2}:\frac{a^2}{c}$; 2) $\frac{mn}{k}:\frac{m^3n^3}{k^3}$; 3) $\frac{3a}{4b}:\frac{9c}{20b}$; 4) $\frac{8m}{9n}:\frac{4k}{18d}$.

Ko‘rsatilgan amallarni bajaring **(123–131)**:

123. 1) $\frac{3bc}{5a^2b}\cdot\frac{10a^3b^2}{9c^2}$; 2) $\frac{6x}{7xy}\cdot\frac{14x^2}{18xy}$; 3) $\frac{35xy}{8y^2}\cdot\frac{16x^2y^3}{21x^4y}$; 4) $\frac{4ab^2}{9b^3}\cdot\frac{45a^4b^3}{32a^5b}$.

124. 1) $\frac{3-6x}{2x^2+4x+8}\cdot\frac{2x+1}{x^2+4x+4}\cdot\frac{8-x^3}{4x^2-1}$; 2) $\frac{a^2+ab}{5a-a^2+b^2-5b}\cdot\frac{a^2-b^2+25-10a}{a^2-b^2}$.

125. 1) $\frac{28a^2b}{33x^2y^3}:\frac{14ab^2}{11x^4y^4}$; 2) $\frac{45cd^2}{27ab^3}:\frac{15c^2d}{18a^2b^2}$;

3) $\frac{26x^2y^3}{35a^2b^4}:\frac{39xy^4}{28a^4b^2}$; 4) $\frac{24a^3b^4}{25c^4d^2}:\frac{36a^4b^2}{35c^3d^4}$.

126. 1) $\dfrac{27a^3-64b^3}{b^2-4}:\dfrac{9a^2+12ab+16b^2}{b^2+4b+4}$;

2) $\dfrac{2a^2+6ac-ab-3bc}{2ab-4a^2+bc-2ac}:\dfrac{2ac+ab+3bc+6c^2}{2ab+bc-4ac-2c^2}$.

127. 1) $\dfrac{8ab}{a^2-4b^2}:\left(\dfrac{a+2b}{a-2b}+1\right)$;

2) $\left(\dfrac{3a+1}{3a-1}-\dfrac{1}{9a^2-1}\right)\cdot\dfrac{3a+1}{3a+1}$;

3) $\left(\dfrac{2a-1}{2a+1}-\dfrac{4a+1}{4a^2-1}\right)\cdot\dfrac{2a+1}{2a}$;

4) $\left(\dfrac{3a+b}{3a-1}-1\right):\dfrac{12ab}{9a^2-b^2}$.

128.* 1) $\dfrac{36a^2-1}{a^2-25}\cdot\dfrac{(a+b)^2}{a^2-25}:\dfrac{6a+1}{(a-5)^2}$;

2) $\dfrac{3ab-4b-9a+12}{9a+12-3ab-4b}:\dfrac{9a-12-3ab+4b}{3ab+4b-9a-12}$.

129.* 1) $\left(\dfrac{2-3a}{2+3a}-\dfrac{3a+2}{3a-2}\right):\left(\dfrac{2+3a}{2-3a}+\dfrac{3a-2}{3a+2}\right)$;

2) $\left(\dfrac{2x+3y}{2x-3y}-\dfrac{2x-3y}{2x+3y}\right):\left(\dfrac{2x-3y}{2x+3y}+\dfrac{2x+3y}{2x-3y}\right)$.

130. 1) $\dfrac{x^2}{x^2-1}+\dfrac{1}{x+1}:\left(\dfrac{1}{2-x}+\dfrac{2}{x^2-2x}\right)$, bunda $x=1{,}7$;

2) $\dfrac{a^3b+2a^2b-3ab}{a^3+5a^2+6a}:\dfrac{a^2-1}{a^2+3a+2}$, bunda $a=19{,}8$, $b=23{,}05$.

131. 1) $\dfrac{5a}{3(4-a)}+\dfrac{a+4}{8-3a}\cdot\left(\dfrac{a-1}{a+4}-\dfrac{a-3}{a-4}\right)$, bunda $a=2{,}8$;

2) $\dfrac{9a^2-24ab+16b^2-25}{3a-4b-5}$, bunda $a=\dfrac{1}{9}$, $b=2\dfrac{1}{3}$.

132.* 1) a ning nechta butun qiymatida $\dfrac{a^4-16}{a^3-4a}:\dfrac{a^3+4a}{a+7a^2}$ ifodaning qiymati butun son bo‘ladi?

2) n ning qanday butun qiymatlarida $\dfrac{4n-5}{2n-1}$ kasr natural son bo‘ladi?

133.* 1) n ning qanday butun qiymatlarida $\dfrac{2n^2-n+1}{n+1}$ kasr butun son bo‘ladi?

2) n ning qanday natural qiymatlarida $\dfrac{2n^2-3n+2}{2n-1}$ kasr butun son bo‘ladi?

134. 1) n ning qanday butun qiymatlarida $\frac{n^2-7n+6}{n^2-1}$ kasr butun son bo'ladi?

2) n ning qanday natural qiymatlarida $\frac{2n^2+5n-7}{2n+1}$ kasr butun son bo'ladi?

Amallarni bajaring **(135–136)**:

135. 1) $\frac{3a-5}{16+24a+9a^2}\cdot\frac{(3a+4)^2}{9a^2-25}$;

2) $\frac{16x^2y^2-49}{16x^2y^2+8axy+a^2}\cdot\frac{4xy+a}{4xy-7}$;

3) $\frac{25a^2-40a+16}{5a+3}:\frac{(5a-4)^2}{25a^2-9}$;

4) $\frac{4x^2-12xy+9y^2}{4x+y}:\frac{2x-3y}{16x^2-y2}$.

136.* 1) $\frac{1-4a^2}{(1+2ax)^2-(2a+x)^2}\cdot\frac{x+x^2}{1-x}$;

2) $\frac{3x^2+3xy}{4xy+6ay}\cdot\left(\frac{x}{ax+ay}+\frac{3}{2x+2y}\right)$;

3) $\left(\frac{4a^2+1}{4a-1}-a\right)\cdot\left(\frac{3-2a}{2a+2}-1\right)$;

4) $\frac{1-a^2}{1+3b}\cdot\frac{1-9b^2}{a+a^2}\cdot\left(1+\frac{a}{1-a}\right)$.

Masalalar yechish

Kasrlarni ko'paytiring **(137–142)**:

137. 1) $\frac{102}{21}\cdot\frac{49}{17}$; 2) $\frac{144}{36}\cdot\frac{39}{13}$; 3) $\frac{225}{45}\cdot\frac{32}{256}$; 4) $\frac{441}{21}\cdot\frac{625}{25}$.

138. 1) $\frac{7}{65}\cdot 13$; 2) $16\cdot\frac{3}{96}$; 3) $\frac{7}{125}\cdot 625$; 4) $961\cdot\frac{5}{31}$.

139. 1) $\frac{a^2b^3}{c^2}\cdot\frac{c}{ab^2}$; 2) $\frac{m^3b^2}{k^3}\cdot\frac{k^2}{m^2b}$; 3) $\frac{a}{12b}\cdot\frac{9c}{4a}$; 4) $\frac{20m}{36k}\cdot\frac{9k}{4m}$.

140. 1) $\left(\frac{4a}{3b}\right)^2\cdot\frac{27b^2}{8a^3}$; 2) $\left(\frac{2a^2}{3b}\right)\cdot\frac{21b^3}{8a^4}$; 3) $\left(\frac{a^2k}{bc^2}\right)\cdot abc$; 4) $ab^2c^2\cdot\left(\frac{d}{abc}\right)^2$.

141. 1) $\frac{5a^3b}{27c^3}\cdot\frac{3c}{4a^2b}$; 2) $\frac{4xy^2}{5a^2b}\cdot 10ab^2$; 3) $\frac{x-2}{a-b}\cdot\frac{9b^2}{a^2-b^2}$; 4) $\frac{m^2-n^2}{4m}\cdot\frac{16m^3}{m+n}$.

142. 1) $\frac{a^2-1}{4b^2}\cdot\frac{b}{a-1}$; 2) $\frac{1-a^2}{a+1}\cdot\frac{4b^2}{b^3}$; 3) $\frac{a+b}{3b}\cdot\frac{9b^2}{a^2-b^2}$; 4) $\frac{m^2-n^2}{4m}\cdot\frac{16m^3}{m+n}$.

Kasrlarni bo'ling **(143–146)**:

143. 1) $\frac{4}{3}:\frac{5}{3}$; 2) $\frac{11}{7}:\frac{11}{2}$; 3) $\frac{9}{10}:\frac{2}{5}$; 4) $\frac{14}{13}:\frac{28}{39}$.

144. 1) $\frac{7}{a}:\frac{3}{a}$; 2) $\frac{13}{c}:\frac{m}{11}$; 3) $\frac{a}{3}:\frac{5}{6}$; 4) $\frac{4}{35}:\frac{b}{12}$.

145. 1) $\frac{b}{a}:\frac{b}{a}$; 2) $\frac{3a}{4b}:\frac{a}{3b}$; 3) $\frac{3a}{4b}:\frac{6a}{bc}$; 4) $\frac{4m}{3n^2}:\frac{12m^3}{n}$.

146. 1) $\frac{a^2b}{c}:\frac{a}{c^2}$; 2) $\frac{m^3n^3}{k^3}:\frac{mn}{k}$; 3) $\frac{9c}{16b}:\frac{3a}{24b}$; 4) $\frac{5k}{18d}:\frac{8m}{9n}$.

Ko‘rsatilgan amallarni bajaring **(147–148)**:

147. 1) $\frac{a^2-2ab+b^2}{a^2-ab+b^2}:\frac{8a-8b}{a^3+b^3}$;

2) $\frac{a^2+2ab+b^2}{a^2+ab+b^2}\cdot\frac{a^3-b^3}{7a+7b}$;

3) $\frac{n^3-m^3}{n^2-m^2}:\frac{n^2+nm+m^2}{n^2+2nm+m^2}$;

4) $\frac{m^2+2mn+n^2}{p^3+c^3}\cdot\frac{p+c}{2m+2n}$.

148*. 1) $\frac{64x^2y^2}{x^2-4}\cdot\frac{(x+2)^2}{x^2-4}\cdot\frac{(x-2)^2}{8xy+1}$;

2) $\frac{x-6}{x^2+6x+9}\cdot\frac{x^2+4x+4}{(x^2+2)(x-2)}\cdot\frac{x^3-9x}{(x-6)(x+2)}$;

3) $\frac{bm^2-bn^2}{m^2+2mn+n^2}:\frac{bm^2-2bmn+bn^2}{3m+3n}$;

4) $\frac{ab-4b-2a+8}{2a+8-ab-4b}:\frac{2a-8-ab+4b}{ab+4b-2a-8}$.

Soddalashtiring **(149–170)**:

149. $\left(\frac{5m}{m+3}-\frac{14m}{m^2+6m+9}\right):\frac{5m+1}{m^2-9}+\frac{3(m-3)}{m+3}$.

□ Qavs ichidagi kasrlarni umumiy maxrajga keltiramiz:

$$\frac{5m}{m+3}-\frac{14m}{m^2+6m+9}=\frac{5m}{m+3}-\frac{14m}{(m+3)^2}=\frac{5m^2+15m-14m}{(m+3)^2}=\frac{5m^2+m}{(m+3)^2}=\frac{m(5m+1)}{(m+3)^2}.$$

Natijani $\frac{5m+1}{m^2-9}$ ga bo‘lishni bajaramiz:

$$\frac{m(5m+1)}{(m+3)^2}:\frac{5m+1}{(m+3)(m-3)}=\frac{m(5m+1)}{(m+3)^2}\cdot\frac{(m+3)(m-3)}{(5m+1)}=\frac{m(m-3)}{m+3}.$$

Endi kasrlarni qo‘shamiz, ularning maxrajlari bir xil:

$$\frac{m(m-3)}{m+3}+\frac{3(m-3)}{m+3}=\frac{(m-3)(m+3)}{(m+3)}=m-3.$$

Javob: $m-3$. ■

150. 1) $\left(m^2-\frac{1+m^4}{m^2-1}\right):\frac{m^2+1}{m+1}$; 2) $\left(\frac{8xy+3}{8xy+1}-\frac{8xy+1}{8xy+3}\right):\frac{12xy+3}{32xy+12}$.

151. 1) $\left(b^2-\frac{1+b^4}{b^2+1}\right):\frac{1-b}{1+b^2}$; 2) $\left(\frac{1}{(a+b)^2}+\frac{1}{(a-b)^2}\right):\left(\frac{a}{a+b}+\frac{b}{a-b}\right)$.

152. $\left(\frac{3a}{a-4}+\frac{10a}{a^2-8a+16}\right):\frac{3a-2}{a^2-16}-\frac{4(a+4)}{a-4}$.

153. $\left(\frac{2x}{x-5}+\frac{x}{x^2-10x+25}\right):\frac{2x-9}{x^2-25}-\frac{5(x+5)}{x-5}$.

154. $\left(\frac{4a}{4-a^2}-\frac{a-2}{4+2a}\right)\cdot\frac{4}{a+2}-\frac{a}{2-a}$.

155. 1) $\frac{x^3-8}{x^2+2x+4}-\frac{x^2-4}{x-2}$; 2) $\frac{27x^3-64}{9x^2+12x+16}-\frac{9x^2-16}{3x-4}$.

156. 1) $\frac{x^3+8}{x^2-2x+4}$; 2) $\frac{27x^3+64}{9x^2-12x+16}$.

157. $\frac{x}{1-x}-\frac{1-x^2}{1+x^2}\cdot\left(\frac{1}{(x-1)^2}-\frac{x}{1-x^2}\right)$.

158. $\frac{a^2+ab+b^2}{a^3-b^3}-\frac{a^2-ab+b^2}{a^3+b^3}$.

159. 1) $\frac{x^3+y^3}{x^2-xy+y^2}-\frac{x^2-y^2}{x+y}$; 2) $\frac{8x^3+27y^3}{4x^2-6xy+9y^2}-\frac{4x^2-9y^2}{2x+3y}$.

160. $\frac{5x+6}{x^2-4}-\frac{x}{x^2-4}:\frac{x}{x-2}-\frac{x+2}{x-2}$.

161. $\left(\frac{1}{a(a+1)}+\frac{1}{(a+1)(a+2)}\right)\cdot\frac{a^2+2a}{8}$.

162. 1) $\frac{x^3-2x^2}{3x+3}:\frac{x^2-4}{3x^2+6x+3}$; 2) $\left(\frac{3x+1}{3x-1}-\frac{1}{9x^2-1}\right)\cdot\frac{3x+1}{3x+2}$.

163. $\left(\frac{1}{m^2-m}-\frac{1}{m-1}\right)\cdot\frac{m}{m+2}+\frac{m}{m^2-4}$.

164*. $(a^3-3a^2b+3ab^2-b^3)\cdot(a+b):\left(\dfrac{a^3+b^3}{a+b}-ab\right)$.

165. $\dfrac{x^3+y^3}{x+y}:(x^2-y^2)+\dfrac{24}{x+y}-\dfrac{xy}{x^2-y^2}$.

166. $\dfrac{a^3-b^3}{a^2-b^2}\div\dfrac{a^2+ab+b^2}{a^3+3a^2b+3ab^2+b^3}$.

167. $\left(2a+\dfrac{2ab}{a-b}\right)\cdot\left(\dfrac{ab}{a+b}-a\right):\dfrac{4,5a^2}{a^2-b^2}$.

168. $\dfrac{a^2}{a^2-1}+\dfrac{1}{a+1}:\left(\dfrac{1}{2-a}+\dfrac{2}{a^2-2a}\right)$.

169. $\left(\dfrac{2}{1-x^2}-\dfrac{2}{(x-1)^2}\right)\cdot(1-x)^2-\dfrac{4}{1+x}$.

170*. $\dfrac{abc}{bc+ac-ab}-\left(\dfrac{a-1}{a}+\dfrac{b-1}{b}-\dfrac{c-1}{c}\right):\left(\dfrac{1}{a}+\dfrac{1}{b}-\dfrac{1}{c}\right)$.

5-§. Algebraik kasrlar ustida birgalikda bajariladigan amallar

Algebraik kasrlar ustida birgalikda bajariladigan amallar: umumiy ko‘paytuvchini qavsdan tashqariga chiqarish, qisqa ko‘paytirish formulalari, guruhlash usullari, kasrlarni qisqartirish, kasrlarni umumiy maxrajga keltirish, kasrlarni qo‘shish va ayirish, ko‘paytirish va bo‘lish. Mazkur amallar turli tartibda birgalikda bajarilishi mumkin. Misollarda ko‘ramiz.

1-masala. Ifodani soddalashtiring:

$$\left(\frac{4a}{4-a^2}-\frac{a-2}{4+2a}\right)\cdot\frac{4}{a+2}-\frac{a}{2-a}.$$

□ Avval qavs ichidagi ifodani soddalashtiraylik:

$$\frac{4a}{4-a^2}-\frac{a-2}{4+2a}=\frac{4a}{(2-a)(2+a)}+\frac{2-a}{2(2+a)}=\frac{8a+(2-a)^2}{2(2-a)(2+a)}=$$

$$\frac{8a+4-4a+a^2}{2(2-a)(2+a)}=\frac{4+4a+a^2}{2(2-a)(2+a)}=\frac{(2+a)^2}{2(2-a)(2+a)}=\frac{2+a}{2(2-a)}.$$

Endi ko‘paytirish, so‘ngra ayirish amallarini bajaramiz:

$$\frac{2+a}{2(2-a)}\cdot\frac{4}{2+a}-\frac{a}{2-a}=\frac{2}{2-a}-\frac{a}{2-a}=\frac{2-a}{2-a}=1.$$ ■

2-masala. Amallarni bajaring: $\left(\dfrac{2x}{x-5}+\dfrac{x}{x^2-10x+25}\right):\dfrac{2x-9}{x^2-25}-\dfrac{5(x+5)}{x-5}$.

□ Bu holda avval qavs ichidagi ifodani soddalashtiramiz:

$$\frac{2x}{x-5}+\frac{x}{x^2-10x+25}=\frac{2x}{x-5}+\frac{x}{(x-5)^2}=\frac{2x(x-5)+x}{(x-5)^2}=\frac{x(2x-9)}{(x-5)^2}.$$

Endi bo'lishni va ayirishni bajaramiz:

$$\frac{x(2x-9)}{(x-5)^2}:\frac{2x-9}{x^2-25}=\frac{x(x+5)}{x-5};\quad \frac{x(x+5)}{x-5}-\frac{5(x+5)}{x-5}=\frac{(x+5)(x-5)}{x-5}=x+5. \blacksquare$$

3-masala. Ota va o'g'il S gektar yerdan hosil yig'ib olishi kerak. Ota bir o'zi hosilni a kunda, o'g'li esa b kunda yig'ib olishi mumkin. Agar ota va o'g'il birga ishlasa, hosilni necha kunda yig'ib oladi?

□ Ota bir kunda $\frac{S}{a}$ ga, o'g'li esa $\frac{S}{b}$ ga yerdan hosil yig'a oladi. Agar ular birgalikda ishlasa, bir kunda $\left(\frac{S}{a}+\frac{S}{b}\right)$ ga yerdan hosil yig'ib olishadi. Ular hosilni t kunda yig'ib olishadi desak, unda ushbu $\left(\frac{S}{a}+\frac{S}{b}\right)\cdot t=S$ tenglik o'rinli bo'ladi. Bundan $\left(\frac{1}{a}+\frac{1}{b}\right)t=1$, $\frac{a+b}{ab}\cdot t=1$, $t=\frac{ab}{a+b}$ (kun). ■

?

171. 1. Algebraik kasrlar ustida birgalikda qanday amallar bajarish mumkin?
2. Fikringizni misollarda tushuntiring.

Ko'rsatilgan amallarni bajaring **(172–175)**:

172. 1) $\left(\frac{a}{3}+\frac{a}{2}\right)\cdot\frac{1}{a^2}$; 2) $\frac{a^2}{5}\left(\frac{3}{a^2}+\frac{3}{a}\right)$; 3) $\frac{a+b}{a-b}\left(\frac{a}{3}-\frac{b}{3}\right)$;

4) $\frac{ab}{a+b}\left(\frac{1}{b}+\frac{1}{a}\right)$; 5) $\left(2+\frac{2}{a}\right)$; 6) $b:\left(b+\frac{2}{b}\right)$.

173. 1) $\left(1-\frac{1}{a}\right):\left(1+\frac{1}{a}\right)$; 2) $\left(a-\frac{a}{b}\right):\left(a+\frac{a}{b}\right)$;

3) $\left(\frac{a}{b}+\frac{b}{a}+2\right):\left(\frac{1}{b}+\frac{1}{a}\right)$; 4) $\left(\frac{m}{n}+\frac{n}{m}-2\right):\left(1+\frac{m+n}{m-n}\right)$.

174. 1) $\left(1+\frac{a-b}{a+b}\right)\left(2-\frac{2b}{a-b}\right)$; 2) $\left(1+\frac{a+b}{a-b}\right)\left(2-\frac{2a}{a+b}\right)$;

3) $\left(\frac{4}{a-b}-\frac{3}{a+b}\right)\cdot\frac{a-b}{a+7b}$; 4) $\left(\frac{4}{c}+\frac{4}{c+d}\right)\cdot\frac{c}{12(2c+d)}$.

175. 1) $\frac{a^2+ab}{a^2-b^2}:\left(\frac{a}{a-b}-\frac{b}{a+b}\right)$; 2) $\frac{ab+b^2}{a^2-b^2}:\left(\frac{a}{a+b}-\frac{b}{a-b}\right)$;

3) $\left(\frac{c-d}{c}-\frac{2c}{c+d}\right)\cdot\frac{d-c}{c^2+d^2}$; 4) $\left(\frac{2c}{c-d}-\frac{d+c}{c}\right)\cdot\frac{c+d}{c^2+d^2}$.

Amallarni bajaring **(176–180)**:

176. 1) $\frac{3a-b}{3ab}\cdot\left(\frac{1}{3a}+\frac{1}{b}\right):\left[\frac{9a^2+b^2}{3ab}\cdot\left(\frac{1}{3a}-\frac{1}{b}\right)\right]$;

2) $\frac{a-2b}{2ab}\cdot\left(\frac{1}{a}+\frac{1}{2b}\right):\left[\frac{a^2+4b^2}{2ab}\cdot\left(\frac{1}{a}-\frac{1}{2b}\right)\right]$.

177. 1) $\frac{a^2+3ab}{6b}:\left(a^2-9b^2\right)\left[\frac{\left(a+3b^2\right)}{12ab}-1\right]$; 2) $\frac{9a^2+3ax}{2x}:\left(9a^2-x^2\right)\left[\frac{\left(3a+x^2\right)}{12x}-1\right]$.

178. 1) $\left(2a+\frac{18ab}{a-9b}\right)\cdot\left(\frac{9ab}{a+9b}-a\right):\frac{4.5a^2}{a^2-81b^2}$;

2) $\left(\frac{4(a+b)^2}{ab}-16\right)\cdot\left(\frac{(a+b)^2-ab}{ab}\right):\frac{a^3-b^3}{ab}$.

179. 1) $\left(\frac{a}{a-a^2}-1\right):\left(a-\frac{1-2a^2}{1-a}+1\right)$; 2) $\left(a-\frac{a}{a-a^2}\right):\left(\frac{a}{a-1}+\frac{1}{a^2-2a+1}\right)$.

180.* 1) $\left(\frac{20b}{4-25b^2}-\frac{5b-2}{4+10b}\right)\cdot\frac{4}{5b+2}-\frac{5b}{2-5b}$;

2) $\left(\frac{5a+4x}{5a}-\frac{4x-3y}{4x}\right)\cdot\frac{25a^2}{16x^2+15ay}:\frac{5a}{32x}$.

181. Avtobus soatiga v km tezlik bilan harakat qilib, s km yo'l bosib o'tdi. Agar avtobusning tezligi soatiga u km bo'lsa, shu vaqt ichida avtobus qancha yo'l bosib o'tadi?

182. Motorli qayiqning turg'un suvdagi tezligi soatiga v km, daryo oqimining tezligi esa v_1 km. Qayiq oqim bo'yicha harakat qilib, s km o'tdi. Motorli qayiq oqimga qarshi shu vaqt ichida qancha masofani bosib o'tgan?

183. (*Qadimgi masala.*) Ikki buyumdan birining 10 tasi bir dinor va ikkinchisining 15 tasi bir dinor. Bir dinorga ikkala buyumdan bir xil miqdorda necha donadan sotib olish mumkin?

Kasrlarni umumiy maxrajga keltiring **(184–185)**:

184. 1) $\frac{3a}{a^3-64}$, $\frac{a-4}{a^2+4a+16}$ va $\frac{1}{a-4}$; 2) $\frac{3}{x-2}$, $\frac{x+1}{x^3-8}$ va $\frac{x+2}{x^2+2x+4}$.

185. 1) $\frac{a}{a^2-16}, \frac{a-4}{x^2-4a+16}$ va $\frac{2}{a-4}$; 2) $\frac{5}{x+2}, \frac{x-1}{x^3+8}$ va $\frac{x-2}{x^2-2x+4}$.

Masalalar yechish

Amallarni bajaring **(186–187)**:

186. 1) $\frac{a+4}{6}+\frac{6+a}{12}+\frac{a-4}{2}$;

2) $\frac{b+7}{4}+\frac{6b-1}{3}+\frac{2b-1}{8}$;

3) $\frac{a+4}{42}-\frac{a-5}{14}+\frac{a-6}{6}$;

4) $\frac{b}{39}-\frac{3b-1}{13}+\frac{2b+1}{3}$.

187. 1) $\frac{y}{n-2}-\frac{z}{2-n}$;

2) $\frac{p-2q}{3p-q}-\frac{4q-p}{q-3p}$;

3) $\frac{3m}{3-4n}-1+\frac{4m-3}{4n-3}$;

4) $7-\frac{2a}{3-2b}-\frac{2(a-9)}{2b-3}\cdot\left(\frac{3-14b}{3-2b}\right)$.

Amallarni bajaring **(188–189)**:

188. 1) $\frac{a^2-2ab+b^2}{a^2-ab+b^2}\cdot\frac{a^3+b^3}{7a-7b}$;

2) $\frac{n^3-m^3}{n^2-m^2}\cdot\frac{n^2+2nm+m^2}{n^2+nm+m^2}$;

3) $\frac{a^2-2ab+b^2}{a^2+ab+b^2}:\frac{7a+7b}{a^3-b^3}$;

4) $\frac{m^2+2nm+n^2}{p^3+c^3}:\frac{2m+2n}{p+c}$.

□ $\frac{m^2+2mn+n^2}{p^3+c^3}:\frac{2m+2n}{p+c}=\frac{(m+n)^2}{(p+c)(p^2-pc+c^2)}\cdot\frac{p+c}{2(m+n)}=\frac{m+n}{2(p^2-pc+c^2)}$. ■

189*. 1) $\frac{ap^2-aq^2}{p^2+2pq+q^2}\cdot\frac{7p+7q}{ap^2+2apq+aq^2}$;

2) $\frac{ab-4b-2a+8}{2a+8-ab-4b}\cdot\frac{ab+4b-2a-8}{2a+8-ab+4b}$;

3) $\left(\frac{x+y}{x-y}+\frac{x-y}{x+y}\right):\left(\frac{x-y}{x+y}-\frac{x+y}{x-y}\right)$;

4) $\left(\frac{2-a}{2+a}-\frac{a+2}{a-2}\right):\left(\frac{2+a}{2-a}+\frac{a-2}{a+2}\right)$.

Amallarni bajaring **(190–194)**:

190. 1) $\left(\frac{2x}{3(a+b)}+\frac{x}{a+b}\right):\left(\frac{5x}{2(a+b)}-\frac{x}{a+b}\right)$;

2) $\left(\frac{a}{a+b}+\frac{2a}{3a+3b}\right):\left(\frac{3a}{2a+2b}-\frac{5a}{4a+4b}\right)$.

191. 1) $\left(\frac{b+x}{b^2+ab}+\frac{x-a}{ab+a^2}\right)\cdot\frac{ab}{2x}$;

2) $\left(\frac{b+x}{b^2+ab}+\frac{x-a}{ab+a^2}\right):\left(\frac{3ab}{2a+2b}-\frac{5ab}{4a+4b}\right)$.

192. 1) $\frac{1}{3x-2}-\frac{6x}{9x^2-4}+\frac{1}{3x+2}$; 2) $\frac{1}{3x-2}-\frac{4}{9x^2-4}+\frac{1}{3x+2}$.

193. 1) $\left(\frac{a}{2}-\frac{a}{3}\right)\cdot\frac{1}{a^2}$; 2) $\frac{a^2}{3}\cdot\left(\frac{2}{a}+\frac{2}{a^2}\right)$; 3) $\frac{a^2}{3}\cdot\left(\frac{2}{a}+\frac{2}{a^2}\right)$;

4) $\frac{ab}{a-b}\left(\frac{1}{b}+\frac{1}{a}\right)$; 5) $2:\left(3-\frac{4}{a}\right)$; 6) $a:\left(a-\frac{1}{a}\right)$.

194*. 1) $\left(1+\frac{1}{a}\right):\left(1-\frac{1}{a}\right):\frac{a+1}{a-1}$; 3) $\left(\frac{b}{a}+\frac{a}{b}+2\right):\left(\frac{1}{b}+\frac{1}{a}\right)\cdot\frac{1}{a+b}$;

2) $\left(a+\frac{a}{b}\right):\left(a-\frac{a}{b}\right):\frac{b+1}{b-1}$; 4) $\left(\frac{m}{n}+\frac{n}{m}-2\right)\cdot\left(1-\frac{m+n}{m-n}\right)\cdot\frac{m}{2(m-n)}$.

195. $M=\dfrac{\dfrac{1}{a}-\dfrac{1}{b+c}}{\dfrac{1}{a}+\dfrac{1}{b+c}}\cdot\left(1+\dfrac{b^2+c^2-a^2}{2bc}\right):\dfrac{b+c-a}{abc}$.

□ Avval qavs oldidagi kasrning ko'rinishini o'zgartiramiz:

$$\frac{\dfrac{1}{a}-\dfrac{1}{b+c}}{\dfrac{1}{a}+\dfrac{1}{b+c}}=\frac{b+c-a}{b+c+a};$$

Endi qavs ichidagi ifodani o'zgartiramiz:

$$1+\frac{b^2+c^2-a^2}{2bc}=\frac{2bc+b^2+c^2-a^2}{2bc}=\frac{(b+c)^2-a^2}{2bc}=\frac{(b+c-a)(b+c+a)}{2bc};$$

Oxirida hamma amallarni bajarib, M ni topamiz:

$$M=\frac{b+c-a}{(b+c+a)}\cdot\frac{(b+c-a)(b+c+a)}{2\cancel{bc}}\cdot\frac{a\cancel{bc}}{b+c-a}=\frac{(b+c-a)\cdot a}{2}.$$ ■

Soddalashtiring **(196 – 207)**:

196. $\dfrac{2\left(x^4+4x^2-12\right)+\left(x^4+11x^2+30\right)}{x^2+6}$.

197*. $\dfrac{\dfrac{a-b}{2a-b}-\dfrac{a^2+b^2+a}{2a^2+ab-b^2}}{\left(4b^4+4ab^2+a^2\right)\cdot\left(2b^2+a\right)}\cdot\left(b^2+b+ab+a\right)$.

198. $\dfrac{(2p-a)^2+2q^2-3pq}{\dfrac{2}{p}+a^2}:\dfrac{4p^2-3pq}{2+pq^2}$.

199. $\dfrac{3a^2+2ax-x^2}{(3x+a)(a+x)}-2+10\cdot\dfrac{ax-3x^2}{a^2-9x^2}$.

200*. $\left[\left(\dfrac{1}{a}+\dfrac{1}{b+c}\right):\left(\dfrac{1}{a}-\dfrac{1}{b+c}\right)\right]:\left(1+\dfrac{b^2+c^2-a^2}{2bc}\right)$.

201. $N=\left[\left(\dfrac{x^2}{y^3}+\dfrac{1}{x}\right):\left(\dfrac{x}{y^2}-\dfrac{1}{y}+\dfrac{1}{x}\right)\right]:\dfrac{(x-y)^2+4xy}{1+\dfrac{y}{x}}$.

□ Avval o‘rta qavs ichidagi ifodani hisoblaymiz:

$$\left(\frac{x^2}{y^2}+\frac{1}{x}\right):\left(\frac{x}{y^2}-\frac{1}{y}+\frac{1}{x}\right)=\frac{x^3+y^3}{xy^3}:\frac{x^2-xy+y^2}{xy^2}=\frac{(x+y)(x^2-xy+y^2)}{xy^3}\cdot\frac{xy^2}{x^2-xy+y^2}=\frac{x+y}{y}.$$

Endi bo‘luvchi (kasr) ni ko‘ramiz:

$$\frac{(x-y)^2+4xy}{1+\dfrac{y}{x}}=\frac{x(x^2-2xy+y^2+4xy)}{x+y}=\frac{x(x+y)^2}{x+y}=x(x+y).$$

Nihoyat, N ni hisoblaymiz:

$$N=\frac{x+y}{y}:x(x+y)=\frac{x+y}{y\cdot x\cdot(x+y)}=\frac{1}{xy}.$$ ■

202. $\left(\dfrac{3}{2x-y}-\dfrac{2}{2x+y}-\dfrac{1}{2x-5y}\right):\dfrac{y^2}{4x^2-y^2}$.

203. $\left(x^2+2x-\dfrac{11x-2}{3x+1}\right):\left(x+1-\dfrac{2x^2+x+2}{3x+1}\right)$.

204. $\left(6a^2+5a-1+\dfrac{a+4}{a+1}\right):\left(3a-2+\dfrac{3}{a+1}\right)$.

205. $\dfrac{(a-b)^2+ab}{(a+b)^2-ab}:\dfrac{a^5+b^5+a^2b^3+a^3b^2}{(a^3+b^3+a^2b+ab^2)(a^3-b^3)}$.

206. $\left(2-x+4x^2+\dfrac{5x^2-6x+3}{x-1}\right):\left(2x+1+\dfrac{2x}{x-1}\right)$.

207. $M=\left[\frac{2-b}{b-1}+2\cdot\frac{a-1}{a-2}\right]:\left[b\cdot\frac{a-1}{b-1}+a\cdot\frac{2-b}{a-2}\right]$.

Avval o‘rta qavslar ichidagi ifodalarni soddalashtiramiz:

$$\frac{2-b}{b-1}+2\cdot\frac{a-1}{a-2}=\frac{(2-b)(a-2)+(a-1)(b-1)}{(b-1)(a-2)}=$$

$$=\frac{2a-4-ab+2b+2ab-2a-2b+2}{(b-1)(a-2)}=\frac{ab-2}{(b-1)(a-2)};$$

$$b\cdot\frac{a-1}{b-1}+a\cdot\frac{2-b}{a-2}=\frac{(ab-b)(a-2)+(2a-ab)(b-1)}{(b-1)(a-2)}=$$

$$=\frac{a^2b-2ab-ab+2b+2ab-2a-ab^2+ab}{(b-1)(a-2)}=\frac{(a^2b-ab^2)-(2a-2b)}{(b-1)(a-2)}=$$

$$=\frac{ab(a-b)-2(a-b)}{(b-1)(a-2)}=\frac{(ab-2)(a-b)}{(b-1)(a-2)}.$$

Nihoyat, M ni hisoblash uchun birinchi o‘rta qavs uchun chiqarilgan ifodani, ikkinchi o‘rta qavs uchun chiqarilgan ifodaga bo‘lamiz:

$$M=\frac{ab-2}{(b-1)(a-2)}:\frac{(ab-2)(a-b)}{(b-1)(a-2)}=\frac{1}{a-b}.$$

Javob: $\frac{1}{a-b}$.

208*. Soddalashtiring: $\frac{p^3+4p^2+10p+12}{p^3-p^2+2p+16}\cdot\frac{p^3-3p^2+8p}{p^2+2p+6}$.

Rivojlantiruvchi mashqlar*

1. Ifodani soddalashtiring:

1) $\frac{8x^2y^2}{x^2-y^2}:\left(\frac{x}{x-y}-\frac{x}{x+y}\right)$;

2) $\left(\frac{1}{4x^2}-\frac{1}{xy}+\frac{1}{y^2}\right)\left(\frac{1}{2x-y}-\frac{1}{y-2x}\right)-\frac{1}{xy^2}$;

3) $\frac{x^3+y^3}{x+y}:\left(x^2-y^2\right)+\frac{2y}{x+y}-\frac{xy}{x^2-y^2}$;

4) $\left(\frac{x+4}{3x+3}-\frac{1}{x+1}\right):\frac{1+x}{3}-\frac{2}{1-x}$.

2. Ayniyatni isbotlang:

1) $\frac{1}{x(x+1)}+\frac{1}{(x+1)(x+2)}+\frac{1}{(x+2)(x+3)}+\frac{1}{(x+3)(x+4)}=\frac{4}{x(x+4)}$;

2) $\frac{1}{x(x-1)}+\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{4}{x(x-4)}$.

3. Agar bolalarga olmalarni a donadan bersak, b dona olma ortib qoladi. Agar c donadan bersak, d dona olma yetmaydi. Bolalar necha nafar?

4. 1-quvur basseynni a soatda, 2-quvur b soatda, 3-quvur esa c soatda to'ldiradi. Uchala quvur baravar ochib qo'yilsa, basseyn necha soatda to'ladi?

5. Ifodaning qiymati a ga bog'liq emasligini ko'rsating:

1) $\left(\frac{4(a-2)}{a^2-a-6}+\frac{a-3}{4-a^2}\right)\cdot\frac{a^2-4}{1-a}+\frac{2}{a-3}$;

2) $\frac{3}{a-2}+\frac{3a+12}{25-a^2}:\left(\frac{2a-1}{a^2-25}-\frac{a-1}{2a^2+9a-5}\right)$.

6. Tenglamani yeching: $\frac{7}{x^2-1}+\frac{8}{x^2-2x+1}=\frac{37-9x}{x^3-x^2-x+1}$

Ifodani soddalashtiring **(7–8)**:

7. 1) $\frac{a-\frac{a^2}{a+1}}{a-\frac{a}{a+1}}$; 2) $\frac{\frac{a-b}{a+b}+\frac{b}{a}}{\frac{a}{a+b}-\frac{a-b}{a}}$; 3) $\frac{a-\frac{6a-9}{a}}{\frac{3}{a}-1}$;

4) $\frac{\frac{x}{x-y}-\frac{y}{y-z}}{\frac{y}{x-z}-\frac{x}{y-z}}$; 5) $1-\frac{1}{1-\frac{1}{a-3}}$; 6) $\frac{a}{a-\frac{1}{1-\frac{a}{1+a}}}$.

8. 1) $\left(1-\frac{1}{a}+\frac{1}{a^2}-\frac{1}{a^3}\right):\left(1+\frac{1}{a}+\frac{1}{a^2}+\frac{1}{a^3}\right)$;

2) $\left(\frac{a}{b-c}+\frac{b-c}{b}-\frac{ac}{b^2-bc}\right):\left(\frac{a}{b-c}-\frac{b-c}{b}-\frac{ac}{b^2-bc}\right)$.

9. Ayniyatni isbotlang:

1) $1+\frac{1}{a}+\frac{1+a}{ab}+\frac{(1+a)(1+b)}{abc}+\frac{(1+a)(1+b)(1+c)}{abcd}=\frac{(1+a)(1+b)(1+c)(1+d)}{abcd}$;

2) $\frac{1}{1-a}+\frac{1}{1+a}+\frac{2}{1+a^2}+\frac{4}{1+a^4}+\frac{8}{1+a^8}+\frac{16}{1+a^{16}}=\frac{32}{1-a^{32}}$.

10. Ifodani soddalashtiring:

1) $\frac{3a+b}{a+b}+\frac{a^2-2ab+b^2}{a}\cdot\left(\frac{a}{(a-b)^2}+\frac{a}{b^2-a^2}\right)$;

2) $\frac{1-2b^2-2ab}{b^2-a^2}+\left(\frac{a^2+ab+b^2}{a^3+b^3}-\frac{b^3-a^3}{a^2-ab+b^2}\right)\cdot\frac{a^2-ab+b^2}{a^3-b^3}$.

11. 1) $\frac{a}{(a-b)(a-c)}+\frac{b^2}{(b-a)(b-c)}+\frac{c^2}{(c-a)(c-b)}$;

2) $\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-a)(b-c)}+\frac{1}{c(c-a)(c-b)}$.

12. n ning qanday butun qiymatlarida $\frac{2n^2+n-9}{n+3}$ kasr butun son bo‘ladi?

□ Suratdagi ko‘phadning darajasi maxrajdagi ko‘phad darajasidan katta. Shu ma’noda bu kasr “noto‘g‘ri” kasrdir. Noto‘g‘ri kasrdan uning butun qismini ajratish uchun surat ustun (burchak) usulida maxrajga bo‘linadi.
Bu yerda ham xuddi shunday yo‘l tutamiz: berilgan kasrning suratmi maxrajiga bo‘lamiz va undan butun qismini ajratamiz:

$$\begin{array}{r|l} 2n^2+n-9 & n+3 \\ \underline{-\,2n^2+6n} & \overline{2n-5} \\ -5n-9 & \\ \underline{-\,-5n-15} & \\ 6 & \end{array}$$

Kasrning butun qismi $(2n-5)$ ifoda n ning ixtiyoriy butun qiymatida butun sondir. Demak, $\frac{6}{n+3}$ kasr n ning qanday butun qiymatlarida butun son bo‘lishini aniqlashimiz kerak. Maxrajdagi $n+3$ surat 6 ning bo‘luvchisiga teng bo‘lsagina $\frac{6}{n+3}$ kasr butun son bo‘ladi. $n+3$ ifodani 6 ning bo‘luvchilariga tenglab, hosil bo‘lgan tenglamadan n ni topamiz. Javobni jadval ko‘rinishida berish qulay:

$n+3$	1	2	3	6	−1	−2	−3	−6
n	−2	−1	0	3	−4	−5	−6	−9

Javob: $\frac{2n^2+n-9}{n+3}$ kasrning qiymati $n=-1;\ -2;\ -4;\ -5;\ -6;\ -9;\ 0;\ 3$ bo'lgandagina butun son bo'ladi.

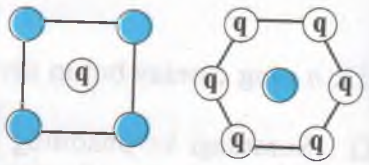
13. Kasrning suratini maxrajiga ustun (burchak) usulida bo'lib, uni ko'phad va "to'g'ri" kasr yig'indisi ko'rinishida tasvirlang: 1) $\frac{3n^2-10n-3}{n-4}$;

2) $\frac{n^3+n^2-n+5}{n+2}$.

14. Kasrning qiymati n ning qanday butun qiymatlarida butun son bo'ladi?

1) $\frac{2n^2+7n+3}{2n-1}$; 2) $\frac{2n^3+n^2-3n-4}{n-2}$.

15. 1) 45 ta ko'k va 40 ta qizil fishka bor. Ularni stol ustiga shunday joylashtirishmoqchi:
yashil fishkalar kvadratning uchlariga qo'yiladi, uning markaziga esa qizil fishka qo'yiladi, qizil fishkalar muntazam oltiburchak uchlariga, uning markaziga esa ko'k fishka qo'yiladi.



Fishkalarni shu usulda joylashtirsak, hamma fishkalar ishlatiladimi? Qancha kvadrat va qancha oltiburchak hosil bo'ladi?
2) Shu masalani ko'k fishkalar 43 ta, qizil fishkalar 42 ta bo'lgan hol uchun ham hal qiling.

16. Yer maydoni kvadrat shaklida bo'lib, uning uchlarida qadimiy archalar bor. Archalarni kesmasdan yer maydoni shaklini saqlagan holda uning yuzini ikki barobar kattalashtiring.

17. (*Qadimiy masala.*) Otaning 5 nafar o'g'li va kvadrat shakldagi 80 tanob yeri bor. Ota o'g'illariga yerni bo'lib berdi: katta o'g'ilga 40 tanob, 2-o'g'ilga 20 tanob, 3-o'g'ilga 10 tanob, Hasan va Husanlarning har biriga 5 tanobdan yer tegdi. Har bir o'g'il o'z yeriga uy qurdi. "Har bir yer maydoni esa bir xil ko'rinishda bo'lsin", – dedi ota. Rasmga qarab o'g'illarning yerlari chegaralarini chizing. (tanob – yer maydonining qadimiy o'lchov birligi bo'lib, 1 tanob yer $=900\text{m}^2=9$ sotix).

18. Halima aya yangi uzilgan shaftolilarni laganda o'g'illari xonasiga qo'ydi. Birinchi o'g'il kelib, shaftolilarning uchdan birini yeb chiqib ketdi. Ikkinchi o'g'il kelib, qolgan shaftolilarning uchdan birini yeb chiqib ketdi. 3-o'g'il ham

shunday qildi. Keyin uchalasi xonaga birgalikda kirishdi. Qarashsa, laganda 8 ta shaftoli qolibdi. 1. Dastlab laganda nechta shaftoli bor edi? 2. Har bir o'g'il barobar miqdorda shaftoli yeyishi uchun qolgan 8 ta shaftolini qanday bo'lishib olishlari kerak?

19. (*Qadimgi masala*). Alida 3 ta, Valida 5 ta shirin kulcha bor. Solihda bunday kulchalar yo'q, ammo unda 8 ta bir xil tanga bor. Uchala bola kulchalarni barobar bo'lib yeyishdi. Solih 8 ta tangani ularga "Shirin kulchalar uchun rahmat", – deb berdi. Bu tangalarni Ali va Vali qanday bo'lib olishlari kerak?

20. Tezyurar A poyezd B yuk poyezdining ketidan kelyapti. B poyezd A poyezdni o'zidan oldinga o'tkazib yuborishi lozim. Asosiy yo'lga bir nechta vagon sig'adigan shoxobcha ulangan. Bu shoxobchaga A yoki B dagi vagonlarning hammasi sig'maydi, bir nechtasi sig'adi, xolos. A poyezdning B dan oldinga o'tib olishiga yordam bering.

21. Rasmdagi 12 ta nuqtani 5 ta kesma bilan qalamni qog'ozdan uzmay, bir kesmadan ikkinchi marta yurmay tutashtiring.

22. 6 kishi – 3 nafar ota va uch nafar bola – daryoning u qirg'og'iga o'tishmoqchi. Qayiqqa faqat 2 nafar odam sig'adi. Bolalar o'z otalari yonida yoki faqat bolalar davrasida bo'lishlari mumkin. Bola begona kishi bilan o'z otasi yonida bo'lsagina qola oladi. Ular qanday qilib daryoning u qirg'og'iga o'ta oladilar?

23. 4 ta buyum bor: piyola, kosa, stakan, ko'za. 4 xil ichimlik bor: sut, o'rik sharbati, limonad va suv. Har bir idishda qaysidir ichimlik bor. Piyolada suv va sut yo'qligi ma'lum. Limonad solingan idish ko'za va sharbatli idish bilan, stakan esa sutli idish va kosa bilan yonma-yon turibdi. Qaysi idishda qanday ichimlik borligini aniqlang.

24. Pallali tarozida massalari har xil bo'lgan to'rtta tosh yordamida 1 kg, 2 kg, 3 kg, ..., 40 kg massalarni o'lchash mumkin bo'lsa, shu toshlar massalarini toping.

25. Kasrni qisqartiring va $x=-2$ bo'lganda uning son qiymatini toping:

$$\frac{x^8+x^6+x^4+x^2+1}{x^4+x^3+x^2+x+1}.$$

26. Kasrni qisqartiring: $\dfrac{x^8+x^6y^2+x^4y^4+x^2y^6+y^8}{x^4+x^3y+x^2y^2+xy^3+y^4}$.

27. Agar $ad-bc=1$ bo'lsa, $\dfrac{a^2+b^2}{ae+bd}$ kasr qisqarmas ekanini isbotlang, bunda a, b, c, d – butun sonlar.

○ Qavslarni ochib ushbu $(ac+bd)^2+(ad-bc)^2=(a^2+b^2)(c^2+d^2)$ ayniyatni bevosita isbotlash mumkin. Faraz qilaylik, (a^2+b^2) va $(ac+bd)$ biror natural son n ga $(n\neq 1)$ bo'linsin. U holda

$(a^2+b^2)(c^2+d^2)-(ac+bd)^2=(ad-bc)^2=1$
ham n ga bo'linishi kerak. Ziddiyatga keldik. Demak, da'vo to'g'ri ekan. ●

28. Ifodani ko'paytuvchilarga ajrating:
$[(a-c)^2+(b-d)^2]\ (a^2+b^2)\ -(ad-bc)^2$.

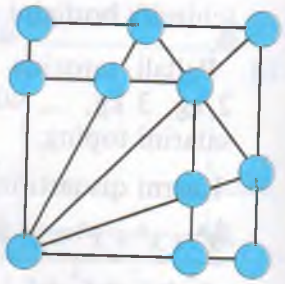
29. Qandaydir 4 xonali sonni 9 ga ko'paytirganda ko'paytmaning raqamlari teskari tartibda yozilib qoldi. U qanday son ekan?

□ Izlanayotgan son $\overline{1000a+100b+10c+d}=\overline{abcd}$. Shartga ko'ra $9\cdot\overline{abcd}=\overline{dcba}$, bundan $10\overline{abcd}=\overline{abcd}+\overline{dcba}$. Demak, $abcd+dcba=abcd0$. Avvalo, yig'indini tahlil qilib, $a=1$, $d=9$ ekanini topamiz. $b=0$ va $b=1$ imkoniyatdan $b=0$ bo'lgan hol mos keladi. Shu bilan birga, $c+1+b=d$, ya'ni $c=d-1=8$. Demak, $a=1$, $b=0$, $c=8$, $d=9$.
Javob: 1 089. ■

30. 49 ta natural son $a_1, a_2, \ldots, a_{49}$ lar yig'indisi 999 ga teng. Ularning eng katta umumiy bo'luvchisi (EKUB)ning eng katta qiymati nechaga teng bo'lishi mumkin?

□ EKUK $(a_1, a_2, \ldots, a_{49})=d$ deylik; $ak\geq d$; $k=1, 2, \ldots, 49$ ekani ravshan. U holda $a_1+a_2+\ldots+a_{49}=999$ yig'indi d ga bo'linadi, ya'ni $d\cdot 999=33\cdot 37$ sonning bo'luvchisidir. $ak\geq d$ bo'lgani uchun $(k=1, 2, \ldots, 49)$ $999=a_1+a_2+\ldots+a_{49}\geq 49d$, bundan $d\leq\frac{999}{49}<21$; d son 999 ning bo'luvchisi bo'lgani uchun u 1,3,9 qiymatlarni qabul qilishi mumkin. d o'zining eng katta qiymatiga $d=9$ bo'lganda erishadi, chunki 999 ni har biri 9 ga bo'linadigan 49 ta qo'shiluvchilar yig'indisi ko'rinishida tasvirlash mumkin: $999=9+9+\ldots+9+567$.
Javob: EKUBning eng katta qiymati 9 ga teng.

31. 1, 2, ..., 10, 11 sonlarni rasmdagi doirachalarga shunday qo'yingki, 10 ta kesmaning har biridagi uchta son yig'indisi o'zaro teng bo'lsin.



32. Tenglamani yeching:

1) $2-\cfrac{2-\cfrac{7}{4x-1}}{2+\cfrac{1}{1-x}}=1$; 2) $2009+\cfrac{a-\cfrac{3}{2x+5}}{a+\cfrac{2}{3-x}}=2010$.

Ko'phadni ko'paytuvchilarga ajratib, tenglamaning butun ildizlarini toping **(33–34)**:

33. 1) $x^3-6x^2+11x-6=0$; 2) $x^3-5x^2-17x-21=0$;
3) $x^3-12x^2+9x+22=0$; 4) $2x^3+x^2-5x+2=0$.

34. 1) $x^4-10x^3+5x^2+40x-36=0$; 2) $x^4-2x^3+2x-1=0$;
3) $x^4+2x^3-x-2=0$; 4) $x^4+x^3-7x^2-13x-6=0$.

35. Tenglamani natural sonlarda yeching:

1) $x+\dfrac{1}{y+\dfrac{1}{z}}=\dfrac{10}{7}$;

2) $x+\dfrac{1}{y+\dfrac{1}{z}}=\dfrac{68}{21}$.

36. Tenglamaning chap qismini ko'paytuvchilarga ajrating va nolga teng bo'lish shartidan foydalanib, uni yeching:

1) $x^3+2x^2-9x-18=0$;
2) $y^3-3y^2-4y+12=0$;
3) $2x^3-3x^2-18x+27=0$;
4) $6x^3+7x^2-x-2=0$.

37. Tenglik ayniyat ekanini isbotlang:

1) $(2x+9)^2-4x^2(x-6)^2=3(4x+3)(2x-3)^2$;
2) $64(5a+1)^2-(25a^2+8)^2=5a\cdot(8-5a)(5a+4)^2$.

38. Quyidagi tasdiqlarning qaysi biri to'g'ri, qaysi biri noto'g'ri? Sababini tushuntiring. Mos misollar tuzing:

1) natural sonning kvadrati ixtiyoriy raqam bilan tugashi mumkin;
2) natural sonning kubi ixtiyoriy raqam bilan tugashi mumkin;
3) natural sonning to'rtinchi darajasi faqat 0; 1; 5; 6 raqamlarining biri bilan tugashi mumkin;
4) natural sonning beshinchi darajasi bu son qaysi raqam bilan tugagan bo'lsa, o'sha raqam bilan tugaydi.

39. Temperatura Kelvin, Selsiy, Farengeyt shkalasi bo'yicha o'lchanishi mumkin. Kelvin shkalasi bo'yicha temperatura Selsiy shkalasi bo'yicha temperatura bilan $t°K=t°C+273°$ formula bilan bog'langan. Selsiy temperaturasidan Farengeyt temperaturasiga o'tish esa $t°F=1{,}8t°C+32°$ formula bilan ifodalanadi. Kelvin temperaturasidan Farengeyt temperaturasiga o'tish formulasini toping.

40. A va B shaharlar orasidagi masofa 256 km. A dan B ga qarab 66 km/s tezlik bilan yuk poyezdi yo'lga chiqdi. Oradan 20 minut o'tgach, B dan A ga 90 km/s tezlik bilan tezyurar poyezd yo'lga chiqdi.

Yuk poyezdi yo'lga chiqqanidan qancha vaqtdan so'ng tezyurar poyezd bilan uchrashadi?

Bu masalani yechish uchun tenglamalarni quyidagicha tuzish mumkin:

a) $66x+90\left(x-\dfrac{1}{3}\right)=256$;
b) $256-66\cdot\dfrac{1}{3}=(66+90)\cdot\left(x-\dfrac{1}{3}\right)$;
c) $\dfrac{x}{60}-\dfrac{256-x}{90}=\dfrac{1}{3}$;
d) $256-90x=66\cdot\left(x+\dfrac{1}{3}\right)$.

1) har bir tenglamada x nimani bildiradi?
2) har bir tenglamada qanday miqdorlar tenglashtirilgan?

$(a^2+b^2)(c^2+d^2)$ mazmuniga mos tenglama tuzish – masala shartini «matema-
ham n ga o'tkazish – masalaning matematik modelini tuzish demakdir.

28. Bu masalani hal qilish uchun turli tenglama, turli matematik model tuzish mumkinligini yuqorida ko'rdik.

41. Hisoblang:

1) $\left(1+\frac{1}{5}\right)\left(1+\frac{1}{5^2}\right)\left(1+\frac{1}{5^4}\right)\ldots\left(1+\frac{1}{5^{16}}\right)\left(1+\frac{1}{5^{32}}\right)$;

2) $(1+a)(1+a^2)(1+a^4)\ldots(1+a^{2n-2})(1+a^{2n})$.

42. a, b, c – musbat sonlar va $\frac{a}{b+c}=\frac{b}{a+c}=\frac{c}{a+b}$ bo'lsa, u holda $a=b=c$ bo'lishini isbotlang.

○ Shartga ko'ra, $\frac{a}{b+c}=\frac{b}{a+c}$, bundan $a^2+ac=b^2+bc$ yoki $a^2-b^2+ac-bc=$ $=(a-b)(a+b+c)=0$; a, b, c lar musbat bo'lgani uchun, $a=b$. Xuddi shunday, $b=c$ ekani ko'rsatiladi. Demak, $a=b=c$. ●

43. n ning $\frac{n^5+3}{n^2+1}$ kasr butun son bo'ladigan barcha butun qiymatlarini toping.

Ko'rsatma: $\frac{n^5+3}{n^2+1}=n^3-n+\frac{n+3}{n^2+1}$ ekanidan foydalaning.

44. Har bir butun n soni uchun $\frac{n^5}{120}-\frac{n^3}{24}+\frac{n}{30}$ ifodaning qiymati ham butun son bo'lishini isbotlang.

○ Berilgan ifodani shunday yozib olish mumkin: $\frac{n^5}{120}-\frac{n^3}{24}+\frac{n}{30}=\frac{n^5-n^3+4n}{120}=$ $=\frac{(n-2)(n-1)n(n+1)(n+2)}{120}$.

Suratda 5 ta ketma-ket butun sonning ko'paytmasi turibdi. Bu sonlarning bittasi, albatta, 5 ga, bittasi 4 ga, bittasi 3 ga va kamida ikkitasi 2 ga bo'linadi. U holda $(n-2)(n-1)n(n+1)(n+2)$ ko'paytma $5\ 4\ 3\ 2=120$ ga qoldiqsiz bo'linadi. ●

45. Agar a, b, c natural sonlar uchun $a^2+b^2=c^2$ tenglik o'rinli bo'lsa;

1) a va b sonlardan hech bo'lmaganda bittasi 3 ga bo'linishini;

2) a, b, c sonlardan hech bo'lmaganda bittasi 5 ga bo'linishini isbotlang.

○ 1) Sonning kvadrati 3 ga bo'linsa, qoldiqda 0, bo'linmasa qoldiqda 1 chiqadi. Agar a ham, b ham 3 ga bo'linmasa, u holda (a^2+b^2) ni 3 ga bo'lganda qoldiq 2 chiqadi. $a^2+b^2=c^2$ bo'lgani uchun c^2 ni 3 ga bo'lganda qoldiqda

yoki 0 yoki 1 chiqishi kerak. Zidlikka keldik. Demak, a va b sonlardan kamida bittasi 3 ga bo'linishi kerak.

2) 5 ga bo'linmaydigan sonlarni $5k\pm1$ yoki $5k\pm2$ ko'rinishida yozish mumkin. Demak, 5 ga bo'linmaydigan sonning kvadratini 5 ga bo'lganda qoldiqda 1 yoki 4 chiqadi. Agar a ham, b ham 5 ga bo'linmasa, u holda (a^2+b^2) ni 5 ga bo'lganda qoldiqda $1+1=2$, $(1+4)-5=0$, $(4+4)-5=3$ chiqadi. $a^2+b^2=c^2$ bo'lgani uchun 5 ga bo'lganda qoldiqda 2 chiqmaydi, ya'ni faqat 0,1,4 sonlardan biri chiqadi. Qaralayotgan holda $a^2+b^2=c^2$ ni 5 ga bo'lganda qoldiqda faqat nol chiqadi, ya'ni c^2 soni 5 ga, demak, c soni ham 5 ga qoldiqsiz bo'linadi. Da'volar isbotlandi. ●

46. $a_0, a_1, a_2, ..., a_{100}$ – natural son. Agar $a_1>a_0$, $a_2=3a_1-2a_0$, $a_3=3a_2-2a_1$, ..., $a_{100}=3a_{99}-2a_{98}$ ekani ma'lum bo'lsa, $a_{100}>2^{99}$ ekanini isbotlang.

○ Masala shartiga ko'ra: $a_1-a_0>0$, $a_2-a_1=2(a_1-a_0)$, $a_{100}-a_{99}=2(a_{99}-a_{98})$. Bundan a_2-a_1, a_3-a_2, $a_{100}-a_{99}$ ayirmalarning musbatligi kelib chiqadi. Shu bilan birga:

$$\frac{a_2-a_1}{a_1-a_0}=\frac{a_3-a_2}{a_2-a_1}=...=\frac{a_{100}-a_{99}}{a_{99}-a_{98}}=2.$$

Bu 99 ta tenglikni bir-biriga ko'paytirib, topamiz: $\frac{a_{100}-a_{99}}{a_1-a_0}=2^{99}$, bundan

$a_{100}=a_{99}+2^{99}\ (a_1-a_0)$.

Ammo $a_{99}>0$, $a_1-a_0\geq1$. Demak, $a_{100}>2^{99}$. ●

47. Ixtiyoriy butun son n uchun $\frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}$ son ham butun bo'lishini isbotlang.

○ Berilgan ifodani M bilan belgilaylik va uni quyidagicha yozib olaylik:

$$M=\frac{n(n^2-1)(n^2-4)}{5}+\frac{5n^3-4n}{5}+\frac{n(n^2-1)}{3}+\frac{n}{3}+\frac{7n}{15}=$$

$$=n^3+\frac{(n-2)(n-1)n(n+1)(n+2)}{5}+\frac{(n-1)n(n+1)}{3},$$

bunda oxirgi ikkita qo'shiluvchining, mos ravishda 5 ga va 3 ga bo'linishi ravshan. ●

48. n^3+3n^2-n-3 son ixtiyoriy toq n da 48 ga bo'linishini isbotlang.

○ Berilgan ifodani M deb belgilaylik. $M=n^3+3n^2-n-3=n^2(n+3)-(n+3)=(n^2-1)(n+3)=(n-1)(n+1)(n+3)$. Shartga ko'ra, n toq son va $n=2k+1$, k – natural son, deb yozish mumkin. U holda $M=2k\cdot(2k+2)\cdot(2k+4)=8\cdot k(k+1)(k+2)$. 3 ta ketma-ket natural sonlar ko'paytmasining 3 ga, 2 ga va demak, 6 ga bo'linishi ravshan. U holda M soni $8\cdot2\cdot3=48$ ga bo'linadi. ●

49. $x + y + z = xyz$ tenglamani natural sonlarda yeching.

□ Faraz qilaylik, $x \le y \le z$ bo'lsin, u holda $x + y + z \le 3z$ va $x + y + z = xyz$ bo'lgani uchun $xyz \le 3z$ yoki $xy \le 3$ kelib chiqadi. Agar $x = y = z$ bo'lsa, u holda $z^3 = 3z$ yoki $z^2 = 3$, bu tenglik butun z lar uchun bajarilmaydi. Demak, x, y, z sonlardan hech bo'lmaganda ikkitasi bir-biriga teng emas, shuning uchun $xy < 3$, ya'ni $xy = 2$ yoki $xy = 1$. Agar $xy = 2$ bo'lsa, u holda $x = 1$, $y = 2$ va berilgan tenglamadan $z = 3$ ni topamiz. Agar $xy = 1$ bo'lsa, u holda $x = y = 1$ va berilgan tenglamadan $2 + z = z$ tenglikka kelamiz. Bu tenglik noto'g'ri. x, y, z lar tenglamada simmetrik qatnashyapti. Demak, qolgan yechimlarni $x = 1$, $y = 2$, $z = 3$ yechimdan o'rin almashtirishlar yordamida hosil qilamiz. ■

50. Ifodaning son qiymatini toping: $1!3 - 2!4 + 3!5 - 4!6 + \ldots - 2012!2014 + 2013!$

□ $n! \cdot (n+2) = n! \cdot (n+1+1) = n! \cdot (n+1) + n! = (n+1)! + n!$ ayniyatdan foydalanamiz. $n! = 1 \cdot 2 \cdot \ldots \cdot n$ ekanini eslatib o'taylik. Demak, berilgan ifodani shunday yozish mumkin: $2! + 1! - 3! - 2! + 4! + 3! - 5! - 4! + \ldots + 2012! + 2011! - 2013! - 2012! + 2013! = 1$. ■

51. Stolda ustida 8 ta bir xil stakanda suv bor. Ixtiyoriy 2 ta stakanni olib, ulardagi suvni biridan ikkinchisiga quyib, suv miqdorini tenglashga ruxsat etiladi. Bu amaliyot yordamida hamma stakanlardagi suv miqdorini o'zaro tenglash mumkinligini isbotlang.

V bobga doir sinov mashqlari (testlar)

1. Kasrni qisqartiring: $\dfrac{48a^2 - 120ab + 75b^2}{16a^2 - 25b^2}$.

A) $\dfrac{3(4a-5b)}{4a+5b}$; B) $\dfrac{3(4a+5b)}{4a-5b}$; C) $\dfrac{4(4a-5b)}{4a+5b}$; D) $\dfrac{2(4a-5b)}{4a+5b}$.

2. Kasrni qisqartiring: $\dfrac{7a^2(ab^2+9a)}{3a(21a+7ab)}$.

A) $\dfrac{a(b^2+9)}{3(3+b)}$; B) $\dfrac{a^2(b^2+9)}{3(3+b)}$; C) $\dfrac{a(b^2+9)}{3(3-b)}$; D) $\dfrac{b^2+9}{3(3+b)}$.

3. Kasrni qisqartiring: $\dfrac{8a^2 - 12a^2 + 6a - 1}{4a^2 - 4a + 1}$.

A) $2a+1$; B) $2a-1$; C) $2a+2$; D) $2a-2$.

4. Amallarni bajaring: $\dfrac{3}{a+b} - \dfrac{4}{a-b} + \dfrac{6b}{a^2-b^2}$.

A) $\dfrac{1}{b-a}$; B) $\dfrac{1}{a-b}$; C) $\dfrac{2}{a-b}$; D) $\dfrac{3}{b-a}$.

5. Ifodani soddalashtiring: $\frac{a^2+9}{a^3-27}-\frac{1}{a-3}$.

A) $-\frac{3a}{a^3-27}$; B) $\frac{3a}{a^3-27}$; C) $\frac{a}{a^3-27}$; D) $-\frac{a}{a^3-27}$.

6. Kasrlarni ko‘paytiring: $\frac{9a^2-16b^2}{6a+8b}\cdot\frac{6a^2}{12b-3a}$.

A) $-a^2$; B) a^2; C) $\frac{a^2}{3a-4b}$; D) $\frac{6}{3a+4b}$.

7. Kasrlarni bo‘ling: $\frac{9a^2-12ab+b^2}{3a-2}:\frac{(3a-2b)^2}{9a^2-4}$.

A) $3a+2b$; B) $3a-2b$; C) $2a+3b$; D) $2a-3b$.

8. Amallarni bajaring: $\left(\frac{3a+4b}{3a-4b}-\frac{3a-4b}{3a+4b}\right)\cdot\left(\frac{3a+4b}{3a-4b}-1\right)$.

A) $\frac{6a}{3a+4b}$; B) $\frac{6a}{3a-4b}$; C) $\frac{5a}{3a+4b}$; D) $\frac{5a}{3a-4b}$.

9. Amallarni bajaring: $\left(\frac{3x+2}{3x-2}-\frac{4}{9x^2-4}\right)\cdot\frac{3x+2}{3x+4}$.

A) $\frac{3x}{3x-2}$; B) $\frac{x}{3x-2}$; C) $\frac{2x}{3x-2}$; D) $\frac{-x}{3x-2}$.

10. Amallarni bajaring: $\left(\frac{a}{a^2-4}-\frac{8}{a^2+2a}\right)\cdot\frac{a^2-2a}{4-a}+\frac{a+8}{a+2}$.

A) $\frac{12}{a+2}$; B) $\frac{11}{a+2}$; C) $\frac{10}{a+2}$; D) $\frac{13}{a+2}$.

11. $P(x)=2x^4-3x^3-x^2-4$ ko‘phadni $x-2$ ikkihadga bo‘lganda qoldiq nechaga teng bo‘ladi?

A) 0; B) 1; C) 2; D) -1.

12. a ning qanday qiymatlarida $P(x)=2x^3-x^2+ax-15$ ko‘phadni $(x+3)$ ikkihadga qoldiqsiz bo‘linadi?

A) -26; B) -25; C) 25; D) -24.

13. a ning qanday qiymatlarida $P(x)=4x^3-ax^2+3x-5$ ko‘phadni $(x+1)$ ikkihadga bo‘lganda qoldiq 5 ga teng bo‘ladi?

A) -17; B) -16; C) -18; D) -15.

14. $P(x)=2x^3-4x^2+6x-3$ ko‘phadni $(x-1)$ ikkihadga bo‘lganda qoldiq nechaga teng bo‘ladi?

A) 1; B) 2; C) 0,5; D) 1,5.

15. Ko'phadni ko'phadga burchak usuli orqali bo'ling: $(8a^3-27b^3):(2a-3b)$.

A) $4a^2+6ab+9b^2$; B) $4a^2-6ab+9b^2$;
C) $4a^2+12ab+9b^2$; D) $4a^2-6ab+9b^2$.

16. Ko'phadni ko'phadga burchak usuli yordamida bo'ling:
$(2a^3+15b^2+6ab^2-5a^2b):(2a-5b)$.

A) a^2-3b^2; B) a^2+3b^2; C) $a+3b$; D) a^2+3b.

17. Amallarni bajaring: $\left(\frac{4a}{4-a^2}-\frac{a-2}{4+2a}\right)\cdot\frac{4}{a+2}-\frac{a}{2-a}$.

A) 1; B) 2; C) 3; D) -1.

18. Soddalashtiring: $\frac{5x+6}{x^2-4}-\frac{x}{x^2-4}:\frac{x}{x-2}-\frac{x+2}{x-2}$.

A) -1; B) 1; C) 2; D) 0,5.

19. Soddalashtiring: $a^2b^2\cdot\left(\frac{1}{(a+b)^2}\cdot\left(\frac{1}{a^2}+\frac{1}{b^2}\right)+\frac{2}{(a+b)^3}\cdot\left(\frac{1}{a}+\frac{1}{b}\right)\right)$.

A) 1; B) 2; C) -1; D) 1,5.

20. Amallarni bajaring: $2-\frac{x-a}{x+a}-\frac{x}{x-a}+\left(\frac{1}{a^2}-\frac{1}{ax}+\frac{2}{x^2}\right):\left(\frac{1}{a^2}-\frac{1}{x^2}\right)$.

A) 1; B) 2; C) 1,5; D) 0.

21. a va b ning shunday qiymatlarini topingki, x ning barcha joiz qiymatlari uchun $\frac{ax^2-x+b}{2x-1}=2x+1$ tenglik o'rinli bo'lsin.

A) $a=2;\ b=1$; B) $a=4;\ b=-1$;
C) $a=-2;\ b=1$; D) $a=1;\ b=2$.

22. a va b ning shunday qiymatlarini topingki, x ning barcha joiz qiymatlari uchun $\frac{ax^2-5x+b}{x-2}=2x-1$ tenglik o'rinli bo'lsin.

A) $a=2;\ b=-2$; B) $a=-2;\ b=-7$; C) $a=-2;\ b=-6$; D) $a=1;\ b=6$.

23. $\frac{x}{y}=2$ bo'lsa, $\frac{x^2-3xy+4y^2}{x^2-xy+y^2}$ ifodaning son qiymatini toping.

A) $\frac{2}{3}$; B) $\frac{4}{3}$; C) $\frac{3}{2}$; D) $\frac{3}{5}$.

TARIXIY MASALALAR

1. *Eyler masalasi.*

Tenglikning to‘g‘riligini ko‘rsating: $a^3+b^3+\left[\frac{b(2a^3+b^3)}{a^3-b^3}\right]^3=\left[\frac{a(a^3+2b^3)}{a^3-b^3}\right]^3$.

2. *Eyler masalasi.*

Quyidagi tenglikning to‘g‘riligini ko‘rsating:

1) $\frac{1}{2}\left[\frac{a^2-b^2}{a-b}+(a-b)\right]=a$; 2) $\frac{1}{2}\left[\frac{a^2-b^2}{a-b}-(a-b)\right]=b$.

TARIXIY MA’LUMOT

Qisqa ko‘paytirish formulalari, kasrlarga oid ma’lumot qadimgi risolalarda uchraydi. Masalan, al-Karajining «Al-Faxri», Misr olimi Abu Komil (850 – 930) ning «Kitob al-jabr val-muqobala» asarlarida ham algebraik kasrlar o‘rganilgan. Abu Komil al-Xorazmiydan keyin algebraga doir kitob yozgan birinchi olimdir. Abu Komil o‘z asarida $\left(\frac{a}{b}\right)\cdot b=a$, $\frac{a}{b}=\frac{a^2}{ab}$, $\frac{a}{b}\cdot\frac{b}{a}=1$, $\frac{a}{b}+\frac{b}{a}=\frac{a^2+b^2}{ab}$ kabi sodda munosabatlarga ham e’tibor qaratadi.

Algebraik kasrlarga I.Nyutonning “Umumiy arifmetika” kitobida ham yetarlicha o‘rin berilgan.

“$\frac{a}{b}$ kasr a ni b ga bo‘lish natijasida hosil bo‘lgan kattalikdir. Xuddi shuningdek, $\frac{ab-bb}{a+x}$ kattalik $ab-bb$ ni $a+x$ ga bo‘lish natijasida hosil bo‘ladi”, – deydi Nyuton.

TADQIQOT UCHUN MASHQLAR*

Tadqiqot uchun. IV bob 5-§ da Bezu teoremasi isbotlangan edi. Bu teoremadan shunday natija kelib chiqadi. a son $P(x)$ ko'phadning ildizi bo'lishi uchun $P(x)$ning $(x-a)$ga qoldiqsiz bo'linishi zarur va yetarlidir. Ya'ni:
1) agar a son $P(x)$ ko'phadning ildizi bo'lsa, u holda $P(x)$ $(x-a)$ ga qoldiqsiz bo'linadi;
2) agar $P(x)$ ko'phad $(x-a)$ ga qoldiqsiz bo'linsa, u holda $x=a$ son $P(x)$ ning ildizi bo'ladi. (Agar $P(a)=0$ bo'lsa, $x=a$ son $P(x)$ ko'phadning ildizi deyilishini eslatib o'tamiz).
Yuqorida keltirilgan natija koeffitsiyentlari butun sonlardan iborat bo'lgan tenglamalarni yechishda nihoyatda foydalidir.

Misol. $4x^4 + 9x^3 - 2x^2 - 9x - 2=0$ tenglamani yeching. Bu tenglamaning koeffitsiyentlari va ozod hadi butun sonlar: 4, 9, −2, −9, −2. Tenglamani chap qismidagi ko'phadni ko'paytuvchilarga ajratish mumkin. $x=1$ tenglamaning ildizi ekanligini tenglamadagi x o'rniga 1 ni qo'yib bevosita tekshirib ko'rish mumkin. Demak,
$P_4(x)=4x^4+9x^3-2x^2-9x-2$ ko'phad $(x-1)$ ga bo'linadi. Bo'lishni ustun usulida bajaring. U holda, $P_4(x)=(x-1)\cdot(4x^3+13x^2+11x+2)$ tenglikni olamiz.
$P_3(x)=4x^3 + 13x^2 + 11x + 2$ ko'phadning ildizi $x=-1$ ekanligi ravshan:
$4(-1)^3 + 13(-1)^2+11(-1)+ 2-2-4-11=15-15=0$
$P_3(x)$ ko'phadni $(x+1)$ ga ustun usulda bo'ling va $P_3(x)=(x+1)\cdot(4x^2+9x+2)$ tenglikni hosil qiling.
$P_2(x)=4x^2+9x+2$ ko'phad $(x+2)$ ga qoldiqsiz bo'linishini ko'rish oson: $P_2(-2)=0$. (O'zingiz tekshiring!) U holda $P_2(x)=(x+2)(4x+1)$ tenglikka kelamiz. Demak, $P_4(x)$ ko'phad ko'paytuvchilarga quyidagicha ajraladi:
$P_4(x)=4x^4+9x^3-2x^2-9x-2=(x-1)(x+1)(x+2)(4x+1)$.

Bundan berilgan tenglamaning ildizlari: $x=1$, $x=-1$, $x=-2$, $x=-\frac{1}{4}$ ekanini topamiz. Tenglamani boshqa usulda yechish ham mumkin. Faraz qilaylik, berilgan tenglamaning ildizi $x=a$ bo'lsin. U holda $P_4(a)=0$, ya'ni $4a^4 + 9a^3 - 2a^2 - 9a - 2=0$. Bundan $a\cdot(4a^3 - 9a^2 - 2a - 9)=2$. Bu tenglikning chap qismi a ga bo'linadi, u holda tenglamaning o'ng qismi ham a ga bo'linishi lozim. 2 ning bo'luvchilari: −1, −2, 1, 2. Bu sonlarni $P_4(x)$ ko'phaddagi x o'rniga galma-gal qo'yib $P_4(x)$ ning qiymatlarini hisoblaymiz. $P_4(1)=0$, $P_4(-1)=0$, $P_4(-2)=0$ ekanini darhol topamiz, ammo $P_4(2)\neq 0$. Shunday qilib, $P_4(x)$ ko'phad $(x-1)$, $(x+1)$, $(x+2)$ ga (demak, ularning ko'paytmasiga ham) bo'linadi. $P_4(x)$ ko'phadni $(x-1)(x+1)(x+2)=(x^2-1)(x+2)=x^3+2x^2-x-2$ ko'phadga ustun usulida bo'lib, bo'linmada

$4x+1$ ni hosil qilamiz. $P_4(x)=0$ tenglamaning 3 ta butun va bitta kasr ildizi bor ekan.

Butun ildizlarni topishda qo'llanilgan bu usul umumiy holda ham to'g'ri bo'lib, uni shunday bayon qilish mumkin:

Agar butun koeffitsiyentli tenglamaning butun ildizi mavjud bo'lsa, bu ildiz ozod hadning bo'luvchisidir.

1. O'zingizga qulay usuldan (yuqoridagi tasdiq, Bezu teoremasi, ko'phadni ko'paytuvchilarga ajratish va h. k.) foydalanib, tenglamaning butun ildizlarini toping:

1) $x^3-6x^2+11x-6=0$;
2) $x^3-5x^2-17x-21=0$;
3) $x^3-12x^2+9x+22=0$;
4) $2x^3+x^2-5x+2=0$;
5) $2x^3-5x^2-22x-15=0$;
6) $x^4-10x^3+5x^2+40x-36=0$;
7) $x^4-2x^3+2x-1=0$;
8) $2x^3-x-2=0$;
9) $x^4+x^3-7x^2-13-6=0$;
10) $x^5-3x^4-5x^3+15x^2+4x-12=0$.

Tadqiqot uchun. Qisqa ko'paytirish formulalaridan biri ushbu

$$a^3-b^3=(a-b)(a^2+ab+b^2) \quad (1)$$

ayniyatdir. (1)ga ko'ra a^3-b^3 ikkihad ikkita ko'paytuvchilarning ko'paytmasiga ajraladi. Ko'paytuvchilardan biri $a-b$ ikkihad, ikkinchisi esa (a^2+ab+b^2) – uchhaddir. (1) ni

$$(a^3-b^3):(a-b)=(a^2+ab+b^2) \quad (2)$$

yoki

$$(a^3-b^3):(a^2+ab+b^2)=(a-b) \quad (3)$$

kabi yozib olish mumkin, bunda $a\neq b$. (2) formula a^3-b^3 ikkihadni $a-b$ ikkihadga bo'lishni, (3) formula esa a^3-b^3 ikkihadni uchhadga bo'lishni ifodalaydi. III bobda ko'phadni birhadga bo'lishni o'rgangan edik. Demak, ko'phadni ikkihadga, uchhadga, ..., ko'phadga bo'lish masalasini qo'yish mumkin. Ana shu masala bilan tanishaylik.

Ko'phadni ko'phadga ustun usulida bo'lish

Ko'phadni ko'phadga ustun usulida bo'lish uchun:

– bo'linuvchi va bo'luvchini ularga kirgan ayni bir harf darajasining ko'rsatkichi kamayish tartibida yozib olinadi;

– bo'linuvchining birinchi (eng katta daraja ko'rsatkichli) hadini bo'luvchining birinchi hadiga bo'lib, bo'linmaning birinchi hadi hosil qilinadi;

– bo'linmaning topilgan bu birinchi hadi bo'luvchiga ko'paytiriladi va ko'paytma bo'linuvchidan ayiriladi;

– hosil bo'lgan ayirma (birinchi qoldiq)ning birinchi hadini bo'luvchining birinchi hadiga bo'linadi, natijada bo'linmaning ikkinchi hadi hosil qilinadi;

– bo‘linmadagi ikkinchi had bo‘luvchiga ko‘paytiriladi va ko‘paytma birinchi qoldiqdan ayiriladi, natijada ikkinchi qoldiq hosil bo‘ladi;
– bu jarayon shu tarzda davom ettiriladi. Bu jarayon chekli qadamdan so‘ng to‘xtaydi.

Agar qoldiqning eng katta daraja ko‘rsatkichli hadi bo‘luvchining birinchi hadiga bo‘linmasa, u holda bo‘lish qoldiqlidir.

1-misol. Ko‘phadni ko‘phadga bo‘ling: $(15-3a^3+5a^2-9a):(5-3a)$.

□ 1) bo‘linuvchi va bo‘luvchini a ning daraja ko‘rsatkichi bo‘yicha tartiblab yozamiz:

$$\begin{array}{r|l} -3a^3+5a^2-9a+15 & -3a+5 \\ \underline{-3a^3+5a^2} & \underline{} \\ -9a+15 & a^2+3 \\ \underline{-9a+15} & \\ 0 & \end{array}$$

2) $-3a^3 : (-3a)=a^2$ — bo‘linmaning birinchi hadi;
3) a^2 ni $(-3a+5)$ ga ko‘paytiramiz, natijani bo‘linuvchidan ayiramiz;
4) Ayirma (birinchi qoldiq)ning birinchi hadi $(-9a)$ ni $(-3a)$ ga bo‘lamiz: $(-9a):(-3a)=3$.
5) 3 ni $(-3a+5)$ ga ko‘paytiramiz, natijani birinchi qoldiqdan ayiramiz: nol chiqadi. Bu misolda qoldiq nolga teng bo‘ldi.
$(-3a+5)(a^2+3)$ ko‘paytmani bevosita hisoblab, bo‘lishning to‘g‘ri bajarilganiga ishonch hosil qilamiz.

Javob: (a^2+3). ■

2-misol. Ko‘phadni ko‘phadga bo‘ling: $(a^4+2a^2-6):(a^2-3)$.

□
$$\begin{array}{r|l} a^4+2a^2+6 & a^2-3 \\ \underline{a^4-3a^2} & \underline{} \\ -5a^2+6 & a^2+5 \\ \underline{-5a^2-15} & \\ 21\ \text{(qoldiq)}. & \end{array}$$

Demak, $a^4+2a^2+6=(a^2-3)(a^2+5)+21$. ■

Bu holda oxirgi qoldiqda a ning nolinchi darajasi qatnashyapti deyishimiz mumkin: $21=21\cdot a^0$. Ammo bo‘luvchida a ning darajasi ikkiga teng. 21 soni a^2 ga bo‘linmaydi. Bu misolda bo‘lish qoldiqli bo‘ldi.

2. Ko‘phadni ko‘phadga bo‘ling

1) $(a^2+2ab+b^2):(a+b)$;
2) $(a^2-2ab+b^2):(a-b)$;
3) $(a^3+b^3):(a+b)$;
4) $(a^3-b^3):(a-b)$;
5) $(a^2-b^2):(a+b)$;
6) $(a^6-b^6):(a^4+a^2b^2+b^4)$;
7) $(a^6+b^6):(a^2+b^2)$;
8) $(a^3+3a^2b+3ab^2+b^3):(a+b)$.

3. Ko‘phadni ko‘phadga bo‘ling

1) $(a^3-3a^2b+3ab^2-b^3):(a-b)$;

2) $(8a^3 - 27b^3) : (4a^2 + 6ab + 9b^2)$;
3) $(8a^3 + 27b^3) : (2a + 3b)$;
4) $(a^4 - ab^3 - a^2b^2 + a^3b) : (a^2 - b^2)$;
5) $(a^5 + a^2b^3 - a^5 + a^3b^2) : (a^3 + b^3)$;
6) $(2a^3 - 15b^2 + 6ab^2 - 5a^2b) : (2a - 5b)$;
7) $(3a^5 + a^4 - 3a^3 - 3a^2 + 2) : (-a^2 + 1)$.

4. Ko'phadni ko'phadga bo'ling. Natijani $P(x) = S(x) \cdot q(x) + r(x)$ ko'rinishida yozing, bunda $P(x)$ — bo'linuvchi, $S(x)$ — bo'luvchi, $q(x)$ — to'liqsiz bo'linma, $r(x)$ — qoldiq.
1) $(3x^4 + 5x^3 - 2x + 6) : (x^2 + 1)$;
2) $(2x^4 - 2x^3 + 3x^2 - 2x) : (2x^2 + 3)$;
3) $(5x^3 + 3x^2 - 6x + 2) : (x - 3)$;
4) $(2x^3 + 5x^2 - 12x + 4) : (x + 4)$;
5) $(x^5 - x^3 + 4x^2 + 3x + 4) : (x^3 + 4)$;
6) $(x^5 - x^4 - 3x^2 + 3x - 7) : (x^3 - 3)$.

5. a ning qanday qiymatlarida:
1) $x^2 + ax + 8$ ko'phad $(x - 2)$ ga;
2) $x^2 - 12x + a$ ko'phad $(x + 3)$ ga qoldiqsiz bo'linadi.

6. a ning qanday qiymatlarida:
1) $x^2 + ax + 8$ ko'phadni $(x - 2)$ ga bo'lganda qoldiq 16 ga teng bo'ladi?
2) $x^2 + ax + 8$ ko'phadni $(x + 3)$ ga bo'lganda qoldiq 10 ga teng bo'ladi?

Tadqiqot uchun. ***Mukammal sonlarning ba'zi xossalari***

O'zidan boshqa barcha bo'luvchilarining yig'indisiga teng son *mukammal son* deyilishini bilasiz. Masalan, 6, 28, 496, 8128 sonlar mukammal sonlardir. Agar p va $(2^p - 1)$ sonlar tub sonlar bo'lsa, u holda $2^{p-1} \cdot (2^p - 1)$ son mukammal son bo'ladi. Bu tasdiqni buyuk yunon matematigi Evklid (e.o. III asr) isbotlagan.

$2^{12} \cdot (2^{13} - 1) = 33\ 550\ 336$;
$2^{16} \cdot (2^{17} - 1) = 8\ 589\ 869\ 056$ (10 ta raqam);
$2^{18} \cdot (2^{19} - 1) = 137\ 438\ 691\ 328$ (12 ta raqam) — mukammal sonlardir.

Mukammal sonlarning ba'zi xossalarini keltiramiz.

1. Mukammal sonlar "uchburchakli" sonlardir, ya'ni ularni $1 + 2 + 3 + \ldots + n$ ko'rinishida tasvirlash mumkin. Masalan, $6 = 1 + 2 + 3$.
1) 28; 2) 496 mukammal sonlarni $1 + 2 + \ldots + n$ ko'rinishida tasvirlang;
3) 28 uchun n nechaga teng? 4) 496 uchun-chi?

2. 6 dan boshqa hamma mukammal sonlar toq sonlarning kublari yig'indisi ko'rinishida tasvirlanishi mumkin. Masalan, $28 = 1^3 + 3^3$.
1) 496; 2) 8128 mukammal sonlarni $1^3 + 3^3 + \ldots + (2n - 1)^3$

ko‘rinishida tasvirlang; 3) 496 uchun $(2n-1)$ nechaga teng? 4) 8128 uchunchi?

3. Mukammal son barcha bo‘luvchilari teskarilarining yig‘indisi 2 ga teng. Masalan, 6 ning barcha bo‘luvchilari: 1, 2, 3, 6. Bu sonlarning teskarilari: $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}$. Ularning yig‘indisi esa $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1+\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{6}\right)=1+1=2.$

Bu xossaning to‘g‘riligini 28 va 496 mukammal sonlar uchun tekshiring.

4. Mukammal sonlar 2 ning darajalari yig‘indisi ko‘rinishida tasvirlanishi mumkin. Masalan,

$6=2+2^2$; $28=2^2+2^3+2^4$.

1) 496; 2) 8128 mukammal sonlarni 2 ning darajalari yig‘indisi ko‘rinishida ifodalang.

5. 6 dan boshqa mukammal son raqamlarining yig‘indisini topib, hosil bo‘lgan sonning yana raqamlari yig‘indisini topsak, v. h., oxirgi yig‘indi 1 ga teng bo‘ladi. Masalan, $28\to2+8=10\to1+0=1$; $496\to4+9+6=19\to1+9=10\to1+0=1$.

Bu xossaning to‘g‘riligini: 1) 8128; 2) 33 550 336; 3) 8 589 869 056 mukammal sonlar uchun tekshirib ko‘ring.

Tadqiqot uchun. ***O‘zini hosil qiluvchi sonlar***

1. 153 soni shunday xususiyatga ega:

$153=1^3+5^3+3^3$. 153 sonida 3 ta raqam bor: 1; 5; 3.

Ularning daraja ko‘rsatkichlari 3 – raqamlar soniga teng.

1) 370; 2) 371; 3) 407 sonlarning ham shu xususiyatga ega ekanini ko‘rsating.

2. 4 xonali 9474 sonida 4 ta raqam bo‘lgani uchun uni $9^4+4^4+7^4+4^4$ ko‘rinishda yozib ko‘raylik-chi, bu yig‘indi, haqiqatan ham, 9474 ga teng bo‘larmikan?

Hisoblab ko‘ramiz: $9^4=6561$; $4^4=256$; $7^4=2401$,

$6561+256+2401+256=9474$.

Bunday xossaga ega bo‘lgan sonlar *o‘zini hosil qiluvchi sonlar* deyiladi.

1) 1634; 2) 8208; 3) 4150;

4) 4151 sonlar o‘zini hosil qiluvchi sonlar ekanini ko‘rsating.

3. 54 748 soni o‘zini hosil qiluvchi sonmi? Tekshirib ko‘ramiz. Bu son 5 xonali bo‘lgani uchun uning har bir raqamini 5-darajaga ko‘taramiz va hosil bo‘lgan darajalarni qo‘shamiz: $5^5+4^5+7^5+4^5+8^5=3\,125+1\,024+16\,807+1\,024+32\,768=54\,748$. Demak, 54 748 o‘zini hosil qiluvchi son ekan. 1) 92 727; 2) 93 084; 4) 194 979; 5) 548 834 sonlar o‘zini hosil qiluvchi sonlarmi? Tekshirib ko‘ring.

Tadqiqot uchun. ***Ratsional sonni yana qanday tasvirlash mumkin?***

Har qanday musbat ratsional sonni $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ..., \frac{1}{n}, ...$ (1) ketma-ketlikning har xil hadlarining chekli yig'indisi ko'rinishida ifodalash mumkin.

○ Odatda, $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{n}...$ qatorni *garmonik qator* deyishadi. Uning n-hadi $\frac{1}{n}$ o'zining chap va o'ng tomonidagi $\frac{1}{n-1}$ va $\frac{1}{n+1}$ hadlarning o'rta garmonik miqdoridir:

$$\frac{2\cdot\frac{1}{n-1}\cdot\frac{1}{n+1}}{\frac{1}{n-1}+\frac{1}{n+1}}=\frac{\frac{2}{n^2-1}}{\frac{n+1+n-1}{n^2-1}}=\frac{2}{2n}=\frac{1}{n}.$$

Eslatib o'tamizki, agar a va b musbat sonlar bo'lsa, ularning o'rta garmonik miqdori $H_2=\frac{2}{\frac{1}{a}+\frac{1}{b}}=\frac{2ab}{a+b}$ kabi aniqlanadi. $\frac{a}{b}$ musbat ratsional son bo'lsin ($a\in N$, $b\in N$). $\frac{a}{b}$ ni (1) ketma-ketlikning a ta bir xil takrorlanuvchi hadlarning yig'indisi ko'rinishida yozish mumkin:

$$\frac{a}{b}=\frac{1}{b}+\frac{1}{b}+...+\frac{1}{b}.$$

Birinchi qo'shiluvchini o'zgarishsiz qoldirib, qolgan $(a-1)$ ta qo'shiluvchini ushbu

$$\frac{1}{n}=\frac{1}{n+1}+\frac{1}{n(n+1)}$$

ayniyat yordamida ko'rinishini o'zgartiramiz. So'ngra takrorlanuvchi qo'shiluvchilarga yana shu ayniyatni qo'llaymiz va hokazo. Hamma qo'shiluvchilar har xil bo'lguncha bu jarayonni davom ettiramiz.

Masalan: $\frac{3}{7}=\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=\frac{1}{7}+\frac{1}{8}+\frac{1}{56}+\frac{1}{8}+\frac{1}{56}=\frac{1}{7}+\frac{1}{8}+\frac{1}{56}+\left(\frac{1}{8}+\frac{1}{56}\right)=$

$=\frac{1}{7}+\frac{1}{8}+\frac{1}{56}+\frac{1}{9}+\frac{1}{72}+\frac{1}{57}+\frac{1}{3192}$. ●

1-masala. Kishining boshidagi soch tolalari soni 380 000 tadan ortiq emas. *A* shaharda 3 400 000 kishi yashaydi. Bu shaharda boshidagi soch tolalari soni bir xil bo'lgan kamida 8 nafar kishi borligini isbotlang.

Juda g'alati masala-ku! Biror formulaga, qoidaga "tushadimi" bu masala? Uni yechish yo'li qanday?

Bunday masalalarni yechish ***Dirixle prinsipi*** ga asoslangan. Uni sodda holda (va hazil tariqasida) shunday bayon etish mumkin: m ta yashikka n ta quyon joylashtirilgan va $n > m$ bo'lsa, u holda kamida bitta yashikdagi quyonlar soni bittadan ortiq. Bu tasdiq nemis matematigi P.Dirixle tomonidan aytilgan. Uni ba'zan "yashiklar prinsipi" ham deyishadi. Ko'pincha, umumlashgan Dirixle prinsipidan foydalanishadi: agar quyonlar soni $n > m \cdot k$ bo'lsa, u holda aqalli bitta yashikdagi quyonlar soni k tadan ko'p.

2-masala. Sinfda 30 ta o'quvchi bor. Ularning kamida 3 tasining tug'ilgan kuni bir oyga to'g'ri kelishini isbotlang.

□ 1 yildagi oylar soni 12 ta, sinfdagi o'quvchilar soni esa 30 ta. O'quvchilarning tug'ilgan oylarini 12 ta oyga "taqsimlab" chiqamiz. $30 : 12 = 2$ (6 qoldiq). 6 o'quvchining tug'ilgan oyi 12 ta oyga yana "tarqatiladi". Demak, kamida 3 nafar o'quvchining tug'ilgan kuni ma'lum bir oyda bo'ladi. Agar o'quvchilar "quyonlar" rolida, oylar "yashiklar" rolida desak, masalani shunday ta'riflash mumkin: 30 ta quyonni 12 ta yashikka joylashtirilgan bo'lsa, kamida bitta yashikdagi quyonlar soni 2 tadan ortiq, chunki $30 > 12 \cdot 2$. Shunday qilib, berilgan masala Dirixle prinsipi yordamida darhol yechiladi. ■

Endi 1-masalani o'zingiz yecha olasiz.

3-masala. n ta butun son berilgan. Ularning ichida yig'indisi n ga bo'linadigan bir nechta son topilishini isbotlang. (n ga bo'linadigan son bitta bo'lishi ham mumkin).

○ $a_1, a_2 \ldots, a_n$ — berilgan n ta butun son bo'lsin. Boshqa n ta sonni qaraymiz: $a_1, a_1 + a_2, a_1 + a_2 + a_3, \ldots, a_1 + a_2 + \ldots + a_n$.

Ikkita hol bo'lishi mumkin:

1) bu sonlarning aqalli bittasi n ga bo'linadi, u holda da'vo isbotlandi.

2) bu sonlarning birortasi ham n ga bo'linmaydi. Jami sonlar n ta bo'lgani uchun bu sonlarni n ga bo'lganda 0 dan farqli qoldiqlar soni $n - 1$ dona bo'ladi. Demak, Dirixle prinsipiga ko'ra kamida ikkita son borki, ularni n ga bo'lganda bir xil (o'zaro teng) qoldiqlar chiqadi. Bu sonlarning ayirmasi esa n ga qoldiqsiz bo'linadi. Bu ayirma berilgan sonlarning bir nechtasining yig'indisidan iborat bo'ladi. Da'vo isbotlandi. ●

Bu masalada "quyonlar" n ta: $a_1 + a_2, a_1 + a_2 + a_3, \ldots, a_1 + a_2 + \ldots + a_n$. "Yashiklar" esa qoldiqlar (turli xil qoldiqlar $n-1$ ta edi), ular $n-1$ ta.

Dirixle prinsipidan foydalanib, quyidagi masalalarni hal qiling:

1. n ta butun son berilgan. Ulardan bir nechtasini hamma vaqt shunday tanlab olish mumkinki, tanlangan sonlar orasiga "+" va "−" ishorasini qo'yib, n ga bo'linadigan sonni hosil qilish mumkinligini isbotlang.

2. $\frac{m}{n}$ oddiy kasrni o'nli kasrga aylantirganda davrdagi raqamlar soni ("davrning uzunligi") $(n-1)$ tadan ortiq bo'la olmasligini ko'rsating.

3. Tomoni uzunligi 1 ga teng kvadrat ichida uzunliklari yig'indisi 10 ga teng bo'lgan bir nechta aylana bor. Bu aylanalarning kamida 4 tasini kesib o'tuvchi to'g'ri chiziq mavjudligini isbotlang.

4. Tomoni uzunligi 1 ga teng kvadrat ichida radiuslari yig'indisi 0,51 ga teng bo'lgan bir nechta doira bor. Kvadratning tomonlaridan biriga parallel va kamida 2 ta doirani kesuvchi to'g'ri chiziq mavjudligini isbotlang.

Rivojlantiruvchi mashqlar*

1. Quyidagi xossaga ega bo'lgan eng kichik natural son n ni toping:
1) uning o'nli martabali sanoq sistemasidagi yozuvi 6 raqami bilan tugaydi;
2) agar shu oxirgi 6 raqamini birinchi o'ringa olib yozilsa, hosil bo'lgan son berilgan sondan 4 marta katta bo'ladi.

□ Masala shartini qanoatlantiruvchi son m bo'lsin. U holda $m=\overline{a_1a_2...a_{n-1}6}$ va $k=4m=\overline{6a_1a_2...a_{n-1}}$. m sonning oxirgi raqami 6 bo'lgani uchun $k=4m$ sonning oxirgi raqami 4 bo'ladi, ya'ni $a_{n-1}=4$. Bu sonni m ga qo'yib, m ni 4 ga ko'paytirsak, $a_{n-2}=8$ ekanini bilib olamiz. m sonda a_{n-2} o'rniga 8 ni qo'yib, 4 m ni hisoblasak, $a_{n-3}=3$ ligini topamiz, bu jarayonni 4 m sonda 6 ni hosil qilguncha davom ettiraveramiz. Natijada, 153 846 sonni hosil qilamiz. Bu son masala shartini qanoatlantiradigan eng kichik natural sondir.
Javob: 153 846. ■

2. n ning 2^n-1 son 7 ga bo'linadigan barcha natural qiymatlarini toping.
□ $n=3k$; $n=3k+1$; $n=3k+2$ hollarni alohida qaraymiz.
1) $n=3k$ bo'lsin, k – natural son. U holda (2^n-1) ni shunday yozib olish mumkin: $2^n-1=2^{3k}-1=8^k-1$, ammo 8^k-1 son $8-1=7$ ga bo'linadi. (a^n-b^n) ifoda $(a-b)$ ga bo'linishini eslang). Demak, n soni 3 ga karrali bo'lsa, ya'ni $n=3k$ bo'lsa, 2^n-1 son 7 ga bo'linadi.
2) $n=3k+1$ bo'lsin. Bu holda $2^n-1=2^{3k+1}-1=2\cdot 8^k-1=2(7+1)^k-1$, ammo $(7+1)^k$ sonni 7 ga bo'lganda 1 qoldiq chiqishi ravshan. Demak, $2\cdot(7+1)^k$ sonni 7 ga bo'lganda 2 qoldiq chiqadi. Ya'ni $2^n-1=2\cdot(7+1)^k-1$ sonni 7 bo'lganda 1 qoldiq chiqadi. Shunday qilib, $n=3k+1$ bo'lganda 2^n-1 son 7 ga bo'linmaydi.

3) $n=3k+2$ bo‘lsin. Bu holda $2^n-1=2^{3k+2}-1=4\cdot(7+1)^k-1$ va $4\cdot(7+1)^k-1$ sonni 7 ga bo‘lganda 3 qoldiq chiqadi. Xulosa: 2^k-1 son n soni 3 ga karrali bo‘lganda va faqat shu holda 7 ga bo‘linadi.

Javob: $n=3k$, k – natural son. ■

3. Agar $\frac{x+z}{2}=y$ bo‘lsa, $x^4+2x^3z-2xz^3-z^4-4x^2y^2+4y^2z^2=0$ ekanligini isbotlang.

4. Agar a, b, c, d sonlar $b^2=ac$, $c^2=bd$ munosabatlarni qanoatlantirsa, $(a-c)^2+(b-c)^2-(a-d)^2$ ifodani soddalashtiring.

5. 1, 2, 3, 4, 5 raqamlarni bo‘sh kataklarga shunday joylashtiringki: 1) raqamlar hech bir qator va hech bir ustunda takrorlanmasin; 2) < (“...dan kichik”), >(“...dan katta”) belgilar bilan bog‘langan kvadratchalardagi sonlar shu belgilar ma’nosiga mos kelsin.

□	□	□	□	<	□	3	□	□	<	□	□
□	□	>	2	□	□	□	1	□	>	□	□
□	3	□	2	□	□	□	>	□	□	□	□
□	<	5	4	□	□	□	□	□	>	2	<	□
□	□	□	□	□	□	□	<	□	□	□	□

6. 1) a, b, c, d harflarning qanday qiymatlarida $5x^3-32x^2+75x-71=a(x-3)^3+b(x-2)^2+c(x-2)+d$ tenglik ayniyat bo‘ladi?
2) $3x^3+7x^2+9x+6$ ko‘phadni ay^3+by^2+cy+d ko‘phad ko‘rinishida tasvirlang, bunda $y=x+1$.

7. Ixtiyoriy butun n da: 1) $(2n+3)(3n-7)-(n+1)(n-1)$;
2) $(7n+8)(n-1)+(3n-2)(n+2)$ ifodalar 5 ga bo‘linadimi?

8. 1) $(10n+5)^2=100n\cdot(n+1)+25$ ayniyatni isbotlang. Bundan foydalanib, 5 raqami bilan tugaydigan natural sonni kvadratga oshirish qoidasini bayon qiling va uni daftaringizga yozib qo‘ying.
2) Topgan qoidangizga ko‘ra: a) 95^2; b) 125^2; d) 75^2; e) 105^2 sonlarni toping.

9. Ayniyatni isbotlang:
1) $(ax-2(a+2))(a(x-1)+2)+2(4-a^2)+3a^2x=ax\cdot(ax-2)$;
2) $(3-b(c-1))(bc+4(b+1))+bc(bc+3b+1)=4b\cdot(b+4)+12$;
3) $(a+b)^2(a-b)-2ab(b-a)-6ab(a-b)=(a-b)^3$;
4) $(a+b)(a-b)^2+2ab(a+b)-2ab(-a-b)=(a+b)^3$;

5) $(a^2+b^2)(a^4-a^2b^2+b^4)-(a^3-b^3)(a^3+b^3)=2b^6$;
6) $(a^2+ab+b^2)(a^2-ab+b^2)=a^4+a^2b^2+b^4$.

10. Agar $b+c=10$ bo'lsa, $(10a+b)(10a+c)=100a(a+1)+bc$ ekanini isbotlang. Bu formuladan foydalanib,
1) $52\cdot58$; 2) $59\cdot51$; 3) $67\cdot63$; 4) $74\cdot76$.
ko'paytmalarni hisoblang.

11. 1) agar $ab+c^2=0$ bo'lsa, $(a+c)(b+c)+(a-c)(b-c)=0$ ekanini;
2) agar $a+b=9$ bo'lsa, $(a+1)(b+1)-(a-1)(b-1)=18$ ekanini isbotlang.

12. 1, 2, ..., 13, 14 sonlarni aylana bo'ylab shunday joylashtiringki, bunda har ikkita qo'shni sonlar orasidagi farq 3, 4 yoki 5 ga teng bo'lsin.

13. $\sqrt{-x^2+2x}+\sqrt{8-x^2+2x}=4$ tenglamani yeching.
Ko'rsatma: $\sqrt{1-(x-1)^2}+\sqrt{9-(x-1)^2}=4$ ekanidan foydalaning.

14. $\frac{77}{60}<\frac{1}{1996}+\frac{1}{1997}+...+\frac{1}{9975}<\frac{25}{12}$ (1) tengsizlikni isbotlang.

□ Ushbu munosabatlarning o'rinli ekanligi ravshan:

$$1995\cdot\frac{1}{2\cdot1995}<\frac{1}{1996}+\frac{1}{1997}+...+\frac{1}{2\cdot1995}<1995\cdot\frac{1}{1995};$$

$$1995\cdot\frac{1}{3\cdot1995}<\frac{1}{2\cdot1995+1}+...+\frac{1}{3\cdot1995}<1995\cdot\frac{1}{2\cdot1995};$$

$$1995\cdot\frac{1}{4\cdot1995}<\frac{1}{3\cdot1995+1}+...+\frac{1}{4\cdot1995}<1995\cdot\frac{1}{3\cdot1995};$$

$$1995\cdot\frac{1}{5\cdot1995}<\frac{1}{4\cdot1995+1}+...+\frac{1}{5\cdot1995}<1995\cdot\frac{1}{4\cdot1995}.$$

Bu tengsizliklarni hadma-had qo'shib (1) tengsizlikni hosil qilamiz. ■

15. Ixtiyoriy musbat a, b, c sonlar uchun

$$\frac{a^3}{b+2c}+\frac{b^3}{c+2a}+\frac{c^3}{a+2b}\ge\frac{a^2+b^2+c^2}{3}$$

tengsizlikning o'rinli ekanini isbotlang.

□ O'rta arifmetik va o'rta geometrik miqdorlar orasidagi tengsizlikdan foydalanamiz:

$$\frac{a^3}{b+2c}+\frac{(b+2c)a}{9}\ge2\cdot\sqrt{\frac{a^3}{b+2c}\cdot\frac{(b+2c)a}{9}}=2\cdot\frac{a^2}{3}=\frac{2a^2}{3},$$

$$\frac{b^3}{c+2a}+\frac{(c+2a)b}{9}\ge\frac{2}{3}b^2,\quad\frac{c^3}{a+2b}+\frac{(a+2b)c}{9}\ge\frac{2}{3}c^2.$$

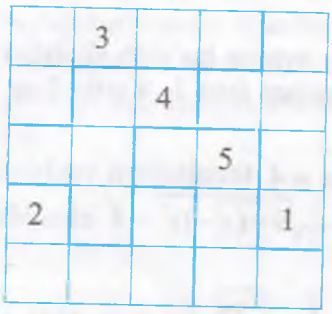
Bu tengsizliklarni hadma-had qo'shib, ushbu tengsizlikni hosil qilamiz:

$$\frac{a^3}{b+2c}+\frac{b^3}{c+2a}+\frac{c^3}{a+2b}+\frac{1}{3}(ab+bc+ca)\ge\frac{2}{3}(a^2+b^2+c^2).$$

Ammo $a^2+b^2+c^2 \ge ab+bc+ca$. U holda

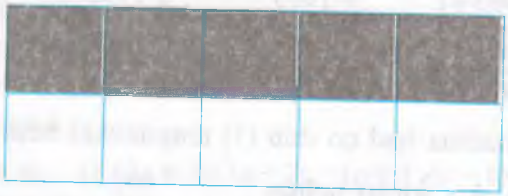
$$\frac{a^3}{b+2c}+\frac{b^3}{c+2a}+\frac{c^3}{a+2b}\ge\frac{2}{3}(a^2+b^2+c^2)-\frac{1}{3}(ab+bc+ca)\ge\frac{1}{3}(a^2+b^2+c^2). \blacksquare$$

16. Kvadrat to'q ko'k rangli quyuq chiziqlar bilan 5 qismga ajratilgan. Bu qismlarga 1, 2, 3, 4, 5 raqamlarini shunday joylashtiringki, raqamlarning har biri har bir qismda bittadan bo'lsin.

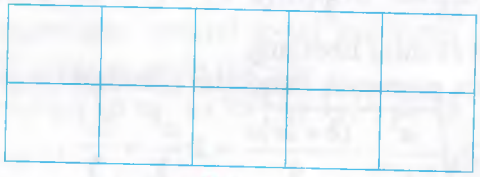


17. Dilafruz to'g'ri to'rtburchakning har birini perimetri 6 bo'lgan 5 ta qismga ajratdi.

Sirojiddin xuddi shunday to'g'ri to'rtburchakning har birini perimetri 12 bo'lgan ikkita qismga ajratdi.



Feruza ham shunday to'g'ri to'rtburchakning har birini perimetri 8 bo'lgan uchta qismga ajrata oladimi?



18. Kvadrat to'q ko'k rangli chiziqlar bilan 5 qismga ajratilgan. Bu qismlarga *A*, *B*, *C*, *D*, *E* harflarini shunday joylashtiringki, ularning har biri katakchalarda bittadan bo'lsin.

19. 1, 2, ...,
ring:

20. Bo'sh ka
sonlar yi
lar yig'in
bo'yicha
pastidagi

21. Har bir
jadvaldag

$72\ d^3$	
$3\ d$	
$:4\ d^2$	
$:9\ d^2$	
$\cdot 0{,}5$	
-2	

				A
E	B			
		D		
			C	

19. 1, 2, ..., 9 raqamlarni kvadratlarga to'g'ri tenglik hosil bo'ladigan qilib joylashtiring:

20. Bo'sh kataklarga 1, 2, ..., 9 raqamlarni shunday joylashtiringki, yo'llar bo'yicha sonlar yig'indisi o'ng ustundagi mos songa teng bo'lsin. Ustunlar bo'yicha sonlar yig'indisi pastki yo'ldagi (qatordagi) mos songa teng bo'lsin. Diagonallar bo'yicha sonlar yig'indisi esa mos ravishda o'ng ustunning eng tepasidagi va eng pastidagi songa teng bo'lsin. (Raqamlar takrorlanishi mumkin).

				25
		8		25
	9			28
			2	18
9				28
31	30	11	27	25

21. Har bir ustunda ko'rsatilgan amallarni bajaring. Chiqqan javob yonidagi harfni jadvaldagi javobga mos sonning tagiga yozing. Hosil bo'lgan so'zni o'qing.

$72\ d^3$	$144m^2$	$2{,}8\ c$	$3{,}2\ a$	$54\ k$	$90\ b^2$
$3\ d$	$-44\ m^2$	$:4$	$\cdot 5\ a$	$\cdot 2\ k$	$:15\ b$
$:4\ d^2$	$:10\ m$	$\cdot 10$	$+4\ a^2$	$:9\ k^2$	$+4\ b$
$:9\ d^2$	$\cdot 3$	$:2\ c$	$:10\ a^2$	$-2\ k^2$	$:2\ b$
$\cdot 0{,}5$	$:2\ m$	$\cdot 3$	$\cdot 5$	$:5\ k^2$	$\cdot 1{,}4$
-2	$+15$	$-3{,}5$	$-2,\ (6)$	$+23$	$+5$
e	*b*	*l*	*a*	*r*	*g*

7, (3)	7	12	1	30	25	7, (3)

VII SINF ALGEBRA KURSINI TAKRORLASH UCHUN MASHQLAR

Sonli ifodaning qiymatini toping **(1–3)**:

1. 1) $\left(5\frac{3}{4}-4\frac{8}{9}\right)\cdot 2+67\frac{1}{2}:2\frac{1}{7}$;

2) $\left(4\frac{1}{10}-3\frac{4}{15}\right)\cdot\frac{5}{6}+4\frac{1}{10}:1\frac{1}{5}$;

3) $1\frac{1}{4}+\frac{5}{12}:\left(\frac{1}{3}\cdot 2\frac{1}{2}-\frac{7}{8}\right)$;

4) $7\frac{5}{13}\cdot 2-1\frac{2}{5}\cdot 6+4\cdot 2\frac{4}{13}-3\cdot 1\frac{1}{5}$.

2. 1) $\dfrac{(0,5:1,25+\frac{7}{5}:1\frac{4}{7}-\frac{3}{11})\cdot 3}{(1,5+\frac{1}{4}):18\frac{1}{3}}$;

2) $\dfrac{(1,26:1,8+1\frac{8}{7}\cdot 1\frac{13}{15}-0,7)\cdot 1,75}{(2,5+1,3):1,9\cdot 7,5}$.

3. $\dfrac{\left(13,75+9\frac{1}{6}\right)\cdot 1,2}{\left(10,3-8\frac{1}{2}\right)\cdot\frac{5}{9}}+\dfrac{\left(6,8-3\frac{3}{5}\right)\cdot 5\frac{5}{6}}{\left(3\frac{2}{3}-3\frac{1}{6}\right)\cdot 56}-27\frac{1}{6}$.

Soddalashtiring **(4–5)**:

4. 1) $\left(\dfrac{a^2-4}{a^2+4}\right)^2+\left(\dfrac{4a}{a^2+4}\right)^2$;

2) $\dfrac{a^3+b^3}{a^2-ab+b^2}\cdot(a-b)\cdot\dfrac{a^3-b^3}{a^2+ab+b^2}\cdot(a+b)$.

5. 1) $\dfrac{3}{2}\left(\dfrac{2a}{3}-\dfrac{a}{7}\right)-\dfrac{12(a-5)}{7}+a+\dfrac{1}{3}\left(\dfrac{a}{2}-5a\right)$;

2) $2-\dfrac{x-a}{x+a}-\dfrac{x}{x+a}+\left(\dfrac{1}{a^2}-\dfrac{1}{ax}+\dfrac{2}{x^2}\right):\left(\dfrac{1}{a^2}-\dfrac{1}{x^2}\right)$.

6. $\left(\dfrac{x+2b}{x-2b}+\dfrac{x+2a}{x-2a}\right):\dfrac{x}{2}$ ifodani $x=\dfrac{4ab}{a+b}$ bo‘lganda soddalashtiring va $a=3$, $b=4$ bo‘lganda son qiymatini hisoblang.

7. Algebraik ifodaning son qiymatini toping:

1) $3\frac{3}{4}a-1\frac{1}{8}b+10,5$, bunda $a=1\frac{1}{4};b=2\frac{2}{3}$;

2) $6a+3\frac{1}{6}b-6,3$, bunda $a=1\frac{5}{6};b=7\frac{2}{5}$;

3) $\frac{3a-4b+c}{3a+4b-c}$, bunda $a=4{,}5$; $b=2{,}5$; $c=0{,}5$.

4) $\frac{(2a+b)\cdot c}{(2a-b)^2}$, bunda $a=2\frac{1}{5}$; $b=1\frac{2}{5}$; $c=15$.

8. Amallarning xossalaridan foydalanib, qulay usulda hisoblang:

1) $3\frac{16}{17}\cdot 2\frac{2}{67}+3{,}7\cdot 13{,}32+6{,}3\cdot 13{,}32$;

2) $28{,}75\cdot 52\frac{4}{19}-42\frac{4}{19}\cdot 28{,}75-20\frac{2}{7}\cdot 1\frac{6}{71}$.

9. Qavslarni oching va soddalashtiring:

1) $9\cdot(3a+2b)-4\cdot(a+3b)-5\cdot(2a-3b)$;
2) $2{,}5\cdot(2a-3b)+3{,}2\cdot(7a+3{,}5b)-3\cdot(4{,}5a-3{,}5b)$;
3) $6\cdot(1{,}5a-2{,}5b)-10\cdot(2{,}3a+1{,}7b)+8\cdot(2{,}5a+3{,}5b)$;
4) $\frac{17}{2}\cdot(4a+5b)-\frac{5}{4}\cdot\left(\frac{12}{25}a+\frac{4}{15b}\right)-2\frac{1}{7}\left(\frac{7}{15}a+3\frac{4}{15}b\right)$.

10. Qavslarni oching:

1) $(2a+(3b-4c))+4d$;
2) $5a+(4b-(3c-d)$;
3) $3a-(4b+(2c-(3d-2k)))$;
4) $4a-((2b-4c)-8d)$.

11. Tenglamani yeching:

1) $2(x-3)-3(x+2)+14=2x-4$;
2) $3(x-4)-2(3-x)-8=7$;
3) $\frac{3x-1}{5}-\frac{5x+1}{6}=\frac{x+1}{8}-3$;
4) $\frac{2-x}{5}+\frac{3-x}{4}+1{,}5=0{,}1\cdot(7-x)$.

12. Ushbu $\frac{x-2}{3}+\frac{9x-1}{9}=\frac{4x+5}{3}$ tenglama yechimga ega emasligini ko‘rsating.

13. a sonning $\frac{2x}{33}+\frac{a}{11}=(x+19)-\frac{31x}{33}$ tenglama cheksiz ko‘p ildizga ega bo‘ladigan qiymatlarini toping:

14. Tenglamani yeching:

1) $\frac{2}{3}x+\frac{3+x}{6}-2=\frac{3x-1}{8}$;
2) $\frac{5-x}{4}+\frac{x+2}{3}-1=\frac{13x-1}{12}$;
3) $\frac{x-3}{4}-\frac{1-2x}{5}=\frac{33}{20}$;
4) $0{,}2(5y-2)=0{,}3(2y-1)-0{,}9$.

15. Tenglamaning yechimga ega emasligini ko‘rsating:

1) $\frac{3x+4}{11}+\frac{6x-z}{22}=\frac{12x+8}{22}$;
2) $\frac{x-7}{7}+\frac{6x-5}{21}=\frac{3x+20}{7}$.

16. Uchta firmada 576 nafar ishchi bor. Ikkinchi firmada birinchisidagiga qaraganda 5 marta ko'p, uchinchi firmada esa birinchi va ikkinchi firmalarda nechta ishchi bo'lsa, shuncha ishchi bor. Har bir firmada nechtadan ishchi bor?

17. Uchta kichik korxonada 1 177 ta mahsulot tayyorlandi. Ikkinchi kichik korxonada birinchi kichik korxonaga qaraganda 3 marta ko'p, uchinchi kichik korxonada esa ikkinchidagidan 2 marta kam mahsulot tayyorlandi. Har bir kichik korxonada nechtadan mahsulot tayyorlangan?

18. Teng yonli uchburchakning perimetri 30 sm ga teng. Agar uning yon tomoni asosidan 6 sm ortiq bo'lsa, uchburchak tomonlari uzunliklarini toping.

19. Neft omboridan uch kunda 11 000 t neft tarqatildi. Ikkinchi kuni birinchi kundagining 3/8 qismicha, uchinchi kuni esa dastlabki ikki kunda qancha tonna tarqatilgan bo'lsa, shuncha tarqatildi. Neft omboridan birinchi kuni necha tonna neft tarqatilgan?

20. Bug'doy va paxta ekilgan maydonning yuzi 4 : 5 nisbatda. Agar hammasi bo'lib yer maydoni 4500 ga bo'lsa, necha gektar yerga paxta ekilgan?

21. Firma har birining tannarxi b so'mdan bo'lgan a ta kostyum tayyorladi. Ularning har birini bir xil bahoda sotib, c so'm foyda ko'rdi. Firma kostyumlarni necha so'mdan sotgan?

22. "Sog'lom bola yili" munosabati bilan bolalar bog'chasiga 36 ta uch g'ildirakli va ikki g'ildirakli velosipedlar sovg'a qilindi. Agar hamma velosipedlarning g'ildiraklari 93 ta bo'lsa, uch va ikki g'ildirakli velosipedlardan nechtadan sovg'a qilingan?

23. Birhadni standart shaklida yozing:

1) $2\frac{1}{3}a\cdot 3\frac{4}{7}b^3\cdot(-1\frac{3}{7}a^2)\cdot\frac{3}{11}c^2b$;

2) $3\frac{7}{12}bc^2\cdot(-3\frac{4}{25})a^2\cdot b\cdot(-3.4)cab$;

3) $x^2\cdot(-3)^3y^2zx(-1)^4yz^2x(-\frac{5}{9})yz$;

4) $(-2)^3\cdot z^3\cdot(-\frac{5}{7})y^2x^2(-2\frac{5}{8})\cdot zyx$.

24. Ko'phadni standart shaklga keltiring:

1) $6ab\frac{1}{2}ac+4aac-8b\frac{1}{2}a^2+20ca^2\frac{1}{5}-b3a^2c+2baa$;

2) $2a^2b+a^3+3a^2b-ab^2+4ab^2-2ab^2+5a^2b$;

3) $\frac{1}{2}a^2-\frac{1}{3}a^3+\frac{1}{2}a+\frac{1}{4}a^3-\frac{3}{4}a^2+0.4a$;

4) $6a\frac{1}{2}b+\frac{2}{3}a(\frac{3}{4}b^2)-\frac{1}{2}b(-2a)-\frac{1}{4}a^2\cdot(16ab)$.

25. Ko‘phadni birhadga ko‘paytiring:

1) $(4a^4-3a^3-2a^2+5a-6)\cdot 3a$;
2) $(5a^3-3a^2-7a+3)\cdot 5a^2$;
3) $5a^3\cdot(3{,}7a^2-4a+3{,}5)$;
4) $-3a^4\cdot(2a^3-3{,}5a^2+4a-2{,}5)$.

26. Ko‘phadni ko‘phadga ko‘paytiring:

1) $(a-2b)(a^2+2ab+4b^2)$;
2) $(a+2b)(a^2-2ab+4b^2)$;
3) $(4x+5y)(16x^2-20xy+25y^2)$;
4) $(4x-5y)(16x^2+20xy+25y^2)$.

27. Ko‘phadni birhadga bo‘ling:

1) $(13a^5+8a^3):(-2a^2)+(5{,}5a^3+3a^2):3{,}5$;
2) $(19b^7-18b^5):(2b^4)-(16b^3-4b^2):(2b)$.

28. Ko‘paytuvchilarga ajrating:

1) $a^4-6a^3+9a^2$;
2) $(3-2a)2-16$;
3) $a^4-6a^3+9a^2$;
4) $(4b-1)^2-(2b-3)^2$;
5) $(2a+b)^2-(a-b)^2$;
6) $4(a+2b)^2-9(2a-b)^2$.

29. Kasrni qisqartiring:

1) $\dfrac{a^2-4}{a^2-4a+4}$;
2) $\dfrac{9x^2-4}{3x-2}$;
3) $\dfrac{4x^2+4x+1}{2x+1}$;
4) $\dfrac{2x+3}{4x^2-9}$.

Amallarni bajaring **(30–35)** :

30. 1) $\dfrac{b-3}{5}+\dfrac{b+7}{10}+\dfrac{b+3}{2}$;
2) $\dfrac{a^2-5a+4}{16-a^2}$.

31. 1) $\dfrac{a}{a^2-4}-\dfrac{2}{4-a^2}$;
2) $\dfrac{4x^2}{2x+3y}-\dfrac{12xy}{2x+3y}+\dfrac{9y^2}{2x+3y}$;
3) $\dfrac{a+b}{ab}-\dfrac{a+c}{ac}$;
4) $\dfrac{1}{14x^3}-\dfrac{1}{21x^2y}+\dfrac{1}{4xy^2}$.

32. 1) $\dfrac{x^2-y^2}{6xy}\cdot\dfrac{12x^2y}{x-y}$;
2) $\dfrac{a^2+4a}{a^2-16}:\dfrac{4a+16}{a^2-4a}$.

33. 1) $\left(\dfrac{a}{a-1}\right):\left(1-\dfrac{a}{a-1}\right)$;
2) $\dfrac{4-a^2}{2+b}\cdot\dfrac{4-b^2}{2a+a^2}\cdot\left(1+\dfrac{2a}{2-a}\right)$.

34. 1) $\left(\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}\right):\left(\dfrac{a^2+b^2}{a^2-b^2}+\dfrac{a^2-b^2}{a^2+b^2}\right)$;
2) $\left(\dfrac{a+4b}{2b}+\dfrac{6b}{4b-a}\right)\cdot\left(1-\dfrac{a^2-2ab+4b^2}{a^2-4b^2}\right)$.

35. 1) $\dfrac{3}{2}\left(\dfrac{2a}{3}-\dfrac{a}{7}\right)-\dfrac{12(a-5)}{7}+a+\dfrac{1}{3}\left(\dfrac{a}{2}-5a\right)$;

2) $2-\dfrac{x-a}{x+a}-\dfrac{x}{x-a}+\left(\dfrac{1}{a^2}-\dfrac{1}{ax}+\dfrac{2}{x^2}\right):\left(\dfrac{1}{a^2}-\dfrac{1}{x^2}\right)$.

36. Ahmad A qishloqdan B qishloqqacha velosipedda 14 km/soat tezlik bilan, qaytishda esa 10 km/soat tezlik bilan yurdi. Agar Ahmad qaytishda 1soat ortiq vaqt sarflagan bo'lsa, qishloqlar orasidagi masofani toping.

37. Tarkibi kumush va misdan iborat qotishmaning massasi 3,5kg. Undagi kumush tarkibi mis tarkibining $16\frac{2}{3}\%$ ini tashkil etadi. Qotishmadagi kumush massasini toping.

38. Firma reja bo'yicha bir nechta mahsulotni 10 kun muddat ichida tayyorlashi kerak edi. Lekin u har kuni qo'shimcha 2 tadan mahsulot tayyorlab, muddatiga bir kun qolganda faqat topshiriqni bajaribgina qolmasdan, balki rejadan ortiq yana 3 ta mahsulot tayyorladi. Firma reja bo'yicha nechta mahsulot tayyorlashi kerak edi?

39. 3 ta qopda 120 kg un bor. 1-qopdagi un 2-qopdagi unning 3/5 qismiga, 3-qopdagi un esa 2-qopdagi unning 80 % iga teng. Har bir qopda necha kilogramm un bor?

40. Umumiy ko'paytuvchini qavsdan tashqariga chiqaring:

1) $(a-2b)a^2+ab(2b-a)$; 2) $(4x+2y)y+(2x+y)x$;

3) $4(2x-3)+x(4x-6)$; 4) $5(4x-8x^2)-3(2x-1)$.

Amallarni bajaring **(41–42)**:

41. 1) $(2x+3y)^2$; 2) $(4x+7y)^2$; 3) $(x+2y)^2$; 4) $\left(a+\frac{1}{2}b\right)^2$; 5) $(3x-2y)^2$.

42. 1) $(7y-4x)^2$; 2) $(2x-y)^2$; 3) $\left(\frac{1}{2}a-b\right)^2$;

4) $(2x+3y)(2x-3y)$; 5) $\left(a+\frac{1}{2}b\right)\left(a-\frac{1}{2}b\right)$.

43. Ko'phadni ikkihadga ustun usuli bilan bo'ling:

1) $2x^2-3x-2$ ni $x-2$ ga; 2) $3x^2+5x-2$ ni $x+2$ ga.

Kasrni qisqartiring **(44–45)**:

44. 1) $\frac{a^2+6a+9}{a^2-9}$; 2) $\frac{9x^2-16}{3x+4}$; 3) $\frac{x^2-3x+2}{x^2-1}$;

4) $\frac{x^2+3xy}{9y^2-x^2}$; 5) $\frac{x^2-16}{x^2-5x+4}$; 6) $\frac{y^2-3y-4}{y^2-1}$.

45. 1) $\frac{a^8-a^4}{a^4+a^2}$; 2) $\frac{x^3-8}{x^2+2x+4}$; 3) $\frac{x^4-3x^2+2}{x^2-1}$.

46. Kasrlarni umumiy maxrajga keltiring:

1) $\frac{a}{a+b}$ va $\frac{b}{a-b}$; 2) $\frac{2a}{2a-3b}$ va $\frac{3b}{2a+3b}$.

47. Isbotlang:

Hech qanday butun son n da:

1) $(n+8)(n-4)-(n+3)(n-2)+27$ ifodaning qiymati 3 ga bo'linmaydi;
2) $(2n+1)(n+5)-2(n+3)(n-3)-(5n+13)$ ifodaning qiymati 6 ga bo'linmaydi;
3) $(n+1)(n-1)-(n-6)(n+2)$ ifodaning qiymati 4 ga bo'linmaydi.

48. Ifodaning qiymati a ga bog'liq emasligini ko'rsating:

1) $(a-3)(a^2-8a+5)-(a-8)(a^2-3a+5)$;
2) $(a^2-3a+2)(2a+5)-(2a^2+7a+17)(a-4)$.

49. a, b, c sonlarning ixtiyoriy qiymatlarida

$(b+c-2a)(c-b)+(c+a-2b)(a-c)-(a+b-2c)(a-b)=0$

ekanini isbotlang.

50. Ayniyatni isbotlang:

1) $(x^4+x^3)(x^2+x)=x^4(x+1)^2$;
2) $(y^4+y^2)(y^2-x)=y^3(y^2+1)(y-1)$;
3) $(c^4-c^2+1)(c^4+c^2-1)=c^8+c^4+1$;
4) $(x-3)(y-5)+1=xy-5x-3y+16$;
5) $(a-4)(a+2)+4=(a+1)(a-3)-1$.

51. Ikki velosipedchi bir vaqtda bitta yo'ldagi qishloqdan bir-biriga qarab yo'lga chiqdilar. Birinchisi 15 km/soat, ikkinchisi esa 12 km/soat, tezlik bilan harakat qilmoqda. Agar qishloqlar orasidagi masofa 40,5 km/soat bo'lsa, qancha vaqtdan keyin uchrashuv sodir bo'ladi?

52. Ikki velosipedchi bir yo'ldagi ikkita qishloqdan bir vaqtda bir xil yo'nalishda yo'lga chiqdi. Ikkinchi velosipedchi oldinda, birinchisi orqada bormoqda. Birinchi velosipedchining tezligi 15 km/soat, ikkinchisiniki esa 12 km/soat. Agar qishloqlar orasidagi masofa 4,5 km/soat bo'lsa, birinchi velosipedchi ikkinchisini qancha vaqtda quvib yetadi?

53. Kasrni qisqartiring: 1) $\frac{a^2-16}{a^2-8a+16}$; 2) $\frac{4x^2-9}{2x+3}$.

Amallarni bajaring **(54–59)**:

54. 1) $\frac{b+3}{5}+\frac{7+b}{10}+\frac{b-3}{2}$; 2) $\frac{a^2+5a-4}{16-a^2}+\frac{7a}{8a+2a^2}$.

55. 1) $\frac{a}{a^2-1}-\frac{1}{1-a^2}$;

2) $\frac{4x^2}{2x-3y}+\frac{12xy}{3y-2x}+\frac{9y^2}{2x-3y}$.

56. 1) $\frac{x^2-y^2}{6xy}\cdot\frac{12x^2y}{x+y}$;

2) $\frac{a^2+4a}{a^2-16}:\frac{4a+16}{a^2-4a}$.

57. 1) $\left(\frac{a}{a+1}+1\right):\left(1-\frac{a}{a+1}\right)$;

2) $\frac{1-a^2}{1+b}\cdot\frac{1-b^2}{a+a^2}\cdot\left(1+\frac{a}{1-a}\right)$.

58. 1) $\left(\frac{9m^2-3n^2}{4mn}-\frac{m-4n}{5n}\right):\left(\frac{2m+n}{3m}-\frac{5n^2-3m^2}{16m^2}\right)$;

2) $\left(\frac{a+4b}{2b}+\frac{6b}{4b-a}\right):\left(1-\frac{a^2-2ab+4b^2}{a^2-4b^2}\right)$.

59. 1) $\frac{3}{2}\left(\frac{2a}{3}-\frac{a}{7}\right)-\frac{12(a-5)}{7}+a+\frac{1}{3}\left(\frac{a}{2}-5a\right)$;

2) $2-\frac{x-a}{x+a}-\frac{x}{x-a}+\left(\frac{1}{a^2}-\frac{1}{ax}+\frac{2}{x^2}\right):\left(\frac{1}{a^2}-\frac{1}{x^2}\right)$.

60. Sayyoh daryo bo'yida joylashgan bir oromgohdan velosipedda yo'lga chiqib, boshqa bir oromgohga tayinlangan vaqtda yetib bormoqchi bo'ldi. Dastlabki 1 soatda u 10,5 km yo'l bosdi. Agar qolgan masofani ham shunday tezlik bilan o'tsa, manzilga mo'ljallagan vaqtdan 1 soat kechikishini hisoblab bildi. Sayyoh qolgan yo'lni soatiga 15 km tezlik bilan o'tdi va manzilga belgilangan vaqtdan yarim soat oldin yetib keldi. Oromgohlar orasidagi masofani toping.

61. Ikki xonali sonning o'nliklar xonasidagi raqam birliklar xonasidagi raqamdan 4 marta katta. O'quvchi 507 ni shu ikki xonali songa ko'paytirmoqchi edi. Ammo u ikki xonali sonning raqamlari o'rnini almashtirib yozib qo'ydi. Natijada, u topgan ko'paytma masalaning javobidan 27378 ga kichik chiqdi. To'g'ri javob nechaga teng ekan?

62. Tarkibi kumush va misdan iborat qotishmaning massasi 10,5 kg. Undagi kumush tarkibi mis tarkibining $16\frac{2}{3}\%$ ini tashkil qiladi. Qotishmadagi mis massasini toping.

63. 3 ta qopda 120 kg un bor. 1-qopdagi un 2-qopdagi unning 1 qismiga, 3-qopdagi un esa 2-qopdagi unning 80 % iga teng. Har bir qopda necha kilogramm un bor?

64. Ahmad A qishloqdan B qishloqqacha velosipedda 14 km/soat, tezlik bilan,

qaytishda esa 10 km/soat tezlik bilan yurdi. Agar Ahmad qaytishga 1 soat ortiq vaqt sarflagan bo'lsa, qishloqlar orasidagi masofani toping.

65. Vertolyot ikki qishloq orasidagi masofani shamol yo'nalishida 1,5 soatda, shamol yo'nalishiga qarshi esa 2 soatda uchib o'tadi. Agar shamolning tezligi 10 km/soat bo'lsa, shu qishloqlar orasidagi masofa qancha?

66. Firma reja bo'yicha bir nechta mahsulotni 10 kun muddat ichida tayyorlashi kerak edi. Lekin u har kuni rejaga qo'shimcha 2 tadan mahsulot tayyorlab, muddatiga bir kun qolganda faqat topshiriqni bajaribgina qolmasdan, balki rejadan yana 3 ta mahsulot ortiq tayyorladi. Firma reja bo'yicha nechta mahsulot tayyorlashi kerak edi?

67. 1) $\left(\frac{x^2}{x+y}-\frac{x^3}{x^2+2xy+y^2}\right):\left(\frac{x}{x+y}-\frac{x^2}{x^2-y^2}\right)$;

2) $\left(\frac{c-d}{c^2+dc}-\frac{c}{d^2+cd}\right):\left(\frac{d^2}{c^2+cd^2}+\frac{1}{c+d}\right)$.

68. 1) $\left(\frac{2n}{k+2n}-\frac{4n^2}{k^2+4nk+4n^2}\right):\left(\frac{2n}{k^2-4n^2}+\frac{1}{2n-k}\right)$;

2) $\left(\frac{2q}{2q+m}-\frac{2q^2}{4q^2+4mq+m^2}\right):\left(\frac{2q}{4q^2-m^2}+\frac{1}{m-2q}\right)$.

69. 1) $\frac{3}{2}\left(\frac{2a}{3}-\frac{a}{7}\right)-\frac{12(a-5)}{7}+a+\frac{1}{3}\left(\frac{a}{2}-5a\right)$;

2) $2-\frac{x-a}{x+a}-\frac{x}{x-a}+\left(\frac{1}{a^2}-\frac{1}{ax}+\frac{2}{x^2}\right):\left(\frac{1}{a^2}-\frac{1}{x^2}\right)$.

70. Tenglamani yeching:

1) $(2x-3)(x+5)-(3-x)(5-2x)=-30$;

2) $5(x-1)^2-2(x+3)^2=3(x+2)^2$.

71. Avtomobil shahardan qishloqqacha bo'lgan masofani 80 km/soat tezlik bilan bosib o'tdi. Orqaga qaytishda u masofaning 75 % ini avvalgi tezlik bilan, qolgan yo'lni esa 60 km/soat tezlik bilan bosib o'tdi va shuning uchun ham qaytishda yo'lga shahardan qishloqqa borishdagiga qaraganda 10 minut ortiq vaqt sarf qildi. Shahardan qishloqqacha bo'lgan masofani toping.

72. Oralaridagi masofa 340 km bo'lgan ikki bekatdan bir vaqtda bir-biriga qarab ikki poyezd yo'lga chiqdi. Ulardan birining tezligi ikkinchisinikidan

5 km/soat ortiq. Agar harakat boshlanganidan 2 soat o'tgandan keyin poyezdlar orasidagi masofa 30 km ekanligi ma'lum bo'lsa, ularning tezligini toping.

73. Ifodaning son qiymatini toping:

1) $\left(\frac{1}{3}x+2a\right)\left(2a-\frac{1}{3}x\right)-\left(\frac{2}{9}x+\frac{4}{7}a\right)\left(7a-\frac{1}{2}x\right)+1\frac{5}{9}ax$, bunda $a=\frac{5}{16}$; $x=28$;

2) $(3x+2y)(9x^2-6xy+4y^2)-8(x^3+y^3)$, bunda $x=0{,}1$; $y=4$.

Amallarni bajaring **(74–80)**:

74. $\left(\frac{2-3a}{2+3a}-\frac{3a+2}{3a-2}\right):\left(\frac{2+3a}{2-3a}+\frac{3a-2}{3a+2}\right)$.

75. $\left(\frac{4a+5b}{4a-5b}-\frac{4a-5b}{4a+5b}\right):\left(\frac{4a+5b}{4a-5b}-1\right)$.

76. 1) $(x^2-1)\left(\frac{1}{x-1}-\frac{1}{1+x}+1\right)$;

2) $\left(\frac{x+y}{x-y}-\frac{x-y}{x+y}\right):\left(\frac{x-y}{x+y}+\frac{x+y}{x-y}\right)$.

77. 1) $\left(1+a-\frac{a^2+3}{a+1}\right)(1-a^2)$;

2) $\left(\frac{2-a}{2+a}-\frac{a+2}{a-2}\right):\left(\frac{2+a}{2-a}+\frac{a-2}{a+2}\right)$.

78. 1) $\frac{a-5}{a^2+6a+9}\cdot\frac{(a+3)^2}{a^2-25}$;

2) $\frac{a^2-49}{a^2+2ab+b^2}\cdot\frac{a+b}{a-7}$.

79. 1) $\frac{b^2-8b+16}{b+3}:\frac{(b-4)^2}{b^2-9}$;

2) $\frac{a^2-2a+1}{2a+1}:\frac{a-1}{4a^2-1}$.

80. 1) $\left(\frac{a+1}{2a-2}+\frac{6}{2a^2-2}-\frac{a+3}{2a+2}\right)\cdot\frac{4a^2-4}{3}$;

2) $\left(\frac{b}{a^2+ab}+\frac{2}{a+b}+\frac{a}{b^2+ab}\right):\frac{a^2-b^2}{4ab}$.

81. Ifodani soddalashtiring:

1) $\left(\frac{a+1}{2a-2}-\frac{1}{2a^2-2}\right)\cdot\frac{2a+2}{a+2}$;

2) $\left(\frac{a+b}{a-b}-\frac{a-b}{a+b}\right):\left(\frac{a+b}{a-b}-1\right)$.

O'ZINGIZNI TEKSHIRIB KO'RING!

Takrorlashga doir sinov mashqlari (testlar)

I BOB.

1. Hisoblang. $19{,}6\cdot 2\frac{1}{2}+\left(5{,}25\cdot 1\frac{1}{5}-4{,}5\cdot\frac{4}{4}\right)+14:4\frac{1}{5}+\frac{1}{12}\cdot 8$.

A) 49,82; B) 50; C) 51,82; D) 48,75.

2. Hisoblang. $\left(3\frac{1}{3}-1\frac{5}{6}\right):2\frac{1}{7}-1\frac{1}{3}\cdot 2{,}4+\left(14-15\frac{1}{8}:2\right)$.

A) $2\frac{15}{16}$; B) $3\frac{15}{16}$; C) $4\frac{11}{15}$; D) $3\frac{11}{16}$.

3. Hisoblang. $\dfrac{\left(2-1\frac{3}{4}\cdot\frac{2}{7}\right)\left(1\frac{1}{3}-2\frac{1}{2}:3\frac{3}{4}\right)}{2\frac{1}{3}:\left(\frac{1}{4}-\frac{5}{6}+\frac{7}{8}\right)}$.

A) $\frac{5}{8}$; B) 0,14; C) $\frac{1}{8}$; D) 0,75.

4. Hisoblang. $\dfrac{14:\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right)}{\left(3\frac{1}{6}\cdot 6-5\frac{1}{2}\cdot 2\frac{5}{11}\right):4\frac{1}{3}}$.

A) 11,8; B) -12; C) 15; D) 16.

5. Hisoblang. $\left(1\frac{1}{2}+2\frac{2}{3}-3\frac{3}{4}\right)\cdot 3\frac{3}{5}+1\frac{1}{9}\cdot 3{,}6-7\frac{1}{3}:1\frac{2}{9}$.

A) $-0{,}5$; B) $\frac{2}{3}$; C) 1,5; D) $-\frac{5}{9}$.

6. Ifodani soddalashtiring va uning $a=\frac{1}{3}$ bo'lgandagi qiymatini toping:

$$4(0{,}5a-6)-14a+21.$$

A) $-9a+7$; 4; B) $-12a-3$; -7; C) $15a+7$; 12; D) $11a+6$; 9.

7. Ifodani soddalashtiring va uning $a=-16$ bo'lgandagi qiymatini toping:
$10(0,7-3a)+14a+13$.
A) $-12a+20$; 275; B) $15a+30$; 210;
C) $-16a+20$; 276; D) $44a+20$; -620.

8. Qavslarni oching va soddalashtiring. $-15a-(9a-(12a+9))$.
A) $36a+9$; B) $21a-24$; C) $24a-9$; D) $-12a+9$.

9. Qavslarni oching va soddalashtiring. $8a-(4b+3a)-(4a-3b)$.
A) $a-b$; B) $a+b$; C) $2a-b$; D) $a+2b$.

10. To'g'ri to'rtburchakning perimetri p ga, asosi a ga teng. Uning yuzini topish uchun ifoda tuzing.
A) $\frac{ap+2a^2}{4}$; B) $\frac{ap-2a^2}{2}$; C) $\frac{ap-4a^2}{2}$; D) $\frac{(p+a)a}{2}$.

11. Uchburchakning bir tomoni uzunligi a ga teng, u 2-tomondan b sm qisqa, 3-tomonidan esa c sm uzun. Shu uchburchakning perimetrini hisoblash uchun ifoda tuzing.
A) $3a+b-2c$; B) $a+b+c$; C) $3a+b-c$; D) $3a+b+c$.

12. $a=10$, $b=6,8$, $c=4,5$ bo'lsa, $S=2(ab+ac+bc)$ ifodaning son qiymatini toping.
A) 98,6; B) 213; C) 143,6; D) 287,2.

13. To'g'ri burchakli parallelepipedning bo'yi $a=5$ dm, eni $b=4,8$ dm, balandligi $c=7$ dm ga teng. Uning hajmi $V=abc$ ni hisoblang.
A) 168 dm^3; B) 148 dm^3; C) 84 dm^3; D) 145 dm^3.

14. Ifodani soddalashtiring va uning $a=10,5$, $b=8,5$ bo'lgandagi son qiymatini toping: $5(24a-3b)-4(a-2b)$.
A) $7a-8b$; 5,5; B) $8a-7b$; 24,5; C) $10a+3b$; D) $16a-23b$.

15. $R=6$, $r=3$ bo'lsa, $\frac{1}{3}\cdot(R^2+Rr+r^2)$ ifodaning son qiymatini toping.
A) 18; B) 63; C) 21; D) 27.

16. To'g'ri to'rtburchakning yuzi S ga, balandligi h ga teng. Uning perimetrini hisoblang uchun ifoda tuzing:
A) $2(h^2+S)$; B) $\frac{h}{S}+h$; C) $\frac{S+h^2}{2}$; D) $2\left(\frac{S}{h}+h\right)$.

II BOB. **1.** $(1,2x-1,6)+3,8x=9,6-(5x-3,7)$ tenglamaning ildizi x_0 bo'lsa, $5(x_0+1,51)$ ifodaning son qiymatini toping.
A) 15; B) -10; C) 12; D) 3.

2. $0,6x-1,4=(3,5x+1,7)-(2,7x-3,4)$ tenglamaning ildizi x_0 bo'lsa, $(50,5+x_0):2$ ifodaning son qiymatini toping.
A) $-30,5$; B) 9; C) 10,5; D) $-9,4$.

3. $(4,5x+9)-(6,2-3,1y)=7,2y+2,8$ tenglamaning ildizi x_0 bo'lsa, $(28+x_0):0,7$ ifodaning son qiymatini toping.

A) 6,5; B) -28; C) 40; D) 0.

4. $(5,3x-0,8)-(1,6-4,7x)=2x-(x-0,3)$ tenglamaning ildizi x_0 bo'lsa, $(2x+9,4)2,5$ ifodaning son qiymatini toping.

A) 18; B) 10; C) -20; D) 25.

Tenglamani yeching **(5–8)**:

5. $(0,7x-2,1)-(0,5-2x)=0,9(3x-1)+0,1$.

A) yechimga ega emas; B) 0; C) 1; D) -2.

6. $5\,(2x-4)=2\cdot(5x-10)$.

A) yechimi yo'q; B) cheksiz ko'p yechimga ega; C) 1; D) 2.

7. $3(0,4x-2)+5,6=0,4(3x+1)+2$.

A) -11; B) 10; C) yechimi yo'q; D) 12.

8. $3x+(x-2)-2x=2(2x-0,5)-2x-1$.

A) 5; B) -1; C) 0; D) cheksiz ko'p yechimga ega.

9. Teng yonli uchburchakning perimetri p sm ga teng, Yon tomoni asosidan a sm uzun. Shu uchburchakning tomonlari uzunligini toping.

A) $\frac{p+a}{3}, \frac{p+a}{3}, \frac{p-2a}{3}$; B) $\frac{p}{3}$; C) $\frac{2(a+p)}{3}$; D) $\frac{p+2a}{3}$.

10. Kema daryo oqimi bo'yicha t soatda necha kilometr suzsa, oqimga qarshi s soatda shuncha suzadi. Daryo oqimining tezligi v km/s bo'lsa, kemaning turg'un suvdagi tezligini toping.

A) $\frac{v}{s+t}$; B) $\frac{(s+t)a}{s-t}$ km/s; C) $(s-t)a$ km/s; D) $(s+t)a$ km/s.

11. Ikkita yengil mashina bir xil tezlik bilan ketayapti. Agar birinchi mashina tezligini a km/s ga oshirsa, ikkinchi mashina tezligini a km/s ga kamaytirsa, birinchi mashina t soatda bosib o'tgan yo'lni ikkinchisi s soatda o'tadi. Mashinalarning tezligini toping.

A) $sa+ta$; B) $\frac{(s+t)a}{2}$ km/s; C) $\frac{(s+t)a}{s-t}$ km/s; D) as km/s.

12. Rahima qog'ozga biror sonni yozdi. Dugonasi bu songa a ni qo'shdi, ikkinchi dugonasi esa o'sha sondan b ni ayirdi. Natijada, yig'indining ayirmadan p marta kattaligi ma'lum bo'ldi Rahima qog'ozga qanday son yozgan edi?

A) $(a-b)(p-1)$; B) $(p-1)ab$; C) $(a+b)p$; D) $\frac{a+pb}{p-1}$.

13. Bir tarvuz ikkinchisiga qaraganda a kg yengil, uchinchidan esa n marta yengil. Birinchi va uchinchi tarvuzlarning birgalikdagi massasi ikkinchisidan k marta og'ir. Har bir tarvuzning massasini toping.

A) $\frac{ak}{n-k+1}$ kg, $\frac{(n+1)a}{n-k+1}$ kg, $\frac{5ak}{n-k+1}$ kg; B) $\frac{ak+nk}{n-k}$ kg; 5 kg;

C) $\frac{(n-k)a}{k-n+1}$ kg; 3 kg; D) aniqlab bo'lmaydi.

14. Ikkita qopning har birida a kg dan shakar bor edi. Birinchi qopdan ikkinchisiga qaraganda k marta ko'p shakar olishdi. Shundan so'ng, birinchi qopda ikkinchisiga qaraganda n marta kam shakar qoldi. Har qopda necha kilogrammdan shakar qolgan?

A) $\frac{ak}{n}$ kg, $\frac{an}{k}$ kg; B) $\frac{an(k-1)}{kn-1}$ kg, $\frac{a(k-1)}{kn-1}$ kg;

C) $\frac{ank}{kn+1}$ kg, $\frac{an}{kn+1}$ kg; D) to'g'ri javob berilmagan.

15. Agar o'ylagan songa 15 ni qo'shib, yig'indini 2,5 ga ko'paytirsak va ko'paytmadan 12 %i 18 bo'lgan sonni ayirsak, ayirma o'ylagan sonning o'ziga teng bo'lib qoldi. Qanday son o'ylangan?

A) 35,5; B) 78; C) 75; D) 80.

16. x ning qanday qiymatida $7x-3$ ifodaning qiymati $12x+1$ ifoda qiymatidan 2 marta kichik bo'ladi?

A) $x=1{,}8$; B) $x=-3{,}2$; C) $x=4$; D) $x=3{,}5$.

17. 1-savatda 2-siga qaraganda k marta kam uzum bor edi. 1-savatga a kg uzum qo'shilgach, unda 2-savatdagi uzumdan b kg ko'p uzum bo'ldi. Dastlab 1-savatda qancha uzum bor edi?

A) $\frac{a+b}{k-1}$ kg; B) $\frac{a-b}{k+1}$ kg; C) $(a-bk)$ kg; D) $(bk+a)$ kg.

III BOB. **1.** Hisoblang: $\frac{4^5\cdot 3^8}{6^9}+\frac{8\cdot 2^5}{4^4}$.

A) $1\frac{2}{3}$; B) 2; C) 1,5; D) 4.

2. Hisoblang: $\frac{(c^2)^4\cdot(d^8)^2}{(d^3)^4\cdot(c^3)^3(cd)^2}$.

A) cd; B) d^2; C) c^2; D) $\frac{c}{d}$.

3. Birhadning son qiymatini toping: $\frac{1}{72}a^3b^2c$, bunda $a=-2$, $b=-3$, $c=21$.

A) -18; B) 27; C) -21; D) 28.

4. Birhadni standart shaklda yozing: $3^3a^2b\left(-\frac{1}{3}\right)^4aab$.

A) $-\frac{1}{3}a^2b^4$; B) $\frac{1}{9}a^2b^2$; C) $\frac{1}{3}a^3b$; D) $-\frac{1}{3}a^4b^2$.

5. Birhadlarni ko'paytiring: $\left(-\frac{35}{36}a^2b^3c^4\right)\left(\frac{18}{7}abc\right)$.

A) $-\frac{5}{2}a^3b^4c^5$; B) $\frac{5}{2}a^4b^3c^5$; C) $\frac{2}{5}a^5b^4c^5$; D) $0{,}4a^3b^2c^3$.

6. Ko'phadni standart shaklda yozing. $3a\cdot4b^2-0{,}86b\cdot4b^2-2ab\cdot3b+b\cdot3b^2-1$.

A) $0{,}86b^3-6a^2b-1$; B) $6ab^2+0{,}8b^3-1$;
C) $5ab^2-0{,}6b^3+1$; D) to'g'ri javob berilmagan.

7. Ko'phadni standart shaklda yozing. $5x^3\cdot2y^2-5x\cdot3xy-7x^2y+6x^3y^2$.

A) $16x^3y^2-8x^2y$; B) $-5x^2y+6x^3y^2$;
C) $16x^3y^2-22x^2y$; D) $15x^3y^2+32x^3y$.

8. Ko'phadlar yig'indini toping: $(6x^2+8x-11)+(-4x^2-7x+10)$.

A) $-2x^2+x+1$; B) $2x^2+15x-21$;
C) $2x^2-15x+21$; D) $2x^2+x+1$.

9. Ko'phadni birhadga ko'paytiring: $(7a^2-3a+4)\cdot9a^2$.

A) $63a^5-27a^4+36a^3$; B) $63a^6-27a^3+36$;
C) $64a^5-28a^4+36a^3$; D) $72a^5-27a^4+36a^3$.

10. Ifodani avval soddalashtiring, so'ngra uning $a=2$ bo'lgandagi qiymatini toping: $12(2-3a)+35a-9(a+1)$.

A) $-80a+15$; -145; B) $-10a+15$; -5;
C) $-9a+33$; 15; D) $10a+25$; 5.

11. Ko'phadlarni ko'paytiring. $(a^2-b^2)(a^2+b^2)(a^4+b^4)$.

A) a^6+b^6; B) a^8+b^8; C) a^8-b^8; D) a^6-b^6.

12. Ko'phadlarni ko'paytiring. $(a+b)(a^2-ab+b^2)(a-b)(a^2+ab+b^2)(a^6+b^6)$.

A) $a^{10}-b^{10}$; B) a^8+b^8; C) $a^{12}+b^{12}$; D) $a^{12}-b^{12}$.

13. Bo'lishni bajaring: $(24a^6b^4-6a^4b^6+a^2b^3):(8a^2b^3)$.

A) $3a^4b-0{,}75a^2b^3+0{,}125$; B) $3a^5b^2-0{,}5a^3b^2+8$;
C) $3a^4b+0{,}8a^2b^3+8$; D) $3a^4b^2-0{,}75a^2b^4+0{,}125$.

14. Ifodaning son qiymatini toping: $(15a^6+22{,}5a^2):7{,}5a^3-24a^6:(-8a^2)$,

bunda $a=-2$.

A) -46; B) 44; C) 48; D) 66.

15. Ifodani soddalashtiring: $(35a^8+21a^4) : 7a^4-8a(3{,}5a^3+\frac{1}{2a})$.

A) $-28a^4+4a^2$; B) $5a^4+3$; C) $-23a^4-1$; D) $23a^4+4$.

16. Ifodani soddalashtiring: $a(a-5)-(a-14)(a+2)$.

A) a^2-28; B) $28-6a$; C) $-7a+26$; D) $7a+28$.

17. Ifodaning son qiymatini toping: $4a^2b-ab^2-3a^2b+ab^2-ab+6$, bunda $a=-3$, $b=2$.

A) 30; B) 32; C) -40; D) -36.

IV BOB.

1. Umumiy ko'paytuvchini qavsdan tashqariga chiqaring $2a^2b^2-6ab^2+2a^2b$.

A) $2ab(ab-3b+a)$; B) $ab(2ab-6b+2a)$;
C) $2a^2(b^2+b)-6ab^2$; D) $2b^2(a^2-3a)+2a^2b$.

2. Umumiy ko'paytuvchini qavsdan tashqariga chiqaring $12xy^2z^2-8x^2yz^2-2x^2y^2z$.

A) $4z^2(3xy^2-2x^2y)$; B) $2xyz(6yz-4xz-xy)$;
C) $12xy^2z^2-2x^2yz(4z+y)$; D) $2y^2z(6xz-x^2)-8x^2yz^2$.

3. Ko'paytuvchilarga ajrating. $x(y-z)+3(z-y)$.

A) $(y-z)(3-y)$; B) $(x+3)(y+z)$;
C) $(z-y)(3-x)$; D) $(z-y)(3-z)$.

4. Ko'paytuvchilarga ajrating. $a(x-2)-b(x-2)-c(2-x)$.

A) $(2-x)(a-b+c)$; B) $(x-2)(a+b-c)$;
C) $(x-2)(a+b+c)$; D) $(x-2)(a-b+c)$.

5. Ko'paytuvchilarga ajrating. $ab-ac-bc+b^2$.

A) $(a+b)(b-c)$; B) $(a-b)(b-c)$;
C) $(a-c)b+b^2$; D) $-c(a+b)+ab$.

6. Ko'paytuvchilarga ajrating. $35a^2-21ab-10a+6b$.

A) $(5a+3b)(7a+2)$; B) $(5a-3b)(7a-2)$;
C) $(3b-5a)(2+7a)$; D) $(3a-5b)(7a+4)$.

7. Tenglamani yeching $(6x+5)^2-(6x-2)^2=105$.

A) -2; B) 2; C) 1; D) -4.

8. Tenglamani yeching $121x^2-(8-11x)^2=112$.

A) 5; B) -2; C) -3; D) 1.

9. Ko'paytuvchilarga ajrating: $27a^3 - 8b^3$.

A) $(3a-2b)(9a^2+6ab+4b^2)$; B) $(3a+2b)(9a^2-6ab+4b^2)$;
C) $(3a+2b)(9a^2+6ab+4b^2)$; D) $(3a-2b)(9a^2-6ab-4b^2)$.

10. Ko'paytuvchilarga ajrating: $(a^2+16)^2-64a^2$.

A) $(a-4)^3(a+4)$; B) $(a-4)^2(a+4)^2$;
C) $(a-4)^2(a+4)^3$; D) $(a+4)^4(8a)^2$.

11. Hisoblang $(28{,}9)^2-28{,}9\cdot 5{,}4-28{,}9\cdot 3{,}5$.

A) 0,578; B) 5,78; C) 578; D) 57,8.

12. Hisoblang $51{,}3\cdot 2{,}5+2{,}4\cdot 48{,}7+51{,}3\cdot 2{,}4+48{,}7\cdot 2{,}5$.

A) 250; B) 487; C) 280; D) 490.

13. Hisoblang $(61^3+39^3) : (61^2-61\cdot 39+39^2)$.

A) 100; B) 140; C) 61^2+39^2; D) 32.

14. $(79^3-29^3) : (79^2+79\cdot 29+29^2)$.

A) 841; B) 50; C) 60; D) 108.

15. Ifodaning son qiymatini toping: $a^4-a^3b-7a+7b$, bunda $a=3$, $b=-1$.

A) -86; B) 84; C) 80; D) 90.

16. Ko'paytuvchilarga ajrating: $a^2x+b^2x-5a^2y-5b^2y$.

A) $(a^2-b^2)(y-5x)$; B) $(x+5y)a^2$;
C) $(x-5y)b^2$; D) $(a^2+b^2)(x-5y)$.

17. Ikkihad yig'idisining kvadratini ko'phad shaklida yozing: $(3x+5b)^2$.

A) $9x^2+30bx+25b^2$; B) $9x^2+25bx+25b^2$;
C) $9x^2+30bx+15b^2$; D) $9x^2+25bx+16b^2$.

18. Ikkihad ayirmasining kvadratini ko'phad shaklida yozing: $(7ax-4by)^2$.

A) $49a^2x^2-56abxy+16b^2y^2$; B) $49a^2x^2-52abxy+16b^2y^2$;
C) $49a^2x^2-56abxy+9b^2y^2$; D) $49a^2x^2-52abxy+9b^2y^2$.

19. Kvadratlar ayirmasi formulasidan foydalanib, ko'paytirishni bajaring:
$(9x^2+4y^2)(3x-2y)(3x+2y)$.

A) $81x^4-16y^4$; B) $81x^4-4y^4$; C) $81x^4-9y^4$; D) $81x^4-25y^4$.

20. Ko'paytirishni bajaring: $(4x-5y)(4x+5y)(16x^2+25y^2)$.

A) $256x^4-625y^4$; B) $256x^4-441y^4$;
C) $256x^4-169y^4$; D) $225x^4-625y^4$.

21. Ko'paytirishni bajaring: $\left(\frac{1}{4}x^2+16y^2\right)\left(\frac{1}{2}x-4y\right)\left(\frac{1}{2}x+4y\right)$.

A) $\frac{1}{16}x^4-256y^4$; B) $\frac{1}{16}x^4-225y^4$; C) $\frac{1}{16}x^4-169y^4$; D) $\frac{1}{25}x^4-256$.

22. Ko'paytirishni bajaring: $(0{,}09x^2+0{,}01y^2)(0{,}3x+0{,}1y)(0{,}3x-0{,}1y)$.

A) $0{,}0081x^2-0{,}0001y^2$; B) $0{,}0081x^2+0{,}001y^2$;
C) $0{,}008x^2+0{,}0001y^2$; D) $0{,}008x^2-0{,}001y^2$.

23. Tenglamani yeching: $(1-3x)(1+3x+9x^2)+9x(3x^2-5)+46x=8$.

A) 7; B) 7,5; C) 6,5; D) 6,8.

24. Tenglamani yeching: $(1-5x)(1+5x+25x^2)+25x(5x^2-1)+27x=11$.

A) 5; B) 5,5; C) 4,5; D) 5,8.

V BOB.

1. Kasrni qisqartiring $\frac{ab+1+a+b}{ab+a}$.

A) $\frac{a+1}{a}$; B) $\frac{a}{a+1}$; C) $\frac{a-1}{b}$; D) $\frac{a-1}{a+1}$.

2. Kasrni qisqartiring $\frac{ax+ay-x^2-xy}{ab+ac-bx-cx}$.

A) $\frac{x-y}{b-c}$; B) $\frac{x+y}{b+c}$; C) $\frac{b+c}{x-y}$; D) $\frac{x^2}{b-c}$.

3. Kasrni qisqartiring $\frac{a^4-2a^2+1}{a^3-a^2-a+1}$.

A) $-a+3$; B) $a-2$; C) $a+1$; D) $2a+1$.

4. Kasrni qisqartiring $\frac{a^3+ab^2-2a^2b}{a^3-ab^2}$.

A) $\frac{b-a}{a+2b}$; B) $\frac{a+b}{2a-3b}$; C) $\frac{a+b}{a^2-b}$; D) $\frac{a-b}{a+b}$.

5. Kasrni qisqartiring $\frac{a^4-b^4}{a^4+2a^3b+2a^2b^2+2ab^3+b^4}$.

A) $\frac{a-b}{a+b}$; B) $\frac{a+b}{a-2b}$; C) $\frac{a+2b}{a-b}$; D) $\frac{a^2+b^2}{a^2-b^2}$.

6. Amallarni bajaring: $\frac{x+1}{(x-1)^2}-\frac{1}{x-1}+\frac{1}{x}$.

A) $\frac{x^2-1}{x^2+1}$; B) $\frac{x^2+1}{x(x-1)^2}$; C) $\frac{1-x^2}{x+2}$; D) $\frac{2-x^2}{1+x^2}$.

7. Amallarni bajaring: $\frac{1}{a+b}-\frac{a+b}{a^2-ab+b^2}+\frac{4ab}{a^3+b^3}$.

A) $\frac{a^3-b^3}{a^2+b^2}$; B) $\frac{a^3-b^3}{ab}$; C) $\frac{ab}{a^3+b^3}$; D) $\frac{ab}{a^3-b^3}$.

8. Amallarni bajaring: $\frac{a-c}{a^2-ac+c^2}-\frac{2}{c-a}-\frac{3c^2}{a^3-c^3}$.

A) $\frac{a^3+c^3}{a^2+c^2}$; B) $\frac{6a^3}{a^3+c^3}$; C) $\frac{a^3+c^3}{3a^2}$; D) $\frac{3a^2}{a^3-c^3}$.

9. Kasrlarni ko‘paytiring: $\frac{a^2+4a+4}{2a-2}\cdot\frac{a^2-a}{3a+6}$.

A) $\frac{a(a+2)}{6}$; B) $\frac{a(a-4)}{8}$; C) $\frac{a(a-2)}{12}$; D) $\frac{6-a}{a(a+2)}$.

10. Kasrlarni ko‘paytiring: $\frac{a-a^4}{a+2}\cdot\frac{1}{(2a-a^2)^2}\cdot\frac{a^2-4}{a-1}$.

A) a^2+a+1; B) $\frac{a^2+a+1}{2-a}$; C) $\frac{a^2-a-1}{a+2}$; D) $\frac{3a}{(a+2)(a-1)}$.

11. Kasrlarni bo‘ling: $\frac{4x-y}{x^2+xy}:\frac{4x^2-xy}{2x^2-2y^2}$.

A) $\frac{x-y}{x^2+1}$; B) $\frac{x+y}{x-y}$; C) $\frac{2(x-y)}{x^2}$; D) $\frac{2(x+y)}{3xy}$.

12. Kasrlarni bo‘ling: $\frac{(x-2)^2}{(x-1)^2}:\frac{(4-2x)^3}{(3-3x)^2}$.

A) $\frac{2-x}{2+x}$; B) $\frac{27}{8(2+x)}$; C) $\frac{6(2-x)}{17}$; D) $\frac{9}{8(2-x)}$.

13. Amallarni bajaring: $\frac{x^4-y^2}{x^2+xy+y^2}:\frac{x^2+y^2}{x^3-y^3}\cdot\frac{1}{(y-x)^3}$.

A) $\frac{x+y}{y-x}$; B) $\frac{x-y}{x+y}$; C) $\frac{x^2-y}{x+y^2}$; D) $\frac{x^2+y^2}{x-y}$.

14. Amallarni bajaring: $\left(a-5+\frac{a^2+7}{a+5}\right)\cdot\left(\frac{a}{a-3}-\frac{a}{a+3}\right):\frac{6a}{25-a^2}$.

A) $10-3a$; B) $2(5-a)$; C) $5a-7$; D) $10+9a$.

15. Amallarni bajaring: $\frac{3a+b}{a+b}+\frac{a^2-2ab+b^2}{a}\cdot\left(\frac{a}{(a-b)^2}+\frac{a}{b^2-a^2}\right)$.

A) a^2-b^2; B) $3a+5b$; C) 3; D) $7ab$.

16. Amallarni bajaring: $\left(1-\frac{1}{a}+\frac{1}{a^2}-\frac{1}{a^3}\right):\left(1+\frac{1}{a}+\frac{1}{a^2}+\frac{1}{a^3}\right)$.

A) $\frac{2a+1}{3a-1}$; B) $\frac{1-a^2}{1+a}$; C) $\frac{a^2+1}{a-1}$; D) $\frac{a-1}{a+1}$.

17. Kasrni qisqartiring: $\frac{27a^2-36ab+12b^2}{9a^2-4b^2}$.

A) $\frac{3(3a-2b)}{3a+2b}$; B) $\frac{3(3a+2b)}{3a-2b}$; C) $\frac{4(3a+2b)}{3a-2b}$; D) $\frac{2(3a+2b)}{3a-2b}$.

18. Kasrni qisqartiring: $\frac{6a(9a-ab^2)}{5a^2\cdot(7ab+21a)}$.

A) $\frac{6(3-b)}{35a}$; B) $\frac{6(3-b)}{36a}$; C) $\frac{5(3-b)}{35a}$; D) $\frac{6(9-b)}{35a}$.

19. Amallarni bajaring: $\frac{4}{a+b}-\frac{5}{a-b}+\frac{8b}{a^2-b^2}$.

A) $\frac{1}{b-a}$; B) $\frac{2}{a-b}$; C) $\frac{3}{b-a}$; D) $\frac{5}{a-b}$.

20. Amallarni bajaring: $\frac{a^2+4}{a^3-8}-\frac{1}{a-2}$.

A) $-\frac{2a}{a^3-8}$; B) $\frac{3a}{a^3-8}$; C) $\frac{4a}{a^3-8}$; D) $-\frac{a}{a^3-8}$.

21. $P(x)=x^4+2x^3-3x^2-5$ ko'phadni $x-2$ ga bo'lganda qoldiq nechga teng bo'ladi?

A) 15; B) 16; C) 14; D) 15,5.

22. $P(x)=x^3+3x^3-5x-7$ ko'phadni $(x-3)$ ikkihadga bo'lganda qoldiq nechga teng bo'ladi?

A) 32; B) 31; C) 30; D) 33.

23. a ning qanday qiymatlarida $P(x)=x^3-3x^2-ax+16$ ko'phadni $(x+2)$ ikkihadga bo'lganda qoldiq 12 ga teng bo'ladi?

A) 8; B) 7; C) 9; D) 8,5.

24. a ning qanday qiymatlarida $P(x)=x^3-3x^2-ax^2+16$ ko'phad $x+2$ ga qoldiqsiz bo'linadi?

A) 2; B) 2,5; C) 3; D) 1,5.

SINFDAN TASHQARI ISHLAR UCHUN MASHQLAR

1. Agar uch xonali sondan undagi raqamlarni teskari tartibda yozishdan hosil bo‘lgan uch xonali son ayirilsa, hosil bo‘lgan ayirmaning moduli 9 ga va 11 ga bo‘linishini isbotlang .

2. Ikkita ketma-ket toq natural son kvadratlari ayirmasining moduli shu sonlar yig‘indisining ikkilanganiga teng bo‘lishini isbotlang.

3. Agar ikki xonali x son raqamlari orasiga shu sonning o‘zini yozilsa, u holda hosil bo‘lgan to‘rt xonali son dastlabki ikki xonali sondan 66 marta katta bo‘ladi. x ni toping.

4. $16^5 + 21^5$ ifodaning qiymati 33 ga bo‘linishini isbotlang.

5. $333^{555} + 555^{333}$ sonli ifodaning qiymati 37 ga bo‘linishini isbotlang.

6. $11^{11} + 12^{12} + 13^{13}$ ifodaning qiymati 10 ga bo‘linishini isbotlang.

7. n ixtiyoriy natural son bo‘lsa, $n^3 + 3n^2 + 5n + 3$ ifodaning qiymati 3 ga bo‘linishini isbotlang.

8. 2003^{2003} soni qanday raqam bilan tugaydi?

9. Besh xonali son 9 ga ko‘paytirilganda yana besh xonali son hosil bo‘ladi. Ammo bu sonda avvalgi son raqamlari teskari tartibda joylashgan. Avvalgi sonni toping.

10. Isbot qiling: 1) $\frac{171717}{252525}=\frac{1717}{2525}=\frac{17}{25}$; 2) $\frac{313131}{757575}=\frac{3131}{7575}=\frac{31}{75}$.

11. Tenglik to‘g‘rimi:

$$2b^2+(a^4+a^3b+a^2b^2+ab^3+b^4)(a-b)=(a^4-a^3b-a^2b^2-ab^3+b^4)(a+b)?$$

12. Tenglikning to‘g‘riligini isbotlang:

1) $(n-2)(n-1)n(n+1)+1=(n^2-n-1)^2$;

2) $(n+1)(n+2)(n+3)+1=(n^2+3n+1)^2$.

13. $2000\cdot 2001\cdot 2002\cdot 2003 + 1$ soni biror x sonning kvadrati bo‘lishini isbot qiling va x ni toping.

14. Ko‘paytuvchilarga ajrating:

1) a^2-2a-3; 2) $b^2-7b+12$; 3) a^3+a^2-12;

4) x^3-7x+6; 5) m^2-7m+6; 6) m^2-m-2.

15. Kasrni qisqartiring:

1) $\frac{a^4+a^3+4a^2+3a+3}{a^3-1}$; 2) $\frac{2a^2-5ab+b^2}{2a^2-ab-3b^2}$;

3) $\frac{a^2+b^3+c^2+2ab+2bc+2ca}{a^2-b^2-c^2-2bc}$.

16. Ifodani soddalashtiring va uning harflarning berilgan qiymatlaridagi son qiymatini toping:

1) $x^2-\dfrac{x^3-4xy^2}{x^2(x-2y)+y^2}\cdot\dfrac{x^2-2xy+y^2}{x-2y}$, bunda $x=-\dfrac{1}{2}$, $y=-5$;

2) $\left(\dfrac{mn}{m^2-n^2}+\dfrac{n}{2n-2m}\right)\dfrac{m^2-n^2}{n}$, bunda $m=6\dfrac{1}{2}$, $n=-1{,}5$;

3) $\dfrac{5m+5n}{m-n}\cdot\left(\dfrac{m}{m+n}+\dfrac{m}{n-m}-\dfrac{2mn}{m^2-n^2}\right)$, bunda $m=3\dfrac{1}{2}$, $n=-\dfrac{1}{2}$.

17. Amallarni bajaring

1) $\left(\dfrac{a-1}{3a+(a-1)^2}-\dfrac{1-3a+a^2}{a^3-1}-\dfrac{1}{a-1}\right):\dfrac{a^2+1}{1-a}$;

2) $\left(\dfrac{2p}{2p+q}-\dfrac{4p^2}{4p^2+4pq+q^2}\right):\left(\dfrac{2p}{q^2-4p^2}+\dfrac{1}{2p-q}\right)$.

18. Amallarni bajaring

1) $\left(\dfrac{2a}{2a+b}-\dfrac{4a}{4a^2+4ab+b^2}\right):\left(\dfrac{2a}{4a^2-b^2}+\dfrac{1}{b-2a}\right)$;

2) $\left(\dfrac{1}{1-a}-1\right):\left(a-\dfrac{1-2a^2}{1-a}-1\right)$.

19. Amallarni bajaring

1) $\dfrac{a+b}{a^2-2ab+b^2}+\dfrac{a-b}{a^2+2ab+b^2}+\dfrac{2a}{b^2-a^2}$;

2) $\dfrac{a-5}{a^2-18a+81}+\dfrac{5-3a}{18a-81-a^2}+\dfrac{131+2a}{(9-a)^2}$.

20. Amallarni bajaring

1) $\dfrac{8y^2+14}{y^2-1}-\dfrac{y+7}{y^2+y+1}$;

2) $\dfrac{2y-1}{y^2-2y+4}+\dfrac{7}{y+2}+\dfrac{9y^2-11y+26}{y^3+8}$;

3) $\dfrac{1}{1-2}+\dfrac{3x-2}{9x^2+6x+4}+\dfrac{18x}{8-27x^3}$.

21. Amallarni bajaring

1) $\dfrac{1x}{x}+\dfrac{1}{x-1}-\dfrac{2}{x+1}$

2) $\dfrac{2}{x+1}-\dfrac{3}{x+2}+\dfrac{1}{x-3}$;

3) $\dfrac{x}{x^2+1}-\dfrac{1}{x}+\dfrac{2x}{(x^2+1)^2}$;

4) $\dfrac{1}{x^2}-\dfrac{1}{1+x^2}-\dfrac{1}{(1+x^2)^2}$.

22. Amallarni bajaring

1) $\frac{4}{1+x^4}+\frac{2}{1+x^2}+\frac{1}{1+x}+\frac{1}{1-x}$;

2) $\frac{a^2-bc}{(a+b)(a+c)}+\frac{b^2}{(b+c)(a+b)}+\frac{c^2-ab}{(a+c)(b+c)}$.

23. Diametri 5 mm bo'lgan po'lat simdan balandligi 122 mm bo'lgan burama silindrik prujina tayyorlash kerak. Agar prujina o'ramlari orasidagi bo'shliq yuk tushmagan vaqtda 88 mm ga teng bo'lishi lozim bo'lsa, prujina o'ramlari sonini toping.

24. Temir yo'l stansiyasiga yaqin joydagi dalahovlisida yashovchi bir kishi, odatda, dalahovlidan 18 minutda stansiyaga poyezd jo'nashiga yetib kelardi. Bir kuni u kishi yo'lga chiqishdan oldin hovlida bir necha minut ushlanib qoldi. Shundan keyin u tezligini odatdagidan 1,2 marta oshirgani bilan baribir poyezdga 2 minut kechikdi. U yo'lga chiqishdan oldin uyida necha minut ushlanib qolgan?

25. Hovuzni ertaga soat 16 gacha to'ldirish uchun bugun soat 13 da bitta quvurni ochishdi. Lekin hovuzni soat 12 gacha to'ldirish kerak bo'lib qolgani uchun biror vaqtdan so'ng xuddi shunday ikkinchi quvur ochib qo'yildi. Ikkinchi quvurni soat nechada ochishgan?

26. Elektropoyezd svetofor yonidan 5 sekundda, uzunligi 150 m bo'lgan platforma yonidan esa 15 sekundda o'tdi. Elektropoyezdning uzunligi va uning tezligi qancha?

27. Fermer xo'jaligidan shaharga boradigan yo'l avval tekis bo'lib, so'ngra esa tepalikka ko'tariladi. Fermer xizmatchisi velosipedda yo'lning tekis qismini 12 km/soat tezlik bilan bosib o'tdi. Tepalikka esa 3 km/soat tezlik bilan piyoda ko'tarildi va yo'lga chiqqanidan 1 soat-u 40 minutdan keyin shaharga yetib keldi. Qaytishda tepalik yo'lni 15 km/soat tezlik bilan, tekis yo'lni esa 12 km/soat tezlik bilan bosib o'tdi va shahardan chiqqanidan 58 minut o'tganidan keyin qishloqqa yetib keldi. Xo'jalikdan shahargacha necha kilometr?

28. A qishloqdan B bekatga qarab piyoda kishi yo'lga chiqdi. Oradan 1 soat-u 24 minut o'tgandan keyin xuddi shu yo'nalishda A dan velosipedchi yo'lga chiqdi va 1 soat o'tgandan keyin velosipedchining piyoda kishiga yetishiga 1 km bor edi. Oradan yana bir soat o'tganidan keyin esa velosipedchi B ga yetib borishi uchun piyoda kishiga qaraganda ikki marta kam masofa qoldi. Agar AB masofa 27 km ga teng ekanligi ma'lum bo'lsa, piyoda kishining va velosipedchining tezliklarini toping.

29. Velosipedchi ma'lum tezlik bilan harakat qilib, A bekatdan B bekatga belgilangan vaqtda yetib keldi. Agar u shu tezligini 3 km/soat ga oshirganda edi, belgilangan joyga muddatidan bir soat oldin yetib kelardi, agar u tezligini

2 km kamaytirganida edi, u holda manzilga bir soat kechikkan bo'lar edi. A va B bekatlar orasidagi masofani, velosipedchining tezligini va uning harakat vaqtini aniqlang.

30. Otlarni boqish uchun ma'lum vaqtga pichan g'amlashdi. Agar otlar ikkita kam bo'lganida edi, u holda bu pichan yana 10 kunga yetgan bo'lar edi; agar otlar ikkita ko'p bo'lganida edi, u holda pichan belgilangan muddatga 6 kun qolganida tugar edi. Otlar nechta bo'lgan va pichan necha kunga mo'ljallangan?

31. Birinchi quvur hovuzni ikkinchi quvur uning $\frac{2}{3}$ qismini to'ldirishga ketgan vaqtning yarmida to'ldiradi. Ikkinchi quvurning o'zi hovuzni birinchisiga qaraganda 6 soat kech to'ldiradi. Har bir quvur hovuzni alohida-alohida qancha vaqtda to'ldiradi?

32. Mis bilan rux qotishmasida mis ruxdan 640 g ortiq. Qotishmadan undagi misning $\frac{6}{7}$ qismini va 60 % ruxni ajratib olishganidan keyin qotishmaning massasi 200 g ga teng bo'lib qoldi. Dastlab qotishmaning massasi qancha bo'lgan?

33. A qishloqdan piyoda kishi, shu vaqtning o'zida B qishloqdan unga qarab velosipedchi yo'lga chiqdi. Ular uchrashganlaridan keyin piyoda B ga qarab yo'lini davom ettirdi, velosipedchi esa orqasiga qaytib, u ham B ga qarab jo'nadi. Piyoda B ga velosipedchidan 2 soat kech yetib kelganligini, piyodaning tezligi esa velosipedchinikidan 3 marta kam ekanligi ma'lum. Harakat boshlanganidan to piyoda bilan velosipedchi uchrashguncha qancha vaqt o'tgan?

34. Suzuvchi daryo oqimiga qarshi suzib borayotib, daryoda oqib kelayotgan bo'sh qayiqni ko'rib qoldi. U qayiq bilan uchrashganidan keyin yana oqimga qarshi suzishni t minut davom ettirdi va orqasiga qayrilib, uchrashuv joyidan s metr masofada qayiqni quvib yetdi. Daryo oqimining tezligini toping.

35. A qishloqdan B qishloqqacha bo'lgan 11,5 km uzunlikdagi yo'l avval tepalikka ko'tariladi, keyin tekislik bo'ylab va nihoyat, pastlikka qarab boradi. Piyoda kishi A dan 5 gacha bo'lgan yo'lga 2 soat-u 54 minut, qaytishidagi yo'lga esa 3 soat-u 6 minut vaqt sarf qildi. U tepalikka 3 km/soat tezlik bilan ko'tarildi, tekis yo'lda 4 km/soat tezlik bilan yurdi, pastga esa 5 km/soat tezlik bilan tushdi. Yo'lning tekislik bo'ylab o'tadigan qismi necha kilometrni tashkil qiladi?

36. Ota o'g'liga yechish uchun 30 ta masalani taklif etdi va u bilan bunday kelishdi: har bir to'g'ri yechilgan masala uchun otasi o'g'liga 7 tanga to'laydi;

har bir noto‘g‘ri yechilgan masala uchun o‘g‘li otasiga 12 tanga to‘laydi, Pirovardida kelishuvga muvofiq otasi o‘g‘liga 77 tanga to‘ladi. Nechta masala to‘g‘ri yechilgan?

37. 56 ta ko‘zguni tashish uchun bir kishi aravakashni bunday shart bilan yolladi: aravakash butun yetkazib kelingan har bir ko‘zgu uchun 1 tangadan oladi, yo‘lda sinib qolgan har bir ko‘zgu uchun 6 tanga to‘laydi. Hisob-kitobda aravakash hech narsa olmasligi ma’lum bo‘ldi. Nechta ko‘zgu butun qolgan?

38. Turli qishloqlarda yashovchi ikki o‘rtoq ko‘rishishni istab, bir kunning o‘zida yo‘lga chiqish chiqish va kuniga 28 km dan yurishga kelishib olishdi. Ularning biri betobligi sabab kelishilganidan 3 kun kechikib yo‘lga chiqdi va kechikilgan vaqtni yetkazib olish maqsadida kuniga 49 km dan yura boshladi. Shunday qilib bu o‘rtoqlar o‘zlari mo‘ljallangan kuni uchrashishdi. Qishloqlar orasidagi masofani toping.

39. 5 ta lagan va 7 ta kosa sotib olindi va hammasiga 56 000 so‘m to‘landi. Boshqa gal shu narxlarda 10 ta lagan va 3 ta kosa sotib olindi va 79 000 so‘m to‘landi. Har bir lagan necha so‘mdan va har bir kosa necha so‘mdan sotib olingan?

40. Ona qizi bilan gilam to‘qishmoqda. Agar ular doimo birga ishlaydigan bo‘lishsa, gilam 15 haftada tayyor bo‘lishini hisoblashdi. Aslida esa ular faqat birinchi 8 hafta davomida birgalikda ishlashdi, shundan so‘ng faqat qizi ishlay boshladi va u 28 haftadan so‘ng gilamni to‘qib bo‘ldi. Ona-bola alohida-alohida ishlaganlarida ularning har biri gilamni necha haftada to‘qib bo‘lishlari mumkin?

41. Xattot bir kunda 14 sahifa, uning o‘g‘li esa faqat 8 sahifa matnni ko‘chirib yozishi mumkin. Birinchi 6 kun davomida matnni o‘g‘lining yolg‘iz o‘zi ko‘chirdi. Shundan so‘ng ishni belgilangan muddatda tugatish maqsadida otasi ham yordam qildi. Ish tugaganidan so‘ng ularning har biri bir xil miqdordagi sahifani ko‘chirganliklari ma’lum bo‘ldi. Ular jami necha sahifa matn ko‘chirganlar?

42. 5 ta zanjir bo‘lagini bir-biriga ulab, butun zanjir hosil qilish uchun eng kamida nechta halqani ochib ulash kerak?

43. 1) 12 ta stulni uch qatorga joylashtiring. Har bir qatorda 5 tadan stul bo‘lsin.
2) 12 ta stulni uch qatorga shunday joylashtiringki, bir qatorda 6 ta, qolgan ikkita qatorda 4 tadan stul bo‘lsin.
3) 12 ta stulni to‘rt qatorga shunday joylashtiringki, har bir qatorda 4 tadan stul bo‘lsin.
4) 10 ta stulni xonaning to‘rt tomoniga joylashtiring. Har bir tomondagi stullar soni bir xil bo‘lsin.

44. Alisher, Nigora, Erkin, Yulduz va Nozim 5 xil sport to‘garagiga – shaxmat, velosiped, futbol, stol tennisi va badminton mashg‘ulotlariga qatnashadilar. Quyidagi ma’lumotlar asosida kim qanday sport to‘garagiga qatnashishini aniqlang:
– qizlarga o‘zlarining sevimli sport turi bilan shug‘ullanishlari uchun koptok kerak emas;
– Alisher har yili stol tennisi musobaqasiga qatnashadi;
– qizlardan biri har kuni velosipedda uchadi;
– Erkin futbol o‘ynashni yoqtirmaydi;
– Nozim badminton bilan shug‘ullanmaydi;
– Nigora velosiped uchishni bilmaydi.

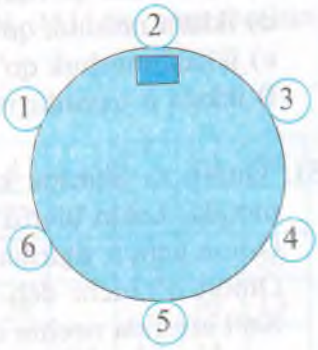
45. Shahnoza Po‘latdan yosh, Komil Po‘latdan yosh, Tohir Shahnozadan yosh, lekin eng yoshi emas. Ularning ismini yoshiga qarab, kattadan kichik tartibida yozib chiqing.

46. Feruza, Fozil, Shoira, Sherzod va Salima dam olish kuni madaniy hordiq chiqarishga borishibdi. Ulardan biri teatrga, ikkinchisi kinoga, uchinchisi sirkka, to‘rtinchisi hayvonot bog‘iga, beshinchisi muzeyga onasi, xolasi, otasi, akasi yoki opasi bilan birga boribdi. Quyidagi ma’lumotlardan foydalanib, kim qayerga kim bilan borganligini aniqlang:
– qizlardan biri hayvonot bog‘iga bormoqchi;
– Fozilning akasi birga boramiz deb so‘z bergan;
– Kim opasi bilan borsa, u muzeyni ko‘rib kelmoqchi;
– Feruza kinoga bormaydi;
– Shoira onasi bilan bormaydi;
– Salimaning xolasi teatrni yoqtiradi;
– Sherzod otasi bilan sirkka bormaydi.
Javoblarni quyidagi jadvalga yozing:

	qayerga	kim bilan
Feruza		
Fozil		
Shoira		
Sherzod		
Salima		

47. Stol atrofida 3 nafar qiz va 3 nafar o‘g‘il bola o‘tirishibdi. Quyidagi ma’lumotlardan foydalanib, kim qaysi stulda o‘tirganligini toping:
– Malika Mahmudning yonida o‘tiribdi;
– Saodat ikki o‘g‘il bola orasida o‘tiribdi;

– Anvar Mahmudning qarshisida o‘tiribdi;
– Malika Zilolaning o‘ng yonida o‘tiribdi;
– Bugun Zilolaning tug‘ilgan kuni;
– Po‘lat Mahmudning yonida o‘tiribdi.

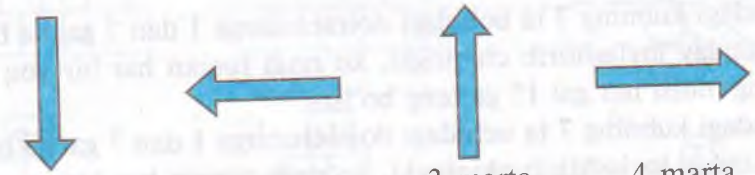


48. *a*) va *b*) rasmlarda ko‘rsatilgan kataklar ichidagi quyonlar yonidagi katakka sakrab o‘tishi mumkin. Har bir gorizontal, vertikal va diagonallardagi kataklarda ikkitadan quyon bo‘lishi uchun ulardan qaysinisi qaysi katakka sakrab o‘tishi kerak? Eng kamida nechta quyon sakrashi kerak? Har bir quyon bittadan ortiq sakrashi mumkin emas.

a)

b)

49. Nodir har gal strelkani soat millari yo‘nalishida rasmda ko‘rsatilgan kabi aylantirmoqda. 19-marta aylantirgandan so‘ng strelka qaysi tomonga yo‘nalgan bo‘ladi? 2015-martada-chi?



1-marta 2-marta 3-marta 4-marta

50. Hovlida ikkita it va ikkita mushuk bor. Mosh ismli mushuk ikkala itdan ham qo‘rqadi. Baroq ismli mushuk Olapar ismli itdan qo‘rqadi, Qoplon ismli it bilan esa do‘st. Quyidagi gaplardan qaysi biri noto‘g‘ri?
a) ikkala mushuk qaysidir itdan qo‘rqadi;

b) bitta itdan qo'rqmaydigan mushuk ham bor;
d) ikkala mushuk qo'rqadigan it bor;
e) ikkala mushuk qo'rqmaydigan it bor;
f) ikkala it qaysidir mushukka qo'rqinch soladi.

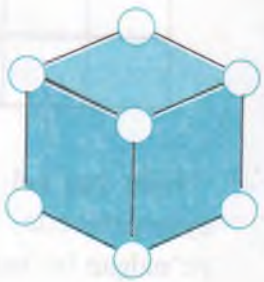
51. Omon va Shavkat ko'l atrofidagi daraxtlarni bir yo'nalishda yurib sanashmoqda. Lekin ular o'z hisoblarini turli daraxtlardan boshlashdi. Shu sababli Omon uchun yigirma to'rtinchi daraxt, Shavkat uchun oltinchi daraxt bo'ldi. Omon o'ninchi deb sanagan daraxt Shavkat uchun ellik to'rtinchi bo'ldi. Ko'l atrofida nechta daraxt bor?

52. Ozoda va Zokirning uylari anhor bo'yida joylashgan. Ozodaning uyidan chap tomonda 47 ta, o'ng tomonida 23 ta uy bor. Zokirning uyi chap va o'ng tomonidagi uylar soni bir-biriga teng. Ozoda va Zokirning uylari orasida nechta uy bor?

53. 1) 1 dan 5 gacha sonlarni doirachalarga shunday joylashtiringki, har bir to'g'ri chiziq bo'yicha sonlar yig'indisi 8 ga teng bo'lsin.

2) 1 dan 5 gacha sonlarni doirachalarga shunday joylashtiringki, har bir to'g'ri chiziq bo'yicha sonlar yig'indisi 9 ga teng bo'lsin.

54. 1. Rasmda kubning 6 ta yog'idan 3 ta yog'i, 8 ta uchidan 7 ta uchi ko'rsatilgan. Shu 7 ta uchidagi doirachalarga 1 dan 7 gacha bo'lgan raqamlarni joylashtirib chiqing. Bunda kubning ko'rinib turgan har bir yog'idagi 4 ta doiracha ichidagi sonlar yig'indisi har gal 13 ga teng bo'lsin.



2. Rasmdagi kubning 7 ta uchidagi doirachalarga 1 dan 7 gacha bo'lgan sonlarni shunday joylashtirib chiqingki, ko'rinib turgan har bir yoq uchlaridagi sonlar yig'indisi har gal 14 ga teng bo'lsin.
3. Rasmdagi kubning 7 ta uchidagi doirachalarga 1 dan 7 gacha bo'lgan sonlarni shunday joylashtirib chiqingki, ko'rinib turgan har bir yoq uchlaridagi sonlar yig'indisi har gal 15 ga teng bo'lsin.
4. Rasmdagi kubning 7 ta uchidagi doirachalarga 1 dan 7 gacha bo'lgan sonlarni shunday joylashtirib chiqingki, ko'rinib turgan har bir yoq uchlaridagi sonlar yig'indisi har gal 16 ga teng bo'lsin.

55. 1. Bo'sh kataklarga shunday sonlarni joylashtiringki, har bir gorizontal, vertikal va diagonal bo'yicha sonlar yig'indisi 34 ga teng bo'lsin (*a rasm*).
2. Bo'sh kataklarga shunday sonlarni joylashtiringki, har bir gorizontal, vertikal va diagonal bo'yicha sonlar yig'indisi 65 ga teng bo'lsin (*b rasm*).

3. Bo‘sh kataklarga shunday sonlarni joylashtiringki, har bir gorizontal, vertikal va diagonal bo‘yicha sonlar yig‘indisi 111 ga teng bo‘lsin (*d rasm*).

a)

16		3	13
5	11		
	7		12
4		15	

b)

22			11	19
6	14		25	
	23	1		12
		15		21
13	16			10

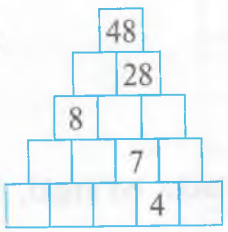
d)

35	1			19	24
	23	7		23	
31		2	22		20
8	28			10	15
30		34	12	14	
4	36		13		11

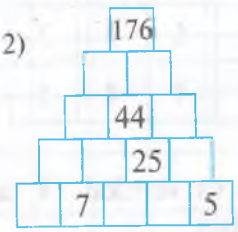
56. Doirani to‘rt bo‘lakka shunday bo‘lingki, har bir bo‘lakdagi sonlar yig‘indisi 21 ga teng bo‘lsin.

57. Sonli piramidani to‘ldiring. Bunda bo‘sh kataklarga shunday sonlarni qo‘yingki, har bir yuqoridagi katak ichidagi son uning ostidagi ikki sonning yig‘indisiga teng bo‘lsin (1—6-rasmlar).

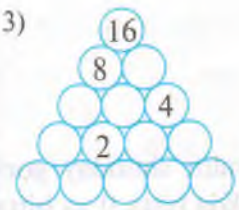
1)



2)



3)



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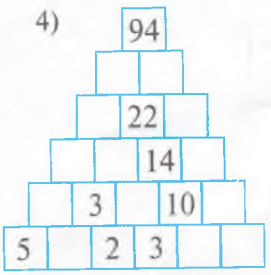
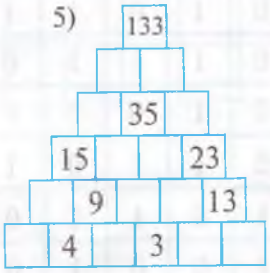
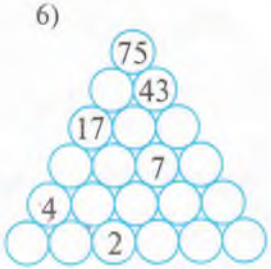


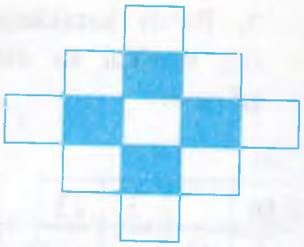
5)



6)

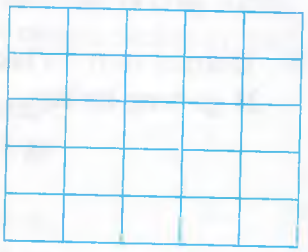


58. 1 dan 9 gacha bo‘lgan 9 ta sonni rasmdagi bo‘sh kataklarga shunday joylashtiringki, natijada unda har bir gorizontal qator, har bir vertikal qator va oltita diagonal qatordagi uchta son yig‘indisi har gal bir xil bo‘lsin.



59. 1 dan 16 gacha bo‘lgan 16 ta sonni 16 ta katakdan iborat kvadratga shunday joylashtiringki, har bir gorizontal va vertikal yo‘l bo‘yicha sonlar yig‘indisi bir xil bo‘lsin.

60. 1, 2, 3, 4, 5 raqamlarini rasmdagi kataklarga shunday joylashtiringki, har bir gorizontal va vertikal yo‘lda shu raqamlar faqat bir marta uchrasin.



61. Jadvalni shunday ikki bo‘lakka bo‘lingki, har bir bo‘lakda 1 dan 8 gacha bo‘lgan sonlar bo‘lsin.

7	6	7	1
1	2	6	8
2	5	3	3
8	4	4	5

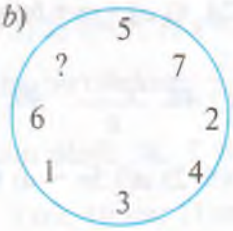
62. Jadvalni shunday gorizontal va vertikal 18 ta bo‘lakka bo‘lingki, har bir bo‘lakda bitta 0 va bitta 1 bo‘lsin.

1	0	1	0	1	1
0	0	1	1	1	0
1	0	1	0	0	0
0	0	1	1	0	1
1	1	0	1	1	0
0	1	0	0	1	0

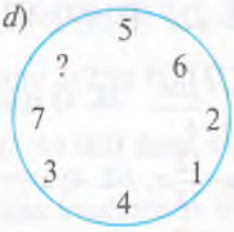
63. Doira ichidagi so‘roq belgisi o‘rniga qanday son qo‘yilishi kerak?

a)

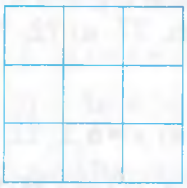
b)

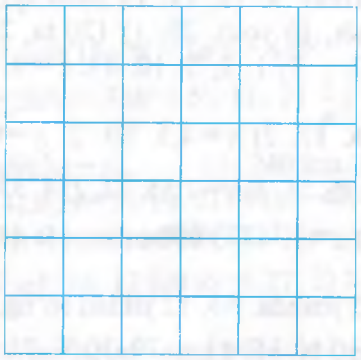


d)

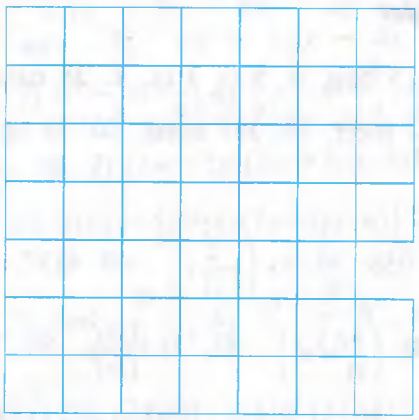


64. Rasmda berilgan 2x2, 3x3 va 6x6 o‘lchamli uchta kvadratdan 7x7 o‘lchamli bitta kvadrat hosil qiling. Qirqish uzunligi eng qisqa bo‘lsin.





65. Bitta katagi olib tashlangan 7x7 o‘lchamli kvadratni 8 ta bir xil bo‘lakka ajratib qirqing.



JAVOBLAR

I bob. **8.** 2) 9 000 000 000 000. **24.** 4) $ak+cp+bn$ so'm. **26.** *b)* $\frac{5}{12}$; **27.** b) $a \neq -\frac{1}{2}$. **35.** $\frac{15a+20b}{a+b}$. **38.** a) $r(2a-\pi r)$ **40.** $\frac{kp+t}{k}$. **46.** 2) 55,625. **50.** 4) $4\frac{5}{6}x+3\frac{77}{90}y$. **57.** $15a-\frac{12}{13}x$. **62.** 4) $x=-9{,}8$. **65.** 2) $a(0{,}5p-a)$ **66.** 2) 10; 4) 29,4; **69.** 4) $a \neq 3$. 10) $t \neq 40$.

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4. 5 216, 6 125; 5 736, 6 375; 5 264, 4 625; 5 784, 4 875. **6.** Mumkin. **7.** 0. **8.** 66 sekund. **9.** 11. **11.** 19 nafar qiz, 11 nafar o'g'il. **12.** Buxoroda XV asrda. **19.** 32 yosh, 16 yosh. **25.** 1) 120 ta, 2) 216 ta. **27.** *a)* 72; 73; 74; 75, *b)* 91; 92; 93; 94. **43.** -1; 1; 3; 5; 10. **49.** $c=9$.

II bob. **17.** 2) $x=4{,}5$. **20.** 2) $x=6$. **21.** 4) $x=6{,}3$. **22.** 2) $x=2{,}4$. **29.** 2) $a=7$, 4) $a=96$. **31.** 3) $x=2{,}55$. **32.** 2) 4. **33.** 2) 1. **34.** 2) $1\frac{13}{16}$. **35.** 2) 6, 4) 20 %. **38.** 2) $x=72$. **40.** 4) $a=1$. **43.** 3 000 so'm. **45.** 45 %. **46.** 7,6 soatda. **47.** 84 soatda. **48.** $\frac{1}{5}$ qismi. **52.** 45 yoshda. **53.** 12 yildan so'ng. **57.** 72 litr. **58.** 52 ta. **60.** 300 metr. **62.** Mis 17 N, rux 7 N. **65.** 4 km. **70.** 30 %. **71.** 12 ta. **77.** $\frac{bk-a}{kn-1}$ kg. **80.** 3. **86.** 442 ta. **87.** 15 ta. **88.** 103 dm. **102.** 1) 500 kg. **105.** 48 km/soat. **106.** 2) 22°. **110.** 4 ta bulbul, 3 ta shox.

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3. 94 ta. **5.** 7 qop va 5 qop. **6.** 5 ta 3 ta. **7.** 28 nafar. **8.** 36 ta g'oz. **9.** 26. **15.** $15\frac{5}{13}$ %. **17.** $233\frac{1}{3}$ metr. **19.** 301 dona. **20.** 38 ta. **22.** 10,89; 9,99; 21,78; 8,19; 18,15.

III bob. **20.** 2) 4 633 058. **30.** 4) $\left(-\frac{2}{11}y\right)^{10}$. **40.** 4) 3^{17}. **48.** 4) 9. **54.** 2) $x=324$. **72.** 4) $(27cd)^3$. **75.** 4) $\left(\frac{4}{11}m^9n^8\right)^2$. **87.** 2) $\frac{(2b)^4}{(3c)^4}$. **96.** $2n=9$. **101.** 2) $x=82$. **112.** 2) $-28p^4q^4$. **128.** 2) $\frac{56}{3}x^{15}y^8$. **129.** 4) $54x^{14}y^9$. **141.** 6) $\overline{xyzt}=1000x+100y+10z+t$. **150.** 2) $-x-y-2$; 19. **161.** 2) $11x^2-x$, 4) $8k^3$. **177.** 2) $3x^3-x$. **192.** a) 10 sm; 30 sm. b) 8m; 6m. **193.** 2) $999\frac{37}{64}$. **209.** 2) $7a^4t^3-3{,}5\,a^4t^2$. **222.** 2) $4a^3+4a^2-a-1$.

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33. 13 ta. **34.** 15. **35.** 1) $x=3$; 2) $x=9$; 3) $x=2$; 4) $x=3$. **37.** 1) Bug'doyning bir o'lchami $\frac{4}{13}$ dirham, arpaning bir o'lchami $\frac{2}{13}$ dirham; 2) 2 va 4; 3) O'g'illari $5\frac{5}{6}$ dirhamdan, ukasi $4\frac{1}{6}$ dirham oladi; **38.** 7 ta o'g'il; 49 000 dona kumush pul. **39.** Tomoni uzunligi: 7 sm; 8 sm; 10 sm; 11 sm bo'lgan kvadratlarni bitta usulda; tomoni 9 sm bo'lgan kvadratni 5 xil usulda yasash mumkin. Bu cho'plardan boshqa kvadratlar yasab bo'lmaydi. **41.** 7^a sinfda 6 nafar; 7^b sinfda 4 nafar qiz bor. **42.** 3 ta daftar va 8 ta qalam yoki 7 ta daftar va 1 ta qalam. **45.** 1) $3+9=12$; $34=12$; $9-5=4$; 2) $7+14=21$; $37=21$; $14-3=11$; 3) $8x=24$, $x=3$; $8+y=24$, $y=16$; $16-3=z$, $z=13$. **46.** 1) $12\cdot6-9=63$; 2) $11\cdot4-8=36$.

IV bob. **2.** 2 $(a+b)$. **3.** $2(a+b+c)$. **4.** 2) $3ab(15a^3b^2-3a^2b+4ab^2-7)$. **8.** 2) $(t-5)(a+2b+4)$. **10.** 2) $-3x^2+3x$; 4) $a^3+3a^2b+3ab^2+b^3$. **11.** 2) $\frac{1}{6}x^2-\frac{1}{6}xy-y^2$; 4) $a^2+ab-2b$. **12.** 2) $6xyz(3z-4y)$; 4) $3ab^2(2a-b)$. **13.** 240 km. **14.** 2 soatdan keyin. **15.** 2) 9 520; 4) 5. **17.** 2) $c(a+b)$, 4) $x(2y-3)$. **19.** 2) $7x^2(x^2-4)$, 4) $9xyz(z-6y)$. **20.** 2) 0,86; 4) 0,7619. **21.** 2) $(b+4)(a-b)$; 4) $(y-3)(a+b)$. **22.** 2) $(p-q)(x+y-z)$; 4) $(x^2+2)(m-n)^2$. **23.** 2) 24; 4) 1; 6) 0. **24.** 50 sek. **26.** 12 400 so'm. **27.** 2) $\frac{a}{5}(3b_1+2b_2)\cdot$ **29.** 2) $(a-b)(1+p)$; 4) $(x-y)(1+2m)$. **30.** 2) $(m-n)(m-n+1)$; 4) $(a-1)(4b+1)$. **31.** 2) $(q-1)(4p+1)$; 4) $(p-1)(3a-2)$. **32.** 2) $(a+2b)(x-3y)$; 4) $(a-3b)(x+5y)$. **34.** 2) $(2nk+5m)(3mk-7n^2)$; 4) $(a-b-c)(x^2-y)$. **35.** 2) 1 700; 4) $90\frac{5}{6}$. **36.** 2) -6; 2; 4) $\frac{4}{3}$; 6) $(2x-3)(x+3)(x-3)$. **40.** 2) $(x-y)(x-3y)$; 8) $(a+2)a+5)$. **44.** 1) $(x+3)(x-4)$; 6) $(x+4)(3x-1)$. **45.** 2) 4 128,94. **48.** 2) $m^2-\frac{2}{3}m+\frac{1}{9}$; 4) $\frac{4}{9}x^4-x^3+\frac{9}{4}$. **50.** 2) -2. 55. 90 sm; 60 sm. **56.** 2) $-8xy$; 4) $8xy$. **58.** 2) $(2m-5n)(2m+5n)$; 4) $(6a-3b)(6a+3b)$. **63.** 2) 0. **64.** 2) -2; 2. 68. 4) $\frac{7}{2}$. **73.** 2) $(4a+5b)^2$; 4) $(7x-3y)^2$; 6) $\left(\frac{a}{3}-3b\right)^2$. **74.** 2) 1,006; 4) 0,998. **77.** 2) $241\frac{3}{19}$. **80.** 2) $4m^2-\frac{1}{4}n^2$; 4) $a^2-\frac{9}{16}b^2$. **87.** 1) $-\frac{1}{2}$; 2) $-\frac{5}{4}$; 4) $\frac{1}{2}$. **95.** 2) $(2a-1)(a+b+c)$. **96.** $(2+a)(a^2+1)a$. **111.** 2) $-\frac{16}{3}$; 4) 15; $\frac{4}{5}$. **114.** 2) -3; $\frac{9}{5}$; 4) $-\frac{1}{2}$; 2. **117.** 2) $(x-y)(3x-3y+7)$; 4) $2(a-b)^2$. **123.** 2) -3; $\frac{9}{5}$; 4) $-\frac{1}{2}$; -1. **128.** 3) 0,75; 4) $-2,5$. **131.** 1) $a=b=1$; 2) $a=2$.

7. 1 991+1+9=2 001, $A=1$; $B=9$; $C=2$; $D=0$. **9.** 13; 13 va 22 yoshda. **10.** 37 nafar o'quvchi. **17.** 2 401. **18.** 25 m; 50 m; 25 m. **21.** $(x^2+x+1)(x^2-x+2\,014)$. **26.** 17 yoshda. **27.** 192 kg olma, 40 kg nok, 448 kg olxo'ri, 320 kg olcha. **28.** 1) 2 ta; 2) 3 ta. **29.** 1) d^3-3d; 2) d^2-2. **31.** 2) $(a+b)(b+c)(a+c)$. **32.** 1. **49.** I va II navli cheylon 2:3 nisbatda aralashtirilgan. **50.** I xumchada 84 dona, II xumchada 27 dona, III xumchada 28 dona dur bo'lgan.

V bob. **3.** $\frac{x-y}{x+y}$. **4.** $\frac{a^2+b^2}{a-b}$. **5.** $\frac{a^3-b^3}{a^2+b^2}$. **6.** $\frac{(a+b)^2}{(a-b)^2}$. **7.** 2) $\frac{5}{11}$; 4) 1,5. **8.** 2) $1\frac{9}{13}$; 4) $\frac{40}{21}$. **9.** 2) $\frac{9}{22}$; 4) $\frac{3}{5}$. **10.** 2) $\frac{1}{2}$; 4) -12. **12.** 2) $\frac{5c}{16b}$; 4) $\frac{2a}{3b}$. **13.** 2) $\frac{1}{23(m+n)}$; 4) $-\frac{1}{3(x-y)}$. **14.** 2) $\frac{1}{m-n}$; 4) $\frac{3}{8n}$. **16.** 2) $\frac{a+b}{2a-2b}$; 4) $\frac{1+n}{1-n}$. **18.** 2) $\frac{p}{p-q}$; 4) $\frac{x}{y}$. **20.** 2) $\frac{3a-2b}{2a-3b}$; 4) $\frac{1-2ab}{a^2b(2b-a)}$. **22.** 2) $\frac{b-5}{4a(3+b)}$; 4) $\frac{b(4+b)}{2(b-3)}$. **23.** 2) $d-3$; 4) $\frac{1}{n(b^2-c^2)}$. **24.** 2) $\frac{1}{b-6}$; 4) $\frac{1}{2p+1}$. **26.** 2) $m+n$; 4) $\frac{1}{9-2x}$. **27.** 2) $\frac{4y-3x}{4y+3x}$. **28.** 2) 64. **29.** 2) $a=-2$, $b=6$. **30.** $\frac{ab}{a-b}$. **31.** 0,5 kg; 1,5 kg. **32.** 5:2. **33.** 2) $\frac{7-a}{7+a}$; 4) $\frac{b}{4a^2+2a+1}$. **34.** 1) $\frac{x^2+y^2}{x}$; 4) $\frac{1}{3a(3a+2)}$. **35.** 2) $\frac{x^2+3}{x+1}$; 4) $\frac{a^{11}+1}{a^{11}}$. **36.** 2) $a=-2$; $b=-6$. **37.** 2) 1; 4) $-\frac{1}{8}$ **38.** 2) $\frac{x-1}{x^2-x+1}$. **39.** $\frac{49}{99}$. **40.** $\frac{10}{19}$. **41.** 20. **42.** 0,5. **43.** 40. **45.** -1. **46.** $\frac{a-b}{a+b}$. **48.** $\frac{3}{2}\left(1-\frac{1}{3^{64}}\right)$. **49.** $\frac{1}{1-a}\left(1-a^{64}\right)$. **51.** 1) $\frac{c-a}{abc}$; 4) $\frac{b(d-a)}{acd}$. **52.** 2) $\frac{8x^2}{12xy}$, $\frac{60}{12xy}$ va $\frac{15y^2}{12xy}$; 4) $\frac{3ax^2}{9x^3}$ va $\frac{b}{9x^3}$. **55.** 2) $\frac{4x(3x+2y)}{9x^2-4y^2}$ va $\frac{5y(3x-2y)}{9x^2-4y^2}$; 4) $\frac{3a(3a+4b)}{9a^2-16b^2}$ va $\frac{4b(3a-4b)}{9a^2-16b^2}$. **56.** 2) 0; 4) $\frac{7x}{6x+6y}$. **57.** 2) $\frac{96a(1+b)}{(2b-3c)^2\cdot 12a(1+b)}$, $\frac{2a(3c+2b)\cdot 12a(1+b)}{(2b-3c)^2\cdot 12a(1+b)}$ va $\frac{(2b+3c)^2}{(2b-3c)^2\cdot 12a(1+b)}$. **59.** $\frac{30(a+b)}{30a^2-30b^2}$, $\frac{1}{30a^2-30b^2}$ va $\frac{6(a-b)}{30a^2-30b^2}$. **64.** 2) $\frac{2m}{3n^2}$; 4) $\frac{x-y}{m+n}$. **65.** 2) $\frac{6m+b}{2q^2}$; 4) $\frac{n}{a}$. **66.** 2) $\frac{8}{ab}$. **67.** 2) $\frac{30}{77}$; 4) $\frac{c+3ad}{12a}$. **68.** 2) $\frac{3ad-b}{15d}$; 4) $\frac{11a+3}{a}$.

69. 2) $\frac{4c^2+2c-4}{c^2}$; 4) $\frac{ab-b^2k+a^2}{b^2}$. **70.** 2) $\frac{k+n}{mnk}$; 4) $\frac{bd-ab}{acd}$; 6) $\frac{3n^2+2m}{mn^3}$.

72. 2) $\frac{acd^2+ad+ca}{c^2d^2}$; 4) $\frac{bcd^2-bc+bd}{c^2d^2}$. **74.** 2) $\frac{2x}{3(x-1)}$; 4) $\frac{20y+25x}{10(y-3)}$. **76.** 2) $\frac{5b^2+3a^2}{ab(x+y)}$;

4) $-\frac{a+b+y}{ab}$; **77.** 2) 6. **78.** 2) $\frac{2x-4}{x^2-9}$; 4) $\frac{2x^2-13x+2}{x^2-16}$. **80.** 2) $\frac{4-7(m+n)}{(m+n)^2}$;

4) $\frac{4a^2-20a-50}{(a+5)^2\cdot(a-5)}$. **81.** 2) $\frac{9a-1}{6(9a+1)}$; 4) $\frac{2ab}{1-8a^3b^3}$. **83.** 2) $\frac{1}{4a-3}$. **84.** 2) $\frac{1}{x-3}$.

86. 2) $\frac{9}{2a+3}$. **87.** 2) 0. **89.** 2) $\frac{m}{n(m-n)}$. **90.** 2) $\frac{1}{3a-2b}$.

92. 2) $-\frac{1}{x+2}$. **93.** $\frac{ab}{a+b}$. **94.** $\frac{ab}{b-a}$. **95.** 2) 12. **96.** 6. **97.** 2) $-\frac{3}{26}$.

108. $2\frac{8}{11}$ soatda. **109.** 40 soatda. **110.** 2) $-\frac{3}{19}$. **111.** 2) $-\frac{9}{8}$. **113.** 2) $\frac{13}{22}$; 4) $\frac{6}{5}$. **114.** 2) $\frac{3}{4}$;

4) 81. **115.** 2) $\frac{k}{mb}$, 4) $\frac{4mk}{5nd}$. **117.** 2) $4x^2ya$; 4) $\frac{b}{2a}$. **118.** 2) $\frac{b}{4(1-a)}$; 4) $\frac{1}{4m^2(m-n)}$.

119. 2) $\frac{7}{2}$; 4) $\frac{3}{2}$. **120.** 2) $\frac{65}{cm}$; 4) $\frac{16}{56}$. **121.** 2) $\frac{6}{5}$; 4) $\frac{1}{3nm^2}$.

123. 2) $\frac{2x}{y^2}$; 4) $\frac{5b}{8}$. **125.** 2) $\frac{ad}{bc}$; 4) $\frac{14b^2d^2}{15ac}$. **127.** 2) $\frac{3a}{3a-1}$; 4) $\frac{3a+b}{a}$.

129. 2) $\frac{12xy}{4x^2+9y^2}$. **130.** 2) $b(a-1)$; 433,34. **132.** 2) 0; -1.

133. 2) 1. **134.** 1. **135.** 2) $\frac{4xy+7}{4xy+a}$; 4) $(2x-3y)(4x-y)$. **137.** 1) 2; 2) 12; 4) 525.

142. 2) $\frac{4(1-a)}{b}$; 4) $4m^2(m-n)$. **150.** $\frac{1}{1-m}$. **152.** $a+4$. **154.** 1. **156.** $x+2$.

158. $\frac{2b}{a^2-b^2}$. **159.** $2y$. **160.** $\frac{x^2}{4-x^2}$. **162.** $\frac{x^2(x+1)}{x+2}$. **164.** $(a-b)(a+b)^2$. **166.** $(a+b)^2$.

170. 1. **172.** 2) $\frac{3}{5}(1+a)$; 4) 1; 6) $\frac{b^2}{b^2+2}$. **181.** $u\frac{s}{v}$. **182.** $\frac{v-v_0}{v+v_0}s$. **183.** 6 tadan.

186. 2) $\frac{60b+31}{24}$; 4) $\frac{18b+26}{39}$. **188.** $n+m$. **190.** 1) $\frac{5}{9}$; 2) $\frac{4}{3}$. **191.** 1) $\frac{1}{2}$; 2) $4(a+b)x$.

192. 1) 0; 2) 0. **194.** 2) 1; 4) -1. **196.** $3x^2+1$. **197.** $\frac{b+1}{b-2a}$. **198.** $p-q$.

199. 1. **200.** $\frac{2bc}{(b+c-a)^2}$. **202.** $\frac{2y}{5y-2x}$. **203.** $3x-2$. **204.** $2a+3$. **205.** $a-b$.

206. $2x-1$. **208.** p.

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7. 2) $\frac{a^2+b^2}{b^2}$; 4) $\frac{z}{x+y-z}$; 6) $-a$. **8.** $\frac{a-1}{a+1}$; 2) $\frac{a+b-c}{a-b+c}$. **10.** 1) 3; 2) $\frac{a+b}{a-b}$. **11.** 1) 1; 2) $\frac{1}{abc}$. **15.** 1) 10 ta kvadrat va 5 ta oltiburchak. **16.** 27 dona; 2-o'g'il 3 ta; 3-o'g'il 5 ta shaftoli olishi kerak. **19.** Ali 1 tanga, Vali 7 tanga olishi kerak. **24.** 1 kg; 3 kg; 9 kg; 27 kg.

Sinfdan tashqari ishlar uchun masalalarga javoblar

8. 3 raqami bilan. **9.** 10 989. **11.** To'g'ri. **14.** 2) $(b-3)(b-4)$; 4) $(x-1)(x-2)(x+3)$; 6) $(m+1)(m-2)$. **15.** 2) $\frac{a-b}{a+b}$. **16.** 2) 2. **17.** 2) $\frac{2p(2p-q)}{2p+q}$. **20.** 2) 0. **22.** 1) $\frac{8}{1-x^8}$; 2) 0. **23.** 10 o'ram. **24.** 5 minut. **25.** Ertalab soat 8 da. **26.** 75 m, 15 km/soat. **27.** 12 km. **28.** 5 km/soat, 11 km/soat. **29.** 60 km, 12 km/soat, 5 soat. **30.** 8 ta ot, 30 kunda. **31.** 3 soat, 9 soat. **32.** 1 040 gramm. **33.** 1 soat. **34.** $\frac{s}{2t}$ m/s. **35.** 4 km.

MUNDARIJA

O'quv nashri

Mirfazil Abdilxaqovich Mirzaxmedov, Gappar Nasritdinov,
Faridon Rahimovich Usmonov, Mavjuda Murzayevna Shaniyozova,
Feruza Soidovna Rahimova, Shohida Rahmatovna Aripova

ALGEBRA

Aniq fanlarga ixtisoslashtirilgan davlat umumta'lim maktablarining 7-sinfi uchun darslik

«O'zbekiston milliy ensiklopediyasi»
Davlat ilmiy nashriyoti,
Toshkent–2014

Muharrirlar: *A. Zulfiqorov, M. Nurmetova*
Badiiy muharrir *A. Yaqubjonov*
Kompyuterda sahifalovchi: *A. Sulaymonov, A. Qo'nishev*

Nashriyot litsenziyasi AI № 160, 14.08.2009-y.

22.12.2014-y.da bosishga ruxsat etildi. Qog'oz bichimi 70 × $100^{1}/_{16}$.
«Times» garniturasi. Kegli 11. Ofset usulida bosildi.
Shartli b.t. 19,35. Nash h.t. 16,40. Adadi 7 144 nusxa
416-buyurtma.

«O'zbekiston milliy ensiklopediyasi» Davlat ilmiy nashriyoti, 2014.
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100128, Toshkent, Shayxontohur ko'chasi, 86-uy.

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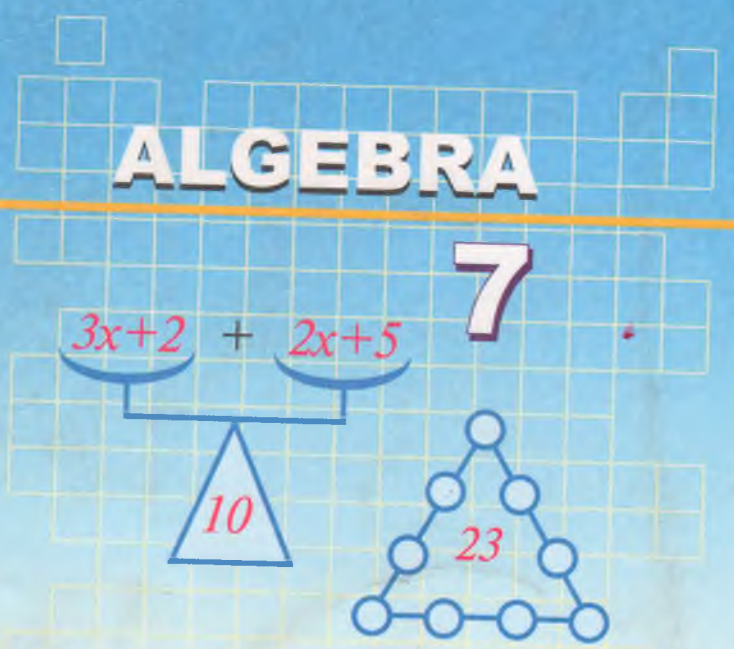

$$(a-b)^3=a^3-b^3-3ab(a-b)$$

Sotuvga chiqarish ta'qiqlanadi

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