PERFECT STUDY

BOYMURODOV DOSTONBEK

MATEMATIKA

2016 – 2017 – YILGI DAVLAT TEST SINOVIDA MATEMATIKA FANIDAN TUSHGAN BA'ZI TESTLAR ISHLANMASI

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1 – masala. Hisoblang:

$$(1+\mathrm{tg}7°)(1+\mathrm{tg}8°)(1+\mathrm{tg}37°)(1+\mathrm{tg}38°)$$

A) 4 B) 2 C) 16 D) 8

Yechish: $\mathrm{tg}37°$ va $\mathrm{tg}38°$ larni mos ravishda $\mathrm{tg}(45°-8°)$, $\mathrm{tg}(45°-7°)$ ko'rinishda yozib olamiz. U holda oxirgi yozgan ikkita ifodaga yig'indi formulasini qo'llab quyidagilarga ega bo'lamiz:

$$tg37^\circ = tg\left(45^\circ - 8^\circ\right) = \frac{1-tg8^\circ}{1+tg8^\circ} \qquad (1.1)$$

$$tg38^\circ = tg\left(45^\circ - 7^\circ\right) = \frac{1-tg7^\circ}{1+tg7^\circ}. \qquad (1.2)$$

(1.1) va (1.2) ni berilgan ko'paytmaga qo'yib ba'zi soddalashtirishlarni bajarsak,

$$\left(1+tg7^\circ\right)\left(1+tg8^\circ\right)\left(1+tg37^\circ\right)\left(1+tg38^\circ\right)=$$

$$=\left(1+tg7^\circ\right)\left(1+tg8^\circ\right)\left(1+\frac{1-tg8^\circ}{1+tg8^\circ}\right)\left(1+\frac{1-tg7^\circ}{1+tg7^\circ}\right)=$$

$$=\left(1+tg7^\circ\right)\cdot\left(1+tg8^\circ\right)\cdot\frac{2}{1+tg8^\circ}\cdot\frac{2}{1+tg7^\circ}=4$$

bo'ladi.

Javob: 4(A)

2 – masala. $\{x|x \in N, 2 \le x^2 \le 34\}$ to'plamni nechta usul bilan ikkita kesishmaydigan qism – to'plamlarga ajratish mumkin?

A) 8 B) 9 C) 4 D) 16

Yechish: 2016 – yilgi matematikadan test sinovlarida to'plamga oid, asosan, ikki turdagi masala ko'p uchraydi: to'plamning qism – to'plamlari sonini va to'plamni ikkita kesishmaydigan qism – to'plamlarga ajratish usullari sonini topish.

Biz quyida aynan shu turdagi masalalarni yechish formulasini keltiramiz:

Aytaylik, $\mathcal{A}$ to'plamning elemementlari soni $n(\ge 1)$ ta bo'lsa, u holda uning

- qism – to'plamlari soni 2^n ga;
- bo'sh bo'lmagan qism to'plamlari soni $2^n - 1$ ga;
- ikkita kesishmaydigan qism – to'plamlarga ajratish usullari soni 2^{n-1} ga teng.

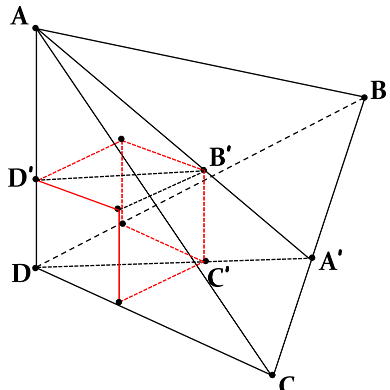
U holda yuqoridagi formulaga ko'ra

$$\{x|x \in N, 2 \le x^2 \le 34\} = \{2;3;4;5\}, n = 4$$

to'plamni $2^{4-1} = 8$ ta usul bilan ikkita kesishmaydigan qism – to'plamlarga ajratish mumkin.

Javob: 8(A)

3 – masala. ABCD tetraedrning D uchidagi barcha yassi burchaklari to'g'ri. Shu tetraedrga kub shunday ichki chizilganki, kubning bitta uchi D nuqtada, unga qarama – qarshi uchi esa ABC yoqda yotibdi. Agar $DA = a, DB = b$ va $DC = c$ bo'lsa, kub qirrasining uzunligini toping.



1 – *chizma*

Yechish: Kubning B' uchi ABC yoqda yotsin (1 – *chizma*). A va B' nuqtalarni tutashtiruvchi chiziq CB qirrani A' nuqtada kesib o'tadi. Avvalo, D, C' va A' (bunda C' nuqta kubning uchlaridan biri) nuqtalar bir to'g'ri chiziqda yotishini ko'rsatamiz. DA' va D'B' chiziqlar AD ga perpendikulyar, ya'ni

$$DA' \perp AD, D'B' \perp AD. \qquad (3.1)$$

(3.1) dan kelib chiqadiki, DA' va D'B' chiziqlar o'zaro parallel, ya'ni

$$DA' \parallel D'B'. \qquad (3.2)$$

Boshqa tomondan D'B' va DC' chiziqlar kubning qarama – qarshi yoqlarining parallel diagonallari, ya'ni

$$D'B' \parallel DC'. \qquad (3.3)$$

(3.1) va (3.2) dan DC' va DA' chiziqlarning bir to'gri chiziqda yotishi kelib chiqadi.

Endi, kubning qirrasini topamiz. DA′ kesma tetraedr asosining D to'g'ri burchagi bissektrisasi bo'lgani uchun

$$|DA'| = \frac{cb}{c+b}\sqrt{2} \qquad (3.4)$$

ga teng bo'ladi. Kubning qirrasini ***x*** deb olsak,

$$|D'B'| = x\sqrt{2}, |AD'| = a - x \qquad (3.5)$$

bo'lib, AD′B′ uchburchak bilan ADA′ uchburchak o'xshash bo'lganligi uchun ushbu

$$\frac{|D'B'|}{|DA'|} = \frac{|AD'|}{|AD|} \qquad (3.6)$$

tenglik o'rinli. (3.4) va (3.5) ni (3.6) ga qo'ysak,

$$\frac{x\sqrt{2}}{\frac{cb}{c+b}\sqrt{2}} = \frac{a-x}{a} \Rightarrow x = \frac{abc}{ab+ac+bc}$$

bo'ladi.

Javob: $\frac{abc}{ab+ac+bc}$

4 – masala. a va b natural sonlarning eng katta umumiy bo'luvchisi 6 ga teng bo'lsa, $a + 5b$ va b sonlarning eng katta umumiy bo'luvchisi nimaga teng?

A) 4 B) 6
C) *bir qiymatli aniqlab bo'lmaydi* D) 1

Yechish: a ni b ga bo'lgandagi qoldiq $a + 5b$ ni b ga bo'lgandagi qoldiqqa teng bo'lganligi uchun ikkita sonning eng katta umumiy bo'luvchis (EKUB)ini topishning Yevklid algoritmiga ko'ra [qarang: Akademik litseylar uchun darslik, 1 – qism, 29 – 30 – betlar] EKUB(a,b) bilan EKUB($a + 5b$,b) teng bo'ladi. Demak, EKUB($a + 5b$,b) = 6.

Javob: 6(B)

Izoh: Aytaylik, a va b natural sonlarning EKUBi d ga teng bo'lsa, istalgan n natural son uchun $a + nb$ va b, $b + an$ va a sonlarning EKUBi ham d ga teng bo'ladi, ya'ni

$$\text{EKUB}(a, b) = \text{EKUB}(a + nb, b) =$$
$$= \text{EKUB}(b + an, a) = d.$$

5 – masala. $[200; 500]$ kesmada 2, 3, 5, 7 ga bo'linganda qoldiq 1 ga teng bo'ladigan natural sonlar nechta?

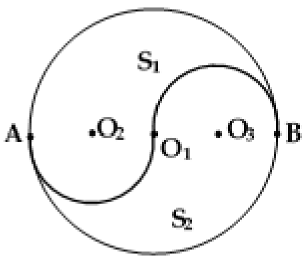
A) 3 B) 2 C) 4 D) 1

Yechish: n natural sonni 2, 3, 5 va 7 ga bo'lganda qoldiq 1 ga teng bo'lsa, u holda n ni 2, 3, 5, 7 sonlarning EKUKiga bo'lganda ham qoldiq 1 ga teng bo'ladi va aksincha, ya'ni n ni 2, 3, 5, 7 sonlarning EKUKiga bo'lganda qoldiq 1 ga teng bo'lsa, n sonni 2, 3, 5 va 7 ga bo'lganda ham qoldiq 1 ga teng bo'ladi. Shuning uchun $[200; 500]$ kesmadagi 210 = EKUK(2,3,5,7) ga bo'lganda qoldiq 1 qoladigan sonlar miqdorini topish yetarli:

$$n = 211, n = 421 \Rightarrow 2 \text{ ta.}$$

Javob: 2 (B)

6 – masala. Rasmda AB katta aylana diametri, O_1 katta aylana markazi, O_2 va O_3 kichik aylanalar markazlari bo'lib, ular uchun $AO_1{:}O_1O_2{=}O_2O_3{:}O_3B$ tenglik o'rinli. S_1 va S_2 sohalar perimetri yig'indisini ifodalaydigan son S_1 soha yuzini ifodalaydigan sondan 25% ga kichik bo'lsa, katta aylananing uzunligini toping.



A) $\frac{64}{3}\pi$ B) $\frac{32}{3}\pi$ C) 16π D) 32π

Yechish: O_2 markazli aylananing radiusini r deb olaylik. $AO_1{:}O_1O_2 = O_2O_3{:}O_3B$ shartdan O_3 markazli aylananing ham radiusi r ga va katta aylananing radisui esa $2r$ ga tengligi kelib chiqadi. Bundan esa S_1 va S_2 sohalarning perimetri teng bo'lishadi.

S_1 soha katta va ikkita kichik aylanalarning yarmidan tashkil topganligi uchun uning perimetri

$$L_1 = 2\pi \cdot r + \pi r + \pi r = 4\pi r$$

ga, yuzasi esa

$$A_1 = \frac{\pi(2r)^2}{2} = 2\pi r^2$$

ga teng bo'ladi. Shartga ko'ra $2L_1 = \frac{3}{4}S_1$ bo'lganligi uchun ushbu

$$2 \cdot 4\pi r = \frac{3}{4} \cdot 2\pi r^2 \Rightarrow r = \frac{16}{3}$$

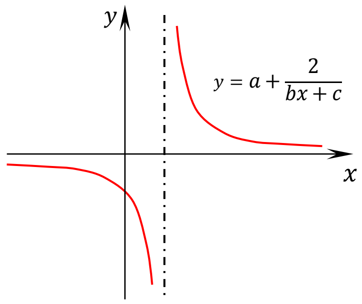
tengliklar o'rinli. Demak, katta aylananing uzunligi

$$L = 4\pi r = \frac{64}{3}\pi$$

ga teng.

Javob: $\frac{64}{3}\pi$(A)

7 – masala. Rasmda $y = a + \frac{2}{bx+c}$ funksiyaning grafigi tasvirlangan. quyidagilardan qaysi biri doimo o'rinli?



A) $c^3 - b^3 > 0$ B) $abc > 0$

C) $bc > 0$ D) $ac^3 + b > 0$

Yechish: $y = a + \frac{2}{bx+c}$ funksiya kasr – chiziqli funksiya bo'lgani uchun uning grafigi rasmdagidek giperboladan iborat. Ma'lumki, giperbolaning ikkita assimptotasi ma'vjud: vertikal va gorizontal. Quyida $y = a + \frac{b}{cx+d}$ giperbolaning ba'zi xossalari keltirilganki, bu xossalardan aynan shu turdagi masalalarni yechishda foydalanish mumkin:

> Giperbola
> - $b \cdot c < 0$ bo'lsa, o'suvchi;
> - $b \cdot c > 0$ bo'lsa kamayuvchi bo'ladi.
>
> Gioerbolaning $x = -\frac{c}{b}$ chiziq vertikal, $y = a$ chiziq gorizontal assimptotasi bo'ladi.

Rasmga va keltirilgan xossalarga ko'ra ushbu sistemani tuzib olamiz:

$$\begin{cases} 2 \cdot b > 0 \\ -\frac{c}{b} > 0 \\ a = 0 \end{cases}.$$

Bundan

$$a = 0, b > 0, c < 0 \qquad (7.1)$$

bo'ladi. Endi (7.1) dan foyfdalanib, masalada keltirilgan javob variantlari orasidan D variantda keltirilgan $ac^3 + b > 0$ tengsizlik doimo o'rinli ekanligini tekshirib ko'rish mumkin.

Javob: $ac^3 + b > 0$ (D)

8 – masala. To'g'ri burchakli uchburchak gipotenuzasiga tushirilgan balandligi $h = 4$ ga, to'g'ri burchak bissektrisasi $l = 5$ ga teng. Uchburchakning yuzini toping.

A) 52 B) $57\frac{1}{7}$ C) $42\frac{1}{7}$ D) 28

Yechish: To'g'ri burchakli ABC uchbrchakning katetlarini $AC = a, BC = b$ va gipotenuzasini $AB = c$ deb olaylik (2 – *chizma*). U holda $l = \frac{2ab}{a+b}\cos\frac{\angle C}{2}$ tenglikka ko'ra

$$l = \frac{ab}{a+b}\sqrt{2} \Rightarrow l(a+b) = ab\sqrt{2} \qquad (8.1)$$

bo'ladi. (8.1) ning har ikkala tomonini kvadratga ko'tarib, kerakli joylarga uchburchak yuzasi S uchun o'rinli bo'lgan ushbu

$$2S = ab = ch \qquad (8.2)$$

tengliklarni qo'yib, quyidagilarga ega bo'lamiz:

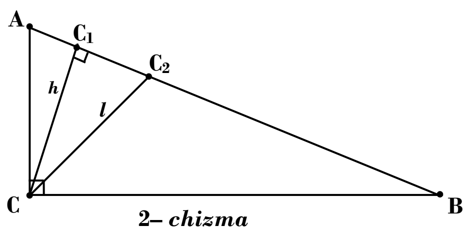
$$l^2(a+b)^2 = \left(2S\sqrt{2}\right)^2 \Rightarrow a^2 + b^2 + 2ab = \frac{8S^2}{l^2}, c^2 + 4S = \frac{8S^2}{l^2} \Rightarrow \left(\frac{2S}{h}\right)^2 + 4S = \frac{8S^2}{l^2} \Rightarrow$$

$$S = \frac{l^2h^2}{2h^2 - l^2}.$$

Demak, gipotenuzasiga tushirilgan balandligi va bissektrisasi mos ravishda h va l bo'lgan to'g'ri burchakli uchburchakning yuzi

$$S = \frac{l^2h^2}{2h^2 - l^2} \qquad (8.3)$$

formula orqali hisoblanadi. U holda $h = 4, l = 5$ qiymatlar uchun, uchburchakning yuzini (8.3) formula orqali hisoblasak, $S = \frac{5^2 \cdot 4^2}{2 \cdot 4^2 - 5^2} = 57\frac{1}{7}$ ga teng bo'ladi.



2– *chizma*

Javob: $57\frac{1}{7}$(B)

9 – masala. To'g'ri burchakli uchburchakning gipotenuzasiga tushirilgan balandligi uni ikkita to'g'ri burchakli uchburchakka ajratadi. Hosil bo'lgan uchburchaklarga ichki chizilgan aylanalarning radiuslari 7 va 24 ga teng bo'lsa, berilgan uchburchakning gipotenuzasiga tushirilgan balandligini toping.

A) 25 B) 56 C) 48 D) $48\sqrt{2}$

Yechish:

1 – *usul*: ACD va BCD uchburchaklar ikkita burchagi bo'yicha o'xshash: $\Delta ACD \sim \Delta BCD$ (3 – *chizma*). Ularning o'xshashlik koeffitsiyenti ularga ichki chizilgan aylanalar radiuslarining nisbatiga teng bo'ladi, ya'ni

$$k = \frac{7}{24}. \tag{9.1}$$

U holda (9.1) ga ko'ra

$$\frac{AC}{BC} = k = \frac{7}{24} \Rightarrow AC = 7x, BC = 24x.$$

ABC uchburchakda Pifagor teoremasidan AB gipotenuzani topib olamiz: $AB = \sqrt{(7x)^2 + (24x)^2} = 25x$. U holda CD kesma (ABC uchburchakning gipotenuzasiga tushirilgan balandligi) va BD kesma (ABC uchburchakda BC katetning AB gipotenuzadagi proyeksiyasi) uchun ushbu

$$CD = \frac{AC \cdot BC}{AB} = \frac{168x}{25}, BD = \frac{BC^2}{AB} = \frac{576x}{25} \tag{9.2}$$

tengliklar o'rinli.

Endi x ni BCD uchburchakka ichki chizilgan aylana radiusini topish formulasi yordamida topamiz:

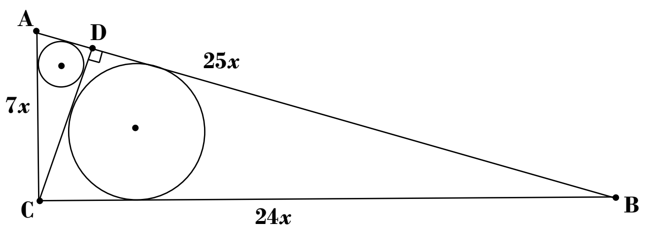
$$24 = \frac{CD + BD - BC}{2} = \frac{72x}{25} \Rightarrow x = \frac{25}{3}. \tag{9.3}$$

U holda (9.3) da topilgan x ning qiymatini (9.2) ga qo'yib, talab qilingan natija, ya'ni CD ning uzunligiga ega bo'lamiz:

$$CD = \frac{168x}{25} = 56.$$

Javob: 56(B)

2 – *usul*: ACD va BCD uchburchaklarga ichki chizilgan aylanalarning radiuslarini mos ravishda r_1 va r_2 deb olsak, u holda ABC uchburchakka ichki chizilgan aylananing radiusi uchun ushbu



3 – *chizma*

$$r = \sqrt{r_1^2 + r_2^2} \tag{9.4}$$

tenglikni isbotsiz keltiramiz. U holda yuqorida sanab o'tilgan uchburchaklarning har biriga ichki chizilgan aylananing radiusini topish formulasini yozib, hosil bo'lgan tengliklarni hadma – had qo'shib quyidagiga ega bo'lamiz:

$$\left.\begin{array}{l} \Delta ACD: \quad r_1 = \frac{AD + CD - AC}{2} \\ \Delta BCD: \quad r_2 = \frac{BD + CD - CB}{2} \\ \Delta ABC: \quad r = \frac{AC + BC - AB}{2} \end{array}\right] \Rightarrow$$

$$\Rightarrow r_1 + r_2 + r = CD = h. \tag{9.5}$$

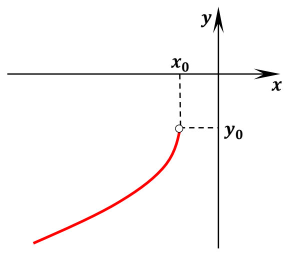
Demak, 2 – chizmadagi ikkita kichkina to'g'ri burchakli uchburchaklarga ichki chizilgan aylanalarning radiuslari r_1 va r_2 bo'lsa, u holda berilgan uchburchakka ichki chizilgan aylananing radiusi (9.4) formuladan, gipotenuzasiga tushirilgan balandligi esa (9.5) formuladan topiladi.

Bizda $r_1 = 7, r_2 = 24$. U holda $r = 25 \Rightarrow h = 56$.

Javob: 56(B)

Izoh: Yuqorida keltirilgan 2 – misolga tipik jihatdan o'xshash bo'lgan barcha masalalarni 1 – usuldan yozma ish yozganda, 2 – usuldan esa test yechishda foydalanish mumkin.

10 – masala. Rasmda $y = a\sqrt{bx + c} + d$ funksiyaning grafigi tasvirlangan. Quyidagilardan qaysi biri doimo o'rinli:



4 – *chizma*

A) $bc > d$ B) $a > c$ C) $ab < 0$ D) $abc > d$

Yechish: Bu masalani yechishdan oldin, $y = a\sqrt{bx + c} + d$ funksiyaning muhim bo'lgan ba'zi xossalarini isbotsiz keltiramiz:

I. Funksiya aniqlanish sohasida
- $a \cdot b > 0$ bo'lsa, o'suvchi
- $a \cdot b < 0$ bo'lsa, kamayuvchi

bo'ladi;

II. Funksiya argumentning $x_0 = -\frac{c}{b}$ qiymatida
- $a > 0$ bo'lsa, o'zining eng kichik qiymati $y_0 = d$ ga
- $a < 0$ bo'lsa, o'zining eng katta qiymati $y_0 = d$ ga

erishadi.

U holda keltirilgan xossalarga va 4 – chizmaga ko'ra quyidagi sistemani tuzib olamiz:

$$\begin{cases} a \cdot b > 0 \\ -\frac{c}{b} < 0 \\ a < 0 \\ d < 0 \end{cases} \quad . \qquad (10.1)$$

Bu sistemaga ko'ra a, b, c, d koeffitsiyentlarning ishorasi quyidagicha bo'ladi:

$$a < 0, b < 0, c < 0, d < 0. \qquad (10.2)$$

Endi (10.2) dan foydalanib, masalada keltirilgan javob variantlari orasidan A variantda keltirilgan $bc > d$ tengsizlik doimo o'rinli ekanligini tekshirib ko'rish mumkin.

Javob: $bc > d$(A)

11 – masala. $|x^2 - 5ax| = 15a$ tenglama ikkita haqiqiy yechimga ega bo'ladigan, a ning natural qiymatlari yig'indisini toping.

A) 3 B) 4 C) 2 D) 10

Yechish:

1 – *usul*: $|x^2 - 5ax| = 15a$ tenglama noldan farqli yechimga ega bo'lishi uchun, avvalo $a > 0$ bo'lishi kerak. Endi berilgan tenglamani yechamiz:

$$x^2 - 5ax = \pm 15a \Rightarrow x^2 - 5ax + 15a = 0, x^2 - 5ax - 15a = 0.$$

Hosil bo'lgan

$$x^2 - 5ax + 15a = 0 \qquad (11.1)$$

va

$$x^2 - 5ax - 15a = 0 \qquad (11.2)$$

kvadrat tenglamalarning diskriminantini mos ravishda $\mathcal{D}_1$ va $\mathcal{D}_2$ deb olib, ularni hisoblaymiz:

$$\mathcal{D}_1 = 25a^2 - 60a \text{ va } \mathcal{D}_2 = 25a^2 + 60a.$$

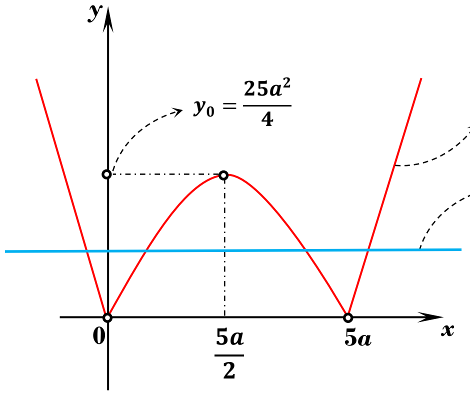
$a(> 0)$ ning istalgan qiymatida $\mathcal{D}_2 > 0$ ekani ravshan. Bu degani (11.2) tenglama ikkita turli haqiqiy ildizlarga ega. Eslatib o'tamizki, (11.1) va (11.2) tenglamalarning ildizlari berilgan modulli tenglamaning ildizi ham bo'ladi. Masala shartiga ko'ra tenglama ikkita haqiqiy yechimga ega bo'lishi uchun, majbur $\mathcal{D}_1$ manfiy bo'lishi kerak (O'ylang!).

$$\mathcal{D}_1 < 0,25a^2 - 60a < 0 \Longrightarrow 0 < a < 2{,}4. \quad (11.3)$$

(11.3) ga ko'ra a ning natural qiymatlari 1 va 2 bo'lib, ularning yig'indisi 3 bo'ladi.

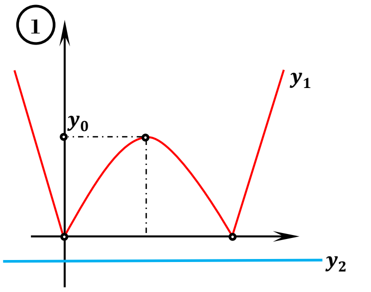
Javob: 3(A)

2 – *usul*: $y_1 = |x^2 - 5ax|$ va $y_2 = 15a$ funksiyalarning grafiklarini bitta koordinatalar tekisligida chizib olamiz(4 – chizma).

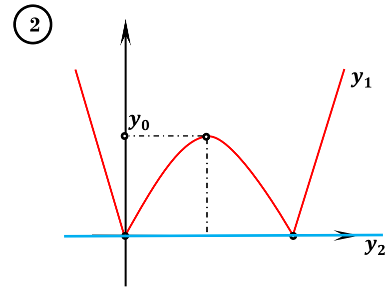


5 – *chizma*

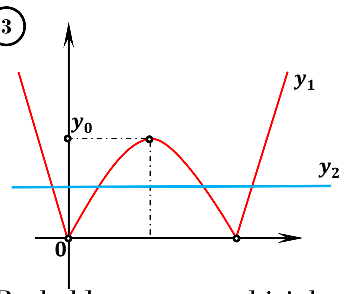
U holda y_2 to'g'ri chiziq y_1 siniq egri chiziqni quyidagi holatlarda kesib o'tish mumkin:



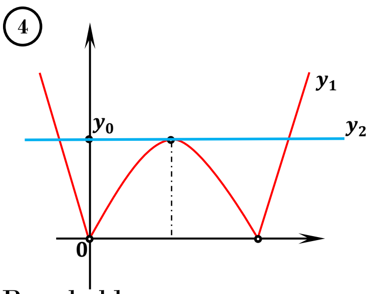
Bu holda y_1 va y_2 chiziqlar umumiy nuqtaga ega emas. Shuning uchun berilgan tenglama yechimga ega emas: $15a < 0 \Longrightarrow a < 0$.



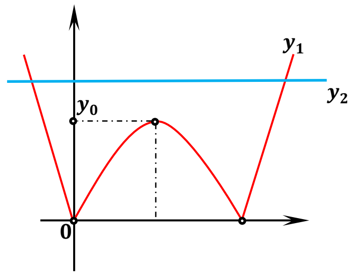
Bu holda $15a = 0, a = 0$ bo'lib, berilgan tenglama $x = 0$ yagona yechimga ega.



Bu holda y_1 va y_2 chiziqlar to'rtta umumiy nuqtaga



Bu holda y_1 va y_2 chiziqlar uchta umumiy

ega. Shuning uchun berilgan tenglama to'rtta yechimga ega bo'lib, ulardan bittasi manfiy, qolgan uchtasi musbat: $0 < 15a < y_0$.	nuqtaga ega. Shuning uchun berilgan tenglama uchta yechimga ega bo'lib, ulardan biri manfiy, qolgan ikkitasi musbat: $0 < 15a = y_0$.
5  Bu holda y_1 va y_2 chiziqlar ikkita umumiy nuqtaga ega. Shuning uchun berilgan tenglama ikkita yechimga ega bo'lib, ulardan biri musbat va ikkinchisi manfiy: $15a > y_0 > 0$.	

Masala shartiga ko'ra berilgan tenglama ikkita yechimga ega bo'lishi uchun $15a > y_0 > 0$ bo'lishi kerak. Bundan

$$15a > y_0 > 0,\ 15a > \frac{25a^2}{4} > 0 \Rightarrow a \in (0; 2{,}4).$$

a ning natural qiymatlari 1 va 2 bo'lib, ularning yig'indisi 3 ga teng.

Javob: 3(A)

12 – masala. R radiusli sferaga muntazam to'rtburchakli piramida ichki chizilgan. Uning uchidagi yassi burchak α ga teng bo'lsa, piramida yon sirtining yuzini toping.

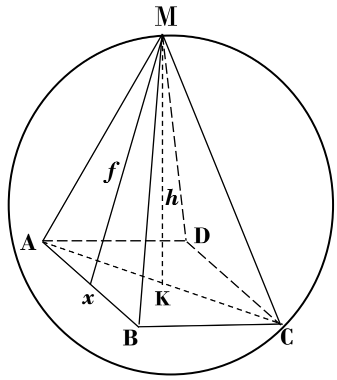
Yechish:

$ABCD$ kvadratning tomonini x, $ABCDM$ piramidaning apofemasini f, balandligini esa $MK = h$ olaylik(6 – *chizma*). Unda piramidaning yon sirti

$$S_{yon} = 4 \cdot \frac{x \cdot f}{2} = 2xf \qquad (12.1)$$

formuladan topiladi. U holda (12.1) formulaga ko'ra piramidaning yon sirtini topish uchun x va f noma'lum miqdorlarni masala shartida berilgan R va α ma'lum miqdorlar orqali ifodalash kerak bo'ladi.

$ABCD$ kvadratning diagonali $AC = 2CK = x\sqrt{2}$ bo'ladi. Teng yonli MBA uchburchakdan



6 – *chizma*

$$MB = MA = \frac{x}{2\sin\frac{\alpha}{2}},\ f = \frac{x}{2}\operatorname{ctg}\frac{\alpha}{2}$$

bo'lib, bularni (12.1) ga qo'ysak, piramidaning yon sirti uchun ushbu

$$S_{yon} = 2xf = x^2 \operatorname{ctg}\frac{\alpha}{2} \qquad (12.1)$$

tenglikni hosil qilamiz. Bundan ko'rinadiki, hamma gap noma'lum x ni R va α orqali ifodalashda qolayapti.

AKM uchburchakdan

$$AM^2 = AK^2 + MK^2 \Rightarrow h^2 = \frac{x^2}{4\sin^2\frac{\alpha}{2}} - \frac{x^2}{2} =$$

$$= \frac{x^2}{4} \cdot \frac{\cos\alpha}{\sin^2\frac{\alpha}{2}} \Rightarrow h = \frac{x}{2} \cdot \frac{\sqrt{\cos\alpha}}{\sin\frac{\alpha}{2}}$$

bo'lib, sferaning katta aylanasi piramidaning diagonal kesimi AMC uchburchakka tashqi chizilgan aylana bo'lganligi uchun, uchburchakka tashqi chizilgan aylana radiusini topish formulasidan quyidagilarni hosil qilamiz:

$$R = \frac{MA \cdot MC \cdot AC}{4 \cdot \frac{AC \cdot MK}{2}} = \frac{MA^2}{2h} = \frac{\frac{x^2}{4\sin^2\frac{\alpha}{2}}}{2 \cdot \frac{x}{2} \cdot \frac{\sqrt{\cos\alpha}}{\sin\frac{\alpha}{2}}} \Rightarrow$$

$$\Rightarrow x = 4R\sin\frac{\alpha}{2}\sqrt{\cos\alpha}.$$

Natijada (12.1) ga ko'ra so'ralgan natijaga ega bo'lamiz:

$$S_{yon} = x^2 \operatorname{ctg}\frac{\alpha}{2} = 16R^2 \sin^2\frac{\alpha}{2} \cdot \cos\alpha \cdot \operatorname{ctg}\frac{\alpha}{2} =$$

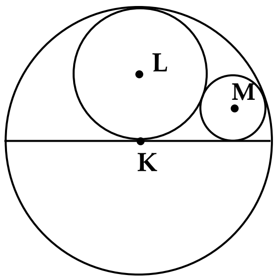
$$= 4R^2 \sin 2\alpha \Rightarrow S_{yon} = 4R^2 \sin 2\alpha.$$

Javob: $4R^2 \sin 2\alpha$

Izoh: 2016 – yilning test savollarida asosan, α ning quyidagi qiymatidagi savollar uchraydi:

1. $\alpha = 15° \Longrightarrow S_{yon} = 2R^2$
2. $\alpha = 30° \Longrightarrow S_{yon} = 2\sqrt{3}R^2$
3. $\alpha = 45° \Longrightarrow S_{yon} = 4R^2$.

13 – masala. AB kesma K aylananing diametri bo'lsin. L aylana K aylanaga hamda AB to'g'ri chiziqqa K aylananing markazida urinadi; M aylana K va L aylanaga hamda AB to'g'ri chiziqqa urinadi (chizmaga qarang). Agar M aylana radiusi $\frac{1}{2}$ ga teng bo'lsa, L aylana radiusini toping.



A) *aniqlab bo'lmaydi* B) 1 C) 2 D) 3

Yechish:

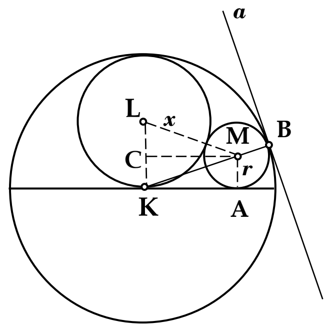
Mayli, M va K larning o'zi bilan shu nomli M va K aylanalarning markazlarini, B bilan M va K aylanlarning urinish nuqtasini belgilab olaylik(7 – chizma). Avvalo, M, K va B nuqtalarning bir to'g'ri chiziqda yotishini ko'rsatamiz.

Darhaqiqat, B nuqtadan har ikki aylanaga urinma o'tkazib olsak, ular ustma – ust tushadi. Uni a deb olsak,

$$MB \perp a, KB \perp a$$

bo'lib, natijada M, K va B nuqtalarning ko'rsatilgan tartiba bitta to'g'ri chiziqda yotishi kelib chiqadi.

M aylananing radiusini r, L aylananing radiusini x desak, ravshanki K aylananing radiusi



7 – chizma

$R = 2x$ bo'ladi. $MA \perp KA, MC \perp LK$ qilib MA va MC chiziqlar o'tkazib olamiz. Natijada

$$ML = x + r, CK = r, MK = 2x - r \Rightarrow LC = x - r$$

bo'lib, LMC uchburchakda Pifagor teoremasiga ko'ra

$$MC = \sqrt{ML^2 - LC^2} = 2\sqrt{xr}.$$

U holda MCK uchburchakda yana Pifagor teoremasiga ko'ra

$$MK^2 = MC^2 + CK^2 \Rightarrow (2x - r)^2 = 4xr + r^2 \Rightarrow$$

$$\Rightarrow x = 2r$$

tenglikni hosil qilamiz. Va demak, M, K va L aylanalarning radiuslari quyidagicha munosabatda bo'ladi:

$$R_M = r, R_L = 2r, R_K = 4r. \qquad (13.1)$$

Bizning misolda $R_M = \frac{1}{2} \Longrightarrow$(13.1) ga ko'ra $R_L = 1$.

Javob: 1(B)

14 – masala. Agar

$$\alpha + \beta + \gamma = \pi, \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2} = -\frac{1}{4}$$

bo'lsa, $\cos\alpha + \cos\beta + \cos\gamma$ ni toping.

A) 1 B) -1 C) 0 D) $\frac{3}{2}$

Yechish:

Ushbu va shunga o'xshash boshqa masalalarni yechishda foydalansa bo'ladigan quyidagi formulalarni isbotsiz keltiramiz:

$\alpha + \beta + \gamma = \pi$ bo'lsa,

1. $\operatorname{tg}\alpha + \operatorname{tg}\beta + \operatorname{tg}\gamma = \operatorname{tg}\alpha \cdot \operatorname{tg}\beta \cdot \operatorname{tg}\gamma$

2. $\cos\alpha+\cos\beta+\cos\gamma=4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}+1$

3. $\sin\alpha+\sin\beta+\sin\gamma=4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

tengliklar o'rinli.

U holda yuqorida keltirilgan 2 – formulaga ko'ra

$$\cos\alpha+\cos\beta+\cos\gamma=4\cdot\left(-\frac{1}{4}\right)+1=0$$

bo'ladi.

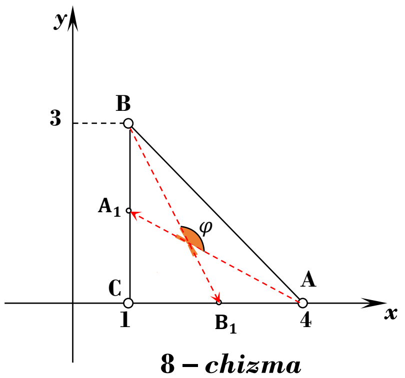
Javob: 0(C)

15 – masala. Uchburchakning uchlari to'g'ri burchakli dekart koordinatalar sistemasida quyidagicha berilgan: $A(4;0), B(1;3), C(1;0)$. O'tkir burchaklar medianalar orasidagi o'tmas burchak kosinusini toping.

A) $-\frac{4}{7}$ B) $-\frac{4}{5}$ C) $-\frac{5}{7}$ D) $-\frac{3}{5}$

Yechish:

Uchburchakni to'g'ri burchakli dekart koordinatalar sistemasida chizib olamiz:



8 – *chizma*

Natijada C burchagi to'g'ri bo'lgan to'g'ri burchakli ABC uchburchak hosil bo'ladi (8 – chizma).

BB_1 va AA_1 chiziqlar mos ravishda AC va BC tomonlarning medianalari bo'lsin. Ular orasidagi o'tmas burchakni φ bilan belgilab olamiz. Uning kosinusini topish uchun $\overrightarrow{AA_1}$ va $\overrightarrow{BB_1}$ vektorlar orasidagi burchakning kosinusini topish yetarli (O'ylang!):

$$B(1;3),\ B_1\left(\frac{5}{2};0\right)\longrightarrow\overrightarrow{BB_1}=\left(\frac{3}{2};-3\right), \qquad (15.1)$$

$$A(4;0),\ A_1\left(1;\frac{3}{2}\right)\longrightarrow\overrightarrow{AA_1}=\left(-3;\frac{3}{2}\right). \qquad (15.2)$$

Endi (15.1) va (15.2) ga ko'ra

$$\cos\varphi=\frac{\overrightarrow{AA_1}\cdot\overrightarrow{BB_1}}{\left|\overrightarrow{AA_1}\right|\cdot\left|\overrightarrow{BB_1}\right|}=$$

$$=\frac{-\frac{9}{2}-\frac{9}{2}}{\frac{9}{4}+9}=-\frac{4}{5}\longrightarrow\cos\varphi=-\frac{4}{5}.$$

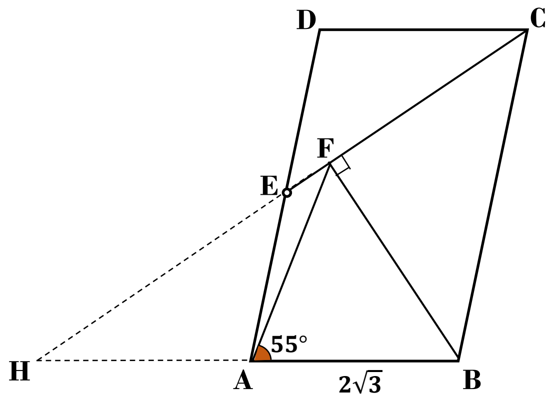
Javob: $-\frac{4}{5}$ (B)

16 – masala. $ABCD$ parallelogrammda E nuqta AD tomonning o'rtasi, F nuqta CE to'g'ri chiziqqa B nuqtadan tushirilgan perpendikulyarning asosi. Agar $AB=2\sqrt{3}$ va $\angle BAF=55°$ bo'lsa, ABF uchbuchakning yuzini toping.

A) $5\sin 55°$ B) $6\sin 55°$

C) $3\sin 55°$ D) $4\sin 55°$

Yechish:



9 – *chizma*

CE va AB to'g'ri chiziqlarning kesishish nuqtasi H bo'lsin (9 – chizma). Unda

$$\left.\begin{matrix}\left.\begin{matrix}AB\parallel DC\\ H\in AB\end{matrix}\right\}\longrightarrow AH\parallel DC\\ AE=ED\end{matrix}\right\}\Longrightarrow\Delta AEH=\Delta CDE\rightarrow$$

$$\longrightarrow AH=2\sqrt{3}$$

bo'ladi. ΔBFH da $\angle F=90°$ ekanini e'tiborga olsak, AF kesma BH gipotenuzaga tushirilgan mediana bo'ladi. Demak,

$$AF=\frac{BH}{2}=2\sqrt{3}.$$

Va nihoyat, ikki tomoni va ular orasidagi burchakka ko'ra uchburchak yuzini topish formulasidan foydalanib, ABF uchburchak yuzini topamiz:

$$S_{ABF}=\frac{1}{2}\cdot AF\cdot AB\cdot\sin 55°=6\sin 55°.$$

Javob: $6\sin 55°$(B)

17 – masala. Quyidagi ifodalarni soddalashtiring:

1. $\frac{(x-b)(x-c)}{(a-b)(a-c)}+\frac{(x-a)(x-c)}{(b-a)(b-c)}+\frac{(x-a)(x-b)}{(c-a)(c-b)}$;
2. $\frac{(x+b)(x+c)}{(a-b)(a-c)}+\frac{(x+a)(x+c)}{(b-a)(b-c)}+\frac{(x+a)(x+b)}{(c-a)(c-b)}$;
3. $a^2\frac{(x-b)(x-c)}{(a-b)(a-c)}+b^2\frac{(x-a)(x-c)}{(b-a)(b-c)}+c^2\frac{(x-a)(x-b)}{(c-a)(c-b)}$;
4. $\frac{a+x}{a(a-b)(a-c)}+\frac{b+x}{b(b-a)(b-c)}+\frac{c+x}{c(c-a)(c-b)}$.

Yechish:

1) Berilgan ifodani $f(x)$ funksiya deb olinsa, ya'ni

$$f(x)=\frac{(x-b)(x-c)}{(a-b)(a-c)}+\frac{(x-a)(x-c)}{(b-a)(b-c)}+\frac{(x-a)(x-b)}{(c-a)(c-b)}\quad(17.1)$$

u holda (17.1) ifodaning chap tomonini soddalashtirib chiqsak, kvadrat uchhad hosil bo'ladi deb faraz qilish mumkin (O'ylang!). Bizga ma'lumki, kvadrat uchhadning grafigi parabola bo'ladi va parabola x ning ikkitadan ortiq qiymatlarida teng qiymatlarni qabul qila olmaydi, ya'ni bir – biridan farqli a, b, c sonlarda $f(x)$ parabola uchun

$$f(a)=f(b)=f(c)$$

munosabat bajarilmaydi. Lekin, (17.1) ga ko'ra

$$f(a)=1, f(b)=1, f(c)=1\qquad(17.2)$$

bo'ladi. (17.1) va (17.2) dan quyidagi xulosaga kelish mumkin:

> Aytaylik, $f(x)$ parabola x ning uchta turli qiymatlarida teng qiymatlar qabul qilsa, u o'zgarmas funksiya bo'ladi, ya'ni $y=mx^2+nx+p$ parabolada $m=n=0$ bo'ladi.

Demak, (17.2) ga ko'ra (17.1) ifoda x ning istalgan qiymatida 1 ga teng qiymat qabul qiladi:

$$\frac{(x-b)(x-c)}{(a-b)(a-c)}+\frac{(x-a)(x-c)}{(b-a)(b-c)}+\frac{(x-a)(x-b)}{(c-a)(c-b)}=1.\ (*)$$

2)

$$\frac{(x+b)(x+c)}{(a-b)(a-c)}+\frac{(x+a)(x+c)}{(b-a)(b-c)}+\frac{(x+a)(x+b)}{(c-a)(c-b)}\quad(17.3)$$

ifoda x ning $-a, -b$ va $-c$ ga teng qiymatlarida 1 ga teng qiymatni qabul qilmoqda. Shuning uchun (17.3) ifodaga yuqoridagi fikr va mulohazalarni qo''lab, uning ham x ning istalgan qiymatida 1 ga teng bo'lishini hosil qilamiz:

$$\frac{(x+b)(x+c)}{(a-b)(a-c)}+\frac{(x+a)(x+c)}{(b-a)(b-c)}+\frac{(x+a)(x+b)}{(c-a)(c-b)}=1.\ (**)$$

3) Berilgan ifodani $f(x)$ deb olamiz:

$$f(x)=a^2\frac{(x-b)(x-c)}{(a-b)(a-c)}+b^2\frac{(x-a)(x-c)}{(b-a)(b-c)}+c^2\frac{(x-a)(x-b)}{(c-a)(c-b)}.\quad(17.4)$$

Va bu ifodani ham soddalashtirsak, kvadrat uchhad hosil bo'ladi deb faraz qilish mumkin. Endi

$$h(x)=f(x)-x^2\qquad(17.5)$$

funksiyani kiritib olamiz, bevosita tekshirib ko'rish mumkinki,

$$h(a)=0,\ h(b)=0,\ h(c)=0\qquad(17.6)$$

bo'ladi. $h(x)$ ning kvadrat uchbhad ekanligini va (17.6) ni hisobga olgan holda x ning istalgan qiymatida $h(x)=0$ tenglikning bajarilishiga ishonch hosil qilish mumkin. Demak, (17.4) va (17.5) ga ko'ra

$$a^2\frac{(x-b)(x-c)}{(a-b)(a-c)}+b^2\frac{(x-a)(x-c)}{(b-a)(b-c)}+c^2\frac{(x-a)(x-b)}{(c-a)(c-b)}=x^2\ (***)$$

bo'ladi.

4)

$$f(x)=\frac{a+x}{a(a-b)(a-c)}+\frac{b+x}{b(b-a)(b-c)}+\frac{c+x}{c(c-a)(c-b)}\ (17.7)$$

desak va undan foydalanib

$$h(x)=f(x)-\frac{x}{abc}\qquad(17.8)$$

funksiyani tuzib olsak, aytish mumkinki, $h(x)$ chiziqli funksiya bo'ladi (O'ylang!). Tekshirib ko'rish mumkinki,

$$h(-a)=0, h(-b)=0, h(-c)=0\qquad(17.9)$$

tengliklar o'rinli bo'ladi. $h(x)$ ning chiziqli funksiya ekanligidan (17.9) ga ko'ra x ning istalgan qiymatida

$$h(x) = f(x) - \frac{x}{abc} = 0$$

bo'lishi kelib chiqadi. Demak,

$$\frac{a+x}{a(a-b)(a-c)} + \frac{b+x}{b(b-a)(b-c)} + \frac{c+x}{c(c-a)(c-b)} = \frac{x}{abc}. \quad (****)$$

18 – masala. 2017 – yilgi test savollarida oldingi masalada keltirilgan 4 ta ifodaga oid quyidagi savollar uchraydi:

1) $\frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-a)(x-c)}{(b-a)(b-c)} + \frac{(x-a)(x-b)}{(c-a)(c-b)}$ funksiyaning biror nuqtadagi hosilasini va qiymatini topish;
2) $a^2\frac{(x-b)(x-c)}{(a-b)(a-c)} + b^2\frac{(x-a)(x-c)}{(b-a)(b-c)} + c^2\frac{(x-a)(x-b)}{(c-a)(c-b)}$ funksiyaning biror nuqtadagi hosilasini va qiymatini topish;
3) $\frac{1}{a(a-b)(a-c)} + \frac{1}{b(b-a)(b-c)} + \frac{1}{c(c-a)(c-b)}$ ifodani soddalashtirish;
4) $\frac{1}{(a-b)(a-c)} + \frac{1}{(b-a)(b-c)} + \frac{1}{(c-a)(c-b)}$ ifodani soddalashtirish;

Shular va shu kabi boshqa masalalarni yuqorida keltirilgan (*), (**), (***), (****) formulalardan foydalanib ishlash mumkin. Masalan, $\frac{1}{(a-b)(a-c)} + \frac{1}{(b-a)(b-c)} + \frac{1}{(c-a)(c-b)}$ ifodani soddalashtirish uchun (****) da $x = 0$ deyish yetarli:

$$\frac{1}{(a-b)(a-c)} + \frac{1}{(b-a)(b-c)} + \frac{1}{(c-a)(c-b)} = 0.$$

19 – masala. *SABC* uchburchakli piramidaning *S* uchidagi yassi burchaklari to'g'ri burchak. *SO* – piramida balandligi. *AOB* va *BOC* uchburchaklar yuzalari mos ravishda 16 va 4 ga teng. *ASB* uchburchak yuzasining *BSC* uchburchak yuzasiga nisbatini toping.

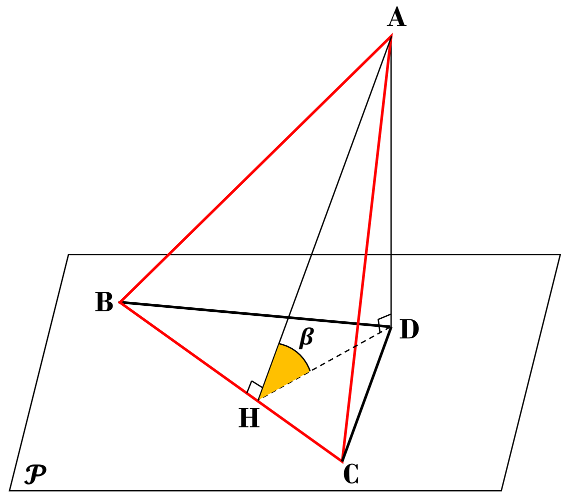
A) *aniqlab bo'lmaydi* B) 3 C) 2 D) $\sqrt{2}$

Yechish:

Bu masalani ishlash uchun ko'pburchak ortogonal proyeksiyasining yuzi haqidagi teoremadan foydalanamiz:

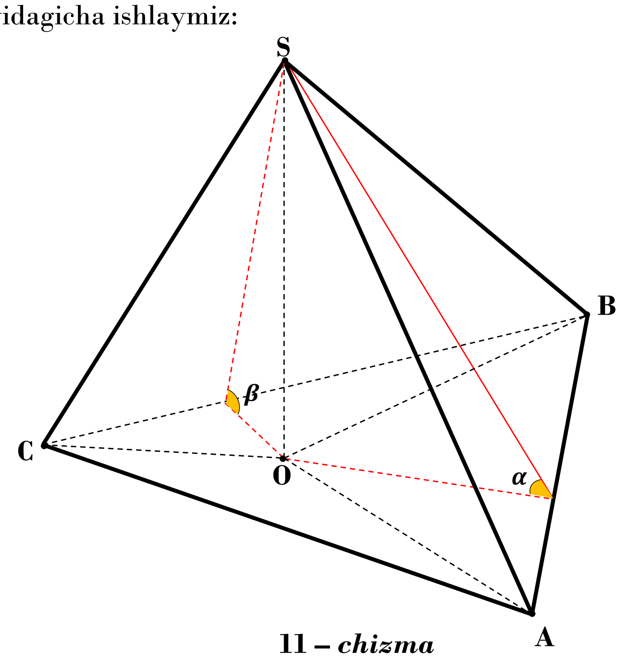
ABC uchburchakning $\mathcal{P}$ tekislikdagi ortogonal proyeksiyasi BCD uchburchak bo'lsin (10 – chizma). (ABC) va (BCD) tekisliklar orasidagi burchak β bo'lsa, u holda ABC va BCD uchburchaklar yuzi orasida quyidagi munosabat o'rinli:

$$S(ABC)\cos\beta = S(BCD). \quad (19.1)$$



10 – *chizma*

Endi shu qoidadan foydalanib masalani quyidagicha ishlaymiz:



11 – *chizma*

ASB va BSC yon yoqlarning ABC asosga og'ish burchaklari mos ravishda α va β bo'lsin (11 – chizma). U holda (19.1) ga ko'ra ushbu

$$\left.\begin{aligned} S(ASB)\cos\alpha = S(AOB)\\ S(BSC)\cos\beta = S(BOC)\end{aligned}\right\} \Rightarrow \begin{cases}\cos\alpha = \frac{S(AOB)}{S(ASB)}\\ \cos\beta = \frac{S(BOC)}{S(BSC)}\end{cases} \quad (19.2)$$

tengliklar o'rinli bo'ladi. Boshqa tomondan piramida yon yoqlarining S uchidagi burchaklari to'g'ri bo'lganligi uchun ABC uchburchakning (BSC) uchburchak tekisligidagi proyeksiyasi BSC yoqning o'zi bo'ladi. Demak, (19.1) ga ko'ra

$$S(ABC)\cos\beta = S(BSC) \Rightarrow \cos\beta = \frac{S(BSC)}{S(ABC)} \quad (19.3)$$

munosabatning bajarilishi ravshan (O'ylang!).

Xuddi shunday,

$$S(ABC)\cos\alpha = S(ASB) \Rightarrow \cos\alpha = \frac{S(ASB)}{S(ABC)} \quad (19.4)$$

bo'ladi. Va nihoyat, (19.2), (19.3) va (19.4) lardan quyidagilarga ega bo'lamiz:

$$\left.\begin{aligned}\frac{S(AOB)}{S(ASB)} = \frac{S(ASB)}{S(ABC)}\\ \frac{S(BOC)}{S(BSC)} = \frac{S(BSC)}{S(ABC)}\end{aligned}\right\} \longrightarrow \frac{S(AOB)}{S(AOB)} = \left(\frac{S(ASB)}{S(ASC)}\right)^2. \quad (19.5)$$

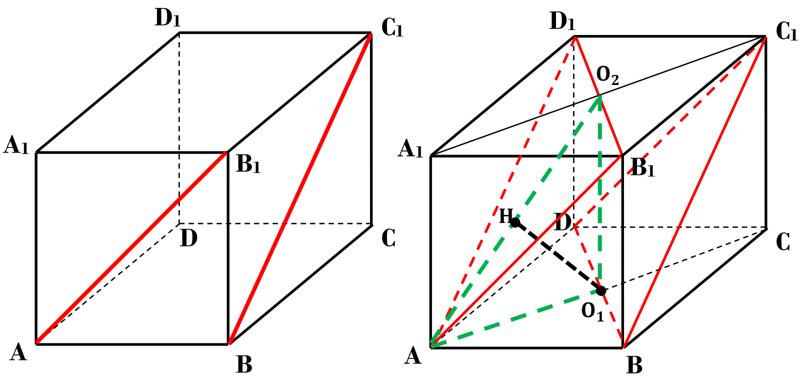
Masala shartiga ko'ra $S(AOB) = 16, S(BOC) = 4$ bo'lib, (19.5) dan masalaning yechimiga ega bo'lamiz: $\frac{S(ASB)}{S(ASB)} = 2$.

Javob: 2(C)

20 – masala. Qirrasi $5\sqrt{3}$ ga teng bo'lgan kubning qo'shni yoqlarining ayqash diagonallari orasidagi masofani toping.

A) $4\sqrt{2}$ B) $3\sqrt{3}$ C) 6 D) 5

Yechish:



a) *b)*

12 – *chizma*

AB_1 va BC_1 kesmalar $ABCDA_1B_1C_1D_1$ kubning ABB_1A_1 va BCC_1B_1qo'shni yoqlarining ayqash diagonallari bo'lsin (12.*a* – chizma). Ular orasidagi masofani topish uchun AB_1 va BC_1 lar yotgan parallel tekisliklar orasidagi masofani topish yetarli (O'ylang!). Shuning uchun berilgan kubda ma'lum bir qo'shimcha chiziqlar o'tkazish orqali BDC_1 va AB_1D_1 kesimlarni yasab olsak, ular yuqorida takidlagan parallel tekisliklar bo'ladi(12.*b* – chizma). Demak, BDC_1 va AB_1D_1 kesimlar orasidagi masofani topamiz: buning uchun kub asosining markazi O_1 nuqtadan AB_1D_1 tekislikka O_1H perpendikulyar o'tkazamiz va u AO_1O_2 to'g'ri burchakli uchburchakning O_1 to'g'ri burchagidan AO_2 gipotenuzasiga tushirilgan balandlikning uzunligiga teng bo'ladi (O'ylang!).

Kubning qirrasini a desak, $AO_1 = \frac{a\sqrt{2}}{2}$, $O_1O_2 = a \Longrightarrow AO_2 = \sqrt{(AO_1)^2 + (O_1O_2)^2} = \sqrt{\frac{3}{2}}a$ bo'ladi. Bundan esa ushbu

$$\boxed{O_1H = \frac{AO_1 \cdot O_1O_2}{AO_2} = \frac{\frac{a\sqrt{2}}{2}\cdot a}{\sqrt{\frac{3}{2}}a} = \frac{a}{\sqrt{3}}} = \boxed{\frac{5\sqrt{3}}{\sqrt{3}} = 5}$$

talab qilingan natijani hosil qilamiz.

Javob: 5(D)

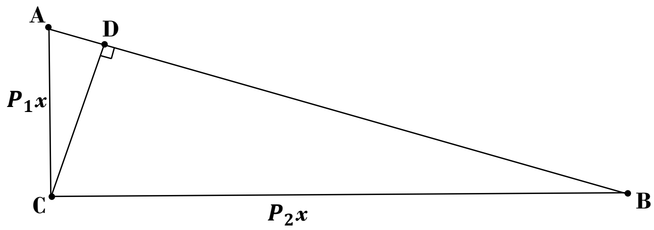
21 – masala. To'g'ri burchakli ABC uchburchak CD balandlik bilan BCD va ACD uchburchaklarga bo'lingan. Shu uchburchaklar yarim perimetrlari mos ravishda 20 va 21 ga teng. ABC uchburchakning yarim perimetrini toping.

A) $24\sqrt{2}$ B) 26 C) 42 D) 29

Yechish:

ADC va BCD uchburchaklar o'xshash (*qarang*: 9 – masala). U holda ADC va BCD uchburchaklarning yarim perimetrlarini mos ravishda P_1 va P_2 deb olinsa, quyidagi munosabat o'rinli bo'ladi:

$$\frac{P_1}{P_2} = \frac{AC}{BC} \Longrightarrow AC = P_1x, BC = P_2x.$$



13 – *chizma*

So'ngra ABC uchburchakda Pifagor teoremasiga ko'ra

$$AB = \sqrt{AC^2 + BC^2} = x\sqrt{P_1^2 + P_2^2}$$

bo'lib, ACD va ABC uchburchaklar o'xshashligidan foydalanib ABC uchburchakning yarim perimetri P ni quyidagi tenglikdan topamiz:

$$\frac{AC}{AB} = \frac{P_1}{P} \Longrightarrow P = \sqrt{P_1^2 + P_2^2}. \qquad (21.1)$$

Bizni masalada $P_1 = 20, P_2 = 21$ bo'lib, (21.1) ga ko'ra $P = 29$ bo'ladi.

Javob: 29(D)

22 – masala. 3^{101} sonini 101 ga bo'lgandagi qoldiqni toping?

A) 27 B) 9 C) 1 D) 3

Yechish:

Bu misolning yechish uchun quyidagi teoremadan foydalanamiz:

> ***Ferma teoremasi:*** $p-$ tub son, $a-$ natural son bo'lsin. Agar a son p ga bo'linmasa, u holda a^{p-1} sonni p ga bo'lgandagi qoldiq 1 ga teng bo'ladi, ya'ni
> $$a^{p-1} \equiv 1(\text{mod } p).$$

Bizning misolda $a = 3$, $p = 101$ bo'lib, Ferma teoremasiga ko'ra $3^{101-1} = 3^{100}$ ni 101 ga bo'lgandagi qoldiq 1 ga teng bo'ladi:

$$3^{100} = 1(\text{mod } 101).$$

U holda $3^{101} = 3^{100} \cdot 3$ sonni 101 ga bo'lsak 3 qoldiq qoladi(O'ylang!).

***Javob*:** 3(D)

23 – masala. ABC uchburchakning BC va AC tomonlarida mos ravishda D va E nuqtalar shunday olinganki, bunda burchak $BAD = 50°$, burchak $ABE = 30°$. Agar burchak $ABC = ACB = 50°$ bo'lsa, burchak BED ni toping.

A) 40° B) 50° C) 70° D) 80°

Yechish:

Bu masalani o'quvchiga yanada tushunarli bo'lishi uchun ikki xil usulda ishlaymiz.

1 – usul.

$\angle BED = x$ deylik (14 – chizma). $\angle BAD = \angle ABD = 50° \Longrightarrow$

$$BD = AD. \qquad (23.1)$$

$\angle A = 80°$ bo'lgani uchun $\angle DAE = 30°, \angle AEB = 70°$ bo'lib, $\angle AED = 70° + x$ bo'ladi. U holda ADE va BED uchburchaklarda sinuslar teoremasiga ko'ra mos ravishda ushbu

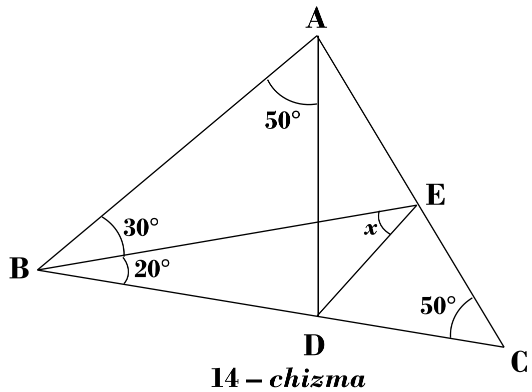
$$\frac{BD}{DE} = \frac{\sin x}{\sin 20°} \qquad (23.2)$$

$$\frac{AD}{DE} = \frac{\sin(70°+x)}{\sin 30°} \qquad (23.3)$$

tengliklarni yozib olamiz va ularning (23.1) ga ko'ra chap qismlari teng bo'lganligi uchun o'ng tomonlarini tenglashtirib, quyidagi trigonometrik tenglamaga ega bo'lamiz:

$$2\sin 20° \sin(70° + x) = \sin x . \Longrightarrow$$

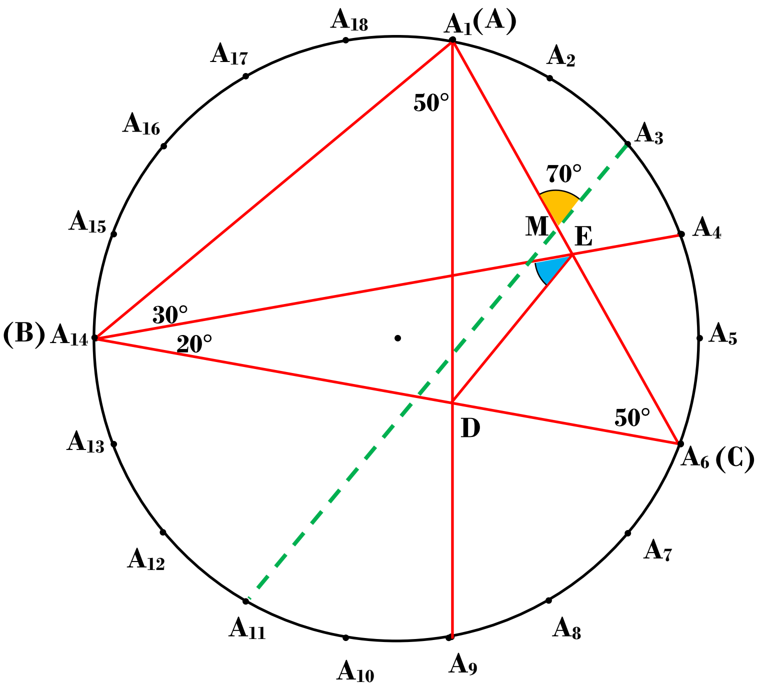
$$\Longrightarrow \cos(50° + x) = 0 \Longrightarrow x = 40°.$$



14 – *chizma*

2 – usul.

Aylanaga ichki chizilgan muntazam o'nsakkizburchakning uchlari uni teng 18 ta bo'lakka bo'lib, har bir yoyning gradus o'lchovi 20° dan bo'lishi ravshan (15 – chizma). Masala



15 – *chizma*

shartida keltirilgan ABC uchburchakning uchlari A_1, A_6 va A_{14} nuqtalarda bo'ladi. Uning uchlaridan chiquvchi AD va BE to'g'ri chiziqlar esa o'nsakkizburchakning A_1A_9 va A_4A_{14} diagonallari bilan ustma – ust tushadi. Ularning kesishish nuqtasini M deylik. DE va A_5A_{11} chiziqlar parallel (O'ylang!). U holda

$$\angle A_1MA_3 = \frac{1}{2}\left(\overset{\frown}{A_1A_3} + \overset{\frown}{A_6A_{11}}\right) = \frac{40°+100°}{2} = 70° \Rightarrow$$

$$\angle DEA_6 = \angle A_1MA_3 = 70°.$$

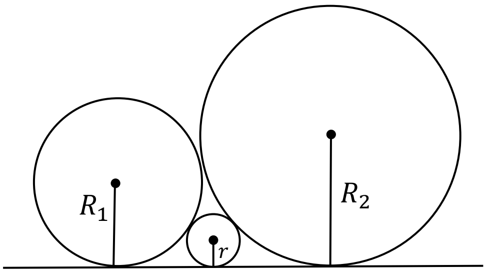
CDE uchburchakning D uchidagi burchagini topsak

$\angle EDC = 180° - (50° + 70°) = 60°$ ga teng bo'lib, BED uchburchakdan izlangan burchak topiladi:

$$x + \angle EBD = \angle EDC \Longrightarrow x = 40°$$

Javob: $40°$(A)

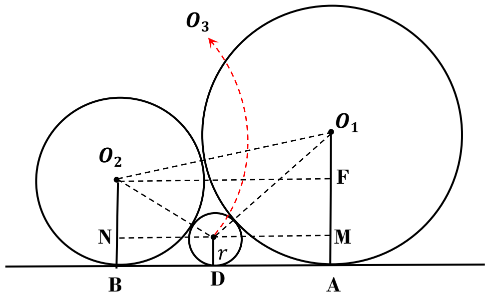
24 – masala. Quyidagi uchta aylana radiuslari uchun qaysi tenglik o'rinli bo'ladi?



A) $\frac{1}{\sqrt{r}} = \frac{\sqrt{R_1}}{\sqrt{R_2}} + \frac{\sqrt{R_2}}{\sqrt{R_1}}$ B) $\frac{1}{\sqrt{r}} = \frac{\sqrt{R_1+R_2}}{R_1 \cdot R_2}$

C) $\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}}$ D) $\frac{1}{\sqrt{r}} = \frac{R_1 \cdot R_2}{\sqrt{R_1+R_2}}$

Yechish:



16 – *chizma*

Aylanalarning markazlarini O_1, O_2, O_3 bilan belgilab olib, O_3 nuqtadan AB ga parallel qilib MN va O_2F to'g'ri chiziqlarni o'tkazamiz. AB to'g'ri chiziq O_1A, O_2B va O_3D radiuslarga perpendikulyar bo'lgani uchun $AM = BN = r$, demak, $O_1M = R_2 - r$ va $O_2N = R_1 - r$. Undan tashqari $O_1O_3 = R_2 + r$ va $O_2O_3 = R_1 + r$. Demak,

$$MO_3 = \sqrt{(R_2 + r)^2 - (R_2 - r)^2} = 2\sqrt{R_2 r};$$

xuddi shunga o'xshash

$$NO_3 = 2\sqrt{R_1 r}.$$

$O_1F = R_2 - R_1$ va $O_1O_2 = R_1 + R_2$ bo'lgani uchun

$$O_2F = \sqrt{(R_2 + R_1)^2 - (R_2 - R_1)^2} = 2\sqrt{R_1 R_2}$$

bo'lib,

$$AB = O_2F = MN = MO_3 + NO_3$$

tenglikka ko'ra

$$2\sqrt{R_1 r} + 2\sqrt{R_2 r} = 2\sqrt{R_1 R_2},$$

bundan

$$\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}}$$

kelib chiqadi.

Javob: $\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}}$(C)

25 – masala. $f(x)=\frac{2}{4^x+2}$ funksiya berilgan. $f\left(\frac{1}{2001}\right)+f\left(\frac{2}{2001}\right)+\cdots+f\left(\frac{2000}{2001}\right)$ ning qiymatini toping.

A) 2000 B) 2001 C) 1001 D) 1000

Yechish:

Avvalo berilgan funksiyada

$$f\left(\frac{k}{n}\right)+f\left(\frac{n-k}{n}\right)=1 \qquad (25.1)$$

bo'lishini ko'rsatamiz:

$$f\left(\frac{k}{n}\right)+f\left(\frac{n-k}{n}\right)=\frac{2}{4^{\frac{k}{n}}+2}+\frac{2}{4^{\frac{n-k}{n}}+2}=$$

$$=\frac{2}{4^{\frac{k}{n}}+2}+\frac{2}{4^{1-\frac{k}{n}}+2}=\frac{2}{4^{\frac{k}{n}}+2}+\frac{2}{\frac{4}{4^{\frac{k}{n}}}+2}=$$

$$=\frac{2}{4^{\frac{k}{n}}+2}+\frac{2\cdot 4^{\frac{k}{n}}}{4+2\cdot 4^{\frac{k}{n}}}=\frac{2}{4^{\frac{k}{n}}+2}+\frac{4^{\frac{k}{n}}}{2+4^{\frac{k}{n}}}=1.$$

Demak, (25.1) ga ko'ra talab qilingan yig'indiga ega bo'lamiz:

$$f\left(\frac{1}{2001}\right)+f\left(\frac{2}{2001}\right)+\cdots+f\left(\frac{2000}{2001}\right)=$$

$$\left(f\left(\frac{1}{2001}\right)+f\left(\frac{2000}{2001}\right)\right)+\left(f\left(\frac{2}{2001}\right)+f\left(\frac{1999}{2001}\right)\right)+$$

$$+\cdots+\left(f\left(\frac{1000}{2001}\right)+f\left(\frac{1001}{2001}\right)\right)=\underbrace{1+1+\cdots+1}_{1000\ ta}=$$

$$=1000.$$

***Javob*:** 1000(D)

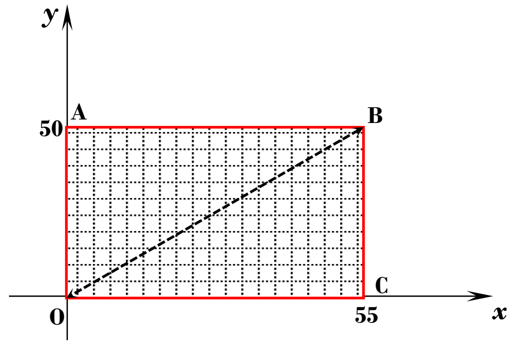
26 – masala. Tomonlari 50 va 55 ga teng bo'lgan to'g'ri to'rtburchak birlik kvadratlarga bo'lingan. Uning diagonali birlik kvadratchalarning uchlari bo'lmish nuqtalarning nechtasidan o'tadi?

A) 2 B) 1 C) 6 D) 5

***Yechish*:**

To'g'ri to'rtburchakning bir uchini koordinata boshi sifatida qabul qilib, uning shu uchidan chiquvchi tomonlari orqali koordinata o'qlarini yo'naltiramiz (17 – chizma). Uning diagonali – OB chiziqning tenglamasi

$$y=\frac{BC}{OC}\cdot x=\frac{50}{55}x=\frac{10}{11}x, 0\le x\le 55 \qquad (26.1)$$



17 – *chizma*

bo'ladi. Masala shartiga ko'ra OB chiziq birlik kvadratchalarning uchlaridan o'tsa, shu nuqtalarda uning x va y koordinatasi butun son bo'ladi. U holda, (26.1) ga ko'ra x ning 0, 11, 22, 33, 44, 55 ga teng qiymatlarida y butun qiymatlarni qabul qiladi. Demak, to'g'ri to'rtburchakning diagonali birlik kvadratchaning uchlari bo'lmish nuqtalarning 6 tasidan o'tadi.

***Javob*:** 6(C)

27 – masala. $[1;25]$ oraliqdagi nechta natural sonlarda $\frac{11n+3}{13n+4}$ kasr qisqaradi.

A) 5 *ta* B) 4 *ta* C) 8 *ta* D) 2 *ta*

***Yechish*:**

$\frac{11n+3}{13n+4}$ kasr qisqarsa, u holda $\frac{13n+4}{11n+3}=1+\frac{2n+1}{11n+3}$ kasr ham qisqaradi. Agar $\frac{2n+1}{11n+3}$ kasr qisqarsa,

$$\frac{11n+3}{2n+1}=5+\frac{n-2}{2n+1}$$

kasr ham qisqaradi. Xuddi shunday $\frac{n-2}{2n+1}$ kasr qisqarsa,

$$\frac{2n+1}{n-2}=2+\frac{5}{n-2}$$

kasr qisqaradi. Va nihoyat $\frac{5}{n-2}$ kasr qisqarishi uchun $n-2$ son 5 ga karrali son bo'lishi kerak. n ning [1;25] oraliqqa tegishli bo'lgan 2, 7, 12, 17, 22 qiymatlarida berilgan kasr qisqaradi. Demak, n ning 5 ta qiymatida berilgan kasr qisqarar ekan.

***Javob*:** 5 ta(A)

28 – masala. $(a^2+b^2+4)x^2+2(a+b+2)x+3=0$ tenglama haqiqiy yechimlarga ega bo'lsa, $3a-b$ ni toping.

A) 3 B) −3 C) −4 D) 4

***Yechish*:**

Tenglamaning diskriminanti $\mathcal{D}$ ni hisoblaylik:

$$\mathcal{D} = 4(a + b + 2)^2 - 12(a^2 + b^2 + 4) =$$

$$= 4(a^2 + b^2 + 4 + 4a + 4b + 2ab - 3a^2 - 3b^2 - 12) =$$

$$= 4(-2a^2 - 2b^2 + 2ab + 4a + 4b - 8) =$$

$$= 4\left(-(a^2 - 2ab + b^2) - (a^2 - 4a + 4) - (a^2 - 4b + 4)\right) =$$

$$= -4((a - b)^2 + (a - 2)^2 + (b - 2)^2) \Rightarrow$$

$$\Rightarrow \mathcal{D} = -4((a - b)^2 + (a - 2)^2 + (b - 2)^2) \le 0$$

Demak, berilgan tenglama haqiqiy yechimga ega bo'lishi uchun $a = b = 2$ bo'lib, $3a - b = 4$.

***Javob*:** 4(D)

29 – masala. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ va $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$ bo'lsa, $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ ni toping.

A) 0,5 B) 2 C) 1 D) 0

***Yechish*:**

$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$ tenglikning ikkala tomonini xyz ga ko'paytirsak, quyidagi ifoda hosil bo'ladi:

$$ayz + bxz + cxy = 0$$

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ tenglikni esa ikkala tomonini kvadratga ko'tarib, quyidagi ifodani hosil qilamiz:

$$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} + 2 \cdot \frac{xy}{ab} + 2 \cdot \frac{xz}{ac} + 2 \cdot \frac{yz}{bc} =$$

$$= \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} + 2 \cdot \frac{cxy + bxz + ayz}{abc} = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}.$$

Demak, $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

***Javob*:** 1(C)

30 – masala. Agar $x - \sqrt{\frac{20}{x}} = 7$ bo'lsa, $\sqrt{5x} - x$ ning qiymatini toping.

A) −2 B) −1 C) 1 D) 2

***Yechish*:**

$x - \sqrt{\frac{20}{x}} = 7$ tenglikda quyidagi almashtirishlarni bajaramiz:

$$x - \sqrt{\frac{20}{x}} = 7 \Rightarrow x\sqrt{x} - 2\sqrt{5} = 7\sqrt{x} \Rightarrow$$

$$\Rightarrow x\sqrt{x} - 5\sqrt{x} = 2\sqrt{5} + 2\sqrt{x} \Rightarrow$$

$$\Rightarrow \sqrt{x}(x - 5) = 2\left(\sqrt{x} + \sqrt{5}\right) \Rightarrow$$

$$\Rightarrow \sqrt{x}\left(\sqrt{x} - \sqrt{5}\right)\left(\sqrt{x} + \sqrt{5}\right) = 2\left(\sqrt{x} + \sqrt{5}\right) \Rightarrow$$

$$\Rightarrow \sqrt{x}\left(\sqrt{x} - \sqrt{5}\right) = 2 \Rightarrow \sqrt{5x} - x = -2.$$

***Javob*:** -2

31 – masala. Uchburchakning balandliklari 4, 5 va 6 ga teng. Unga tashqi chizilgan aylana radiusi R uchun quyidagi munosabatlardan qaysi biri to'g'ri?

A) $R \ge \frac{120}{37}$ B) $R \le \frac{120}{37}$ C) $R > \frac{120}{13}$ D) $R \le \frac{120}{13}$

***Yechish*:**

Uchburchakning balandliklari h_1, h_2, h_3 va ichki chizilgan aylana radiusi r uchun ushbu munosabat o'rinli:

$$\frac{1}{r} = \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3}.$$

U holda

$$\frac{1}{r} = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \Rightarrow r = \frac{60}{37}.$$

Endi $R \ge 2r$ tengsizlikka ko'ra $R \ge \frac{120}{37}$ munosabat kelib chiqadi.

***Javob*:** $R \ge \frac{120}{37}$

32 – masala. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{10} = A$ bo'lsa, A qaysi oraliqqa tegishli?

A) $\left(\frac{23}{10}; \frac{25}{6}\right)$ B) $\left(\frac{15}{11}; \frac{23}{11}\right)$ C) $(6; 7)$ D) $\left(\frac{35}{6}; \frac{47}{6}\right)$

***Yechish*:**

Quyidagi ikkita

$$A_1 = 1 + \frac{1}{2} + \underbrace{\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}}_{8\,ta} = \frac{23}{10}$$

va

$A_2 = 1 + \frac{1}{2} + \underbrace{\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}}_{8\,ta} = \frac{25}{6}$

yig'indilarni qarasak,

$$\frac{1}{10} < \frac{1}{3} \le \frac{1}{3},\ \frac{1}{10} < \frac{1}{4} < \frac{1}{3},\ \frac{1}{10} < \frac{1}{5} < \frac{1}{3},$$

$$\frac{1}{10} < \frac{1}{6} < \frac{1}{3},\ \frac{1}{10} < \frac{1}{74} < \frac{1}{3},\ \frac{1}{10} < \frac{1}{8} < \frac{1}{3},$$

$$\frac{1}{10} < \frac{1}{9} < \frac{1}{3},\ \frac{1}{10} \le \frac{1}{10} < \frac{1}{3}$$

tengsizliklarga ko'ra

$$A_1 < A < A_2$$

bo'lishi kelib chiqadi. Demak, $A \in \left(\frac{23}{10}; \frac{25}{6}\right)$.

***Javob*:** $\left(\frac{23}{10}; \frac{25}{6}\right)$(A)

33 – masala. Nechta natural $n < 100$ soni uchun $\frac{n^3+23}{24} \in \mathbb{N}$ o'rinli bo'ladi?

A) 4   B) 5   C) 9   D) 10

***Yechish*:**

$$\frac{n^3+23}{24} = \frac{n^3-1+24}{24} = \frac{n^3-1}{24} + 1 =$$

$$= \frac{(n-1)(n^2+n+1)}{24} + 1.$$

Oxirgi tenglikdan $\frac{n^3+23}{24} \in \mathbb{N}$ bo'lishi uchun $\frac{(n-1)(n^2+n+1)}{24} \in \mathbb{N}$ bo'lishi kelib chiqadi. Ravshanki, istalgan $n \in \mathbb{N}$ da (n^2+n+1) toq son bo'ladi. shunga asosan quyidagi hollarni ko'rib chiqamiz:

1 – *hol*: n^2+n+1 3 ga karrali son bo'lsin. U holda $n-1$ esa 8 ga karrali son bo'ladi. Bundan esa $n-1 = 8k \Rightarrow n = 8k+1 \Rightarrow n^2+n+1 = 64k^2+24k+3$ kelib chiqadi. Oxirgi ifodadan esa ko'rinib turibdiki, $n = 8k+1$ ko'rinishidagi sonlarda k ning 0, 3, 6, 9, 12 $(n < 100)$ qiymatlaridagina n^2+n+1 son 3 ga karrali bo'ladi (O'ylang!). $\Rightarrow n = 1, n = 25, n = 49, n = 73, n = 97$.

2 – *hol*: n^2+n+1 3 ga karrali son bo'lmasin. U holda $n-1$ ning 24 ga karrali son bo'lishi kelib chiqadi. $\Rightarrow n = 1, n = 25, n = 49, n = 73, n = 97$. Lekin n ning keltirilgan qiymatlarida n^2+n+1 3 ga karrali son bo'liadi (1 – hol ga qarang).

Shunday qilib, n ning 1, 25, 49, 73, 97 ga teng bo'lgan 5 ta qiymatida berilgan kasr natural son bo'ladi.

***Javob*:** 5 (B)

34 – masala. $\begin{cases} a = \frac{2}{37} + \frac{9}{51} - \frac{50}{76} \\ b = \frac{11}{37} - \frac{2}{17} + \frac{4}{38} \end{cases}$ bo'lsa, $\frac{a}{b}$ ni toping.

A) $-0{,}5$ B) -1 C) $-1{,}5$ D) -2

***Yechish*:**

Berilgan sistemani ushbu

$$\begin{cases} a = \frac{2}{37} + \frac{3}{17} - \frac{25}{38} \\ b = \frac{11}{37} - \frac{2}{17} + \frac{4}{38} \end{cases}$$

ko'rinishda yozib olib, sistemaning 1 – tengligini 2 ga, 2 – sini 3 ga ko'paytirib hadma – had qo'shib yuborilsa

$$2a + 3b = \left(\frac{4}{37} + \frac{6}{17} - \frac{50}{38}\right) + \left(\frac{33}{37} - \frac{6}{17} + \frac{12}{38}\right) = 0$$

bo'lib, bundan $\frac{a}{b} = -1{,}5$ kelib chiqadi.

***Javob*:** $-1{,}5$(C)

35 – masala. $26^x + 27 \ge 9\left(6-\sqrt{10}\right)^x + 3\left(6+\sqrt{10}\right)^x$ tengsizlikni yeching.

A) $\left[\log_{6+\sqrt{10}} 9; \log_{6-\sqrt{10}} 3\right]$
B) $\left(\log_{6+\sqrt{10}} 9; \log_{6-\sqrt{10}} 3\right)$
C) $\left(-\infty; \log_{6+\sqrt{10}} 9\right] \cup \left[\log_{6-\sqrt{10}} 3; +\infty\right)$
D) $\left(-\infty; \log_{6+\sqrt{10}} 9\right) \cup \left(\log_{6-\sqrt{10}} 3; +\infty\right)$

***Yechish*:**

$6-\sqrt{10} = \frac{26}{6+\sqrt{10}}$ deb olinsa, berilgan tengsizlik $9\left(\frac{26}{6+\sqrt{10}}\right)^x + 3\left(6+\sqrt{10}\right)^x \le 26^x + 27$ ko'rinishga keladi. $\Rightarrow \left(6+\sqrt{10}\right)^x = t$ deb almashtirish olamiz va

$$3t^2-(26^x+27)t+9\cdot 26^x\le 0\Rightarrow$$

$$t^2-\left(9+\frac{26^x}{3}\right)t+3\cdot 26^x\le 0\Rightarrow$$

$$(t-9)\left(t-\frac{26^x}{3}\right)\le 0\Rightarrow$$

$$\left(\left(6+\sqrt{10}\right)^x-9\right)\left(\left(6+\sqrt{10}\right)^x-\frac{26^x}{3}\right)\le 0.$$

Oxirgi tengsizlikda

$$26^x=\left(6+\sqrt{10}\right)^x\left(6-\sqrt{10}\right)^x$$

deb olinsa,

$$\left(\left(6+\sqrt{10}\right)^x-9\right)\left(\left(6+\sqrt{10}\right)^x-\frac{\left(6+\sqrt{10}\right)^x\left(6-\sqrt{10}\right)^x}{3}\right)\le 0$$

$$\Rightarrow\left(\left(6+\sqrt{10}\right)^x-9\right)\left(\left(6-\sqrt{10}\right)^x-3\right)\ge 0$$

ko'rinishga keladi. U holda ushbu

$$\left[\begin{array}{l}\begin{cases}\left(6+\sqrt{10}\right)^x-9\le 0\\ \left(6-\sqrt{10}\right)^x-3\le 0\end{cases}\\ \\ \begin{cases}\left(6+\sqrt{10}\right)^x-9\ge 0\\ \left(6-\sqrt{10}\right)^x-3\ge 0\end{cases}\end{array}\right.$$

tengsizliklar majmuasiga ega bo'lamiz (O'ylang!). Uni yechsak,

$$\left[\begin{array}{l}\begin{cases}\left(6+\sqrt{10}\right)^x-9\le 0\\ \left(6-\sqrt{10}\right)^x-3\le 0\end{cases}\\ \\ \begin{cases}\left(6+\sqrt{10}\right)^x-9\ge 0\\ \left(6-\sqrt{10}\right)^x-3\ge 0\end{cases}\end{array}\right.\Rightarrow\left[\begin{array}{l}\begin{cases}x\le\log_{6+\sqrt{10}}9\\ x\le\log_{6-\sqrt{10}}3\end{cases}\\ \\ \begin{cases}x\ge\log_{6+\sqrt{10}}9\\ x\ge\log_{6-\sqrt{10}}3\end{cases}\end{array}\right.$$

$$\Rightarrow\left[\begin{array}{l}x\le\log_{6+\sqrt{10}}9\\ x\ge\log_{6-\sqrt{10}}3\end{array}\right.\Rightarrow$$

$$x\in\left(-\infty;\log_{6+\sqrt{10}}9\right]\cup\left[\log_{6-\sqrt{10}}3\,;+\infty\right)$$

bo'ladi.

***Javob*:** $\left(-\infty;\log_{6+\sqrt{10}}9\right]\cup\left[\log_{6-\sqrt{10}}3\,;+\infty\right)$(C)

36 – masala. a,b va c haqiqiy sonlar uchun $a-7b+8c=4$ va $8a+4b-c=7$ tengliklar o'rinli bo'lsa, $a^2-b^2+c^2$ qanday bo'ladi?

A) 0 B) 1 C) 4 D) 7

***Yechish*:**

Berilgan har ikkala ifodani $a+8c=4+7b$ va $8a-c=7-4b$ ko'rinishda yozib olamiz va ularni kvadratga ko'tarib hadma – had qo'shib yuborsak,

$$\left.\begin{array}{l}(a+8c)^2=(4+7b)^2\\ (8a-c)^2=(7-4b)^2\end{array}\right\}+\Rightarrow$$

$$65a^2+65c^2=65+65b^2\Rightarrow a^2-b^2+c^2=1$$

bo'ladi.

***Javob*:** 1(B)