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Variant - 14.

1. $a = 6c_1 + 2$
$b = 6c_2 + 4$ (A)
$a+b = 6(c_1+c_2)+6$
$a+b = 6(c_1+c_2+1)$

2. $\left(\frac{\sqrt{7}+\sqrt{5}}{\sqrt{3}+1} \cdot \frac{\sqrt{7}-\sqrt{5}}{\sqrt{3}-1}\right) : \left(\frac{3\sqrt{3}}{\frac{1}{\sqrt{3}}} - \frac{\sqrt{3}}{9} + \frac{\frac{\sqrt{3}}{1}}{\sqrt{27}}\right)$ (C)
$= \frac{2}{2} : \frac{3\sqrt{3}-\sqrt{3}+\sqrt{3}}{9} = 1 : \frac{3\sqrt{3}}{9} = \frac{9}{3\sqrt{3}} = \sqrt{3}$

3. $3^{18} = x+1$. $3^{18}-1 = x$ $9^9-1 = x$.
$729^3 - 1 = x$ $(728)(729^2+729+1) = x$
$13;14$ (B)

4. $a = \sqrt{2}-1$ $b = \sqrt{2}+1$ $a \cdot b = 1$.
$a+b = 2\sqrt{2}$
$a^3+b^3 = (a+b)^3 - 3ab(a+b) =$ (D)
$= 8 \cdot 2\sqrt{2} - 3 \cdot 2\sqrt{2} = 10\sqrt{2}$

5. $\frac{256m^4-1}{256m^4-128m^3+32m^2-8m+1} =$
$= \frac{(4m)^4-1^4}{256m^4-128m^3+16m^2+16m^2-8m+1} =$
$= \frac{(4m-1)(4m+1)(16m^2+1)}{16m^2(16m^2-8m+1)+16m^2-8m+1} =$ (B)
$= \frac{(4m-1)(4m+1)(16m^2+1)}{(16m^2-8m+1)(16m^2+1)} = \frac{4m+1}{4m-1}$

6. $a_1 = 37$ $d = 1,5$ $a_n = 2b_n$.
$b_1 = 11$ $d = 1,5$ $a_1+(n-1)\cdot d = 2(b_1+(n-1)d)$
$37+1,5(n-1) = 22+3(n-1)$ (C)
$15 = 1,5(n-1)$ $n = 11$ $n-1 = 10$.

7. $\frac{x}{x^2-4} + \frac{x-2}{x^2+2x} = \frac{x+2}{x^2-2x}$ $2x+(x-2)(x-4) = x+2$
(A) $x \neq 2$ $x = 3$

8. $\sqrt{21-\sqrt{21+x}} = x$
$21-\sqrt{21+x} = x^2$.
$21-x^2 = \sqrt{21+x}$
$441-42x^2+x^4 = 21+x$.
$x^4-42x^2-x+420 = 0$.
$x = 4$. (B)

9. $\begin{cases} 2ax = x^{1-2b^3}+9. & 2ax-9 = x^{1-2b^3} \\ 3cx = x^{2b^3-1}-5 & 3cx = \frac{1}{2ax-9}-5. \end{cases}$
$c = \frac{46-10ax}{3x(2ax-9)}$ (A)

10. $\frac{5}{|x+2|+2} < |x+2|-2$.
$5 < (x+2)^2-4$. $(x+2)^2 > 9$
$(x+2-3)(x+2+3) > 0$
$(x-1)(x+5) > 0$
$(-\infty;-5) \cup (1;\infty)$.
(D) $-5;-4;-3;-2;-1;0;1$

11. $a_6+a_{10} = 18$ $a_9+a_{11} = 38$. (D)
$S_{15} = \frac{a_1+a_{15}}{2} \cdot 15 = \frac{18}{2} \cdot 15 = 135$

12. $\begin{cases} \frac{b_1}{1-q} = 3 & \text{①} \\ \frac{b_1^3}{1-q^3} = 81 \end{cases}$ ~~$\frac{1}{2}$~~
$b_1 = 3(1-q)$
$\frac{27(1-q)^3}{1-q^3} = 81$.
$(1-q)^3 = 3(1-q^3)$ $q = -2$
$\boxed{q = -\frac{1}{2}}$ $q = 1$. (A)

13. $2+\operatorname{tg}\alpha\cdot\operatorname{tg}\beta+(\operatorname{tg}\alpha+\operatorname{tg}\beta)$
$\operatorname{ctg}(\alpha+\beta) =$
$= 2+\operatorname{tg}\alpha\cdot\operatorname{tg}\beta+(\operatorname{tg}\alpha+\operatorname{tg}\beta) \cdot$
$\cdot \frac{1-\operatorname{tg}\alpha\operatorname{tg}\beta}{\operatorname{tg}\alpha+\operatorname{tg}\beta} = 3$
(A)

14. $\sin x = \frac{1}{2}$ $\cos x = \frac{\sqrt{3}}{2}$ (C)

$6{,}8 + 26\cos^2 x = 6{,}8 + 2\cdot\frac{3}{4} = 8{,}3$

15. $\cos(x+\pi k) = \cos x$

$\cos(x+\pi k) - \cos x = 0.$

$-2\sin\frac{x+\pi k - x}{2}\sin\frac{x+\pi k + x}{2} = 0$

$\sin\frac{\pi}{2}k = 0$ $\frac{\pi}{2}k = \pi n.$

$\boxed{k = 2n}$ (A)

16. $f(x) = \frac{7\cdot 2^{2x} + 5\cdot 2^{-2x}}{2}$

$g(x) = \frac{7\cdot 2^{2x} - 5\cdot 2^{-2x}}{2}$

$f^2(x) - g^2(x) = \left(\frac{7\cdot 2^{2x} + 5\cdot 2^{-2x} + 7\cdot 2^{2x} - 5\cdot 2^{-2x}}{2}\right)\cdot$

$\left(\frac{7\cdot 2^{2x} + 5\cdot 2^{-2x} - 7\cdot 2^{2x} + 5\cdot 2^{-2x}}{2}\right) =$

$7\cdot 2^{2x}\cdot 5\cdot 2^{-2x} = 35$ (B)

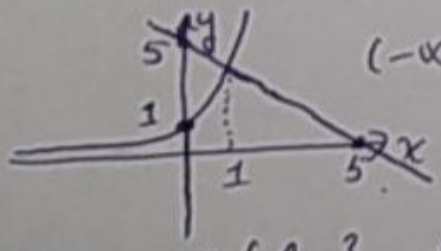
17. $6^x - 6^{-x} = 6$ $(6^x - 6)\cdot 6^x = (a-6)\cdot a =$

$6^x = a.$ $= a^2 - 6a = 1.$

$a - \frac{1}{a} = 6$ $a^2 - 6a - 1 = 0.$ (A)

$a^2 - 6a = 1.$

18. $4^x \le 5 - x.$



$(-\infty; 1]$ (A)

19. $a = \frac{1}{6}\left(\log_2^3 3 - \log_2^3 6 - \log_2^3 12 + \log_2^3 24\right)$

$\log_2 3 = x$ $\log_2 6 = x+1$ $\log_2 12 = x+2$

$\log_2 24 = 3 + x$ (B)

$a = \frac{1}{6}\left(x^3 - (x+1)^3 - (x+2)^3 + (x+3)^3\right)$

$a = 2x + 3 = 2\log_2 3 + 3$ $2^{2\log_2 3 + 3} = 8\cdot 9 = 72.$

20. $3^{\log_3^2 x} + x^{\log_3 x} = 162.$

$3^{\log_3 x\cdot\log_3 x} = x^{\log_3 x}$

$2\cdot x^{\log_3 x} = 162$ $x^{\log_3 x} = 81$

$\log_3 x\cdot\log_3 x = 4$ $\log_3 x = 2$

$\log_3 x = -2$ $x = 9$ $x = \frac{1}{9}.$

$9^4\cdot\frac{1}{9^4} = 1$ (D)

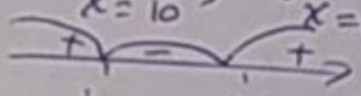
21. $\log_{100} x^2 + \lg^2 x < 6.$

$2\cdot\frac{1}{2}\lg x + \lg^2 x < 6.$

$\lg^2 x + \lg x - 6 < 0.$

$\lg x = -3$ $\lg x = 2.$

$x = 10^{-3}$ $x = 10^2$



$\frac{1}{1000}$ 100 $\left(\frac{1}{1000}; 100\right)$ (C)

$\frac{1+99}{2}\cdot 99 = 4950.$

22. $f^2(x)$ $\frac{1}{f(x)}$ $x = 10.$

$2\cdot f(x)\cdot f'(x)$ | $-\frac{1}{f^2(x)}\cdot f'(x)$

$2\cdot f(10)\cdot f'(10) = 4$ $-\frac{f'(10)}{f^2(10)} = -2.$

$f'(10) = 2.$

23. $f(x) = \frac{x^2}{x^3+1}$ $\left(\sqrt[3]{e-1}; 2\right)$

$f(x) = \frac{3x^2}{3(x^3+1)}$

$F(x) = \frac{1}{3}\ln|x^3+1| + C.$

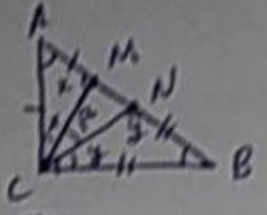
$2 = \frac{1}{3}\ln|e-1+1| + C.$

$C = 2 - \frac{1}{3} = \frac{5}{3}$ (A)

$F(x) = \frac{1}{3}\ln(x^3+1) + \frac{5}{3}.$

24. (D)

25. (D) $|2x-3|=-4$ $\emptyset$

26. 

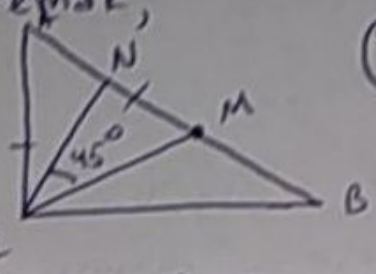
$x+\beta+y=90^\circ$

$2x+2y+30^\circ=120^\circ+180^\circ$

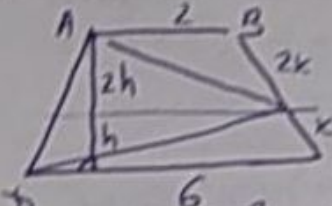
$x+y=135^\circ$.

$\beta=45^\circ$.

Demak,



(D)

27. 

$\frac{2+6}{2}\cdot 3h=48$

$h=4$

$48-\frac{2\cdot 8}{2}-\frac{6\cdot 4}{2}=28$.

28. $A(3;0)$ $B(-1;2)$ $y=x+2$.

$(x;x+2)$.

$(x-3)^2+(x+2)^2=(x+1)^2+(x+2-2)^2$

(A) $x=3$ $y=5$. $R=5$.

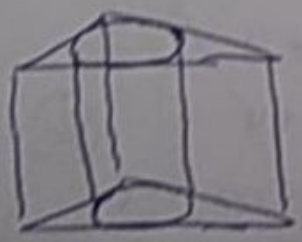
$(x-3)^2+(y-5)^2=25$.

29. $\begin{cases}2x+5y=3\\3x+4y=2\end{cases}$ $M(0;4)$

$x=-\frac{2}{7}$ $y=\frac{5}{7}$.

$\frac{x+\frac{2}{7}}{-\frac{2}{7}-0}=\frac{y-\frac{5}{7}}{\frac{5}{7}-4}$

$23x-2y+8=0$ (A)

30. 

$a=2\sqrt{3}$

$r=\frac{\sqrt{3}}{6}\cdot 2\sqrt{3}=1$.

$H=4$.

$S_t=2\pi r^2+2\pi rH=$

$=2\pi+2\pi\cdot 4=10\pi$

Variant-11

1. $12+99=111$ (B)

2. $\frac{7}{1+\frac{3}{x-2}} = \frac{7(x-2)}{x+1}$

$x \neq 2 \quad x \neq -1$

$2+(-1)=-1$ (B)

3. $xy^2-2y+2y^2-xy = y^2(x+2) -$

$-y(x+2) = (x+2)(y^2-y) = y(y-1)(x+2)$

(B)

4. $\frac{\left(\sqrt[3]{(a^2+4)\cdot\sqrt{1+\frac{4}{a^2}}} - \sqrt[3]{(a^2-4)\sqrt{1-\frac{4}{a^2}}}\right)^2 - \frac{\sqrt[3]{a}}{a}}{\left(\sqrt[3]{\frac{(\sqrt{a^2+4})^3}{a}} - \sqrt[3]{\frac{(\sqrt{a^2-4})^3}{a}}\right)^2 - \frac{\sqrt[3]{a}}{a}} = \frac{\sqrt{a^4-16}-a^2}{\sqrt{a^4-16}-a^2} = 1$

$= \frac{\frac{(\sqrt{a^2+4}-\sqrt{a^2-4})^2}{\sqrt[3]{a^2}}}{\sqrt{a^4-16}-a^2} - \frac{\sqrt[3]{a}}{a} = \frac{2}{\sqrt[3]{a^2}} \cdot \frac{\sqrt[3]{a}}{a} =$

$= \frac{2\sqrt[3]{a}}{a} - \frac{\sqrt[3]{a}}{a} = \frac{\sqrt[3]{a}}{a}$ (A)

5. $\sqrt{6+\sqrt{6+\sqrt{6+\dots}}} = a \qquad \sqrt{6+a}=a$

$6+a=a^2 \quad a^2-a-6=0 \quad \boxed{a=3}$

$\sqrt{3\cdot\sqrt{3\cdot\sqrt{3\cdot}}} = x \qquad \sqrt{3\cdot x}=x$

$3\cdot x = x^2 \quad 3=x$ (A)

6. $x\cdot\frac{t}{2}=1 \quad y\cdot t=1 \quad x=2y$

$(x+y)\cdot 18=1$

$3y\cdot 18=1 \quad y=\frac{1}{54} \quad x=\frac{1}{27}$

$y\cdot t_1=\frac{2}{3} \quad \frac{1}{54}\cdot t_1=\frac{2}{3}$

$t_1=36$ (A)

7. $x^4-2x^3+x^2-9=0$

$(x^2-x)^2=9 \quad x^2-x=3 \quad x^2-x=-3$

$x^2-x-3=0 \quad x^2-x+3=0$

$a=1 \quad b=2 \quad \emptyset$

$1+2=3$ (B)

8. $x^2+(k+2)^2x+2k-4=0$

$x_1<2 \quad x_2<2$

$(x_1-2)<0 \quad (x_2-2)<0$

$(x_1-2)(x_2-2)>0$

$x_1x_2-2(x_1+x_2)+4>0$

$4+2k-4+2(k+2)^2>0$

$4+2k-4+2k^2+8k+8>0$

$2k^2+10k+8>0$

$k^2+5k+4>0$

$(-\infty;-4)\cup(-1;\infty)$

$\boxed{k=1}$ (D)

9. $x^7 \cdot |x^2+8x+7|<0$

$\begin{cases} x<0 & [-8;1] \\ x\neq -7 & -8;-6;-5; \\ x\neq -1 & -4;-3;-2 \end{cases}$ (A)

10. $\begin{cases} 2\leq x\leq 18 \\ -2\leq y\leq 3 \end{cases} \quad \frac{x}{y}$

$\min \to \frac{2}{3} \qquad \max \to \frac{2}{-2}=-1$

$(-\infty;-1]\cup[\frac{2}{3};\infty)$ (D)

11. $1;4;7;\dots;46$

$1;7;13;19;25;\dots$ (D)

$S_{16}=\frac{2\cdot 1+15\cdot 6}{2}\cdot 16=736$

12. $800;775;750;\dots;$

$S_n=\frac{2\cdot 800+(n-1)\cdot(-25)}{2}\cdot n=6300$

$n=9$ (B)

13. $\frac{1}{\cos 20^\circ} - 4\cos 40^\circ = \frac{1-4\cos 40^\circ \cdot \cos 20^\circ}{\cos 20^\circ} =$

$= \frac{1-4\cdot\frac{1}{2}(\cos 60^\circ + \cos 20^\circ)}{\cos 20^\circ} = \frac{1-2\left(\frac{1}{2}+\cos 20^\circ\right)}{\cos 20^\circ} =$

$= \frac{1-1-2\cos 20^\circ}{\cos 20^\circ} = -2$ (D)

14. $\operatorname{tg} 20^\circ + \operatorname{tg} 40^\circ + \operatorname{tg} 60^\circ + \ldots + \operatorname{tg} 160^\circ = 0$

$\operatorname{tg} x = -\operatorname{tg} y \quad x+y = 180^\circ$ (A)

15. $\cos\left(x-\frac{\pi}{2}\right) = 0 \quad \sin x = 0 \quad x = \pi n$ (C)

16. $f(4x) = 4x+7 \quad f(f(1)) = ?$

$f(x) = x+7 \quad f(1) = 8$

$f(8) = 8+7 = 15$ (A)

17. $5^x - 5^y = 3 \quad x+y = 3$

$5^{2x} + 5^{2y} + 25^x \cdot 5^y - 5^x \cdot 25^y =$

$\left(5^x - 5^y\right)^2 + 2\cdot 5^{x+y} + 5^{x+y}\left(5^x - 5^y\right)$

$3^2 + 2\cdot 5^3 + 5^3 \cdot 3 = 634$ (D)

18. $\lg 2019{,}9 - \lg 0{,}00020199 =$

$= \lg \frac{2019{,}9}{0{,}00020199} = \lg 10^7 = 7$ (D)

19. $4^{\log_4^2(x+2)} + 2(x+2)^{\log_4\sqrt{x+2}} = 8.$

$(x+2)^{\log_4(x+2)} + 2(x+2)^{\log_4\sqrt{x+2}} = 8.$

$a^2 + 2a - 8 = 0 \quad a = -4 \quad a = 2.$

$(x+2)^{\log_4\sqrt{x+2}} = 2.$

$\log_4\sqrt{x+2}\cdot\log_4(x+2) = \log_4 2$

$\frac{1}{2}b^2 = \frac{1}{2} \quad b = \pm 1.$

$\log_4(x+2) = 1 \quad x = 2.$

$\log_4(x+2) = -1 \quad x = -2+\frac{1}{4}$

$-\frac{7}{4}$

$2+\left(-\frac{7}{4}\right) = \frac{1}{4}$ (C)

20. $y = \sqrt{\sin\sqrt{x}} - 6x + \ln 2$

$y'\left(\frac{\pi^2}{4}\right) = ?$

$y' = \frac{1}{2}\sin^{-\frac{1}{2}}\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\cos\sqrt{x} - 6$

$y'\left(\frac{\pi^2}{4}\right) = -6$ (A)

21. (A)

22. $\int\left(6 - x^2 f(x)\right)dx = x^2 - 5mx + C$

$6 - x^2 f(x) = 2x - 5m.$

$x = -2 \quad f(-2) = 4.$

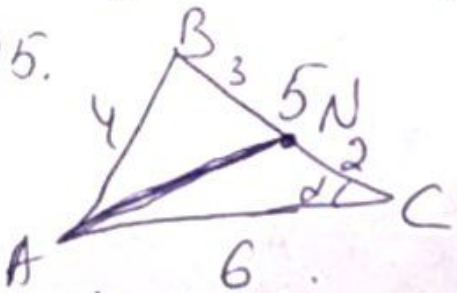
$6 - 4\cdot 4 = -4 - 5m$

$m = 1{,}2$ (C)

23. 2; 3; (A)

24. $6\cdot 5\cdot 4 = 120$ (B)

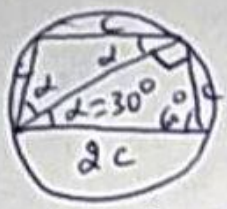
25.



$\begin{cases} AN^2 = 6^2 + 2^2 - 2\cdot 6\cdot 2\cdot\cos\angle \\ 4^2 = 6^2 + 5^2 - 2\cdot 6\cdot 5\cdot\cos\angle \end{cases}$

$AN = \sqrt{22}$ (D)

26.

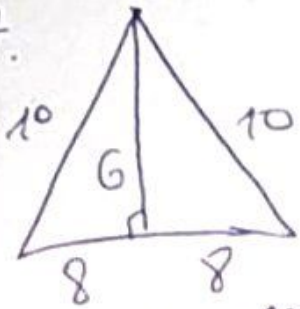


$H=\frac{\sqrt{3}}{2}c$

$2c=2R \quad c=R.$ (C)

$S=\frac{2c+c}{2}\cdot\frac{\sqrt{3}}{2}c=\frac{3\sqrt{3}}{4}R^2$

27.



$R=\frac{10\cdot10\cdot16}{2\cdot16\cdot6}=\frac{25}{3}$ (B)

28. $(2;0)\quad(0;-6)$

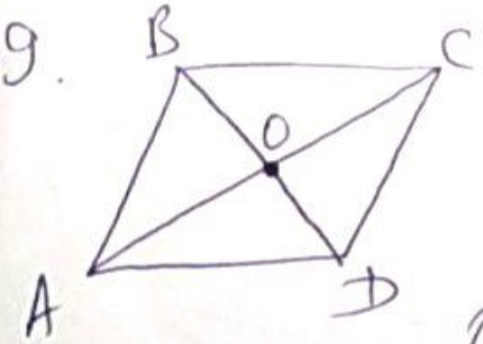
$y=kx+b \qquad k_1\cdot k_2=-1$

$0=2k+b$

$-6=b \qquad k_1=3 \quad k_2=-\frac{1}{3}$

$y=-\frac{1}{3}x \qquad x+3y=0$ (C)

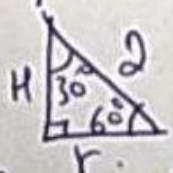
29.



$\overline{AC}=k\cdot\overline{AO}$ (C)

$\overline{AC}=2\cdot\overline{AO} \qquad k=2$

30. $f=2 \quad \alpha=60°.$ (C)



$r=1. \quad S_{y\text{o}N}=\frac{6\cdot\frac{2\sqrt{3}}{3}\cdot2}{2}=4\sqrt{3}.$

$a=2\cdot1\cdot\operatorname{tg}30°=\frac{2\sqrt{3}}{3}.$

$S_6=6\cdot1^2\cdot\operatorname{tg}30°=2\sqrt{3} \qquad S_1=2\sqrt{3}+4\sqrt{3}=6\sqrt{3}.$

Variant-12

1. $\begin{cases} 4a = 3b \\ EKUB(a;b) = 6 \end{cases}$ $\frac{a}{b} = \frac{3}{4}$ $a = 18$ $b = 24$ $18+24=42$ (A)

2. $\frac{2}{3x} = -\frac{3}{4y} = \frac{4}{5z}$ $y < 0$

(C) $10z = 12x$ $z > x > y$

3. $a^2 - b^2 + 8a - 2b + 15 =$
$a^2 + 8a - (b^2 + 2b - 15) =$
$(b+5)(b-3)$
$= (a - b - 5)(a + b - 3)$ (C)

4. $\sqrt{5}a - 3b = 2a - \sqrt{5}b - 15$
$a = -b$ (A)
$-3b = -2b - 15$
$b = 15$ $a = -15$

5. $a^2 - 1 = 8^{17}(2^{49} + 1)$
$a^2 - 1 = 2^{51}(2^{49} + 1)$
$a^2 - 1 = 2^{100} + 2^{51}$
$a^2 = 2^{100} + 2^{51} + 1$
$a = 2^{50} + 1$
$\frac{2^{50} + 1 - 1}{2^{48}} = 2^2$ (B)

6. $v = \frac{S}{t}$ $v_1 = \frac{5}{15} = \frac{1}{3}$
$v_2 = \frac{3}{55}$
$\frac{\frac{3}{55}}{\frac{1}{3}} \cdot 100\% = \frac{9}{55} \cdot 100^{20} = \frac{180}{11}$ (A)

7. $x\sqrt{x} - 8\sqrt{x} = 7 \quad x - \sqrt{x}$
$x\sqrt{x} - \sqrt{x} = 7 + 7\sqrt{x}$
$\sqrt{x}(\sqrt{x}-1)(\sqrt{x}+1) = 7(1+\sqrt{x})$
$x - \sqrt{x} = 7$ (C)

8. $n \cdot (13x - 1) = 17n - n^2$
$13x - 1 = 17 - n$
$x = \frac{18-n}{13} > 0$
$n < 18$ (B)

9. $||x| - 2| \le 1$
$-1 \le |x| - 2 \le 1$
$1 \le |x| \le 3$ (D)
$[-3;-1] \cup [1;3]$

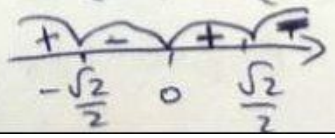
10. $S_5 = 100$
$x + x+1 + x+2 + x+3 + x+4 = 100$
$x = 18$ (C)

11. $\frac{(\sin^2\alpha + tg^2\alpha + 1)(\cos^2\alpha + 1 - ctg^2\alpha)}{(\cos^2\alpha + ctg^2\alpha + 1)(\sin^2\alpha + tg^2\alpha - 1)} =$
$\frac{\left(\sin^2\alpha + \frac{1}{\cos^2\alpha}\right)\left(\cos^2\alpha + \frac{\sin^2\alpha - \cos^2\alpha}{\sin^2\alpha}\right)}{\left(\cos^2\alpha + \frac{1}{\sin^2\alpha}\right)\left(\sin^2\alpha + \frac{\sin^2\alpha - \cos^2\alpha}{\cos^2\alpha}\right)} =$
$= \frac{\sin^2\alpha}{\cos^2\alpha} \cdot \frac{\cos^2\alpha}{\sin^2\alpha} = 1$ (B)

12. $\sin x + \cos x = 1{,}04$
$\sin x > 0 \quad \cos x > 0$ (B)

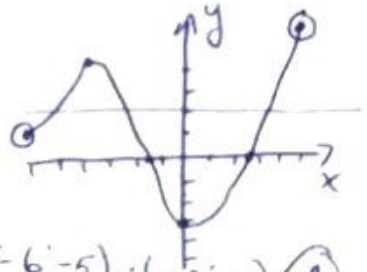
13. $\sin 2x + 2\sin x = \cos x + 1$
$2\sin x(\cos x + 1) = \cos x + 1$
$\sin x = 0 \quad \cos x = -1$
$x = \pi n; \ n \in Z$
$-\pi; 0; \pi$ (C)

14. $y = \frac{3x^2}{(1+x^2)(4x^2+1)} = \frac{1}{1+x^2} - \frac{1}{4x^2+1}$
$y' = \frac{-2x}{(1+x^2)^2} + \frac{8x}{(4x^2+1)^2} = 0 \quad x = \pm\frac{\sqrt{2}}{2} \quad x = 0$



$y\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{3}$ (C)

15. $[-6;5]$
$f(x) < 2$



$[-6;-5) \cup (-2;4)$ (A)

16. $3^{-x} = a \quad 2^x = b$
$216^{-x} = 6^{-3x} =$
$= 3^{-3x} \cdot 2^{-3x} =$
$(3^{-x})^3 (2^x)^{-3} =$
$= a^3 b^{-3}$ (D)

17. $\sqrt{(\log_{16}24-\log_{16}6)\cdot\log_{16}4}=$

$=\sqrt{\log_{16}4\cdot\log_{16}4}=\log_{16}4=\frac{1}{2}$ (D)

18. $y=\ln(x^2-2x-3)<0$.

$x^2-2x-3<1$.

$\begin{cases} x^2-2x-4<0 \\ x^2-2x-3>0 \end{cases}$ (C)

19. $\ln(x+1)\geq 2$ $x+1\geq e^2$

(C) $x\geq e^2-1$

$x=7$.

20. $y=e^{-x}-2x^7$

$y'=-e^{-x}-14x^6$ (A)

21. $f(x)=\frac{2}{3}x^3-ax^2+7ax+5$

$f'(x)=2x^2-2ax+7a>0$

$D<0$ $4a^2-4\cdot 7a<0$

$4a(a-14)<0$

$(0;14)$ (D)

22. $\int_1^a (2x+1)\,dx=4$

$x^2+x\Big|_1^a=4$

$a^2+a-1-1=4$

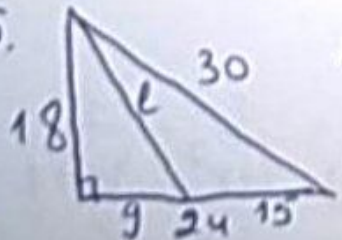
$a^2+a-6=0$ (C)

-3 2 $a>0$

23. 9) 3) (C)

24. $C_{22}^3=\frac{22!}{3!\,19!}=\frac{20\cdot 21\cdot 22}{6}=1540$ (A)

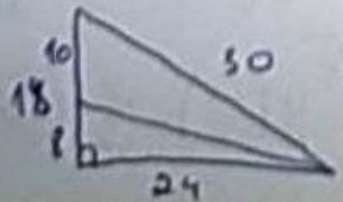
25.



$\ell^2=18\cdot 30-9\cdot 15^5$

$\ell=\sqrt{405}=9\sqrt{5}$

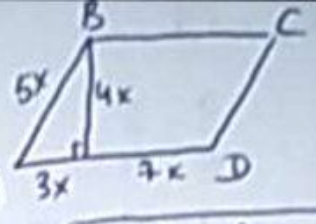
(D)



$\ell^2=30\cdot 24-8\,10$

$\ell=\sqrt{640}=8\sqrt{10}$

26.



$\frac{AC}{BD}=?$

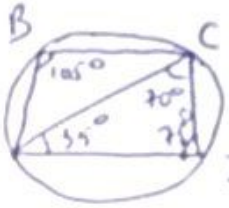
$\cos\angle=\frac{3}{5}$.

$AC=\sqrt{100x^2+25x^2+2\cdot 10x\cdot 5x\cdot\frac{3}{5}}=\sqrt{185}x$

$BD=\sqrt{100x^2+25x^2-2\cdot 10\cdot 5x\cdot\frac{3}{5}}=\sqrt{65}x$.

$\frac{AC}{BD}=\frac{\sqrt{37}}{\sqrt{13}}$ (D)

27.



$\angle ABD=\angle ACD=70^\circ$

(C)

28. $A(8;-4)$ $y=x-4$

$B(0,-4)$ $8+x=0+8$ $x=0$

$C(8;4)$ $-4+y=-4+4$ $y=4$.

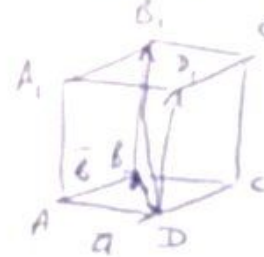
$D(x;y)$

(A)

29. $ABCDA_1B_1C_1D_1$ $\bar{AD}=\bar{a}$ $\bar{AB}=\bar{b}$

$\bar{AA_1}=\bar{c}$ $\bar{DB_1}=\bar{DB}+\bar{DD_1}$

$\bar{AC}=\bar{AD}+\bar{AB}=\bar{a}+\bar{b}$ (C)



$\bar{DB}=\bar{b}-\bar{a}$

$\bar{DD_1}=\bar{AA_1}=\bar{c}$.

$\bar{DB_1}=\bar{b}-\bar{a}+\bar{c}$.

30. $\frac{\sqrt{3}}{2}a=12$ $a=\frac{24}{\sqrt{3}}$

$S=3\cdot a\cdot H=3\cdot\frac{24}{\sqrt{3}}\cdot 4=96\sqrt{3}$

(B)

Variant-13

1. $93m07n$ $:8$ $:6$

$n=2$

$93m072$

$m=0;3;6;9$ 18 (C)

2. $3<a<7$ $3<b<10$

$\frac{1+\frac{a}{b}}{1+\frac{b}{a}} : \frac{\frac{b+a}{b}}{\frac{a+b}{a}} = \frac{a}{b} = \frac{6}{4} = \frac{3}{2}$ (A)

3. $x^2+\frac{2}{x}=3$ $x\neq 1$ $x^2+x=?$

$x^3+2=3x$ $x^3-3x+2=0$

$\begin{array}{r|l} x^3-3x+2 & x-1 \\ \hline x^3-3x+2 & x^2+x-2 \\ \hline 0 & \end{array}$ (C)

$(x-1)(x^2+x-2)=0$ $x^2+x=2$

4. $|a|\neq|b|\neq|c|$

$\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}=-2$ $(\times a)(\times b)(\times c)$

$\begin{cases} \frac{a^2}{b+c}+\frac{ab}{a+c}+\frac{ac}{a+b}=-2a \\ \frac{ab}{b+c}+\frac{b^2}{a+c}+\frac{bc}{a+b}=-2b \\ \frac{ac}{b+c}+\frac{bc}{a+c}+\frac{c^2}{a+b}=-2c \end{cases}$ (+)

$\frac{a^2}{b+c}+\frac{b^2}{a+c}+\frac{c^2}{a+b}+a+b+c=-2a-2b-2c$

$\frac{a^2}{b+c}+\frac{b^2}{a+c}+\frac{c^2}{a+b}=-3(a+b+c)$ (B)

$-3(a+b+c):(a+b+c)=-3$

5. $\sqrt{30+\sqrt{30+\sqrt{30+\dots}}}=a$

$\sqrt{30+a}=a$ $30+a=a^2$ $a=6$. (A)

$\sqrt{6+\sqrt{6+\sqrt{6+\dots}}}=x$ $\sqrt{6+x}=x$

$6+x=x^2$ $x^2-x-6=0$ $x=3$

6. $x+y+z=42,8$

$y=0,8x$

$z=0,425y=0,8\cdot 0,425x$

$x+0,8x+0,425\cdot 0,8x=42,8$

$x=20$

$z=0,8\cdot 0,425\cdot 20$

$z=6,8$

7. $\sqrt{x+16}-\sqrt{x+8}=2$

$\sqrt{x+16}=2+\sqrt{x+8}$

$x+16=4+4\sqrt{x+8}+x+8$

$4=4\sqrt{x+8}$ (B)

$x+8=1$ $x=-7$

8. $x^2-6x+3=0$

$\frac{a^3b^2-a^2b^3}{a^2-b^2}=\frac{a^2b^2(a-b)}{(a-b)(a+b)}=$

$=\frac{a^2b^2}{a+b}=\frac{9}{6}$ (B)

9. $\begin{cases} (x-3)^4\cdot(y-5)=1 \\ (x-3)^5\cdot(y-5)^4=1 \end{cases}$

$\frac{x-5}{y-3}=?$

$(x-3)^{11}=1$

$x=4$

$\frac{4-5}{6-3}=-\frac{1}{3}$ (A)

$y-5=\frac{1}{(x-3)^4}$

$(x-3)^5\cdot\frac{1}{(x-3)^{16}}=1$

$\frac{1}{(x-3)^{11}}=1$

$y-5=1$ $y=6$.

10. $\left|\frac{4-2x}{1+3x}\right|>0$ $4-2x\neq 0$ $x\neq 2$

$1+3x\neq 0$ $x\neq -\frac{1}{3}$ (C)

11. $a_1=-3,1$ $a_{n+1}=a_n+0,9$ $S_{19}=?$

$a_2=a_1+0,9=-2,2$ $d=0,9$.

$S_{19}=\frac{a_1+a_{19}}{2}\cdot 19=(a_1+9d)\cdot 19=$

$=(-3,1+9\cdot 0,9)\cdot 19=95$ (B)

12. $tg3x - tg2x - tgx =$

$= \frac{\sin 3x}{\cos 3x} - \frac{\sin x}{\cos x} - \frac{\sin 2x}{\cos 2x} =$

$= \frac{\sin 2x}{\cos x \cos 3x} - \frac{\sin 2x}{\cos 2x} =$

$= \frac{\sin 2x (\cos 2x - \cos x \cdot \cos 3x)}{\cos x \cos 2x \cos 3x} =$

$= \frac{\sin 2x (\cos 2x - \frac{1}{2}(\cos 4x + \cos 2x))}{\cos x \cos 2x \cos 3x} =$

$= \frac{\sin 2x \cdot \frac{1}{2}(\cos 2x - \cos 4x)}{\cos x \cos 2x \cos 3x} =$ (A)

$= \frac{\sin 2x \cdot \sin 3x \sin x}{\cos x \cos 2x \cos 3x} = tgx\, tg2x\, tg3x$

13. $\left((\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2\right) : \left(4\sin^2\frac{\alpha-\beta}{2}\right) - 3$

$\left(2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)\right) : \left(2(1 - \cos(\alpha-\beta))\right) - 3 =$

(C) $= \left(2(1-\cos(\alpha-\beta))\right) : \left(2(1-\cos(\alpha-\beta))\right) - 3 = 1 - 3 = -2$

14. $2\arcsin\left(-\frac{\sqrt{3}}{2}\right) + \operatorname{arcctg}(-1) + \arccos\frac{1}{\sqrt{2}} + \frac{1}{2}\operatorname{arcctg}(-1)$

$2\cdot\left(-\frac{\pi}{3}\right) + \frac{3\pi}{4} + \frac{\pi}{4} + \frac{1}{2}\cdot\pi = \frac{5\pi}{6}$ (C)

15. $\sqrt{4-x^2}\,(\sin^7 x - \cos^7 x) = 0$

$4 - x^2 \geq 0 \quad [-2;2] \qquad \sin^7 x = \cos^7 x$

$x = \frac{\pi}{4} + \pi n \;;\; n \in \mathbb{Z} \qquad tgx = 1$

$n = 0 \quad \frac{\pi}{4}$ (A)

16. $f(\varphi(x)) = \frac{x+2}{x-3} \qquad \varphi(x) = 2x - 1$

$x = \frac{\varphi(x)+1}{2} \qquad f(\varphi(x)) = \frac{\frac{\varphi(x)+1}{2} + 2}{\frac{\varphi(x)+1}{2} - 3}$

$f(\varphi(x)) = \frac{\varphi(x)+5}{\varphi(x)-5} \qquad f(x) = \frac{x+5}{x-5}$ (C)

17. $2^x = 152 \qquad |x-8| + |x-6| = 8 - x + x - 6 = 2$

$128 < 2^x < 256$ (A)

$2^7 < 2^x < 2^8$

$7 < x < 8.$

18. $\log_{30} 3 = a \qquad \log_{30} 5 = b \qquad \log_{30} 64 =$

$= \log_{30} 2^6 = 6\log_{30} 2 = 6\log_{30}\frac{30}{15} =$

$= 6(1 - \log_{30} 15) = 6(1 - a - b)$ (C)

19. $\log_{0,5}(2x-7) \leq \log_{0,5}(10-x) + 1$

$\log_{0,5}(2x-7) \leq \log_{0,5} 0{,}5(10-x)$

$\begin{cases} 2x - 7 \geq 0{,}5(10-x) \\ 10 - x > 0 \end{cases} \quad \begin{cases} x \geq 4{,}8 \\ x < 10 \end{cases}$

5; 6; 7; 8; 9 (C)

20. $f(3x+2) = 2x^2 - 5x + 8$

$3x + 2 = a \qquad x = \frac{a-2}{3}$

$f(a) = 2\left(\frac{a-2}{3}\right)^2 - 5\cdot\left(\frac{a-2}{3}\right) + 8$

$f(x) = \frac{2}{9}x^2 - \frac{23}{9}x + \frac{110}{9}$

$f'(x) = \frac{4}{9}x - \frac{23}{9}$ (C)

21. $g(x) = 2x - 3$

$f(g(x)) = 3x^2 - 8x + 13$

$x = \frac{g(x)+3}{2}$

$f(g(x)) = 3\cdot\left(\frac{g(x)+3}{2}\right)^2 - 8\,\frac{g(x)+3}{2} + 13$

$f(g(x)) = \frac{3}{4}(g(x))^2 + \frac{1}{2}g(x) + \frac{31}{4}$

$f'(g(x)) = \frac{3}{2}(g(x)) + \frac{1}{2} =$

$= \frac{3}{2}(2x-3) + \frac{1}{2} = 3x - \frac{9}{2} + \frac{1}{2}$

$= 3x - 4$ (B)

22. $\left(\int (x^2 - 5x) f(x)\,dx\right)' = (x^2 - 10x + c)'$

$(x^2 - 5x)\cdot f(x) = 2x - 10$

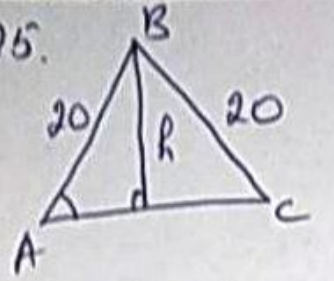
$f(x) = \frac{2(x-5)}{x(x-5)} = \frac{2}{x}$ (B)

23. 1) 3) (A)

(A) 24. $A = \{1; 3; 5; 6; 8; 9; 10; 11\}$

$B = \{5; 6; 7; 8; 10; 11\} \qquad A \cup B = \{1; 3; 5; 6; 7; 8; 9; 10; 11\}$

25.



$\cos A = \frac{2\sqrt{6}}{5}$

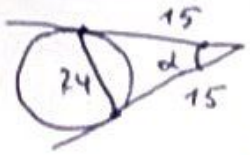
$\sin A = \frac{1}{5} = \frac{h}{20}$ $h = 4$

(B)

26. $2(a+b) = 40$ $\begin{cases} a+b=20 \\ a-b=10 \end{cases}$ (C)

$a = 15$ $b = 5$

27.



$24^2 = 15^2 + 15^2 - 2 \cdot 15^2 \cos\alpha$

$\cos\alpha = -\frac{7}{25}$

$24^2 = R^2 + R^2 + 2R^2 \cdot \frac{7}{25}$

$R = 15$ (C)

28. $A(-4;2)$ $B(6;5)$ $C(1;-4)$

$\frac{x-6}{6-1} = \frac{y-5}{5+4}$ $9x - 5y - 29 = 0$

$k_1 \cdot k_2 = -1$ $k_1 = \frac{9}{5}$ $k_2 = -\frac{5}{9}$

$A(-4;2)$ $y = -\frac{5}{9}x + b$

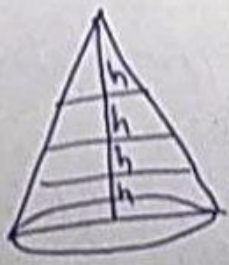
$2 = -\frac{5}{9} \cdot (-4) + b$ $b = -\frac{2}{9}$

$y = -\frac{5}{9}x - \frac{2}{9}$ $5x + 9y + 2 = 0$ (C)

29. $\bar{a}(2;-1;1)$

$|\bar{a}|_{Yoz} = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$ (C)

30.



$\frac{V}{V_1} = \left(\frac{4h}{h}\right)^3$ $V_1 = \frac{V}{64}$

$\frac{V}{V_2} = \left(\frac{4h}{2h}\right)^3$ $V_2 = \frac{V}{8}$

$\frac{V}{V_3} = \left(\frac{4h}{3h}\right)^3$ $V_3 = \frac{27}{64}V$

$V_2 - V_1 = \frac{V}{8} - \frac{V}{64} = \frac{7V}{64}$ (A)